UNIVERSITY OF ZIELONA GÓRA
FACULTY OF MATHEMATICS,
COMPUTER SCIENCE AND ECONOMETRICS
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LÃ MINH HOÀNG

AN APPLICATION OF NEURAL NETWORKS IN MULTI-CRITERIA DECISION ANALYSIS

Thesis supervisor

Dr hab. Zbigniew Świtalski, prof. UZ

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Introduction

Multi-criteria Decision Analysis (MCDA) which can also be called as "Multiple Criteria Decision-Making" or "Multi-criteria Decision Aiding" in English or "Analyse Multicritère" in French is an exciting field which has seen many achievements including increasing number of research papers, books, and theories over the years since the 70s (Vincke, 1992; Roy, 2005).

For many decision makers, MCDA is an excellent tool in getting comprehensive insight information when solving a decision problem especially when there are many different factors which are taken into account (Vincke, 1992; Roy, 2005).

In this thesis, we show to the reader how to incorporate Artificial Neural Networks (ANN) in MCDA . Thus, to prove the feasibility of this incorporation; first, we point out some difficulties which MCDA has when dealing with complicated decision problems, especially problems that are repeated many times. Then we see what advantages ANN can offer to solve these difficulties. In our case study, we use the very popular method of MCDA, namely Analytic Hierarchy Process (AHP), invented by Saaty (1980).

To make more sense to the reader, the contents of this thesis is structured as follow:

In chapter 1 on the following page, the reader is greeted with basic concepts of MCDA and what should we expect from MCDA when using it to solve a decision problem.

Next, in chapter 2 on page 17, we show the concept of Analytic Hierarchy Process (AHP) which is an MCDA method to derive priorities or ratio scales from paired comparisons.

In chapter 3 on page 34, we focus on the details of ANN.

Then, in chapter 4 on page 49, we design a model in which Artificial Neural Network plays a major role in the MCDA process. We also present a case study where we use real data to show how effective Artificial Neural Network can be in learning the decision making process.

Finally, we conclude the thesis with some conclusions which summarizes the points of this thesis.

1 Multi-criteria Decision Analysis

To begin this chapter, we examine the foundation theory of MCDA which is the Decision Theory in section 1.1.

Next, we take a look at the structure of a typical decision situation and how it operates in section 1.2 on the next page and section 1.3 on page 6.

Then we explain to the reader the reasons why people use MCDA for aiding the decision making process in section 1.4 on page 9 and section 1.5 on page 10.

After having a brief description of Decision Theory, decision-making process and why we need MCDA, we continue with the philosophy and paradigms of MCDA in section 1.6 on page 10.

We also list some interesting MCDA methods in section section 1.7.

1.1 The foundation of MCDA: Decision Theory

Decision Theory, in general, is simply a theory about making decisions. However, it is not an easy task to precisely define what Decision Theory is. It is because Decision Theory is an subject that connects many disciplines such as economics, statistics, psychology, politics, social science, philosophy (Wikipedia, 2017*e*). Each discipline has its way of studying and theoretizing on Decision Theory, for example, a psychologist might want to investigate the behavior of people when they are making decisions, an economist is likely to study the payoffs of each decision or a political scientist might want to try to find the optimal voting rule by studying Decision Theory. However, if we abstract Decision Theory from all disciplines, we can see that Decision Theory is all about theoretizing on human activities in situations where there is a goal and options to choose between in order to reach that goal, in other words, decision-making process.

We can generalize the Decision Theory into two definitions: one is broad, and the other one is narrow (Świtalski, 2016):

Broad definition: Decision Theory is the set of all possible disciplines connected with making decisions.

Narrow definition: Decision Theory is the set of theoretical considerations of what is a decision, what is decision situation, what are the elements of a decision situation, what is right or optimal decision and how to make a right or optimal decision.

It is also important to know that Decision Theory has two main branches: Normative Decision Theory and Descriptive Decision Theory. The brief definitions of those two branches can be given as follow:

Normative Decision Theory concerns about constructing rules and methods of making decisions in which an ideal decision maker with perfect information could make optimal decisions.

Descriptive Decision Theory interests in the psychological or behavioral aspect of the decision maker (i.e. how do people in real situations make their decisions).

Also, reader should notice the direction of the thesis in terms of which branch of Decision Theory we will go from here. Because the goal of this study is to introduce a new approach, in this case, Artificial Neural Network which is used to enhance the process of making a decision; therefore, the majority of this study will be in the context of Normative Decision Theory.

Next, in section 1.2 we examine the basic elements of a decision situation.

1.2 Elements of a decision situation

Typically, a decision situation comprises of these elements (Świtalski, 2016):

Decision maker: The decision maker is the one who is responsible for the consequences of their decisions.

Goal: The desired state of our decision problem, usually it is what we want to achieve after solving the decision problem.

Initial state: The beginning state of our decision problem i.e. all the measurements, parameters, conditions or factors we have before starting the decision-making process.

Alternatives: The options or alternative courses of action which are the possibilities of realizing our goal or desired state.

Criterion: A measurement or a standard which allows comparing between different options or alternatives. For example, when we want to buy a new apartment, measurements such as the number of rooms, the number of floors, or the price of the apartment can be used to compare between alternatives. Also, a criterion can also be known as a measurable index, a measurable attribute or a measurable characteristic. A criterion can also be positive or negative, for example, criteria related to time such as time to manufacture, time to delivery or response time are negative criteria because for these criteria we often want their values as small as possible. On the other hand, criteria related to money such as profit, revenue, or return of investment are positive criteria because for these criteria, more is usually better.

Constraints: Limitations stated by the decision problem. For example, a budget to buy a new apartment can not exceed 50.000 US dollars or the response time from a computer system should be below 100 milliseconds.

Consequences: Outcomes or results we get after realizing the goal with the option or alternative we have chosen.

Uncertainty: In a decision situation, uncertainty is an unknown factor that could affect the outcomes of the decision situation. For example, when we are traveling, going by plane is a good decision since it is fast and cheap for a long trip, but we can not know if we are going to be captured by terrorist or not.

Preferences: The view of the decision maker which determines what kind of criterion or alternative the decision maker will use in the decision situation. For example, when choosing a apartment to buy, if the decision maker only prefers the appearance of the apartment then he may choose criteria such as the number of windows, the color of the wall, or what kind of architectural style the apartment is. On the other hands, if the goal of the decision situation is looking for accommodation and the decision maker only prefers apartment then he only considers apartments as the alternatives for the goal.

Decision-making process: The process in which the decision maker makes the evaluation of alternatives with respect to criteria and constraints. The result of this process is the final suggestion to the decision maker so he or she can make the final decision.

The figure 1.1 on the following page shows us the relationship between the elements of a decision situation. As we can see, from the initial state we formulate the decision-making process by determining the criteria and constructing the alternatives from the decision problem.

In the decision-making process, the criteria should be determined with the respect of the decision maker's preferences and the alternatives should be evaluated by the criteria and the constraints of the decision problem. Then, from the result of the decision-making process, the decision maker makes the final decision in order to solve the decision problem thus realizing the goal of the decision situation.

After realizing the goal, we get the consequences or the outcomes of the decision situation, and because of the uncertainty, these results could be affected in an unexpected way.

Next, in the section 1.3, we study the steps of decision-making process.

1.3 The steps of decision-making process

To solve the decision problem, the decision maker should follow the steps as shown in figure 1.2 on page 8 (Świtalski, 2016):

Investigate the situation in detail: The decision maker starts to formulate the decision-making process by investigating the decision situation. The investigation includes determining the goal of the decision problem, collecting data and information related to the decision problem and find the sources of the decision problem.

Construct the decision alternatives: After the investigation, based on the gathered information, the decision maker constructs a set of alternatives. These alternatives are the potential solutions for realizing the goal of the decision problem.

Formulate criteria: The decision maker formulates a set of criteria which is used to determine the value of each alternative. The preferences of the decision maker decide what kind of criterion will be used in the decision-making process.

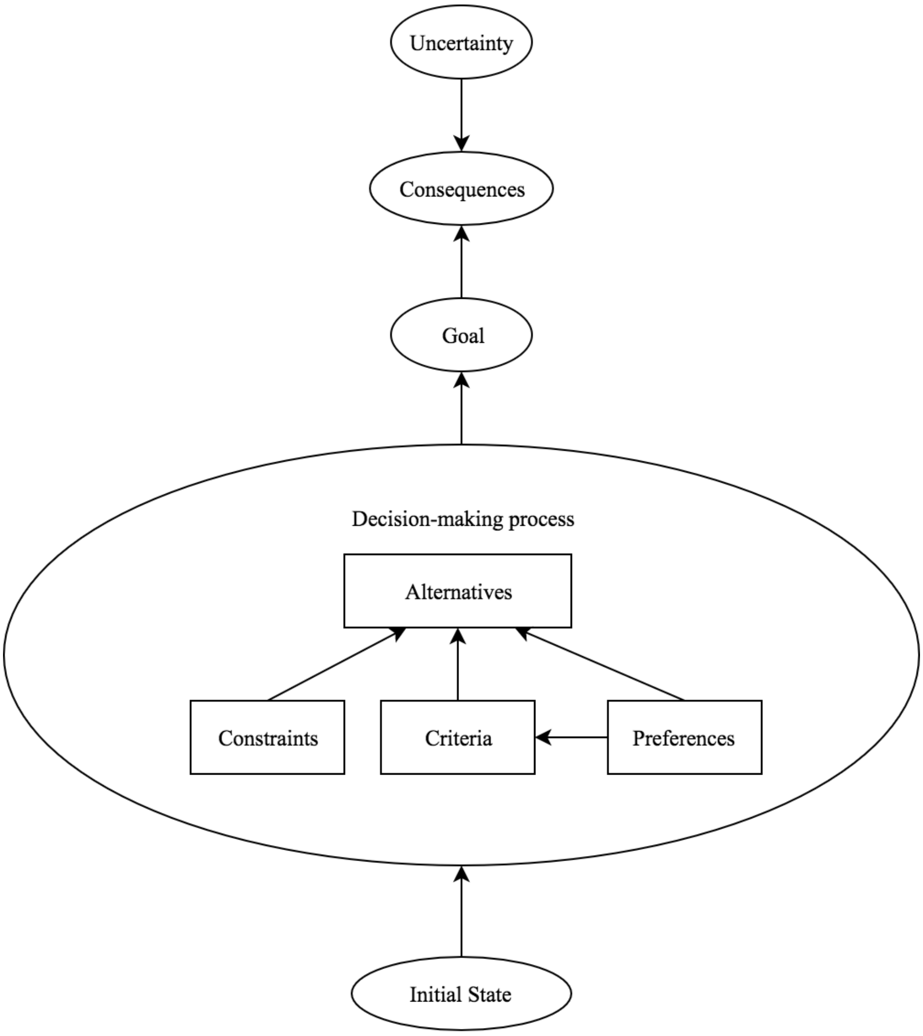

Figure 1.1: Elements of a decision situation

Evaluation of alternatives: By using some methods (refer to section 1.7 on page 12), the decision maker evaluates the alternatives with the formulated criteria. The result of the evaluation helps the decision maker to make the decision which realizes the goal of the decision problem.

Implement the decision: The decision maker constructs a plan of activity based on his or her decision. From this plan, the decision maker implements it and in the end, realizes the goal of the decision problem.

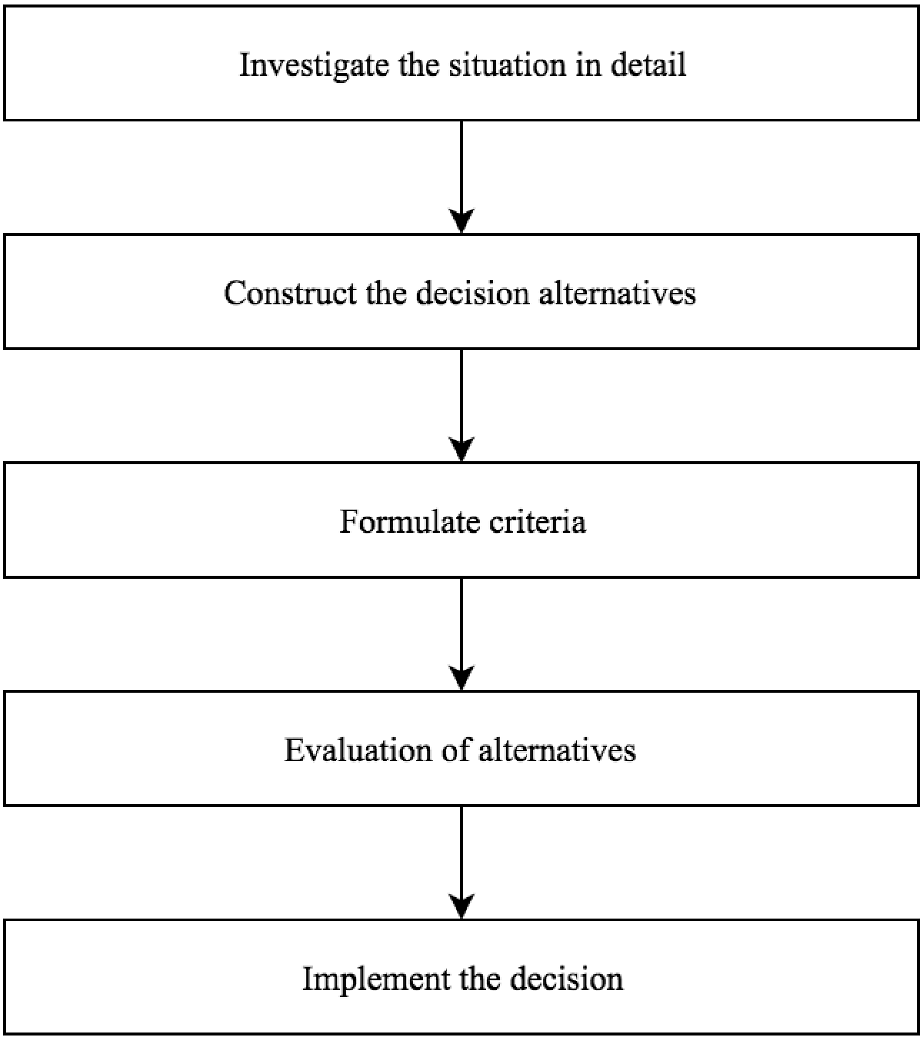

Figure 1.2: Steps of a decision-making process

Depending on the nature of the decision problem, the decision-making process can become a cycle in which the next cycle evaluates the consequences of the decision made from the previous decision cycle and makes corrections. The figure 1.3 on the next page depicts the decision cycle.

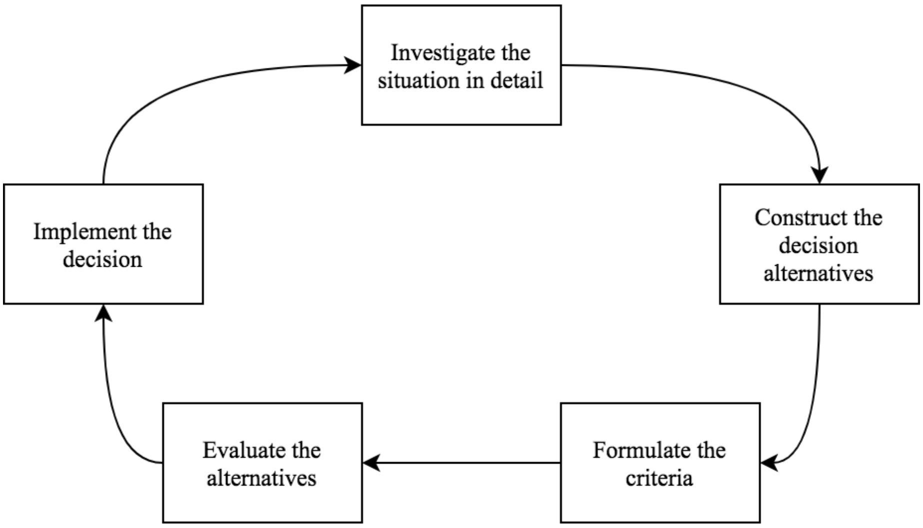

Figure 1.3: Decision cycle

1.4 The necessity of Decision Analysis

In sections 1.1-1.3, we learned that in a decision-making process, the decision maker has the responsibility to choose the right decision in order to realize the goal in that particular situation. However, there are many factors with hidden information in a decision-making process that can hide the true value of each decision thus affect the judgment of the decision maker.

For example, consider the situation when we choose suppliers for providing materials to build a particular product. If we do not spend the time to research the past performance of each supplier, we will never know the true value of each supplier in terms of providing the needed materials. Thus our judgment for choosing the suppliers will not be certain. To overcome this difficulty of uncertain judgment, a decision maker needs to practice Decision Analysis in a decision-making process.

Decision Analysis or Decision Aiding (DA) can be defined as follows (Roy, 2005): DA is an activity in which the decision maker uses explicit and perhaps formalized models to answer the questions asked by the stakeholder who interests in the decision process. These answers help the decision maker to clarify various elements inside the decision situation.

With the above definition, DA's purpose is to establish the decision-making process in a scientifically based way to clear out the uncertainty in the decision maker's judgment (Roy, 2005). It is made possible by providing the decision-making process with working hypotheses and formulations of propositions such as satisfying solutions or possible compromises.

In section 1.5, we present to reader the notions of Mono-criterion approach and Multi-criteria approach, thus the reason why Multi-criteria approach is the preferred tool for many decision makers in decision analysis.

1.5 From Mono-criterion to Multi-criteria

In a decision-making process, a decision maker has two approaches to evaluate the options. They are Mono-criterion approach and Multi-criteria approach. To simply put, Mono-criterion approach is when the decision maker only uses one criterion for determining the value of each option of the decision problem. Multi-criteria approach, on the other hand, evaluates each option on many different criteria associated with various aspects of the decision problem. The reader can find more information about criterion in section 1.6.

When a decision maker is dealing with a DA process, it is very difficult for the decision maker to think with only one single criterion in his or her mind only (Mono-criterion approach). In a multi-actor environment, it is even more unlikely for that single criterion to satisfy the point of view of every actor in the decision-making process because each actor plays a different role in that process which gives different preference to the actor own objectives and value system (Roy, 2005).

For any case, it is essential to consider every point of view dealing with many perspectives of a decision-making process, for example, the financial status, the current human resources, the affected ecosystem, security or quality of services. By regarding each related point of view individually, independently from the others, it is likely to arrive at a clear decision that meets all actors' preferences and requirements (Roy, 2005).

Therefore, it is typical to associating a particular criterion to each related point of view (Multi-criteria approach) and use a suitable scale (qualitative scale or quantitative scale) to evaluate any potential action (Roy, 2005).

As we can see, in many decision-making contexts, using Mono-criterion approach might overlook certain aspects of the decision problem, thus affecting the judgment of the decision maker. By using Multi-criteria approach in such cases, the decision maker can avoid the aforementioned danger of neglecting.

Next, in section 1.6, we explore the three basic concepts of MCDA which gives us a throughout view of the structure of a Decision Analysis process that connects with Multi-criteria to help the decision maker in solving his or her decision problem.

1.6 Basic Concepts of MCDA

Recall from section 1.2 on page 5, we know that from the elements of a decision situation, there is **alternatives** and **criterion** (or **criteria**). Alternatives define the options that the decision maker has to choose as the way to realize the goal in a decision-making process. Criterion (or criteria) acts as a tool to evaluate and compare alternatives. These elements have fundamental roles in the course of analyzing and structuring the decision-making process with MCDA.

The section below presents the definitions of each element so the reader can generalize what an MCDA process is.

1.6.1 Alternatives

In a decision-making process, to make a decision, the decision maker have to choose an action with the purpose of realizing the goal of the decision problem by using the chosen action.

The action in this sense can be defined as a potential action which constitutes to the object of the decision or one of the objectives of the decision analysis process. However, not all actions can be qualified as potential actions. The action must be considered to be possible to implement it or justifies some interests within the decision analysis process (Roy, 2005).

With the definition of potential action, the concept of alternative is often modeled in two ways (Roy, 2005):

Mutual exclusive: Two distinct potential actions must not be conjointly put into realizing the goal of the decision problem.

Non-mutual exclusive: Various potential actions can conjointly be put together to realize the goal of the decision problem.

Many authors believed that potential actions should be mutual exclusive, but it is not always the case (Roy, 2005). In many real world decision analysis contexts, it is appropriate to combine several potential actions into realizing the goal of the decision problem rather than using only one potential action.

In any case, for any decision analysis process, there is always a set of more than two potential actions or alternatives, and this set is not fixed (Roy, 2005). During the decision analysis process, this set can evolve i.e. adding more potential actions as a result of investigating every aspect of the decision problem. For example, the investigation can help in gaining more knowledge about the goal, constraints, the preferences of the stakeholders, the possible outcomes of the actions, and other aspects of the decision problem. Thus, this new knowledge could open new boundary that helps to identify other actions as potential actions.

For the set of all potential actions or alternatives in a decision-making process, we use the symbol A to denote it. To designate a potential action or alternative, we use a. When the number of actions is finite ($|A| = m$) we have:

$$A = \{a_1, a_2, \ldots, a_m\}$$

For each potential action, we can represent it by some variables x_1,x_2,x_3,... and we can write it as follows:

$$a = (x_1, x_2, x_3...)$$

Each variable x_i where $i = 1, 2, ...$ is the score of evaluating the action a by using a certain criterion.

Next, the above notions are used to describe the definition of Criterion in section 1.6.2.

1.6.2 Criteria

The reader should recall from section 1.2 on page 5 that a criterion is used as a tool to determine the value of an alternative. By using the notion of potential action a in section 1.6.1 on the preceding page, we can denote a criterion as a function g which evaluates a potential action or alternative; this function then outputs a $performance$ value associating with the input potential action. We denote this evaluation as $g(a)$(Roy, 2005).

Often, $g(a)$ is a real number. However, it is necessary to define a set X_g of all possible evaluations that the criterion g can produce. This set X_g should also follow a scale system to be accepted by all stakeholders in an MCDA process(Roy, 2005).

Each element $x \in X_g$ is called *degree* or *score* of the scale. We can use any number, verbal statement or pictogram to designate a degree. During the comparison between two alternatives, we compare the two degrees which represent the respective performances of the two alternatives according to criterion g.

There are many types of scales which reader should notice; the following two scales are the most typical (Roy, 2005):

Purely ordinal scale or qualitative scale: The gap between two degrees does not have a clear meaning i.e. we only know the ranking of the degrees, we do not know how close or far from one degree to another.

Quantitative scale: The degrees in this scale are defined by a clear quantity in a way that it gives meanings i.e. we can count the differences between two different degrees. The quantitative scale can also be known as the interval scale.

1.7 MCDA Methods

Decision Makers can use different methods to analyze the alternatives and find the best alternative or the group of best alternatives, some of these methods do not require criteria to have weights or order of importance, while others do require (Świtalski, 2016).

For example, the following methods do not use weights:

Pareto Rule: We choose from the given alternatives the set of alternatives which are non-dominated by any other alternative. Non-dominated alternative is an alternative a_k such that there exists no superior alternative a_i to a_k when a_k is being compared with all other alternatives. The reader should also know that a_i is considered superior than a_k if and only if the scores of a_i are better than or equal the scores of a_k respectively but all of them must not be equal. For example, we have the alternatives a_i and a_k such that $a_i = (x_1, x_2, x_3)$ and $a_k = (x_1^{'}, x_2^{'}, x_3^{'})$. Now, in order for a_i to dominate a_k or $a_i > a_k$, a_i and a_k must satisfy one of the following conditions:

$$(x_1, x_2, x_3) > (x_1^{'}, x_2^{'}, x_3^{'}) \iff (x_1 \geq x_1^{'}) \wedge (x_2 \geq x_2^{'}) \wedge (x_3 > x_3^{'})$$

or

$$(x_1, x_2, x_3) > (x_1^{'}, x_2^{'}, x_3^{'}) \iff (x_1 > x_1^{'}) \wedge (x_2 \geq x_2^{'}) \wedge (x_3 \geq x_3^{'})$$

or

$$(x_1, x_2, x_3) > (x_1^{'}, x_2^{'}, x_3^{'}) \iff (x_1 \geq x_1^{'}) \wedge (x_2 > x_2^{'}) \wedge (x_3 \geq x_3^{'})$$

however, the condition

$$(x_1, x_2, x_3) > (x_1^{'}, x_2^{'}, x_3^{'}) \iff (x_1 \geq x_1^{'}) \wedge (x_2 \geq x_2^{'}) \wedge (x_3 \geq x_3^{'})$$

is not valid because there is no domination when:

$$(x_1 = x_1^{'}) \wedge (x_2 = x_2^{'}) \wedge (x_3 = x_3^{'})$$

in other words, the scores of a_i are better or equal than the scores of a_k but not all of them are equal.

For better understanding, we use table 1 as a simple example to demonstrate the Pareto Rule. The table 1 is a decision table that includes the scores of three alternatives a_1, a_2, and a_3 in a apartment buying problem. In our apartment buying problem, we have three criteria: number of rooms (g_1), floor area of the apartment in square meter (g_2), and the price of the apartment in US dollar (g_3) . We want to buy a apartment such that the bigger the number of rooms and floor area the better the apartment while the price should be as low as possible.

