Ambient Occlusion in 4D

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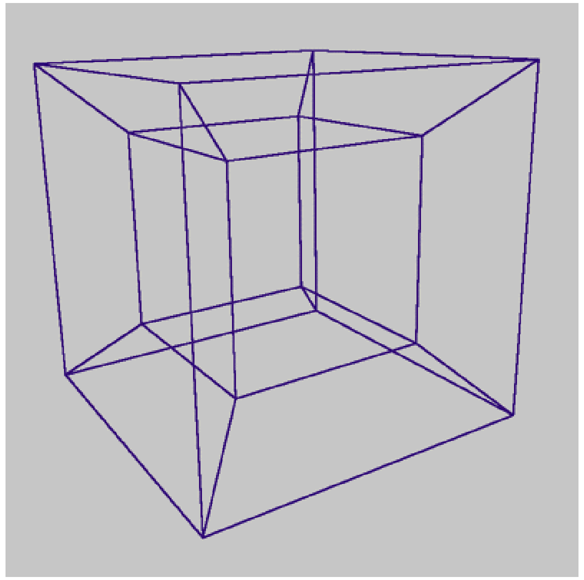
Outline

- Introduction
- Methods
- Implementation
- Results & Discussion
- Conclusion

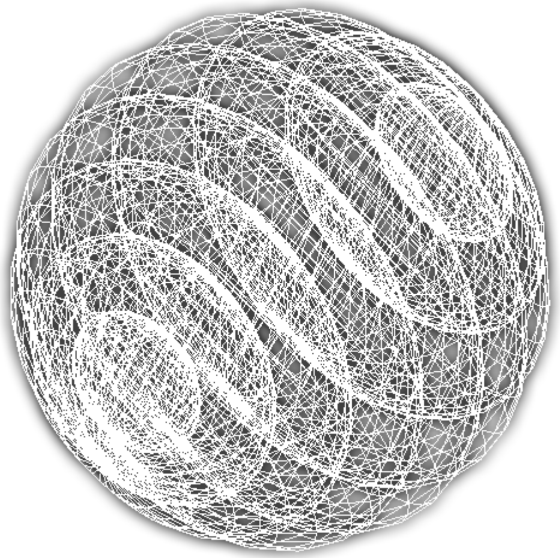
Introduction

Motivation:

- 4D structures are difficult to perceive
- Improve perception of 4D structures using illumination



4-cube represented as a 2D wireframe image



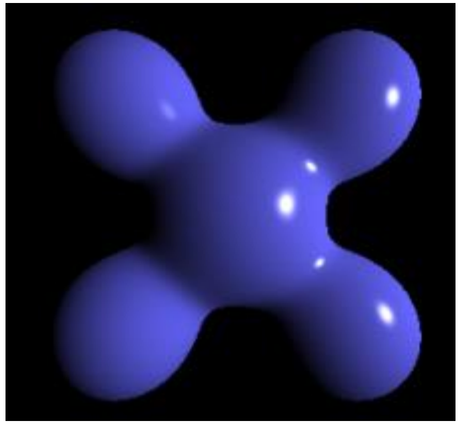
Direct projection of hypersphere in 4D into 3D space, rendered as a 2D image

Source: HOLLASCH, Steven R., 1991. Four-Space Visualization of 4D Objects. MA thesis. Arizona State University; Antipov, Eugene. (7 September 2006). 2D-projection of 3D-projection of Hypersphere of 4D-space. Retrieved July 2, 2021 from https://en.wikipedia.org/wiki/File:Hypersphere.png

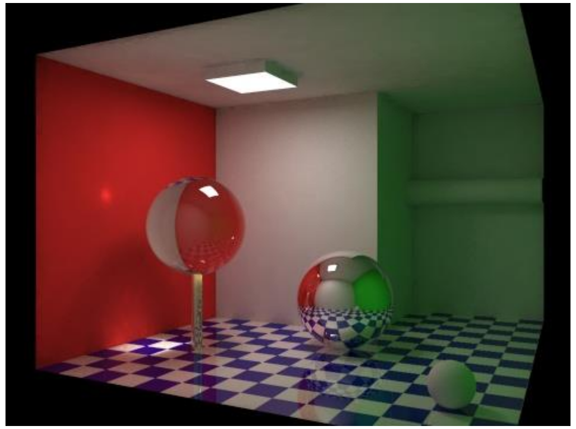
Introduction

Motivation:

- existing illumination methods in 3D visualization:
  - Phong shading
  - global illumination
    - ambient occlusion, approximation of global illumination



Phong shading



global illumination

ambient occlusion

Image source: Smith, Brad. (7 August 2006). Illustration of the Phong reflection model. Retrieved July 2, 2021 from https://commons.wikimedia.org/wiki/File:Phong_components_version_4.png, Barahag. (24 March 2020). 3 spheres in the small room. Retrieved July 2, 2021 from https://en.wikipedia.org/wiki/File:Global_illumination1.png, MITTRING, Martin, 2007. Finding next Gen: CryEngine 2. In: ACM SIGGRAPH 2007 Courses. San Diego, California: Association for Computing Machinery. SIGGRAPH 07. Available from doi: 10.1145/1281500.1281671.

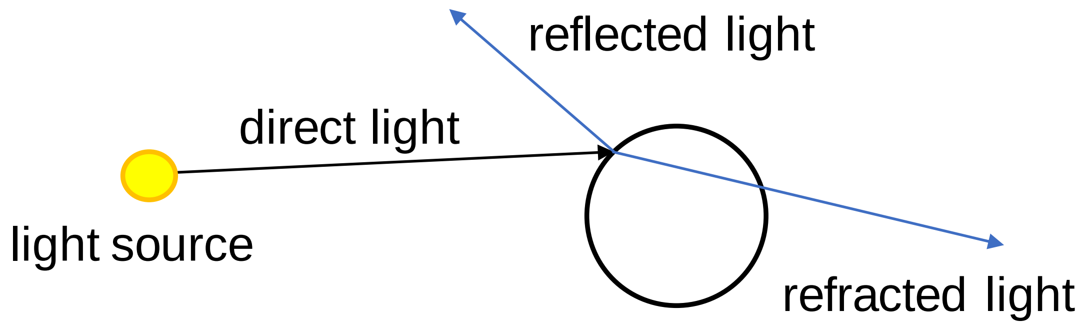
Introduction

Problem:

expand ambient occlusion method to 4D space

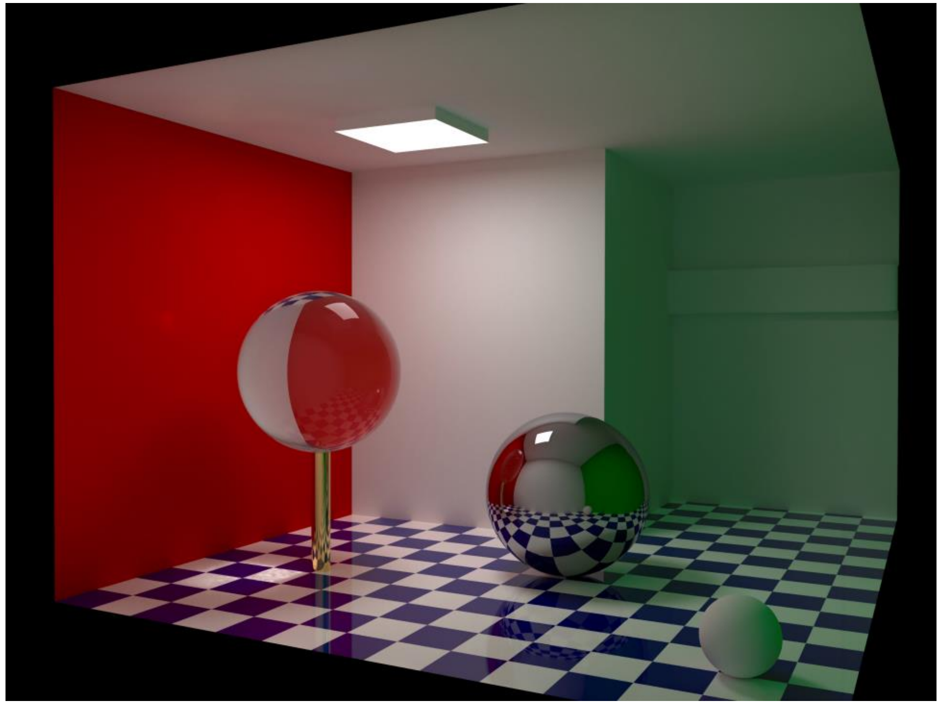
Global Illumination

- Shading technique, uses both direct and indirect light
- Photorealistic results
- Global illumination takes the entire scene into consideration
- Direct light: light coming directly from a light source
- Indirect light: reflected/refracted rays
- Calculated recursively, expensive to calculate

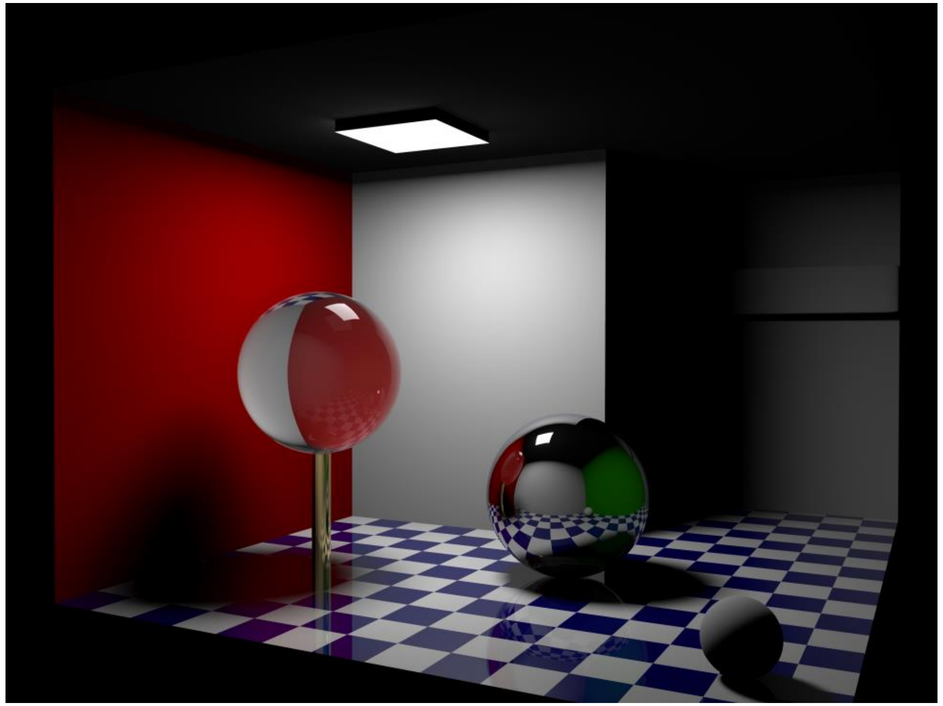


Global Illumination

Example of global illumination:



