Time Series Management

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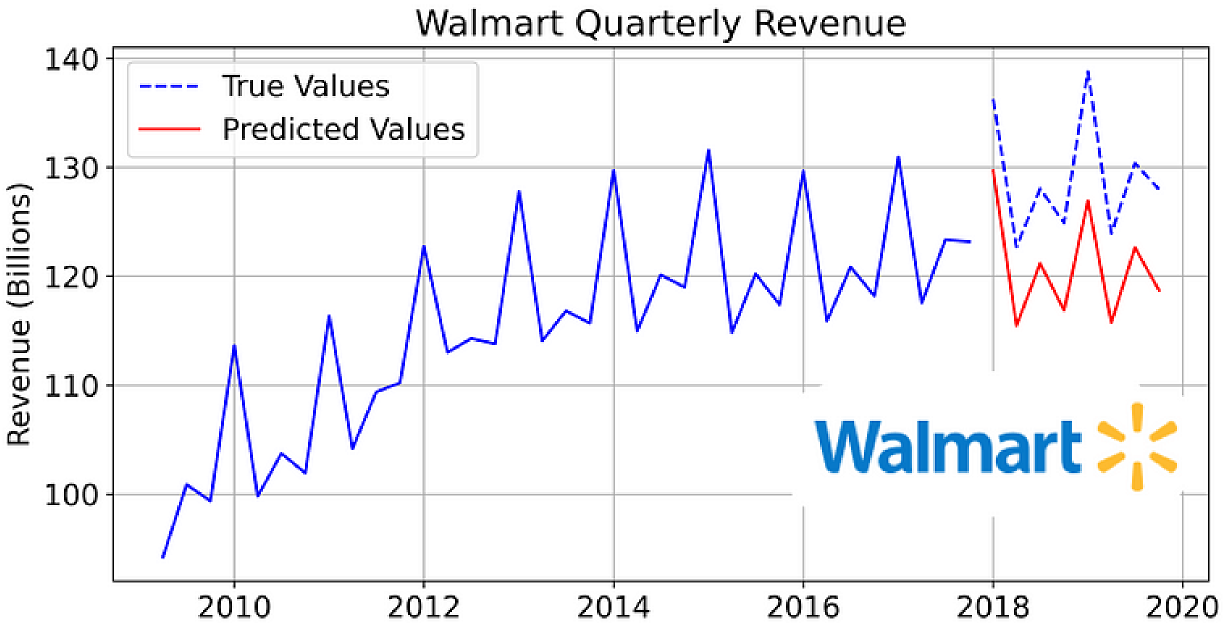
Some of the slides of thes course are taken from the **excellent** Tutorial of **Eammon Keogh**
A Decade of Progress in Indexing and Mining Large Time Series Databases. VLDB 2006.

Syllabus

- Ubiquity of data series collections
- Time series data mining
- Similarity Search
- Metrics
- SAX : Time Series Symbolic Aggreagate approXimation.
- DTW Lower bounding
- Speed up computation
  by lower bounding true Euclidean distance over SAX representation.

Time series data… quick recap

- A **univariate time series** is a sequence of measurements of the same variable collected over time. Most often, the measurements are made at regular time intervals.



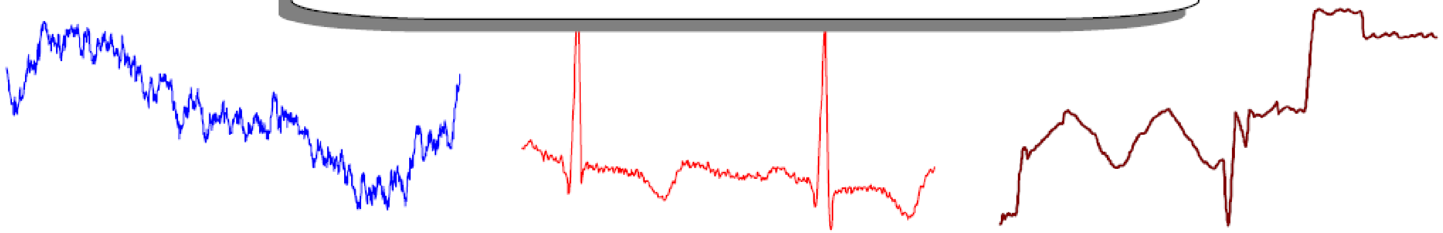
https://www.kaggle.com/code/anushkaml/walmart-time-series-sales-forecasting

Time series are ubiquitous

People measure things...

- *Their blood pressure*
- *George Bush's popularity rating*
- *The annual rainfall in Seattle*
- *The value of their Google stock*

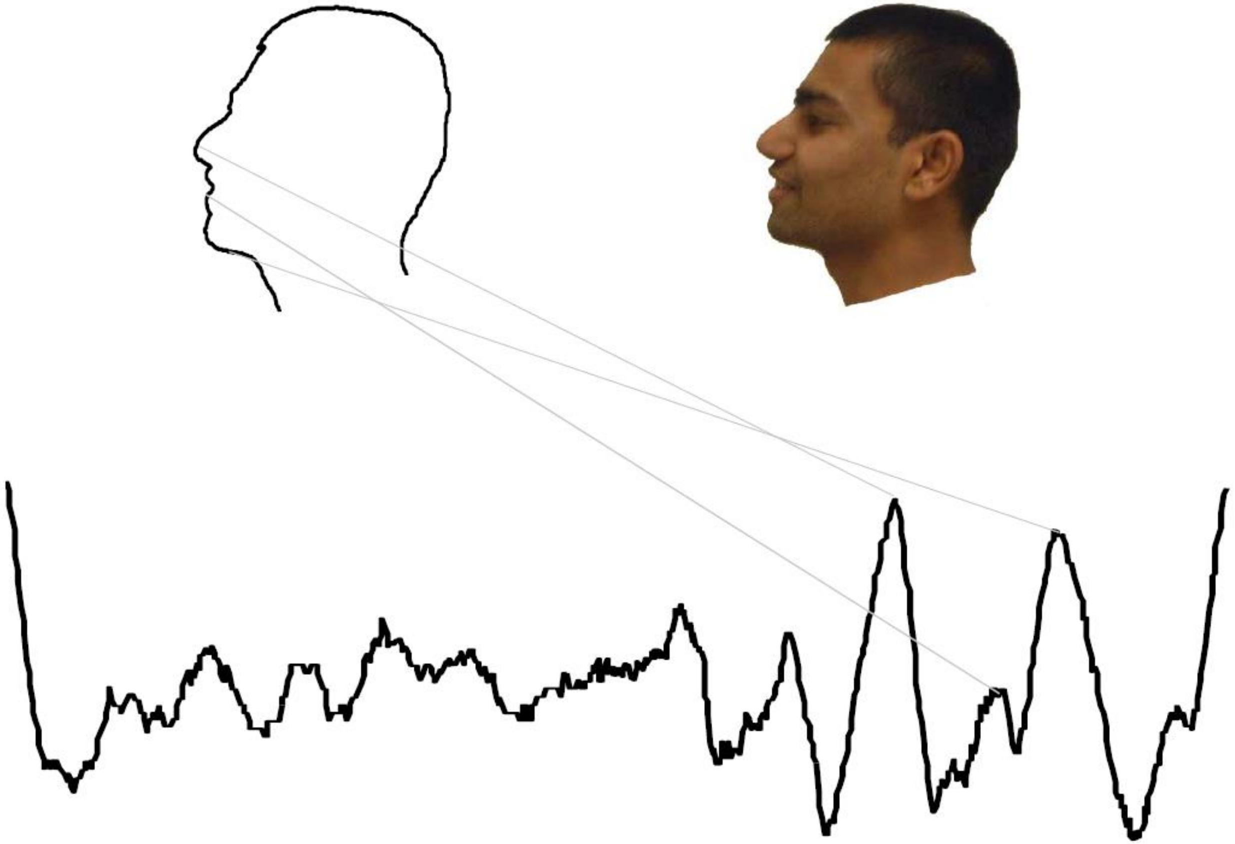
...and things change over time...



Thus time series occur in virtually every medical, scientific and businesses domain

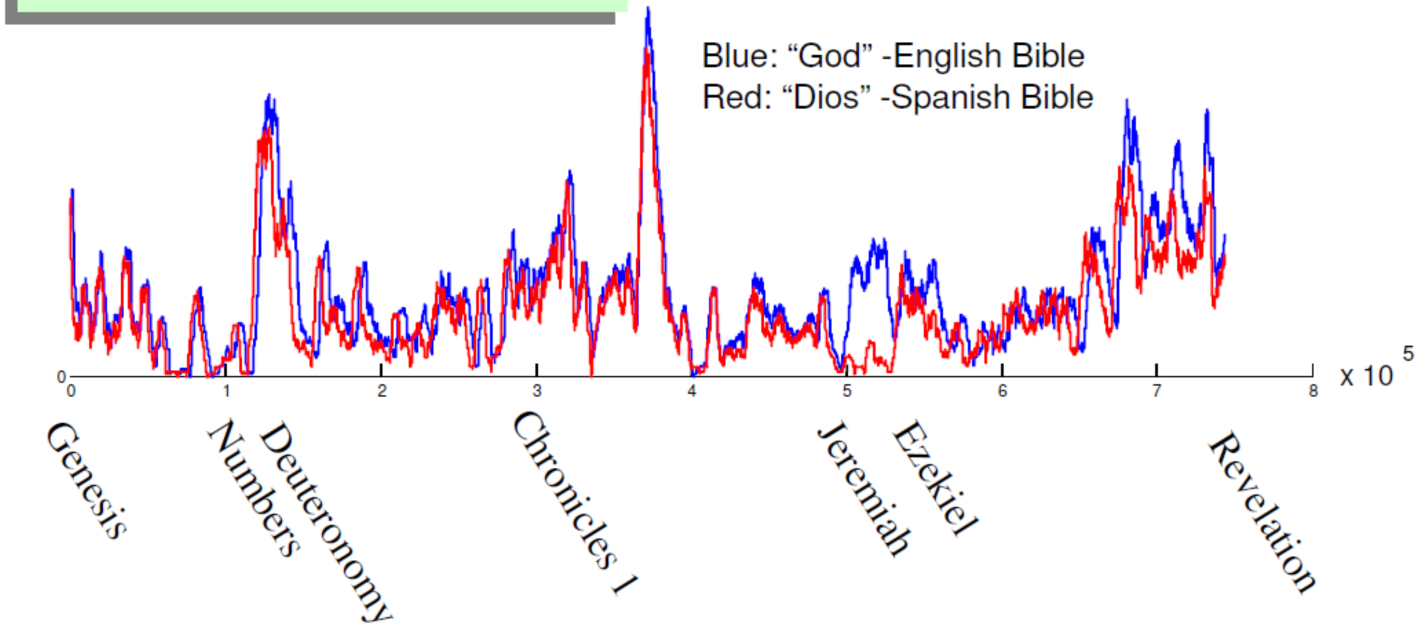
Time series are ubiquitous (1/4)

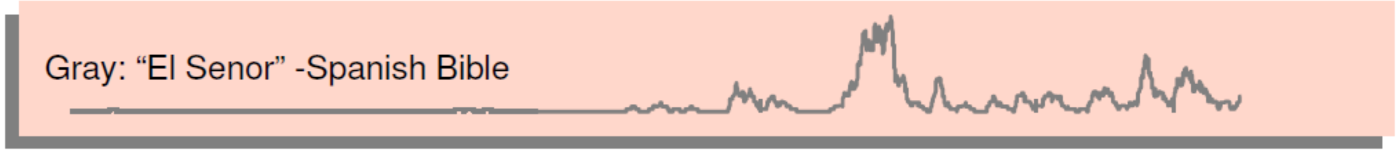
Image data, may best be thought of as time series…



Time series are ubiquitous (2/4)

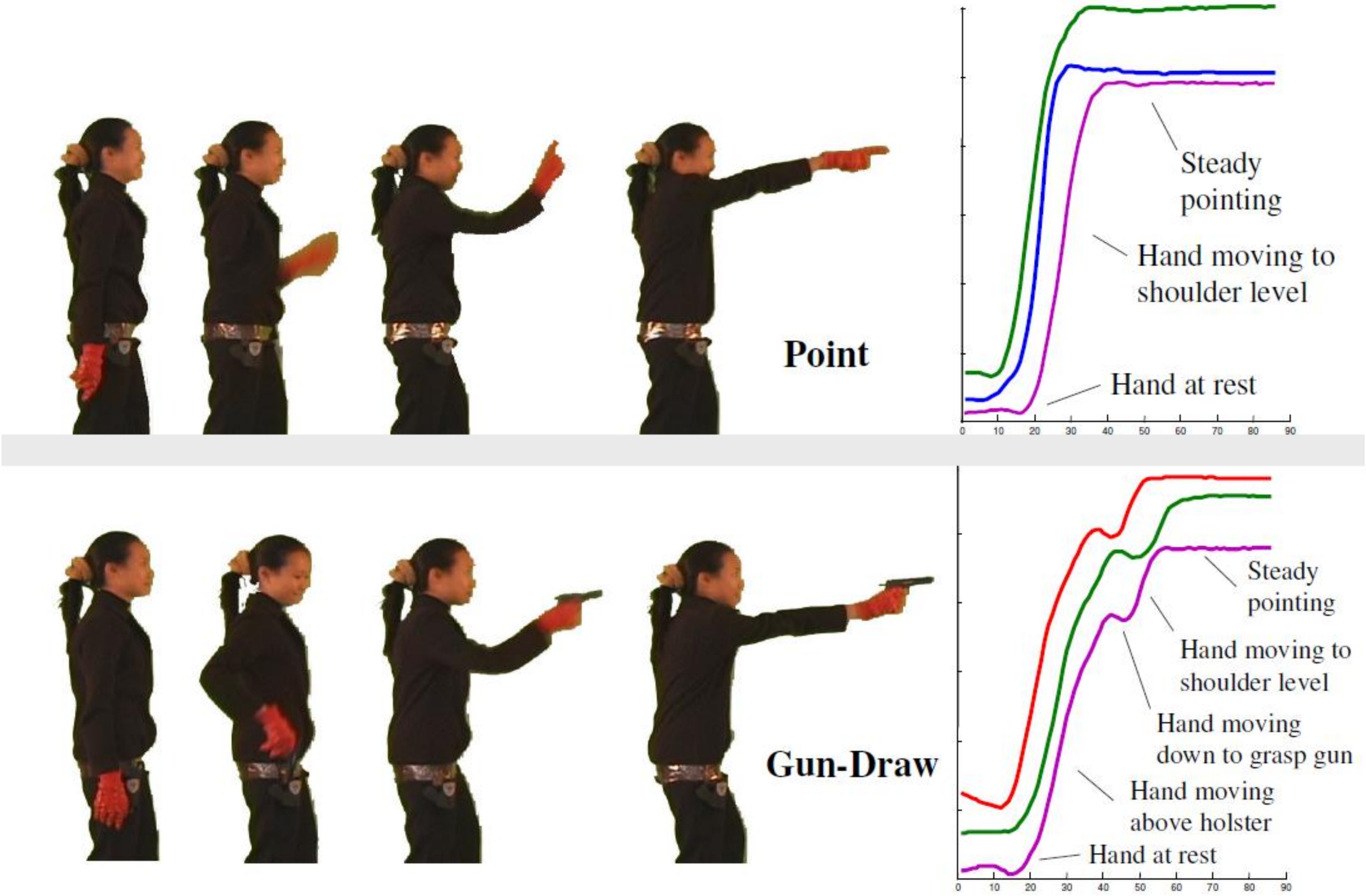
The local frequency of words in the Bible





