



Time Series Management

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Syllabus

- Time series data
- Trend, seasonality, cycles and residuals
- Stationary processes
- Autoregressive processes.
- Moving average processes.
- ACF & PACF
- Fitting $AR(p)$ $MA(q)$ models

Time series data

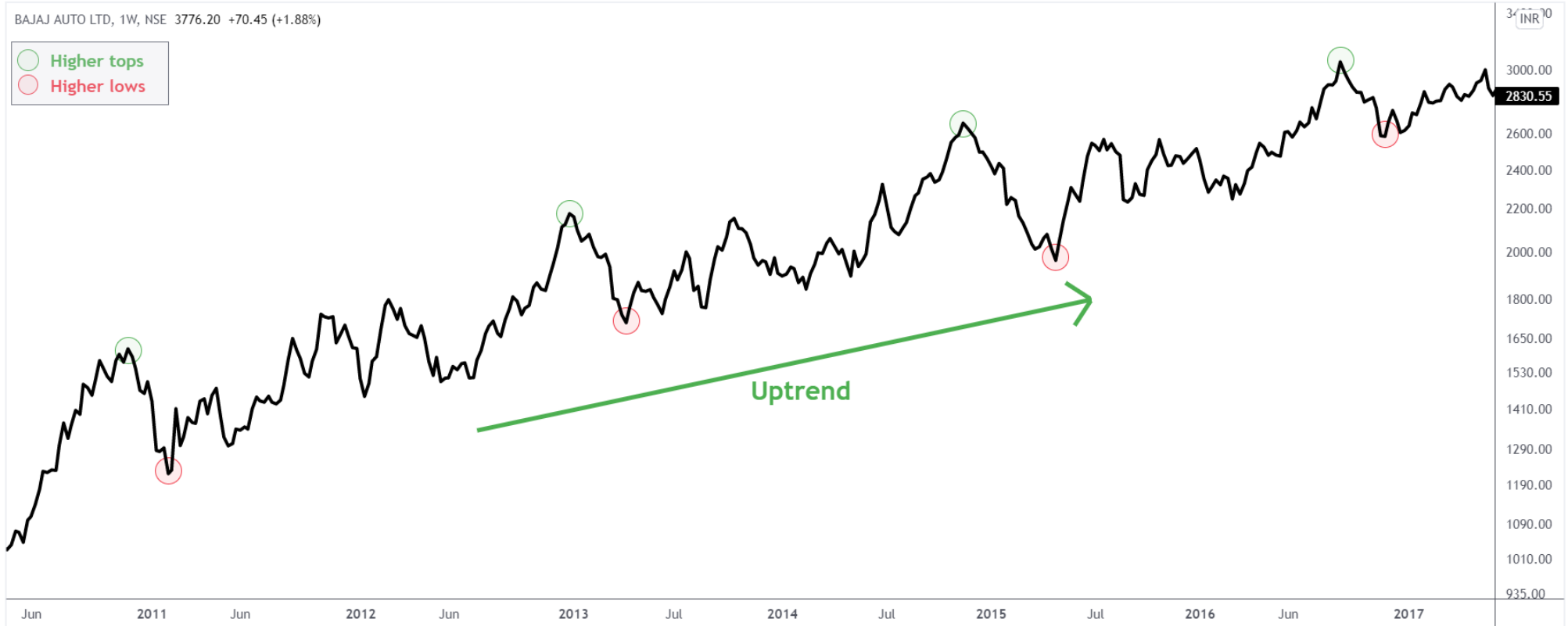
- A **univariate time series** is a sequence of measurements of the same variable collected over time. Most often, the measurements are made at regular time intervals.



