EE444/544 **Spring 2020** **Homework #1**

Due: February 25, to be collected in class.

You must work alone, no collaboration is permitted.

We will discuss the homework in class on Tuesday February 18.

Problem. Consider the standard feedback control system with an interval plant $P \in \mathcal{P}$,

$$\mathcal{P} = \left\{ P(s) = \frac{q_0 s + q_1}{r_0 s^6 + r_1 s^5 + r_2 s^4 + r_3 s^3 + r_4 s^2 + r_5 s + r_6} \right\}$$

where $q_0 \in [0.95\ ,\ 1]$, $q_1 \in [0.35\ ,\ 0.4]$, $r_0 \in [0.9\ ,\ 1]$, $r_1 \in [9\ ,\ 12]$, $r_2 \in [50\ ,\ 100]$, $r_3 \in [120\ ,\ 150]$, $r_4 \in [195\ ,\ 200]$, $r_5 \in [120\ ,\ 130]$, $r_6 \in [-\delta\ ,\ +\delta]$.

(a) By using the 16 plant theorem find the maximum value of δ such that there exists a robustly stabilizing controller of the form

$$C(s) = \frac{K\ -\ 4\ s}{s}$$

for the family of plants $\mathcal{P}$. Determine the corresponding optimal value of K.

(b) Draw the root locus of this system with respect to $K > 0$, by taking the nominal plant as

$$P_o(s) = \frac{s + 0.35}{s^6 + 10s^5 + 60s^4 + 140s^3 + 200s^2 + 125s - \frac{\delta_{\max}}{2}}$$

where $\delta_{\max}$ is as found in part (a), and show the location of the closed loop system poles for the optimal value of K determined above.

(c) Find the gain, phase and delay margin of the nominal system defined in part (b). Also, draw the magnitude of the sensitivity function and compute its peak value.