A scene with global illumination



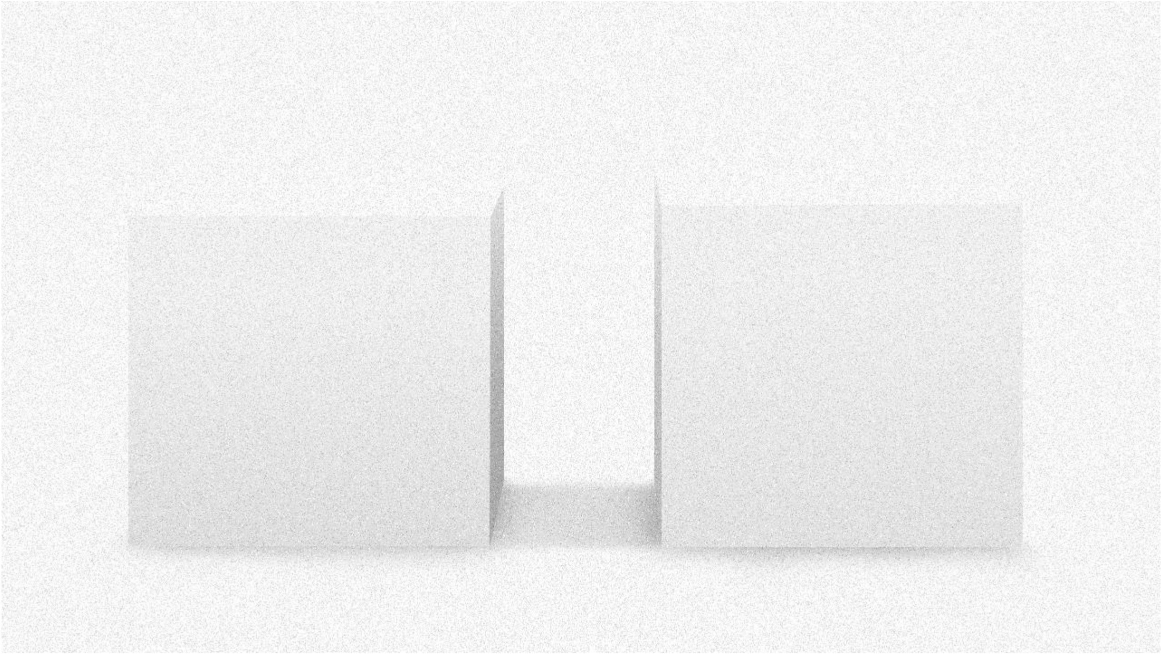
A scene without global illumination

Image source: Barahag. (24 March 2020). Three spheres lit only by direct lighting algorithms. Retrieved July 2, 2021 from https://en.wikipedia.org/wiki/File:Direct_lighting.png, Barahag. (24 March 2020). 3 spheres in the small room. Retrieved July 2, 2021 fromhttps://en.wikipedia.org/wiki/File:Global_illumination1.png

Ambient Occlusion

- Approximates global illumination, also takes the entire scene into consideration
- Shading on surfaces independent of light sources
- Examples of ambient occlusion maps:

Ambient occlusion of a scene from *Crysis* using *CryEngine 2*

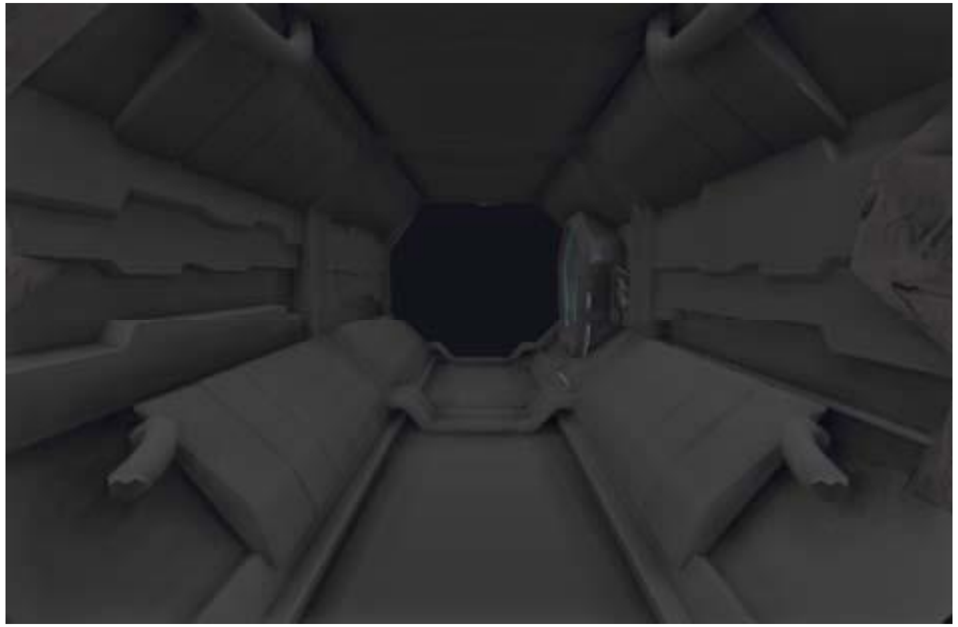


A simple scene of 2 cubes on a plane with ambient occlusion

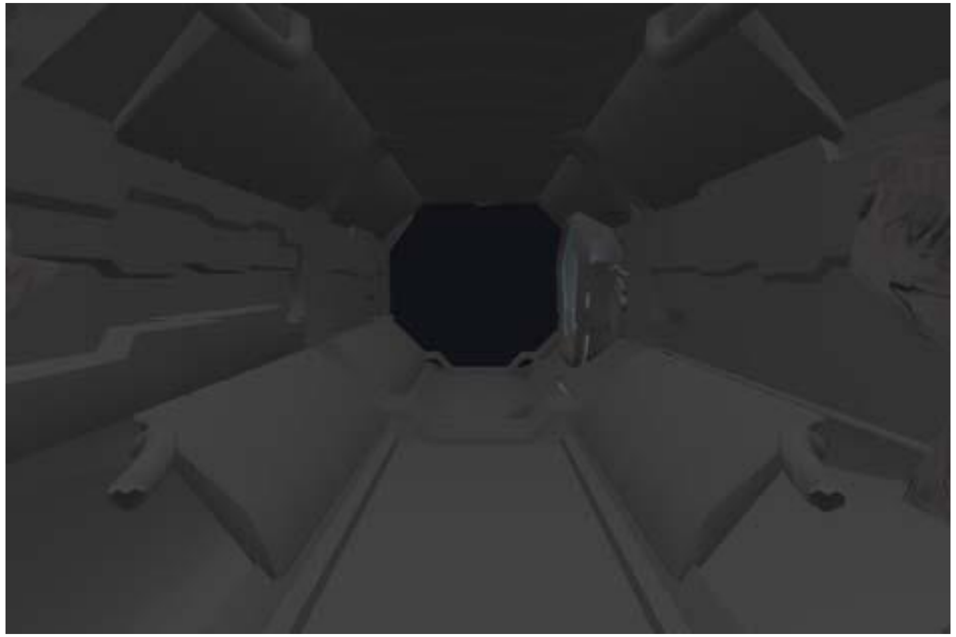
Image source: MITTRING, Martin, 2007. Finding next Gen: CryEngine 2. In: ACM SIGGRAPH 2007 Courses. San Diego, California: Association for Computing Machinery. SIGGRAPH 07. Available from doi: 10.1145/1281500.1281671.

Ambient Occlusion

- Ambient occlusion maps calculated separately from the scene
- Scene combined with ambient occlusion afterwards



A scene with ambient occlusion



A scene without ambient occlusion

Image source: MITTRING, Martin, 2007. Finding next Gen: CryEngine 2. In: ACM SIGGRAPH 2007 Courses. San Diego, California: Association for Computing Machinery. SIGGRAPH 07. Available from doi: 10.1145/1281500.1281671.