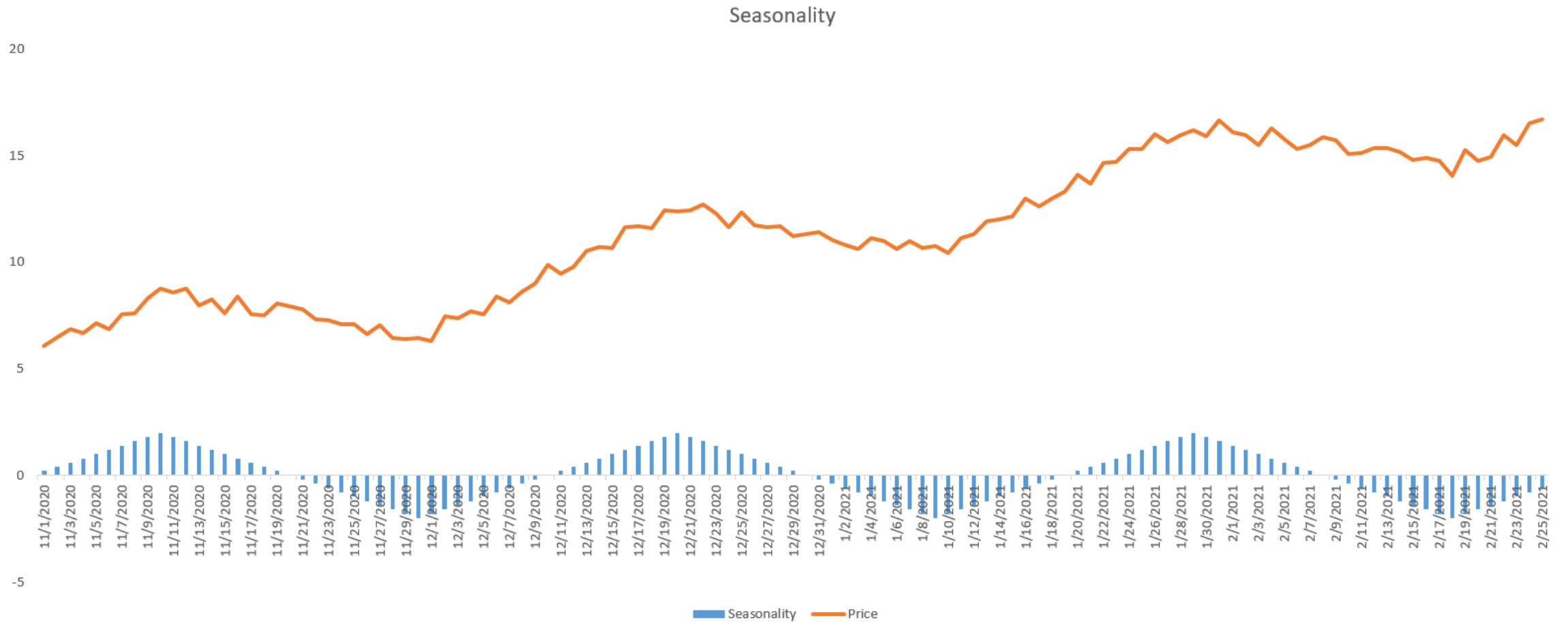
<https://www.kaggle.com/code/anushkaml/walmart-time-series-sales-forecasting>