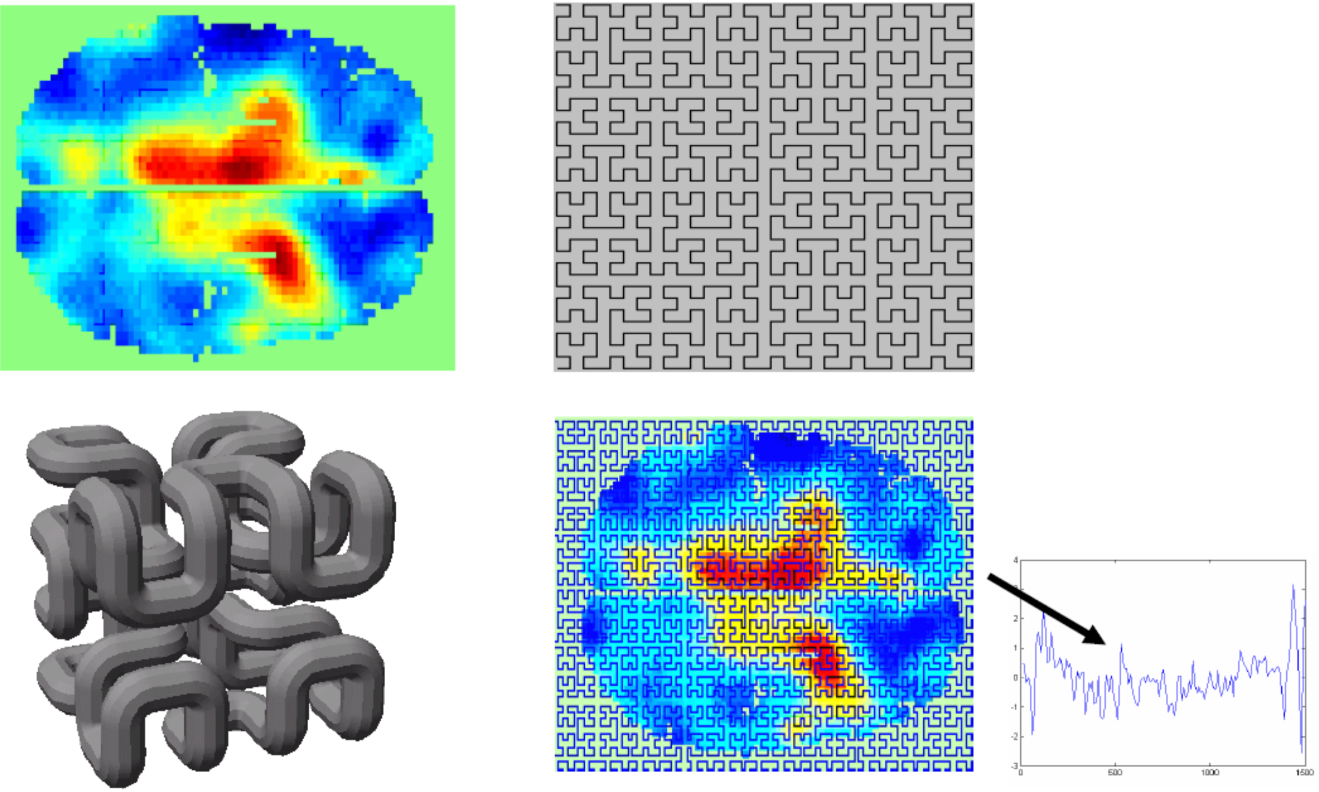
Time series are ubiquitous (3/4)

Video data, may best be thought of as time series…



Time series are ubiquitous (4/4)

Brain scans (3D voxels), may best be thought of as time series..



Wang, Kontos, Li and Megalooikonomou ICASSP 2004

Why is Working With Time Series is so difficult? (1/3)

1 Hour of EKG data: 1 Gigabyte.

Typical Weblog: 5 Gigabytes per week.

Space Shuttle Database: 200 Gigabytes and growing.

Macho Database (Canadian Astronomy Data Centre): 3 Terabytes, updated with 3 gigabytes a day.

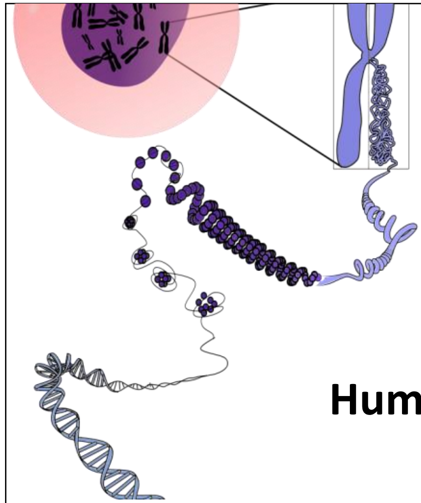
Why is Working With Time Series is so difficult? (2/3)

NASA's Solar Observatory

1.5 TB per day

Large Synoptic Survey Telescope (2019)

~30 TB per night



Human Genome project

130 TB

passenger aircrafts

20 TB per hour

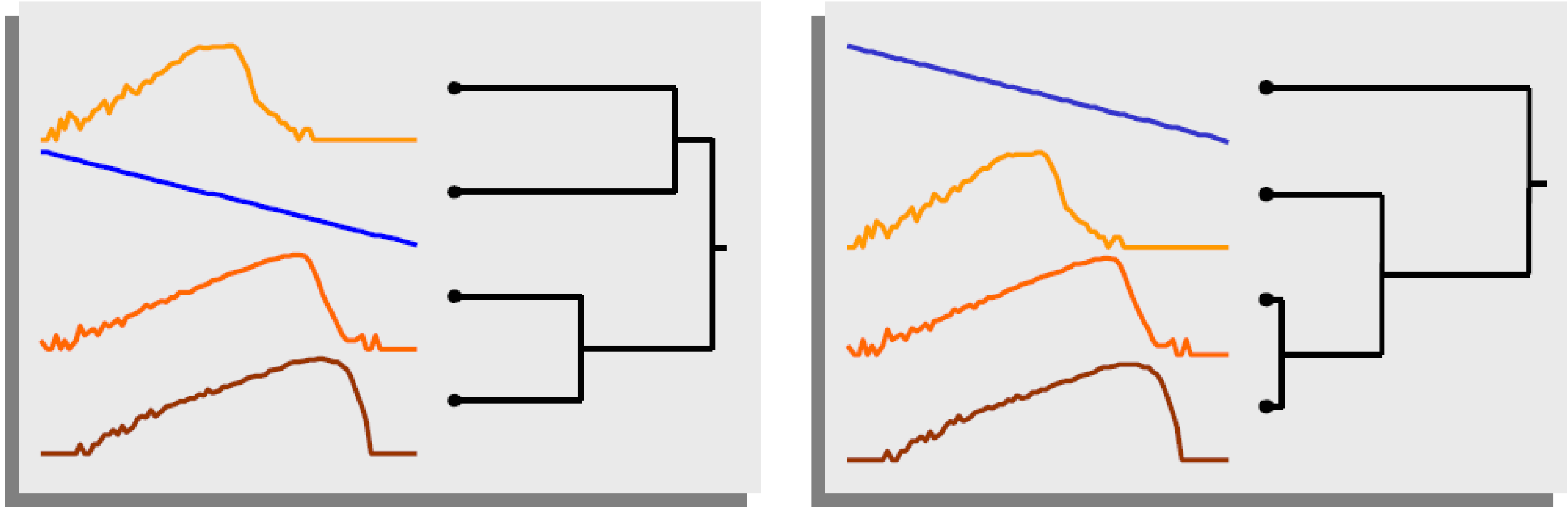
data center and services monitoring

2B data series

4M points/sec

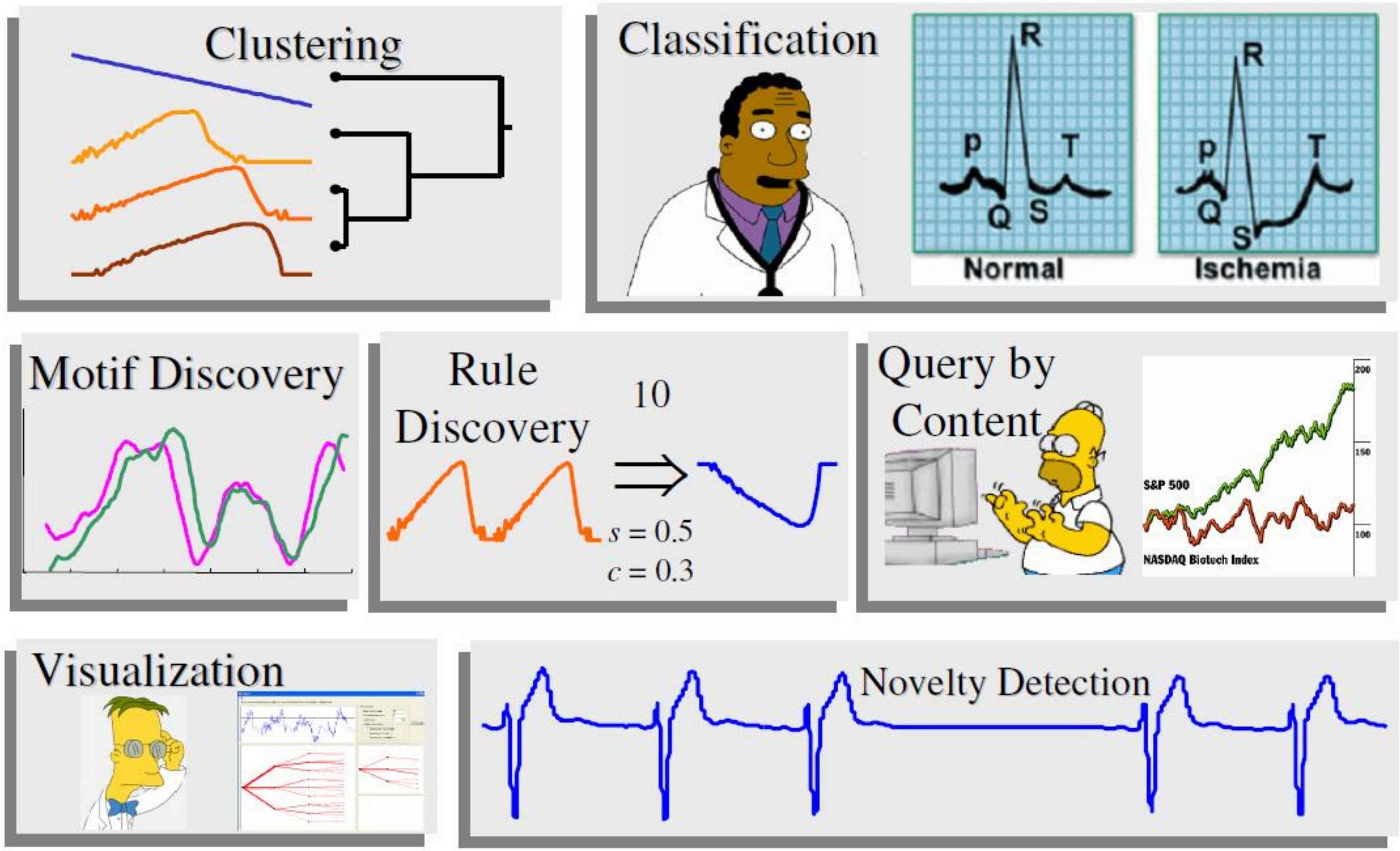
Why is Working With Time Series is so difficult? (3/3)

Answer: We are dealing with subjectivity



The definition of similarity depends on the user, the domain and the task at hand. We need to be able to handle this subjectivity.