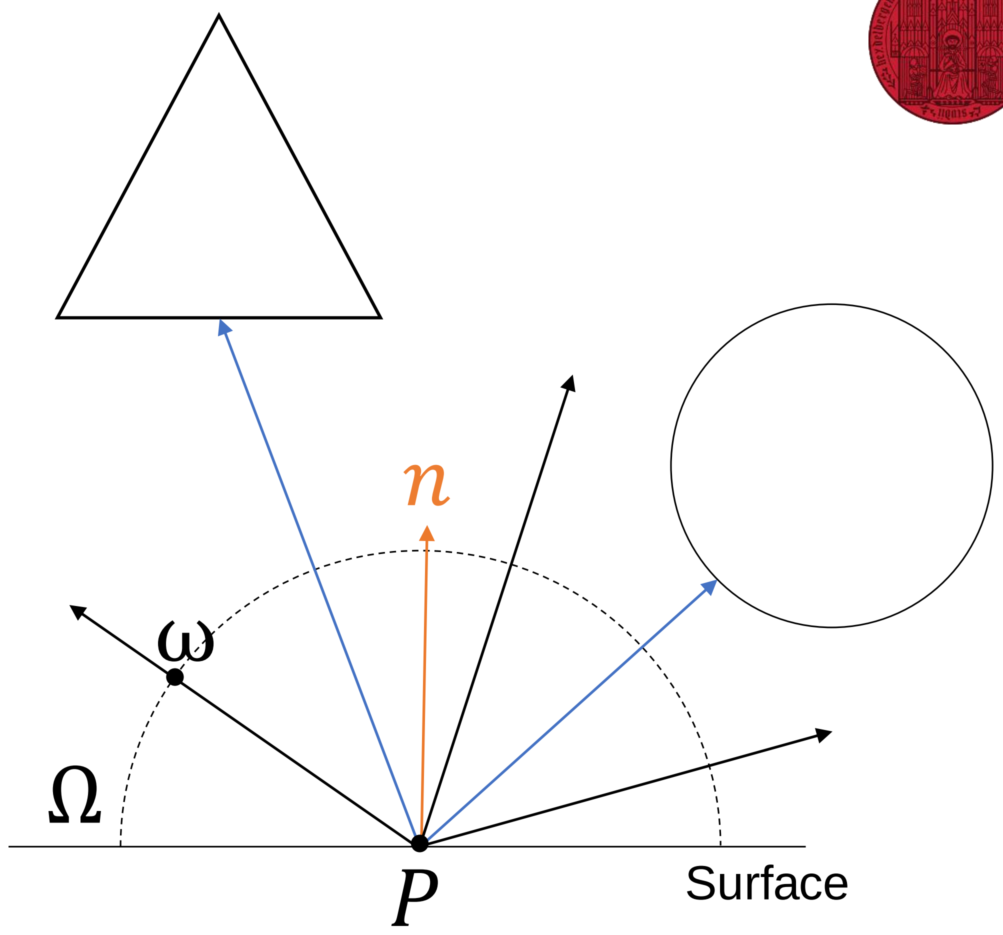
Variations of Ambient Occlusion

- Screen Space Ambient Occlusion (SSAO):
  uses only currently visible scene and depth buffer to calculate occlusion
- Ray-traced Ambient Occlusion:
  uses ray tracing to calculate occlusion

Ambient Occlusion (AO)

$$A(p, n) = \frac{1}{\pi} \int_{w \in \Omega} V(p, w) |w \cdot n| dw$$

p = *point in scene*
n = *surface normal*
Ω = hemisphere
$V(p, \omega)$ = visibility function, returns 1 if there's a ray-object intersection for ray from p to ω, otherwise 0



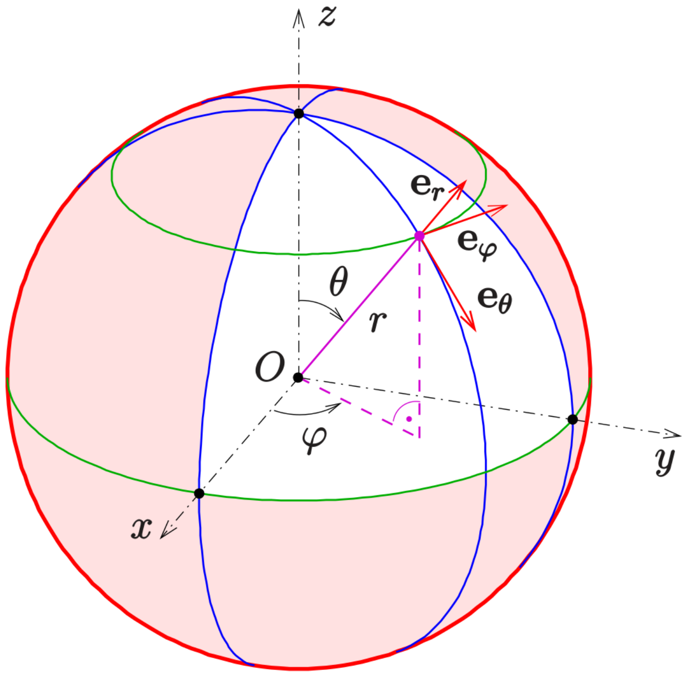
- Ambient occlusion calculates the average of the visibility function
- Practically, AO is calculated using hemispheric sampling

Hemispheric Sampling

- Create a hemisphere centered on a point p
- Shoot rays to random points on the hemisphere
- Points on the hemisphere calculated using spherical coordinates

Point $P = (x, y, z)$ on a sphere with center point c and radius r can be defined as

$$x = c + r\cos\varphi\sin\theta$$
$$y = c + r\sin\varphi\sin\theta$$
$$z = c + r\cos\theta$$

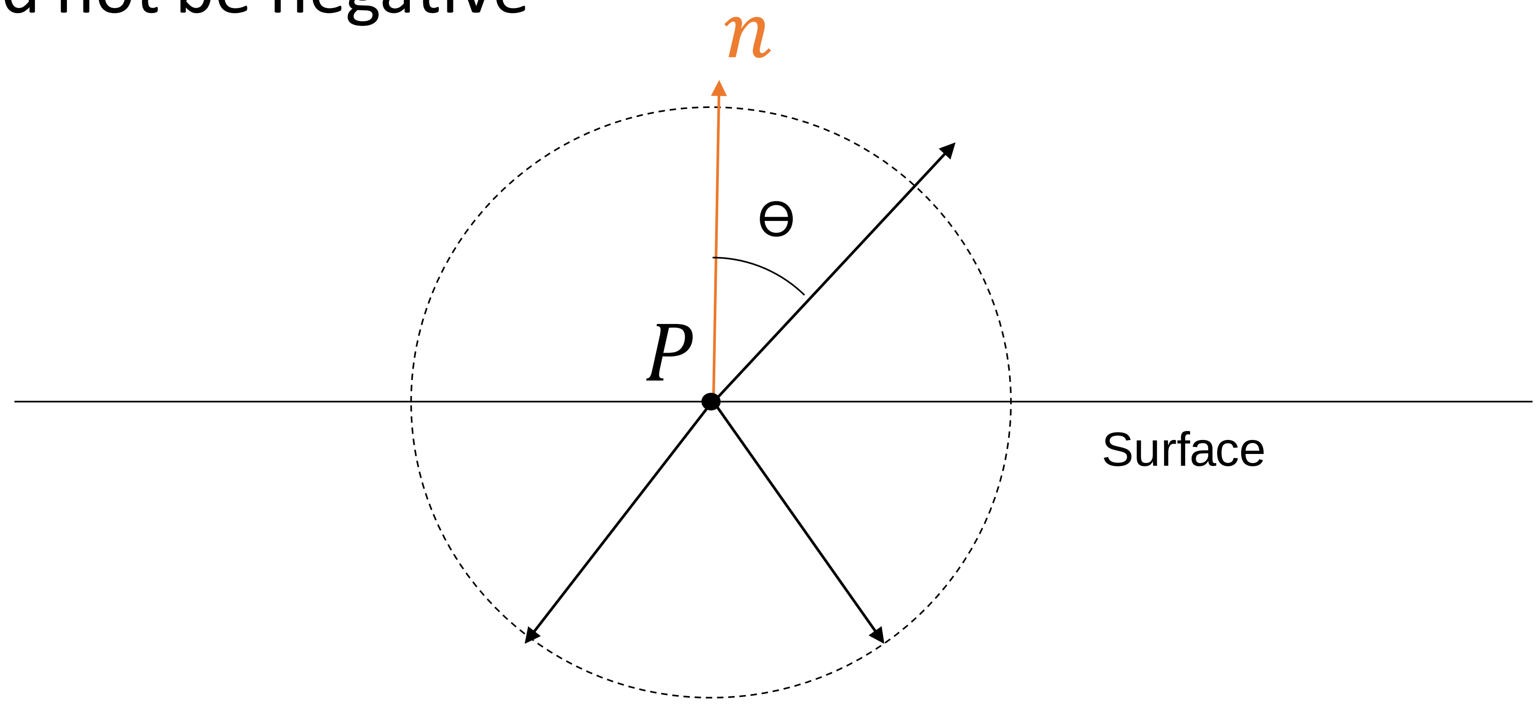


Spherical coordinates in 3D

Image source: Ag2gaeh. (3 October 2015). Kugelkoordinaten: lokale Basis. Retrieved July 2, 2021 from https://commons.wikimedia.org/wiki/File:Kugelkoord-lokale-Basis-s.svg