TS Trend

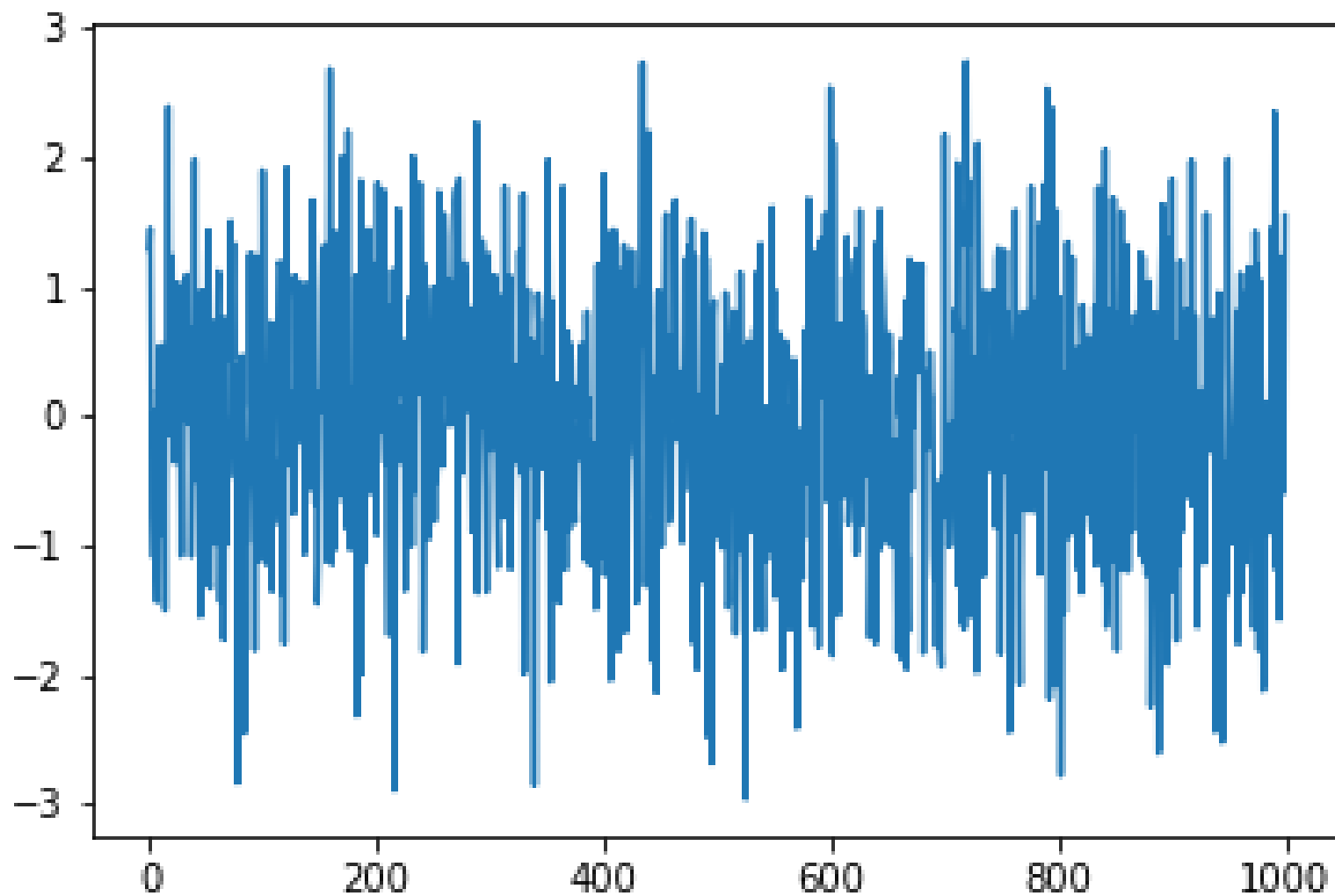
divyant published on TradingView.com, Apr 10, 2022 21:19 UTC+5:30