Problems requiring Similarity Search

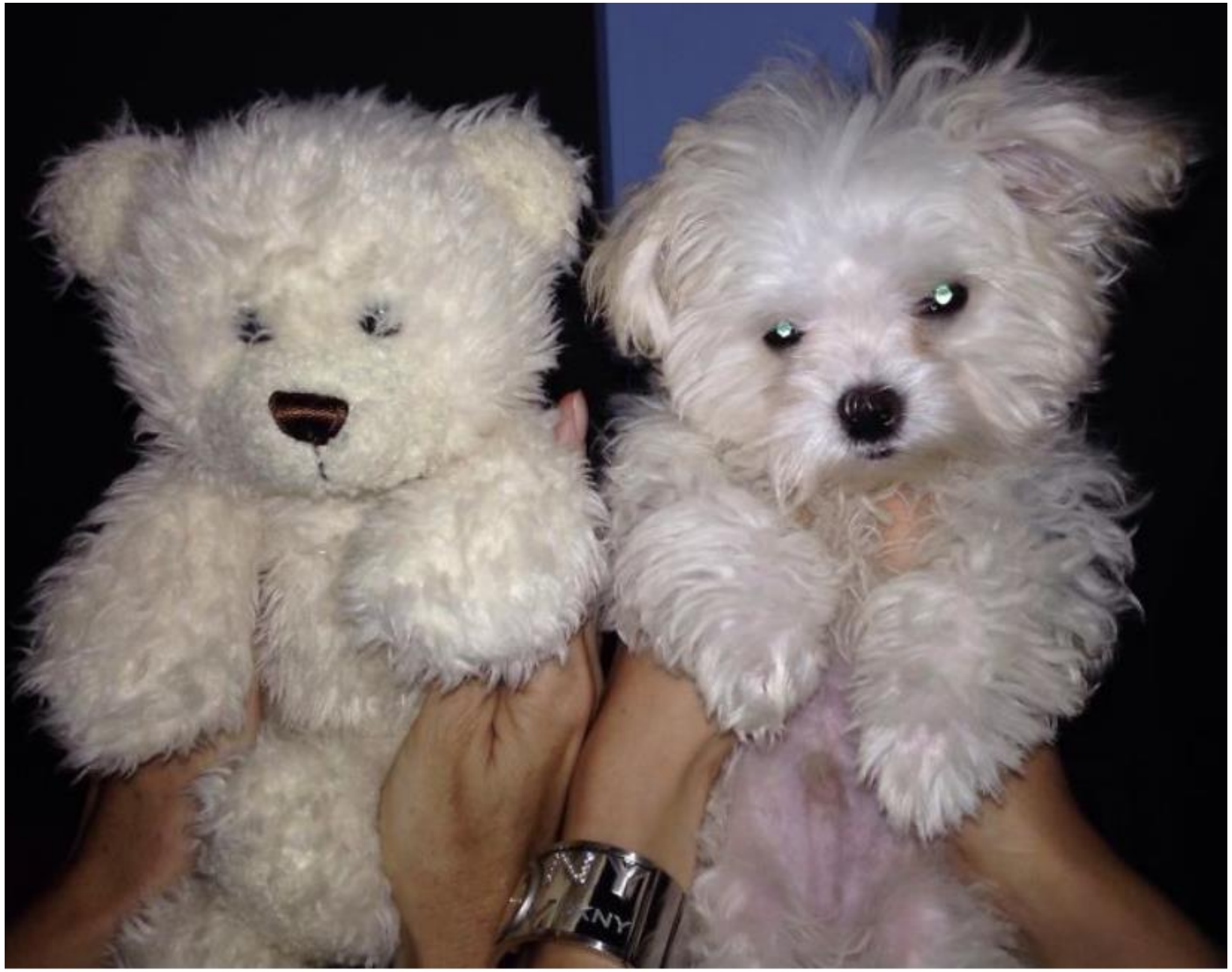


Important Data Mining questions

How do we define similarity ?

How do we search large time series collection quickly ?

What is similarity ?



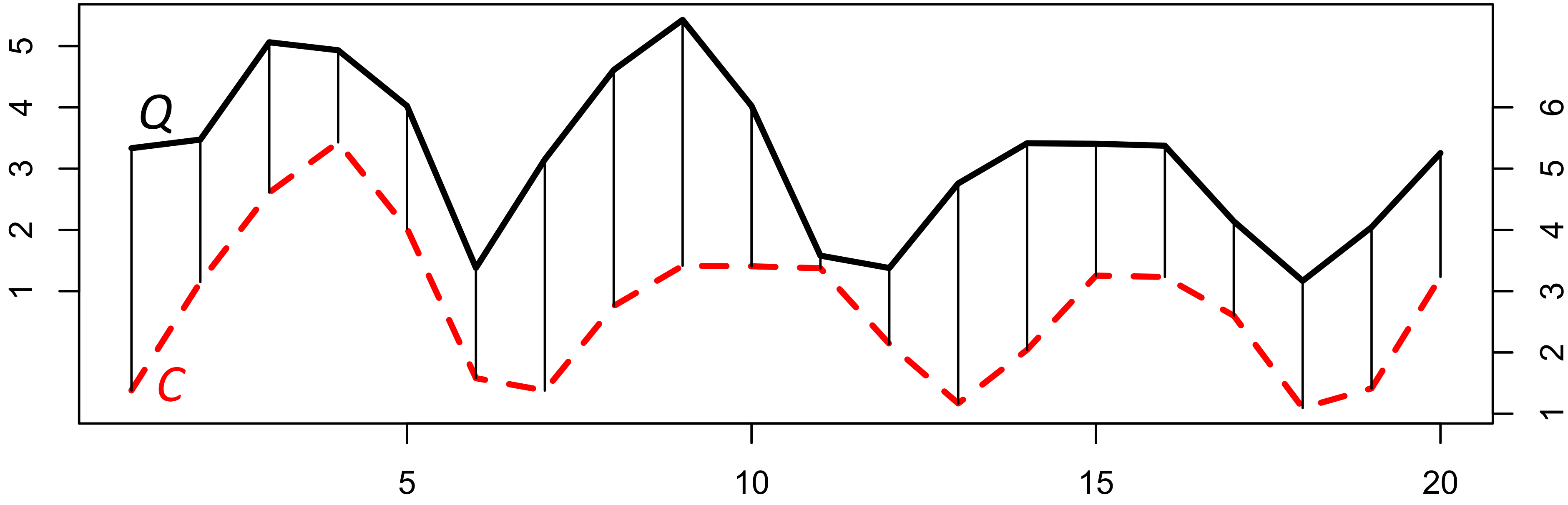
What is similarity ?

The quality or state of being similar; likeness; resemblance; as, a similarity of features.

Similarity is hard to define, but…"We know it when we see it"

The real meaning of similarity is a philosophical question.

Similarity Measure: Euclidean



Given two time series $Q = q_1 \ldots q_n$ and $C = c_1 \ldots c_n$

their Euclidean distance is defined as: $ED(Q,C) = \sqrt{\sum_{i=1}^{n}(q_i - c_i)^2}$

Preprocessing the data before distance calculations

- Z-Normalization (Amplitude Scaling)
- Linear Trend
- Noise

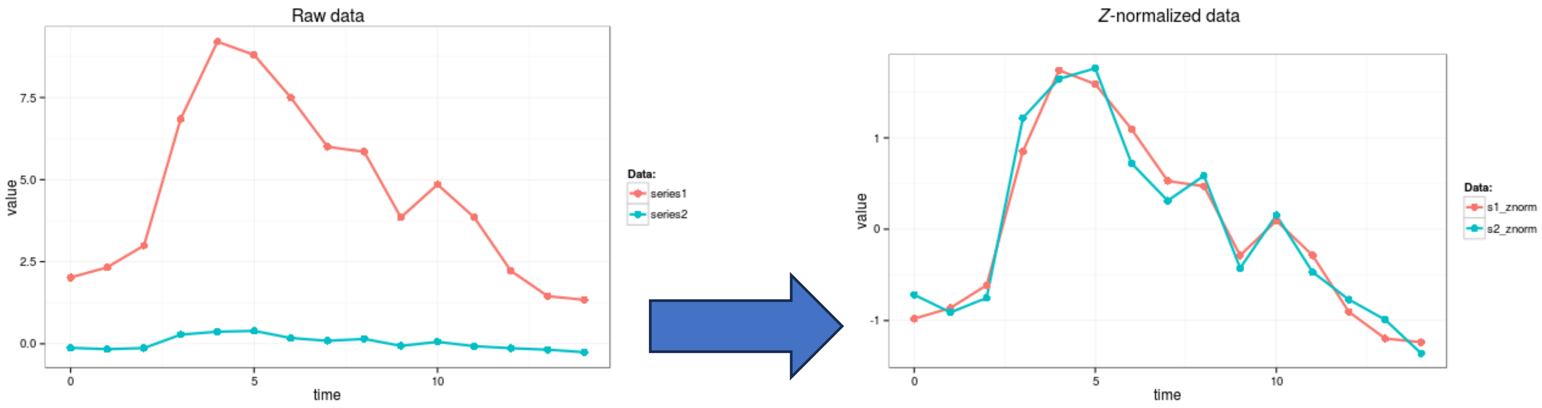
Z Normalization

Z-normalization, also known as

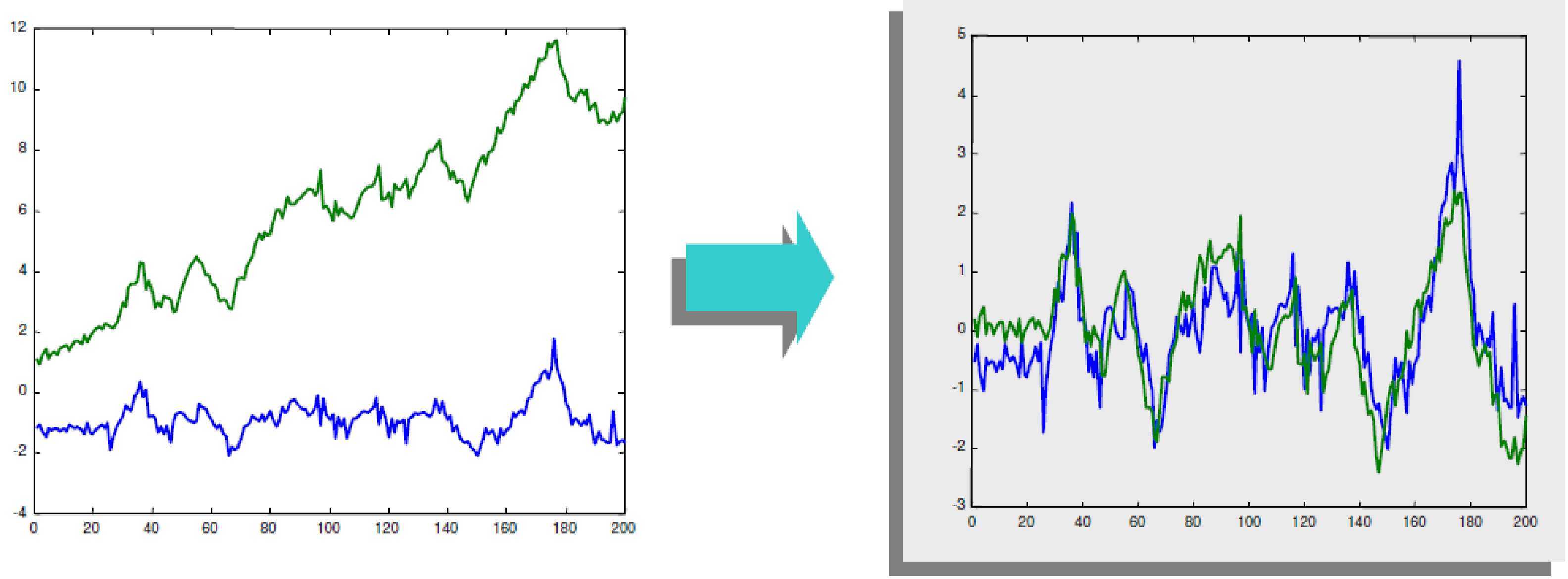
“Normalization to Zero Mean and Unit of Energy”.

The procedure ensures, that all elements of the input vector are transformed into the output vector whose mean is approximately 0 while the standard deviation is in a range close to 1.

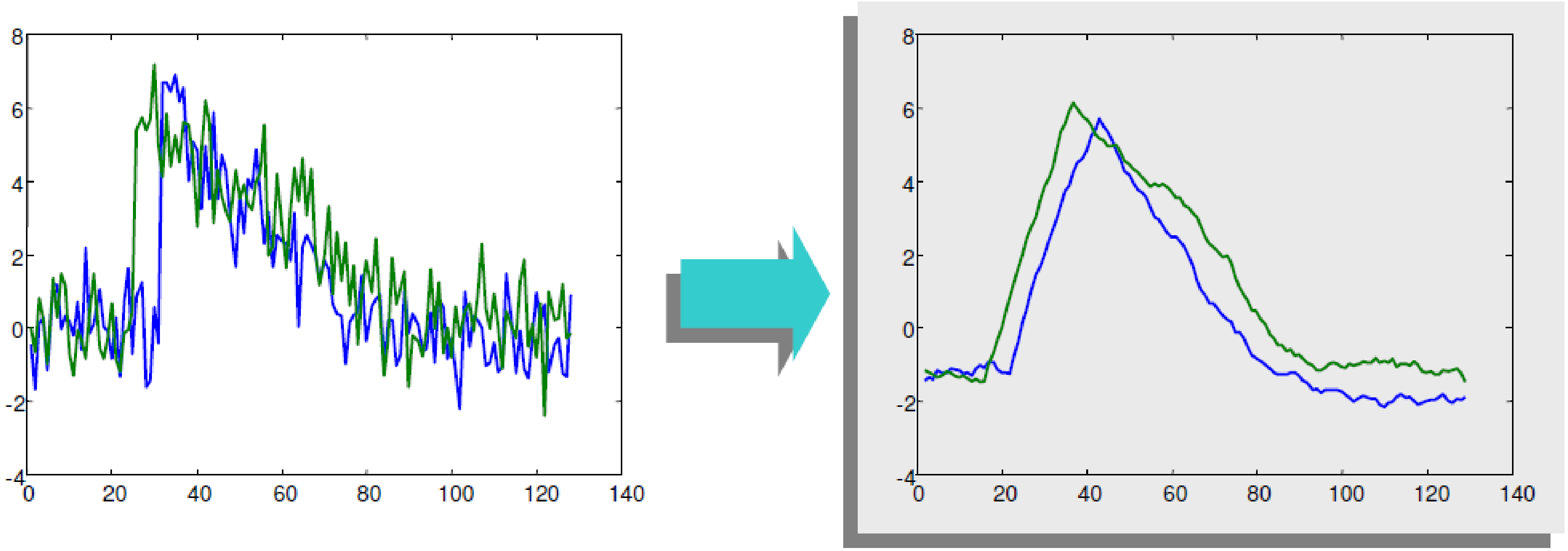
$$x_i^{\prime} = \frac{x_i - \mu}{\sigma}, \text{ where } i \in \mathbb{N}$$



