

Learning to see the hidden world: A perspective on causal representations

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Science and
Technology
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Causal
Learning and AI



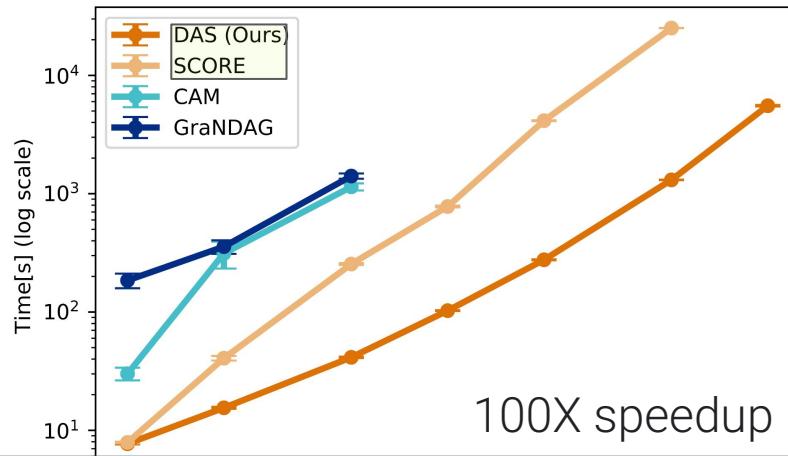
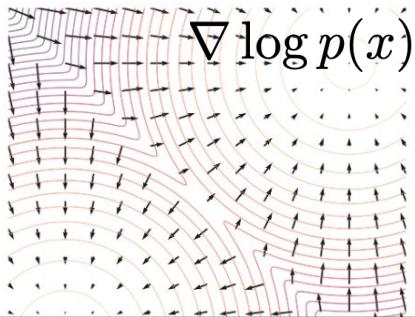
What triggers grooming behaviors in ants as a collective hygiene policy?

Why causal models?

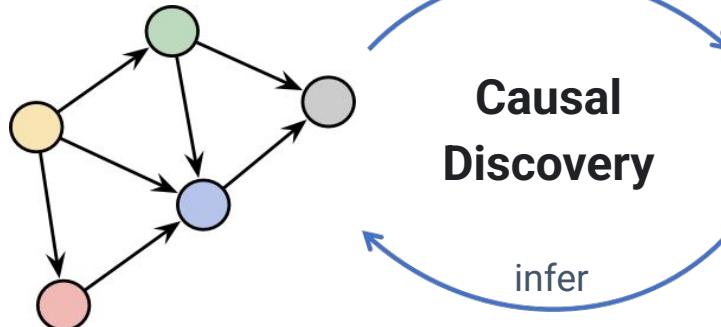
Observational data + **causal model** predict the result of **randomized studies** [1]

$$\begin{aligned} P(X_1, \dots, X_n) &= \prod_i P(X_i | \mathbf{PA}_i) \\ &\downarrow \\ P(X_1, \dots, X_n | do(X_j = x)) &= \prod_{i \neq j} P(X_i | \mathbf{PA}_i) \delta(X_j = x) \end{aligned}$$

Learning the graph

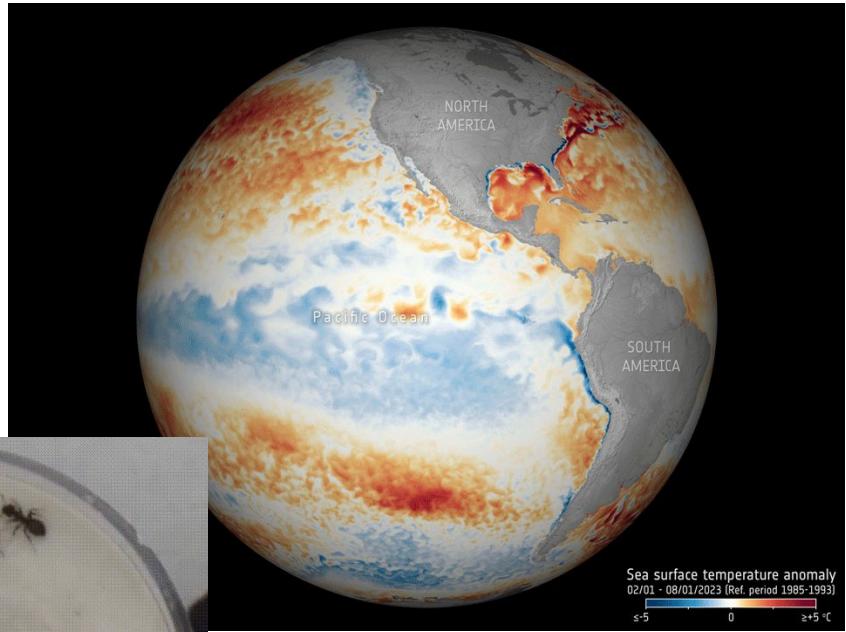
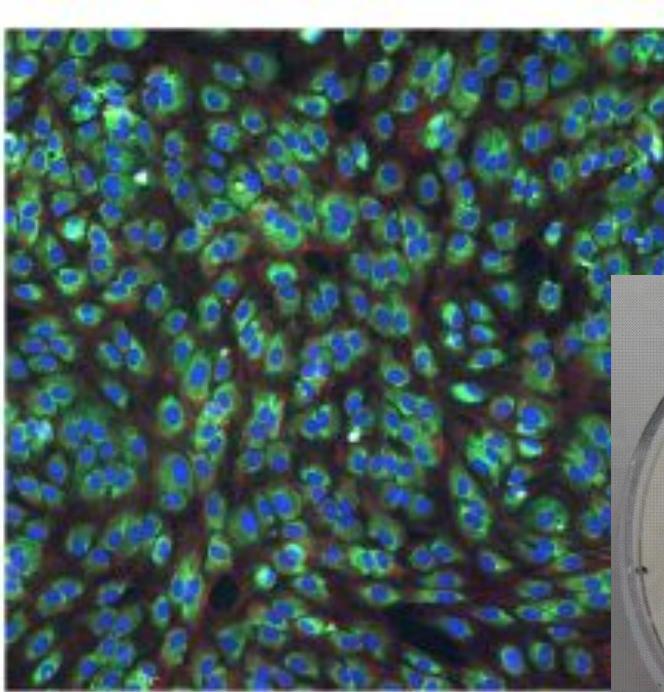


The discovery of the causal order converges linearly as $\exp(-\Theta\left(\frac{nC_m^2}{\log(m)}\right))$



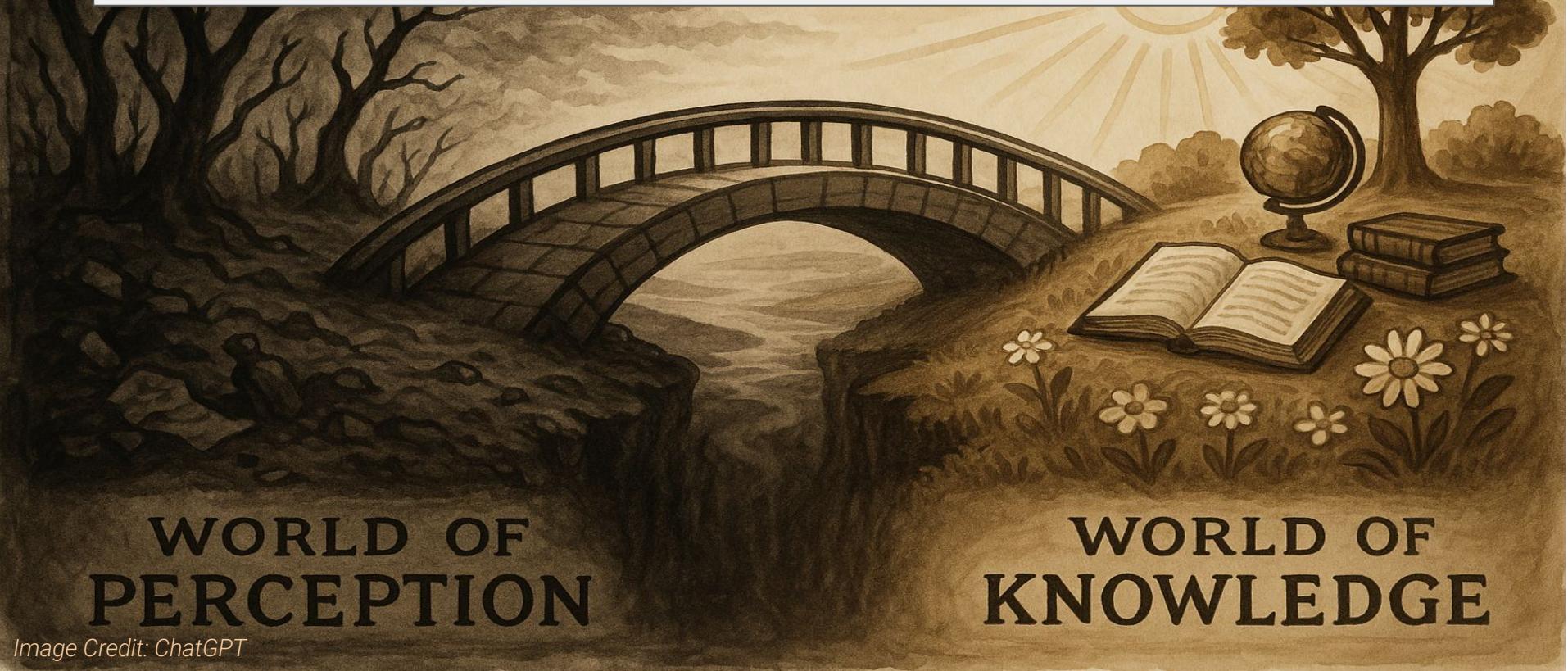
x_1	x_2	x_3	x_4	x_5
0.2	0.1	2.2	1.6	4.3
0.3	0.1	2.1	1.9	5.3
...

Data



[Image Credit: rxrx1 Dataset, www.esa.int/, ISTAnt dataset]

- Prediction-powered causal inference
- Exploratory causal inference
- Beyond “standard” causal models



**WORLD OF
PERCEPTION**

**WORLD OF
KNOWLEDGE**

What do we want?