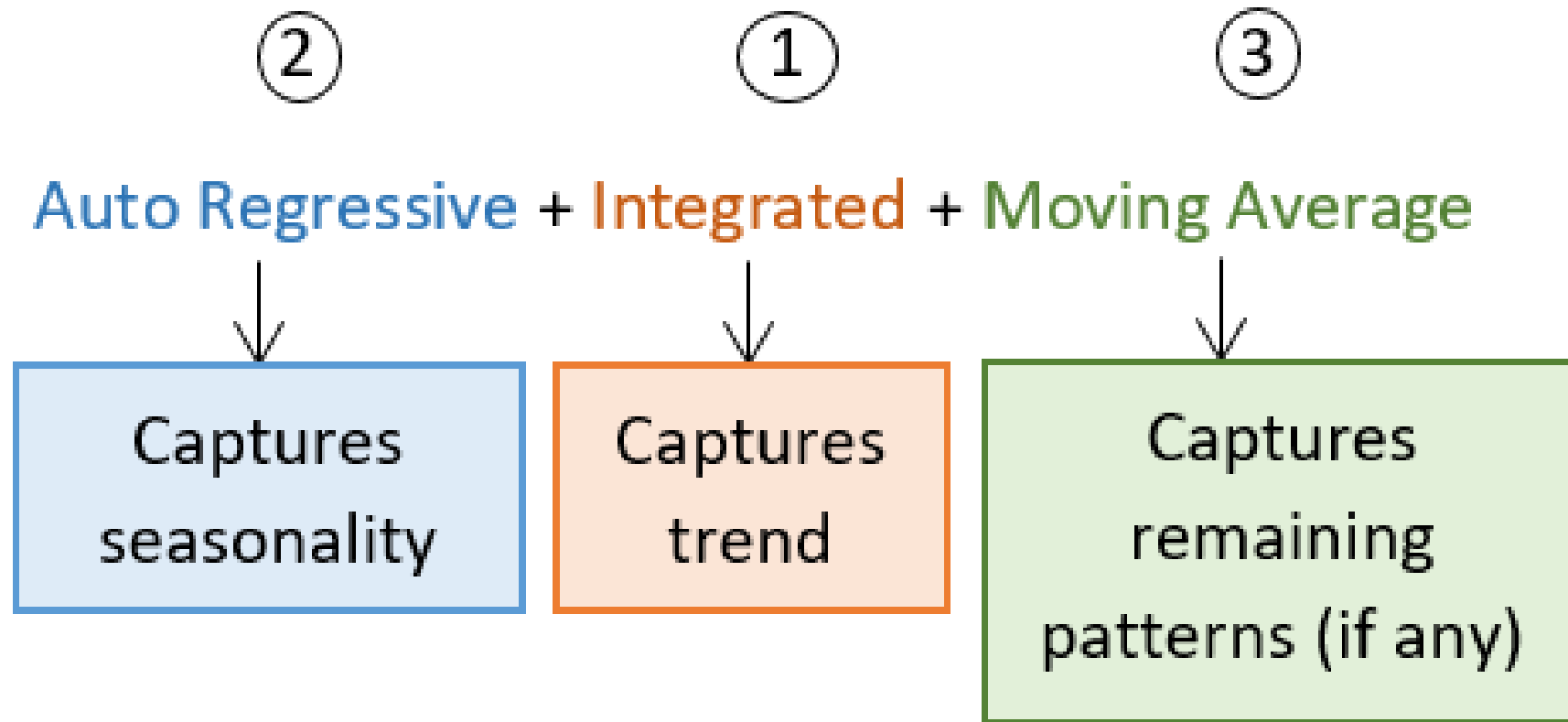
TS Seasonality and Ciclicality



TS Noise



Predictive models



ACF – Autocorrelation function

- **The sample autocorrelation function (ACF)** for a series gives correlations between the series $X_{[t]}$ and lagged values of the series for lags of 1,2,3 and so on. We represent a lagged series with $X_{[t-h]}$, where $h := \text{lag}$.
- Example: $X_{[t]} = [1,2,3,4,\dots]$, $X_{[t-1]} = [2,3,4,\dots]$, $X_{[t-2]} = [3,4,\dots]$, ...

$$\frac{\text{Covariance}(X_{[t]}, X_{[t-h]})}{\text{Std. Dev.}(X_{[t]})\text{Std. Dev.}(X_{[t-h]})}$$