Table 1: Decision table for apartment buying problem

	g_1	g_2	g_3
a_1	3	100	76000
a_2	2	75	88000
a_3	4	100	190000

To apply the Pareto Rule, we check each alternative a_1,a_2, and a_3 to see which one is non-dominated. In table 1 we can see that a_2 is dominated by a_1 because a_1 has more rooms, bigger floor area, and lower price than a_2. Therefore, we exclude a_2 from our non-dominated alternatives set. Continue, we can see that a_1 and a_3 are non-dominated because there is no other alternative that can be considered superior than a_1 and a_3. Finally, we get the set of non-dominated alternatives as follows:

$$P(A) = \{a_1, a_3\}, \tag{1.1}$$

where A is the set of the given alternatives $\{a_1, a_2, a_3\}$ and $P(A)$ is the set of non-dominated alternatives that we get after using Pareto Rule.

Conjunctive Method: We define levels of satisfaction for all criteria then for each criterion we find the set of alternatives which satisfies it. After getting all the sets of satisfying alternatives for each criterion, we intersect all these sets then use the resulting set from the intersection as our chosen set of alternatives. For example, we define the levels of satisfaction for the apartment buying problem in table 1 as follow:

$s_1 = 3, s_2 = 50, s_3 = 100000$. Now, we find all the sets of satisfying alternatives for each criterion g_1,g_2, g_3:

$$C_1(A) = \{a_{1,}a_3\} \tag{1.2}$$

$$C_2(A) = \{a_1, a_2, a_3\} \tag{1.3}$$

$$C_3(A) = \{a_1, a_2\} \tag{1.4}$$

Let us take a look at the set $C_1(A)$. Because only a_1 and a_3 have the number of rooms that is equal or bigger than s_1 ($3 = 3$, $4 > 3$) therefore the set $C_1(A)$ includes a_1 and a_3 but excludes a_2, thus $C_1(A) = \{a_{1,}a_3\}$. Similarly, we compare s_2 and s_3 with the scores of criterion g_2 and g_3 respectively to get the set $C_2(A)$ and the set $C_3(A)$. Finally, we intersect $C_2(A)$, $C_2(A)$, and set $C_3(A)$ to get the final set of alternatives that satisfies all the levels of satisfaction s_1, s_2, s_3:

$$C(A) = C_1(A) \cap C_2(A) \cap C_3(A) = \{a_1\} \tag{1.5}$$

Disjunctive Method: Similar to Conjunctive Method but we choose alternatives which are good for at least one criterion. In other words, instead of intersecting all the sets of satisfying alternatives for each criterion, we find the union of all those sets:

$$D(A) = C_1(A) \cup C_2(A) \cup C_3(A) = \{a_1, a_2, a_3\} \tag{1.6}$$

The next methods require weights or order of importance for criteria:

Lexicographic Method: We order the criteria from the most important to the less important. Then, we choose alternatives which are the best with respect to the first ranking criterion base on the evaluation of the Decision Table. If there is only one such alternative, then we stop the process. If there are more such alternatives (more than one), we continue to choose from these alternatives the ones which are the best with respect to the second ranking criterion. We continue the process unless we reach the last criterion in the ordering or there is only one alternative left. For example, we define the order of importance of the criteria in the apartment buying problem in table 1 on the previous page as follows:

$$g_2 > g_3 > g_1 \tag{1.7}$$

Now, we find which alternative is the best with respect to the criterion g_2. From table 1 on the preceding page we can see that a_1 and a_3 are equal in terms of g_2 therefore both of them are the best alternative thus the set of best alternatives with respect to g_2 is:

$$C_2^*(A) = \{a_{1,}a_3\} \tag{1.8}$$

Because we have two alternatives in the set $C_2^*(A)$, we continue to choose from the set $C_2^*(A)$ the alternatives which are the best with respect to the criterion g_3. Now, from table 1 we can see that a_1 is better than a_3 in terms of g_3, therefore we have:

$$C_3^*(C_2^*(A)) = \{a_1\} \tag{1.9}$$

Finally, because we only have one alternative in the set $C_3^*(C_2^*(A))$ therefore we stop the process and use the set $C_3^*(C_2^*(A))$ as the final result:

$$L(A) = \{a_1\} \tag{1.10}$$

Weighted Average Method: In this method, we change the scale of table 1 on page 13 to the scale of satisfaction. In this scale of satisfaction, an alternative with respect to a criterion has the value of 3 if it is the best comparing to other alternatives, 2 if it is the second and 1 if it is the last. Using the scale of satisfaction, we build a scaled decision table in table 2. Also, we assign a weight to each criterion which shows how important a criterion is compared to other criteria. The reader should also know that the sum of all weights is 1.

Table 2: Scaled decision table

	g_1	g_2	g_3
a_1	2	3	3
a_2	1	1	2
a_3	3	3	1
w	0.2	0.5	0.3

In table 2 we can see that now the values for each criterion is an value created from the ordering or ranking between alternatives. For example, in terms of g_1, a_1 has the value of 2 or the second position in the ranking because its original value in table 1 on page 13 is 3 rooms which is higher than a_2 but lower than a_3. For g_2, because a_1 and a_3 have the same original value which is 100 square meters and it is the best value in term of g_2 in table 1 on page 13; therefore, they have the same value of 3. Finally, for $g3$, by comparing the prices the alternatives, we can see that a_1 has the best value of 3, g_2 has the value of 2, and a_3 has the value of 1.
Now, we calculate the weighted average for each alternative by multiplying the scaled decision table with the weights of criteria and sum table row by row. By comparing the scores and select the biggest one, we have our best alternative as in figure 3 on the following page.

As we can see in table 3 on the next page, the alternative a_1 has

Table 3: Scaled decision table multiplied with weights

	g_1	g_2	g_3	$S(a_i)$
a_1	$2 * 0.2 = 0.4$	$3 * 0.5 = 1.5$	$3 * 0.3 = 0.9$	$(0.4 + 1.5 + 0.9) = 2.8$
a_2	$1 * 0.2 = 0.2$	$1 * 0.5 = 1$	$2 * 0.3 = 0.6$	$(0.2 + 1 + 0.6) = 1.8$
a_3	$3 * 0.2 = 0.6$	$3 * 0.5 = 1.5$	$1 * 0.3$	$(0.6 + 1.5 + 0.3) = 2.4$
w	0.2	0.5	0.3	

the highest score $S(a_1) = 2.8 > 2.4 > 1.8$ therefore a_1 is the best alternative according to the weighted average method.

The above methods are just simple methods, however, there are other methods that are more complicated and can be used in complex decision problem (Wikipedia, 2017*h*):

Analytic hierarchy process (AHP): a method to derive priority or ratio scales from paired comparisons (Wikipedia, 2017*b*).

ELECTRE: this method is a "outranking method" of decision making which has applications in choosing problem, ranking problem, and sorting problem (Wikipedia, 2017*f*).

PROMETHEE: is also another "outranking method". Combining with its descriptive complement method which is geometrical analysis for interactive aid (GAIA), Promethee and Gaia methods can be used both in normative approach and descriptive approach (Wikipedia, 2017*i*).

In the next chapter 2 on the following page, we present to the reader a comprehensive description of AHP method which is used in chapter 4 on page 49 as a basic method for our case study when applying Artificial Neural Networks in MCDA.

2 The Method of Analytic Hierarchy Process

In this section, we get into the details of Analytic Hierarchy Process (AHP) by answering questions such as: What is AHP? Who created it? In which kind of decision problems can we apply AHP?

After getting the general ideas about AHP, we continue with the methodology and core elements of AHP such as: What is the Pairwise comparison? How to make a comparison matrix? How to calculate priority vector? How to use Consistency Index and Consistency Ratio to verify the comparisons?

Explaining AHP using only its definitions can be tough at some points. Therefore, we use an example to explain each aspect of AHP. It is easier to understand the concept of AHP that way.

2.1 Brief History of Analytic Hierarchy Process

Analytic Hierarchy Process (AHP) is created by Professor Thomas L. Saaty as a Multi-Criteria decision-making method. This method is described in the book "Mathematical Principles of Decision Making" or "Principia Mathematica Decernendi" which Saaty wrote in the 70s (Saaty, 1980; Goepel, 2013).

In general, Analytic Hierarchy Process is a method to derive priority or ratio scales (we use the word "priority" and "ratio" interchangeably) from paired comparisons (Saaty, 2008). The AHP method takes input in the form of measurements such as length, size, amount, or subjective opinion such as preferences (Teknomo, 2006). To derive the ratio scales, we need to solve the principal Eigenvectors problem and then to verify the scales we use Consistency Index which is calculated from Eigenvalue.

Because AHP allows small inconsistency in making comparisons (Teknomo, 2006), we can use AHP to solve various decision problems, from making simple and straightforward decisions to complex and high-stakes problems in which human perceptions and judgments are required (Wikipedia, 2017*b*), for example:

1. Choice
2. Ranking
3. Quality management
4. Conflict resolution
5. Etc.

An interesting example of AHP application is suspect identification by witnesses in criminal cases (Mu and Chung, 2013). Usually, candidates for identification are shown altogether or sequential, but in the AHP case, they are shown in pairs. Comparisons between pairs are then made by the witnesses. Studies have shown that by using AHP method, the reliability of identification has been increased from 55% to 83% and the false identification rate has been reduced from 20% to 17% (Mu and Chung, 2013). Also, the consistency index produced by AHP method is a good indicator for telling if the statements made by witnesses are consistent or not.

2.2 The steps of AHP

To solve a decision problem by using AHP, we need to decompose the decision problem into the following steps (Saaty, 2008; Goepel, 2013):

1. From the decision problem, we define the goal of the problem and determine the related elements such as alternatives, criteria, and sub-criteria.
2. We construct a hierarchy based on the decision problem. We put the goal of the decision problem on the first level of the hierarchy. Then we put the criteria (and sub-criteria for each criterion) in the intermediate levels. Finally, at the last level we put the alternatives.
3. On each level of the hierarchy (except the top level), we make pairwise comparisons for the criteria, groups of sub-criteria that belongs to a particular criterion, and the alternatives with respect to each criterion or sub-criterion. We use comparison matrices to represent the results of pairwise comparisons.
4. We calculate the priorities or priority vectors from the comparison matrices. Also, we should calculate the Consistency Index and Consistency Ratio to check if the comparisons are consistent or not.
5. We calculate the ranking of alternatives by aggregating priorities from criteria, sub-criteria and the alternatives with respect to each criterion or sub-criterion.
6. After obtaining the ranking for the alternatives, we should examine the Consistency Index and Consistency Ratio to see if our comparisons are consistent or not. If not, we need to revise our comparisons.

If the comparisons are consistent, we can use the ranking of the alternatives to select the optimal alternative for our decision problem, i.e. the alternative with the highest ranking.

In the next sections, we explain in detail the fundamentals of AHP as follows:

- In sections 2.3 - 2.6, we explain the mathematics behind AHP and the way to check the consistency of the pairwise comparison. For simplicity, we use a very simple example which doesn't have any criteria.
- Next, in section 2.7 on page 26, we show the readers how we can construct the hierarchy of AHP from a multi-criteria decision problem by using an example with criteria and sub-criteria.
- Finally, in section 2.8 on page 28, we show the readers the way to get the final ranking of alternatives by aggregating priorities.

2.3 Pairwise comparison

The most fundamental element of AHP method is the pairwise comparison. In short, it is like asking between two objects, which one do you prefer and how much do you prefer it to the other object.

Suppose we have two cars made by two different manufacturers, one is from Renault, and the other is from Toyota. To make a pairwise comparison between these two cars, we use a relative scale in figure 2.1 to measure how much we prefer a car compare to the other (Saaty, 2008).

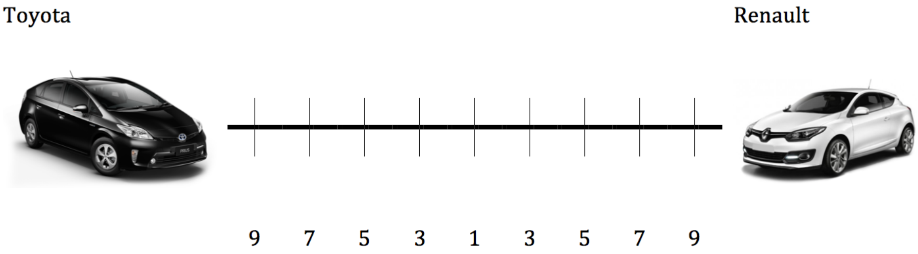

Figure 2.1: Relative scale between two car manufacturers

In figure 2.1 , if we prefer the Toyota car than the Renault car, we choose a number which denotes a preference value between one and nine on the left side of the scale. If we prefer the Renault car otherwise, we choose a number on the right side of the scale.

The preference values or judgement values are used to describe how strong our preference is over a car. The table 4 explains the judgement values (Saaty, 2008)

Table 4: Judgement table

Preference Value	Description
1	Two cars are equal in comparison; no car is more preferred than the other.
3	One car is slightly preferred than the other.
5	One car is strongly preferred than the other.
7	One car is very strongly preferred than the other.
9	On car is absolutely preferred than the other.
2,4,6,8	Intermediate scales between two adjacent values.

For example, if we firmly prefer the Toyota car than the Renault car, we then mark the scale like in figure 2.2 on the next page.

Now, if we add one more car from another manufacturer, our pairwise comparisons become like in figure 2.3 on the following page.

As we can see, the number of pairwise comparisons has been increased from one to three comparisons; it is a combination of the number of objects to be compared. We can calculate the number of comparisons using table 5 on the next page (Teknomo, 2006).

From table 5 on the following page, it suggests that if we compare too many objects (30 cars for example), the number of comparisons will become large

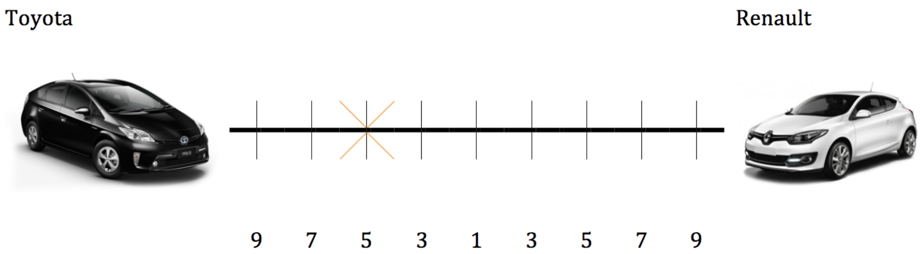

Figure 2.2: Toyota is being preferred on relative scale

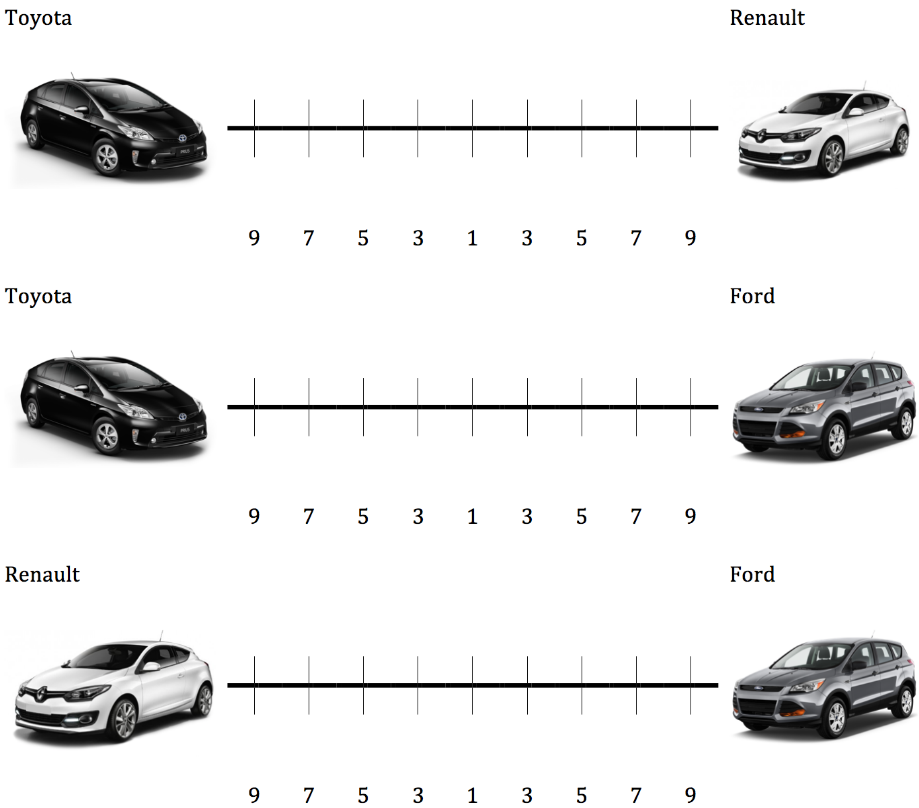

Figure 2.3: Comparisons between three car manufacturers

Table 5: Number of comparisons per number of objects

Number of objects	1	2	3	4	5	6	7	8	n
Number of comparisons	0	1	3	6	10	15	21	28	$\frac{n(n-1)}{2}$

(435 comparisons for 30 cars). With a large number of comparison, it often confuses the person who makes the comparison, and as a result, the likelihood of inconsistent comparisons will be high.

Next, in section 2.4, as we have made our pairwise comparisons between objects, we will aggregate them into a comparison matrix.

2.4 Comparison Matrix

We use the comparison between three cars in the previous section as the example of how to make a comparison matrix from pairwise comparisons.

In our example, we have a middle-aged man named Darek. Darek wants to buy a new car, but he does not know which car he should buy because there are just too many cars to choose. Thankfully, a friend who works in automobile industry recommends Darek three car models from three different manufacturers. These three cars are currently the best, but Darek can only choose one. Therefore he makes the comparisons in figure 2.4 based on his subjective judgement.

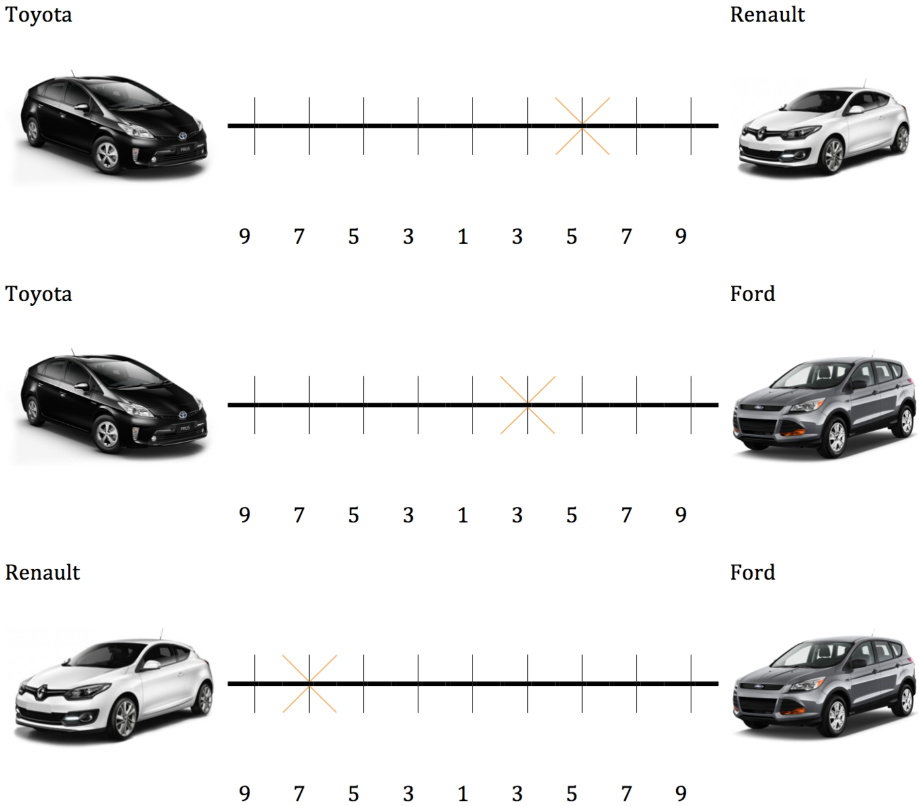

Figure 2.4: Darek's pairwise comparisons

Because we have three pairwise comparisons, in this case, our comparison matrix is a three by three matrix. In this matrix, the values on the diagonal line are one because it is the value of a comparison of two same objects. First, we fill up the upper triangular matrix using the results of our comparisons and then we fill the lower triangular matrix by using the reciprocal values of the upper triangular matrix. We use the following rules to fill up the comparison values (Teknomo, 2006):

1. We put the **preference value** when the preference value is on the **left side** of 1 on the scale.

2. We put the **reciprocal value** of the **preference value** when the preference value is on the **right side** of 1 on the scale.

In our case, because Darek strongly prefers Renault than Toyota, we put $\frac{1}{5}$ in row 1 and column 2 of the matrix. Next, Darek slightly prefers Ford than Toyota; we put $\frac{1}{3}$ in row 1 and column 3 of the matrix. Finally, between Renault and Ford, Darek very strongly prefers Renault then Ford, therefore we put 7 in row 2 and column 3 of the matrix. The comparison matrix is presented in table 6.

Table 6: Upper triangular matrix for Darek's pairwise comparisons

	Toyota	Renault	Ford
Toyota	1	$\frac{1}{5}$	$\frac{1}{3}$
Renault		1	7
Ford			1

We only have set the value for the upper triangular matrix, for the lower triangular matrix we use the reciprocal values of the upper triangular matrix based on the following rule (Teknomo, 2006): If a_{ij} is the element of row i column j of the matrix, then the lower diagonal is filled using this equation:

$$a_{ij} = \frac{1}{a_{ji}} \tag{2.1}$$

Using the rule above we now have the complete comparison matrix in table 7.

Table 7: Complete comparison matrix

	Toyota	Renault	Ford
Toyota	1	$\frac{1}{5}$	$\frac{1}{3}$
Renault	5	1	7
Ford	3	$\frac{1}{7}$	1

Notice that all values in the comparison matrix must be positive, or $a_{ij} > 0$.

In section 2.5 on the next page, we use this comparison matrix to calculate the eigenvalues and eigenvectors to calculate the final priorities or Priority Vector of our alternatives which are the three car models in Darek example.

2.5 Priority Vector

With the complete comparison matrix in the previous section, we can now find the priority vector for our alternatives. However, first, we need to calculate the eigenvalues and eigenvectors of the matrix because the priority vector is the normalized principal Eigenvector of the matrix.

Usually, eigenvalues and eigenvectors of a matrix are calculated by using computer programs such as MATLAB or Maxima due to the increasing complexity in calculation when the size of the matrix is getting bigger. However, for small matrices with size $n \leq 3$ like the one in our example, it is possible to use an approximation method to approximate the normalized principal Eigenvector with low error rate (Teknomo, 2006).

The approximation method consists of several steps. First, we need normalize each column in the comparison matrix. For example, we have table 7 on the preceding page from section 2.4 on page 21. First, we sum all elements of each column of table 7 on the preceding page to get table 8.

Table 8: Comparison matrix with sum by column

	Toyota	Renault	Ford
Toyota	1	$\frac{1}{5}$	$\frac{1}{3}$
Renault	5	1	7
Ford	3	$\frac{1}{7}$	1
Sum by column	9	$\frac{47}{35}$	$\frac{25}{3}$

Next, we normalize each column in table 8 by dividing each element of the column by the sum of that column, the calculations are shown in table 9.

Table 9: Normalized comparison matrix

	Toyota	Renault	Ford
Toyota	$1/9 = \frac{1}{9}$	$\frac{1}{5}/\frac{47}{35} = \frac{7}{47}$	$\frac{1}{3}/\frac{25}{3} = \frac{1}{25}$
Renault	$5/9 = \frac{5}{9}$	$1/\frac{47}{35} = \frac{35}{47}$	$7/\frac{25}{3} = \frac{21}{25}$
Ford	$3/9 = \frac{1}{3}$	$\frac{1}{7}/\frac{47}{35} = \frac{5}{47}$	$1/\frac{25}{3} = \frac{3}{25}$

To get the approximation of the normalized principal Eigenvector we sum each row of the normalized comparison matrix in table 9 and then we divide each sum with the number of alternatives (which is $n = 3$):

$$v = \frac{1}{3}\begin{bmatrix} \frac{1}{9} + \frac{7}{47} + \frac{1}{25} \\ \frac{5}{9} + \frac{35}{47} + \frac{21}{25} \\ \frac{1}{3} + \frac{5}{47} + \frac{3}{25} \end{bmatrix} = \begin{bmatrix} 0.1000 \\ 0.7134 \\ 0.1866 \end{bmatrix} \tag{2.2}$$

Now we have got the priority vector or the approximation of the normalized principal Eigenvector of the comparison matrix. Since the sum of all elements in the priority vector is 1, it gives us the relative weights of all alternatives we have compared.

By having the priority vector and presenting it as percentages, we can see the weight or the priority of each car model in our example as follows:

- The Toyota car is 10%
- The Renault car is 71.34%
- The Ford car is 18.66%

According to these weights, the Renault car has the highest priority (71.34%) among the three car models. Therefore, Darek can easily choose the Renault car as his new car.