Hemispheric Sampling

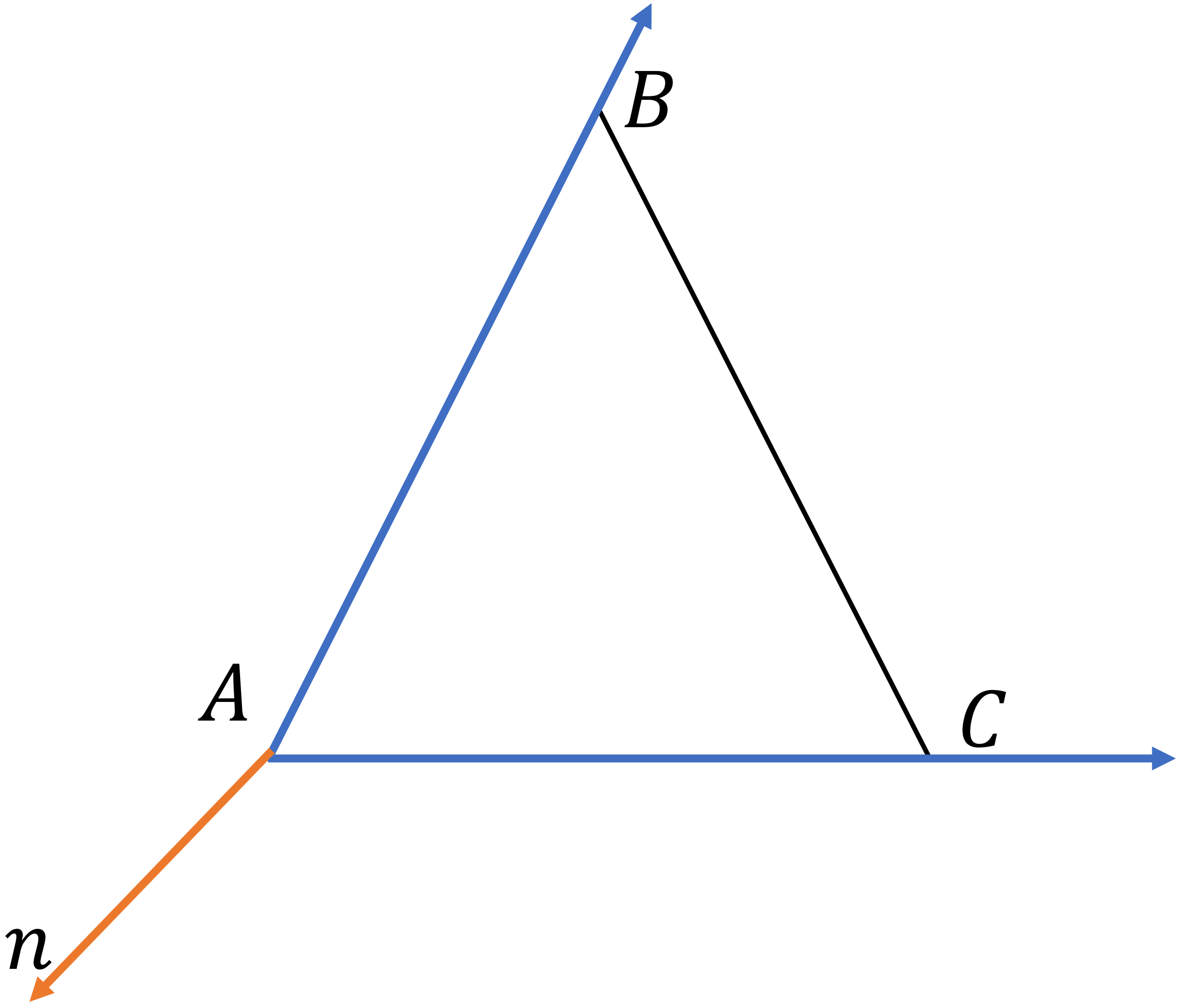
- Problem: Using spherical coordinates we get points on a sphere, not hemisphere. Sampled rays may point under the surface
- Solution: Ray must point to the same direction as surface normal, angle between ray and surface normal cannot be more than 90⁰ → cross product of ray and surface normal should not be negative



Finding Surface Normal

In 3D: find surface normal using cross product of two sides of a triangle

$$n = \frac{(B-A)\times(C-A)}{[(B-A)\times(C-A)]}$$



Ray-Object Intersection in 3D

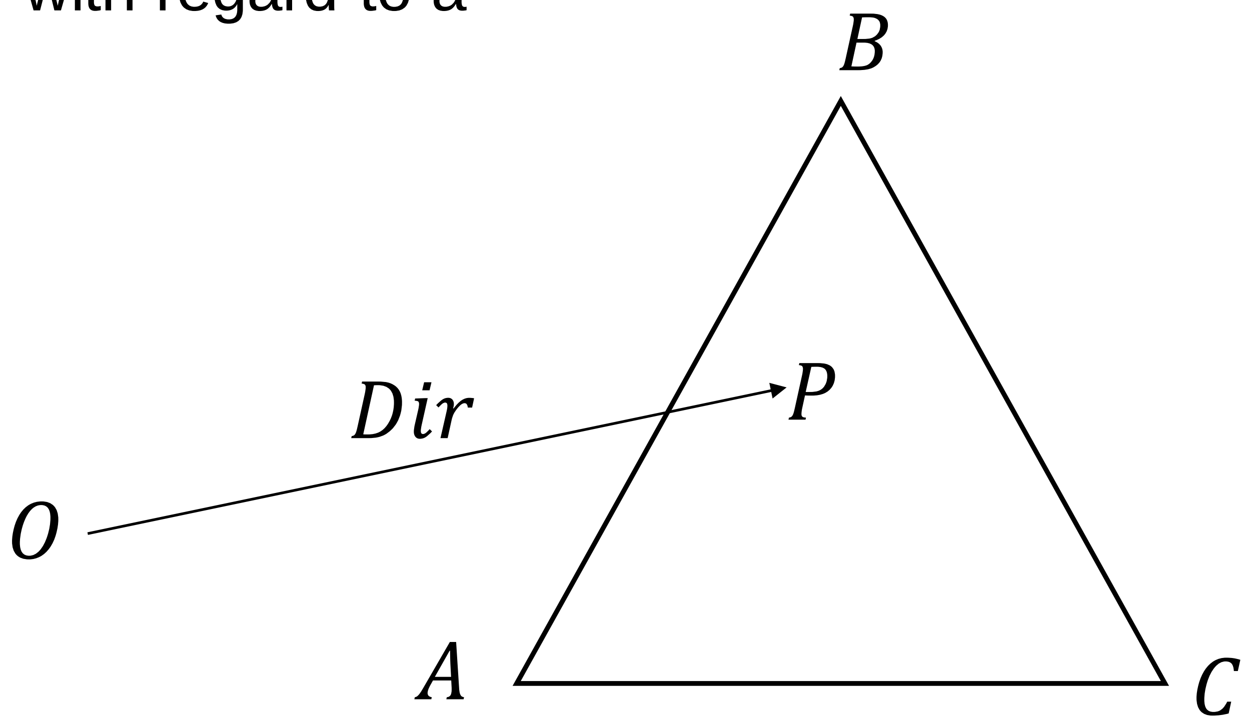
- Ray-object intersection is solved using ray-triangle intersection
- Ray-triangle intersection calculated using barycentric coordinates

$\lambda_{1,}\, \lambda_2,\, \lambda_3$ barycentric coordinates for a point P with regard to a triangle ABC

$$P = \lambda_1 A + \lambda_2 B + \lambda_3 C$$
$$\text{with } \lambda_1 + \lambda_2 + \lambda_3 = 1$$

Barycentric coordinates is calculated using definition of a point on a ray with origin O and direction Dir:

$$P = O + tDir$$



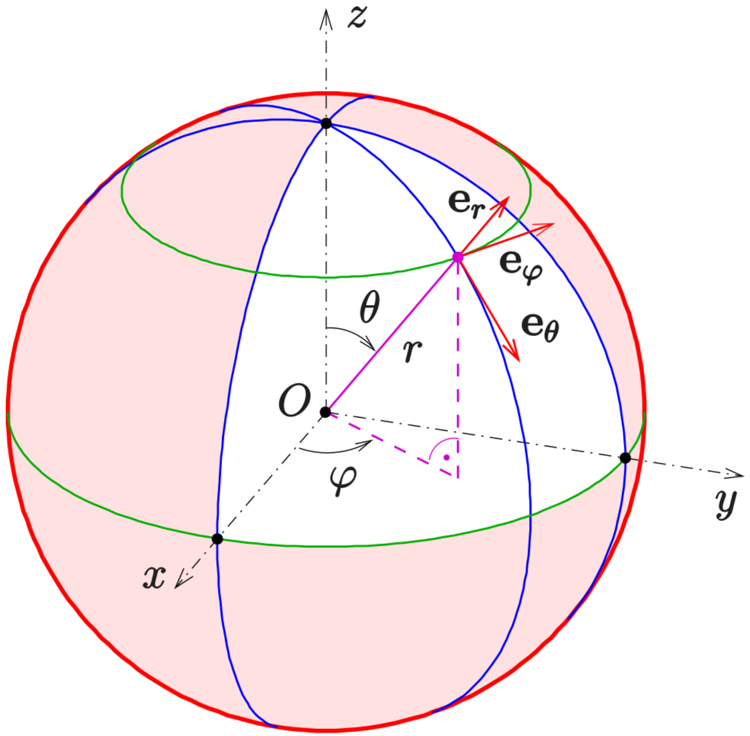
Ambient Occlusion in 4D

Subproblems:

- Hemispheric Sampling using hypersphere
- Ray-Object Intersection in 4D

Hemispheric Sampling in 4D

Generalized spherical coordinates



Spherical coordinates in 3D

$$p = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} r\cos(\phi_1) \\ r\sin(\phi_1)\cos(\phi_2) \\ r\sin(\phi_1)\sin(\phi_2)\cos(\phi_3) \\ \vdots \\ r\sin(\phi_1)\ldots\sin(\phi_{n-2})\cos(\phi_{n-1}) \\ r\sin(\phi_1)\ldots\sin(\phi_{n-2})\sin(\phi_{n-1}) \end{bmatrix}$$

Image source: Ag2gaeh. (3 October 2015). Kugelkoordinaten: lokale Basis. Retrieved July 2, 2021 from https://commons.wikimedia.org/wiki/File:Kugelkoord-lokale-Basis-s.svg

