Lecture March 26, 2020

Thursday, March 26, 2020 3:32 AM

Can you see this?

$$\gamma_{opt} = \inf_{Q \in \mathcal{H}_\infty} \| \underbrace{W - MQ}_{F(s)} \|_\infty$$

$$M(s) = \prod_{i=1}^{n} \left(\frac{s-\alpha_i}{s+\bar{\alpha}_i} \right) \quad Re(\alpha_i) > 0$$

$\alpha_i \neq \alpha_j$ for $i \neq j$

$F(s) \in \mathcal{H}_\infty \quad F(\alpha_i) = W(\alpha_i) = \beta_i$ and try to find the best F satisfying $\|F\|_\infty \leq \gamma$ for the smallest γ.

Nevanlinna-Pick interpolation problem.

$$F_{opt}(s) = \lambda \frac{[s^{n-1} \dots s^0] J \Phi}{[s^{n-1} \dots s^0] \Phi}$$

$\lambda = \pm \gamma_{opt} = \pm \sqrt{\lambda_{max}(A^{-1}B)}$

Φ corresponding eigenvector of $(J V_\alpha^{-1} D_\beta V_\alpha)$

$$J = \begin{bmatrix} 1 & & & 0 \\ & -1 & & \\ & & 1 & \\ 0 & & & -1 & \\ & & & & \ddots \end{bmatrix} \quad D_\beta = \begin{bmatrix} \beta_1 & & 0 \\ & \ddots & \\ 0 & & \beta_n \end{bmatrix} \quad V_\alpha = \begin{bmatrix} \alpha_1^{n-1} & \dots & \alpha_1^0 \\ \vdots & & \\ \alpha_n^{n-1} & \dots & \alpha_n^0 \end{bmatrix}$$

$F \in \mathcal{H}_\infty \quad F(s)$ is analytic and bounded in $\mathbb{C}_+$

$$\|F\|_\infty = \sup_{Re(s)>0} |F(s)| = \sup_\omega |F(j\omega)|$$

by the maximum modulus principle

Homework #3:

① $P(s) = \dfrac{4(s-2)}{(s^2-2s+2)}$ $\quad P(s) = \dfrac{N(s)}{D(s)}$ $\quad N, D \in \mathcal{H}_\infty$ and they do not have common zeros in $\mathbb{C}_+$ includin $\{+\infty\}$.

$$D(s) = \frac{(s^2-2s+2)}{(s+a)(s+b)}$$

$a > 0$
$b > 0$

$$N(s) = \frac{4(s-2)}{(s+a)(s+b)}$$

I could also select $\begin{cases} a = r + j\omega & r > 0 \\ b = r - j\omega & \omega > 0 \end{cases}$

Find $X, Y \in \mathcal{H}_\infty$ satisfying $NX + DY = 1$

Find $X, Y \in \mathcal{H}_\infty$ satisfying $NX + DY = 1$

$$Y = \frac{1 - N(s)X(s)}{D(s)}$$

the zeros of $D(s)$ are $(1 \pm j) \in \mathbb{C}_+$

$$\left\{ \begin{array}{l} X(1+j) = 1/N(1+j) \\ X(1-j) = 1/N(1-j) \end{array} \right\}$$ two interpolation conditions

$X(s) \in \mathcal{H}_\infty$ and X satisfies

$$X(s) = \frac{x_1 s + x_2}{s + r_0}$$

$r_0 > 0$ ... arbitrary

find x_1, x_2 from

Hence X, Y, N, D are computed.

$$\boxed{C = \frac{X + DQ}{Y - NQ} \qquad Q \in \mathcal{H}_\infty}$$

>> minreal

{alternatively by using polynomial root computations for num(Y) den(Y)}

make sure that $(1 \pm j)$ are cancelled in $Y(s)$ and that $Y(s)$ is stable.

(b) • e_{ss} for $R(s) = \frac{1}{s}$ is zero $\Rightarrow$ Controller has a pole at $s = 0$

• e_{ss} for $r(t) = \sin(3t)$ is zero: {Controller has poles at $s = \pm j3$

$$\left. \begin{array}{l} Y(0) - N(0)Q(0) = 0 \\ Y(j3) - N(j3)Q(j3) = 0 \\ Y(-j3) - N(-j3)Q(-j3) = 0 \end{array} \right\}$$

$$Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{(s+3)^2}$$

find q_1, q_2, q_0

(2) $\|W_m T\|_\infty \le 1$ $\qquad T = N(X + DQ_c)$

$$\| \delta (s+1) N(s) (X(s) + D(s) Q_c(s)) \|_\infty \le 1$$

$$\| \delta (s+1) N(s) \left(X(s) + D(s) Q_c(s) \right) \|_\infty \le 1$$

$$\delta_{max} = \frac{1}{\gamma_{opt}} , \quad \gamma_{opt} = \inf_{Q_c \in \mathcal{H}_\infty} \| (s+1) N(s) \left(X(s) + D(s) Q_c(s) \right) \|_\infty$$

$$\hookrightarrow \quad \gamma_{opt} = \inf_{Q \in \mathcal{H}_\infty} \| W - M Q \|_\infty$$

$$M(s) = \left(\frac{s^2 - 2s + 2}{s^2 + 2s + 2} \right) \qquad W(s) = \text{its num. contains } 4(s+1)(s+2)$$

Solve using NevPickNew.m $\quad \gamma_{opt}, \; Q_{opt} \to Q_{c,opt} \to C_{opt}$

OGKY2018 2.4.1

(b) $P_\Delta = P(1 + \Delta_m) \qquad |\Delta_m(j\omega)| < \delta \, |j\omega + 1|$

$$= P(s) + P(s) \Delta_m(s)$$

$$= P(s) + P(s) \cdot \delta (s+1) \qquad 0 < \delta < \delta_{max}$$

an example

Next Lecture: The Nehari problem 2.4.2
Theorem 2.13

Next Tuesday $\underline{9{:}00 - 10{:}15}$