Remove linear trend



Removing Noise

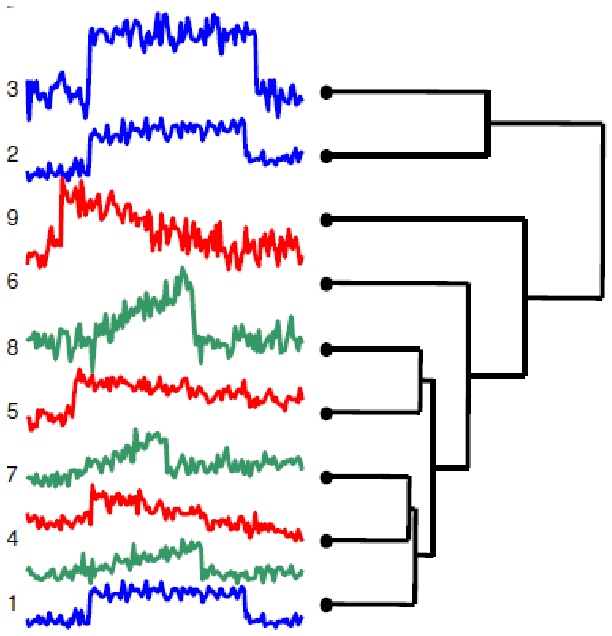


Smoothing function: Remove noise component

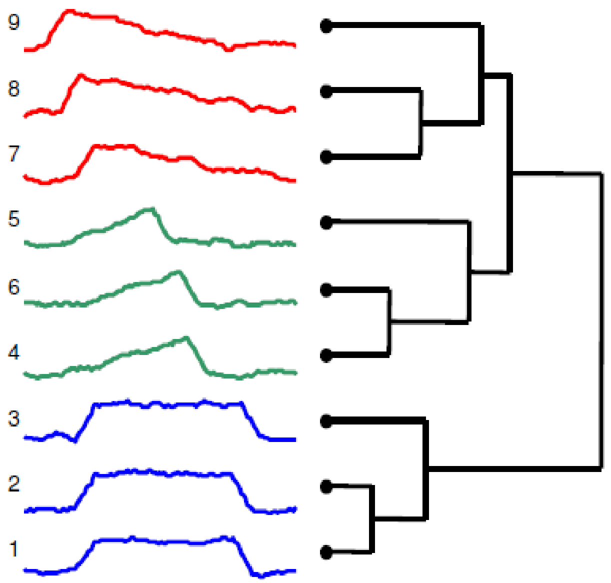
e.g., Average each datapoints value with its neighbors

Importance of data pre-processing

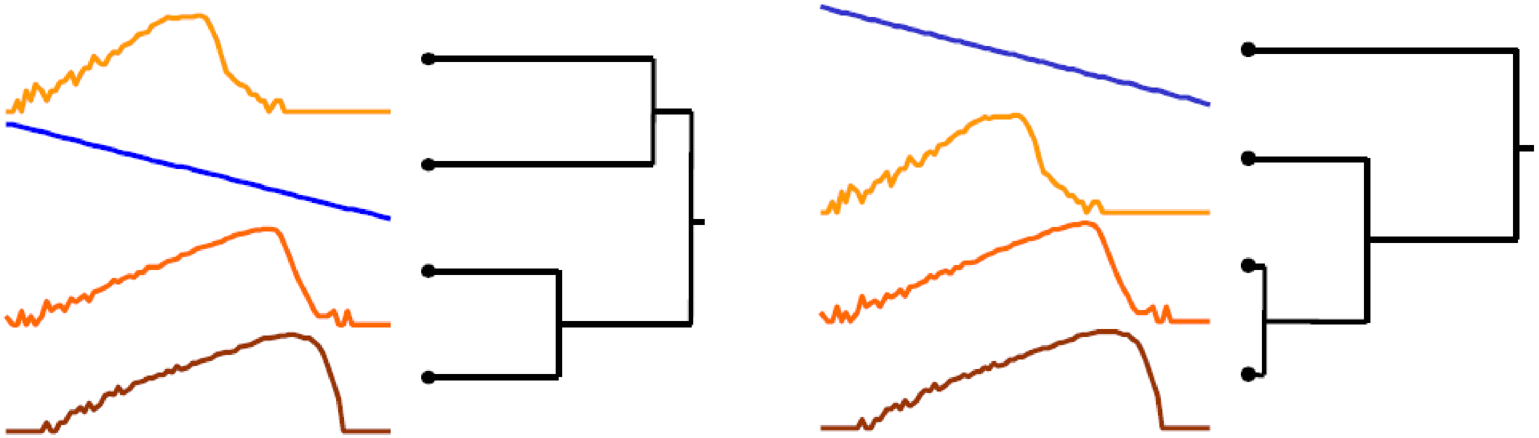
Clustered using Euclidean distance on the raw data.

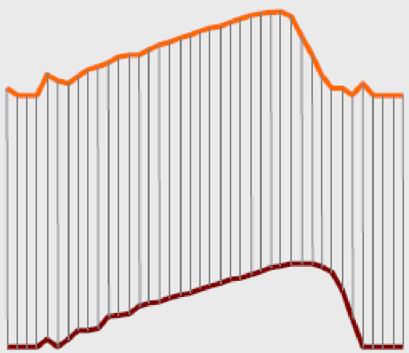


Clustered using Euclidean distance, after removing noise, linear trend, and Z-Normalization



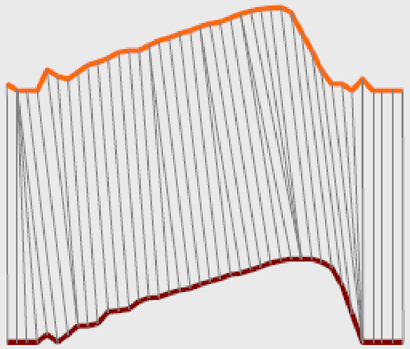
Dynamic Time Warping





Fixed Time Axis

Sequences are aligned "one to one".

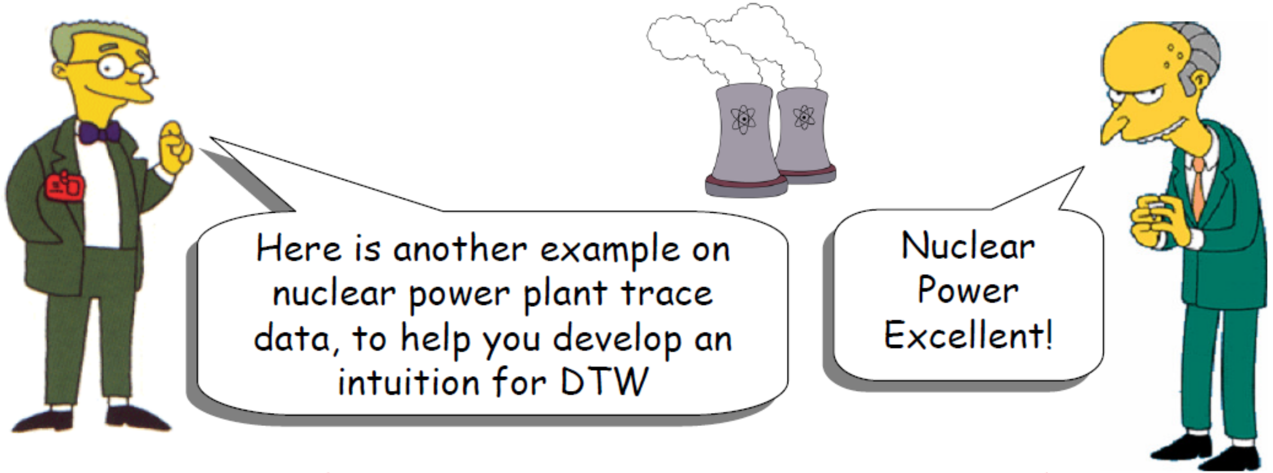


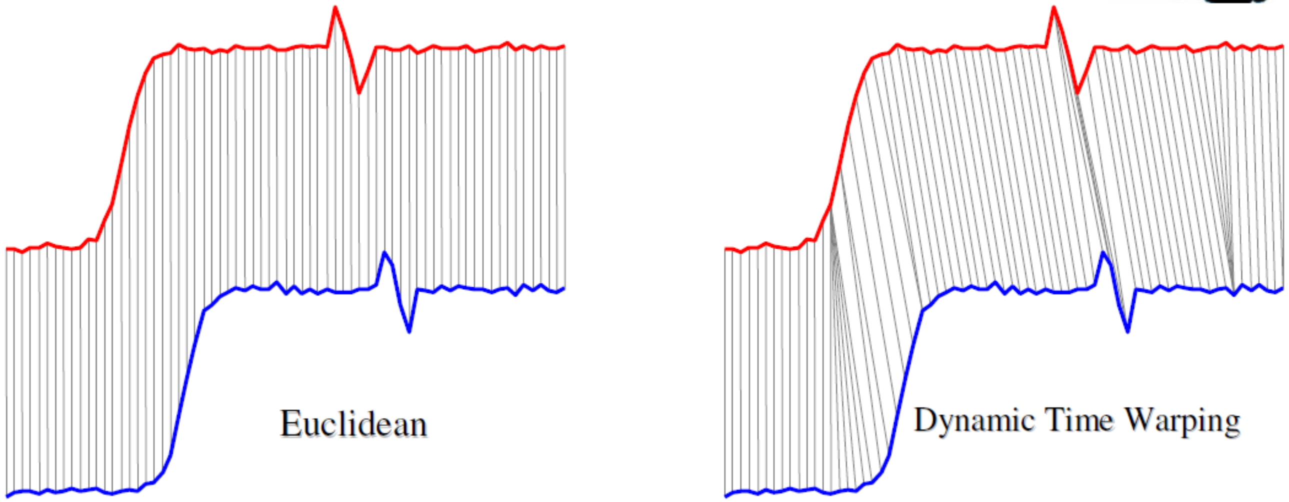
"Warped" Time Axis

Nonlinear alignments are possible.

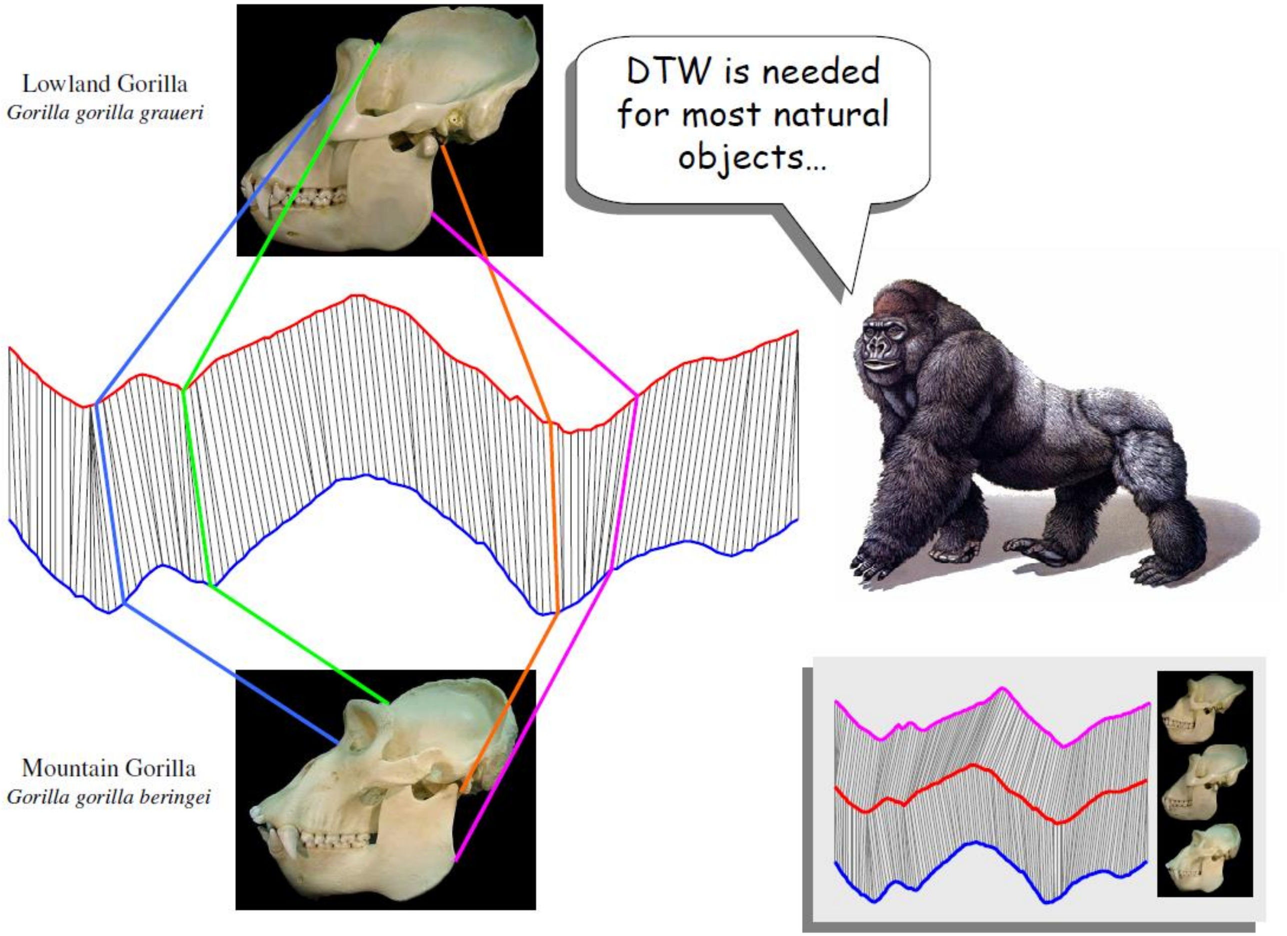
Note: We will first see the utility of DTW, then see how it is calculated.