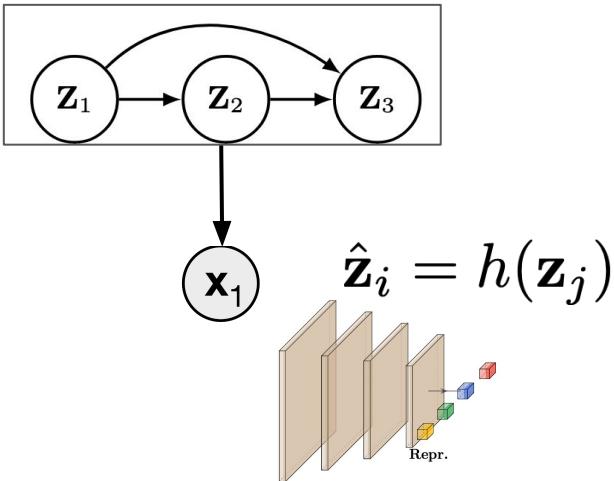
$$P(X_1, \dots, X_n) = \prod_i P(X_i | \mathbf{PA}_i)$$

$$\begin{aligned} P(X_1, \dots, X_n | do(X_j = x)) &= \prod_{i \neq j} P(X_i | \mathbf{PA}_i) \delta(X_j = x) \\ &= P(X_{V \setminus \{j, \mathbf{PA}_j\}} | X_j = x, X_{\mathbf{PA}_j}) P(X_{\mathbf{PA}_j}) \end{aligned}$$

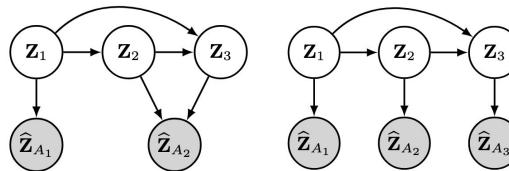
Causal estimand that is statistically identified on causal variables is also identifiable from the representation

Idea: Representation should make it easier/possible to extract causal information with some downstream estimator

Measurement perspective of CRL and identifiability



When is a learned representation a valid proxy for a causal variable? “measurement models” [1]



Conditions for causal validity of downstream estimate [2]:

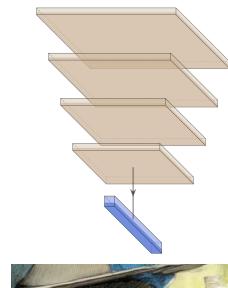
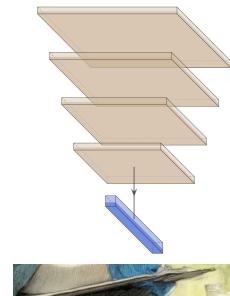
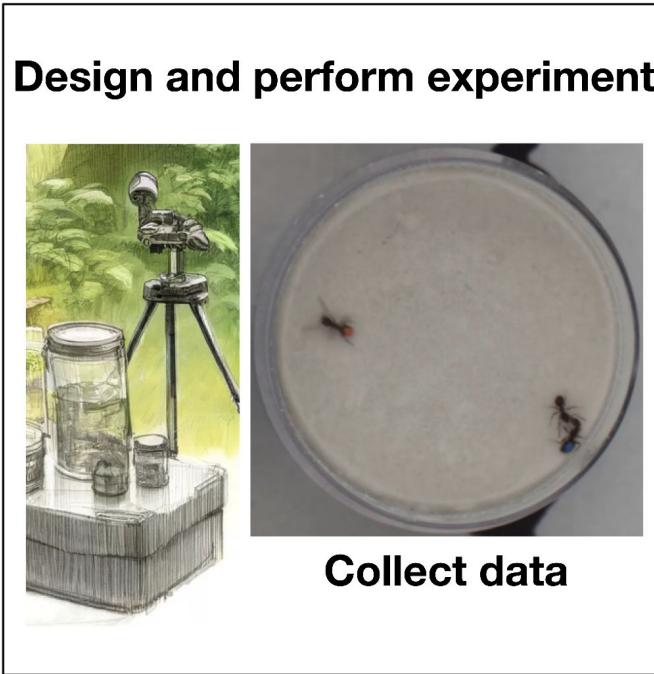
- Know $\hat{\mathbf{z}}_{A_j} \perp\!\!\!\perp \mathbf{z}_i \mid \mathbf{z}_{[N] \setminus \{i\}}$
- Estimate is invariant to h

[1] “Learning the structure of linear latent variable models” R. Silva, R. Scheines, C. Glymour, P. Spirtes, and D. M. Chickering. JMLR, 2006

[2] “The Third Pillar of Causal Analysis? A Measurement Perspective on Causal Representations”, Yao* and Huang*, Cadei, Zhang, L; NeurIPS 2025

[3] “Self-supervised Representation Learning Provably Isolates Content From Style”, von Kügelgen*, Sharma*, Greselle*, Brendel, Schölkopf+, Besserve+, L+ NeurIPS 2021

Exemplary pipeline in experimental ecology



ISTAnt dataset



No grooming
Blue to Focal
grooming

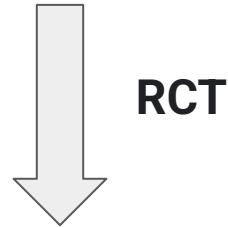


What? First benchmark to estimate causal effects from real world ecological videos collected in a randomized controlled trial.

Why is it unique?

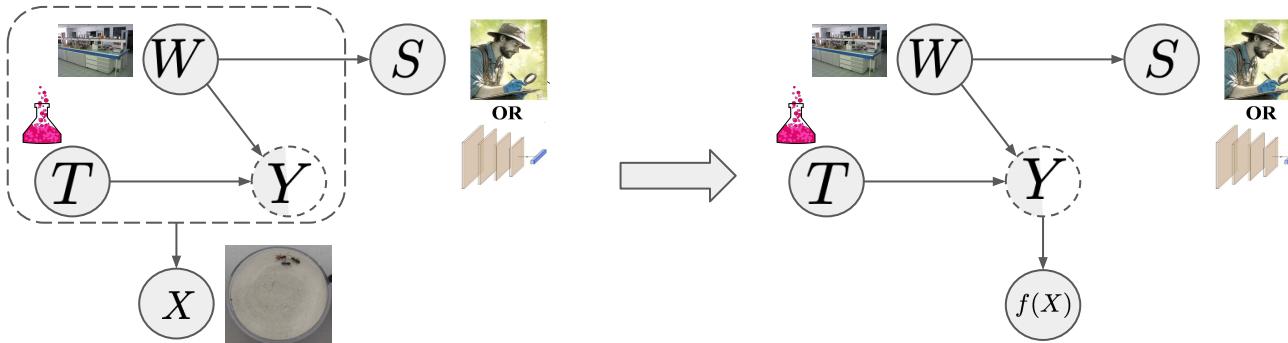
Other benchmarks stop at validating statistical accuracy. We estimate causal effects from real world ecological videos collected in a randomized controlled trial

$$ATE := \mathbb{E}[Y|do(T = 1)] - \mathbb{E}[Y|do(T = 0)].$$



$$AD := \mathbb{E}[Y|T = 1] - \mathbb{E}[Y|T = 0].$$

A measurement perspective on the problem

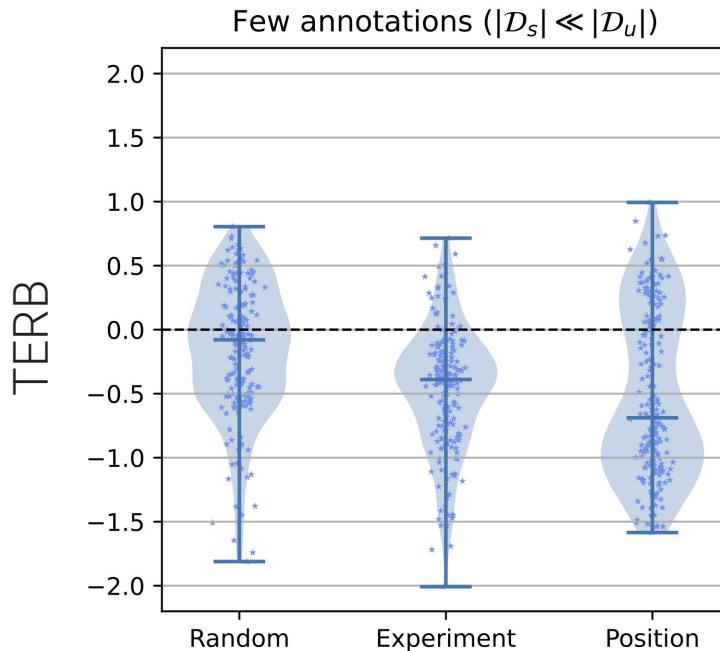


$$TEB := \left(\underbrace{\mathbb{E}_{\mathbf{X}|do(T=1)}[f(\mathbf{X})] - \mathbb{E}_{Y|do(T=1)}[Y]}_{\text{Interventional Bias under Treatment}} \right) - \left(\underbrace{\mathbb{E}_{\mathbf{X}|do(T=0)}[f(\mathbf{X})] - \mathbb{E}_{Y|do(T=0)}[Y]}_{\text{Interventional Bias under Control}} \right)$$

Errors come from:

- Selection bias: which samples are labelled?
- Pre-training data
- Discretization bias

Problem 1: What data to label?



Implication: Sampling choice matters, but random sampling is not always possible

Optimize for invariance in unified CRL

$$\iota(\mathbf{z}_A) = \iota(\tilde{\mathbf{z}}_A) \Leftrightarrow \mathbf{z}_A \sim_{\iota} \tilde{\mathbf{z}}_A.$$

Two vectors have the same projection onto the quotient induced by the equivalence relationship

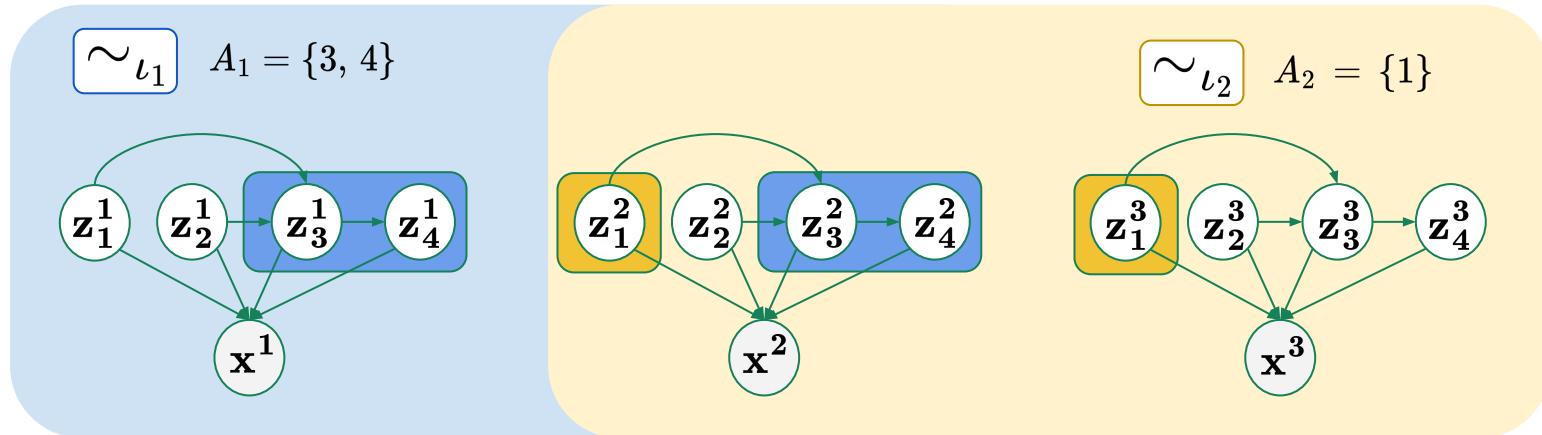
They belong to the same equivalence class

How? Assume access to multiple non-i.i.d. groups of sample. All samples in the same group are *equivalent* in some sense

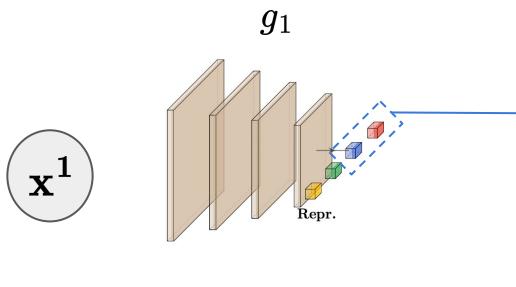
Why? Because you believe isolating these invariances is relevant to a task

Key idea for unified CRL: invariance principle

$$\iota(\mathbf{z}_A) = \iota(\tilde{\mathbf{z}}_A) \Leftrightarrow \mathbf{z}_A \sim_{\iota} \tilde{\mathbf{z}}_A.$$



Key idea for unified CRL: learning

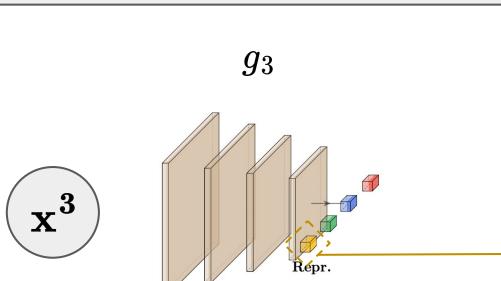


- Invariance Constraint:

$$g_1(\mathbf{x}^1) \underset{\square}{\sim_{\iota_1}} g_2(\mathbf{x}^2) \underset{\square}{\sim_{\iota_2}}$$

$$g_2(\mathbf{x}^2) \underset{\square}{\sim_{\iota_2}} g_3(\mathbf{x}^3) \underset{\square}{\sim_{\iota_3}}$$

Unify using a single “language” **31** different identification results from 28 paper!

