HW3

1 Clustering (10%)

In the lecture we saw the K-means algorithm for clustering. It tries to minimize the Euclidian metric between the examples and some point in space which is named "centroid". Other methods try to minimize dissimilarities between the pairwise data examples. A classical algorithm which was designed to handle pairwise data is the K-medoid. This algorithm seeks to find a set of cluster representatives (named medoid) in the dataset and assign other examples to them. The algorithm randomly picks a k-set of medoids from the data and assigns points to each medoid based on their L_1 distances to that medoid. Then, it iteratively tries to improve the assignment by swapping assigned medoid points with non-medoid points until the energy of the entire system (which is measured by the sum of distances between medoid points and their assigned data points) is minimized.

a. Is K-medoid more robust to noise (or outliers) than the K-means algorithm? Explain your answer.

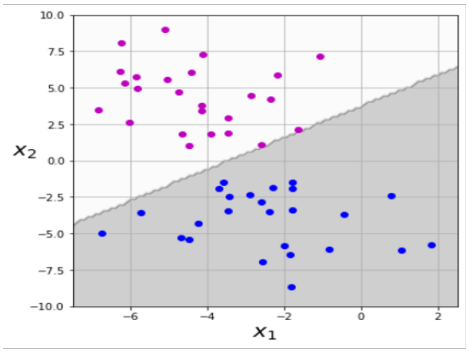
b. Prove that for the 1D case ($x \in \mathrm{R}^1$) of K-means, the centroid (μ) which minimizes the term $\sum_{i=1}^{m}(x_i - \mu)^2$ is the mean of m examples.

Bonus: Prove that the centroid (practically, the medioid) which minimizes the term $\sum_{i=1}^{m}|(x_i - \mu)|$ is the median of m examples given that μ belongs to the dataset.

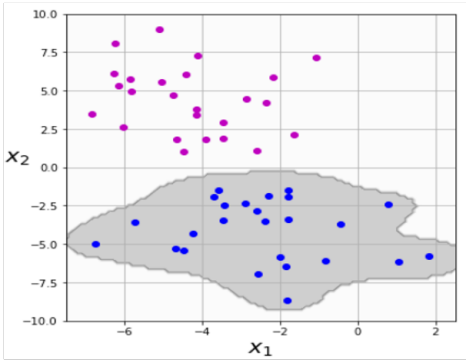
2 SVM (30%)

In the following figures you can see a visualization of SVM running with different settings (kernels and parameters) as follows:

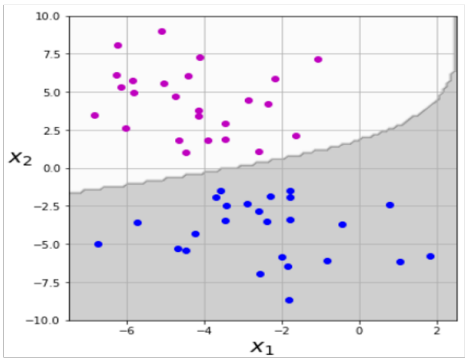
A



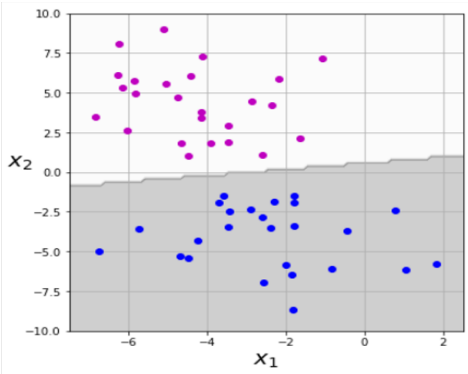
B



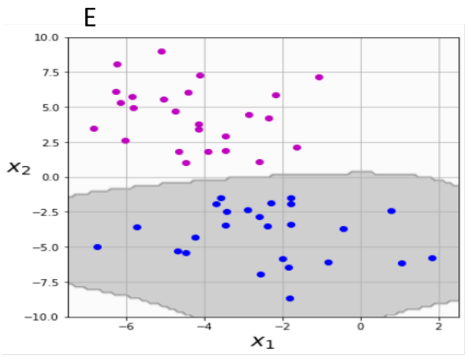
C



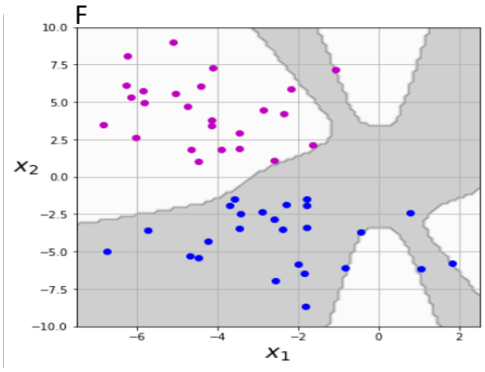
D



E



F



The settings that were used are as following:

1. Linear kernel with $C = 0.01$.
2. Linear kernel with $C = 1$.
3. 2^{nd} order polynomial kernel.

4. 10^{th} order polynomial kernel.