Some real-world example (1/3)

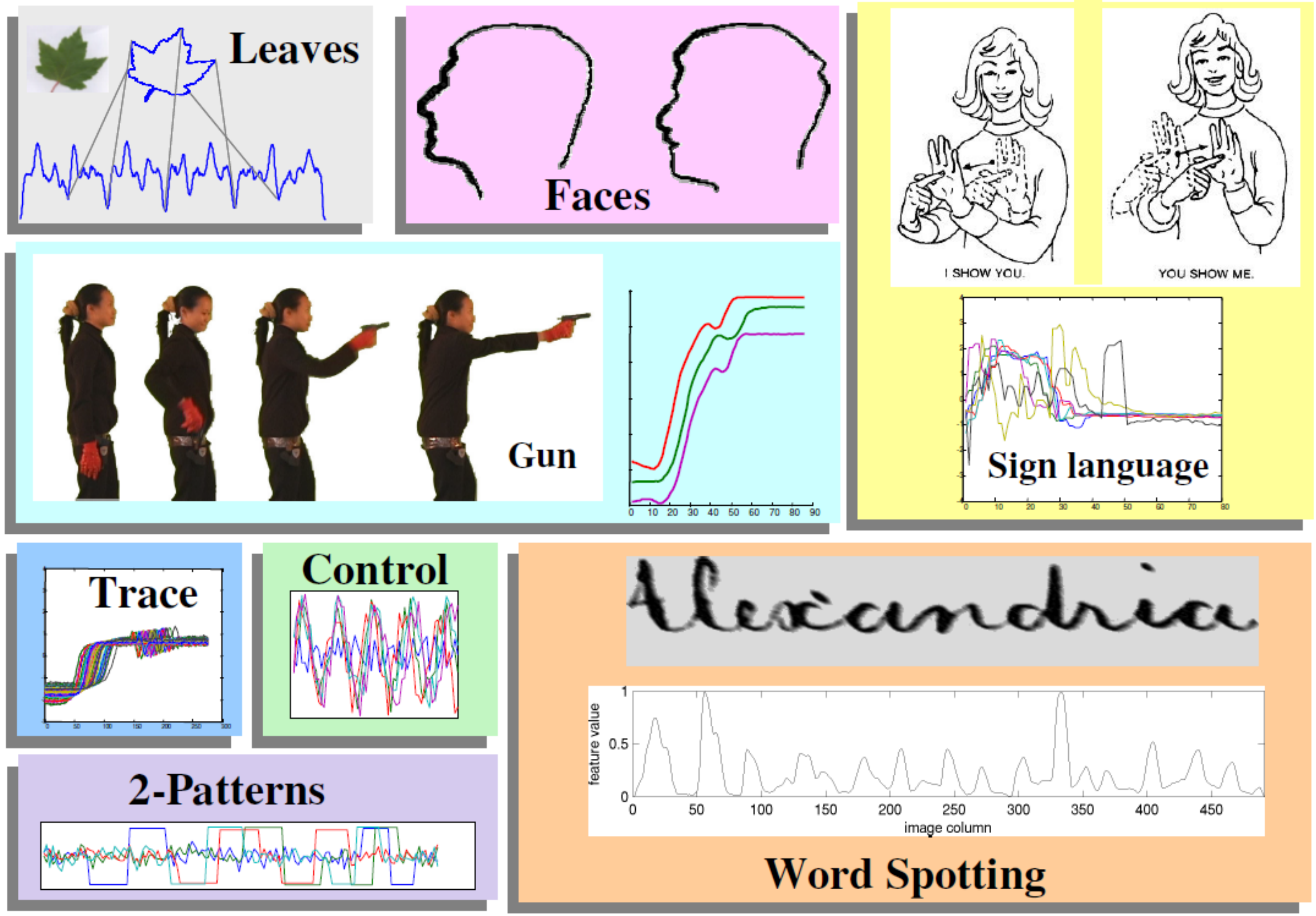

Here is another example on nuclear power plant trace data, to help you develop an intuition for DTW
Nuclear Power Excellent!



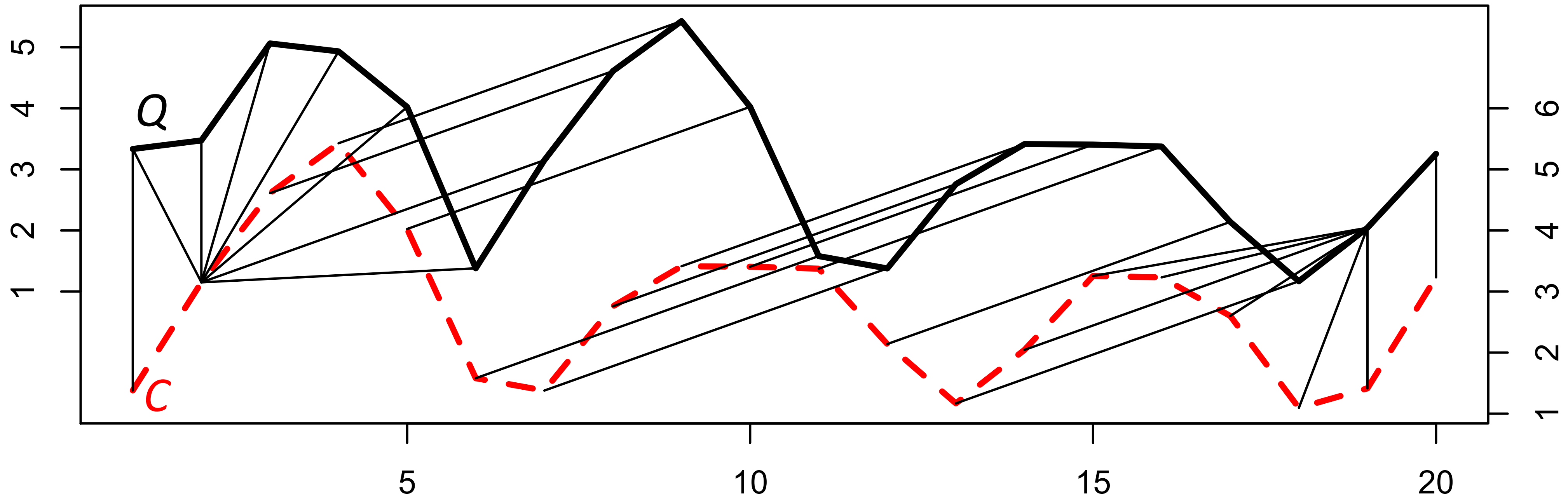
Some real-world example (2/3)



Some real-world example (3/3)



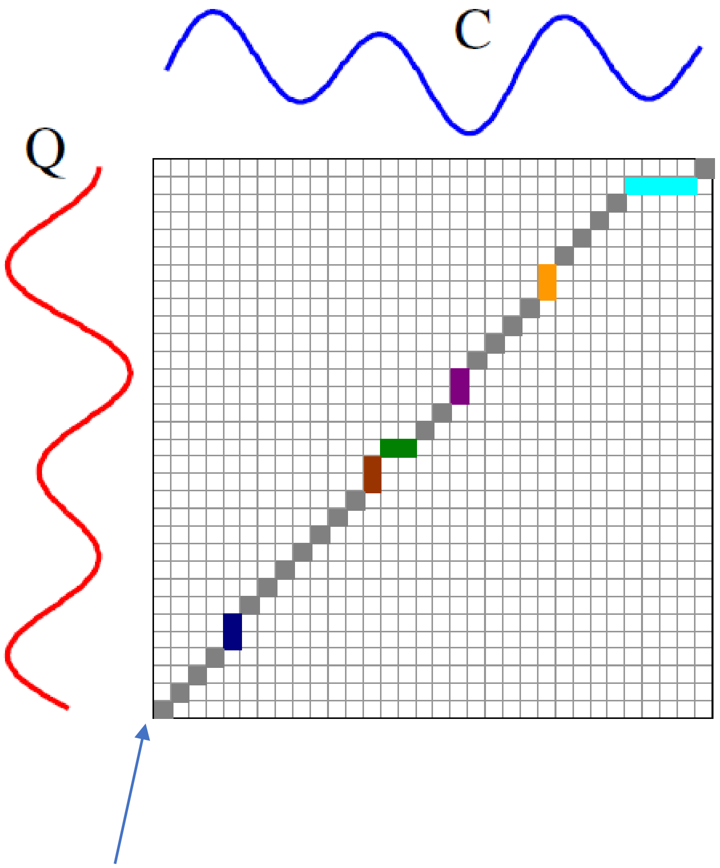
Distance Measure: Dynamic Time Warping (DTW)



Given two time series *Q* = *q₁*…*qₙ* and *C* = *c₁*…*cₙ*

We want to find a pairwise points alignment $w \in (\mathbb{N}x\mathbb{N})^n$, wich minimize the pairwise points distance: $\text{DTW}(Q, C) = \underset{P}{\text{argmin}}(\sqrt{\sum_{i=1}^{|P|}\left(q_{P_i\,[0]} - c_{P_i\,[1]}\right)^2})$

DTW calculation



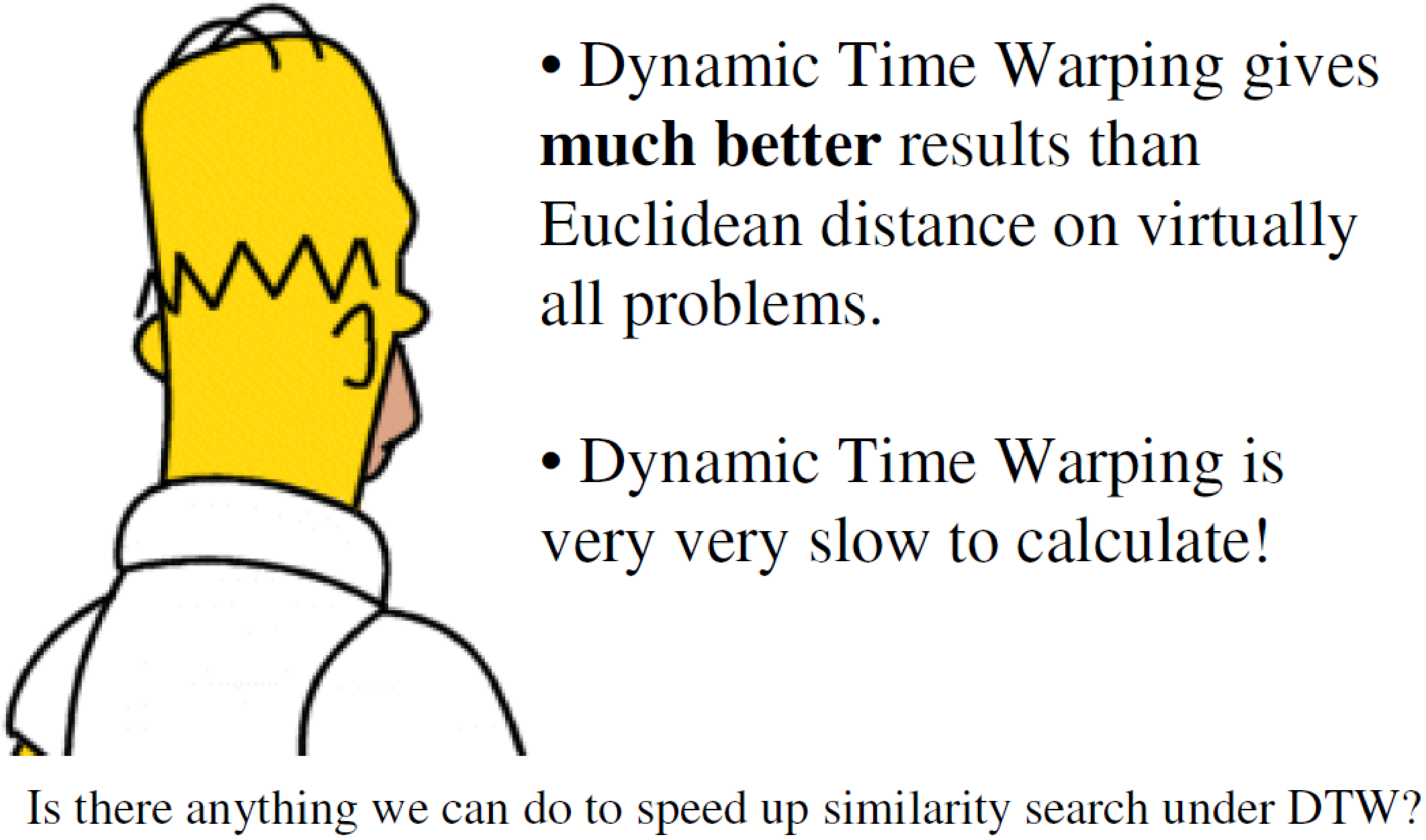
Warping path $P \in (\mathbb{N}x\mathbb{N})^n$

Recursive function of minimum cost path

$$\gamma(P_i) = \sqrt{(q_{P_i\,[0]} - c_{P_i\,[1]})^2 + \min\{\gamma\left(P_{i[0]} - 1\right), \gamma\left(P_{i[1]} - 1\right), \gamma\left(P_{i[0]} - 1\,, P_{i[1]} - 1\right)\}}$$

$\boldsymbol{P_0} = (\mathbf{0}, \mathbf{0})$ and $\boldsymbol{P_n} = (\mathbf{n}, \mathbf{n})$ are always the first and the last element of the warping path rispectively.