However, we need to verify if the comparisons made by Darek is consistent or not. To do this, we need to calculate the Principal Eigenvalue. It can be obtained in our approximation by the summation of products between each element of the priority vector and the sum of columns of the comparison matrix (Teknomo, 2006):

$$\lambda_{max} = 9 * (0.1) + \frac{47}{35} * (0.7134) + \frac{25}{3} * (0.1866) = 3.4129 \tag{2.3}$$

Using this Principal Eigenvalue, we can verify the consistency of our pairwise comparisons by calculating the Consistency Index and Consistency Ratio which is shown in section 2.6.

2.6 Consistency Index and Consistency Ratio

The consistency of pairwise comparison is closely related to the transitive property (Teknomo, 2006). This means the followings: for a person who prefers A than B or $A \succ B$ and he also prefers B than C or $B \succ C$ then according to transitive property, this person should prefer A than C or $A \succ C$. If for any reason, this particular person does not prefer A than C, but he prefers C than A or $C \succ A$, then we can say that his comparisons over A, B, and C are inconsistent with each other.

Let's check this consistency in our example in section 2.4 on page 21. Based on the three comparisons in figure 2.4 on page 21, we can see that:

- Darek prefers the Renault car more than the Ford car or $Renault \succ Ford$.
- Darek prefers the Ford car more than the Toyota car or $Ford \succ Toyota$.
- Darek prefers the Renault car more than the Toyota car or $Renault \succ Toyota$.

In this case, the comparisons made by Darek appear to be consistent:

$$Renault \succ Ford \wedge Ford \succ Toyota \Longrightarrow Renault \succ Toyota \tag{2.4}$$

(satisfies the transitive property)

If Darek had made the last comparison as follows: $Toyota \succ Renault$, then this comparison will be inconsistent with other comparisons thus making Darek judgement in choosing new car inconsistent and not reliable.

However, there is a problem with this check. This check does not take into account of how much Darek prefers each car. Therefore, we cannot know if the values in the comparison matrix are consistent with each other or not. This fact can make the whole hierarchy of AHP (especially multi-level hierarchy) becomes inconsistent because we have to make a pairwise comparison in each level of the hierarchy and the immediately lower level uses the priority vector produced by pairwise comparison of the base level to calculate its global weights (Saaty, 2008).

According to Saaty, for a comparison matrix to be consistent, the largest Eigenvalue should be equal to the size of the comparison matrix, or $\lambda_{max} = n$ (Saaty, 2008). Then, to know how many degrees of consistency a comparison matrix is, Saaty gave us a measurement of consistency, the Consistency Index or CI. To calculate this index, we use the following formula:

$$CI = \frac{\lambda_{max} - n}{n - 1} \tag{2.5}$$

Therefore, applying into our example which has $\lambda_{max} = 3.4129$ and the size of the comparison matrix is $n = 3$; we have the consistency index as follows:

$$CI = \frac{\lambda_{max} - n}{n - 1} = \frac{3.4129 - 3}{2} = 0.20645 \tag{2.6}$$

Now, how can we use this index to determine the consistency of our comparison? Once again, according to Saaty, by comparing it with the appropriate one we can know what is the degree of consistency of the comparison matrix. The proper consistency index is called Random Consistency Index or RI (Saaty, 1980).

Saaty had pre-calculated the Random Consistency Index for comparison matrix using a 1-9 scale based on a sample size of 500 random generated reciprocal matrices in table 10 (Saaty, 1980):

Table 10: Pre-calculated Random Consistency Index

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

To compare the Consistency Index with the corresponding Random Consistency Index, Saaty proposed the Consistency Ratio which is calculated using the following formula (Saaty, 1980):

$$CR = \frac{CI}{RI} \tag{2.7}$$

If the value of Consistency Ratio is smaller or equal to 10%, then the inconsistency in the comparison matrix is acceptable. However, if the Consistency

Ratio is greater than 10%, we need to revise our judgement in the pairwise comparison process.

In our example, we have: $CI = 0.20645$ and RI for $n = 3$ is 0.58, then we have the Consistency Ratio as follows:

$$CR = \frac{CI}{RI} = \frac{0.20645}{0.58} = 0.3559 \tag{2.8}$$

We have $CR = 35\% > 10\%$. Therefore we can see that Darek's subjective judgement in the pairwise comparison process is not consistent. Thus, we recommend a revision for Darek's pairwise comparison between three car models to improve the consistency.

2.7 The Hierarchy of AHP

Now we show an example with criteria and sub-criteria. First, let us define an example of multi-criteria decision problem (decision problem for short) in which we construct an AHP hierarchy. In this decision problem, we want to select a supplier who provides the needed materials for our production line. After some discussions, we have decided that we will select the future supplier based on the following criteria:

- The **price** of the materials.
- The **quality** of the materials.
- The **reliability** of the supplier.

For quality and reliability, we also have added some sub-criteria to them so we can know exactly what aspects of quality and reliability that we are interested in:

- Quality:
  - The **durability** of the materials which determines the length of product life.
  - The **conformance** of the materials which tells if the materials meet the industry standards or not.
- Reliability:
  - The **capital** of the supplier which shows the financial assets of the supplier.
  - The **credibility** of the supplier which tells if the supplier can be trusted or not.
  - The **delivery time** of the supplier which tells if the supplier can provide the materials in time or not.

After some market inquiries, we have the following potential suppliers as our alternatives:

- Supplier **Ava**.
- Supplier **Belle**.

- Supplier **Casey**.

Recall step 2 in section 2.2 on page 18, we construct the hierarchy for our decision problem as in figure 2.5:

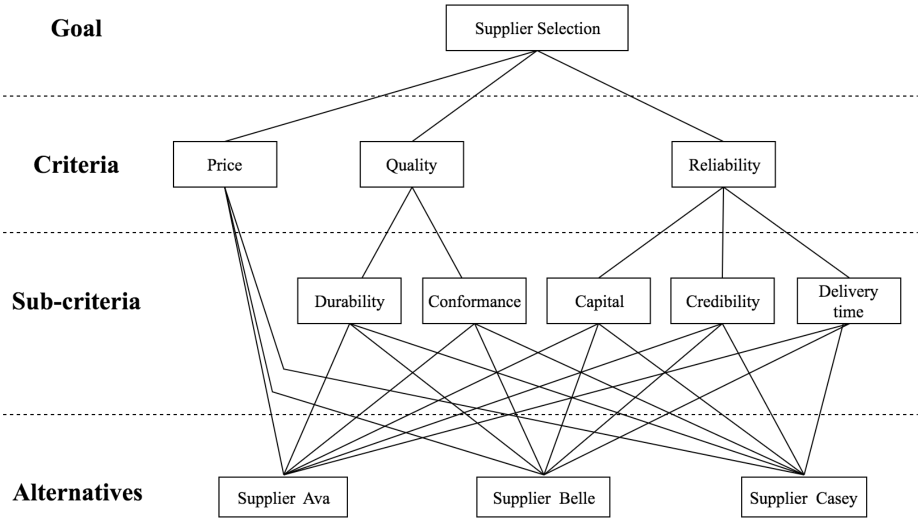

Figure 2.5: AHP Hierarchy for a multi-criteria decision problem

As we can see in figure 2.5, our hierarchy has four levels:

- The top or first level is the goal of our decision problem which is about selecting a supplier for the production line.
- The second level contains the criteria (price, quality, and reliability).
- The third level contains the sub-criteria of quality (durability and conformance) and reliability (capital, credibility, and delivery time).
- The fourth or the last level contains the alternatives (Ava, Belle, and Casey).

Each connection line in figure 2.5 shows the relationship between the elements of the hierarchy:

- The connection lines from the goal to the criteria indicate that we use three criteria (price, quality, and reliability) to compare between alternatives so we can choose the best one to realize the goal of the decision problem (refer to section 1.2 on page 5).
- The connection lines from the criteria to the sub-criteria show to which the criterion in the upper level the sub-criteria belong. In this case, durability and conformance belong to quality while capital, credibility, and delivery time belong to reliability.
- The connection lines from the criteria or the sub-criteria to the alternatives show which criteria or sub-criteria are used to compare the alternatives. We can see that price, durability, conformance, capital, credibility, and delivery time are used to compare our suppliers.

We have constructed the AHP hierarchy for our decision problem, in the next section, we show the readers the comparison process of AHP which helps us to calculate the ranking of the alternatives based on pairwise comparison.

2.8 The comparison process of AHP

Now, according to the steps 3 - 5 in section 2.2 on page 18, we have to make pairwise comparisons, calculate the priority vectors and finally aggregate the priority vectors to find the ranking of the alternatives in our hierarchy. The figure 2.6 on the following page shows us the process of pairwise comparisons and how priority vectors are aggregated to calculate the ranking of the alternatives.

As we can see in figure 2.6 on the next page, the pairwise comparison process runs from the top to the bottom and has two main stages (the stages are separated by the dashed line in the middle). The stages are described as follows:

First stage: In the first stage, we make pairwise comparisons in three groups: the group of criteria, the group of sub-criteria for quality criterion, and the group of sub-criteria for reliability criterion. In each group which is enclosed by the dashed line, we make pairwise comparisons between the criteria or sub-criteria in order to produce the priority vector. The comparisons are based on our preferences on the criteria or sub-criteria (i.e. the more we prefer a particular criterion, the more value of priority that criterion will have). When all priority vectors of the three groups have been calculated, we aggregate these priority vectors to produce the global priority vector which is used in the second stage. We do the aggregation process as follows:

- Let us denote P_x as the weight or the value of a priority for a criterion or sub-criterion x. Suppose the following priority vectors are the results of the pairwise comparisons for the three groups in the first stage (for the calculations of the results, refer to sections 2.3 - 2.5):
  - The priority vector for the group of criteria:
  $$(P_{price}, P_{quality}, P_{reliability}) \tag{2.9}$$
  - The priority vector for the group of sub-criteria for quality criterion:
  $$(P_{durability}, P_{conformance}) \tag{2.10}$$
  - The priority vector for the group of sub-criteria for reliability criterion:
  $$(P_{capital}, P_{credibility}, P_{delivery}) \tag{2.11}$$
- If a criterion has no sub-criteria, then its priority will also be the global priority. However, if a criterion has sub-criteria, then we have to multiply the weight of each sub-criterion with the weight of the corresponding criterion to get the global priorities. Let us denote G_x as the weight or the value of a global priority for a criterion or sub-criterion x. Next, we describe the exact calculations for the global priorities or the global priority vector for the criteria or sub-criteria in table 11 on page 30.

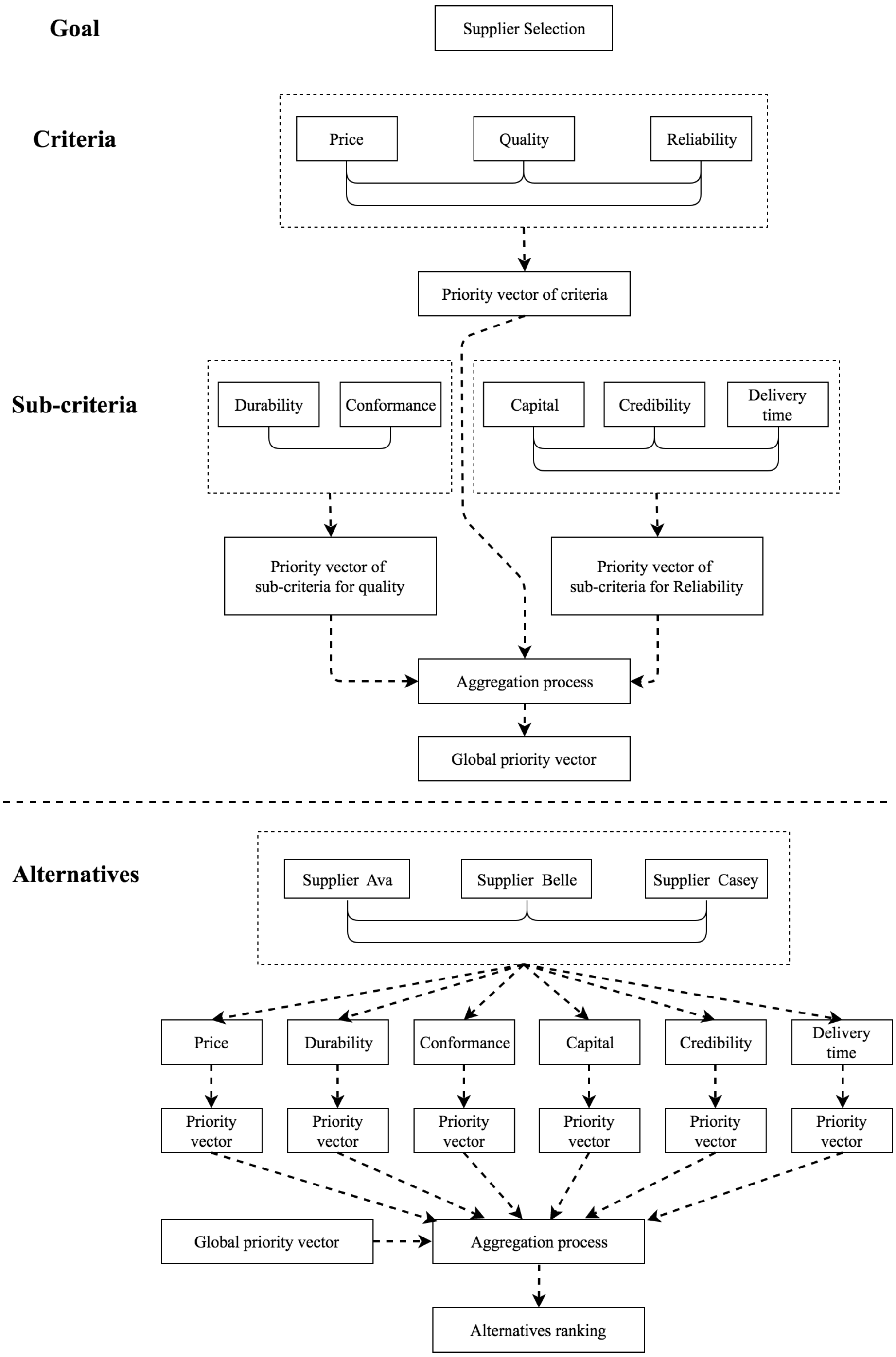

Figure 2.6: AHP pairwise comparison process

Table 11: Global priority vector calculation

Criterion	Sub-criterion	Global priority
P_{price}		$G_{price} = P_{price}$
$P_{quality}$	$P_{durability}$	$G_{durability} = P_{quality} * P_{durability}$
	$P_{conformance}$	$G_{conformance} = P_{quality} * P_{conformance}$
$P_{reliability}$	$P_{capital}$	$G_{capital} = P_{reliability} * P_{capital}$
	$P_{credibility}$	$G_{credibility} = P_{reliability} * P_{credibility}$
	$P_{delivery}$	$G_{delivery} = P_{reliability} * P_{delivery}$

- According to table 11, we now have the global priorities or the global priority vector for the criteria or sub-criteria:

$$(G_{price}, G_{durability}, G_{conformance}, G_{capital}, G_{credibility}, G_{delivery}) \quad (2.12)$$

Second stage: Recall from our hierarchy in figure 2.5 on page 27, the alternatives are compared by using the criterion or sub-criterion of price, durability, conformance, capital, credibility, and delivery time. Therefore we have to make six rounds of pairwise comparisons, in each round we use a different criterion or sub-criterion to compare the alternatives and produce a priority vector. After we had got all the priority vectors, we combine them with the global priority vector we got from the first stage and put all of them in the aggregation process to produce the final ranking of our alternatives. The aggregation process in the second stage is described as follows:

- Let us denote P_x^y as the weight or the value of a priority for an alternative y which has been pairwise compared using a criterion or sub-criterion x. Also, let us use the symbols A, B, and C to denote our alternatives Ava, Belle, and Casey respectively. Suppose the following priority vectors are the results from the six rounds of pairwise comparison we mentioned above:
  - The priority vector for the pairwise comparison using price criterion:

$$(P^A_{price}, P^B_{price}, P^C_{price}) \quad (2.13)$$

  - The priority vector for the pairwise comparison using durability sub-criterion:

$$(P^A_{durability}, P^B_{durability}, P^C_{durability}) \quad (2.14)$$

  - The priority vector for the pairwise comparison using conformance sub-criterion:

$$(P^A_{conformance}, P^B_{conformance}, P^C_{conformance}) \quad (2.15)$$

- The priority vector for the pairwise comparison using capital sub-criterion:
$$(P^A_{capital}, P^B_{capital}, P^C_{capital}) \tag{2.16}$$
- The priority vector for the pairwise comparison using credibility sub-criterion:
$$(P^A_{credibility}, P^B_{credibility}, P^C_{credibility}) \tag{2.17}$$
- The priority vector for the pairwise comparison using delivery time sub-criterion:
$$(P^A_{delivery}, P^B_{delivery}, P^C_{delivery}) \tag{2.18}$$

- We take the global priority vector from the first stage and multiply it with all the priority vectors above to get the weighted priority vectors for the alternatives with respect to each criterion or sub-criterion. Let us denote W^y_x as the weight or the value of the weighted priority for an alternative y which has been pairwise compared using a criterion or sub-criterion x. The table 12 on the next page shows us the exact calculations for the weighted priorities.

- Let us denote R_y as the score of an alternative y. Now, to calculate the ranking of the alternatives, we simply sum all the weighted priorities that belong to a particular alternative and repeat for all alternatives as in table 13 on page 33.

- According to table 13 on page 33, we have the score vector of the alternatives as follows:
$$(R_A, R_B, R_C) \tag{2.19}$$

From the score vector of the alternatives, which is often presented as a vector of percentages, we can see the performance of an alternative based on pairwise comparisons using the criteria or sub-criteria in the hierarchy in figure 2.5 on page 27. Therefore we can know the ranking of our alternatives based on those performances. In the next sections 2.3 - 2.6, we show the readers the mathematics behind the comparison process and also the way to check if our pairwise comparisons are consistent or not.

Table 12: Weighted priority vectors

The priority vector for the alternatives		
$W^{A}_{price} = P^{A}_{price} * G_{price}$	$W^{B}_{price} = P^{B}_{price} * G_{price}$	$W^{C}_{price} = P^{C}_{price} * G_{price}$
$W^{A}_{durability} = P^{A}_{durability} * G_{durability}$	$W^{B}_{durability} = P^{B}_{durability} * G_{durability}$	$W^{C}_{durability} = P^{C}_{durability} * G_{durability}$
$W^{A}_{conformance} = P^{A}_{conformance} * G_{conformance}$	$W^{B}_{conformance} = P^{B}_{conformance} * G_{conformance}$	$W^{C}_{conformance} = P^{C}_{conformance} * G_{conformance}$
$W^{A}_{capital} = P^{A}_{capital} * G_{capital}$	$W^{B}_{capital} = P^{B}_{capital} * G_{capital}$	$W^{C}_{capital} = P^{C}_{capital} * G_{capital}$
$W^{A}_{credibility} = P^{A}_{credibility} * G_{credibility}$	$W^{B}_{credibility} = P^{B}_{credibility} * G_{credibility}$	$W^{C}_{credibility} = P^{C}_{credibility} * G_{credibility}$
$W^{A}_{delivery} = P^{A}_{delivery} * G_{delivery}$	$W^{B}_{delivery} = P^{B}_{delivery} * G_{delivery}$	$W^{C}_{delivery} = P^{C}_{delivery} * G_{delivery}$

Table 13: The ranking of the alternatives

Alternative	Ranking
Ava	$R_A = W^A_{price} + W^A_{durability} + W^A_{conformance} + W^A_{capital} + W^A_{credibility} + W^A_{delivery}$
Belle	$R_B = W^B_{price} + W^B_{durability} + W^B_{conformance} + W^B_{capital} + W^B_{credibility} + W^B_{delivery}$
Casey	$R_C = W^C_{price} + W^C_{durability} + W^C_{conformance} + W^C_{capital} + W^C_{credibility} + W^C_{delivery}$

3 Artificial Neural Networks

When using Multi-Criteria Decision Analysis, especially AHP method, to deal with decision problems which have many criteria or alternatives, the cost of computation to find the best possible decision could increase exponentially. Also, the task in which a decision maker have to make the best possible decision as an output from a system where it takes existing information, and future predictions as the input is a challenging task (Golmohammadi, 2011).

However, there is an interesting approach to approximate the best possible decision from such system without paying too much computation cost; it is by using Artificial Neural Networks (ANN).

ANN is an alternative approach to computing which emulates the remarkable ability of the human mind in reasoning and learning in an environment of uncertainty and imprecision. In fact, ANN simulates the human brain and its ability to learn, recall, and generalize from training data by modeling the essence of the human brain: the networks of biological neurons (Mashrei, 2012). These features make ANN a powerful data modeling tool that is capable of capturing and representing the complex relationships between input and output data (Golmohammadi, 2011).

With such properties, ANN has found its place in many applications such as function approximation, regression analysis, classification, data processing, robotics, and control engineering (Wikipedia, 2017*c*).

Therefore, in this section, we study the following details of ANN: what inspires the model of ANN, what is the fundamental element of ANN, the architecture of ANN, how ANN can learn, and how to train ANN.

3.1 The Biological Neuron

In a sense, artificial neural networks (ANN) simulate the function of the human brain. It is a type of network which nodes are artificial neurons. These artificial neurons are based on the mathematical model of biological neurons.

Biologically, a human brain is a network of about one hundred billion neurons, and each neuron connects to about ten thousand other neurons by using dendrites, and axons. Every single neuron receives electrochemical input signals from other neurons at the dendrites. Then, the cell body of the neuron sums all the strength of all the electrochemical input signals which it receives from other neurons, and if the sum is greater than some threshold level, the neuron will be activated. When the neuron is activated, it transmits or fires an electrochemical signal along the axon and passes this signal to the dendrites of other neurons that are attached to this neuron. It is important to know that a neuron does not fire any signal if the sum of electrochemical inputs does not surpass a particular level. In other words, a neuron can only do two actions: fire a signal or not, there is no other action in between (Clabaugh et al., 2000). The structure of a biological neuron is shown in figure 3.1.

Hence, the entire human brain is a huge network of interconnected electrochemical transmitting neurons. This property gives the human brain the power to perform exceptionally complex tasks. Because of this power, the human brain has become the model for the ANN to be based on in order to solve problems which are simple tasks for an ordinary human but challenging for a conventional

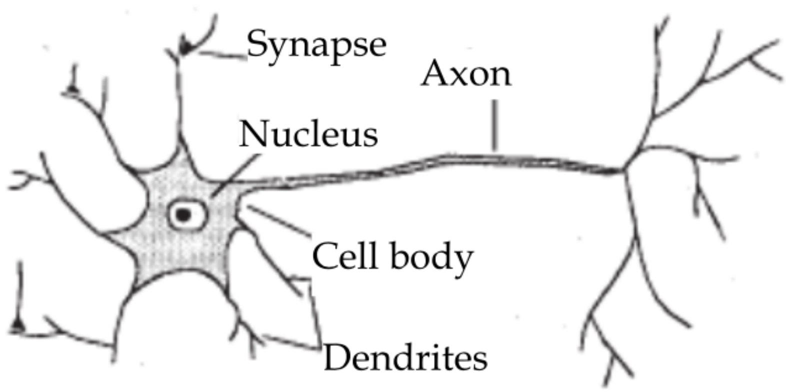

Figure 3.1: A Biological Neuron (Mashrei, 2012)

computer. Some notable examples of these problems are image recognition and predictions based on past knowledge (Clabaugh et al., 2000).

3.2 The Artificial Neuron

To model ANN from the human brain, first, we have to model the most fundamental element, the biological neuron. Similar to the natural counterpart, an artificial neuron is a simple processing unit which consists of numerical input values (the receiving electrochemical input signals), which are multiplied by weights (the strength of the respective electrochemical input signals), and these inputs then be totaled by a sum function. Finally, an activation function calculates the output of the artificial neuron by using the result of the sum function (Mashrei, 2012). A typical artificial neuron is shown in figure 3.2.

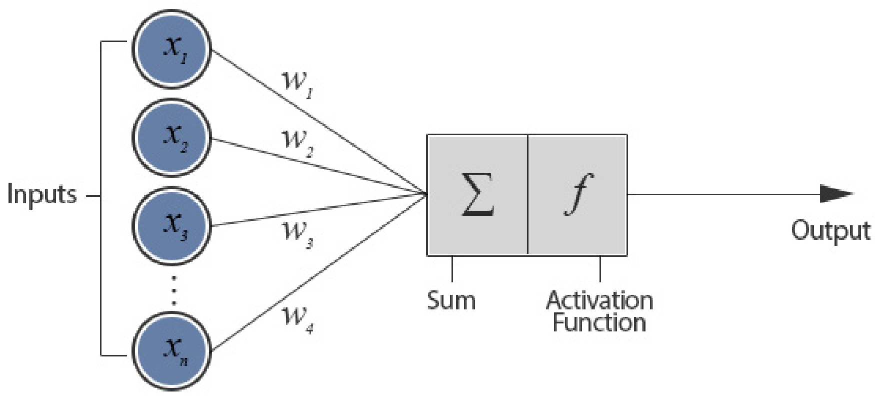

Figure 3.2: An artificial neuron (Jacobson, 2013)

In figure 3.2 on the preceding page, we can see that an artificial neuron has many inputs x_1,x_2,x_3,...,x_n and each input is independently weighted w_1,w_2,w_3,...,w_n when we calculate the sum of all input signals. These weights have the responsibility to amplify or weaken the original input signals. For example, if the input x_1 has the initial value of 1 and the weight w_1 corresponding to x_1 has the value of 0.5, then the value we put into the sum function will be 0.5 because $x_1 w_1 = 1 * 0.5 = 0.5$. After all weighted input signals have been added together, the result is passed into the activation function which determines the output of the artificial neuron. There are several types of activation function such as step function, pure-linear function, log sigmoid function, or tangent sigmoid function (Wikipedia, 2017*a*). The table 14 compares the properties of those activation functions. The plots of those functions are shown in the figures 3.3, 3.4, 3.5 and 3.6.