5. RBF kernel with $\gamma = 0.2$.

6. RBF kernel with $\gamma = 1$.

Match every image (labeled by a capital letter) to its' setting (number). Explain each of your answers.

Notice: An unexplained answer will not be marked even if it was correct!

3 Capability of generalization (20%)

Ockham's razor states that "Non sunt multiplicanda entia sine necessitate". This is known as the law of parsimony and its' translation to English is "Entities are not to be multiplied without necessity". This concept, which is attributed mostly to the English Franciscan friar William of Ockham, basically means that the simplest explanation is usually preferred.
A more modern variation of this concept (and a much more readable one) is attributed mostly to Albert Einstein and it states that "Everything should be made as simple as possible but not simpler". This balance is a major guideline in science in general and in data science in particular.
We saw in the tutorial two methods of choosing a parsimonious model for K-means. In GMM, there is another criterion to do so and it is known as "Akaike information criterion" (AIC). It is composed of two terms and defined as follows:

$$AIC = 2p - 2ln(\hat{L})$$

where p is the total number of learned parameters and $\hat{L}$ is the estimated likelihood given these parameters.

a. What is the scientific term of the balance that Einstein meant to in machine learning aspect?

b. How does each of the terms $(2p, 2ln(\hat{L}))$ in AIC affect the terms of the balance you defined in (a)?

c. What are the two options that are likely to happen if this balance was violated?

d. What are we aiming for with the AIC? Should it be high or low? Explain.

4 EigenFaces (40%)

The aim of PCA is to find a new orthornormal space where we can project our data onto it so the variance of the projected data is maximal. Thus, we get a new "coordinate system" where we can represent our data. The coordinate system is composed of the eigenvectors of the covariance matrix of the centered data. One of the most interesting projects done in this field is known as "EigenFaces",

first published by Sirovich and Kirby in 1987. In this work, many human faces images were collected, centered, flattened into vectors and stacked into a matrix X. The eigenvectors (u_i) of XX^T were extracted and once reshaped back to the image size (vector $\rightarrow$ matrix), the eigenfaces were revealed. These faces that looked like ghosts were the orthonormal basis for the construction of every human face image!
Let's say that we would like to recontruct your face out of the faces used to build the new coordinate system. If your face image (f) is an MXN matrix, then the flattened image is now a vector with MN elements. After cenentering and by using orthonormality, your flattened and centered image can be calculated as:

$$f = \sum_{i=1}^{MN} \langle f, u_i \rangle u_i$$

Due to the fact that our axes are actually *principal axes*, we can approximate your face image as:

$$f \approx \sum_{i=1}^{K} \langle f, u_i \rangle u_i = \sum_{i=1}^{K} c_i u_i$$

where $K << MN$. This approximation of course implies if the eigenvectors are **ordered** by their eignevalues i.e. u_1 has a larger eigenvalue than u_2 which has a larger eigenvalue than u_3 and so on and so forth. Once "corrected back" and reshaped as an MXN matrix, you should see your face. Notice that we treat every pixel as if it were a feature but because all of the pixels have the same range (grayscale values) then scaling is not needed.

Specific task:
In the attached Jupyter notebook you are asked to build the orthonormal basis using PCA and to reconstruct your own face image :). Please make sure that you have updated your `bm-336546` venv with `tutorial11.yml` or at least make sure that the package `pillow` appears when you activate your venv and then type `conda list`. If it is not there, simply type `pip install pillow`.

Further reading (not part of the task):
Eigenfaces are used for the task of face recognition. Each and every individual has his own set of $c_i's$ which are stacked in a vector C and are coupled with the ID. These vectors and the adequate ID's are saved in a database. Once a person needs to be identified by face recognition, his new face image is centered and then projected onto the same eigenvectors and new $c_i's$ are calculated. This new C vector is then run through the database and the Euclidiean metric is measured between this vector and any other C vector in the databse. The coupled ID of the C vector (of the the database) that results in the minimal Euclidiean distance is the ID of the tested person.