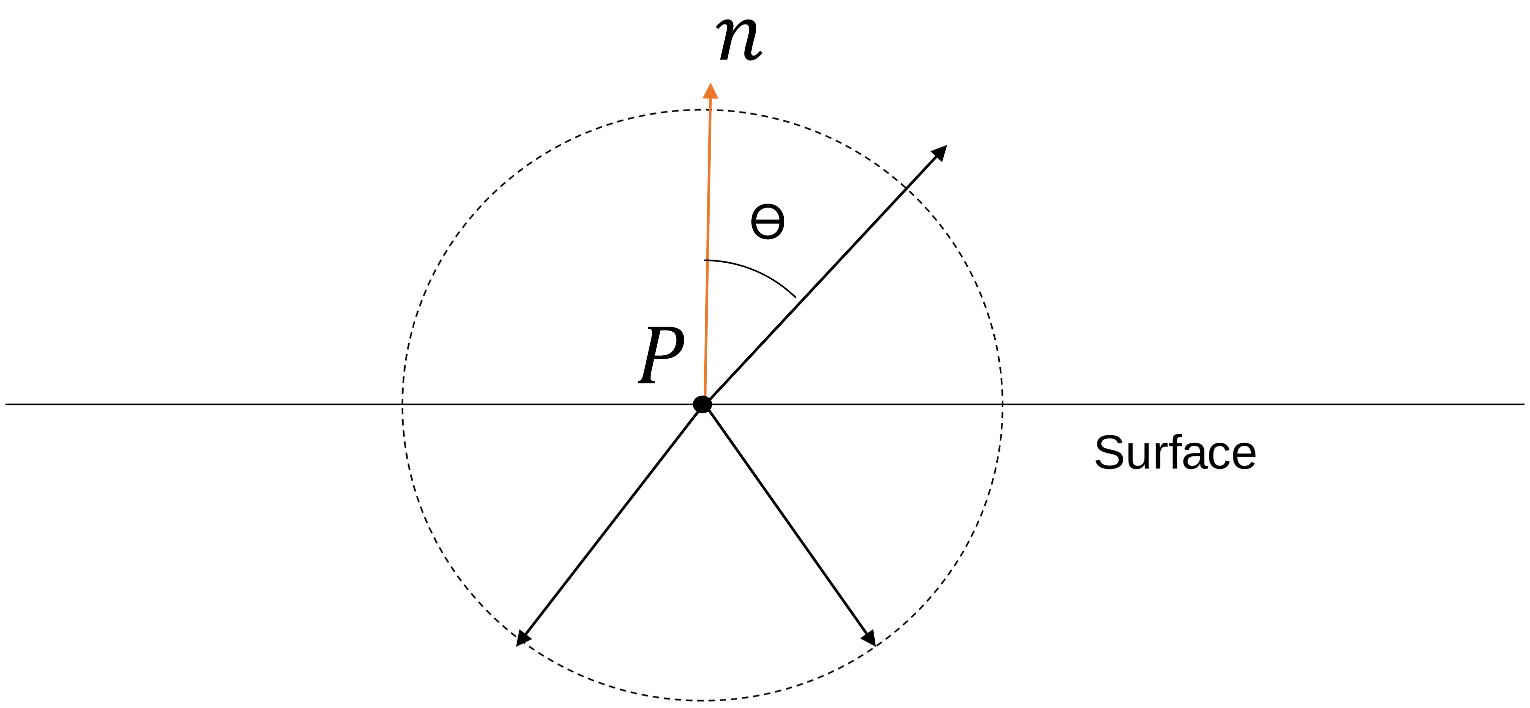
Hemispheric Sampling in 4D

Spherical coordinates for $n = 4$, center point c

$$p = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} c + r\cos(\phi_1) \\ c + r\sin(\phi_1)\cos(\phi_2) \\ c + r\sin(\phi_1)\sin(\phi_2)\cos(\phi_3) \\ c + r\sin(\phi_1)\sin(\phi_2)\sin(\phi_3) \end{bmatrix}$$

Hemispheric Sampling in 4D

- The same problem as in 3D, ray must point to the same direction as surface normal, angle between ray and surface normal cannot be more than 90^o → cross product of ray and surface normal should not be negative



Finding Surface Normal

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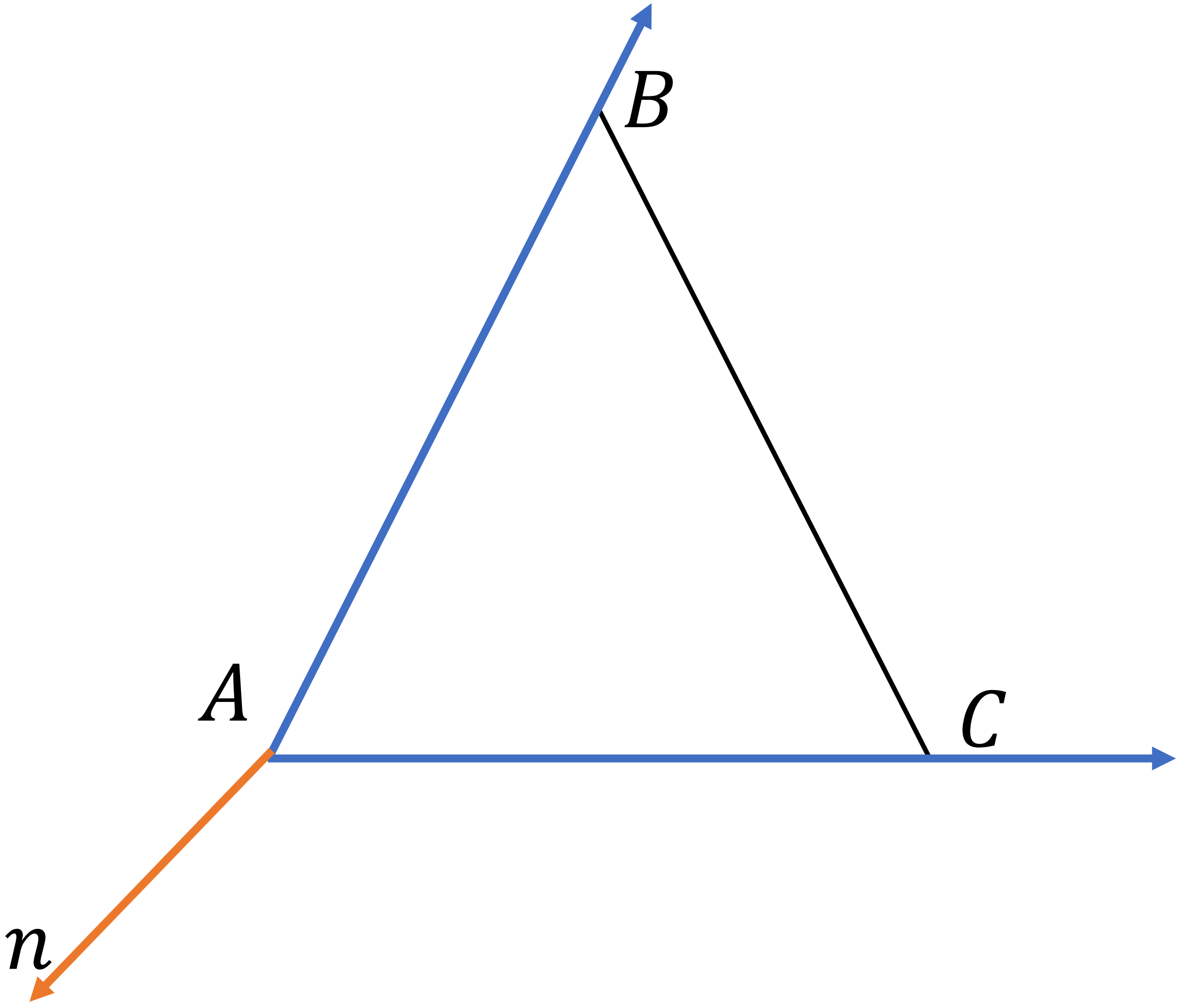
- In 3D: find surface normal using cross product of two sides of a triangle
- In 4D: cross product not defined



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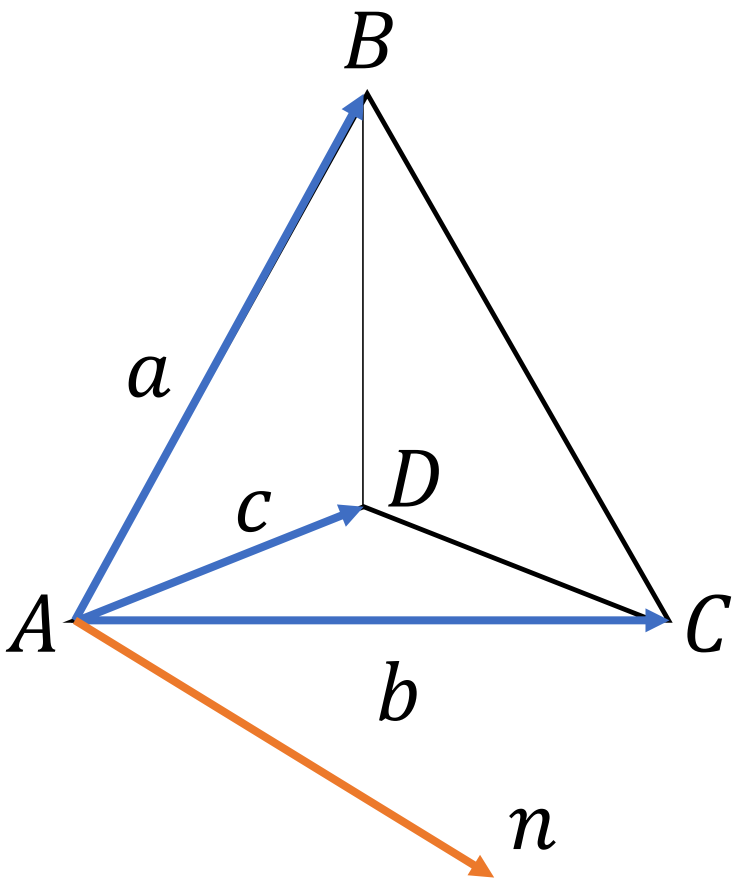
Finding Surface Normal

- Finding surface normal: Finding an orthogonal vector
- In 4D possible to find a vector that is orthogonal to 3 other vectors
- 3 vectors selected from 3 sides of the tetrahedron