Table 14: Activation functions (Wikipedia, 2017*a*)

Name	Equation	Output range
Pure-linear (Identity)	$f(x) = x$	$(-\infty, \infty)$
Step	$f(x) = \begin{cases} 0 & x < T \\ 1 & x \geq T \end{cases}$	$\{0, 1\}$
Log sigmoid (Logistic)	$f(x) = \frac{1}{1+e^{-x}}$	$(0, 1)$
Tangent	$f(x) = tanh(x) = \frac{2}{1+e^{-2x}} - 1$	$(-1, 1)$

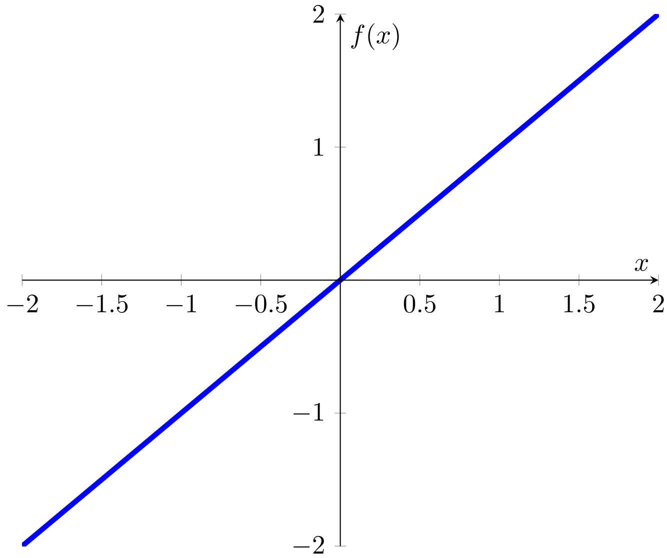

Figure 3.3: The plot of Pure-linear (Identity) function

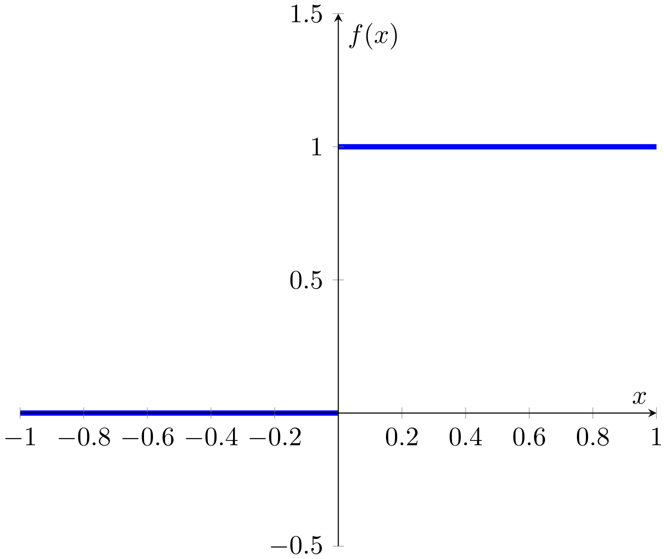

Figure 3.4: The plot of Step function with T = 0

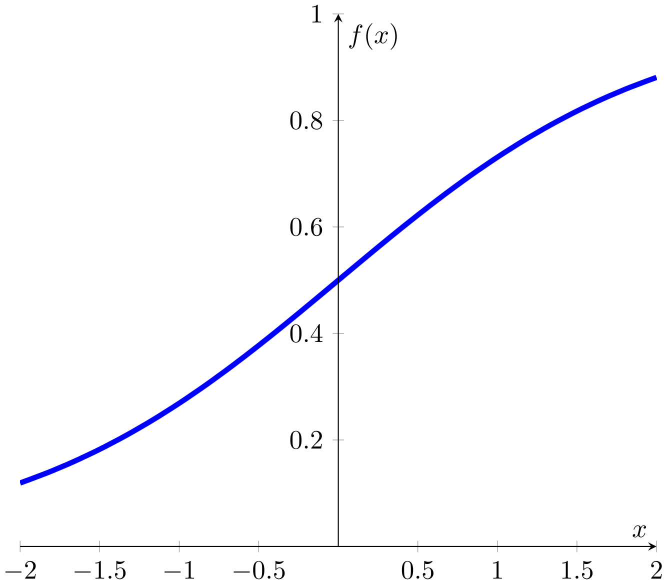

Figure 3.5: The plot of Log sigmoid function

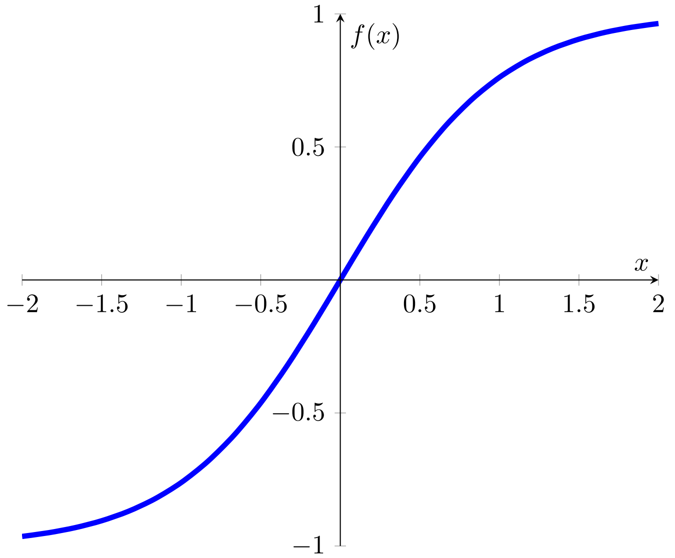

Figure 3.6: The plot of Tangent function

Let us use the step function which is the simplest activation function in table 14 on page 36 to demonstrate how an artificial neuron works. Typically, a step function gives the output of 1 if the input exceeds a specified threshold level, if not then the step function produces the output of 0. For example, we have the following inputs, outputs and threshold level for an artificial neuron with the step function as its activation function:

- Input $x_1 = 0.3$
- Input $x_2 = 0.7$
- Weight $w_1 = 1.3$
- Weight $w_2 = 0.8$
- Threshold $T = 1.0$

According to figure 3.2 on page 35, first we have to calculate the sum of all weighted input signals:

$$\sum = x_1w_1 + x_2w_2 = (0.3 * 1.3) + (0.7 * 0.8) = 0.95$$

Next, we input the calculated sum into the artificial neuron's activation function which is the step function. Because we have specified the threshold $T = 1.0$ for the step function, the calculated sum does not exceed this threshold T, therefore, the step function returns the output of 0, in other words, the artificial neuron does not fire.

0.95<1.0 therefore f(0.95)=0

3.3 Architecture of Artificial Neural Networks

In section 3.2 on page 35, we have taken a look at how artificial neuron works, now we look at the architecture of ANN to see the way an ANN connects artificial neurons together and processes information.

Because the human brain is very complex, a single model of ANN can not cover all the functions of the human brain. Therefore, there are many types of ANN, each type of ANN deals with different aspects of the human brain such as classification or segmentation (Wikipedia, 2017*j*). The following artificial neural networks are the most used types of ANN architecture (Wikipedia, 2017*j*):

Feedforward neural network: This type of network is the most simple type of ANN, the flow of the information moves in just one direction: begin from the input layer, information is passed through the hidden layer then is transferred to the output layer. Also, there are no loops in the network; the calculation always goes forward, never goes back.

Recurrent neural network: Opposite to feedforward network which information can only move from input to output, in recurrent neural network, the information can go in a bi-directional flow. In other words, the information which has been processed in the later stages can be transferred back to earlier stages.

Modular neural network: Studies in biology have shown that the brain functions as a collection of small networks, therefore the concept modular neural network was conceived. In this concept, several artificial neural networks try to cooperate or compete to solve problems.

In this section, we discuss the detail of the feedforward neural network because it is the simplest neural network and also our primary ANN type for the case study of the thesis. First, let us consider the basic architecture of a feedforward neural network in figure 3.7 on the next page.

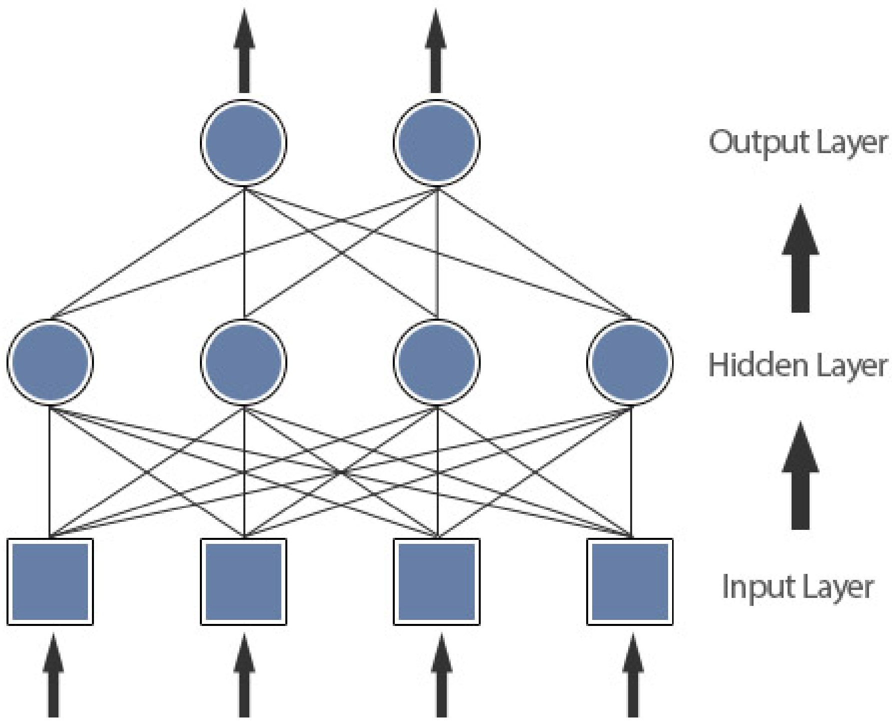

Figure 3.7: Feedforward neural network (Jacobson, 2013)

As we can see, every node in the network is an artificial neuron described in section 3.2 on page 35 and these nodes are arranged in layers (Clabaugh et al., 2000):

Input layer which takes in inputs from the external world.

Hidden layer which can be one hidden layer or multiple hidden layers and does not have any connection with the external world.

Output layer which is responsible for producing the outputs.

In the input layer, because the artificial neurons only have the responsibility to bring the input from the external world into the network, therefore the artificial neurons in the input layer do not have any weight and pass the same input it received from the external world to the next layer.

Each artificial neuron in a layer connects to every artificial neuron on the next layer. Thus, the information is always moving forward from one layer to the next layer. This characteristic is the reason why people call this type of artificial neural network "Feedforward" (Clabaugh et al., 2000).

The readers should also notice that there is no connection between artificial neurons in the same layer.

Often, it is required for a Feedforward neural network, or ANN in general, to have a hidden layer in order to solve problems. The reason is the "Linear separability" of the problem on which the ANN models (Jacobson, 2013). To explain the "Linear separability" and the necessity of the hidden layer, we model the OR function and the XOR function using ANN as follows:

First, let us take a look at the graphs of OR function and XOR function in figure 3.8.

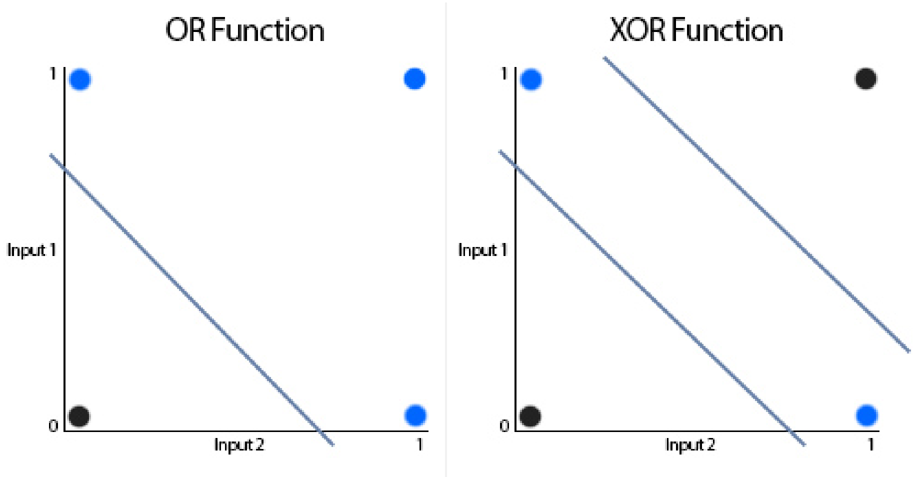

Figure 3.8: OR function and XOR function(Jacobson, 2013)

By looking at figure 3.8, we can see that both OR function and XOR function take two inputs which can be either one or zero. We define the output of OR function and XOR function as follows:

OR function outputs true when at least one of the inputs is one. In other words, OR function is an inclusive disjunction operation on two inputs.

XOR function outputs true only when the inputs are different from each other (i.e. when one input is one and the other is zero). In other words, XOR function is an exclusive disjunction operation on two inputs.

In the graph of OR function, we can see that the outputs of the OR function can be separated by a single straight line. This tells us OR function is "linearly separable", therefore it is possible to model the OR function using ANN without implementing any hidden layer (Jacobson, 2013). In fact, OR function can be modeled with a single artificial neuron like in figure 3.9 on the next page.

For the XOR function, looking at the graph we can see that it is impossible to separate the outputs of XOR function by just using only one straight line. Therefore, we have to use an additional hidden layer to achieve the separation of the outputs of XOR function (Jacobson, 2013). The extra hidden layer is added to the ANN model like in figure 3.10 on page 43.

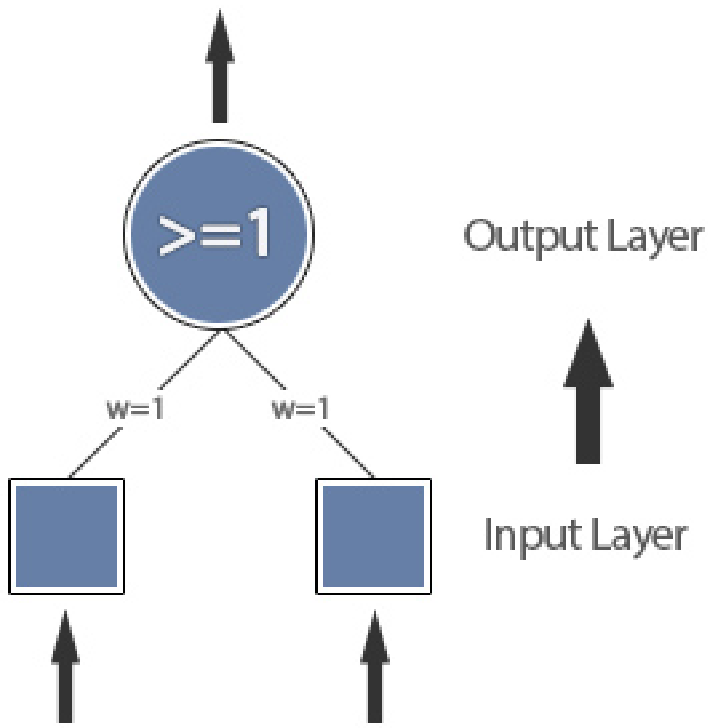

Figure 3.9: OR function modeled by a single artificial neuron (Jacobson, 2013)

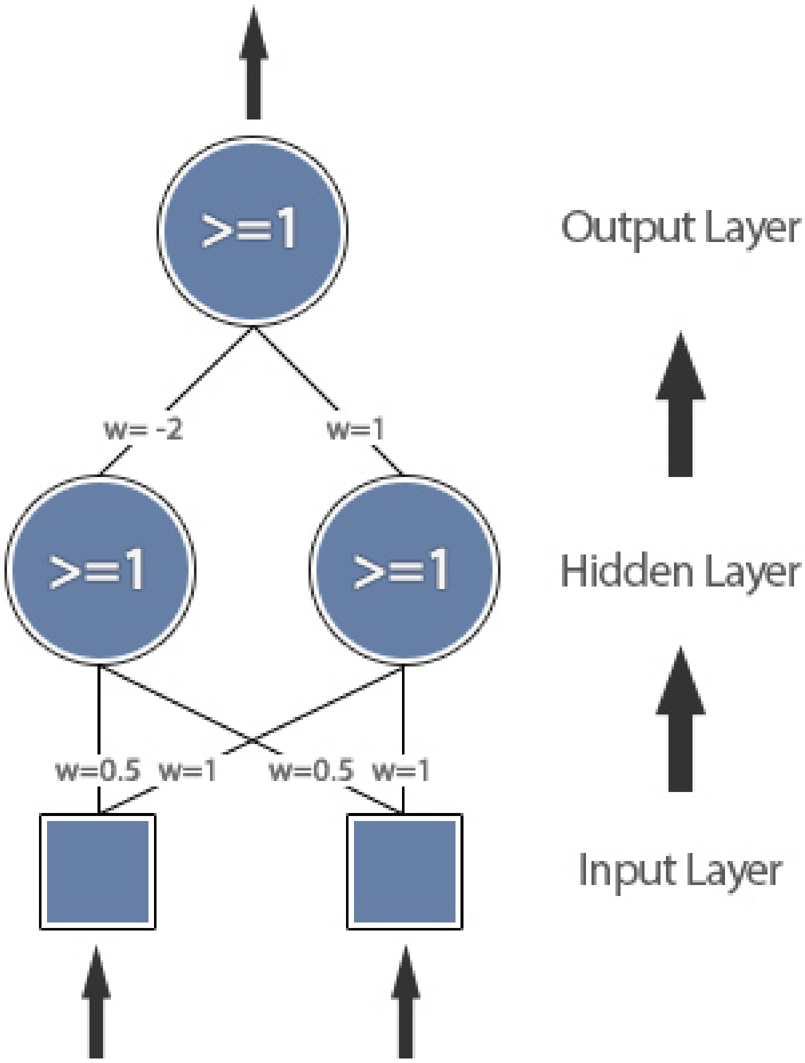

Figure 3.10: XOR function modeled with hidden layer (Jacobson, 2013)

3.4 The Learning Ability of Artificial Neural Networks

In section 3.2 on page 35 and section 3.3 on page 39 we have been introduced to the basic concepts of ANN: what is an artificial neuron and how ANN connects artificial neurons to solve problems. Now, we explain the learning ability of ANN.

First, it is necessary to define what is the meaning of the word "learning" in the context of ANN. We can not say the way ANN learn is the same as how a human should learn because the possibility where machines can have awareness on what they are learning is still not yet decided. (Jacobson, 2014). However, it is possible to train ANN with past experience so the performance of ANN can be improved. Therefore, we define the learning process of ANN as follow: Learning in the context of ANN is the ability to perform better at a given task after being trained with past experience of that given task (Jacobson, 2014).

Now, let us recall from the beginning of section 3.1 on page 34 which has shown us the way we modeled ANN from its natural counterpart, the human brain. Although ANN does not capture all functionalities, ANN has been able to model though simplified the most interesting characteristic of the human brain that is the ability to learn. During the learning process of the human brain, it is believed that the biological neural network of the brain is adjusted in a way that it either increases or decreases the strength of synaptic connections between neurons depending on the subject of the learning process (Jacobson, 2014). This property is the reason why human can easily remember information that is relevant but difficult to remember information that has not been used for a long time; it is because synaptic connections for relevant information are reinforced than less used information (Jacobson, 2014).

ANN models the learning process of the human brain by fine-tuning the weight for each connection between artificial neurons in the artificial neural network. This adjustment simulates the strengthening and weakening of synaptic connections in the human brain therefore it gives ANN the ability to learn (Jacobson, 2014).

The learning ability gives ANN many advantages over traditional methods (computer programs written by software programmers). For example, facial recognition problem is very difficult for a software programmer to find the right set of rules to recognize people's faces accurately; however, it can be solved much easier by using an ANN with its learning ability. ANN also can pick up the underlying relationship in input data. Another example is that ANN can solve classification problem such as loan granting application which ANN learns from past loan data to decide if future loan applications are safe or not (Jacobson, 2014).

3.5 Learning Paradigms

There are three major learning paradigms that can be used to train ANN, each paradigm has their advantages and disadvantages, but overall they share the same purpose: to find the best possible set of weights which ANN uses to accurately map any input to a correct output (Jacobson, 2014).

Supervised Learning: In this paradigm, ANN is provided with the desired output along with the training input as a pair when in training. By using pairs of training input - desired output as training data, we can calculate an error value based on the difference between the output we want and the output produced by ANN. Then, we can use this error value to make corrections to the network by adjusting the network's weights, therefore, improve its performance.

Unsupervised Learning: Opposite to supervised learning, ANN is only provided with a set of inputs, and it has to find the underlying pattern within the provided inputs without any outside intervention. Unsupervised learning is often used by data mining systems and other recommendation systems because of its ability to recognize the underlying relationship in the data or clusters in a population.

Reinforcement Learning: Instead of providing the desired output like in supervised learning, reinforcement learning introduces a reward system: a reward is awarded to the network based on its performance. The objective is to maximize the reward through trial-and-error. This kind of learning paradigm is similar to how animal learns in nature. For example, a dog is likely to remember the trick which the owner has given the dog the most candy (the reward in this case).

3.6 Training ANN with Back-propagation

Back-propagation is a training method for the feed-forward artificial neural network. It belongs to the supervised learning paradigm where pairs of input and output are fed into the network for many cycles until the network can learn the relationship between the input and output (Clabaugh et al., 2000).

The method starts by applying the inputs as an input vector to the input layer of the network. Then, this input vector is passed or propagated through the hidden layer, and an output vector is produced at the end of the network which is the output layer. When the forward propagation is completed, the network evaluates the errors between the output vector generated by the network and the desired outputs. Next, it uses the evaluated errors to adjust the weights of each artificial neuron in the network according to a learning rule which aims to minimize the error. Finally, the network uses the adjusted weights to start a new cycle. This back-propagation cycle or an epoch repeats until the errors between the desired outputs and the actual outputs from the network are minimized (Mashrei, 2012). An diagram of a back-propagation cycle is depicted in figure 3.11 on the following page.

To demonstrate the back-propagation method, let us make a simple example in the classification problem. First, we provide the network with training data which consists of an input vector i and its corresponding desired output d, the training data could be something like in table 15 on the next page.

According to figure 3.11 on the following page, every time we propagate the input vector i through the network, we get the output vector o, we then compare this output vector o with the desired output vector d to get the $Error$ value using the following calculation:

$$Error = \frac{1}{n}\sum_{i=1}^{n}(o_i - d_i)^2 \qquad (3.1)$$

where:

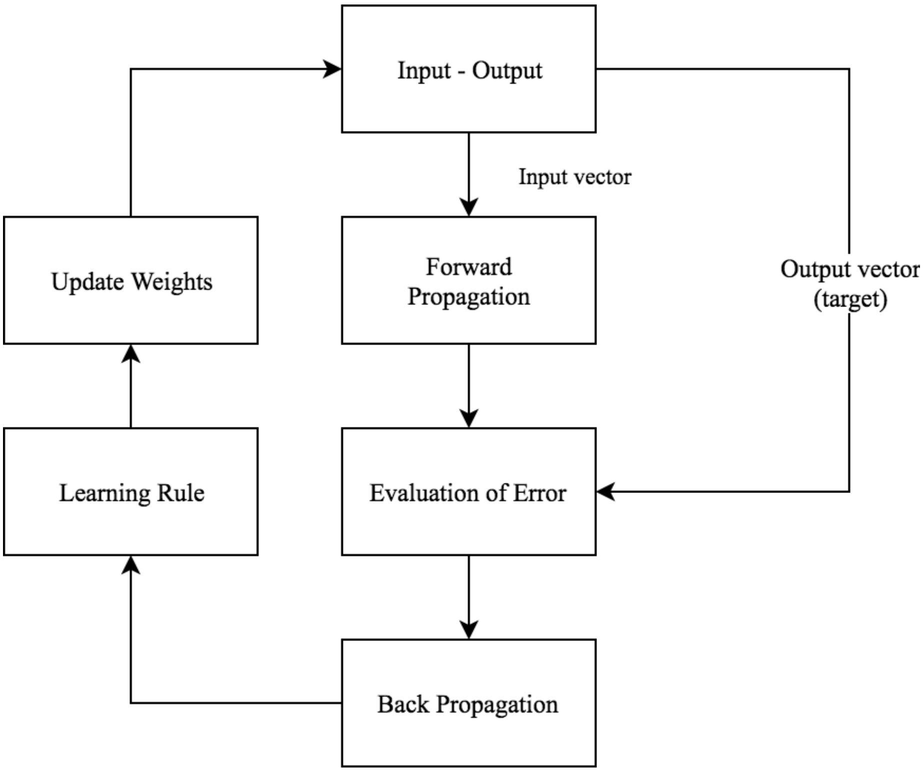

Figure 3.11: Back-propagation cycle (Mashrei, 2012)

Table 15: Example of training data

Input i	Desired output d
$(0, 0)$	0
$(0, 1)$	1
$(1, 0)$	1
$(1, 1)$	0

- n is the number of elements in the output vector.
- $o = (o_1, o_2, ..., o_n)$ is the output vector.
- $d = (d_1, d_2, ..., d_n)$ is the desired output vector.