PACF – Partial Autocorrelation function

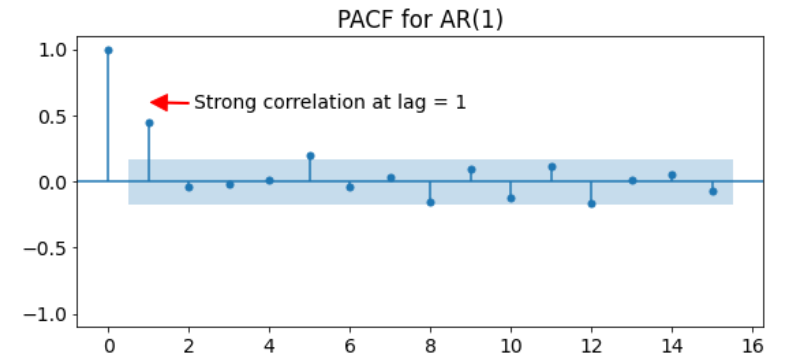
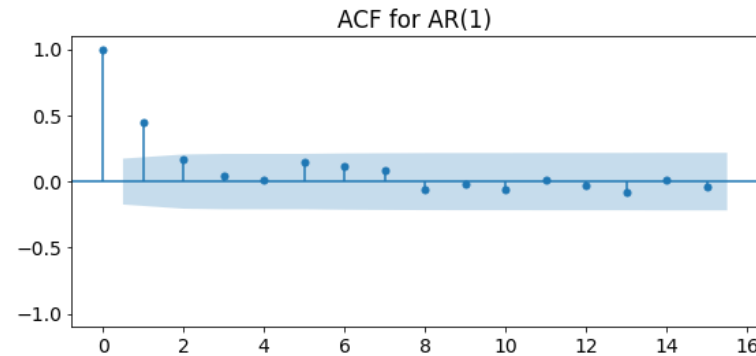
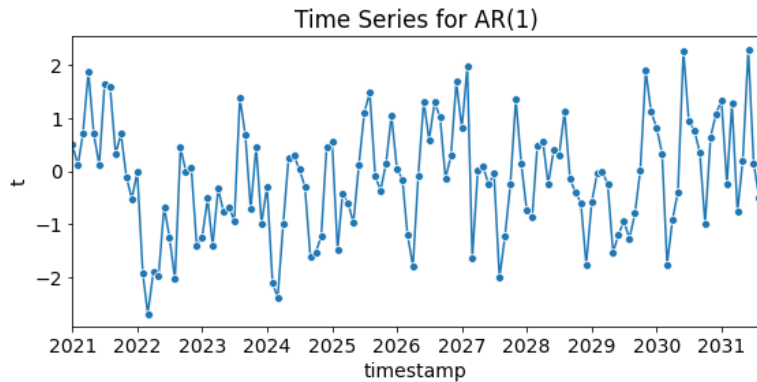
In general, a partial correlation is a conditional correlation, namely the correlation between two variables under the assumption that we know and take into account the values of some other set of variables.

If we assume that $X_t = C + \boldsymbol{\varphi}_1 X_{t-1} + \boldsymbol{\varphi}_2 X_{t-2} + \dots$

We may want to consider the (partial) dependency between X_{t-3} to X_t . Hence, PACF of order 3:

$$\frac{\text{Covariance}(X_{[t]}, X_{[t-3]} | X_{[t-2]}, X_{[t-1]})}{\sqrt{\text{Variance}(X_{[t]} | X_{[t-1]}, X_{[t-2]}) \text{Variance}(X_{[t-3]} | X_{[t-1]}, X_{[t-2]})}}$$

