March 31 Lecture

Tuesday, March 31, 2020 1:41 AM

The Lecture begins formally at 9:00 until then we chat on course topics, and assessments.

$P \to C$ (C,P) is stable $\quad P = \frac{N}{D} \quad C = \frac{X_c}{Y_c} \quad N, D, X_c, Y_c \in \mathcal{H}_\infty$

Char. function $\quad DY_c + NX_c = \mathcal{U} \in \mathcal{H}_\infty \quad \mathcal{U}^{-1} \in \mathcal{H}_\infty$

$$D\underbrace{(Y_c\mathcal{U}^{-1})}_{Y} + N\underbrace{(X_c\mathcal{U}^{-1})}_{X} = 1 \qquad NX + DY = 1 \qquad C = \frac{X + DQ}{Y - NQ}$$

Grading scheme: (444)

- Homework : 35 (5 HW 7pts each)
- Midterm : 35
- Final Exam : 30

2 HWs ① and ② $\to$ 6 ~~7~~ pts each } 42 ~~44~~ pts

3rd HW is still on $\to$ 10 pts

4th $\to$ 10 pts

5th $\to$ 10 pt

New For 544 and 444:

3 quizzes 10 pts } 30pts

1 Final Exam 28 ~~26~~ pts (take home)

Last week: Robustness optimization and Sensitivity Minimization

$$\hookrightarrow \quad \gamma_{opt} = \inf_{Q_c \in \mathcal{H}_\infty} \| T_1 - T_2 Q_c \|_\infty = \inf_{Q \in \mathcal{H}_\infty} \| W - MQ \|_\infty$$

$Q = T_{2o} Q_c$

Model Matching problem: $\quad \gamma_{opt} = \inf_{Q \in \mathcal{H}_\infty} \| W - MQ \|_\infty$

$W \in \mathcal{H}_\infty$

$M \in \mathcal{H}_\infty$, inner $\quad |M(j\omega)| = 1 \quad \forall \omega \qquad$ Find γ_{opt}, $\to Q_{opt} \in \mathcal{H}_\infty$

$F = W - MQ$; $\quad M(s) = \prod_{i=1}^{n} \left(\frac{s - \alpha_i}{s + \bar{\alpha}_i} \right) \quad \begin{cases} F \in \mathcal{H}_\infty \quad F(\alpha_i) = W(\alpha_i) =: \beta_i \\ \|F\|_\infty \text{ should be minimized} \end{cases}$

Nevanlinna-Pick interpolation
Solution is given last week. }

HW#3 new due date April 7

$$F \in \mathcal{H}_\infty \qquad |F(j\omega)| = |\overline{M(j\omega)}\, F(j\omega)| \quad \forall \omega$$

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$|M(j\omega)| = 1 = |\overline{M(j\omega)}| \quad \forall \omega$ <u>Notation</u>: $\overline{M(j\omega)} = M^*$

$|M^*(W - MQ)| = |M^* W - Q|$ because $M^* M = 1$

<u>Example</u> $M(s) = \frac{s-1}{s+1} \qquad M(j\omega) = \frac{j\omega - 1}{j\omega + 1} \qquad \overline{M(j\omega)} = \frac{-j\omega - 1}{-j\omega + 1}$

$= \frac{1}{M(j\omega)}$

$\underline{M^* M = 1 \quad \forall \omega}$

$|F(j\omega)| = |\overline{M(j\omega)}\, W(j\omega) - Q(j\omega)|$

$\|F\|_{\mathcal{H}_\infty} = \|F\|_{\mathcal{L}_\infty}$ $\boxed{\left(\underbrace{\sup_{Re(s) \geq 0} |F(s)|}_{\|F\|_{\mathcal{H}_\infty}} = \underbrace{\sup_{\omega} |F(j\omega)|}_{\|F\|_{\mathcal{L}_\infty}} \right)}$

$\|F\|_{\mathcal{H}_\infty} = \| \underbrace{M^* W - Q}_{\text{a function in } \mathcal{L}_\infty} \|_{\mathcal{L}_\infty}$ (i.e. no poles on Im-axis poles in RHP and LHP)

$M^* W \in \mathcal{L}_\infty \;\rightarrow\; \left(\frac{1}{M(s)} W(s) \right) \quad s = j\omega$

$\frac{W(s)}{M(s)} = R(s) \;\rightarrow$ no poles on Im-axis some poles in RHP and some poles in LHP

(Section 2.4.2 (The Nehari problem) of OGKY2018 book
See moodle

$\gamma_{opt} = \inf_{Q \in \mathcal{H}_\infty} \|R - Q\|_\infty$

$\uparrow \mathcal{L}_\infty$ norm

example $M(s) = \frac{s-1}{s+1}$

$W(s) = \frac{1}{s+2}$

$\frac{W(s)}{} = R(s) = \frac{s+1}{}$ LHP ~~RHP~~ poles $\equiv$ poles of $W \in \mathcal{H}_\infty$

$$\frac{W(s)}{M(s)} = R(s) = \frac{s+1}{(s+2)(s-1)}$$

LHP ~~RHP~~ poles $\equiv$ poles of $W \in \mathcal{H}_\infty$

RHP ~~LHP~~ poles $\equiv$ zeros of $M(s)$

Therefore $\boxed{R(s) = R_s + R_u}$ $\quad R_s \in \mathcal{H}_\infty \quad R_u(-s) \in \mathcal{H}_\infty$

