EE444/544 **Spring 2020** **Homework #2**

Due: March 5, to be collected in class.
You must work alone, no collaboration is permitted.

Problem. Consider the standard feedback control system with the plant

$$P_\delta(s) = \frac{e^{-hs}}{\tau s - 1} \qquad \tau \in [0.2\ ,\ 0.25] \quad h \in [0\ ,\ 0.05] \ \text{ and define } \ P(s) = \frac{1}{0.2\ s - 1}\ , \quad \delta = P_\delta - P.$$

(i) Find an uncertainty weight in the form

$$W_a(s) = \frac{a_1\ s}{(a_2\ s + 1)^2}\ , \quad a_1, a_2 > 0$$

such that $|W_a(j\omega)| > |\delta(j\omega)|$ for all ω, and $|W_a(j\omega)|$ is as small as possible for each ω.

(ii) Now consider the class of uncertain plants

$$\mathcal{P} = \{P_\Delta = P + \Delta\ :\ P_\Delta \text{ has one pole in } \mathbb{C}_+\ , \quad |\Delta(j\omega)| < |W_a(j\omega)| \quad \forall\ \omega\}$$

and find the allowable interval for $\beta > 0$ so that the controller

$$C(s) = \frac{15s + 1}{\beta\ s}$$

is robustly stabilizing (C, P), for all $P_\Delta \in \mathcal{P}$.

(iii) Now pick a value of β in the interval determined above and verify that the feedback system formed by the controller C and the plant $P_1(s) = \dfrac{e^{-0.05s}}{0.25s - 1}$ is stable.

(iv) Let $W_r(s) = \dfrac{1}{s}$, $\gamma_r = 10$; determine if there exists β values in the interval determined in part (ii), satisfying the robust performance condition

$$\left|\frac{W_r(j\omega)}{\gamma_r} S(j\omega)\right| + |W(j\omega)C(j\omega)S(j\omega)| \le 1 \quad \forall\ \omega \qquad \text{where } \ S = (1 + PC)^{-1}.$$

What is the smallest value of $\gamma_r > 0$ such that there exists a feasible β satisfying the robust performance condition? Find the corresponding optimal value of β.