The squared difference between the output vector o and the desired output vector d gives us the sense of how far the desired value for a particular input (Clabaugh et al., 2000). The back-propagation method aims to minimize the $Error$ value for all samples in the training data thus improving the performance of the network i.e. the ability to learn the relationship between the training input and the desired output (Clabaugh et al., 2000).

$$Minimize(Error) \tag{3.2}$$

By using a continuous function such as Pure-linear or Log sigmoid as the activation function (refer to section 3.2 on page 35 and table 14 on page 36), we can express the change of $Error$ value or gradient with respect to the change of weight vectors w as follow (Clabaugh et al., 2000):

$$Gradient = \frac{\delta Error}{\delta w} = \begin{bmatrix} \frac{\partial Error}{\partial w_1} \\ \frac{\partial Error}{\partial w_2} \\ \vdots \\ \frac{\partial Error}{\partial w_m} \end{bmatrix} \tag{3.3}$$

where:

- $\frac{\partial Error}{\partial w_m}$ is the partial derivative of $Error$ value with respect to weight w_m.
- w is the current weight vector.
- m is the number of elements in the weight vector w.

To update the weight vectors every time a training sample is fed into the network, we need to use the learning rule in formula (3.4) which is based on the fact that if we decrease the value of weight vector w in the direction of the gradient, the $Error$ value will decrease as well (Clabaugh et al., 2000).

$$w_{new} = w_{old} - l\frac{\delta Error}{\delta w} \tag{3.4}$$

Where:

- w_{new} is the newly updated weight vector.
- w_{old} is the old weight vector.
- l is the learning rate which should be a small number (about 0.1).
- $\frac{\delta Error}{\delta w}$ is the gradient or the change of $Error$ value with respect to the change of weight vector w.

When the back-propagation cycle repeats for many epochs, by using the formula (3.4), the weight vectors are constantly adjusted so that the $Error$ value decreases to a minimum value (Clabaugh et al., 2000). When $Error$ value is at minimum, it often means that the network is trained and is ready to produce output similar to the desired output vector d when it is presented with corresponding input vector i.

4 Applying ANN in MCDA Using the AHP method

In chapters 1 - 3, we have learned the basic concepts of multi-criteria decision analysis (MCDA), the method of analytic hierarchy process (AHP), and artificial neural network (ANN). These concepts give us the rough idea of how a decision maker makes a decision in the process of MCDA. Then, by exploring the intricacy of the AHP method, we know that a decision maker can use his or her preferences to make judgments on alternatives and rank them. Moreover, we know that ANN is a powerful data modeling tool that can learn the complex relationships between input and output.

However, these concepts seem rather separated, and not much connection between them have been mentioned either. Therefore, in this chapter, we introduce to the reader the proposed model from Golmohammadi (2011) which connects all three concepts mentioned above. This proposed model is used for reducing the decision maker's effort in making future ranking of alternatives. Then, we make a case study using a real-life decision problem to introduce our implementation of this proposed model and to show its feasibility to the decision methods.

4.1 The proposed model of Golmohammadi

According to the study of Golmohammadi (2011), the task in which a decision maker have to make the best possible decision based on the existing information, and future predictions is a challenging task. Golmohammadi wants to create a model that can act as a tool and support the task above, and he chooses ANN and AHP as two crucial components for his proposed model.

The proposed model of Golmohammadi (2011) can use historical data for making the future ranking of alternatives without the judgment effort of the decision maker. In the figure 4.1 on the next page we can see the difference between this proposed model and other traditional models.

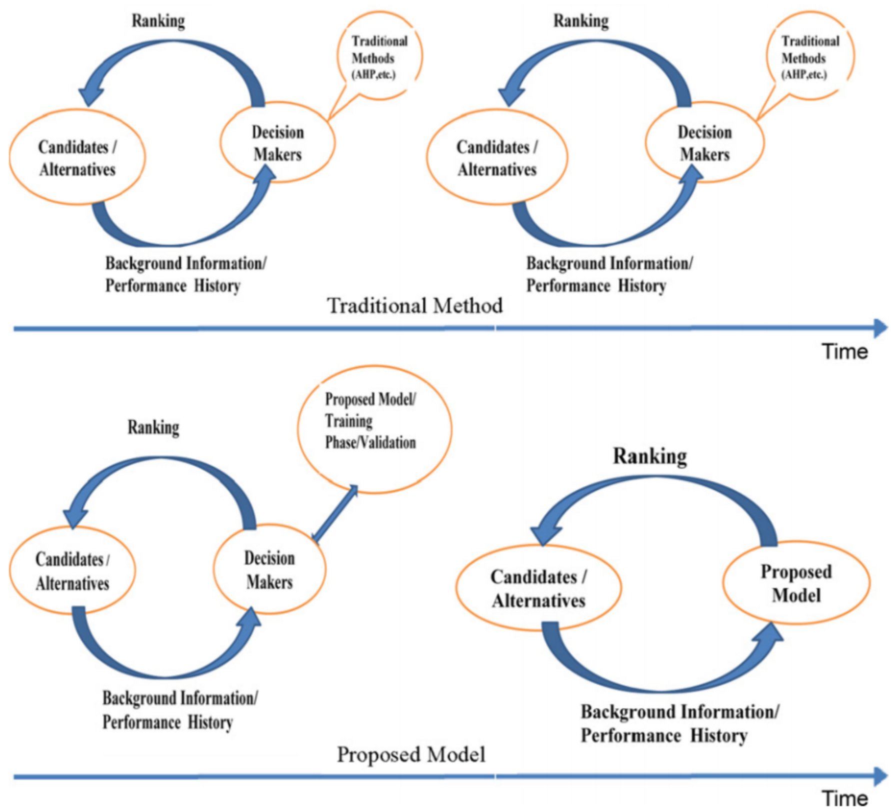

Figure 4.1: The difference between the proposed model and traditional models (Golmohammadi, 2011)

We can see in the figure 4.1 that the main difference is in the second phase of the proposed model and other traditional models. In other traditional models, the decision maker has to repeat the whole process of ranking again while in the proposed model, the decision maker does not make the ranking, the proposed model makes the ranking by itself. From this difference, because the proposed model can make the ranking by itself, the decision maker does not have to spend more effort on the second phase and the following phases after.

In his proposed model, Golmohammadi chooses ANN as the mechanism for which the proposed model can use to make the ranking automatically (thanks to the learning ability of ANN, please refer to section 3.4 on page 44) (Golmohammadi, 2011). Golmohammadi also chooses AHP as the pairwise comparison technique which is responsible for weight calculations which are then applied to the training data for ANN (Golmohammadi, 2011).

4.2 Implementation of The Proposed Model

In the figure 4.2 on page 52, Golmohammadi has described a model design which can be used to implement the proposed model (Golmohammadi, 2011). However, the scope of the thesis does not allow us to follow all the steps from the model design of Golmohammadi; instead, we make a simplified implementation design as in the figure 4.3 on page 53. For example, we don't use fuzzy sets, the defuzzification procedure, and the sensitivity analysis. In our simplified design, we assume that we are going to solve a decision problem which is a ranking problem. For this ranking problem, we use multi-criteria decision analysis with analytic hierarchy process as the main method. For the artificial neuron network which is used to support the MCDA process, we use a feedforward neural network and train it with the back-propagation method.

4.2.1 Making Input

Recall from section 3.6 on page 45 that to train an ANN we need to provide the network with a set of training inputs. This set of training inputs is the data taken form the prior decisions made by the decision maker.

It is also important to normalize the set of training inputs to have the same order of magnitude because if we do not normalize, some training input values may appear to have more significance than they actually do and this difference in order of magnitude between training input values could affect the performance of the ANN (Mashrei, 2012).

4.2.2 Making Output

Also from section 3.6 on page 45, we need a set of training outputs which is the scores for ranking of alternatives corresponding to the training input. This set of training outputs is the desired output vector (refer to section 3.6 on page 45 and table 15 on page 46) which is the ranking produced by the AHP method (refer to chapter 2 on page 17 and section 2.5 on page 23).

For the same reason as in section 4.2.1, we also need to normalize the set of training outputs.

4.2.3 Determine ANN Structure

Because we use a feedforward neural network and train it with the back-propagation method, we only have to care about the number of artificial neurons in each layer of the ANN. For the input layer, the number of neurons is the same as the number of criteria we use in our decision problem. For the output layer, because our decision problem is a ranking problem so we just need one neuron to represent the score for ranking our alternatives.

It is always difficult to decide the number of neurons and the number of layers for the hidden layer of our ANN. There is no rule of thumb to determine the exact number of neurons and number of hidden layers. Usually, the optimal number is found by trial-and-error in which we train the network with various configurations. Then we pick the configuration with the fewest number of layers and neurons that still keep the network at the minimum Mean Squared Errors (refer to section 4.2.5 on page 54) (Mashrei, 2012).

4.2.4 Model Training

From section 3.6 on page 45 and the assumption in the beginning of section 4.2 on the previous page, we know that our ANN is trained using the back-propagation method. We keep training our ANN until the $Error$ value is

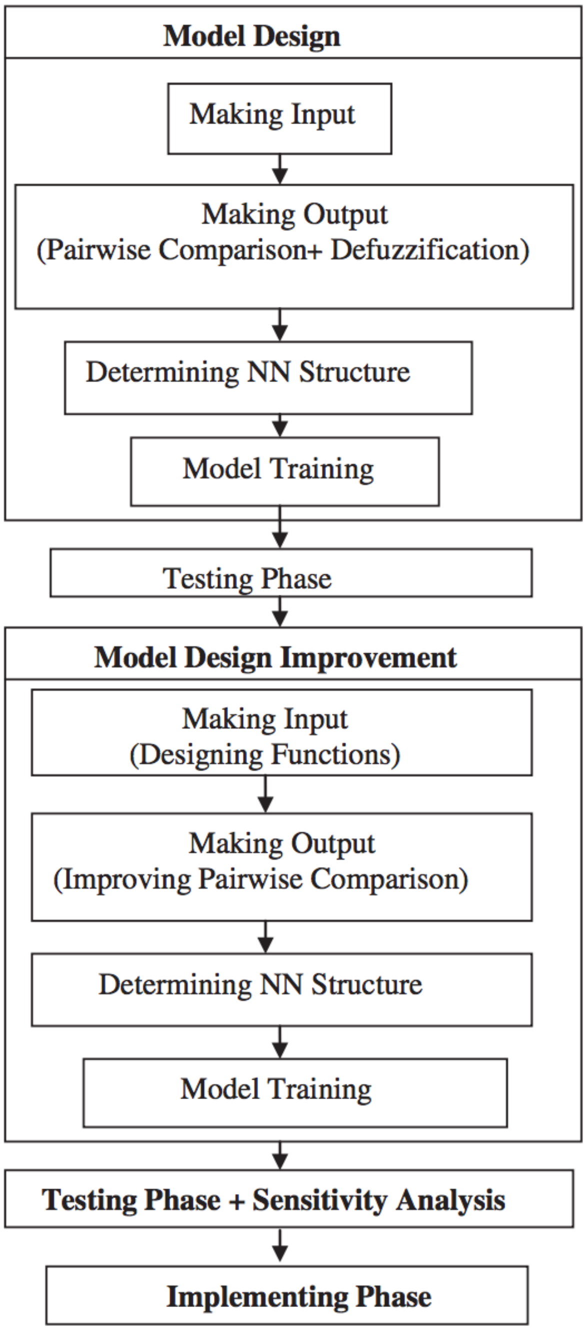

Figure 4.2: Model design steps (Golmohammadi, 2011)

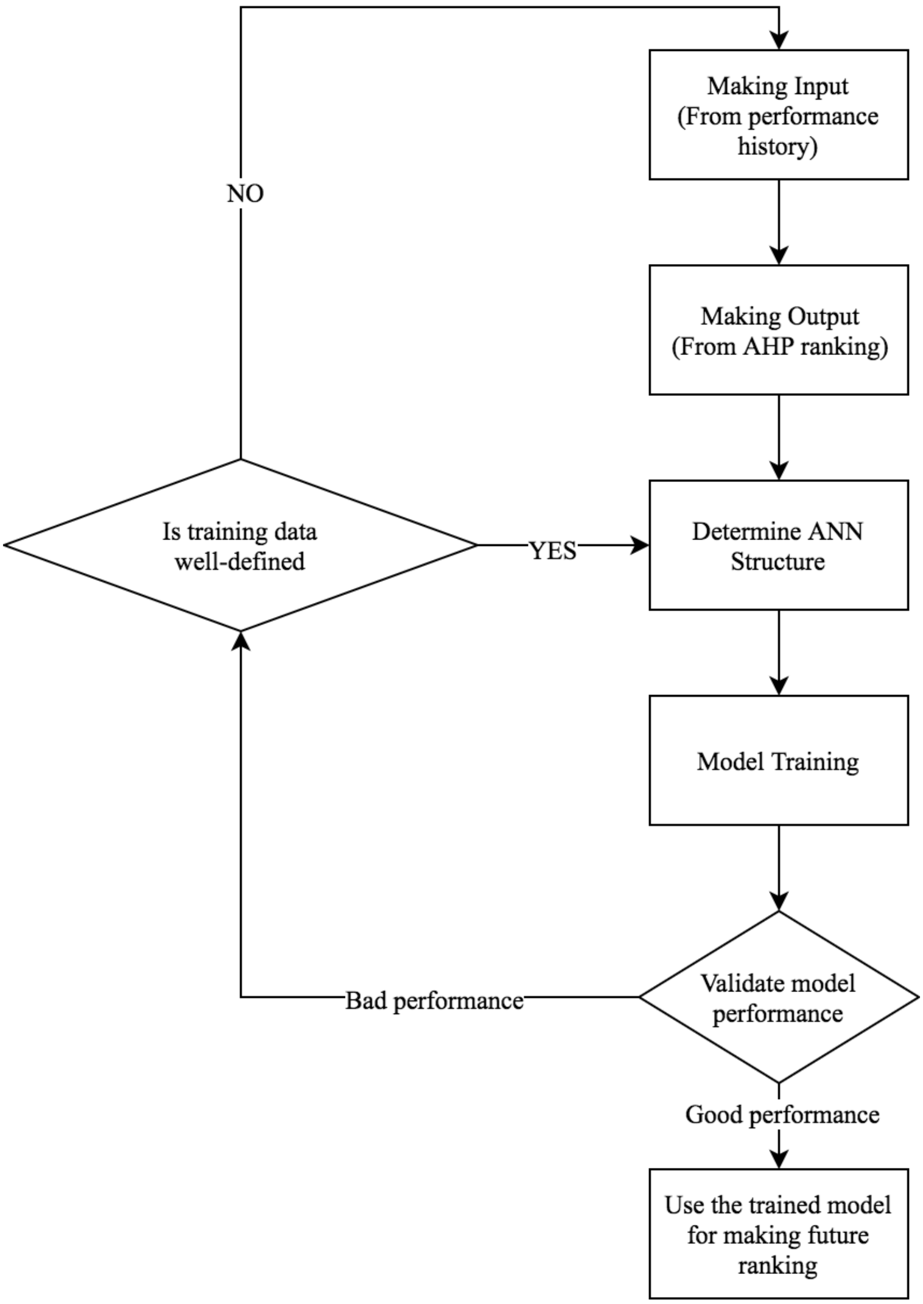

Figure 4.3: Simplified model design steps

at minimum, then we test the performance of our ANN using Mean Squared Error and R-value (refer to section 4.2.5).

4.2.5 Validate model performance

We evaluate the performance of ANN by using Mean Squared Errors (MSE) and R-value:

Mean Squared Error is defined as follow:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(o_i - d_i)^2 \tag{4.1}$$

where:

- n is the number of observations.
- $o = (o_1, o_2, ..., o_n)$ is the output vector produced by the ANN.
- $d = (d_1, d_2, ..., d_n)$ is the desired output vector we want the ANN to learn.

R-value which is denoted as R^2 and is defined as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(o_i - d_i)^2}{\sum_{i=1}^{n}(o_i - \overline{d})^2} \tag{4.2}$$

where:

- $\overline{d} = \frac{1}{n}\sum_{i=1}^{n}(d_i)$ is the average value of the desired output vector d.

When the training process is finished, we test our trained ANN on a set of independent input and output which is separated from the training data. Based on the result, if the performance is good (MSE close to 0 and R-value close to 1) then we can use the trained ANN for future ranking. However, if the performance is bad then we have to investigate where is the problem in our model design.

Often bad performance is caused by bad configuration (Mashrei, 2012). Training an ANN requires a lot of parameters to be properly set such as the number of artificial neurons in the hidden layer, the number of hidden layers, the type of activation function, or did our training data is normalized yet? Therefore, we have to identify what kind of bad configuration is causing the bad performance (Is the set of training inputs and training output well defined yet? Is the number of artificial neurons too much or too few?) in order to correct the right step in the model design in figure 4.3 on the previous page.

4.3 Case Study

In this section, we demonstrate the proposed model of applying Artificial Neural Network (ANN) in multi-criteria decision making problems by applying it for an actual problem based on real data.

4.3.1 Problem definition

The problem is to rank the eco-friendly levels of top twelve best-selling car manufacturers in the United Kingdom by using AHP method and then to use the ranking to choose the most eco-friendly car; we call this problem Eco-Friendly Car Manufacturers Ranking Problem.

To have the list of top twelve best-selling car manufacturers for each year, we use sales data provided by The Society of Motor Manufacturers and Traders of United Kingdom or SMMT (2017). Each year, SMMT releases a sales report which tells us how many cars were sold in the United Kingdom in that year. The report also says the number of cars each car manufacturer has sold. Therefore we know which car manufacturer has the best sale. Base on this information, we select the top twelve best-selling car manufacturers for our problem.

We also need some environment related information such as fuel consumptions and emissions data for each car manufacturer in each year to determine their eco-friendly level. We use the "Car fuel consumptions and emissions 2000-2013" dataset published by the Vehicle Certification Agency of United Kingdom Department for Transport (2013). This dataset contains the environment related information of all car models from the year 2000 to the year 2013 in the United Kingdom. It also has the manufacturer information for each car model. Therefore, we can determine how much impact on the environment each manufacturer could make each year by averaging all the impacts on the environment of all car models each car manufacturer has for each year.

One thing about our data is that it does not contain the actual information of environment impact (information achieved under "real life" driving conditions), it contains only the data that are obtained by specific test conditions which are performed by Vehicle Certification Agency of United Kingdom Department for Transport (2013). Also, the reason why we choose to rank only the top twelve best-selling car manufacturers is to reduce the amount of pairwise comparisons which could get exponentially large if we would have too many alternatives (refer to table 5 on page 20).

Therefore, the ranking produced from the Eco-Friendly Car Manufacturers Ranking Problem tells us the potential environment impact level of cars produced by different manufacturers each year.

Although the datasets from SMMT and VCA have data for more than a decade, we pick only data from the years 2009, 2010, 2011, 2012 and 2013 to use in the demonstration of our problem. This amount of data is sufficient for the training of ANN, and it also helps us to reduce the size of our problem.

4.3.2 Data Sources

Sales Report for Car in United Kingdom: We use the sales report posted by Car Magazine (2017) which takes data from SMMT each year to select out top twelve best-selling car manufacturers. This report contains the total number of cars sold in one year in United Kingdom. It also groups the car sales by manufacturers, therefore we simply sort the report in descending order and pick the top twelve car manufacturers to have our best-selling car manufacturers.

Table 16: UK Car Sales 2009 (Car Magazine, 2009)

Manufacturer	Sales (cars)
Ford	316,369
Vauxhall	237,40
Volkswagen	161,137
Toyota	102,612
Peugeot	102,574
BMW	98,683
Audi	91,172
Citroen	72,450
Mercedes-Benz	72,281
Renault	63,174
Fiat	60,337
Hyundai	56,726

Table 17: UK Car Sales 2010 (Car Magazine, 2011)

Manufacturer	Sales (cars)
Ford	280,364
Vauxhall	247,265
Volkswagen	174,655
BMW	109,418
Peugeot	109,324
Audi	99,828
Renault	95,608
Nissan	89,681
Toyota	87,396
Mercedes-Benz	74,977
Citroen	73,317
Honda	63,652

Table 18: UK Car Sales 2011 (Car Magazine, 2012)

Manufacturer	Sales (cars)
Ford	265,894
Vauxhall	234,710
Volkswagen	179,290
BMW	116,642
Audi	113,797
Nissan	96,269
Peugeot	94,989
Mercedes-Benz	81,873
Toyota	73,589
Citroen	68,464
Renault	68,449
Hyundai	62,900

Table 19: UK Car Sales 2012 (Car Magazine, 2013)

Manufacturer	Sales (cars)
Ford	281,917
Vauxhall	232,255
Volkswagen	183,098
BMW	127,530
Audi	123,622
Nissan	105,835
Peugeot	99,486
Mercedes-Benz	91,855
Toyota	84,563
Hyundai	74,285
Citroen	73,656
Kia	66,629

Table 20: UK Car Sales 2013 (Car Magazine, 2015)

Manufacturer	Sales (cars)
Ford	310,865
Vauxhall	259,444
Volkswagen	194,085
Audi	142,040
BMW	135,583
Nissan	117,967
Mercedes-Benz	109,456
Peugeot	105,435
Toyota	88,648
Citroen	78,358
Hyundai	76,918
Kia	72,090

Car Fuel Consumptions And Emissions 2000-2013: The "Car fuel consumptions and emissions 2000-2013" dataset from VCA is a collection of fuel consumptions and emissions data from car models from all car manufacturers in United Kingdom from year 2000 to year 2013. This dataset has in total of 27 fields of information, we do not use all of those fields, instead we use only the following 4 fields which give us enough information of how much the environment impact each car model has, they are:

CO2: CO2 emissions in grammes per kilometer (g/km).

Noise level: External noise emitted by a car shown in decibels as measured on the A scale of a noise meter (dB (A)).

Combined fuel consumption: average of the urban and extra-urban tests, weighted by the distances covered in each part, in liters per 100 Kilometers (l/100 Km).

Engine capacity: Engine capacity in cubic centimeters (cc).

These 4 fields are used as criteria to determine how much a car model can impact the environment. However, these fields not only show us the environment impact information but also show us how efficient a car model can be in terms of saving the environment. For example, a car model might have high engine capacity but its CO2 emissions and noise level might be not as high as for other car models with the same engine capacity, this tells us that this car model is using superior technology to minimize the amount of CO2 emissions and noise level while maintain high engine capacity.

Notice that the data we use in our case study is averaged data for each car manufacturer. For example, the fuel consumption number for Ford in figure 21 on the next page is the result of averaging all fuel consumption numbers from car models which belong to Ford.

