# Toy Model #4

The equations that define the simple Toy Model are described in Equations 1 and 2.

$$\frac{d\Phi}{dt} = -\beta V + \frac{\Phi^3}{3} - \Phi \qquad = L_\Phi \tag{1}$$

$$\frac{dV}{dt} = \beta\Phi + \alpha \qquad = L_V \tag{2}$$

Then with diffusion added Equations 1 and 2 become 3 and 4 as follows;

$$\frac{d\Phi}{dt} = \mathrm{D}\frac{\mathrm{d}^2\Phi}{\mathrm{dx}^2} + L_\Phi \tag{3}$$

$$\frac{dV}{dt} = \mathrm{D}\frac{\mathrm{d}^2 V}{\mathrm{dx}^2} + L_V \tag{4}$$

Where α and β are variable parameters

## Results 1

Due to there being two parameters it is important to view the interaction between the two. First if you hold α constant and vary β:

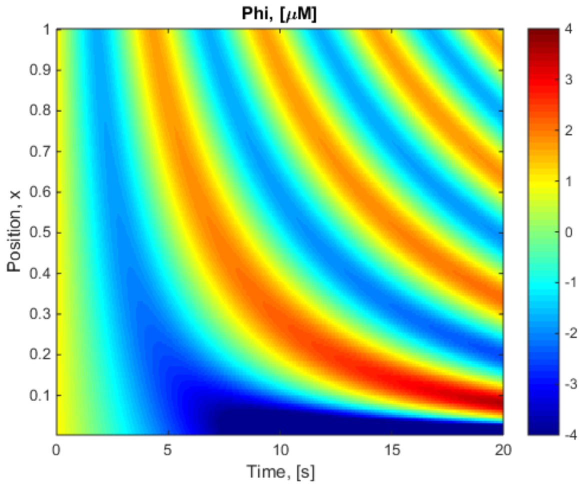


Figure 1: Holding α constant at 0 and varying β linearaly between {0,1}

From here α was changed to 0.5 and again varied β. For Figure 2 the scale has been fixed {-4,4} and thus in the dark blue region which heads to negative infinity is not viewed.