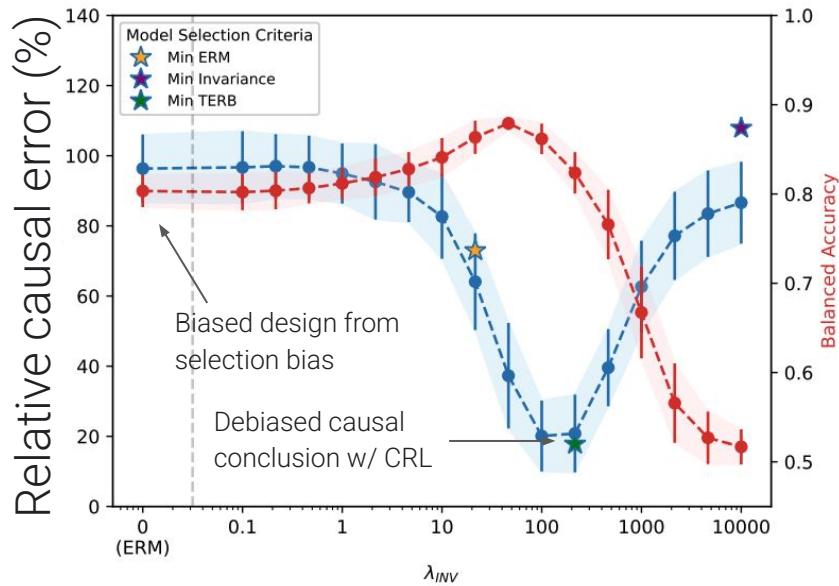


$$I(\mathbf{z}_{A_2}^k, g_k(\mathbf{x}^k) \underset{\square}{}) = H(\mathbf{z}_{A_2}^k) \quad k = 2, 3$$

- **Result:** Smooth encoders satisfying the two constraints block-identify invariant components

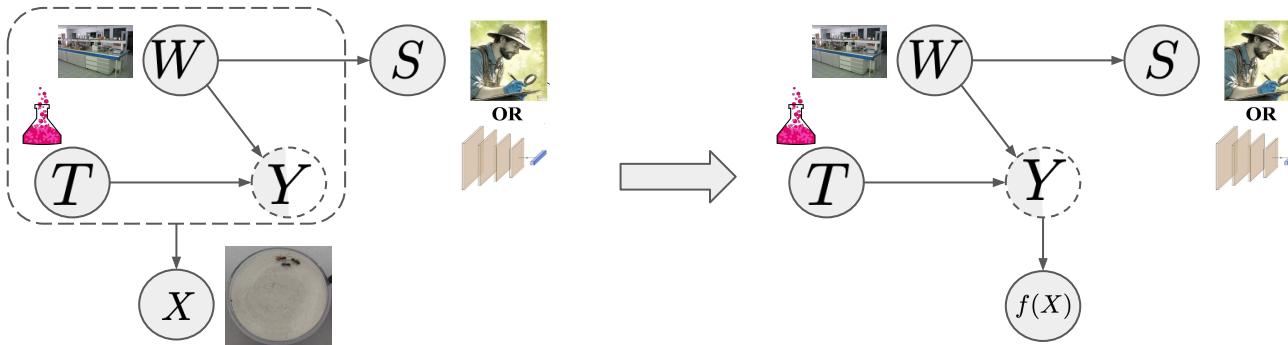
$$\hat{\mathbf{z}}_i = h(\mathbf{z}_j) \quad \boxed{\mathbf{z}_{A_1}^{1,2}} \quad \boxed{\mathbf{z}_{A_2}^{2,3}}$$

Debiasing with CRL and the invariance principle



Idea: Assume invariant representation across experiment settings

A measurement perspective on the problem

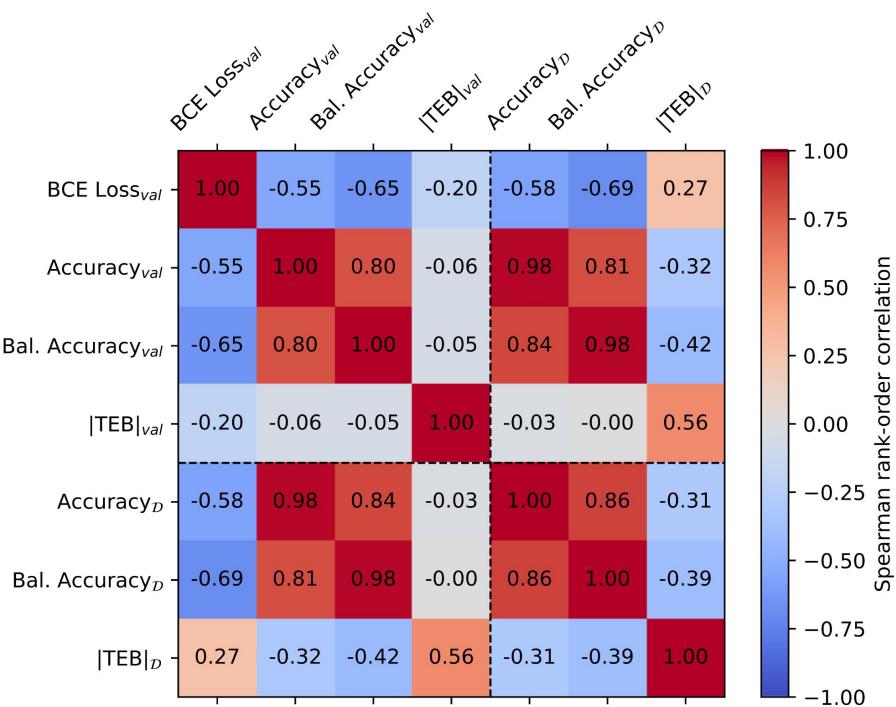
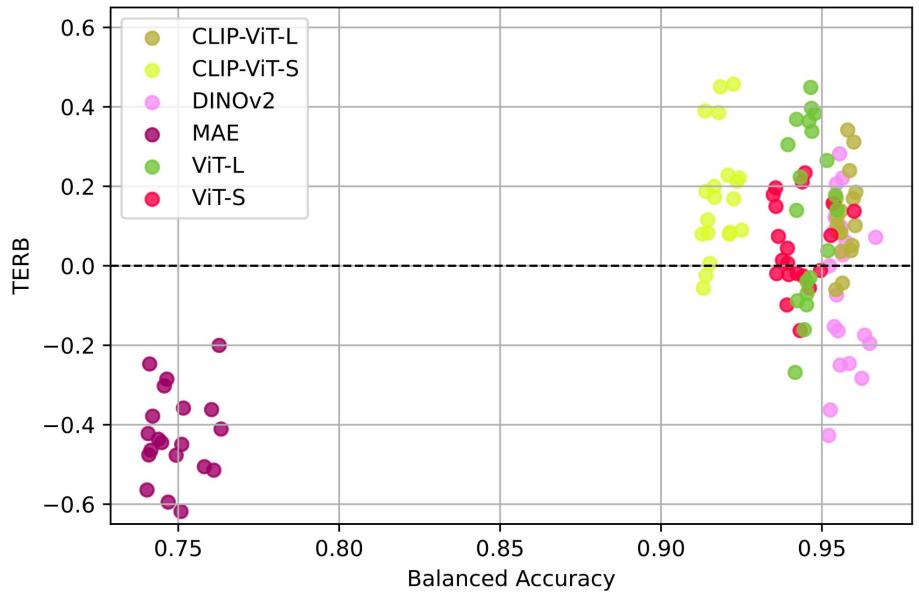


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Errors come from:

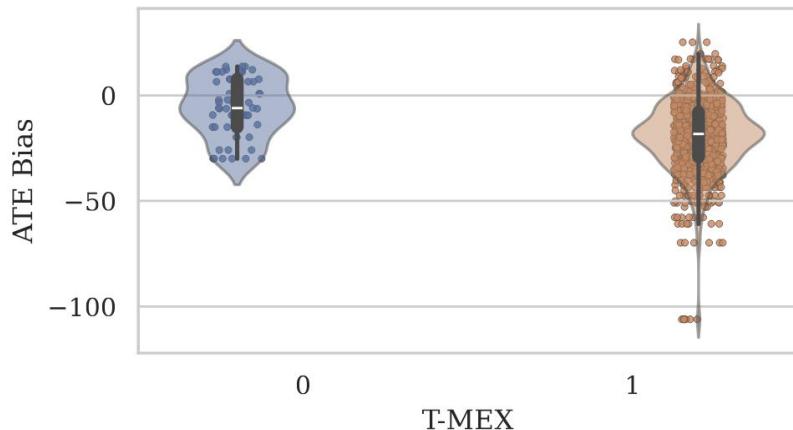
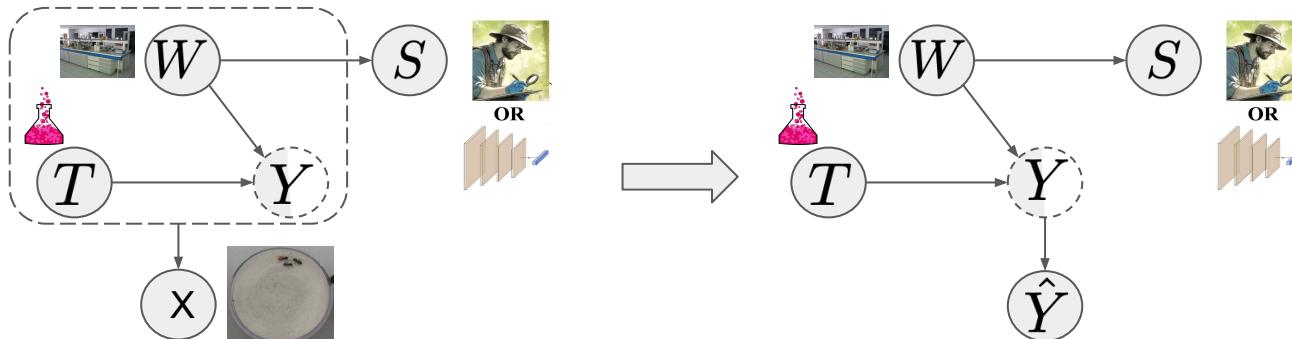
- Selection bias: which samples are labelled?
- Pre-training data
- Discretization bias

