CS4341 HW 4

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1. Detective James is solving a case. The four suspects A, B, C and D made the following statements:
- **If A is guilty then B was an accomplice**
- **If B is guilty then either C was an accomplice or A is innocent**
- **If D is guilty then A is guilty and C is innocent**
- **If D is guilty then A is guilty**

Is D guilty based on these statements?

Assume being guilty is true.

If A is guilty then B was an accomplice: A=>B
If B is guilty then either C was an accomplice or A is innocent: B=>(C v ~A)
If D is guilty then A is guilty and C is innocent: D=>(A ^ ~C)
If D is guilty then A is guilty: D=> A

Goal: D

Convert to CNF:

A => B: ~A V B
B => (C V ~A): ~B V C V ~A
D => (A ^ ~C): (~D V A) ^ (~D V ~C)
D => A: ~D V A
D: D

Proof by resolution with refutation:

1	~A V B	
2	~B V C V ~A	
3	~D V A	
4	~D V ~C	
5	~D	negate of goal
6	~D V B	1, 3
7	~D V C V ~A	2, 6
8	~A V C	1, 2
9	~D V C	8, 9
10	~D V ~A	4, 7

From the proof, we can tell that we can't apply resolution any more, which means this can not be proved.

Truth table:

A	B	C	D	~A v B	~B v C v ~A	~D v (A ^ ~C)	~D v A	KB	Is D guilty
FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	no
FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	no
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	no
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	no
TRUE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	
TRUE	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	no
FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	
FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE	
TRUE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	
TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	
TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	TRUE	FALSE	

No. To make all 4 statements true, under no condition that can make D guilty.

2. If the Congress refuses to vote for new laws then strike would not be finished. Except for the case when it continues for more than a month and a company's CEO retires. The Congress refuses to operate and strike finishes. Therefore strike was going on for more than a month.

X = Congress refuses to vote for new laws
Y = strike would be finished
Z = strike continues for more than a month
M = a company's CEO retires
N = Congress refuses to operate

Goal: Z

If the Congress refuses to vote for new laws then strike would not be finished, except for the case when it continues for more than a month and a company's CEO retires:
$(X \wedge \sim(Z \wedge M)) \Rightarrow \sim Y$
The Congress refuses to operate and strike finishes: $X \wedge Y$

Convert to CNF:

$(X \wedge \sim(Z \wedge M)) \Rightarrow \sim Y$: $(\sim Y \vee \sim X \vee Z) \wedge (\sim Y \vee \sim X \vee M)$
$X \wedge Y$: $X \wedge Y$
Z: Z

Proof by resolution with refutation:

1	~X V ~Y V Z	
2	~X V ~Y V M	
3	X	
4	Y	
5	~Z	negate of goal
6	~Y V Z	1, 3
7	~Y	5, 6
8	FALSE	4, 7

Since we derive false by adding the negate of goal, we can tell that the statements implies strike was going on for more than a month.

Truth Table:

X	Y	Z	M	(~Y V ~X V Z) ^ (~Y V ~X V M)	X ^ Y	KB	Is Z true?
FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	
FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE	
FALSE	TRUE	FALSE	FALSE	TRUE	FALSE	FALSE	
FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	FALSE	
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	FALSE	
TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE	
TRUE	TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	
TRUE	TRUE	TRUE	FALSE	FALSE	TRUE	FALSE	
FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	
FALSE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE	
FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE	
FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	
TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	
TRUE	FALSE	TRUE	TRUE	TRUE	FALSE	FALSE	
TRUE	TRUE	FALSE	TRUE	FALSE	TRUE	FALSE	
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	yes

Yes.The strike was going on for more than a month.

3. If 2 is a prime number then 2 is the smallest prime number. If 2 is the smallest prime number then 1 is not a prime number. 1 is not a prime number. Are the following propositions correct based on the aforementioned statements?

- **2 is the smallest prime number**
- **2 is a prime number**

X = 2 is a prime number

Y = 2 is the smallest prime number

Z = 1 is not a prime number.

If 2 is a prime number, then 2 is the smallest prime number: X => Y

If 2 is the smallest prime number, then 1 is not a prime number: Y => Z

Goal:

2 is the smallest prime number: Y

2 is a prime number: X

Convert to CNF:

X => Y: ~X V Y

Y => Z: ~Y V Z

Y: Y

X: X

Proof by resolution with refutation for "2 is the smallest prime number"

1	~X V Y	
2	~Y V Z	
3	~Y	negate of goal
4	~X	1, 3
5	~X V Z	1, 2

From the proof, we can tell that we can't apply resolution any more, which means this can not be proved.