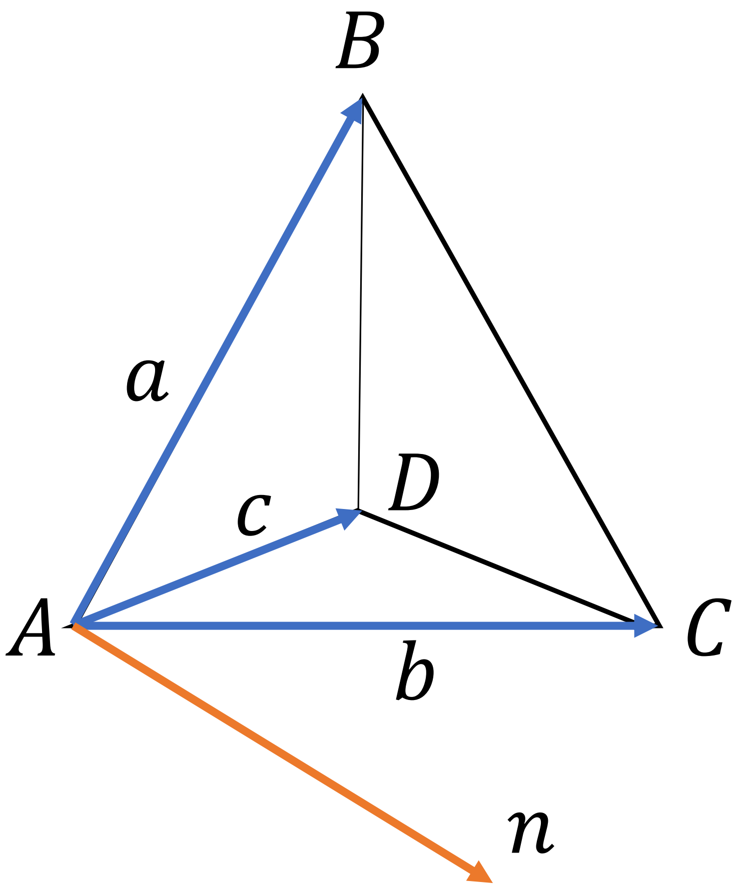
Finding Surface Normal

Use formal determinant to find orthogonal vector

$$a \times b \times c = \begin{vmatrix} x & y & z & w \\ a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \end{vmatrix}, n = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$x = \det \begin{bmatrix} \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{a}_4 \\ \boldsymbol{b}_2 & \boldsymbol{b}_3 & \boldsymbol{b}_4 \\ \boldsymbol{c}_2 & \boldsymbol{c}_3 & \boldsymbol{c}_4 \end{bmatrix} \quad y = \det \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_3 & \boldsymbol{a}_4 \\ \boldsymbol{b}_1 & \boldsymbol{b}_3 & \boldsymbol{b}_4 \\ \boldsymbol{c}_1 & \boldsymbol{c}_3 & \boldsymbol{c}_4 \end{bmatrix}$$

$$z = \det \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_4 \\ \boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_4 \\ \boldsymbol{c}_1 & \boldsymbol{c}_2 & \boldsymbol{c}_4 \end{bmatrix} \quad w = \det \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 \\ \boldsymbol{b}_1 & \boldsymbol{b}_2 & \boldsymbol{b}_3 \\ \boldsymbol{c}_1 & \boldsymbol{c}_2 & \boldsymbol{c}_3 \end{bmatrix}$$



Review: Ray-Object Intersection in 3D

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- Ray-triangle intersection calculated using barycentric coordinates

$\lambda_1, \lambda_2, \lambda_3$ barycentric coordinates for a point P with regard to a triangle ABC

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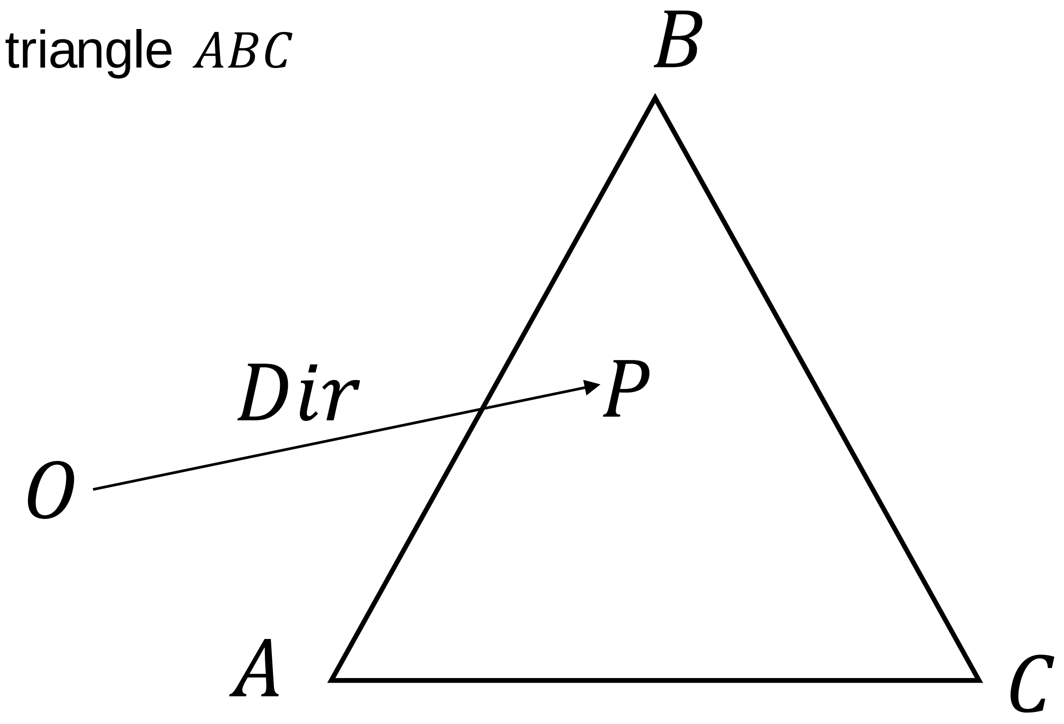
$$\text{with } \lambda_1 + \lambda_2 + \lambda_3 = 1$$



Barycentric coordinates are solved using definition of a point on a ray with origin O and direction Dir:

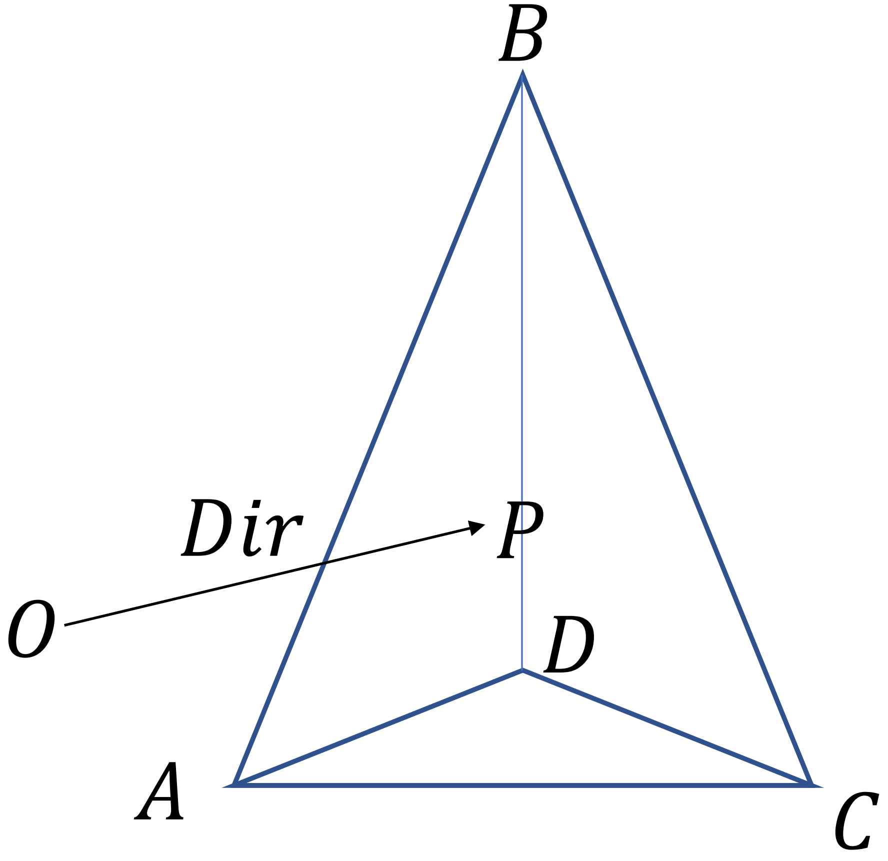
$$P = O + tDir$$

Ray-Object Intersection in 4D

- Same principles like in 3D but with a 3-simplex (tetrahedron)
- Use barycentric coordinate with tetrahedron to find ray intersection point

Barycentric coordinate with 3-simplex (tetrahedron)

$$P = \lambda_1 A + \lambda_2 B + \lambda_3 C + \lambda_4 D$$
$$\text{with } \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$$



Barycentric coordinates in 4D can also be solved using definition of a point on a ray with origin O and direction Dir

$$P = O + tDir$$

Ray-Object Intersection in 4D

Using the barycentric coordinate with 3-simplex (tetrahedron) and definition of point on a ray, we get the following linear system

$$
\begin{aligned}
O + tDir &= \lambda_1 A + \lambda_2 B + \lambda_3 C + \lambda_4 D \\
&= A(1 - \lambda_2 - \lambda_3 - \lambda_4) + \lambda_2 B + \lambda_3 C + \lambda_4 D \\
&= A + \lambda_2 (B - A) + \lambda_3 (C - A) + \lambda_4 (D - A)
\end{aligned}
$$

$$O - A = tDir + \lambda_2 (B - A) + \lambda_3 (C - A) + \lambda_4 (D - A)$$

Ray-Object Intersection in 4D

Linear system solved using Cramer's rule (using determinants)

Cramer's Rule

for $a = (a1, a2)$, $b = (b1, b2)$,
$c = (c1, c2)$ x and y can be calculated as follows:

$$ax + by = c$$

$$x = \det(c\ b)\ /\det(a\ b)$$
$$y = \det(a\ c)\ /\det(a\ b)$$

Ray-Object Intersection in 4D

Solution of the linear system:

$$t = \frac{\det((O-A) \quad (B-A) \quad (C-A))}{\det(Dir \quad (B-A) \quad (C-A) \quad (D-A))}$$

$$\lambda_2 = \frac{\det(Dir \quad (O-A) \quad (C-A))}{\det(Dir \quad (B-A) \quad (C-A) \quad (D-A))}$$