ACF – PACF Examples

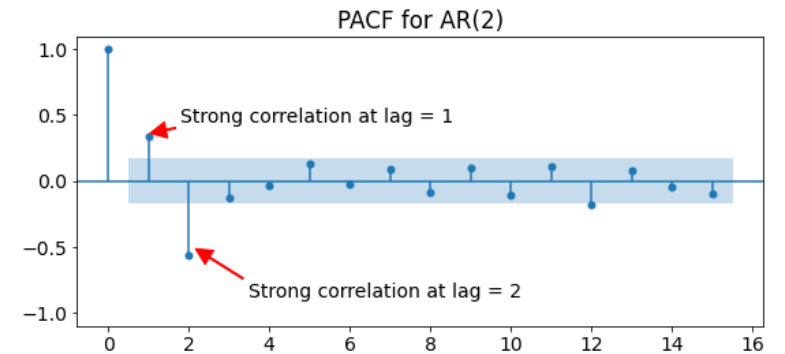
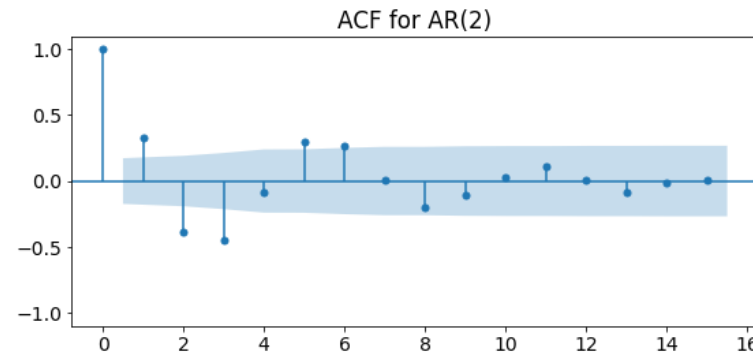
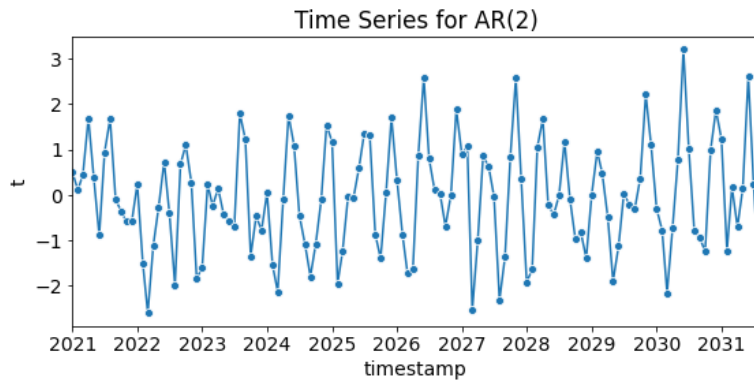


AR($p= 1$)

ACF Tails off (Geometric decay)

PACF Significant at each lag p / Cuts off after lag p

ACF – PACF Examples

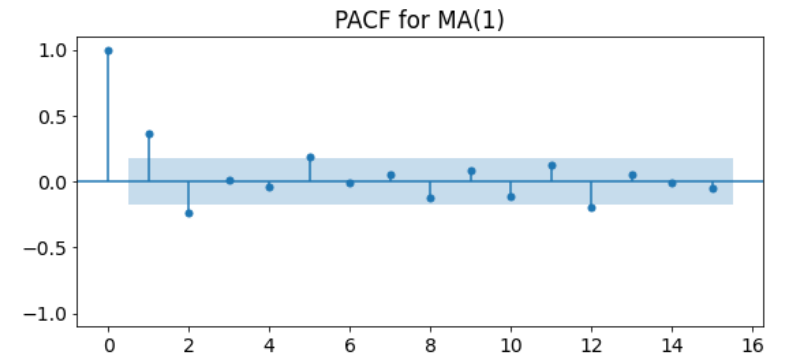
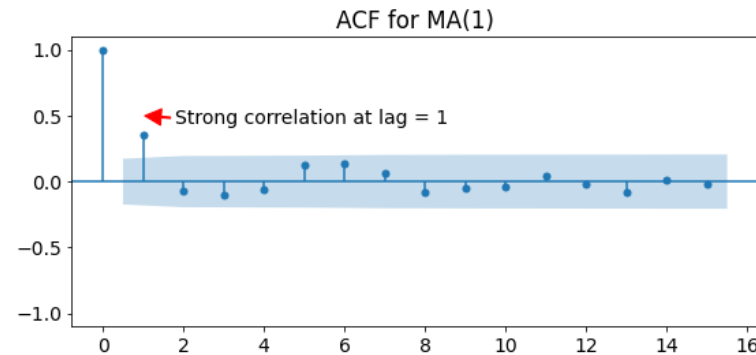
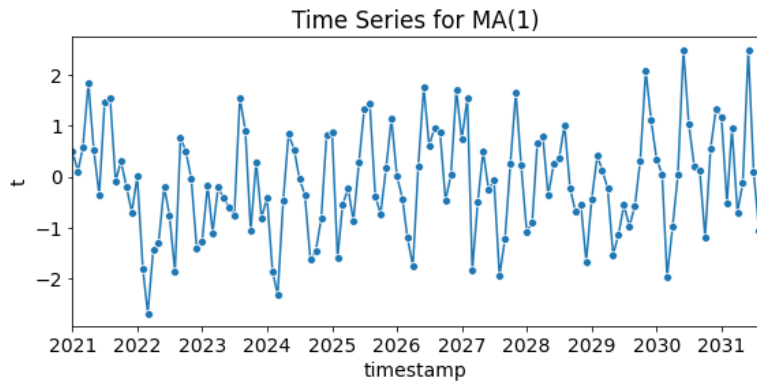