R_u : anti-stable all its poles are in RHP (zeros of $M(s)$)

Given W, M $\longrightarrow$ define $R(s) = \dfrac{W(s)}{M(s)} = R_s + R_u$

$R_s \in \mathcal{H}_\infty \quad R_u(-s) \in \mathcal{H}_\infty$

Matlab command : >> stabsep

$$\|R - Q\|_{\mathcal{L}_\infty} = \| \underbrace{R_u + R_s - Q} \|_\infty \qquad Q - R_s = Q_1$$

$Q \in \mathcal{H}_\infty \quad Q_1 \in \mathcal{H}_\infty$

$$\boxed{\gamma_{opt} = \inf_{Q_1 \in \mathcal{H}_\infty} \| R_u - Q_1 \|_\infty} \longrightarrow \text{The Nehari Problem}$$

Solution of the Nehari Problem (pp. 18–20 of the book)

Summary (Theorem 2.13):

$R_u(s) =$ all its poles are in RHP

$R_u(s) = C(sI - A)^{-1}B$ $\quad \left.\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}\right\} : R_u(s)$

assumption: (A,B) controllable
(C,A) observable

$u \to \boxed{R_u} \to y$

$[B \,\vdots\, AB \,\vdots\, \cdots \,\vdots\, A^{n-1}B]$ has rank n : controllability

$\begin{bmatrix} C \\ CA \\ \vdots \end{bmatrix}$ has rank n : observability

$\begin{bmatrix} \vdots \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$ has rank n : observability

If $R_u(s)$ is given as a transfer function
you can use "tf2ss" in Matlab to get A, B, C.

Step 1: Solve the Lyapunov eqn:

$$\left.\begin{aligned} AW_c + W_c A^T &= BB^T \\ A^T W_o + W_o A &= C^T C \end{aligned}\right\}$$ Matlab command is "lyap"

find W_o and W_c : $n \times n$ matrices

$$\gamma_{opt} = \sqrt{\lambda_{max}(W_c W_o)} \qquad \sigma_{max}^2 = \gamma_{opt}^2$$

Step 2 x_{max} is the corresponding eigenvector

$$\sigma_{max}^2 x_{max} = W_c W_o x_{max}$$) use "eig" command

$$\boxed{\underbrace{W - M Q_{opt}}_{F_{opt}}} \Rightarrow \boxed{Q_{opt} = R(s) - \gamma_{opt} \frac{C(sI-A)^{-1} x_{max}}{B^T(sI+A^T)^{-1} y_{max}}}$$

where $y_{max} = \gamma_{opt}^{-1} W_o x_{max}$

$R = \frac{W}{M}$ $\longrightarrow$ find F_{opt} from Here.

In the handout there is an example (2.14)
and there is an exercise.

and there is an exercise.

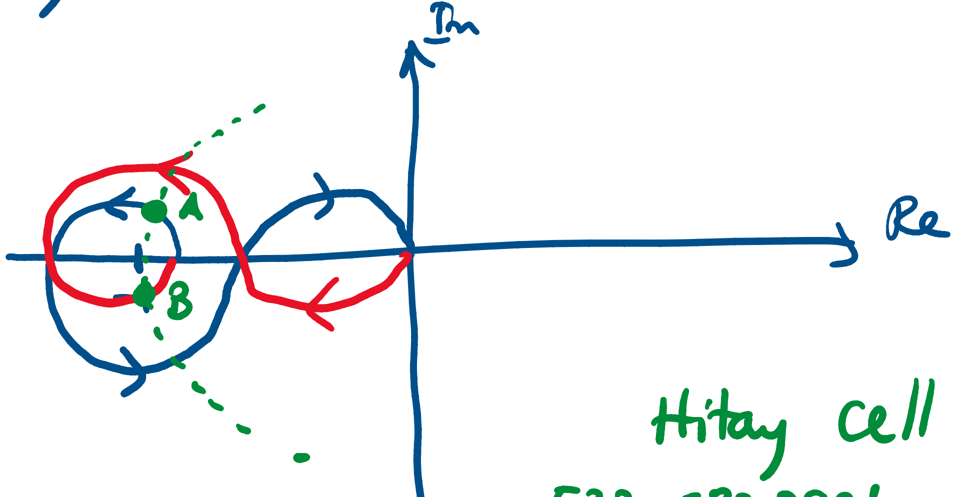
Sensitivity Minimization and
Robustness optimization are solved by
Nevanlinna-Pick or Nehari approach
1-block $\mathcal{H}_\infty$ problems (Model Matching Problem)
Next: Mixed Sensitivity minimization

$$\gamma_{opt} = \inf_{\substack{C \text{ stabilizing} \\ (C,P)}} \left\| \begin{bmatrix} W_1 S \\ W_2 T \end{bmatrix} \right\|_\infty$$

↳ first transform this problem to a 1-block problem (by using Spectral factorization)

(C, P_Δ) stable

"allmargin" → stable ✓



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