Problem 2: model choice



Implication: Models have different TERB and accuracy is not a good indicator of downstream causal performance. TEB on validation data works best

Selecting valid measurements with T-Mex



Conditions for causal validity of downstream estimator:

- $\hat{\mathbf{Z}}_{A_j} \perp\!\!\!\perp \mathbf{z}_i \mid \mathbf{Z}_{[N] \setminus \{i\}}$
- Estimator is invariant to h

Note: T-Mex doesn't need data to follow trial distribution (as long as the conditional indep. still holds)

Problem 3: Postprocessing

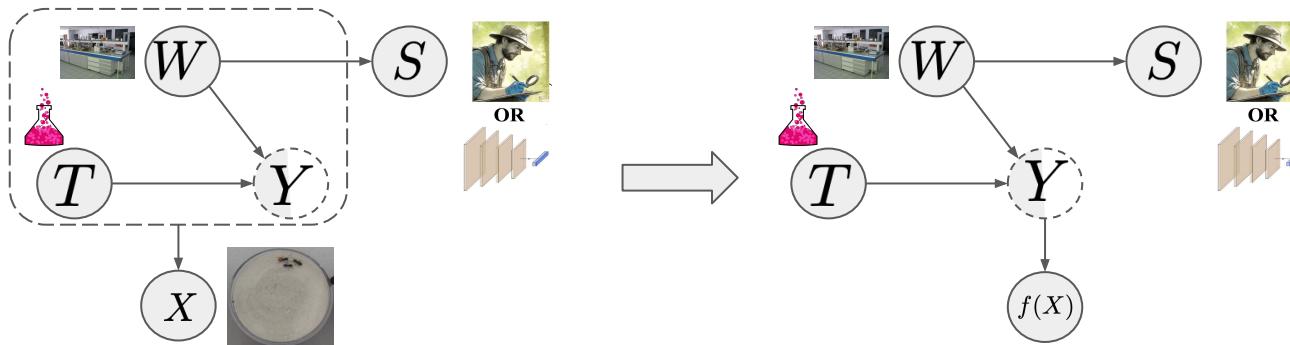
A common choice in ML is to threshold the predictions to make them binary.

Theorem [informal]: This choice can introduce bias.

$$\mathcal{H}_0 : \mathbb{E}[|\text{TEB}(f)|] = \mathbb{E}[|\text{TEB}(\mathbf{1}_{[0.5,1]}(f))|] \quad vs \quad \mathcal{H}_1 : \mathbb{E}[|\text{TEB}(f)|] < \mathbb{E}[|\text{TEB}(\mathbf{1}_{[0.5,1]}(f))|]$$

T-test rejects null hypothesis with $p \sim 10^{-25}$

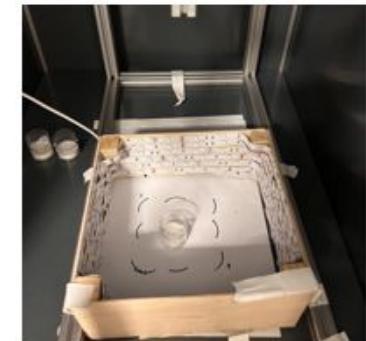
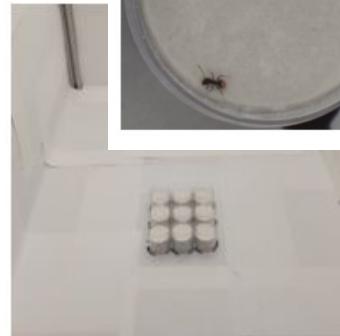
How do we optimize for validity



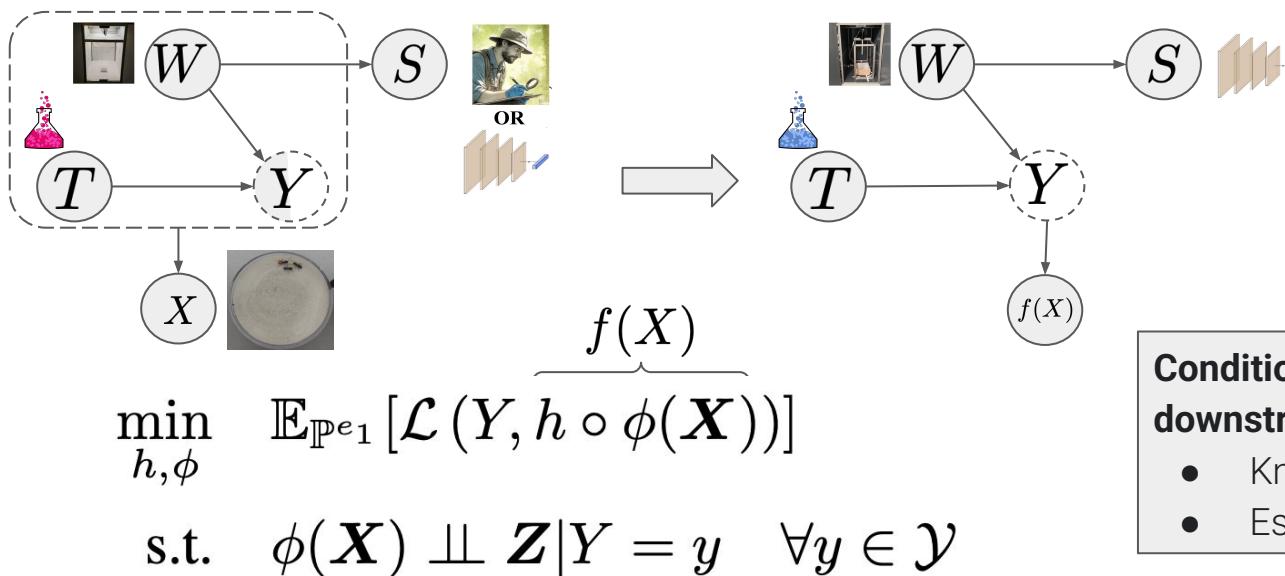
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Thm: Conditional calibration $\mathbb{E}[Y - f(X)|W, T] = 0$ implies valid estimates with correct *confidence intervals* using AIPW. Conditional independence of the measurement model implies conditional calibration.

Zero-shot transfer across experiments



Validity across experiments

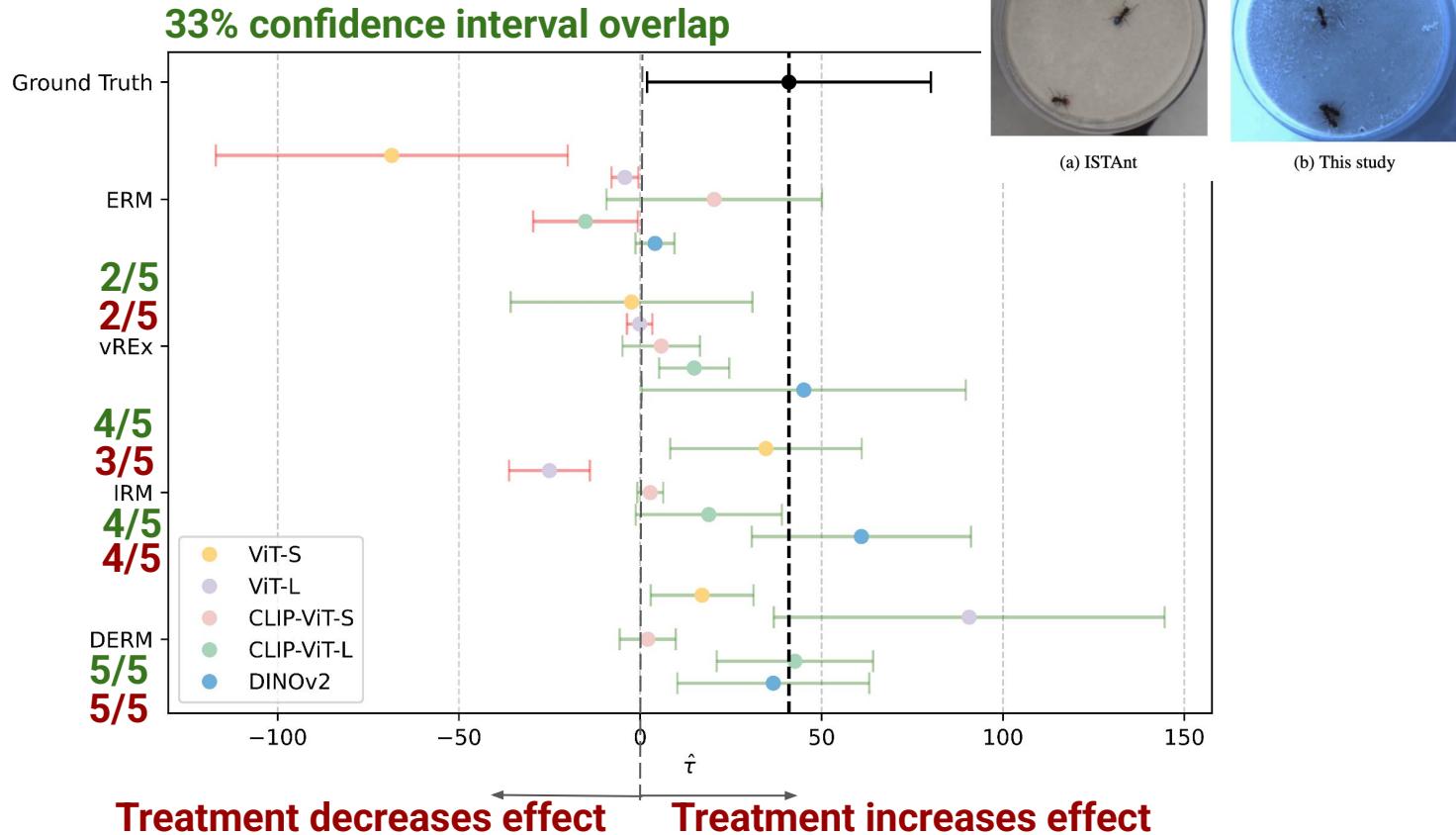


Conditions for causal validity of downstream estimate:

- Know $\hat{\mathbf{z}}_{A_j} \perp\!\!\!\perp \mathbf{z}_i | \mathbf{z}_{[N] \setminus \{i\}}$
- Estimate is invariant to h

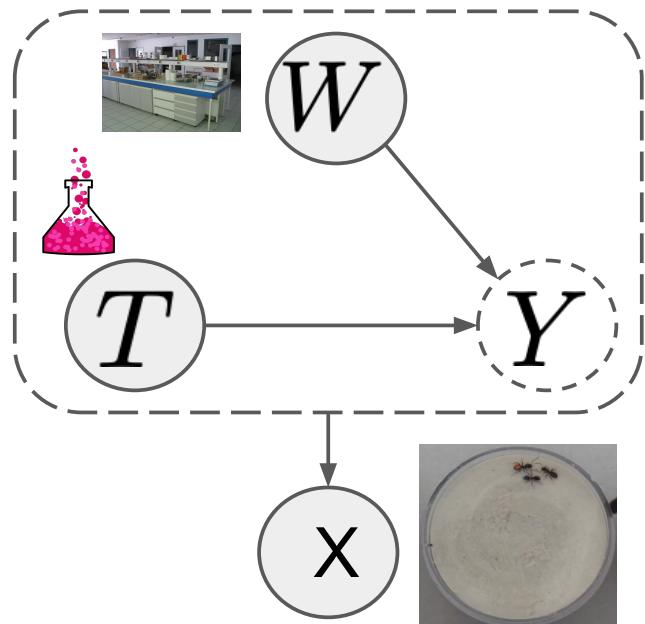
Thm: if a representation is valid on a training experiment and transfers to a target experiment while satisfying $\phi(\mathbf{X}) \perp\!\!\!\perp \mathbf{Z} | Y = y$, then it remains causally valid.