Time complexity – O(n^2)



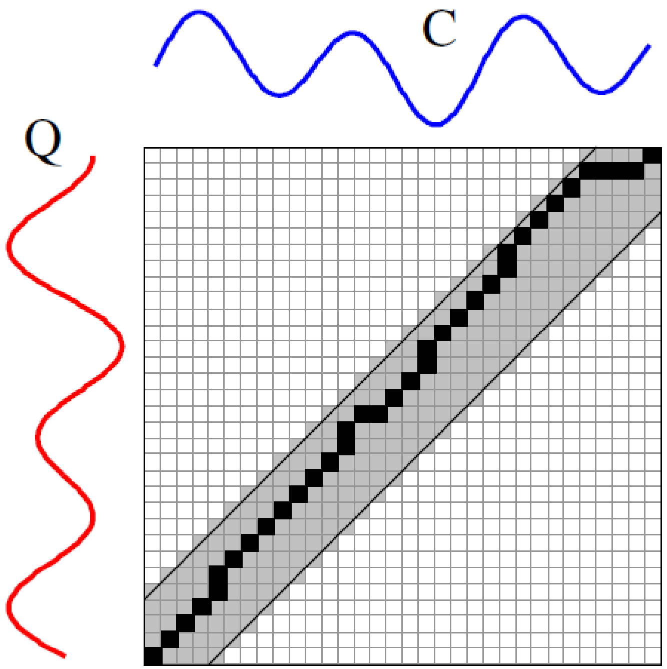
- Dynamic Time Warping gives **much better** results than Euclidean distance on virtually all problems.

- Dynamic Time Warping is very very slow to calculate!

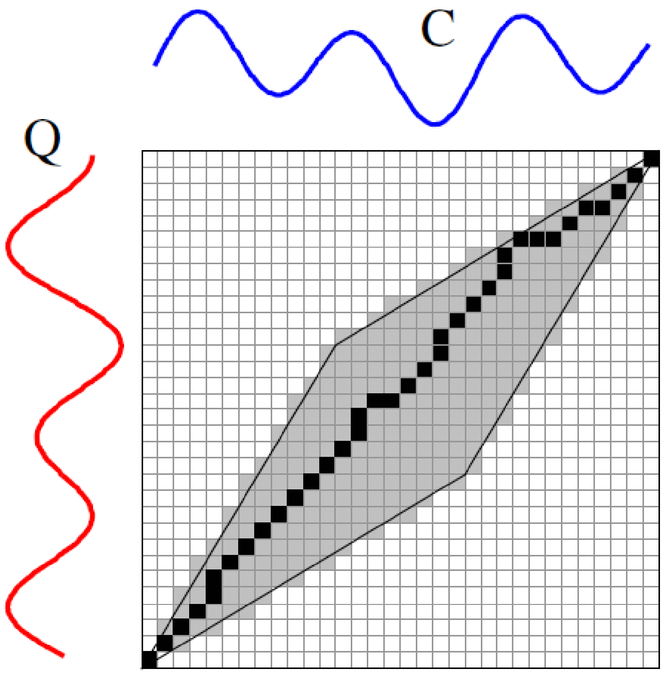
Is there anything we can do to speed up similarity search under DTW?

Global warping path constraints

- Slightly speed up the calculations
- Prevent pathological warpings

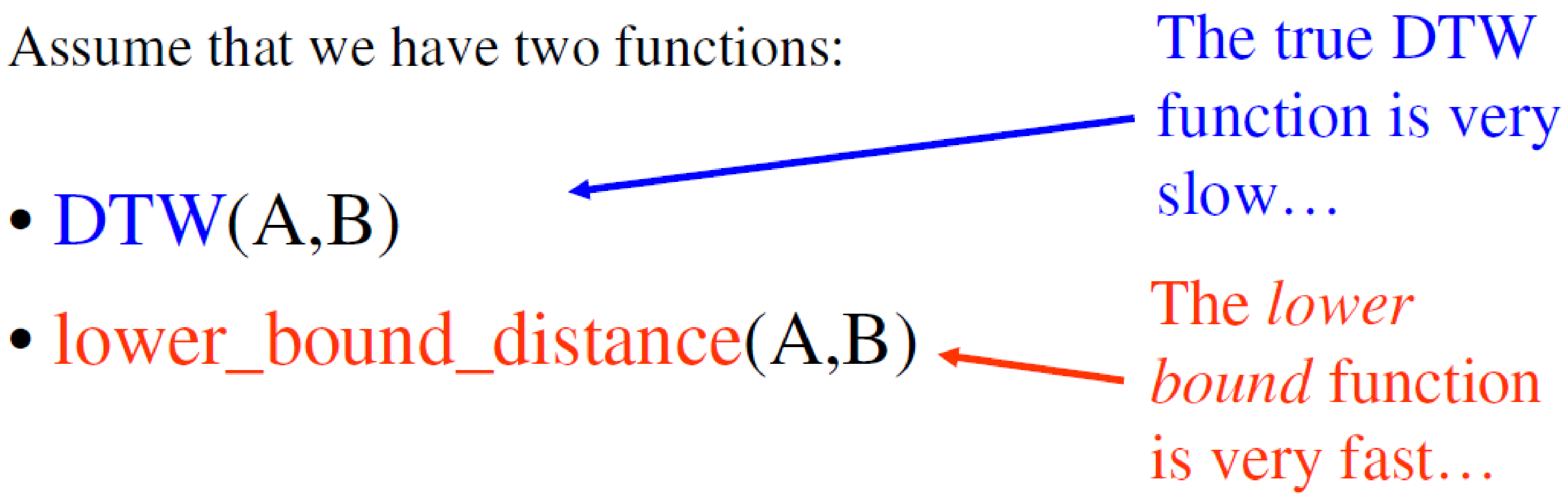


Sakoe-Chiba Band



Itakura Parallelogram

Lower bounding



Assume that we have two functions:

- DTW(A,B)
- lower_bound_distance(A,B)

The true DTW function is very slow…

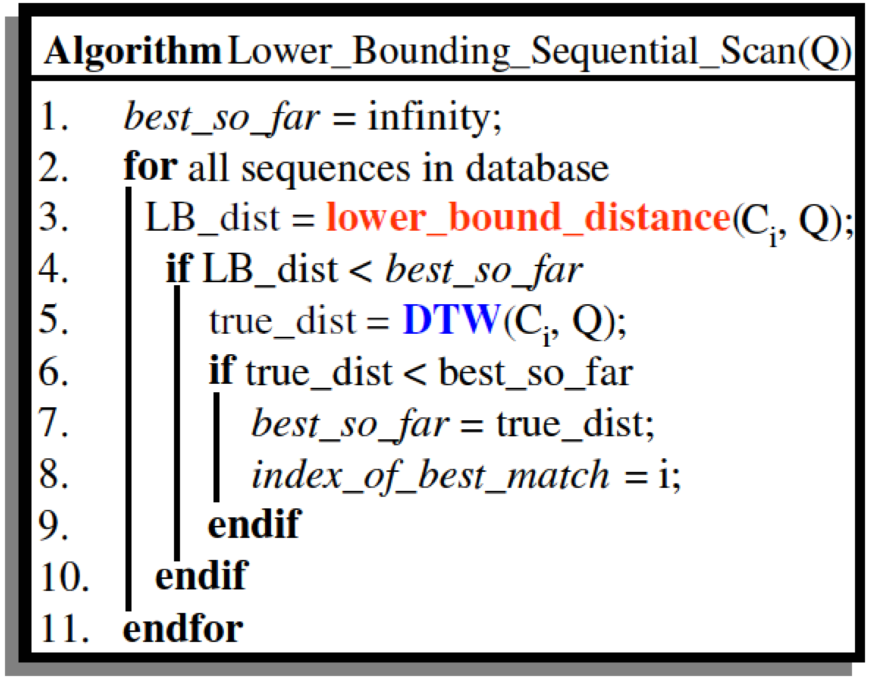
The *lower bound* function is very fast…

By definition, for all A, B, we have

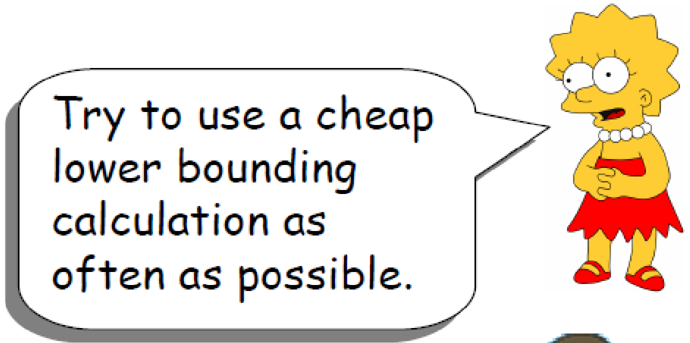
lower_bound_distance(A,B) ≤ DTW(A,B)

Speed up search with lower bounding

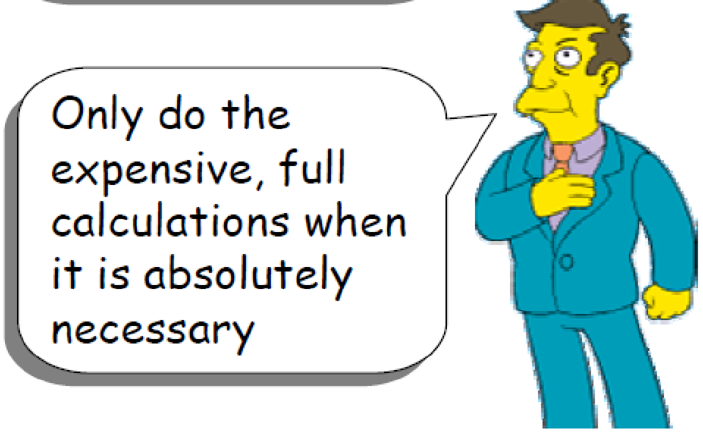
We can speed up similarity search under DTW
by using a lower bounding function



```
Algorithm Lower_Bounding_Sequential_Scan(Q)
best_so_far = infinity;
for all sequences in database
    LB_dist = lower_bound_distance($C_i$, Q);
    if LB_dist < best_so_far
        true_dist = DTW($C_i$, Q);
        if true_dist < best_so_far
            best_so_far = true_dist;
            index_of_best_match = i;
        endif
    endif
endfor
```

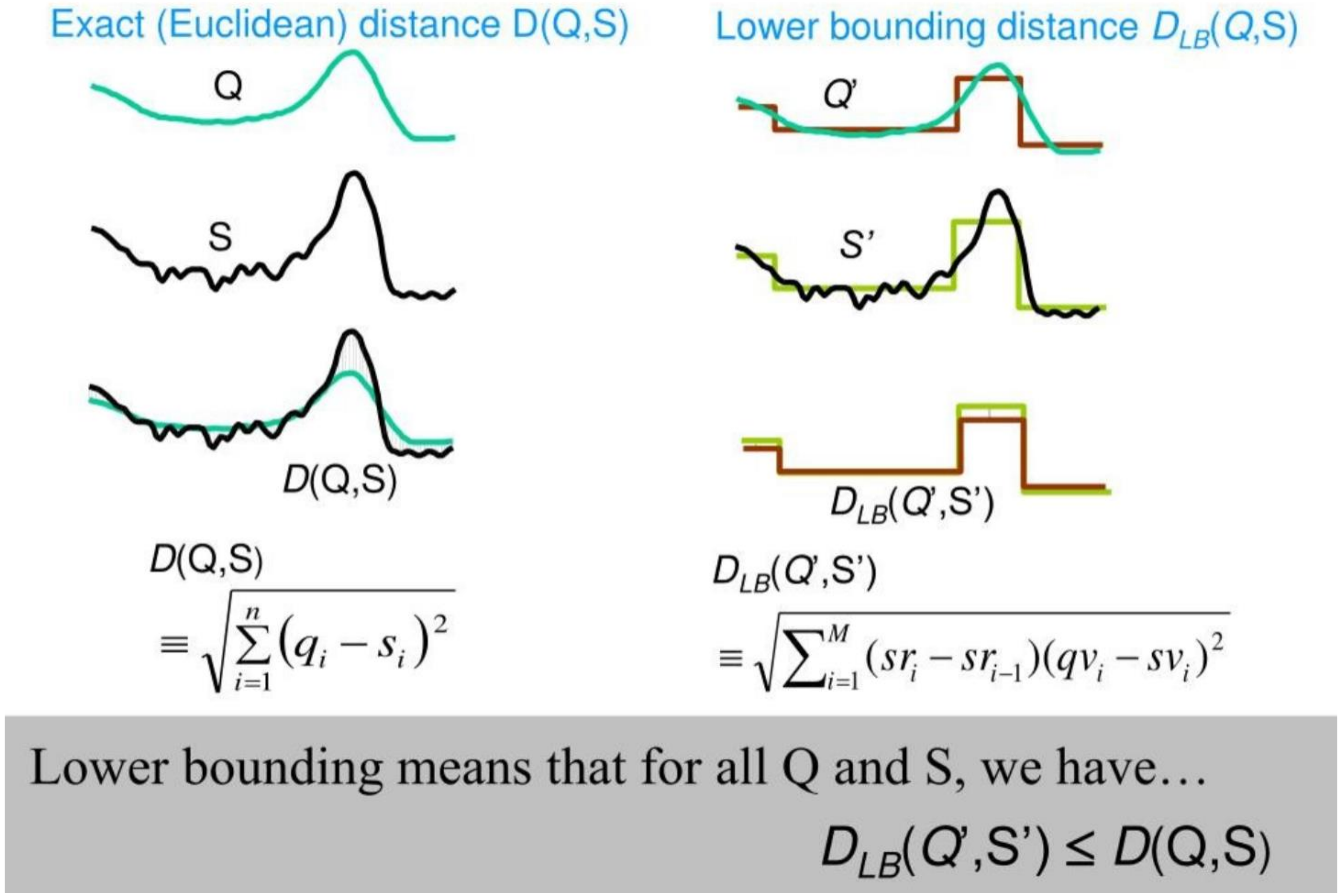


Try to use a cheap lower bounding calculation as often as possible.