AR($p = 2$)

ACF Tails off (Geometric decay)

PACF Significant at each lag p / Cuts off after lag p

ACF – PACF Examples

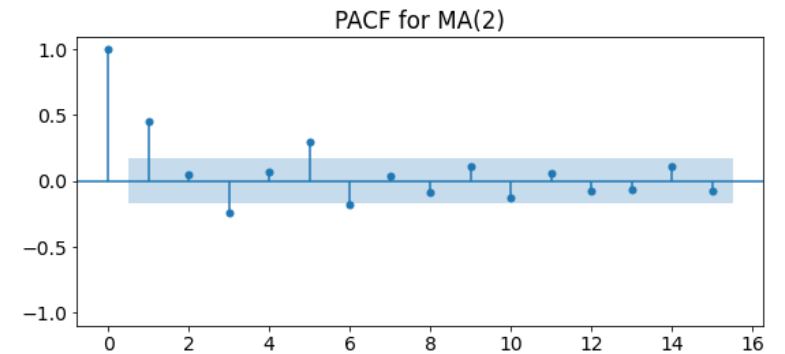
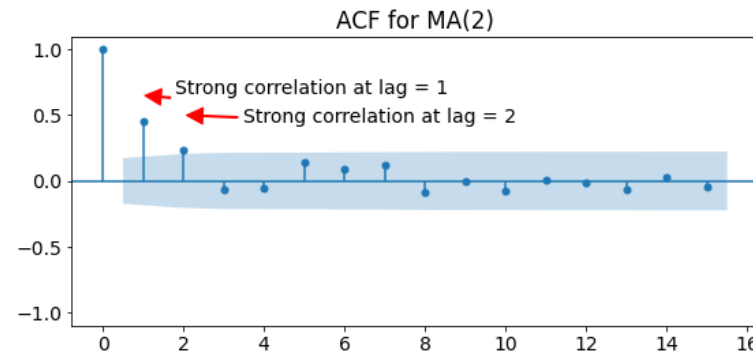
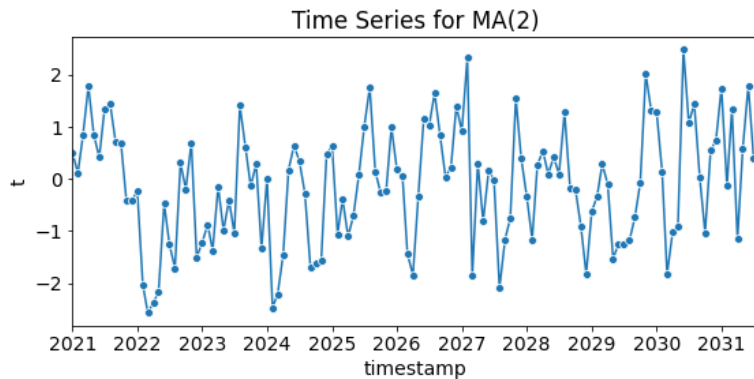


MA($q = 1$)

ACF Significant at lag q / Cuts off after lag q

PACF Tails off (Geometric decay)