Table 21: Car Manufacturers With Environmental Information For Year 2009

Manufacturer	Fuel consumption	Noise level	CO2	Engine capacity
Ford	6.53	70.87	164.06	1,847.74
Vauxhall	7.06	72.56	175.58	1,815.62
Volkswagen	7.20	72.30	182.38	2,004.47
Toyota	6.30	71.03	156.89	1,838.27
Peugeot	6.48	73.43	162.48	1,710.82
BMW	7.36	71.91	182.65	2,735.24
Audi	7.95	72.59	195.12	2,499.68
Citroen	6.57	72.84	165.01	1,734.65
Mercedes-Benz	8.06	72.21	199.32	2,619.47
Renault	6.80	71.31	169.56	1,776.82
Fiat	5.79	72.59	144.67	1,494.11
Hyundai	6.52	71.81	161.81	1,678.40

Table 22: Car Manufacturers With Environmental Information For Year 2010

Manufacturer	Fuel consumption	Noise level	CO2	Engine capacity
Ford	6.49	70.66	162.67	1,860.81
Vauxhall	6.67	72.11	164.64	1,677.48
Volkswagen	6.96	72.10	176.54	1,942.49
BMW	7.32	71.77	180.84	2,752.63
Peugeot	6.21	73.28	154.67	1,660.64
Audi	7.97	72.60	195.13	2,550.52
Renault	6.47	71.91	161.85	1,760.33
Nissan	7.31	70.53	180.61	1,997.75
Toyota	5.94	71.10	147.53	1,853.17
Mercedes-Benz	7.87	72.04	194.87	2,604.66
Citroen	6.20	72.89	156.80	1,719.90
Honda	6.35	70.06	154.69	1,858.30

Table 23: Car Manufacturers With Environmental Information For Year 2011

Manufacturer	Fuel consumption	Noise level	CO2	Engine capacity
Ford	6.47	70.46	160.00	1,856.30
Vauxhall	6.10	71.89	151.17	1,637.36
Volkswagen	6.62	71.64	166.15	1,881.00
BMW	6.96	71.78	171.05	2,633.44
Audi	7.19	72.54	174.72	2,402.03
Nissan	7.00	70.59	173.71	1,930.12
Peugeot	5.71	72.59	141.90	1,624.74
Mercedes-Benz	7.17	71.87	177.23	2,477.43
Toyota	5.81	70.98	142.77	1,777.88
Citroen	5.69	72.60	141.82	1,613.25
Renault	6.23	71.95	155.23	1,722.69
Hyundai	5.87	71.82	144.66	1,662.88

Table 24: Car Manufacturers With Environmental Information For Year 2012

Manufacturer	Fuel consumption	Noise level	CO2	Engine capacity
Ford	6.14	69.99	149.60	1,813.85
Vauxhall	5.80	71.87	143.52	1,636.40
Volkswagen	6.24	71.24	156.70	1,795.22
BMW	6.60	72.01	162.87	2,574.89
Audi	6.65	72.04	161.67	2,309.72
Nissan	6.80	70.49	163.73	1,899.06
Peugeot	5.50	72.36	135.18	1,604.80
Mercedes-Benz	6.70	72.07	166.47	2,486.44
Toyota	5.52	71.31	135.42	1,800.60
Hyundai	5.55	71.92	137.62	1,590.19
Citroen	5.49	72.47	136.33	1,649.97
Kia	5.49	72.09	136.93	1,571.45

Table 25: Car Manufacturers With Environmental Information For Year 2013

Manufacturer	Fuel consumption	Noise level	CO2	Engine capacity
Ford	5.70	69.80	141.16	1,735.91
Vauxhall	5.58	71.66	138.11	1,614.11
Volkswagen	6.06	71.35	151.75	1,779.67
Audi	6.70	72.01	162.52	2,381.38
BMW	6.08	72.18	149.66	2,339.21
Nissan	6.81	70.85	165.59	1,950.59
Mercedes-Benz	6.30	72.45	155.03	2,371.14
Peugeot	5.34	72.15	132.08	1,620.88
Toyota	5.46	71.78	133.03	1,802.49
Citroen	5.25	72.52	130.54	1,619.99
Hyundai	5.49	72.18	136.48	1,618.02
Kia	5.38	72.15	133.39	1,555.73

4.3.3 Solving The Problem Using AHP Method

Define Decision Hierarchy And Global Priorities First, we must structure our Eco-Friendly Car Manufacturers Ranking Problem in a decision hierarchy (refer to section 2.7 on page 26) with appropriate criteria which are: Combined fuel consumption, Noise Level, CO2, and Engine capacity. These criteria must also be assigned with global priorities which should be computed by pairwise comparisons to reflect the following order of importance:

$$CO2 > Noise\ Level > Combined\ fuel\ consumption = Engine\ capacity \tag{4.3}$$

Notice that the order of importance 4.3 comes from the subjective judgment of the decision maker. In our case study, we assume that the personal preferences of the decision maker are as follows: the level of CO2 emissions is the most important factor when choosing a car. The second most important factor is the noise level. Finally, the combined fuel consumption and the engine capacity have the same level of importance.

To compute the global priorities for our criteria in order to follow the above order of importance, we make the pairwise comparisons as in figure 4.4.

	1	2	3	4
1	1	0.20	0.14	1.00
2	5.00	1	0.33	5.00
3	7.00	3.00	1	7.00
4	1.00	0.20	0.14	1

Figure 4.4: Judgement Table For Criteria

In figure 4.4, we have denoted:

1. Combined fuel consumption
2. Noise level
3. CO2
4. Engine capacity

After we had the pairwise comparisons, we can compute the priorities and ranking as in figure 4.5 on the following page. Then when we got the priorities for our criteria, we can define our decision hierarchy with global priorities as in figure 4.6 on the next page.

The calculations and the pairwise comparisons are made by using the AHP Online System which is a online platform for multi-criteria decision making using AHP (Goepel, 2017).

Category		Priority	Rank
1	Fuel Consumption	6.7%	4
2	Noise Level	28.2%	2
3	CO2	58.3%	1
4	Engine Capacity	6.7%	3

Figure 4.5: Criteria Priorities And Ranking

Decision Hierarchy

Level 0	Level 1		Global Priorities
Environment Friendly Car Manufactur	Fuel Consumption	0.0674	6.7 %
	Noise Level	0.2825	28.2 %
	CO2	0.5827	58.3 %
	Engine Capacity	0.0674	6.7 %
			1.0

Figure 4.6: Decision Hierarchy With Global Priorities

Calculate Scores of Alternatives When we have got the global priorities for our criteria, we can proceed to calculate the scores of our Alternatives. We use the data in table 21 on page 62 to demonstrate the calculation of our scores.

First, similar to the process of finding the global priorities for our criteria, we must find the priority vector of the 12 car manufacturers for each criterion. To find the priority vector for each criterion, we make the pairwise comparisons between 12 car manufacturers for each criteria. For example, let us make pairwise comparisons for the criterion fuel consumption. Because we have 12 alternatives therefore we have to make 66 comparisons (refer to table 5 on page 20) as in figure 4.7 on the following page, then we get the priority vector in figure 4.8 on page 71 by solving the principal Eigenvectors problem (refer to chapter 2 on page 17).

After repeating the process for the remaining criteria to find all priority vectors (refer to figures A.1 - A.6 in appendix A), we multiply the priority vector of each criteria with the global priorities to get the consolidated priorities or the scores for ranking our 12 car manufacturers. The results for the year 2009 are in figure 4.9 on page 72. The diagram presenting the scores for year 2009 is in figure 4.10 on page 73. The scores and rankings for year 2010, 2011, 2012 and 2013 are shown in the figures 4.11 on page 73, 4.12 on page 74, 4.13 on page 74, and 4.14 on page 75. The results of pairwise comparisons, the priority vectors, and the consolidated priorities for year 2010, 2011, 2012 and 2013 are shown in the figures A.7 - A.42 in appendix A.

The calculations, the pairwise comparisons, and the ranking diagrams are made by using the AHP Online System (Goepel, 2017).

4.3.4 Artificial Neural Network Training

Making Input and Output: We use the measurements from the tables 21, 22, 23, and 24 as our training inputs (refer to section 4.2.1 on page 51). But we do not use table 25 on page 66 as out training inputs, instead, we use it only for validation.

For the training outputs, we use the scores from the figures 4.10, 4.11, 4.12, 4.13 (refer to section 4.2.2 on page 51). And for the same reason as input, we only use figure 4.14 on page 75 for validation.

Notice that all training data should be normalized to have the same order of magnitude (refer to sections 4.2.1 and 4.2.2 on page 51).

Determine ANN Structure: Because we have 4 criteria, therefore we have 4 artificial neurons in our input layers, as for output layer we only need 1 neuron because the score for ranking for each car manufacturer is just a single numerical value.

For the number of neurons in hidden layer and the number of hidden layers, we choose [9 4], this means we have 2 hidden layers and the first layer has 9 neurons and the second layer has 4 neurons. The reason we choose this configuration is because after several tries, this configuration gives the best performance for our ANN. Also, we choose the Tangent function (refer to table 14 on page 36) as the activation function for our neurons in the hidden layers and we choose the Pure-linear function as the activation function for the neuron in the output layer.

Figure 4.15 on page 75shows the shape of our artificial neural network.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	4.00	5.00	0.25	0.33	6.00	7.00	2.00	8.00	3.00	0.20	0.50
2	0.25	1	2.00	0.14	0.17	3.00	4.00	0.33	5.00	0.50	0.13	0.20
3	0.20	0.50	1	0.13	0.14	2.00	3.00	0.25	4.00	0.33	0.11	0.17
4	4.00	7.00	8.00	1	2.00	9.00	9.00	5.00	9.00	6.00	0.50	3.00
5	3.00	6.00	7.00	0.50	1	8.00	9.00	4.00	9.00	5.00	0.33	2.00
6	0.17	0.33	0.50	0.11	0.13	1	2.00	0.20	3.00	0.25	0.11	0.14
7	0.14	0.25	0.33	0.11	0.11	0.50	1	0.17	2.00	0.20	0.11	0.13
8	0.50	3.00	4.00	0.20	0.25	5.00	6.00	1	7.00	2.00	0.17	0.33
9	0.13	0.20	0.25	0.11	0.11	0.33	0.50	0.14	1	0.17	0.11	0.11
10	0.33	2.00	3.00	0.17	0.20	4.00	5.00	0.50	6.00	1	0.14	0.25
11	5.00	8.00	9.00	2.00	3.00	9.00	9.00	6.00	9.00	7.00	1	4.00
12	2.00	5.00	6.00	0.33	0.50	7.00	8.00	3.00	9.00	4.00	0.25	1

Figure 4.7: Judgement Table For Fuel Consumption in year 2009

In figure 4.7, we have used the following numbering of cars:

1. Ford
2. Vauxhall
3. Volkswagen
4. Toyota
5. Peugeot
6. BMW
7. Audi
8. Citroen
9. Mercedes-Benz
10. Renault
11. Fiat
12. Hyundai

Category		Priority	Rank
1	Ford	8.1%	5
2	Vauxhall	3.1%	8
3	Volkswagen	2.3%	9
4	Toyota	20.0%	2
5	Peugeot	14.9%	3
6	BMW	1.7%	10
7	Audi	1.3%	11
8	Citroen	5.9%	6
9	Mercedes-Benz	1.1%	12
10	Renault	4.3%	7
11	Fiat	26.4%	1
12	Hyundai	11.1%	4

Figure 4.8: Priority vector for Fuel Consumption in year 2009

Decision Hierarchy

Level 0	Level 1	Global Priorities	Ford	Vauxhall	Volkswag en	Toyota	Peugeot	BMW	Audi	Citroen	Mercedes -Benz	Renault	Fiat	Hyundai
Environment Friendly Car Manufactur	Fuel Consumption 0.0674	6.7 %	0.0054	0.0021	0.0015	0.0135	0.0101	0.0011	0.0009	0.004	0.0007	0.0029	0.0178	0.0074
	Noise Level 0.2825	28.2 %	0.0742	0.0087	0.012	0.0562	0.003	0.0224	0.0063	0.0037	0.0165	0.0437	0.0047	0.031
	CO2 0.5827	58.3 %	0.0471	0.0181	0.0132	0.1165	0.0644	0.0098	0.0076	0.0343	0.0063	0.0249	0.1536	0.0871
	Engine Capacity 0.0674	6.7 %	0.0055	0.0029	0.0072	0.0041	0.0011	0.0178	0.0101	0.0015	0.0135	0.0021	0.0007	0.0009
		1.0	**13.2 %**	**3.2 %**	**3.4 %**	**19.0 %**	**7.9 %**	**5.1 %**	**2.5 %**	**4.3 %**	**3.7 %**	**7.4 %**	**17.7 %**	**12.6 %**

Figure 4.9: Consolidated priorities for car manufactures in year 2009

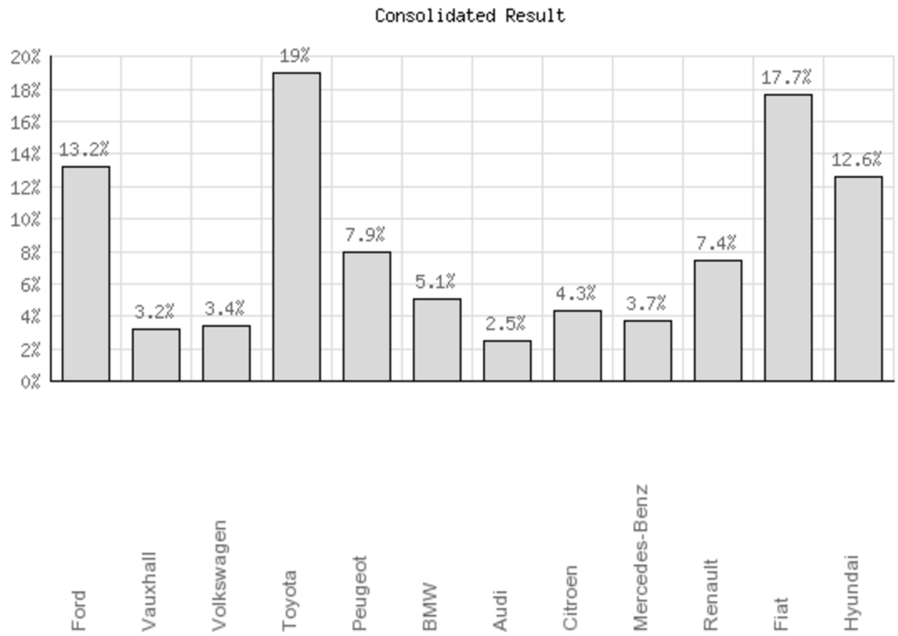

Figure 4.10: Car manufacturers ranking in 2009

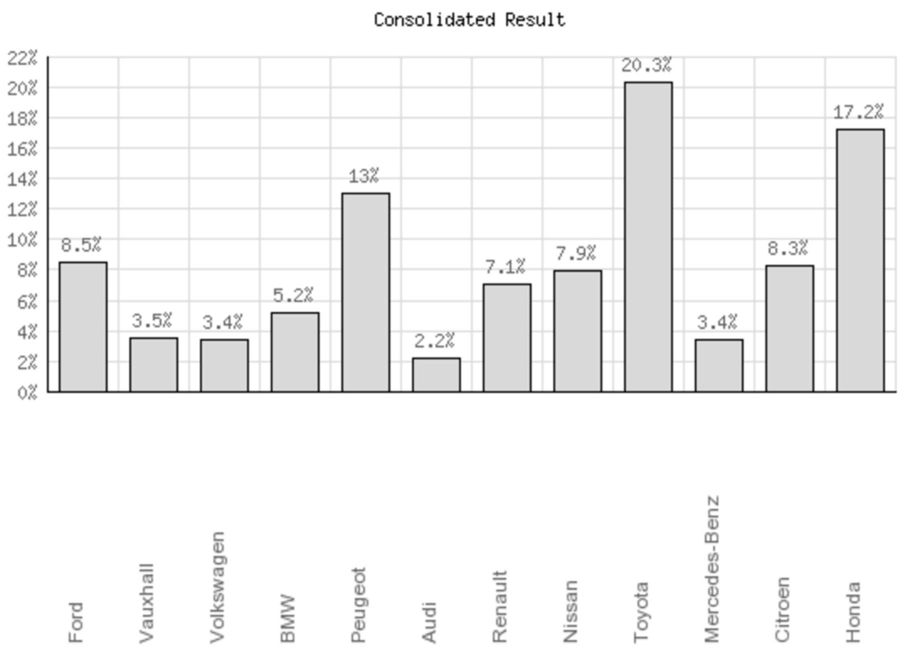

Figure 4.11: Car manufacturers ranking in 2010

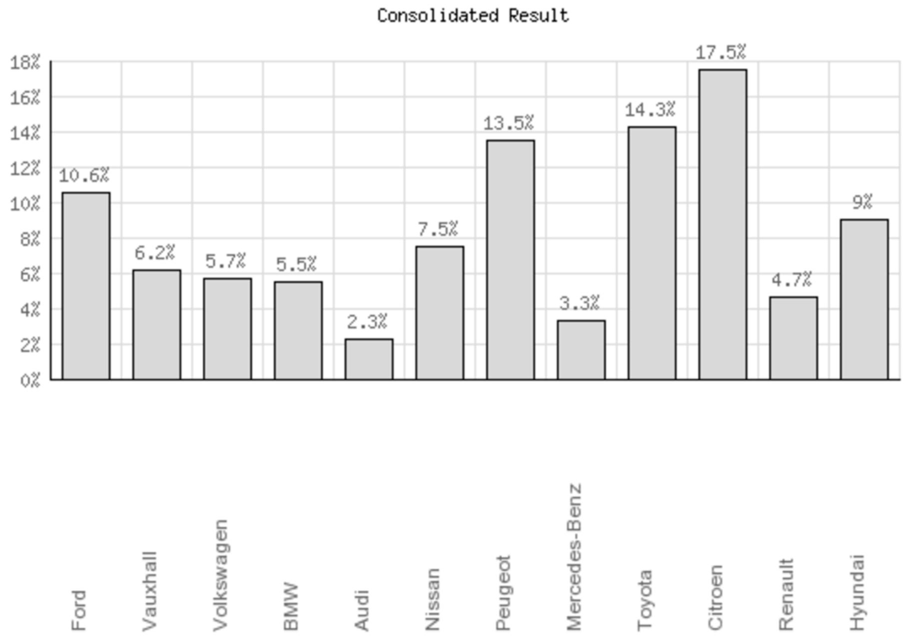

Figure 4.12: Car manufacturers ranking in 2011

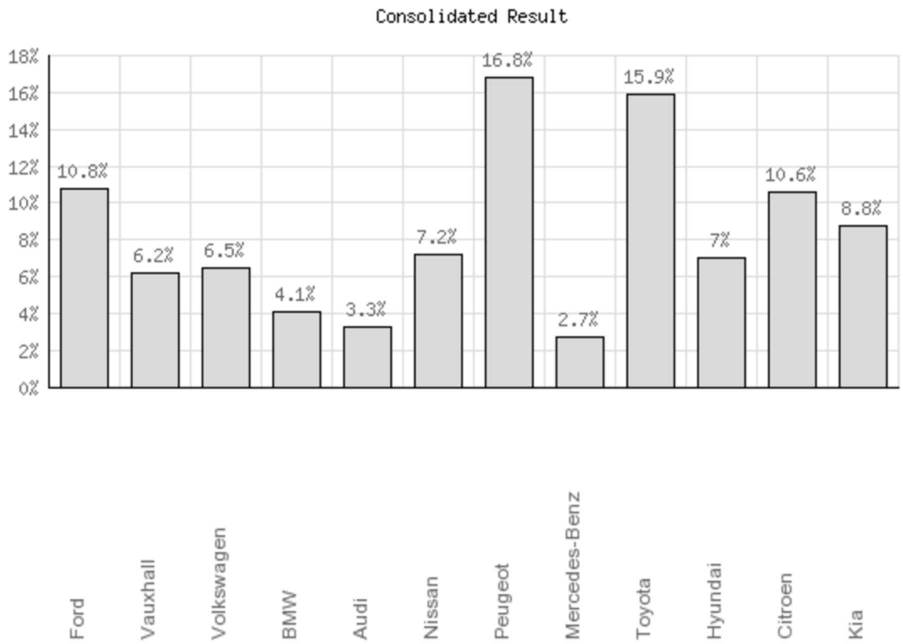

Figure 4.13: Car manufacturers ranking in 2012

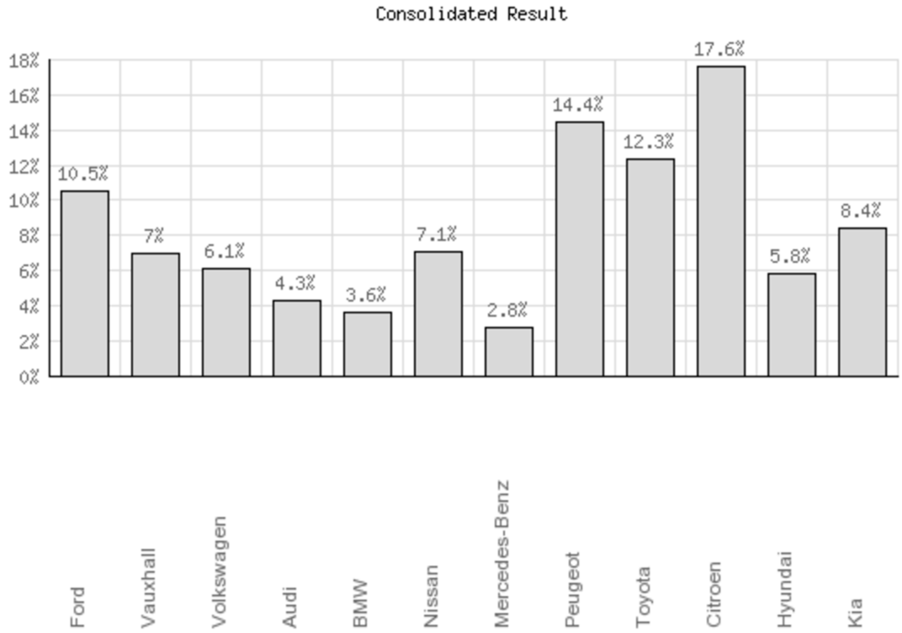

Figure 4.14: Car manufacturers ranking in 2013

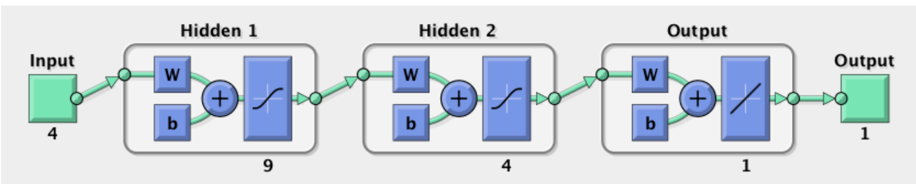

Figure 4.15: Artificial Neural Network Shape

Model Training: We choose MATLAB version R2016b with Neural Network Toolbox as our main tool for creating, training, and calculating the performance of our ANN.

To create an instance of ANN, we use *feedforwardnet* function from Neural Network Toolbox in MATLAB. The feedforwardnet function generates a feedforward neural network consisting of series of layers. The first layer is for connecting the network input, subsequent hidden layers each has connection with previous layer, and the final layer for producing the network output.

We have to prepare these parameters before using feedforwardnet:

hiddenSizes - is a vector of one or more hidden layer sizes, we have chosen [9 4] so we use "[9 4]".

trainFcn - is where we specify our training function, we use "traingdm" which is a network training function that updates weight and bias values according to gradient descent with momentum. We use this so we can control learning rate and momentum value.

The code to create an instance of ANN in MATLAB:

```
% Create a Feedforward Neural Network
hiddenSizes = [9 4];
trainFcn = 'traingdm';
net = feedforwardnet(hiddenSizes,trainFcn);
```

To train an instance of ANN we use the train function from Neural Network Toolbox in MATLAB. The train function trains a neural network using the training parameters we provide to the neural network. Depending on the chosen training function, we put in different type of training parameters, in this case we have chosen "traingdm" as our training function.

We have to provide these parameters to the neural network before using train:

net.layers{1}.transferFcn - we specify the activation function of the neurons in the first hidden layer.

net.layers{2}.transferFcn - we specify the activation function of the neurons in the second hidden layer.

net.initFcn - we specify the initialize function for the neural network.

net.layers{1}.initFcn - we specify the initialize function of the first hidden layer.

net.layers{2}.initFcn - we specify the initialize function of the second hidden layer.

divideFcn - we specify the function to divide training data.

divideMode - we specify the mode for the divide function.

divideParam.trainRatio - we specify ratio of data used for training.

divideParam.valRatio - we specify ratio of data used for validation. .

divideParam.testRatio - we specify ratio of data used for testing.

trainParam.lr - we specify the learning rate of the neural network.

trainParam.mc - we specify the momentum value of the neural network.

net.trainParam.max_fail - we specify the maximum number of times the neural network does not improve after one cycle of training, the neural network stops training if this number is reached.

net.trainparam.epochs - we specify how many training cycles the neural network should be trained.

trainingInputs - is the inputs from training data.

trainingTargets - is the targets or desired outputs from training data.

The code to create and train an instance of ANN in MATLAB:

```
% Set seed so we can get the same result
% everytime we train the ANN with same parameters
RandStream.setGlobalStream (RandStream ('mrg32k3a','Seed', 1234));

% Create a Feedforward Neural Network
hiddenSizes = [9 4];
trainFcn = 'traingdm';
net = feedforwardnet(hiddenSizes,trainFcn);

% Set the parameters
net.layers{1}.transferFcn = 'logsig';
net.layers{2}.transferFcn = 'tansig';
net.initFcn='initlay';
net.layers{1}.initFcn='initnw';
net.layers{2}.initFcn='initnw';
net.divideFcn = 'divideblock';  % Divide data to sequential blocks
net.divideMode = 'sample';  % Divide up every sample
net.divideParam.trainRatio = 60/100;
net.divideParam.valRatio = 20/100;
net.divideParam.testRatio = 20/100;
net.trainParam.lr = 0.35;
net.trainParam.mc = 0.3;
net.trainParam.max_fail = 10000;
net.trainparam.epochs = 100000;

% Train the Network
x = [trainingInputs]';
t = [trainingTargets]';
[net,tr] = train(net,x,t);
```

Validation After our ANN is trained, we use the input data from table 25 on page 66 and the output data from figure 4.14 on page 75 to test the performance of our ANN. The result of R-value is shown in figure 4.16 on the following page.

For the MSE, we compare the the outputs produced from the ANN with the actual scores we have calculated for data in the year 2013 and then calculate the MSE value for the ANN in table 26 on page 80.

We can see that MSE value in table 26 on page 80 is very close to 0 and R-value in figure 4.16 on the next page is very close to 1. When MSE is close to 0, it often means that the error between the desired output and the actual output is very small therefore our ANN model is a good estimator (Wikipedia, 2017*g*). Similarly, when the R-value is close to 1, it often indicates that our ANN model can make good prediction for future input data (Wikipedia, 2017*d*).