Only do the expensive, full calculations when it is absolutely necessary

Lower Bounding Euclidean distance



Exact (Euclidean) distance D(Q,S)

Lower bounding distance $D_{LB}(Q,S)$

D(Q,S)

$$\equiv \sqrt{\sum_{i=1}^{n}\left(q_i - s_i\right)^2}$$

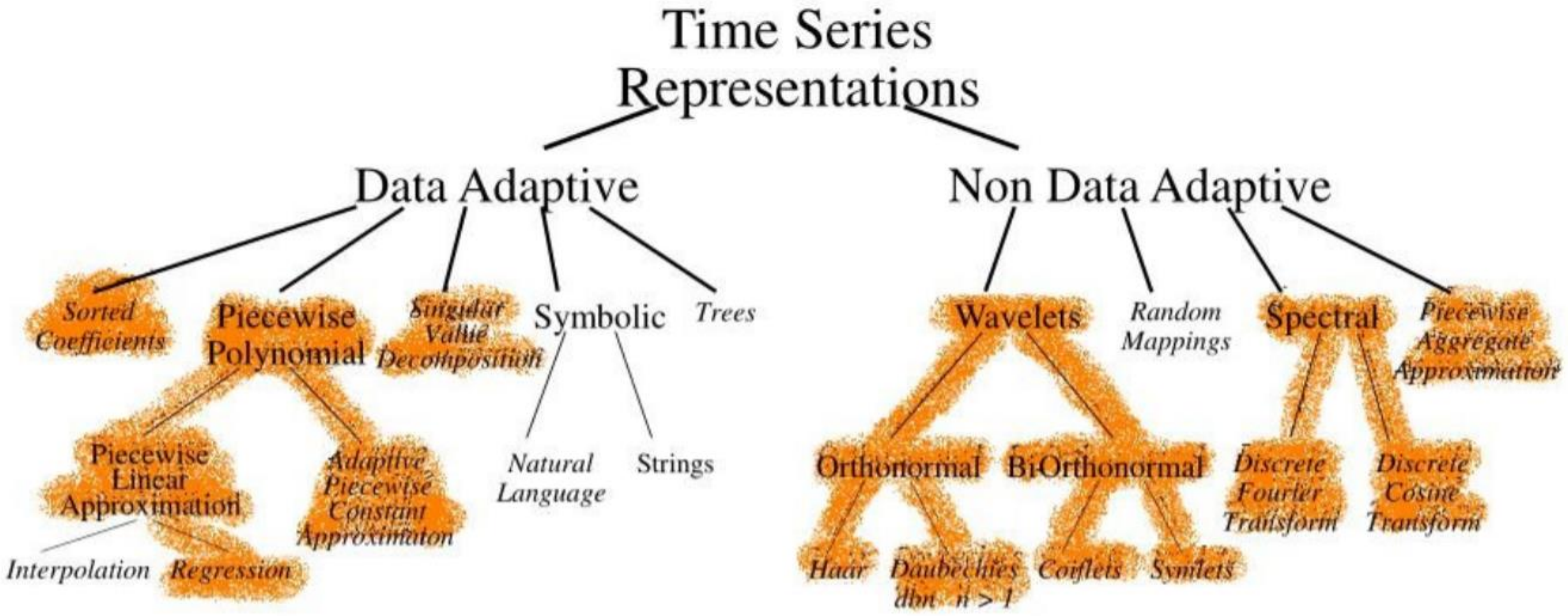
D_{LB}(Q',S')

$$\equiv \sqrt{\sum_{i=1}^{M}(sr_i - sr_{i-1})(qv_i - sv_i)^2}$$

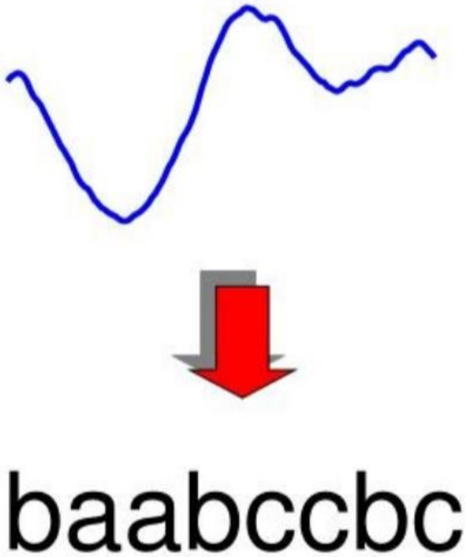
Lower bounding means that for all Q and S, we have…

$$D_{LB}(Q',S') \leq D(Q,S)$$

Lower bounding over discrete TS representation

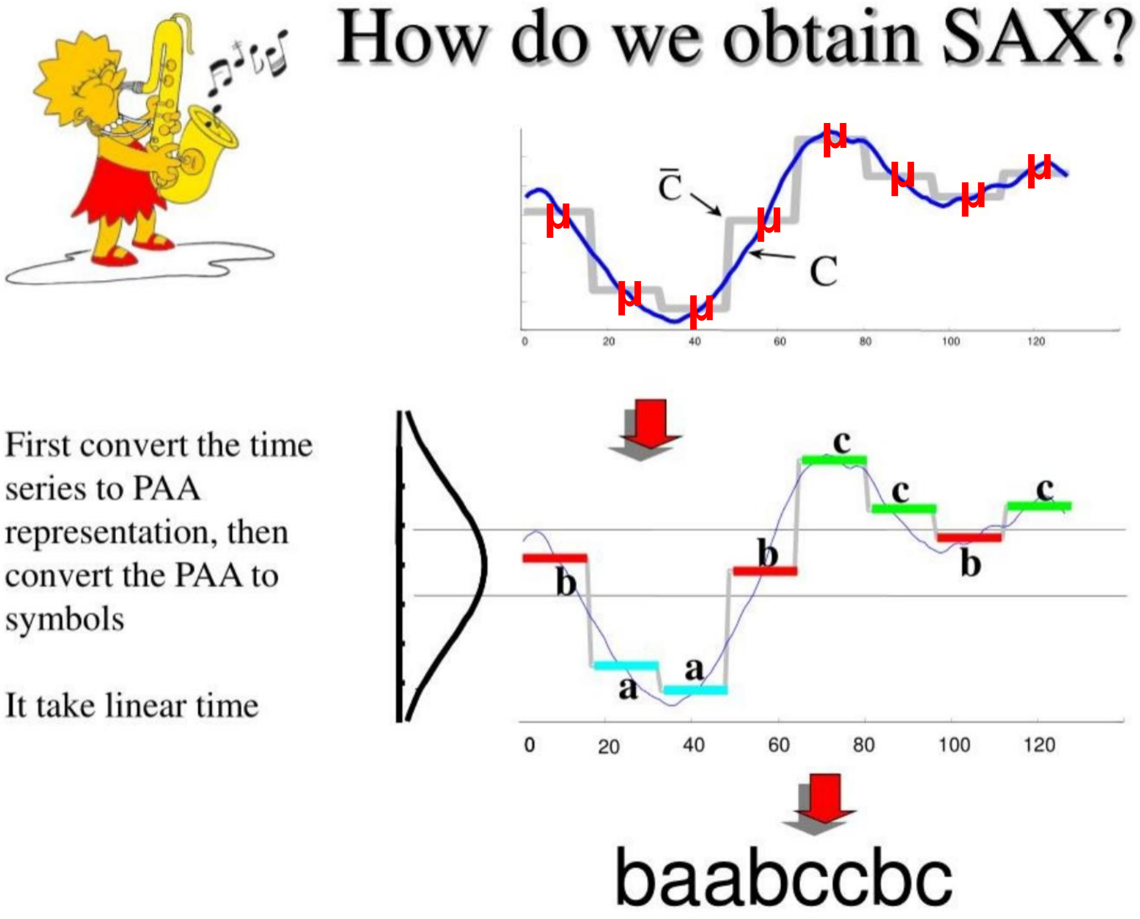


Symbolic Aggregate approXimation (SAX)



baabccbc

Symbolic Aggregate approXimation (SAX)



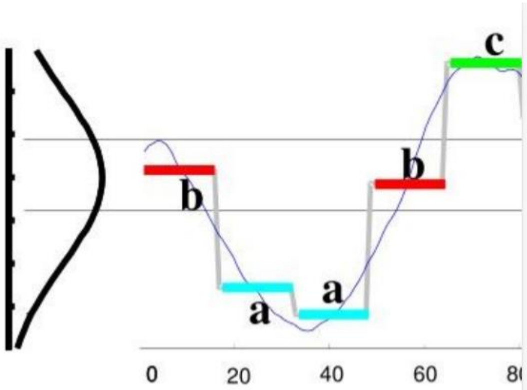
How do we obtain SAX?

First convert the time series to PAA representation, then convert the PAA to symbols

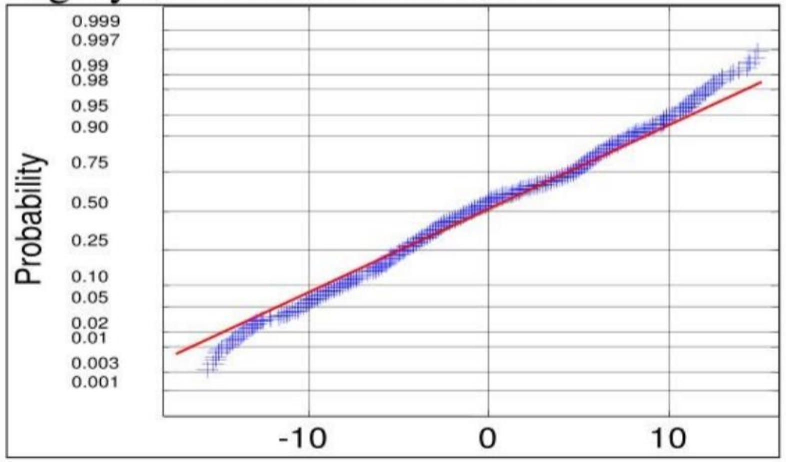
It take linear time

baabccbc

Symbolic Aggregate approXimation (SAX)

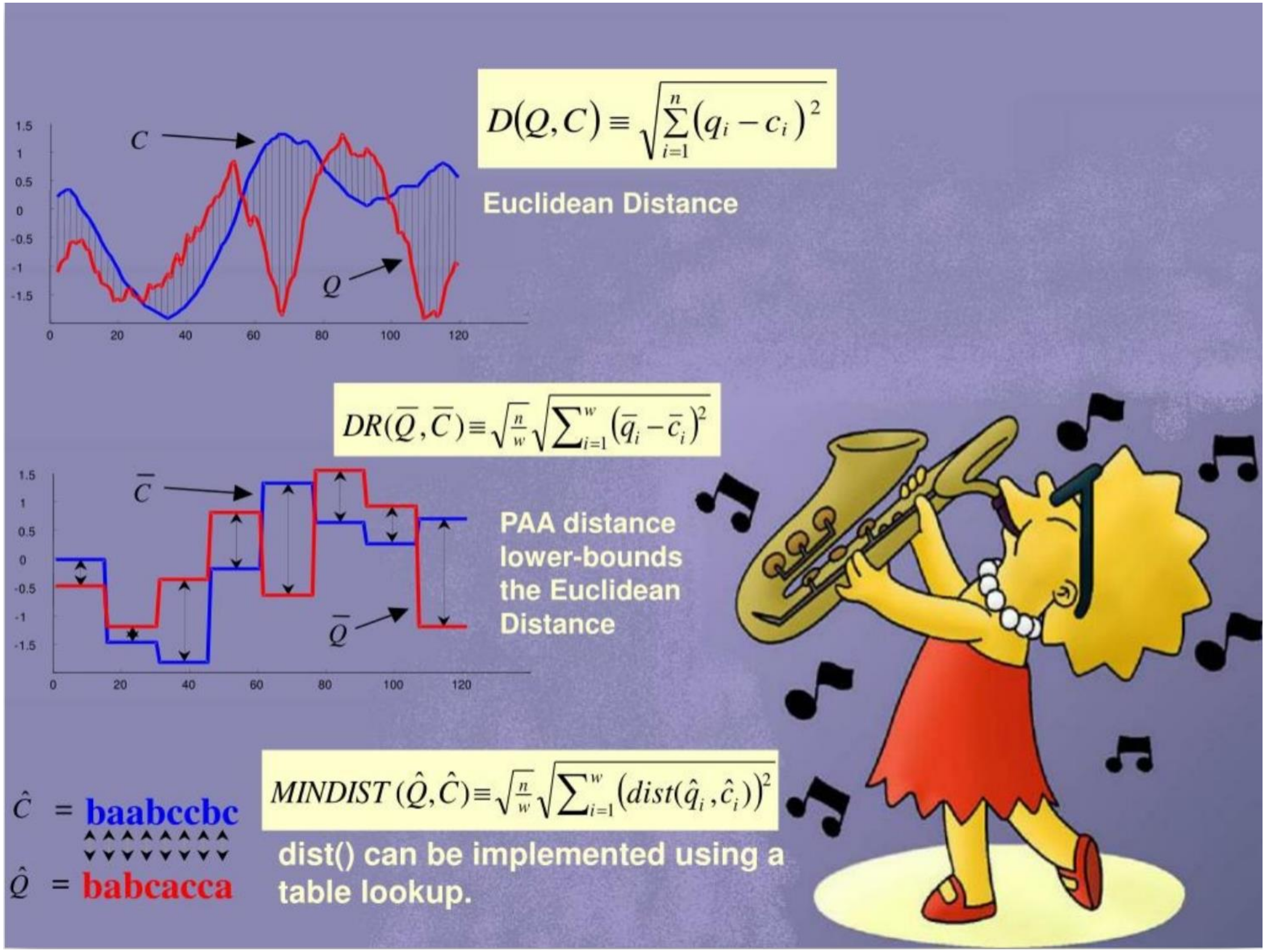


Time series subsequences tend to have a highly Gaussian distribution



A normal probability plot of the (cumulative) distribution of values from subsequences of length 128.