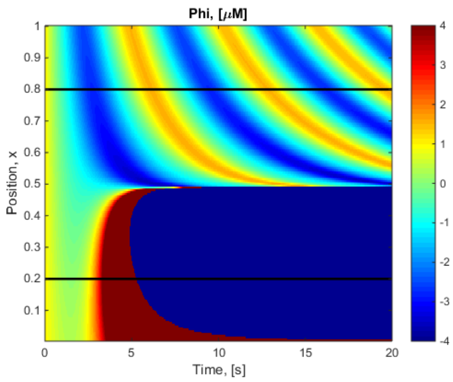


Figure 2: Holding α constant at 0.5 and varying β linearaly between {0,1}

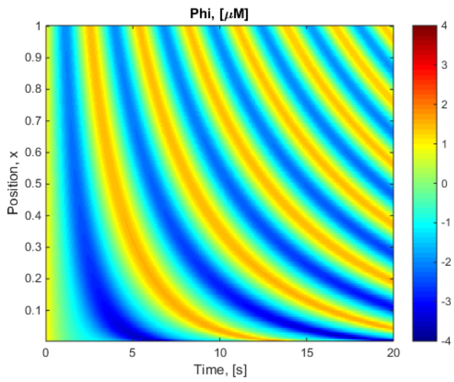


Figure 3: Holding α constant at 0.5 and varying β linearaly between {0.5,2}

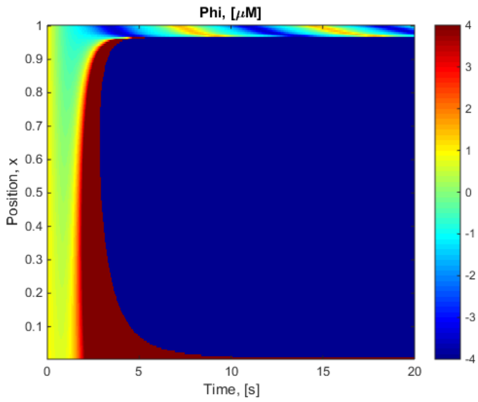


Figure 4: Holding α constant at 1 and varying β linearaly between {0,1}

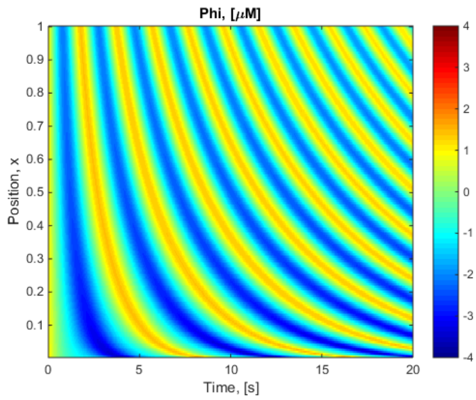


Figure 5: Holding α constant at 1 and varying β linearaly between {0.98,2}

Next holding Beta constant and varying alpha had an equal but opposite effect

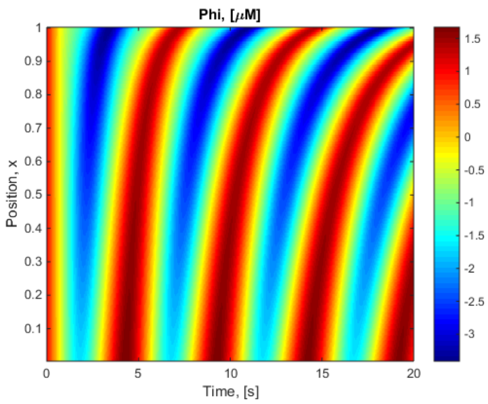


Figure 6: Holding β constant at 1 and varying α linearaly between {0,1}

# Results 3

Creating zero osculation's; to view the diffusion it is important to have a region that initially had no osculation's. To do this the rate of changes of Φ and V to zero for the initial conditions (Φ = V = 1).

$$\frac{d\Phi}{dt} = -\beta V + \frac{\Phi^3}{3} - \Phi = 0 \qquad \& \qquad \frac{dV}{dt} = \beta\Phi + \alpha = 0$$

$$-\beta \times 1 + \frac{1^3}{3} - 1 = 0 \quad \therefore \quad \beta = -\frac{2}{3} \qquad\qquad \beta \times 1 + \alpha = 0 \quad \therefore \quad \alpha = \frac{2}{3}$$

Figure 7 shows Beta and alpha varying over x to obtain zero osculating.

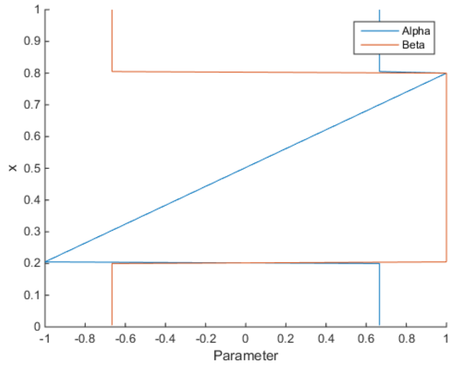


Figure 7: Alpha and Beta varying over x to obtain zero osculation's between 0<x<0.2 and 0.8<x<1.

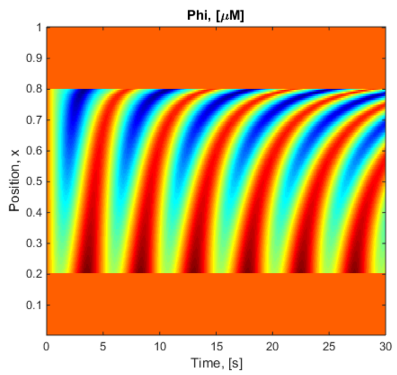


Figure 8: α and β determined by Figure 7 and Equations 1 and 2; Zero Diffusion

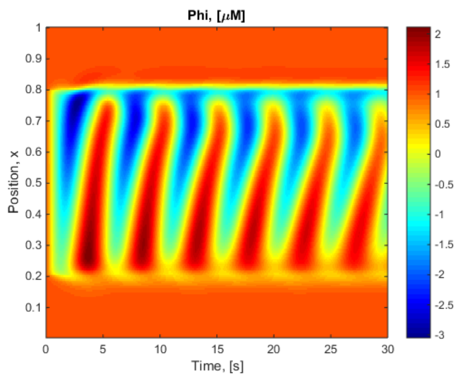


Figure 9: α and β determined by Figure 7 and Equations 3 and 4; Simple Diffusion 200e-6

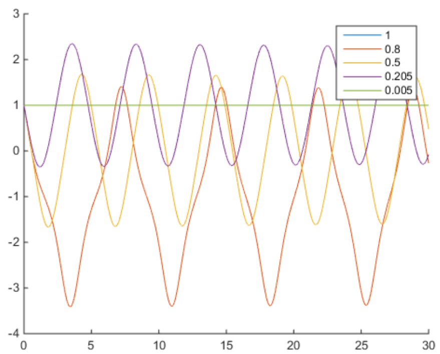


Figure 10: A few x points of Figure 8 potted over time.

# Results 4

Changing the diffusion constant, D.