Although the value of MSE and R-value are very promising, when we look at the differences between the desired output and the actual output in table 26 on page 80 we can see that our ANN's prediction is not perfect. Some differences are quite significant such as Ford, Nissan, Hyundai and Kia. However, when we rank the scores and compare the predicted ranking with the desired ranking in table 27 on page 81, we can see that the predicted ranking is very similar to the desired one. For example, some rankings are identical such as Ford, Vauxhall, Volkswagen, Audi, BMW, Mercedes-Benz, Citroen, and Kia. Still, there are some differences for other manufacturers such as Nissan and Hyundai.

By looking at the MSE value, the R-value, and the similarity between the actual ranking (ranking from AHP method) and the predicted ranking (ranking predicted by our ANN), we can be confident that our ANN has learned the way to make ranking from AHP method and can make good prediction for future data (i.e. the data that is not included in the training data).

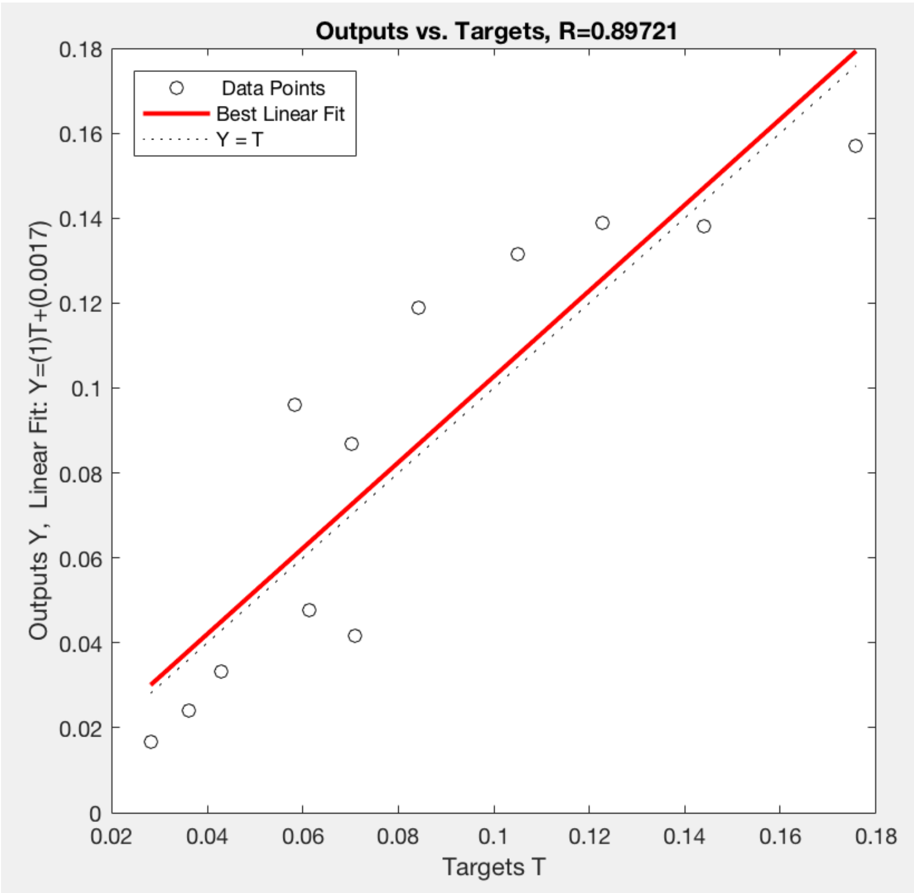

Figure 4.16: R-value graph

Table 26: Mean Squared Error

Manufacturer	AHP-Score	ANN-Score	Difference
Ford	10.51%	13.15%	2.64%
Vauxhall	7.03%	8.68%	1.66%
Volkswagen	6.14%	4.77%	1.37%
Audi	4.29%	3.33%	0.96%
BMW	3.60%	2.39%	1.21%
Nissan	7.10%	4.16%	2.94%
Mercedes-Benz	2.81%	1.66%	1.15%
Peugeot	14.41%	13.81%	0.60%
Toyota	12.28%	13.88%	1.60%
Citroen	17.59%	15.70%	1.89%
Hyundai	5.84%	9.61%	3.77%
Kia	8.42%	11.89%	3.47%
MSE		0.0004728	

Table 27: Rankings of alternatives

Manufacturer	AHP-Rank	ANN-Rank	Difference
Ford	4	4	0
Vauxhall	7	7	0
Volkswagen	8	8	0
Audi	10	10	0
BMW	11	11	0
Nissan	6	9	3
Mercedes-Benz	12	12	0
Peugeot	2	3	1
Toyota	3	2	1
Citroen	1	1	0
Hyundai	9	6	3
Kia	5	5	0

Conclusions

Now we have come to the end of the thesis; we conclude our study with some summarizes and discussions as follows:

Closing the gap between artificial neural network and multi-criteria decision analysis: We have been shown with many great features of the artificial neural network: the inspiration from the biological neuron to the artificial neuron, how artificial neurons connect with each other in layers to create a particular kind of system that can simulate the human brain and its most important function - the ability to learn. With the capacity to learn the underlying complex relationship between input and output, an artificial neural network can even solve challenging classification problems which may be difficult when using traditional computer programs.

With the learning capability of the artificial neural network, we have followed the proposed model from Golmohammadi (Golmohammadi, 2011) and made an attempt to implement this model. Although the implementation is simplified to some extent, we have been shown it with promising results.

As Golmohammadi has suggested (refer to section 4.1 on page 49), his proposed model can use historical data for making the future ranking of alternatives without the judgment effort of the decision maker. Although we can not use this model entirely and replace the traditional decision-making methods (according to figure 4.1 on page 50), it is still an interesting approach to reduce the amount of work for the decision maker by using an artificial neural network to simulate the way the decision maker judge and make decisions.

However, when the validation results of our attempt in implementing Golmohammadi's proposed model has given such a quite good performance, we can be assured that this approach is not just only interesting but also bridges the gap between multi-criteria decision analysis and artificial neural network and opens the gate to many new kinds of application.

Issues with the artificial neural network: There are many problems which have been found during the process of implementing the proposed model of Golmohammadi.

First, the network will not work if we introduce new criteria to our problem in our case study (refer to section 4.3 on page 54). This issue happens because the network has only been trained with the training data that only contains information related to the provided criteria. If we introduce new criteria, the network will perform badly because it does not have the relevant information about the new criteria. In the situation of new criteria being introduced, we have to restart everything from scratch again, in other words, we have to build an entirely new training data which contains information about the new criteria so the artificial neural network can learn.

Second, if the training data are not well-defined (for example, the training data are not normalized properly or there is not enough training data), the performance of the artificial neural network will be deeply affected and can not be useful anymore. Therefore, we have to be extra careful when modeling the training data in order not to waste too much time in training the neural network, in particular in the case when we have a large set of training data, and the architecture of the artificial neural network is complex.

Third, this study only scrapes the tip of the iceberg. Until now, the artificial neural network seems very simple to us because we only pick the most basic and easiest to understand concepts of artificial neural network and machine learning in general. Looking at the code where we created an instance of artificial neural network in MATLAB (refer to section 4.3.4 on page 69), we can see that there are many parameters to be configured. Moreover, yet those settings are for the feedforward neural network, the simplest kind of artificial neural network. Therefore, it is very hard and requires a lot of skill, experience, and patient to configure an optimal artificial neural network.

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A Appendix

This appendix contains the following data:

- Results of pairwise comparisons for fuel consumption, noise level, CO2, and engine capacity in year 2009, 2010, 2011, 2012 and 2013 (figures A.1 - A.3, A.8 - A.11, A.17-A.20, A.26 - A.29, and A.35 - A.38).
- The priority vectors of car manufacturers for each criterion in year 2009, 2010, 2011, 2012 and 2013 (figures A.4 - A.6, A.12 - A.15, A.21 - A.24, A.30 - A.33, and A.39 - A.42).
- The consolidated priorities for car manufacturers in year 2010, 2011, 2012 and 2013 (figures A.7, A.16, A.25, and A.34).

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	8.00	7.00	2.00	9.00	5.00	9.00	9.00	6.00	3.00	9.00	4.00
2	0.13	1	0.50	0.14	5.00	0.25	2.00	4.00	0.33	0.17	3.00	0.20
3	0.14	2.00	1	0.17	6.00	0.33	3.00	5.00	0.50	0.20	4.00	0.25
4	0.50	7.00	6.00	1	9.00	4.00	8.00	9.00	5.00	2.00	9.00	3.00
5	0.11	0.20	0.17	0.11	1	0.13	0.25	0.50	0.14	0.11	0.33	0.11
6	0.20	4.00	3.00	0.25	8.00	1	5.00	7.00	2.00	0.25	6.00	0.50
7	0.11	0.50	0.33	0.13	4.00	0.20	1	3.00	0.25	0.14	2.00	0.17
8	0.11	0.25	0.20	0.11	2.00	0.14	0.33	1	0.17	0.11	0.50	0.13
9	0.17	3.00	2.00	0.20	7.00	0.50	4.00	6.00	1	0.25	5.00	0.33
10	0.33	6.00	5.00	0.50	9.00	4.00	7.00	9.00	4.00	1	8.00	2.00
11	0.11	0.33	0.25	0.11	3.00	0.17	0.50	2.00	0.20	0.13	1	0.14
12	0.25	5.00	4.00	0.33	9.00	2.00	6.00	8.00	3.00	0.50	7.00	1

Figure A.1: Judgement Table For Noise Level in year 2009

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	4.00	5.00	0.25	0.50	6.00	7.00	2.00	8.00	3.00	0.20	0.33
2	0.25	1	2.00	0.14	0.20	3.00	4.00	0.33	5.00	0.50	0.13	0.17
3	0.20	0.50	1	0.13	0.17	2.00	3.00	0.25	4.00	0.33	0.11	0.14
4	4.00	7.00	8.00	1	3.00	9.00	9.00	5.00	9.00	6.00	0.50	2.00
5	2.00	5.00	6.00	0.33	1	7.00	8.00	3.00	9.00	4.00	0.25	0.50
6	0.17	0.33	0.50	0.11	0.14	1	2.00	0.20	3.00	0.25	0.11	0.13
7	0.14	0.25	0.33	0.11	0.13	0.50	1	0.17	2.00	0.20	0.11	0.11
8	0.50	3.00	4.00	0.20	0.33	5.00	6.00	1	7.00	2.00	0.17	0.25
9	0.13	0.20	0.25	0.11	0.11	0.33	0.50	0.14	1	0.17	0.11	0.11
10	0.33	2.00	3.00	0.17	0.25	4.00	5.00	0.50	6.00	1	0.14	0.20
11	5.00	8.00	9.00	2.00	4.00	9.00	9.00	6.00	9.00	7.00	1	3.00
12	3.00	6.00	7.00	0.50	2.00	8.00	9.00	4.00	9.00	5.00	0.33	1

Figure A.2: Judgement Table For CO2 in year 2009

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	3.00	0.50	2.00	6.00	0.20	0.33	5.00	0.25	4.00	8.00	7.00
2	0.33	1	0.25	0.50	4.00	0.14	0.20	3.00	0.17	2.00	6.00	5.00
3	2.00	4.00	1	2.00	7.00	0.25	0.50	6.00	0.33	5.00	9.00	8.00
4	0.50	2.00	0.50	1	5.00	0.17	0.25	4.00	0.20	3.00	7.00	6.00
5	0.17	0.25	0.14	0.20	1	0.11	0.13	0.50	0.11	0.33	3.00	2.00
6	5.00	7.00	4.00	6.00	9.00	1	3.00	9.00	2.00	8.00	9.00	9.00
7	3.00	5.00	2.00	4.00	8.00	0.33	1	7.00	0.50	6.00	9.00	9.00
8	0.20	0.33	0.17	0.25	2.00	0.11	0.14	1	0.13	0.50	4.00	3.00
9	4.00	6.00	3.00	5.00	9.00	0.50	2.00	8.00	1	7.00	9.00	9.00
10	0.25	0.50	0.20	0.33	3.00	0.13	0.17	2.00	0.14	1	5.00	4.00
11	0.13	0.17	0.11	0.14	0.33	0.11	0.11	0.25	0.11	0.20	1	0.50
12	0.14	0.20	0.13	0.17	0.50	0.11	0.11	0.33	0.11	0.25	2.00	1

Figure A.3: Judgement Table For Engine Capacity in year 2009

Category		Priority	Rank
1	Ford	26.3%	1
2	Vauxhall	3.1%	8
3	Volkswagen	4.2%	7
4	Toyota	19.9%	2
5	Peugeot	1.1%	12
6	BMW	7.9%	5
7	Audi	2.2%	9
8	Citroen	1.3%	11
9	Mercedes-Benz	5.8%	6
10	Renault	15.5%	3
11	Fiat	1.7%	10
12	Hyundai	11.0%	4

Figure A.4: Priority vector for Noise Level in year 2009

Category		Priority	Rank
1	Ford	8.1%	5
2	Vauxhall	3.1%	8
3	Volkswagen	2.3%	9
4	Toyota	20.0%	2
5	Peugeot	11.1%	4
6	BMW	1.7%	10
7	Audi	1.3%	11
8	Citroen	5.9%	6
9	Mercedes-Benz	1.1%	12
10	Renault	4.3%	7
11	Fiat	26.4%	1
12	Hyundai	14.9%	3

Figure A.5: Priority vector for CO2 in year 2009

Category		Priority	Rank
1	Ford	8.1%	5
2	Vauxhall	4.3%	7
3	Volkswagen	10.6%	4
4	Toyota	6.0%	6
5	Peugeot	1.7%	10
6	BMW	26.4%	1
7	Audi	15.0%	3
8	Citroen	2.3%	9
9	Mercedes-Benz	20.0%	2
10	Renault	3.1%	8
11	Fiat	1.1%	12
12	Hyundai	1.3%	11

Figure A.6: Priority vector for Engine Capacity in year 2009

Decision Hierarchy

Level 0	Level 1	Global Priorities	Ford	Vauxhall	Volkswag en	BMW	Peugeot	Audi	Renault	Nissan	Toyota	Mercedes -Benz	Citroen	Honda
Environment Friendly Car Manufactur	Fuel Consumption 0.0674	6.7 %	0.004	0.0029	0.0021	0.0011	0.0095	0.0007	0.0055	0.0015	0.0178	0.0009	0.0135	0.0078
	Noise Level 0.2825	28.2 %	0.0422	0.0064	0.0088	0.0228	0.0031	0.0048	0.0166	0.0565	0.0312	0.0121	0.0037	0.0745
	CO2 0.5827	58.3 %	0.0344	0.0252	0.0181	0.0098	0.1169	0.0064	0.0473	0.0132	0.1521	0.0076	0.0647	0.0869
	Engine Capacity 0.0674	6.7 %	0.004	0.0009	0.0054	0.0178	0.0007	0.0101	0.0016	0.0075	0.0021	0.0135	0.0012	0.0028
		1.0	**8.5 %**	**3.5 %**	**3.4 %**	**5.2 %**	**13.0 %**	**2.2 %**	**7.1 %**	**7.9 %**	**20.3 %**	**3.4 %**	**8.3 %**	**17.2 %**

Figure A.7: Consolidated priorities for car manufactures in year 2010

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2.00	3.00	5.00	0.25	7.00	0.50	4.00	0.17	6.00	0.20	0.33
2	0.50	1	2.00	4.00	0.20	6.00	0.33	3.00	0.14	5.00	0.17	0.25
3	0.33	0.50	1	3.00	0.17	5.00	0.25	2.00	0.13	4.00	0.14	0.20
4	0.20	0.25	0.33	1	0.13	3.00	0.17	0.50	0.11	2.00	0.11	0.14
5	4.00	5.00	6.00	8.00	1	9.00	3.00	7.00	0.33	9.00	0.50	1.00
6	0.14	0.17	0.20	0.33	0.11	1	0.13	0.25	0.11	0.50	0.11	0.11
7	2.00	3.00	4.00	6.00	0.33	8.00	1	5.00	0.20	7.00	0.25	0.50
8	0.25	0.33	0.50	2.00	0.14	4.00	0.20	1	0.11	3.00	0.13	0.17
9	6.00	7.00	8.00	9.00	3.00	9.00	5.00	9.00	1	9.00	2.00	4.00
10	0.17	0.20	0.25	0.50	0.11	2.00	0.14	0.33	0.11	1	0.11	0.13
11	5.00	6.00	7.00	9.00	2.00	9.00	4.00	8.00	0.50	9.00	1	3.00
12	3.00	4.00	5.00	7.00	1.00	9.00	2.00	6.00	0.25	8.00	0.33	1

Figure A.8: Judgement Table For Fuel Consumption in year 2010

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	7.00	6.00	3.00	9.00	8.00	4.00	0.50	2.00	5.00	9.00	0.33
2	0.14	1	0.50	0.20	4.00	2.00	0.25	0.13	0.17	0.33	3.00	0.11
3	0.17	2.00	1	0.25	5.00	3.00	0.33	0.14	0.20	0.50	4.00	0.13
4	0.33	5.00	4.00	1	8.00	6.00	2.00	0.25	0.50	3.00	7.00	0.20
5	0.11	0.25	0.20	0.13	1	0.33	0.14	0.11	0.11	0.17	0.50	0.11
6	0.13	0.50	0.33	0.17	3.00	1	0.20	0.11	0.14	0.25	2.00	0.11
7	0.25	4.00	3.00	0.50	7.00	5.00	1	0.20	0.33	2.00	6.00	0.17
8	2.00	8.00	7.00	4.00	9.00	9.00	5.00	1	3.00	6.00	9.00	0.50
9	0.50	6.00	5.00	2.00	9.00	7.00	3.00	0.33	1	4.00	8.00	0.25
10	0.20	3.00	2.00	0.33	6.00	4.00	0.50	0.17	0.25	1	5.00	0.14
11	0.11	0.33	0.25	0.14	2.00	0.50	0.17	0.11	0.13	0.20	1	0.11
12	3.00	9.00	8.00	5.00	9.00	9.00	6.00	2.00	4.00	7.00	9.00	1

Figure A.9: Judgement Table For Noise Level in year 2010

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2.00	3.00	5.00	0.20	7.00	0.50	4.00	0.17	6.00	0.33	0.25
2	0.50	1	2.00	4.00	0.17	6.00	0.33	3.00	0.17	5.00	0.25	0.20
3	0.33	0.50	1	3.00	0.14	5.00	0.25	2.00	0.13	4.00	0.20	0.17
4	0.20	0.25	0.33	1	0.11	3.00	0.17	0.50	0.11	2.00	0.14	0.13
5	5.00	6.00	7.00	9.00	1	9.00	4.00	8.00	0.50	9.00	3.00	2.00
6	0.14	0.17	0.20	0.33	0.11	1	0.13	0.25	0.11	0.50	0.11	0.13
7	2.00	3.00	4.00	6.00	0.25	8.00	1	5.00	0.20	7.00	0.50	0.33
8	0.25	0.33	0.50	2.00	0.13	4.00	0.20	1	0.11	3.00	0.17	0.14
9	6.00	6.00	8.00	9.00	2.00	9.00	5.00	9.00	1	9.00	4.00	3.00
10	0.17	0.20	0.25	0.50	0.11	2.00	0.14	0.33	0.11	1	0.13	0.11
11	3.00	4.00	5.00	7.00	0.33	9.00	2.00	6.00	0.25	8.00	1	0.50
12	4.00	5.00	6.00	8.00	0.50	8.00	3.00	7.00	0.33	9.00	2.00	1

Figure A.10: Judgement Table For CO2 in year 2010

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	6.00	0.50	0.17	7.00	0.25	4.00	0.33	3.00	0.20	5.00	2.00
2	0.17	1	0.14	0.11	2.00	0.11	0.33	0.13	0.25	0.11	0.50	0.20
3	2.00	7.00	1	0.20	8.00	0.33	5.00	0.50	4.00	0.25	6.00	3.00
4	6.00	9.00	5.00	1	9.00	3.00	9.00	4.00	8.00	2.00	9.00	7.00
5	0.14	0.50	0.13	0.11	1	0.11	0.20	0.11	0.20	0.11	0.33	0.17
6	4.00	9.00	3.00	0.33	9.00	1	7.00	2.00	6.00	0.50	8.00	5.00
7	0.25	3.00	0.20	0.11	5.00	0.14	1	0.17	0.50	0.13	2.00	0.33
8	3.00	8.00	2.00	0.25	9.00	0.50	6.00	1	5.00	0.33	7.00	4.00
9	0.33	4.00	0.25	0.13	5.00	0.17	2.00	0.20	1	0.14	3.00	0.50
10	5.00	9.00	4.00	0.50	9.00	2.00	8.00	3.00	7.00	1	9.00	6.00
11	0.20	2.00	0.17	0.11	3.00	0.13	0.50	0.14	0.33	0.11	1	0.33
12	0.50	5.00	0.33	0.14	6.00	0.20	3.00	0.25	2.00	0.17	3.00	1

Figure A.11: Judgement Table For Engine Capacity in year 2010

Category		Priority	Rank
1	Ford	5.9%	6
2	Vauxhall	4.3%	7
3	Volkswagen	3.1%	8
4	BMW	1.7%	10
5	Peugeot	14.1%	3
6	Audi	1.1%	12
7	Renault	8.1%	5
8	Nissan	2.3%	9
9	Toyota	26.4%	1
10	Mercedes-Benz	1.3%	11
11	Citroen	20.1%	2
12	Honda	11.6%	4

Figure A.12: Priority vector for Fuel Consumption in year 2010

Category		Priority	Rank
1	Ford	14.9%	3
2	Vauxhall	2.3%	9
3	Volkswagen	3.1%	8
4	BMW	8.1%	5
5	Peugeot	1.1%	12
6	Audi	1.7%	10
7	Renault	5.9%	6
8	Nissan	20.0%	2
9	Toyota	11.1%	4
10	Mercedes-Benz	4.3%	7
11	Citroen	1.3%	11
12	Honda	26.4%	1

Figure A.13: Priority vector for Noise Level in year 2010

Category		Priority	Rank
1	Ford	5.9%	6
2	Vauxhall	4.3%	7
3	Volkswagen	3.1%	8
4	BMW	1.7%	10
5	Peugeot	20.1%	2
6	Audi	1.1%	12
7	Renault	8.1%	5
8	Nissan	2.3%	9
9	Toyota	26.1%	1
10	Mercedes-Benz	1.3%	11
11	Citroen	11.1%	4
12	Honda	14.9%	3

Figure A.14: Priority vector for CO2 in year 2010

Category		Priority	Rank
1	Ford	5.9%	6
2	Vauxhall	1.3%	11
3	Volkswagen	8.1%	5
4	BMW	26.3%	1
5	Peugeot	1.1%	12
6	Audi	14.9%	3
7	Renault	2.3%	9
8	Nissan	11.1%	4
9	Toyota	3.1%	8
10	Mercedes-Benz	20.0%	2
11	Citroen	1.7%	10
12	Honda	4.1%	7

Figure A.15: Priority vector for Engine Capacity in year 2010

Decision Hierarchy

Level 0	Level 1	Global Priorities	Ford	Vauxhall	Volkswag en	BMW	Audi	Nissan	Peugeot	Mercedes -Benz	Toyota	Citroen	Renault	Hyundai
Environment Friendly Car Manufactur	Fuel Consumption 0.0674	6.7 %	0.0029	0.0055	0.0021	0.0015	0.0007	0.0011	0.0135	0.0009	0.0101	0.0178	0.004	0.0074
	Noise Level 0.2825	28.2 %	0.0743	0.0087	0.0311	0.0227	0.0047	0.0563	0.0037	0.0121	0.043	0.003	0.0063	0.0165
	CO2 0.5827	58.3 %	0.0249	0.0471	0.0181	0.0132	0.0076	0.0098	0.1165	0.0063	0.0871	0.1536	0.0343	0.0644
	Engine Capacity 0.0674	6.7 %	0.004	0.0011	0.0054	0.0178	0.0101	0.0074	0.0009	0.0135	0.0029	0.0007	0.0021	0.0015
		1.0	**10.6 %**	**6.2 %**	**5.7 %**	**5.5 %**	**2.3 %**	**7.5 %**	**13.5 %**	**3.3 %**	**14.3 %**	**17.5 %**	**4.7 %**	**9.0 %**

Figure A.16: Consolidated priorities for car manufactures in year 2011

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.33	2.00	3.00	6.00	4.00	0.17	5.00	0.20	0.14	0.50	0.25
2	3.00	1	4.00	5.00	8.00	6.00	0.25	7.00	0.33	0.20	2.00	0.50
3	0.50	0.25	1	2.00	5.00	3.00	0.14	4.00	0.17	0.13	0.33	0.20
4	0.33	0.20	0.50	1	4.00	2.00	0.13	3.00	0.14	0.11	0.25	0.17
5	0.17	0.13	0.20	0.25	1	0.33	0.11	0.50	0.11	0.11	0.14	0.11
6	0.25	0.17	0.33	0.50	3.00	1	0.11	2.00	0.13	0.11	0.20	0.14
7	6.00	4.00	7.00	8.00	9.00	9.00	1	9.00	2.00	0.50	5.00	3.00
8	0.20	0.14	0.25	0.33	2.00	0.50	0.11	1	0.11	0.11	0.17	0.14
9	5.00	3.00	6.00	7.00	9.00	8.00	0.50	9.00	1	0.33	4.00	2.00
10	7.00	5.00	8.00	9.00	9.00	9.00	2.00	9.00	3.00	1	6.00	4.00
11	2.00	0.50	3.00	4.00	7.00	5.00	0.20	6.00	0.25	0.17	1	0.33
12	4.00	2.00	5.00	6.00	9.00	7.00	0.33	7.00	0.50	0.25	3.00	1