SAX lowerbounding (MINDIST)



$$D(Q,C) \equiv \sqrt{\sum_{i=1}^{n}(q_i - c_i)^2}$$

Euclidean Distance

$$DR(\bar{Q},\bar{C}) \equiv \sqrt{\tfrac{n}{w}}\sqrt{\sum_{i=1}^{w}(\bar{q}_i - \bar{c}_i)^2}$$

PAA distance lower-bounds the Euclidean Distance

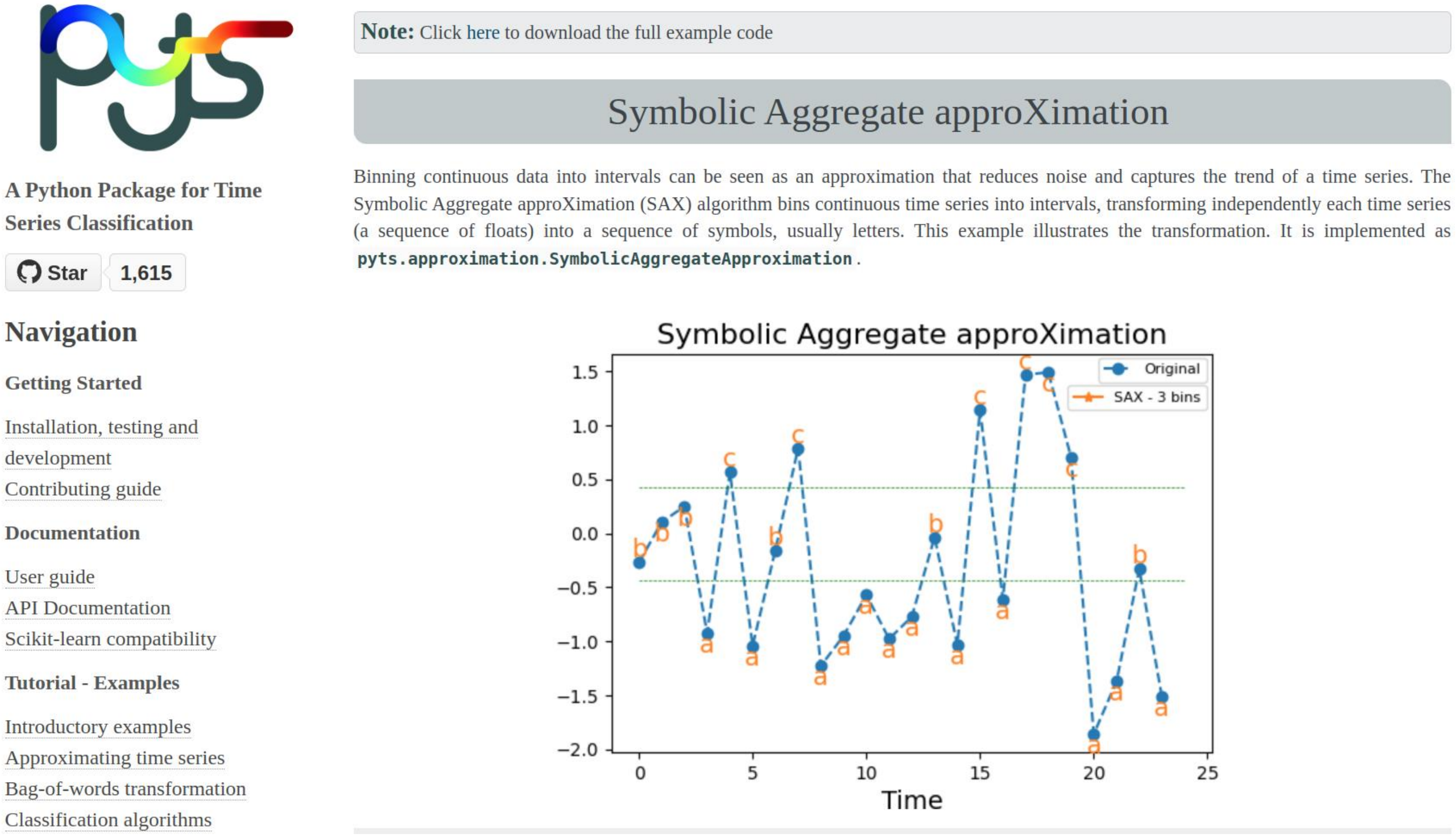
$\hat{C}$ = baabccbc

$\hat{Q}$ = babcacca

$$MINDIST(\hat{Q},\hat{C}) \equiv \sqrt{\tfrac{n}{w}}\sqrt{\sum_{i=1}^{w}(dist(\hat{q}_i,\hat{c}_i))^2}$$

dist() can be implemented using a table lookup.

Implementation (PYTS library)



A Python Package for Time Series Classification

Star 1,615

Navigation

Getting Started

- Installation, testing and development
- Contributing guide

Documentation

- User guide
- API Documentation
- Scikit-learn compatibility

Tutorial - Examples

- Introductory examples
- Approximating time series
- Bag-of-words transformation
- Classification algorithms

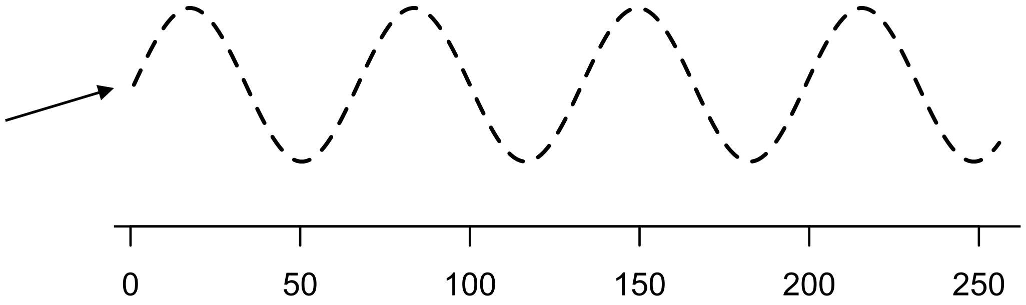
Note: Click here to download the full example code

Symbolic Aggregate approXimation

Binning continuous data into intervals can be seen as an approximation that reduces noise and captures the trend of a time series. The Symbolic Aggregate approXimation (SAX) algorithm bins continuous time series into intervals, transforming independently each time series (a sequence of floats) into a sequence of symbols, usually letters. This example illustrates the transformation. It is implemented as `pyts.approximation.SymbolicAggregateApproximation`.

Symbolic Aggregate approXimation
Original
SAX - 3 bins
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0
5
10
15
20
25
Time
b b b a c a b c a a a a a a b a c a c c c a a b a

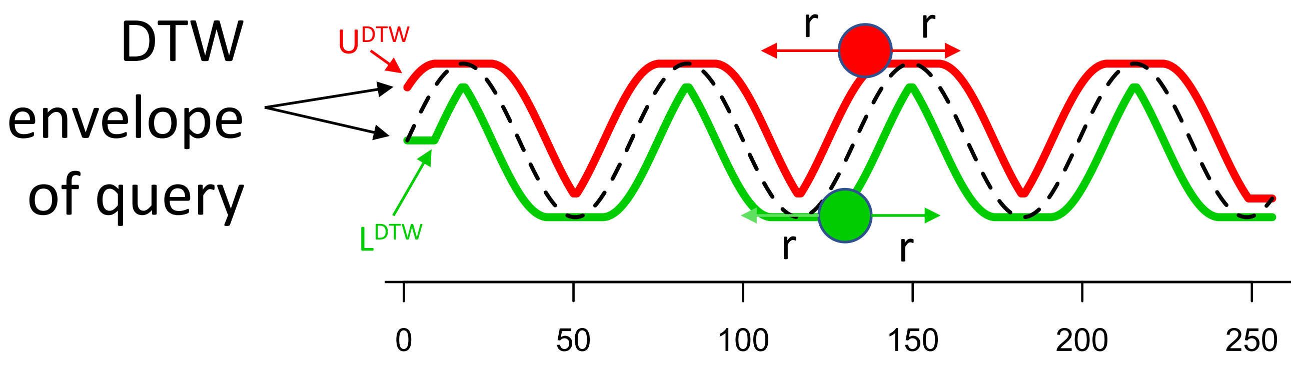
Lower bounding DTW



query

Lower bounding DTW

$\text{dtwENV}_r(\text{query}) = (L^{DTW}, U^{DTW})$

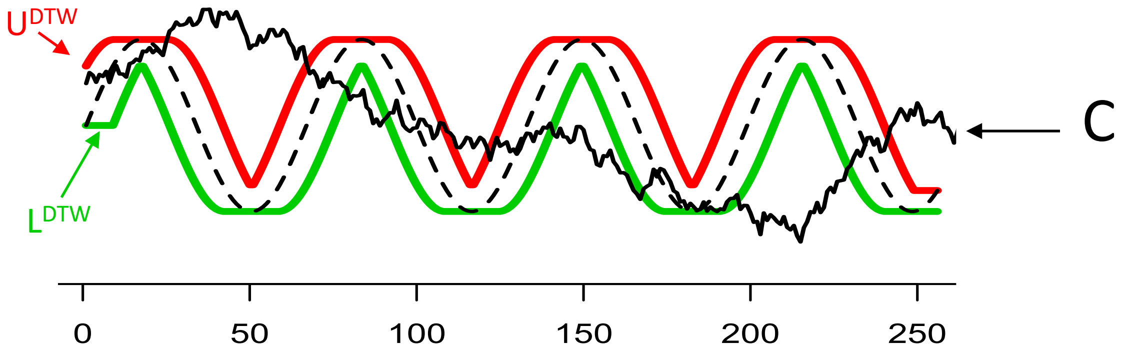


DTW envelope of query

- For each query position, dtwENV bounds the values that can be aligned to a single point in the data series candidates (warping window r).

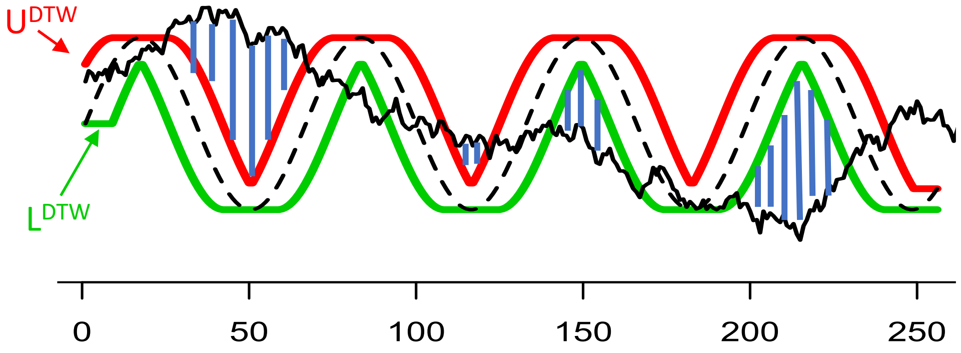
Lower bounding DTW

LB_Keogh(dtwENV$_r$(query), C)



Lower bounding DTW

LB_Keogh(dtwENV$_r$(query), candidate)



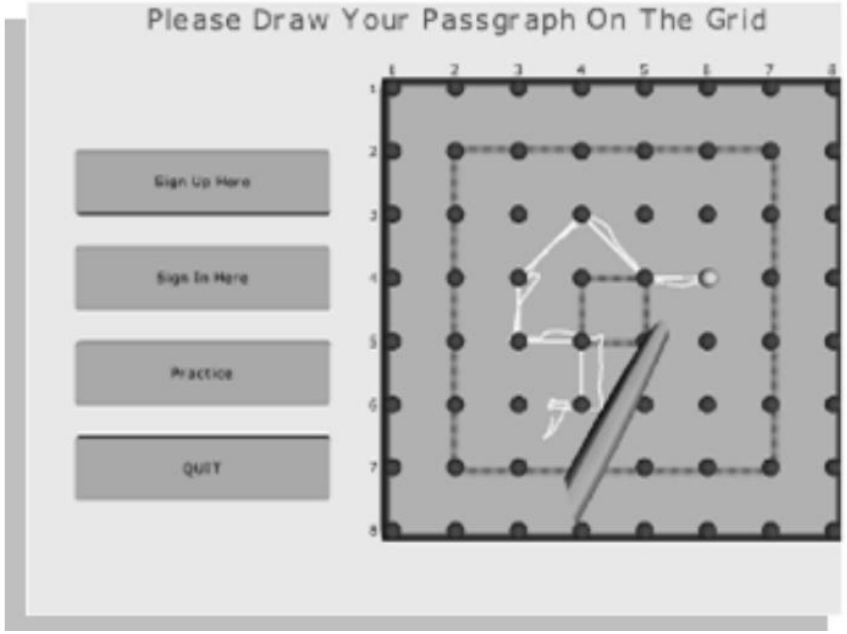
- Lower bounding of the true DTW distance between Query and Candidate

[O(n) time]

Presentation du TD

Haptics DATA

Data are taken from 5 people entering their passgraph (a code to access a system protected by a graphical authentication system) on a touchscreen. The data are the x-axis movement only.



Please Draw Your Passgraph On The Grid

Notebook

- Open the file (Python Notebook):

TS_SimilaritySearch.ipynb

- Instruction are contained in the notebook

References

- Eamonn J. Keogh. A Decade of Progress in Indexing and Mining Large Time Series Databases. VLDB 2006: 1268

- Eamonn J. Keogh, Li Wei, Xiaopeng Xi, Sang-Hee Lee, Michail Vlachos: **LB_Keogh Supports Exact Indexing of Shapes under Rotation Invariance with Arbitrary Representations and Distance Measures. VLDB 2006: 882-893**

- Chotirat Ann Ralanamahatana, Jessica Lin, Dimitrios Gunopulos, Eamonn Keogh, Michail Vlachos & Gautam Das **Mining Time Series Data** **https://link.springer.com/chapter/10.1007/0-387-25465-x_51#Abs1**

- https://tslearn.readthedocs.io/ (tslearn package documentation)