Proof by resolution with refutation for "2 is a prime number"

1	~X V Y	
2	~Y V Z	
3	~X	negate of goal
4	~X V Z	1, 2

From the proof, we can tell that we can't apply resolution any more, which means this can not be proved.

Truth table:

X	Y	Z	~X v Y	~Y v Z	KB	Is X true	Is Y true
FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	no	no
FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	no	no
FALSE	TRUE	FALSE	TRUE	FALSE	FALSE		
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	no	yes
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE		
TRUE	FALSE	TRUE	FALSE	TRUE	FALSE		
TRUE	TRUE	FALSE	TRUE	FALSE	FALSE		
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	yes	yes

No, both two statement cannot be inferred from aforementioned statements.

4. If tomorrow would be a snow storm then Patrick would wear a heavy coat. But Patrick would only wear heavy coat if zipper would be mended. It would be a snow storm tomorrow and the zipper is not mended. Would Patrick wear a heavy coat?

X = Patrick would wear a heavy coat
S = snow storm
Z = zipper would be mended

If tomorrow would be a snow storm then Patrick would wear a heavy coat. But Patrick would only wear heavy coat if zipper would be mended: (S ^ Z) => X
Patrick would only wear heavy coat if zipper would be mended: (~Z) => ~X
It would be a snow storm tomorrow and the zipper is not mended: S ^ ~Z

Goal: X

Convert to CNF:

(S ^ Z) => X: ~S V ~Z V X
(~Z) => ~X: Z V ~X
S ^ ~Z: S ^ ~Z
X: X

Proof by resolution with refutation:

1	X V ~S V ~Z	
2	~X V Z	
3	S	
4	~Z	
5	~X	negate of goal
6	X V ~Z	1, 3
7	~X V X = true	2, 6 on Z

From the proof, we can tell that we can't apply resolution any more, which means this can not be proved.

Truth table:

S	Z	X	~S v ~Z v X	Z v ~X	S ^ ~Z	KB	is X true
FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	
FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE	
FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE	
FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	
TRUE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	no

TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE	FALSE	
TRUE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	

No, based on the truth table, the statements cannot infer Patrick wear a heavy coat.

6. Either Sally and Bob are of the same age or Sally is older than Bob. If Sally and Bob are of the same age then Nancy and Bob are not of the same age. If Sally is older than Bob then Bob is older than Walter. Do those statements imply that either Nancy and Bob are not of the same age or Bob is older than Walter?

X = Sally and Bob are of the same age
Y = Sally is older than Bob
Z = Nancy and Bob are not of the same age
M = Bob is older than Walter

Either Sally and Bob are of the same age or Sally is older than Bob: X v Y
If Sally and Bob are of the same age then Nancy and Bob are not of the same age: X => Z
If Sally is older than Bob then Bob is older than Walter: Y => M

Goal: Z V M

Convert to CNF:

X V Y: X v Y
X => Z: ~X V Z
Y => M: ~Y V M
Z V M: Z V M

Proof by resolution with refutation:

Negate of goal is ~(Z V M) which is ~Z ^ ~M.

1	X V Y	
2	~X V Z	
3	~Y V M	
4	~Z	negate of goal
5	~M	negate of goal
6	~X	2, 4
7	~Y	3, 5
8	X	1, 7
9	FALSE	6, 8

Since we derive false by adding the negate of goal, we can tell that the statements implies that either Nancy and Bob are not of the same age or Bob is older than Walter.

Truth table:

X	Y	Z	M	X v Y	~X v Z	~Y v M	KB	Z v M?
FALSE	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE
FALSE	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	TRUE
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	FALSE	FALSE
FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE	TRUE
TRUE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	FALSE	FALSE
TRUE	FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	FALSE	TRUE	FALSE	FALSE	FALSE	FALSE
TRUE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	FALSE	TRUE
FALSE	FALSE	FALSE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE
FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE
FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	FALSE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE
TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
TRUE	TRUE	FALSE	TRUE	TRUE	FALSE	TRUE	FALSE	TRUE
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

Yes. Based on truth table, those statements imply that either Nancy and Bob are not of the same age or Bob is older than Walter.