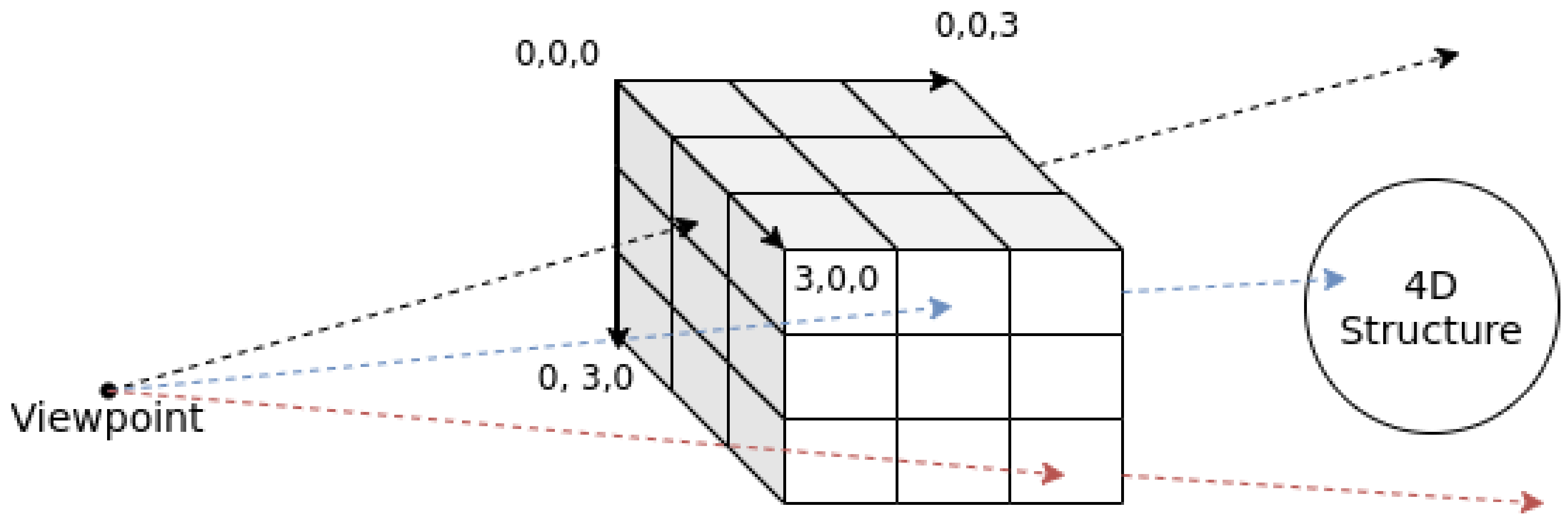
$$\lambda_3 = \frac{\det(Dir \quad (B-A) \quad (O-A))}{\det(Dir \quad (B-A) \quad (C-A) \quad (D-A))}$$

$$\lambda_4 = \frac{\det(Dir \quad (B-A) \quad (C-A) \quad (O-A))}{\det(Dir \quad (B-A) \quad (C-A) \quad (D-A))}$$

Implementation

C++, OpenCL for parallel computing

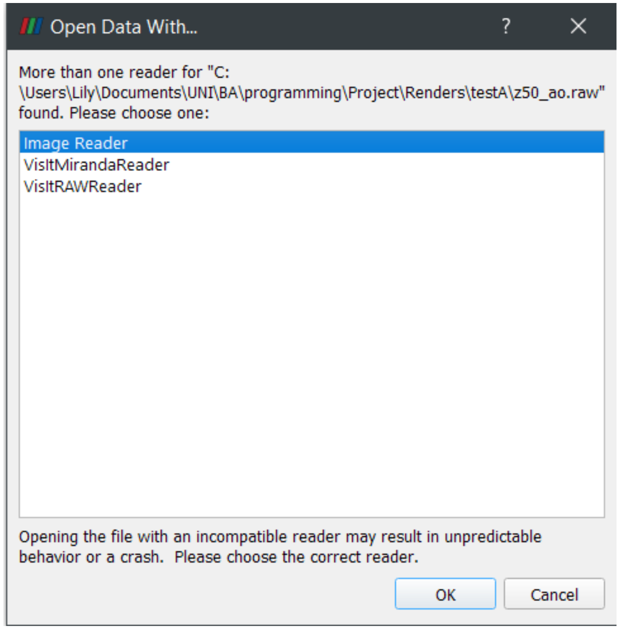
Experimental Result

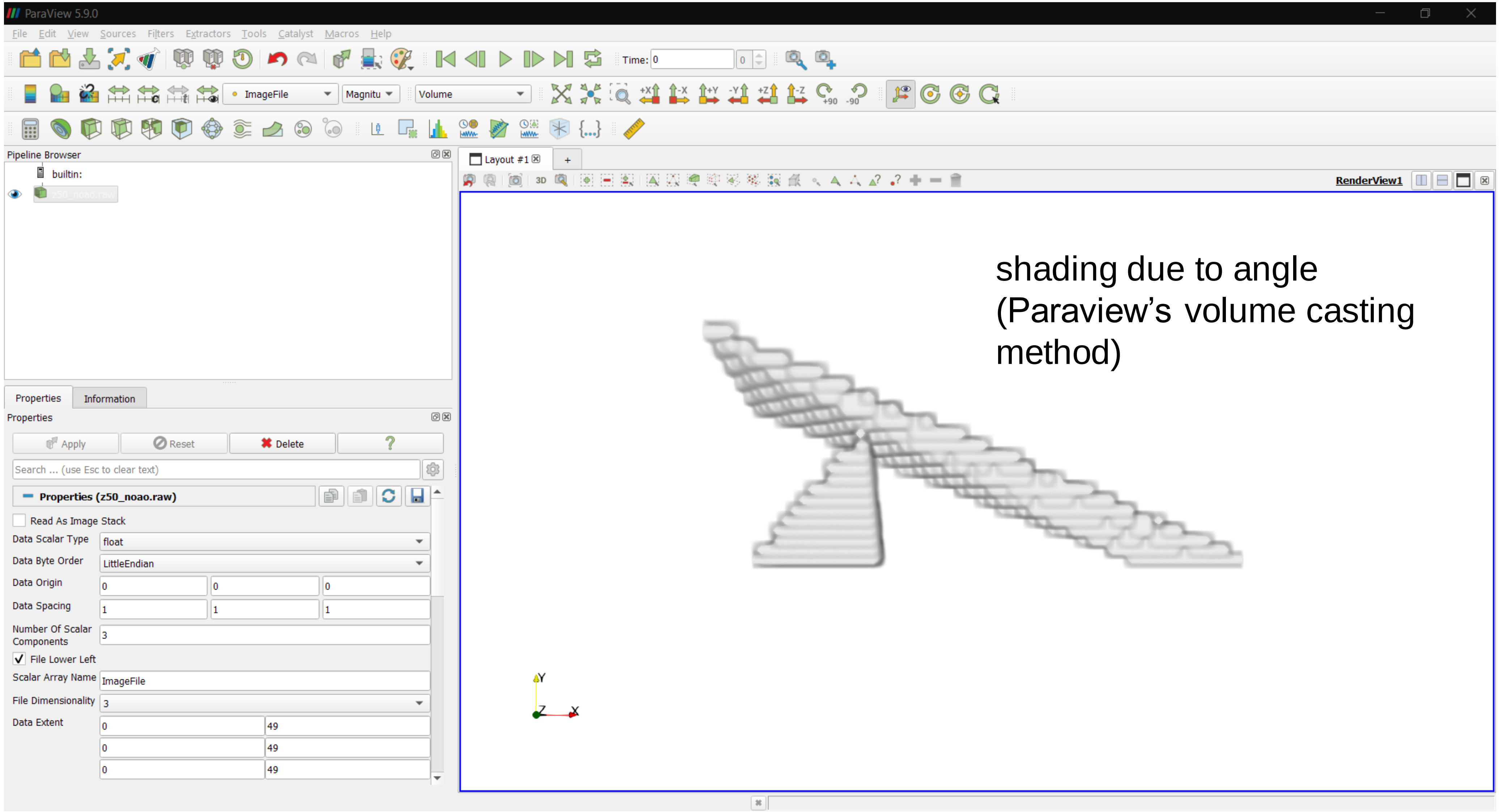


4D scenes projected to 3D image space (50 x 50 x 50)

Experimental Result

Results rendered in ParaView using RAW Image Reader





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Experimental Result

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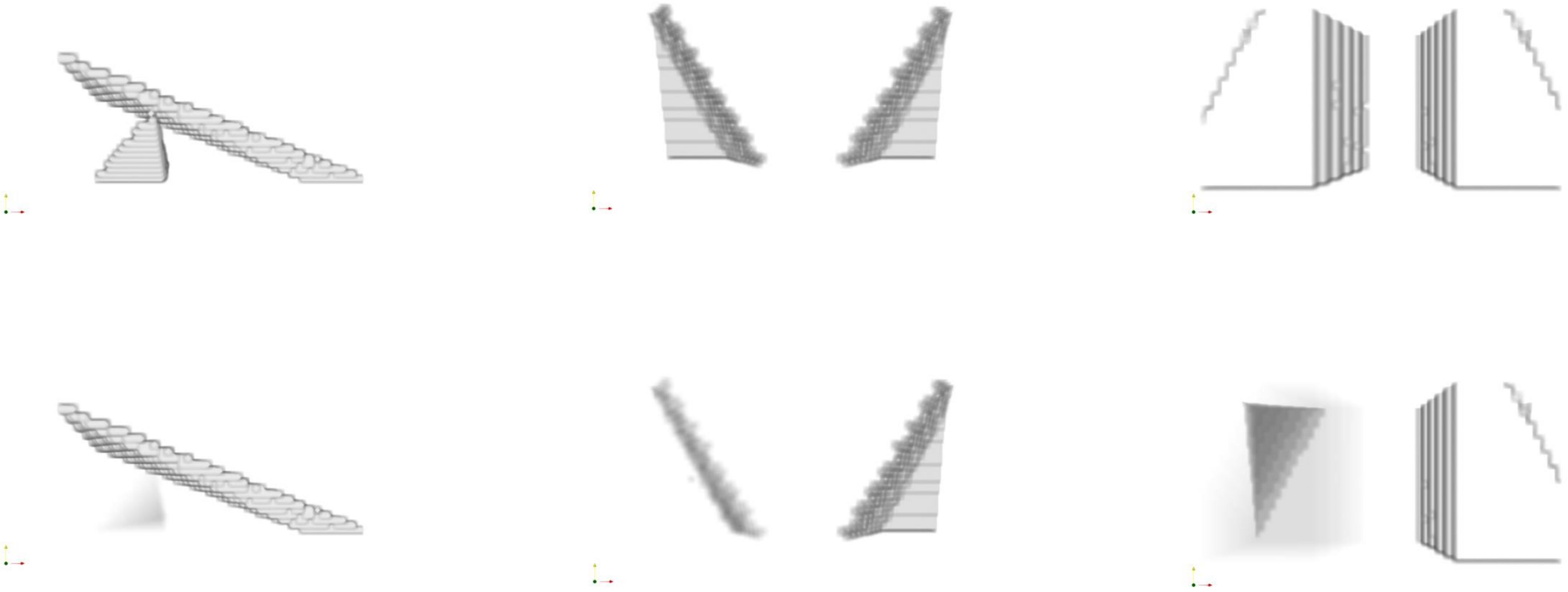
without ambient occlusion

