EE444/544 Spring 2020 Homework #3

Due: April 3rd midnight. You must work alone, no collaboration is permitted. Matlab is allowed; submit your code with the solution.

Problem 1. Consider the unstable plant

$$P(s) = \frac{4\ (s-2)}{(s^2 - 2s + 2)}.$$

(a) Find a characterization of the set of all controllers stabilizing the feedback system (C, P):

$$\mathcal{C}(P) = \left\{ C = \frac{X + DQ}{Y - NQ} \quad : \quad Q \in H_\infty \right\}$$

where $N, D, X, Y, \in H_\infty$ are such that $N(s)X(s) + D(s)Y(s) = 1$ and $P(s) = N(s)/D(s)$.

(b) Find a controller $C(s)$ stabilizing (C, P), and satisfying the following steady state performance conditions:

- steady state error for a unit step reference input is zero
- steady state error for a sinusoidal input of the form $r(t) = \sin(3t)$, $t \geq 0$, is zero.

Implement this feedback system in Simulink and illustrate that performance conditions are satisfied.

Problem 2. For the nominal plant given in Problem 1 consider the following set of uncertain plants:

$$\mathcal{P} = \{ P_\Delta = P(1 + \Delta_m) \ : \ P_\Delta \ \mathit{has}\ 2\ \mathit{poles}\ \mathit{in}\ \mathbb{C}_+ \ , \ |\Delta_m(j\omega)| < |W_m(j\omega)| \ \ , \ \forall\ \omega \ \}$$

where

$$W_m(s) = \delta\ (s+1).$$

(a) Find the largest $\delta > 0$ for which there exists a controller C stabilizing (C, P_Δ) for all $P_\Delta \in \mathcal{P}$; and determine the corresponding optimal controller, C_{opt}.

(b) With the largest δ computed above, pick an arbitrary element $P_\Delta \neq P$ in the set $\mathcal{P}$, and prove that $(C_{\text{opt}}, P_\Delta)$ is indeed stable (find the location of the closed loop system poles and determine the stability margins of this system i.e. gain, phase, delay and vector margins).