ACF – PACF Examples

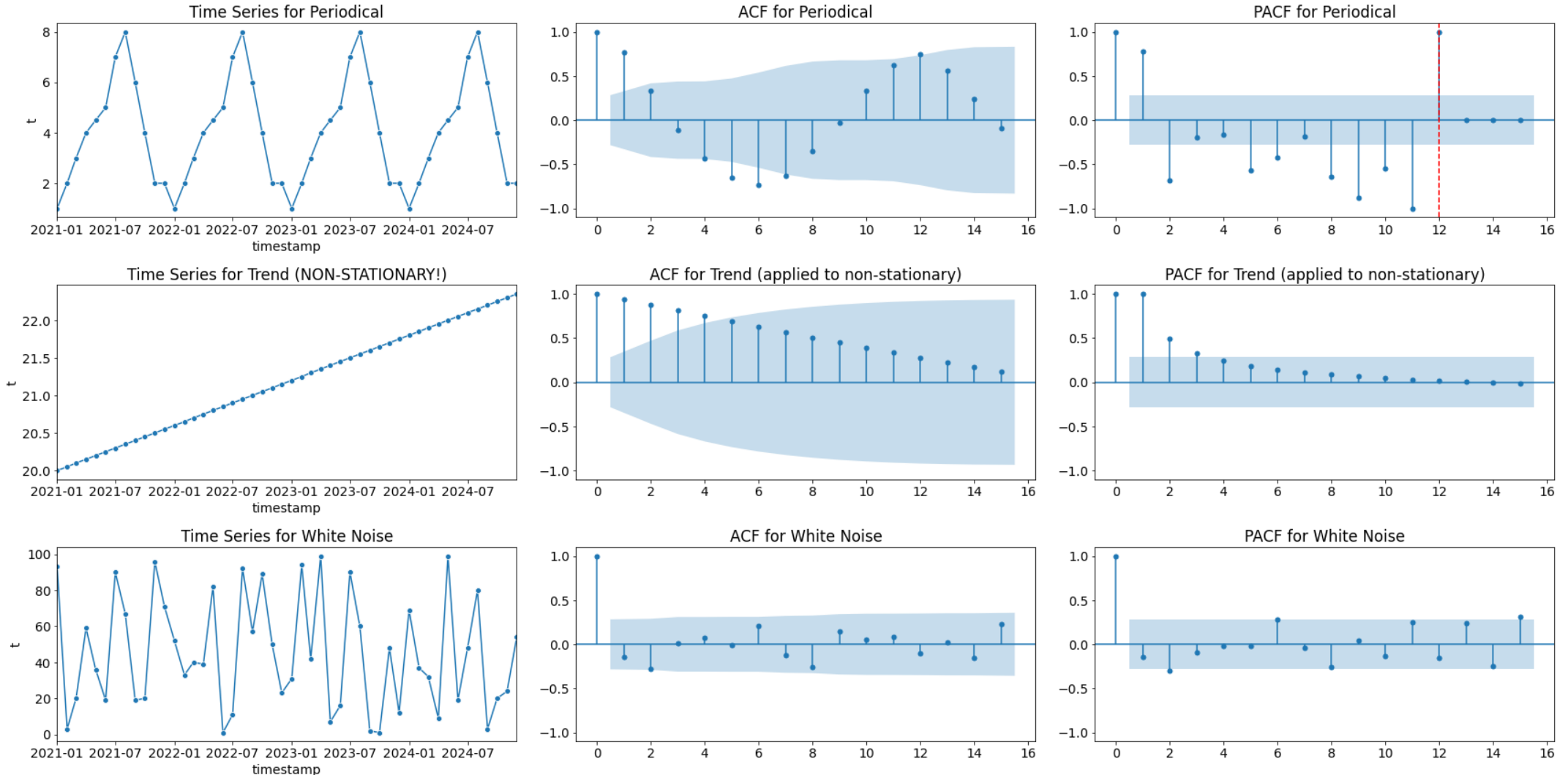


MA($q = 2$)

ACF Significant at lag q / Cuts off after lag q

PACF Tails off (Geometric decay)

ACF – PACF Examples ... continue



References

- Andrew V. Metcalfe, Paul S.P. Cowpertwait, **Introductory Time Series with R** (2009).
- Aileen Nielsen, **Practical Time Series Analysis: Prediction with Statistics and Machine Learning** (2019).