Results

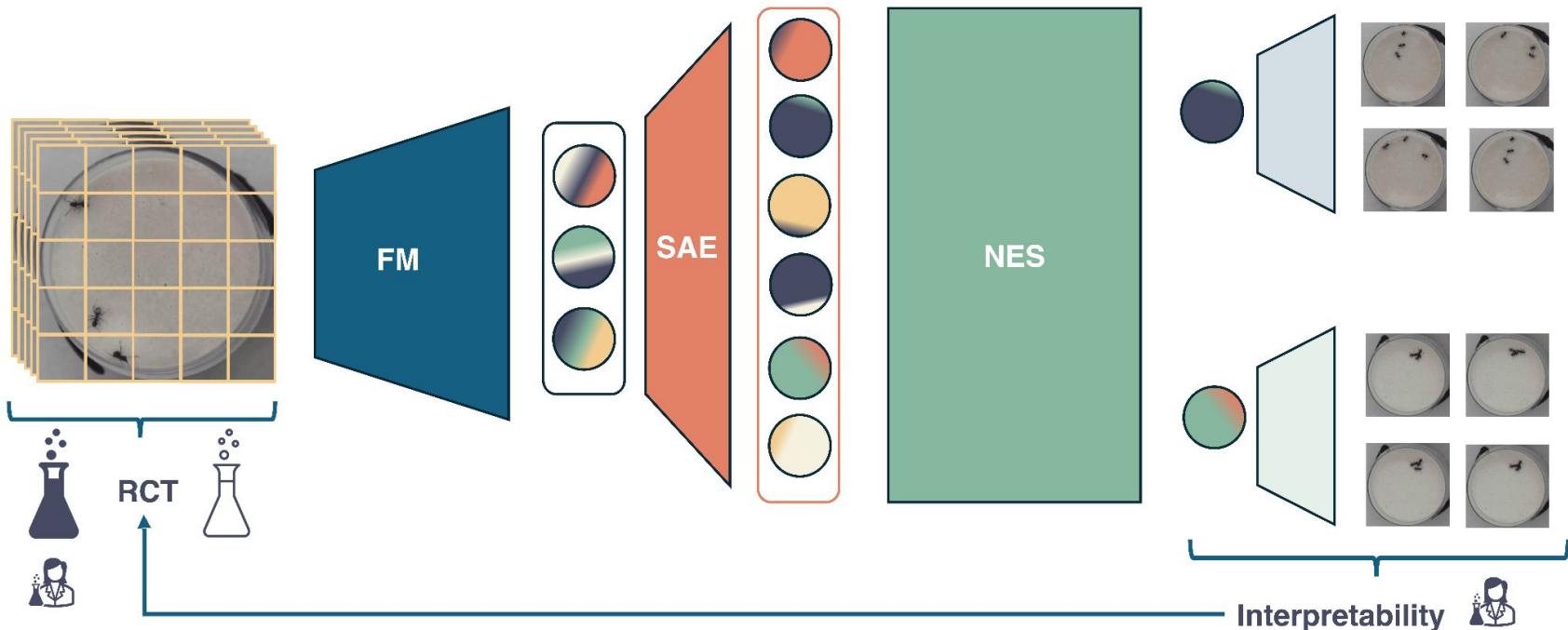


Exploratory causal inference

Exploratory causal inference

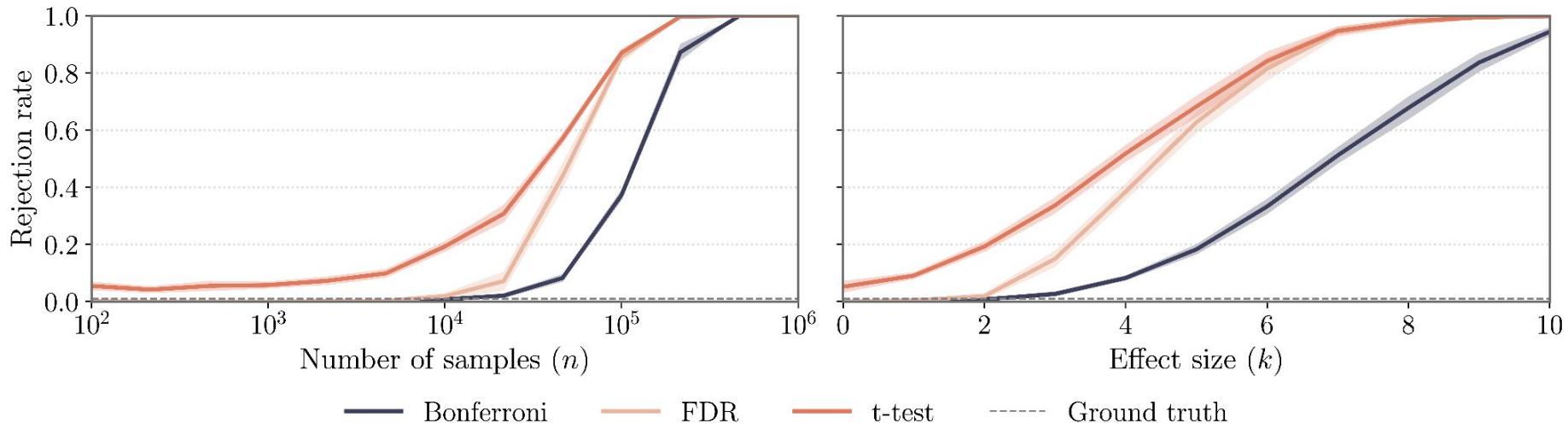


Deep networks as data-driven measurement devices



Hypothesis testing challenges: the paradox of ECI

NES: Consistent recursive testing procedure that corrects the entanglement from previously discovered hypotheses.



Idea: With powerful tests (strong effects or large sample sizes), all correlations are statistically significant. Small entanglement → all neurons are individual effects.

Results on synthetic trials

Most activated images for Neuron 38



Most activated images for Neuron 6051

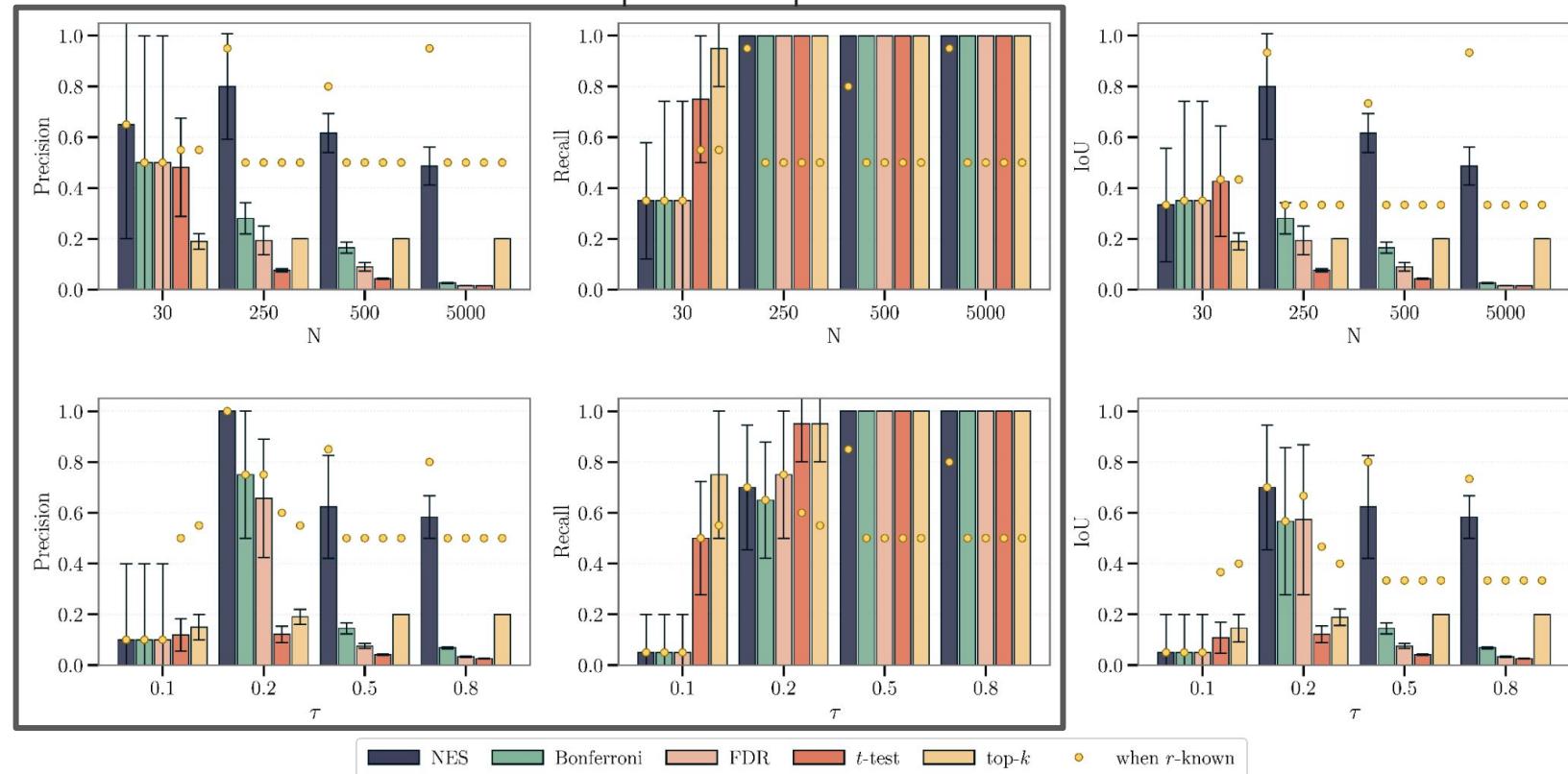


█ stronger activation (left panel)
█ weaker activation (left panel)

█ stronger activation (right panel)
█ weaker activation (right panel)

