Toy Model Stability #4b condenced

27 NOV 2015

$$J = \begin{bmatrix} \frac{\partial(\dot\phi)}{\partial\phi} & \frac{\partial(\dot\phi)}{\partial V} \\ \frac{\partial(\dot V)}{\partial\phi} & \frac{\partial(\dot V)}{\partial V} \end{bmatrix} = \begin{bmatrix} -\phi^2+1 & -\beta \\ \beta & 0 \end{bmatrix}$$

$$\mathrm{Det}(I\lambda - J) = 0 = \mathrm{Det}\begin{bmatrix} \lambda+\phi^2-1 & \beta \\ -\beta & \lambda \end{bmatrix} = \lambda^2 + \lambda(\phi^2-1) + \beta^2 = 0$$

$$\lambda^2 + \lambda(\phi^2-1) = -\beta^2$$

$$\left(\lambda + \frac{\phi^2-1}{2}\right)^2 = -\beta^2 + \left(\frac{\phi^2-1}{2}\right)^2$$

$$\lambda = \pm\sqrt{-\beta^2 + \left(\frac{\phi^2-1}{2}\right)^2} - \frac{\phi^2-1}{2} \qquad \text{let } \frac{\phi^2-1}{2} = a$$

$$= \pm\sqrt{-\beta^2 + a^2} - a \qquad \phi = \frac{\alpha}{\beta}$$

$\beta \neq 0$

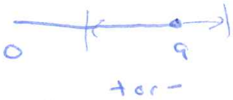
$\alpha > \beta$ then $\phi > 1$ then $a > 0$ — $\beta^2 > a^2$ then $\mathrm{sign}(Re(\lambda)) = -ve$; else $-ve$

$\alpha = \beta$ then $\phi = 1$ then $a = 0$ then $Re(\lambda) = 0$

$\alpha < \beta$ then $0 < \phi < 1$ the $a < 0$ — $\beta^2 > a^2$ then $\mathrm{sign}(Re(\lambda)) = +ve$; else $+ve$.

if $\beta^2 > a^2$ then $\sqrt{\text{negative}}$ then $\mathrm{sign}(Re(\lambda)) = -a$

if $\beta^2 < a^2$ then at max $\pm\sqrt{a^2} \pm a$ $\mathrm{sign}(Re(\lambda)) = 0$ ie



thus again $\mathrm{sign}(Re(\lambda)) = -a$

Thus

- $\beta \neq 0$
- $\alpha = \beta$ $Re(\lambda) = 0$ transition
- $\alpha < \beta$ $\mathrm{Sign}(Re(\lambda)) = +ve$ therefore occulation/unstable
- $\alpha > \beta$ $\mathrm{Sign}(Re(\lambda)) = -ve$ therefore stable.