Figure A.17: Judgement Table For Fuel Consumption in year 2011

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	8.00	4.00	5.00	9.00	2.00	9.00	7.00	3.00	9.00	9.00	6.00
2	0.13	1	0.20	0.25	3.00	0.14	4.00	0.50	0.17	5.00	2.00	0.33
3	0.25	5.00	1	2.00	7.00	0.33	8.00	4.00	0.50	9.00	6.00	3.00
4	0.20	4.00	0.50	1	6.00	0.25	7.00	3.00	0.33	8.00	5.00	2.00
5	0.11	0.33	0.14	0.17	1	0.11	2.00	0.25	0.13	3.00	0.50	0.20
6	0.50	7.00	3.00	4.00	9.00	1	9.00	6.00	2.00	9.00	8.00	5.00
7	0.11	0.25	0.13	0.14	0.50	0.11	1	0.20	0.11	2.00	0.33	0.17
8	0.14	2.00	0.25	0.33	4.00	0.17	5.00	1	0.17	7.00	3.00	0.50
9	0.33	6.00	2.00	3.00	8.00	0.50	9.00	6.00	1	9.00	7.00	4.00
10	0.11	0.20	0.11	0.13	0.33	0.11	0.50	0.14	0.11	1	0.25	0.14
11	0.11	0.50	0.17	0.20	2.00	0.13	3.00	0.33	0.14	4.00	1	0.25
12	0.17	3.00	0.33	0.50	5.00	0.20	6.00	2.00	0.25	7.00	4.00	1

Figure A.18: Judgement Table For Noise Level in year 2011

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.33	2.00	3.00	5.00	4.00	0.17	6.00	0.20	0.14	0.50	0.25
2	3.00	1	4.00	5.00	7.00	6.00	0.25	8.00	0.33	0.20	2.00	0.50
3	0.50	0.25	1	2.00	4.00	3.00	0.14	5.00	0.17	0.13	0.33	0.20
4	0.33	0.20	0.50	1	3.00	2.00	0.13	4.00	0.14	0.11	0.25	0.17
5	0.20	0.14	0.25	0.33	1	0.50	0.11	2.00	0.11	0.11	0.17	0.13
6	0.25	0.17	0.33	0.50	2.00	1	0.11	3.00	0.13	0.11	0.20	0.14
7	6.00	4.00	7.00	8.00	9.00	9.00	1	9.00	2.00	0.50	5.00	3.00
8	0.17	0.13	0.20	0.25	0.50	0.33	0.11	1	0.11	0.11	0.14	0.11
9	5.00	3.00	6.00	7.00	9.00	8.00	0.50	9.00	1	0.33	4.00	2.00
10	7.00	5.00	8.00	9.00	9.00	9.00	2.00	9.00	3.00	1	6.00	4.00
11	2.00	0.50	3.00	4.00	6.00	5.00	0.20	7.00	0.25	0.17	1	0.33
12	4.00	2.00	5.00	6.00	8.00	7.00	0.33	9.00	0.50	0.25	3.00	1

Figure A.19: Judgement Table For CO2 in year 2011

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	5.00	0.50	0.17	0.25	0.33	6.00	0.20	2.00	7.00	3.00	4.00
2	0.20	1	0.17	0.11	0.13	0.14	2.00	0.11	0.25	3.00	0.33	0.50
3	2.00	6.00	1	0.20	0.33	0.50	7.00	0.25	3.00	8.00	4.00	5.00
4	6.00	9.00	5.00	1	3.00	4.00	9.00	2.00	7.00	9.00	8.00	9.00
5	4.00	8.00	3.00	0.33	1	2.00	9.00	0.50	5.00	9.00	6.00	7.00
6	3.00	7.00	2.00	0.25	0.50	1	8.00	0.33	4.00	9.00	5.00	6.00
7	0.17	0.50	0.14	0.11	0.11	0.13	1	0.11	0.20	2.00	0.25	0.33
8	5.00	9.00	4.00	0.50	2.00	3.00	9.00	1	6.00	9.00	7.00	8.00
9	0.50	4.00	0.33	0.14	0.20	0.25	5.00	0.17	1	7.00	2.00	3.00
10	0.14	0.33	0.13	0.11	0.11	0.11	0.50	0.11	0.14	1	0.20	0.25
11	0.33	3.00	0.25	0.13	0.17	0.20	4.00	0.14	0.50	5.00	1	2.00
12	0.25	2.00	0.20	0.11	0.14	0.17	3.00	0.13	0.33	4.00	0.50	1

Figure A.20: Judgement Table For Engine Capacity in year 2011

Category		Priority	Rank
1	Ford	4.3%	7
2	Vauxhall	8.1%	5
3	Volkswagen	3.1%	8
4	BMW	2.3%	9
5	Audi	1.1%	12
6	Nissan	1.7%	10
7	Peugeot	20.0%	2
8	Mercedes-Benz	1.3%	11
9	Toyota	15.0%	3
10	Citroen	26.4%	1
11	Renault	5.9%	6
12	Hyundai	11.0%	4

Figure A.21: Priority vector for Fuel Consumption in year 2011

Category		Priority	Rank
1	Ford	26.3%	1
2	Vauxhall	3.1%	8
3	Volkswagen	11.0%	4
4	BMW	8.0%	5
5	Audi	1.7%	10
6	Nissan	19.9%	2
7	Peugeot	1.3%	11
8	Mercedes-Benz	4.3%	7
9	Toyota	15.2%	3
10	Citroen	1.1%	12
11	Renault	2.2%	9
12	Hyundai	5.9%	6

Figure A.22: Priority vector for Noise Level in year 2011

Category		Priority	Rank
1	Ford	4.3%	7
2	Vauxhall	8.1%	5
3	Volkswagen	3.1%	8
4	BMW	2.3%	9
5	Audi	1.3%	11
6	Nissan	1.7%	10
7	Peugeot	20.0%	2
8	Mercedes-Benz	1.1%	12
9	Toyota	14.9%	3
10	Citroen	26.4%	1
11	Renault	5.9%	6
12	Hyundai	11.1%	4

Figure A.23: Priority vector for CO2 in year 2011

Category		Priority	Rank
1	Ford	5.9%	6
2	Vauxhall	1.7%	10
3	Volkswagen	8.1%	5
4	BMW	26.3%	1
5	Audi	14.9%	3
6	Nissan	11.0%	4
7	Peugeot	1.3%	11
8	Mercedes-Benz	20.0%	2
9	Toyota	4.3%	7
10	Citroen	1.1%	12
11	Renault	3.1%	8
12	Hyundai	2.3%	9

Figure A.24: Priority vector for Engine Capacity in year 2011

Decision Hierarchy

Level 0	Level 1	Global Priorities	Ford	Vauxhall	Volkswagen	BMW	Audi	Nissan	Peugeot	Mercedes-Benz	Toyota	Hyundai	Citroen	Kia
Environment Friendly Car Manufactur	Fuel Consumption 0.0674	6.7 %	0.0029	0.0039	0.0021	0.0015	0.0011	0.0007	0.01	0.0009	0.0077	0.0054	0.0134	0.0177
	Noise Level 0.2825	28.2 %	0.0745	0.0228	0.0422	0.0121	0.0088	0.0565	0.0037	0.0064	0.0312	0.0166	0.0031	0.0048
	CO2 0.5827	58.3 %	0.0249	0.0343	0.0181	0.0098	0.0132	0.0076	0.1536	0.0063	0.1165	0.0471	0.0871	0.0644
	Engine Capacity 0.0674	6.7 %	0.0054	0.0015	0.0029	0.0178	0.0101	0.0074	0.0011	0.0135	0.004	0.0009	0.0021	0.0007
		1.0	**10.8 %**	**6.2 %**	**6.5 %**	**4.1 %**	**3.3 %**	**7.2 %**	**16.8 %**	**2.7 %**	**15.9 %**	**7.0 %**	**10.6 %**	**8.8 %**

Figure A.25: Consolidated priorities for car manufactures in year 2012

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.50	2.00	3.00	4.00	6.00	0.20	5.00	0.25	0.33	0.17	0.14
2	2.00	1	3.00	4.00	5.00	7.00	0.25	6.00	0.25	0.50	0.20	0.17
3	0.50	0.33	1	2.00	3.00	5.00	0.17	4.00	0.20	0.25	0.14	0.13
4	0.33	0.25	0.50	1	2.00	4.00	0.14	3.00	0.17	0.20	0.13	0.11
5	0.25	0.20	0.33	0.50	1	3.00	0.13	2.00	0.14	0.17	0.11	0.11
6	0.17	0.14	0.20	0.25	0.33	1	0.11	0.50	0.11	0.13	0.11	0.11
7	5.00	4.00	6.00	7.00	8.00	9.00	1	9.00	2.00	3.00	0.50	0.33
8	0.20	0.17	0.25	0.33	0.50	2.00	0.11	1	0.13	0.14	0.11	0.11
9	4.00	4.00	5.00	6.00	7.00	9.00	0.50	8.00	1	2.00	0.33	0.25
10	3.00	2.00	4.00	5.00	6.00	8.00	0.33	7.00	0.50	1	0.25	0.20
11	6.00	5.00	7.00	8.00	9.00	9.00	2.00	9.00	3.00	4.00	1	0.50
12	7.00	6.00	8.00	9.00	9.00	9.00	3.00	9.00	4.00	5.00	2.00	1

Figure A.26: Judgement Table For Fuel Consumption in year 2012

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	5.00	3.00	7.00	8.00	2.00	9.00	9.00	4.00	6.00	9.00	9.00
2	0.20	1	0.33	3.00	4.00	0.25	7.00	5.00	0.50	2.00	8.00	6.00
3	0.33	3.00	1	5.00	6.00	0.50	9.00	7.00	2.00	4.00	9.00	8.00
4	0.14	0.33	0.20	1	2.00	0.17	5.00	3.00	0.25	0.50	6.00	4.00
5	0.13	0.25	0.17	0.50	1	0.14	4.00	2.00	0.20	0.33	5.00	3.00
6	0.50	4.00	2.00	6.00	7.00	1	9.00	8.00	3.00	5.00	9.00	9.00
7	0.11	0.14	0.11	0.20	0.25	0.11	1	0.33	0.13	0.17	2.00	0.50
8	0.11	0.20	0.14	0.33	0.50	0.13	3.00	1	0.17	0.25	4.00	2.00
9	0.25	2.00	0.50	4.00	5.00	0.33	8.00	6.00	1	3.00	9.00	7.00
10	0.17	0.50	0.25	2.00	3.00	0.20	6.00	4.00	0.33	1	7.00	5.00
11	0.11	0.13	0.11	0.17	0.20	0.11	0.50	0.25	0.11	0.14	1	0.33
12	0.11	0.17	0.13	0.25	0.33	0.11	2.00	0.50	0.14	0.20	3.00	1

Figure A.27: Judgement Table For Noise Level in year 2012

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.50	2.00	4.00	3.00	5.00	0.14	6.00	0.17	0.33	0.20	0.25
2	2.00	1	3.00	5.00	4.00	6.00	0.17	7.00	0.20	0.50	0.25	0.33
3	0.50	0.33	1	3.00	2.00	4.00	0.13	5.00	0.14	0.25	0.17	0.20
4	0.25	0.20	0.33	1	0.50	2.00	0.11	3.00	0.11	0.17	0.13	0.14
5	0.33	0.25	0.50	2.00	1	3.00	0.11	4.00	0.13	0.20	0.14	0.17
6	0.20	0.17	0.25	0.50	0.33	1	0.11	2.00	0.11	0.14	0.11	0.13
7	7.00	6.00	8.00	9.00	9.00	9.00	1	9.00	2.00	5.00	3.00	4.00
8	0.17	0.14	0.20	0.33	0.25	0.50	0.11	1	0.11	0.13	0.11	0.11
9	6.00	5.00	7.00	9.00	8.00	9.00	0.50	9.00	1	4.00	2.00	3.00
10	3.00	2.00	4.00	6.00	5.00	7.00	0.20	8.00	0.25	1	0.33	0.50
11	5.00	4.00	6.00	8.00	7.00	9.00	0.33	9.00	0.50	3.00	1	2.00
12	4.00	3.00	5.00	7.00	6.00	8.00	0.25	9.00	0.33	2.00	0.50	1

Figure A.28: Judgement Table For CO2 in year 2012

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	5.00	3.00	0.20	0.33	0.50	6.00	0.25	2.00	7.00	4.00	8.00
2	0.20	1	0.33	0.11	0.14	0.17	2.00	0.13	0.25	3.00	0.50	4.00
3	0.33	3.00	1	0.14	0.20	0.25	4.00	0.17	0.50	5.00	2.00	6.00
4	5.00	9.00	7.00	1	3.00	4.00	9.00	2.00	6.00	9.00	8.00	9.00
5	3.00	7.00	5.00	0.33	1	2.00	8.00	0.50	4.00	9.00	6.00	9.00
6	2.00	6.00	4.00	0.25	0.50	1	7.00	0.33	3.00	8.00	5.00	9.00
7	0.17	0.50	0.25	0.11	0.13	0.14	1	0.11	0.20	2.00	0.33	3.00
8	4.00	8.00	6.00	0.50	2.00	3.00	9.00	1	5.00	9.00	7.00	9.00
9	0.50	4.00	2.00	0.17	0.25	0.33	5.00	0.20	1	6.00	3.00	7.00
10	0.14	0.33	0.20	0.11	0.11	0.13	0.50	0.11	0.17	1	0.25	2.00
11	0.25	2.00	0.50	0.13	0.17	0.20	3.00	0.14	0.33	4.00	1	5.00
12	0.13	0.25	0.17	0.11	0.11	0.11	0.33	0.11	0.14	0.50	0.20	1

Figure A.29: Judgement Table For Engine Capacity in year 2012

Category		Priority	Rank
1	Ford	4.2%	7
2	Vauxhall	5.8%	6
3	Volkswagen	3.1%	8
4	BMW	2.2%	9
5	Audi	1.7%	10
6	Nissan	1.1%	12
7	Peugeot	14.9%	3
8	Mercedes-Benz	1.3%	11
9	Toyota	11.4%	4
10	Hyundai	8.0%	5
11	Citroen	19.9%	2
12	Kia	26.3%	1

Figure A.30: Priority vector for Fuel Consumption in year 2012

Category		Priority	Rank
1	Ford	26.4%	1
2	Vauxhall	8.1%	5
3	Volkswagen	14.9%	3
4	BMW	4.3%	7
5	Audi	3.1%	8
6	Nissan	20.0%	2
7	Peugeot	1.3%	11
8	Mercedes-Benz	2.3%	9
9	Toyota	11.1%	4
10	Hyundai	5.9%	6
11	Citroen	1.1%	12
12	Kia	1.7%	10

Figure A.31: Priority vector for Noise Level in year 2012

Category		Priority	Rank
1	Ford	4.3%	7
2	Vauxhall	5.9%	6
3	Volkswagen	3.1%	8
4	BMW	1.7%	10
5	Audi	2.3%	9
6	Nissan	1.3%	11
7	Peugeot	26.4%	1
8	Mercedes-Benz	1.1%	12
9	Toyota	20.0%	2
10	Hyundai	8.1%	5
11	Citroen	14.9%	3
12	Kia	11.1%	4

Figure A.32: Priority vector for CO2 in year 2012

Category		Priority	Rank
1	Ford	8.1%	5
2	Vauxhall	2.3%	9
3	Volkswagen	4.3%	7
4	BMW	26.4%	1
5	Audi	14.9%	3
6	Nissan	11.1%	4
7	Peugeot	1.7%	10
8	Mercedes-Benz	20.0%	2
9	Toyota	5.9%	6
10	Hyundai	1.3%	11
11	Citroen	3.1%	8
12	Kia	1.1%	12

Figure A.33: Priority vector for Engine Capacity in year 2012

Decision Hierarchy														
Level 0	Level 1	Global Priorities	Ford	Vauxhall	Volkswagen	Audi	BMW	Nissan	Mercedes-Benz	Peugeot	Toyota	Citroen	Hyundai	Kia
Environment Friendly Car Manufactur	Fuel Consumption 0.0674	6.7 %	0.0029	0.0039	0.0021	0.0009	0.0015	0.0007	0.0011	0.0134	0.0074	0.0177	0.0054	0.0103
	Noise Level 0.2825	28.2 %	0.0745	0.0312	0.0422	0.0166	0.0064	0.0565	0.0037	0.0121	0.0228	0.0031	0.0048	0.0088
	CO_2 0.5827	58.3 %	0.0249	0.0343	0.0132	0.0076	0.0181	0.0063	0.0098	0.1165	0.0871	0.1536	0.0471	0.0644
	Engine Capacity 0.0674	6.7 %	0.0029	0.0009	0.004	0.0178	0.0101	0.0074	0.0135	0.0021	0.0054	0.0015	0.0011	0.0007
		1.0	**10.5 %**	**7.0 %**	**6.1 %**	**4.3 %**	**3.6 %**	**7.1 %**	**2.8 %**	**14.4 %**	**12.3 %**	**17.6 %**	**5.8 %**	**8.4 %**

Figure A.34: Consolidated priorities for car manufactures in year 2013

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.50	2.00	5.00	3.00	6.00	4.00	0.17	0.25	0.14	0.33	0.20
2	2.00	1	3.00	6.00	4.00	7.00	5.00	0.20	0.33	0.17	0.50	0.20
3	0.50	0.33	1	4.00	2.00	5.00	3.00	0.14	0.20	0.13	0.25	0.17
4	0.20	0.17	0.25	1	0.33	2.00	0.50	0.11	0.13	0.11	0.14	0.11
5	0.33	0.25	0.50	3.00	1	4.00	2.00	0.13	0.17	0.11	0.20	0.14
6	0.17	0.14	0.20	0.50	0.25	1	0.33	0.11	0.11	0.11	0.13	0.11
7	0.25	0.20	0.33	2.00	0.50	3.00	1	0.11	0.14	0.11	0.17	0.13
8	6.00	5.00	7.00	9.00	8.00	9.00	9.00	1	3.00	0.50	4.00	2.00
9	4.00	3.00	5.00	8.00	6.00	9.00	7.00	0.33	1	0.25	2.00	0.50
10	7.00	6.00	8.00	9.00	9.00	9.00	9.00	2.00	4.00	1	5.00	3.00
11	3.00	2.00	4.00	7.00	5.00	8.00	6.00	0.25	0.50	0.20	1	0.33
12	5.00	5.00	6.00	9.00	7.00	9.00	8.00	0.50	2.00	0.33	3.00	1

Figure A.35: Judgement Table For Fuel Consumption in year 2013

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	4.00	3.00	6.00	9.00	2.00	9.00	7.00	5.00	9.00	9.00	8.00
2	0.25	1	0.50	3.00	6.00	0.33	8.00	4.00	2.00	9.00	7.00	5.00
3	0.33	2.00	1	4.00	7.00	0.50	9.00	5.00	3.00	9.00	8.00	6.00
4	0.17	0.33	0.25	1	4.00	0.20	6.00	2.00	0.50	7.00	5.00	3.00
5	0.11	0.17	0.14	0.25	1	0.13	3.00	0.33	0.20	4.00	2.00	0.50
6	0.50	3.00	2.00	5.00	8.00	1	9.00	6.00	4.00	9.00	9.00	7.00
7	0.11	0.13	0.11	0.17	0.33	0.11	1	0.20	0.14	2.00	0.50	0.25
8	0.14	0.25	0.20	0.50	3.00	0.17	5.00	1	0.33	6.00	4.00	2.00
9	0.20	0.50	0.33	2.00	5.00	0.25	7.00	3.00	1	8.00	6.00	4.00
10	0.11	0.11	0.11	0.14	0.25	0.11	0.50	0.17	0.13	1	0.33	0.20
11	0.11	0.14	0.13	0.20	0.50	0.11	2.00	0.25	0.17	3.00	1	0.33
12	0.13	0.20	0.17	0.33	2.00	0.14	4.00	0.50	0.25	5.00	3.00	1

Figure A.36: Judgement Table For Noise Level in year 2013

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	0.50	3.00	5.00	2.00	6.00	4.00	0.17	0.20	0.14	0.33	0.25
2	2.00	1	4.00	6.00	3.00	7.00	5.00	0.20	0.25	0.17	0.50	0.33
3	0.33	0.25	1	3.00	0.50	4.00	2.00	0.13	0.14	0.11	0.20	0.17
4	0.20	0.17	0.33	1	0.25	2.00	0.50	0.11	0.11	0.11	0.14	0.13
5	0.50	0.33	2.00	4.00	1	5.00	3.00	0.14	0.17	0.13	0.25	0.20
6	0.17	0.14	0.25	0.50	0.20	1	0.33	0.11	0.11	0.11	0.13	0.11
7	0.25	0.20	0.50	2.00	0.33	3.00	1	0.11	0.13	0.11	0.17	0.14
8	6.00	5.00	8.00	9.00	7.00	9.00	9.00	1	2.00	0.50	4.00	3.00
9	5.00	4.00	7.00	9.00	6.00	9.00	8.00	0.50	1	0.33	3.00	2.00
10	7.00	6.00	9.00	9.00	8.00	9.00	9.00	2.00	3.00	1	5.00	4.00
11	3.00	2.00	5.00	7.00	4.00	8.00	6.00	0.25	0.33	0.20	1	0.50
12	4.00	3.00	6.00	8.00	5.00	9.00	7.00	0.33	0.50	0.25	2.00	1

Figure A.37: Judgement Table For CO2 in year 2013

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	5.00	0.50	0.14	0.20	0.25	0.17	2.00	0.33	3.00	4.00	6.00
2	0.20	1	0.17	0.11	0.11	0.13	0.11	0.25	0.14	0.33	0.50	2.00
3	2.00	6.00	1	0.17	0.25	0.33	0.20	3.00	0.50	4.00	5.00	7.00
4	7.00	9.00	6.00	1	3.00	4.00	2.00	8.00	5.00	9.00	9.00	9.00
5	5.00	9.00	4.00	0.33	1	2.00	0.50	6.00	3.00	7.00	8.00	9.00
6	4.00	8.00	3.00	0.25	0.50	1	0.33	5.00	2.00	6.00	7.00	9.00
7	6.00	9.00	5.00	0.50	2.00	3.00	1	7.00	4.00	8.00	9.00	9.00
8	0.50	4.00	0.33	0.13	0.17	0.20	0.14	1	0.25	2.00	3.00	5.00
9	3.00	7.00	2.00	0.20	0.33	0.50	0.25	4.00	1	5.00	6.00	8.00
10	0.33	3.00	0.25	0.11	0.14	0.17	0.13	0.50	0.20	1	2.00	4.00
11	0.25	2.00	0.20	0.11	0.13	0.14	0.11	0.33	0.17	0.50	1	3.00
12	0.17	0.50	0.14	0.11	0.11	0.11	0.11	0.20	0.13	0.25	0.33	1

Figure A.38: Judgement Table For Engine Capacity in year 2013

Category		Priority	Rank
1	Ford	4.3%	7
2	Vauxhall	5.8%	6
3	Volkswagen	3.1%	8
4	Audi	1.3%	11
5	BMW	2.2%	9
6	Nissan	1.1%	12
7	Mercedes-Benz	1.7%	10
8	Peugeot	19.9%	2
9	Toyota	11.0%	4
10	Citroen	26.3%	1
11	Hyundai	8.0%	5
12	Kia	15.3%	3

Figure A.39: Priority vector for Fuel Consumption in year 2013

Category		Priority	Rank
1	Ford	26.4%	1
2	Vauxhall	11.1%	4
3	Volkswagen	14.9%	3
4	Audi	5.9%	6
5	BMW	2.3%	9
6	Nissan	20.0%	2
7	Mercedes-Benz	1.3%	11
8	Peugeot	4.3%	7
9	Toyota	8.1%	5
10	Citroen	1.1%	12
11	Hyundai	1.7%	10
12	Kia	3.1%	8

Figure A.40: Priority vector for Noise Level in year 2013

Category		Priority	Rank
1	Ford	4.3%	7
2	Vauxhall	5.9%	6
3	Volkswagen	2.3%	9
4	Audi	1.3%	11
5	BMW	3.1%	8
6	Nissan	1.1%	12
7	Mercedes-Benz	1.7%	10
8	Peugeot	20.0%	2
9	Toyota	14.9%	3
10	Citroen	26.4%	1
11	Hyundai	8.1%	5
12	Kia	11.1%	4

Figure A.41: Priority vector for CO_2 in year 2013

Category		Priority	Rank
1	Ford	4.3%	7
2	Vauxhall	1.3%	11
3	Volkswagen	5.9%	6
4	Audi	26.4%	1
5	BMW	14.9%	3
6	Nissan	11.1%	4
7	Mercedes-Benz	20.0%	2
8	Peugeot	3.1%	8
9	Toyota	8.1%	5
10	Citroen	2.3%	9
11	Hyundai	1.7%	10
12	Kia	1.1%	12

Figure A.42: Priority vector for Engine Capacity in year 2013