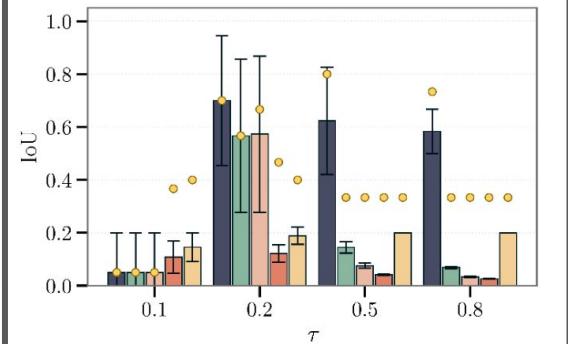
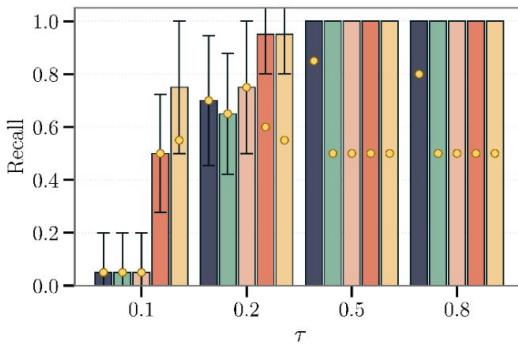
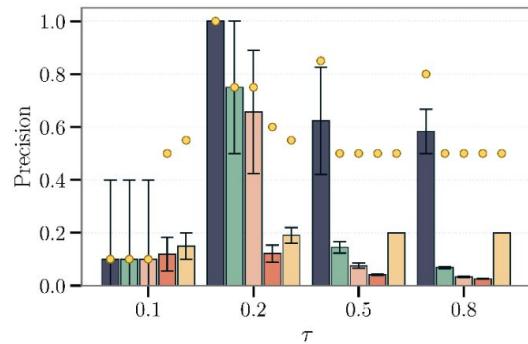
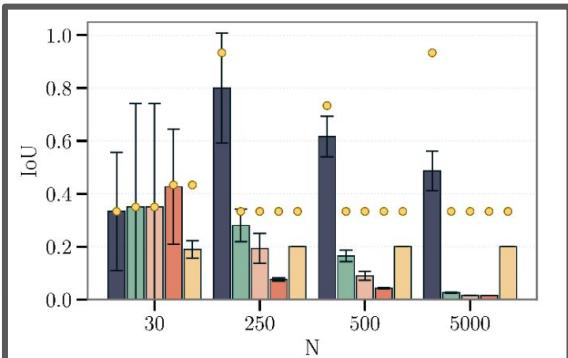
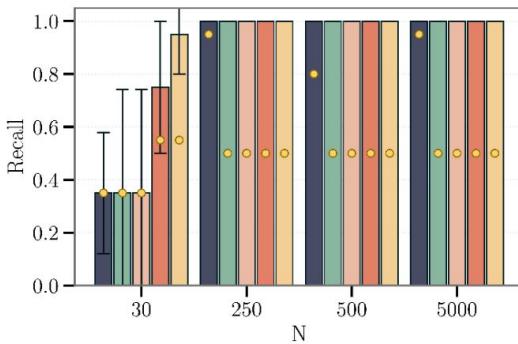
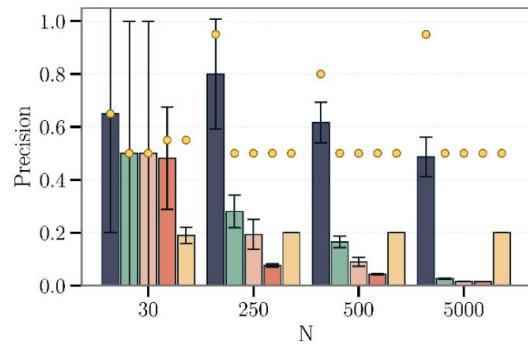
Results on synthetic trials

Paradox of ECI: Precision collapses w/ powerful tests



Results on synthetic trials

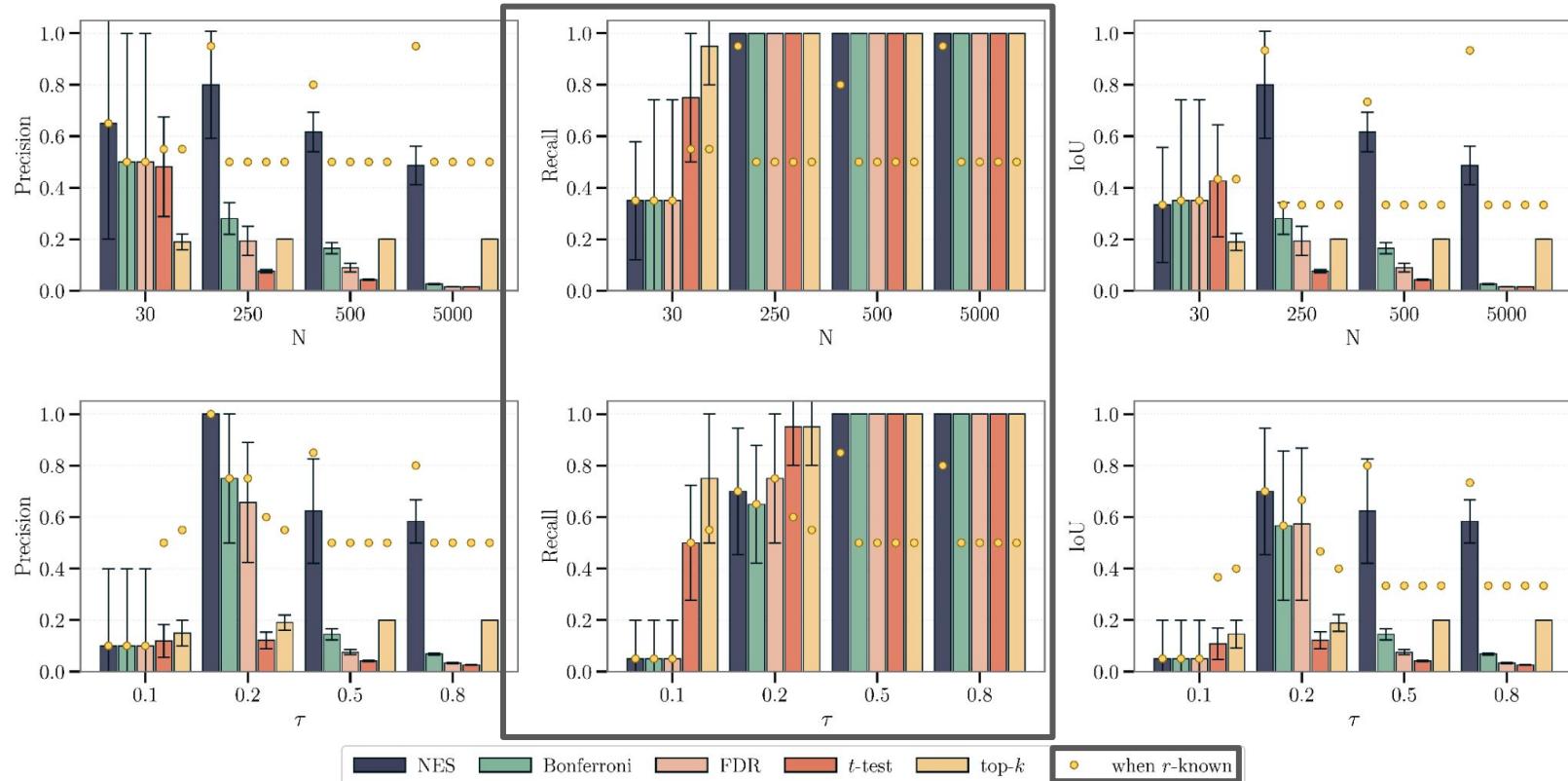
NES is robust and works well



█ NES █ Bonferroni █ FDR █ t -test █ top- k ● when r -known

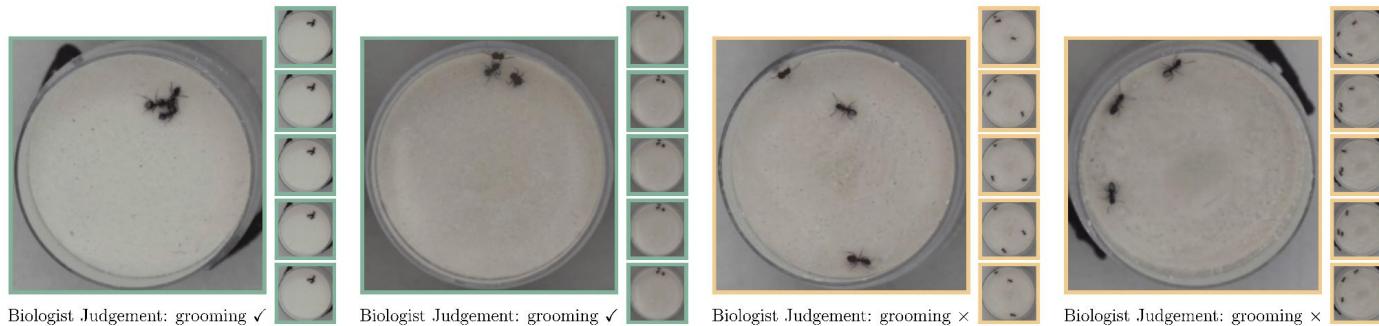
Results on synthetic trials

If r is known, NES does not miss effects

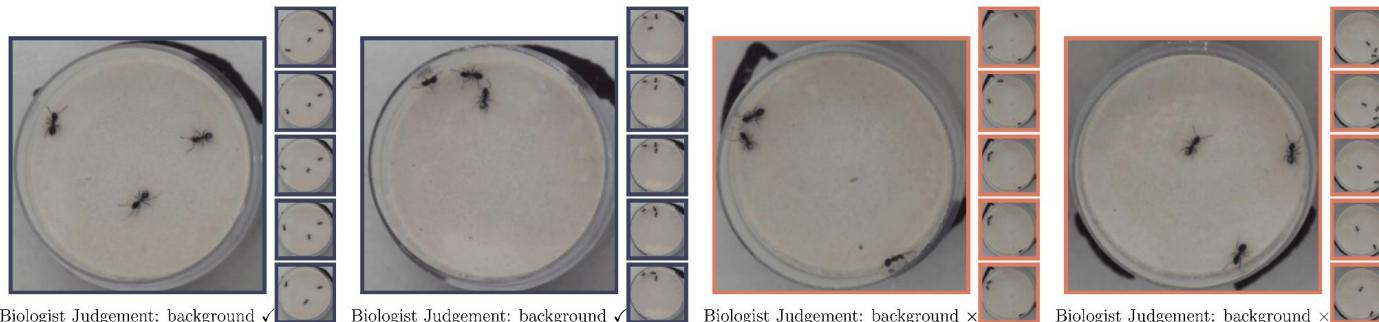


Results on ISTAnt

Qualitative Interpretation for Neuron 394



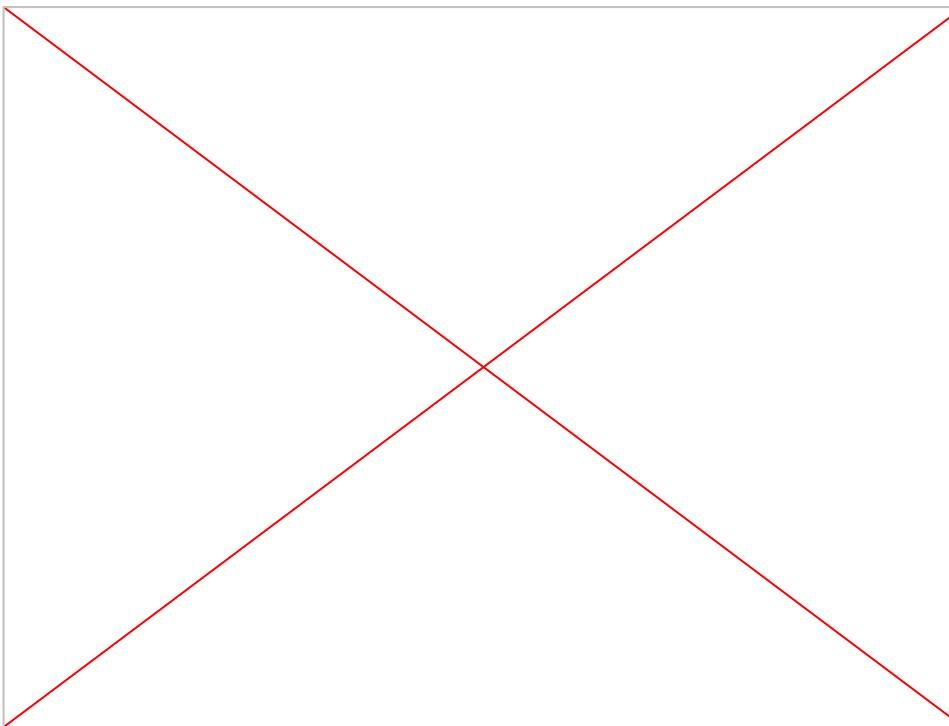
Qualitative Interpretation for Neuron 550