with ambient occlusion



2 tetrahedra

2 structure out of 2 tetrahedra

Cube-like structures with an indent

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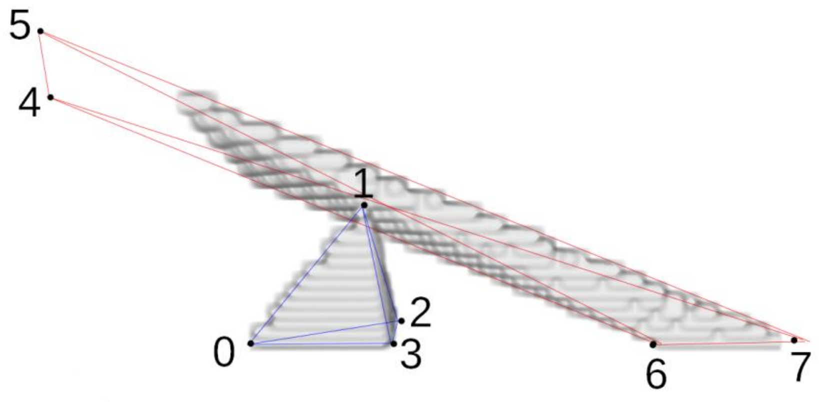
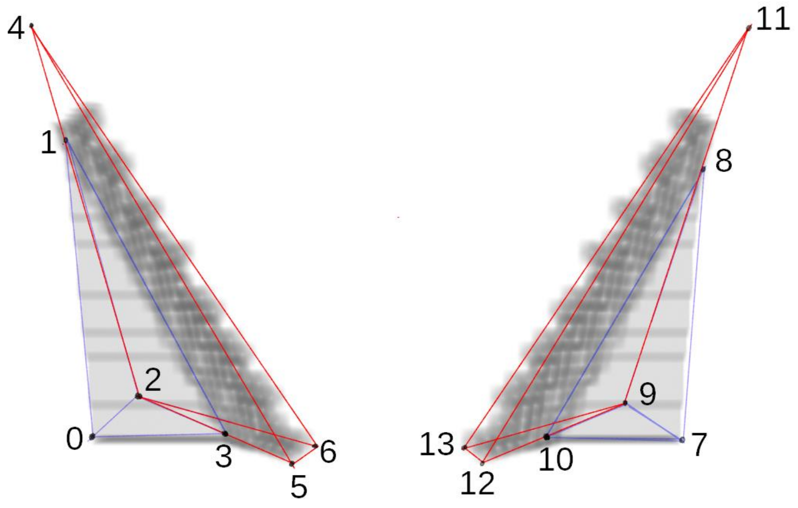
Unexpected Result in Our Implementation

- Entirely occluded tetrahedra, values either 0 or 1.
- Left object always affected differently than the right

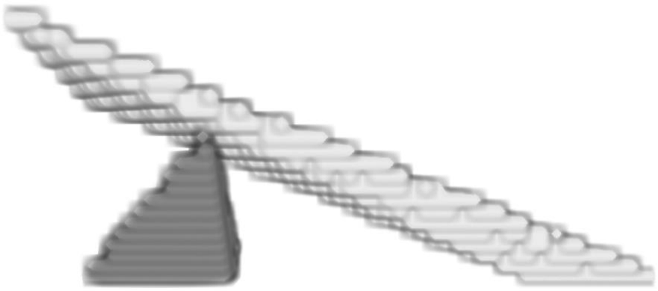
Theoretical Result

without ambient occlusion, marked vertices





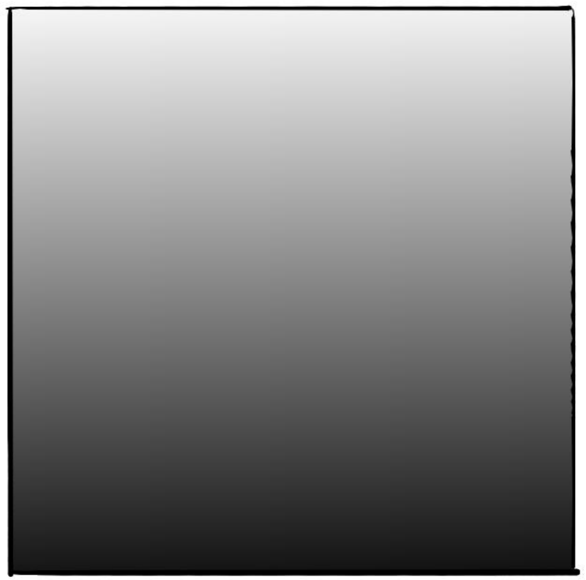
with theoretical ambient occlusion



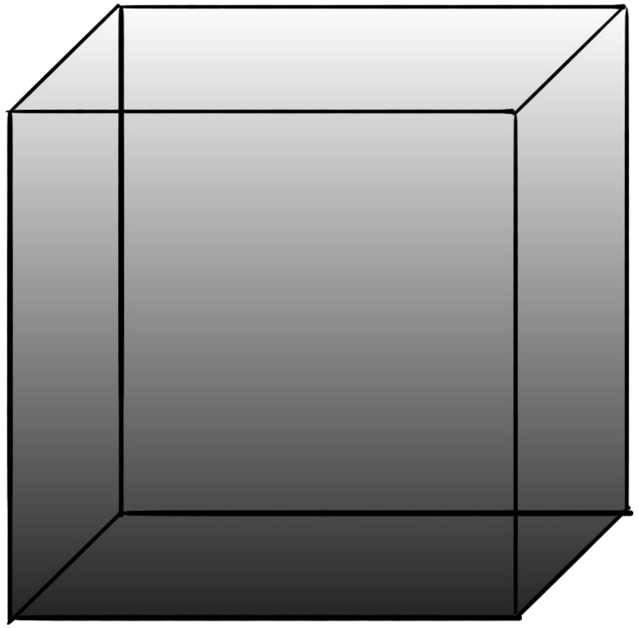
2 tetrahedra

2 structure out of 2 tetrahedra

Theoretical Result



In 3D: surface of an object is a plane,
occlusion is calculated plane-wise



In 4D: surface of an object is a volume,
occlusion is calculated volume-wise

Insufficient Test Data

- Test data uses structures build out from tetrahedra
- Test data does not represent surfaces of 4D structures
- Cannot observe true characteristics of ambient occlusion in 4D

Finding Better Test Data

- Test data should represent surfaces of 4D structures
- Surface of a 4D structure is a volume
- Volume represented as tetrahedral mesh
- Finding surface of a 4D structure is difficult

Finding Surfaces of 4D Structures

- Use isosurface to find surface of a 4D structure
- Isosurface represents points of constant values in a volume
- Possible solution: marching cubes in 4D

Marching Cubes

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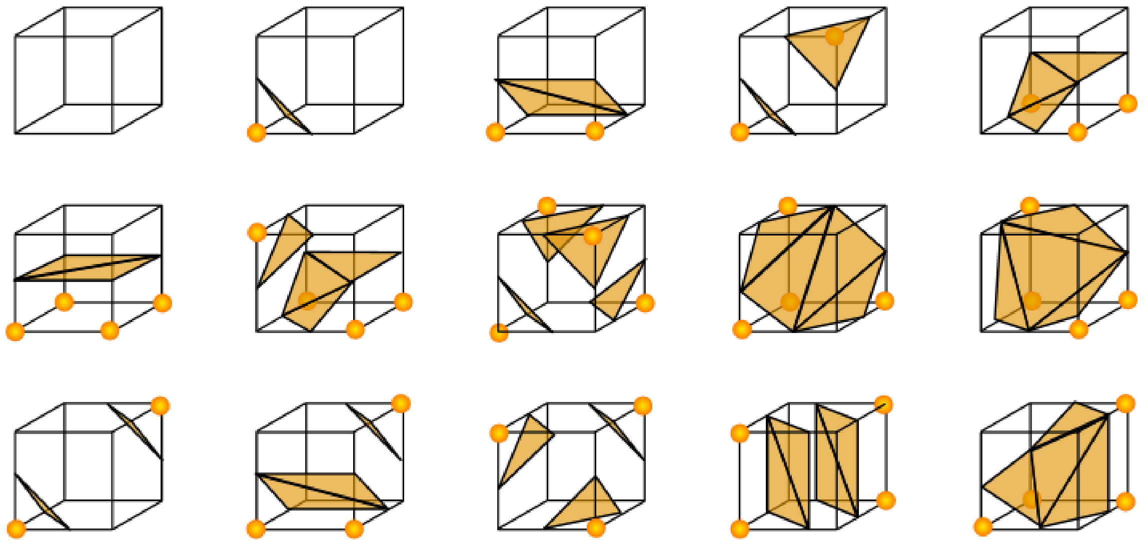
In 3D:

- Extract polygonal mesh of an isosurface of a discrete 3D structure
- Uses pre-defined polygonal configurations in a cube saved as a look-up table

In higher dimensions:

- Introduced by Bhaniramka et. al., 2000
- Generates d-simplex mesh for dimensions $\mathrm{d} \geq 3$
- Look-up table generation, size of look-up table 2^{2^d}



15 configurations from the original marching cubes algorithm in 3D

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Image source: Jmtrivial. (16 October 2006). Marching cubes cases. Retrieved July 1, 2021 from https://commons.wikimedia.org/wiki/File:MarchingCubes.svg

Conclusion

- It is possible to extend ambient occlusion to 4D directly
- Our implementation has unexpected results
- Tetrahedral mesh of surfaces of 4D structures required to fully observe characteristics of ambient occlusion
- Tetrahedral mesh can be generated using marching cubes

- Future work:
  - Illumination in 4D including specular value

Sources

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Thank you for your attention!