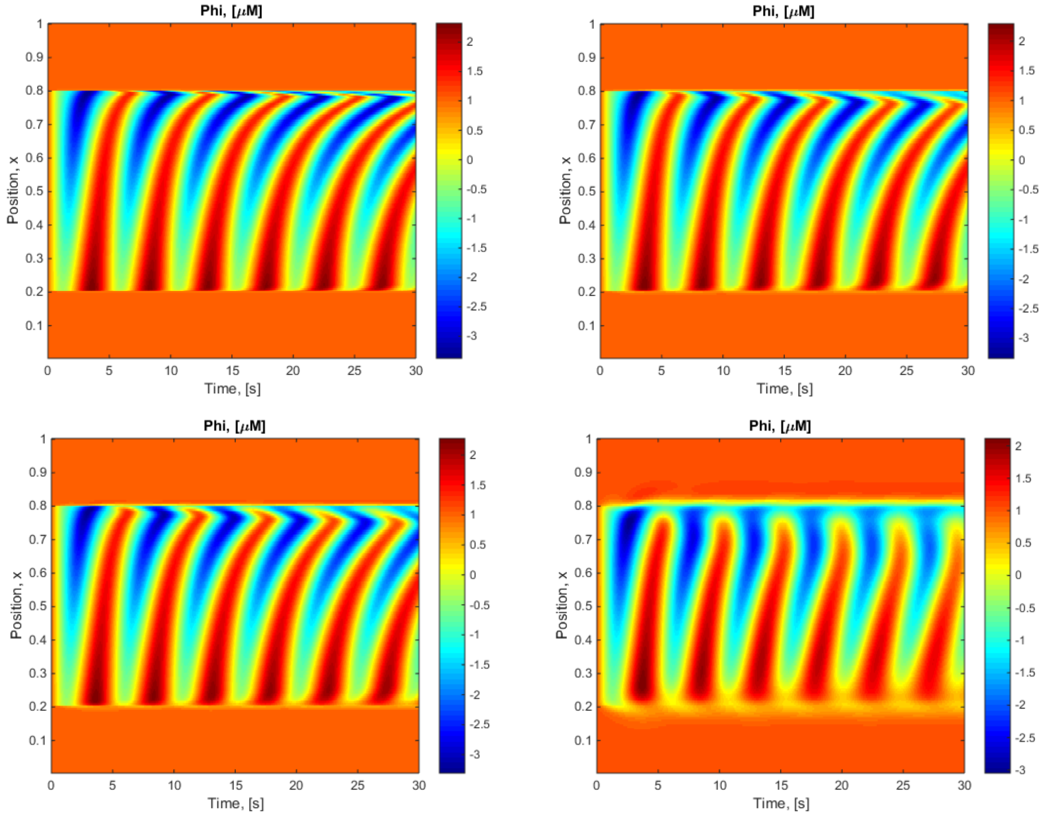


Figure 11-14: D = {1e-6, 5e-6, 10e-6, 100e-6}

# Results 5

Simulated over a long period of time

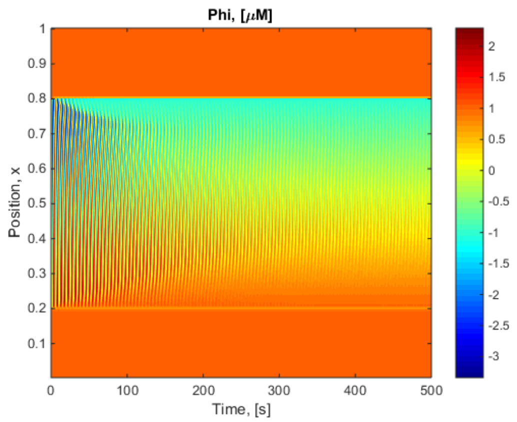


Figure 15: Diffusion, D = 5e-6 and long time, t={0:500}

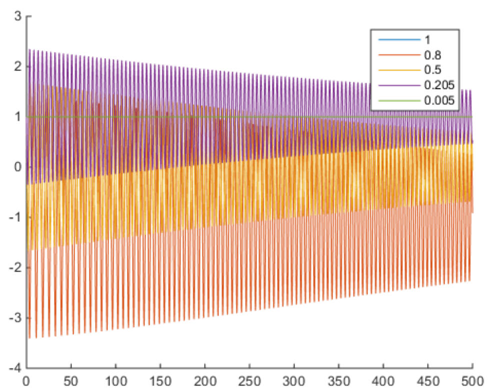


Figure 16: x points from Figure 15 plotted over time.

## Discussion

- Increasing alpha from 0 to 1 shows that there is an increasing instability in the results.
- At low diffusion constants the waves do not protrude into the unexpected region
- however they do form the triangle pattern of diffusion.