■ Max-Activating (Neuron 394) ■ Non-Activating (Neuron 394)
■ Max-Activating (Neuron 550) ■ Non-Activating (Neuron 550)

Beyond “standard” causal models

Temporal dynamics



Dynamical systems as causal models

System of coupled differential equations *modeling physical mechanisms* responsible for time evolution

$$\frac{dx}{dt} = f(x), \quad x \in \mathbb{R}^d$$

Future states are “caused” by immediate past

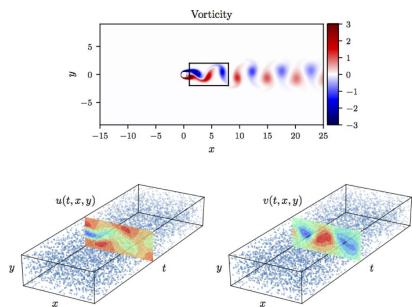
$$x(t + dt) = x(t) + dt \cdot f(x(t))$$

Unclear to which extent these can be learned from data for non-linear systems

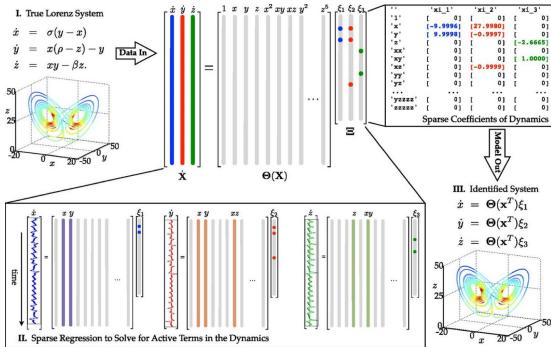
For the equation to have causal meaning, f must be a causal mechanism (i.e., interventions are well defined and align with physical experiments)

ML methodologies

PINNs



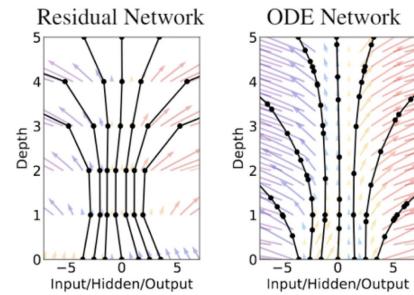
SINDy



- Constrain evolution prediction to **follow physics**
- Embeds physics **implicitly via loss**

Brunton et al., 2016

NeuralODE

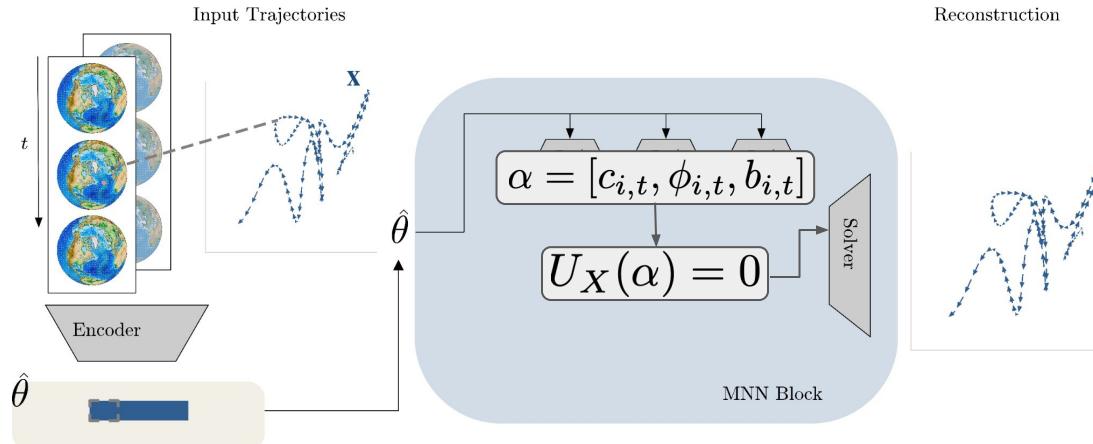


- Discover governing equation** regressing time derivatives onto basis functions
- No learned representations**
- Reconstruct time derivatives**

- Embed **network in solver** to predict next states
- Train by **reconstruction error**

Model Architecture

Parameterize equation as a combination of **learnable** coefficients parameterized by deep neural networks. Trained fully end-to-end.



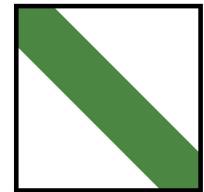
$$U_X : \sum_i \underbrace{c_i(t, X) u^{(i)}}_{\text{linear terms}} + \sum_k \underbrace{g_k(t, u, u', \dots; \phi(t, X))}_{\text{nonlinear terms}} = b(t, X)$$

Direct least squares solution

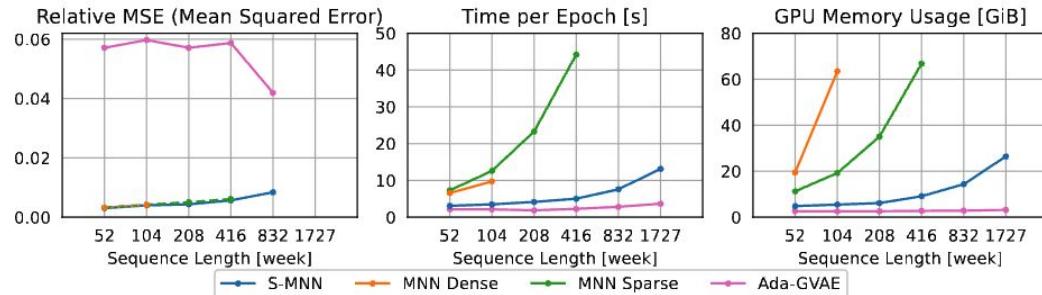
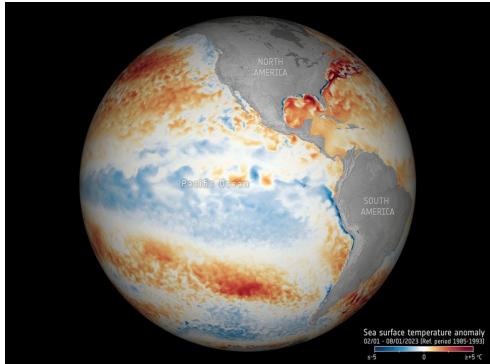
	Time Complexity	Space Complexity
Dense Solver	$O(T^3)$	$O(T^2)$
Sparse Solver	$O(T^2)$	$O(T^2)$

- Linear system $\sum_i \textcolor{teal}{c}_{i,t} u^{(i)} = \textcolor{teal}{b}_t \rightarrow \textcolor{teal}{A}z = \textcolor{teal}{b}$
$$z = (A^T A)^{-1} A^T b$$

$$z = M^{-1} \beta$$
- M is a banded symmetric matrix \rightarrow solver with linear time and space complexity
- In both cases, we also need to derive the backward gradients for GPU implementation

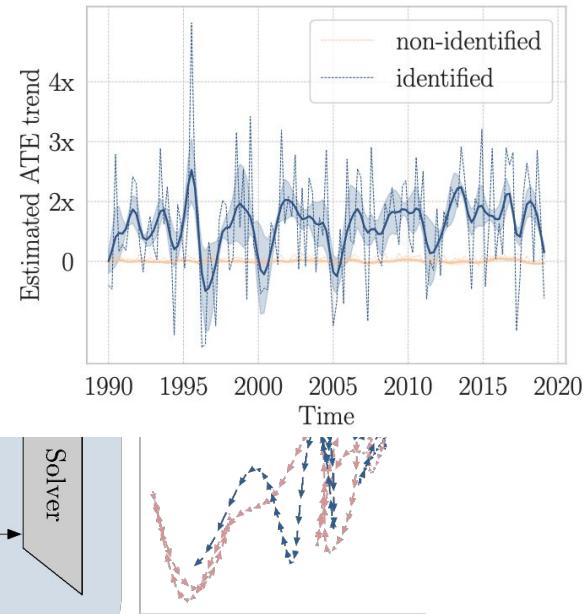
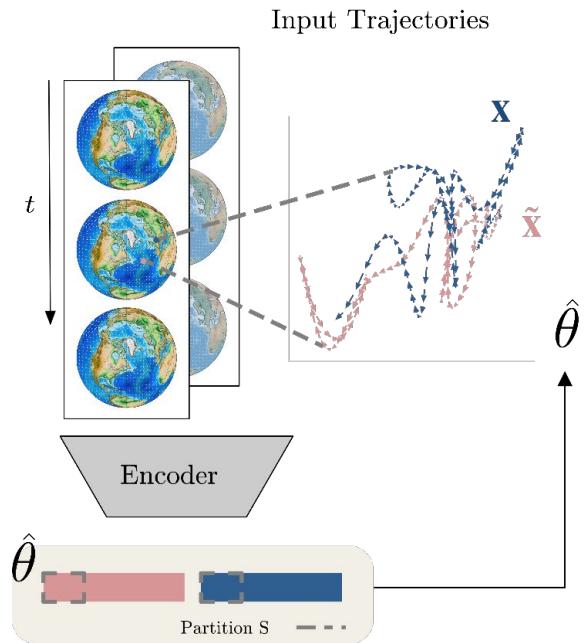


Long term sea surface temperature forecasting



Implication: Better solver immediately improves scalability without affecting accuracy

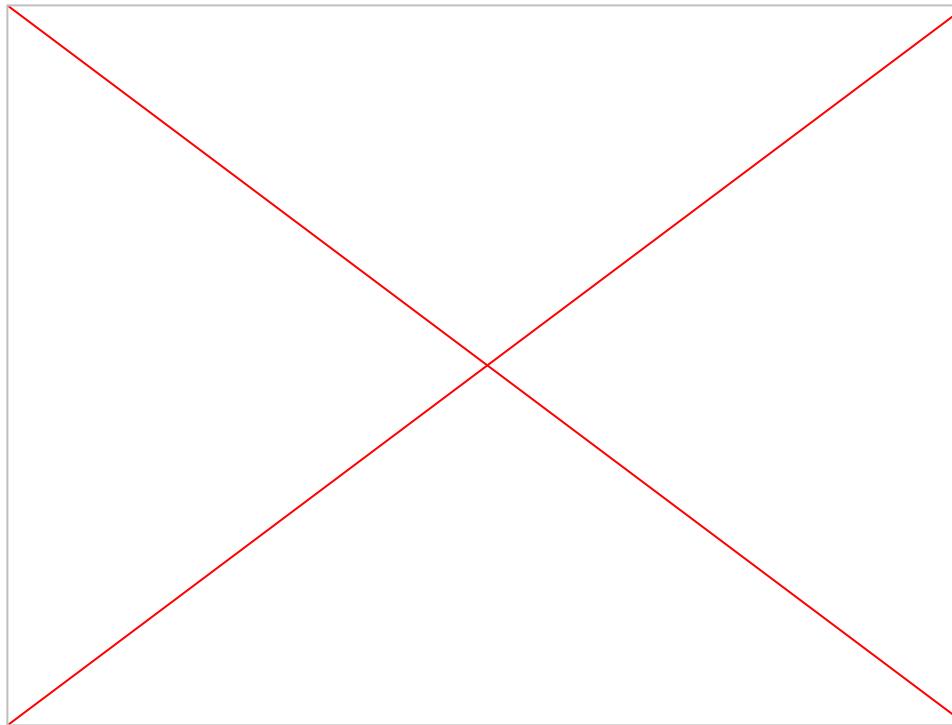
CRL recipes also extend to dynamical systems



"Marrying Causal Representation Learning with Dynamical Systems for Science", Yao, Muller, L; NeurIPS 2024

"The arctic has warmed nearly four times faster than the globe since 1979", Rantanen et al., Nature Communications Earth & Environment 2022.

PDE extension: Ginzburg-Landau Reaction Diffusion



Application: Modeling cell differentiation

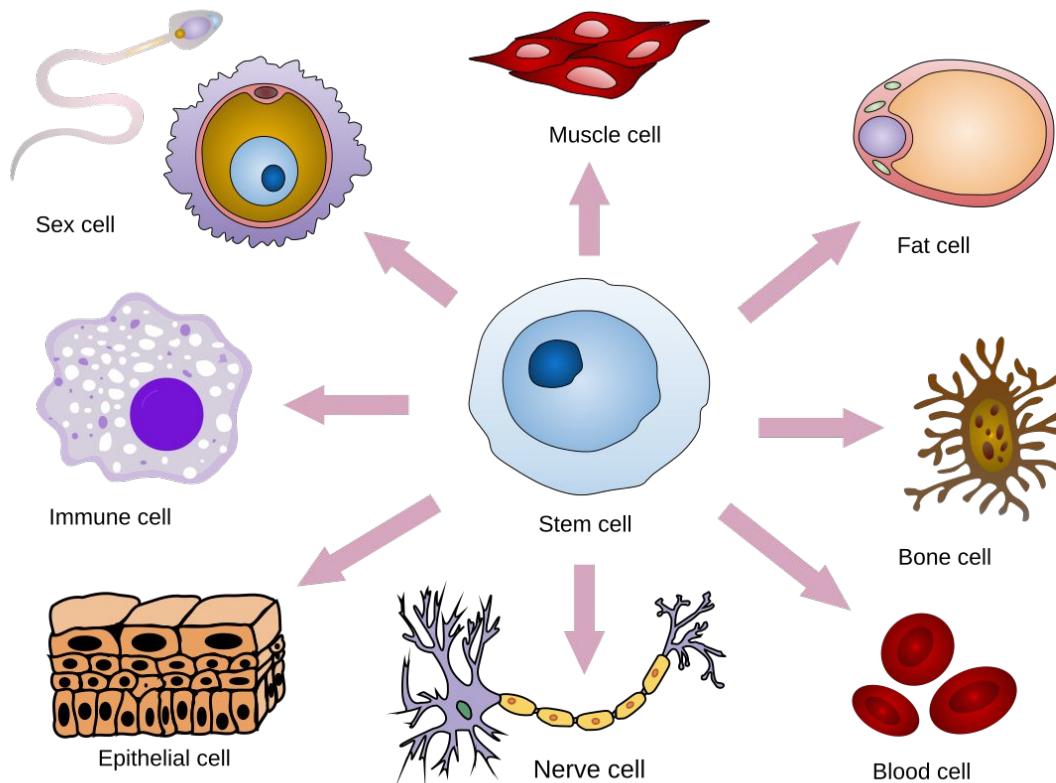
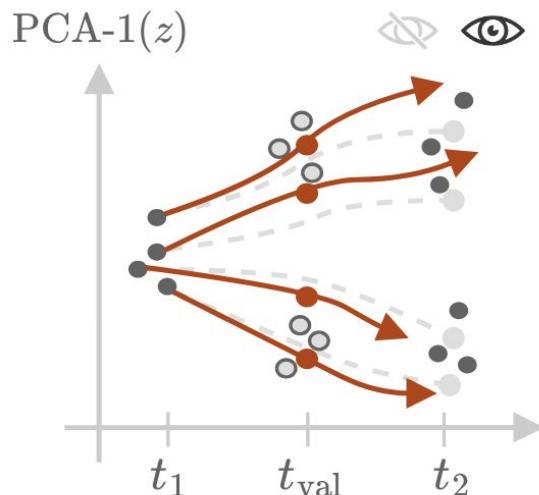


Image credit: Wikipedia (Cellular Differentiation)

Application: Modeling cell differentiation



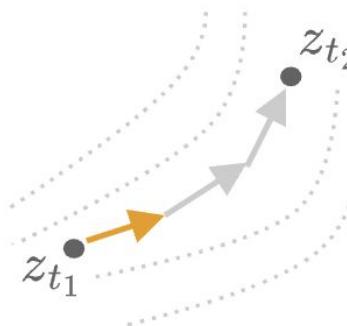
Validation: $\text{EMD}(\mathbf{q}_{t_{\text{val}}}^{\theta}, \mathbf{p}_{t_{\text{val}}})$

The legend indicates that a solid red circle represents the validation set ($q_{t_{\text{val}}}^{\theta}$) and an open circle represents the training set ($p_{t_{\text{val}}}$).

NODE / FM predicts

a velocity

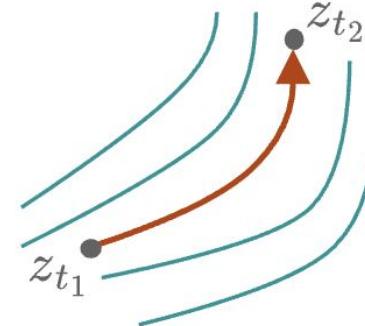
$$\dot{z} = f_{\theta}(z_{t_1}, t_1)$$



Cell-MNN predicts

a linear ODE

$$\dot{z} = A_{\theta}(z_{t_1}, t_1) z$$



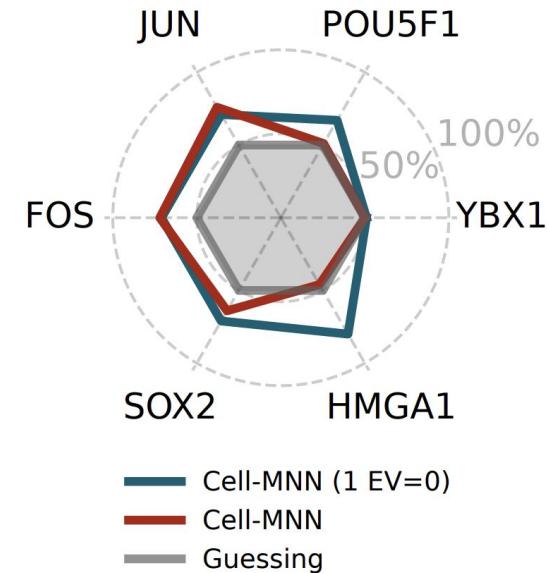
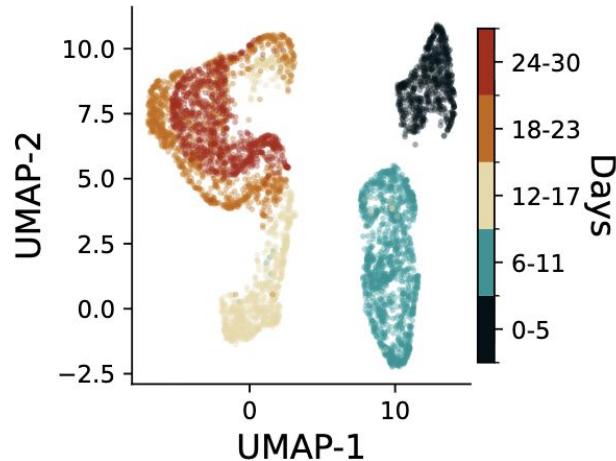
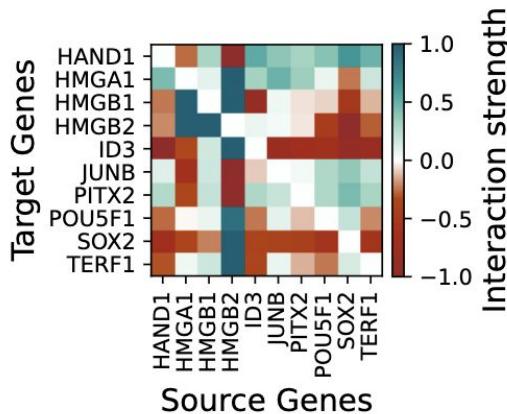
analytical solve

$$z_{t_2} = \exp[A_{\theta}(z_{t_1}, t_1) \cdot (t_2 - t_1)] z_{t_1}$$

Performance

Method	Cite	EB	Multi	Average ↓
TrajectoryNet [50]	—	0.848	—	—
WLF-UOT [39]	—	0.800 ± 0.002	—	—
NLSB [27]	—	0.777 ± 0.021	—	—
SB-CFM [52]	1.067 ± 0.107	1.221 ± 0.380	1.129 ± 0.363	1.139 ± 0.077
[SF] ² M-Sink [53]	1.054 ± 0.087	1.198 ± 0.342	1.098 ± 0.308	1.117 ± 0.074
[SF] ² M-Geo [53]	1.017 ± 0.104	0.879 ± 0.148	1.255 ± 0.179	1.050 ± 0.190
I-CFM [52]	0.965 ± 0.111	0.872 ± 0.087	1.085 ± 0.099	0.974 ± 0.107
DSB [14]	0.965 ± 0.111	0.862 ± 0.023	1.079 ± 0.117	0.969 ± 0.109
I-MFM [24]	0.916 ± 0.124	0.822 ± 0.042	1.053 ± 0.095	0.930 ± 0.116
[SF] ² M-Exact [53]	0.920 ± 0.049	0.793 ± 0.066	0.933 ± 0.054	0.882 ± 0.077
OT-CFM [52]	0.882 ± 0.058	0.790 ± 0.068	0.937 ± 0.054	0.870 ± 0.074
DeepRUOT [57]*	0.845 ± 0.167	0.776 ± 0.079	0.919 ± 0.090	0.846 ± 0.071
OT-Interpolate*	0.821 ± 0.004	0.749 ± 0.019	0.830 ± 0.053	0.800 ± 0.044
OT-MFM [24]	0.724 ± 0.070	0.713 ± 0.039	0.890 ± 0.123	0.776 ± 0.099
Cell-MNN (ours)*	<u>0.791 ± 0.022</u>	0.690 ± 0.073	0.742 ± 0.100	0.741 ± 0.050

Discovering GRN



Concluding remarks

Discussion

- ML has a great opportunity in powering causal analysis with key applications in scientific discovery but causal questions are subtle
- Chasing predictive accuracy may not lead to more accurate causal conclusions. In AI4Science, scientific questions should be part of the benchmark (especially if they are causal)
- Seeing the hidden world requires assumptions that the statistical causality language can describe
- **CRL:** Representation that makes it easier/ possible to extract causal information with some downstream estimator

Thanks!



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Causal
Learning and