1) Physics and Computation of Aero-Optics [1]

- The timescale for optical propagation is negligibly short relative to flow timescales, and hence optical popagation can be solved in a frozen flow field at each time instant.
- **OPL:** Optical Path Length.
- **OPD:** Optical Path Difference.
- **SR:** Strehl Ratio.
- $OPD_{(x,y,t)} = \underbrace{OPD_{(x,y)}}_{\text{steady-lensing}} + \underbrace{\left[A_{(t)}x + B_{(t)}y\right]}_{\text{beam jitter}} + \underbrace{OPD_{(x,y,t)}}_{\text{beam shaping}}$
- **steady-lensing:** function only of the time-averaged density field and impose a steady distortion such as defocus.
- **beam jitter:** does not change spatial distribution from the outgoing beam but simply redirects it in directions defined by $A_{(t)}$ and $B_{(t)}$.
- **beam shaping:** causes the beam to change its shape and intensity distribution.
- Given the OPD profile after the beam passes the turbulence region, one can solve its free space propagation using Fourier optics to obtain the exact far-field projection.
- **SR:** measure of beam quality relative to a diffraction-limited beam at each instant.
- Computation of aero-optics consist of two essential parts: solutions of the aberrating flow field via CFD and propagation of the optical beam through the aberrating flow to the target.
- Beam propagation is computed by a combination of ray tracing with Fourier optics.
- To accurately compute the index of refraction field, one must capture turbulence scales over all optically relevant wave numbers and frequencies.
- Euler equations do not describe the correct physics of refractive index fluctuation in a turbulent flow and must rely on numerical dissipation to mimic the effect of physical viscosity.
- Steady RANS calculates all turbulent scales, resulting in an ensemble-average (time-averaged) density field from which the steady-lensing effects can be evaluated but not the beam jitter and beam shaping.
- A hybrid RANS/LES model reduces computational costs for wall-bounded flows at high Reynolds numbers.
- A lack of flow resolution will cause errors in the computation of the optical phase when the beam is traced through the turbulence flow.
- Optical effects are negligible at a small scale in turbulence flow.
- Experiments suggested that the majority of optically active structures in compressible boundary layers are located in the outer region of the boundary layer, moving at $0.82 - 0.85$ times the free stream velocity.

- The predominant mechanism for density fluctuations in the compressible boundary layers is thought, to be adiabatic heating/cooling due to velocity fluctuations via the strong Reynolds analogy, as pressure fluctuations inside the boundary layers are much smaller than temperature fluctuations.
- Optical distortions by the fully developed wake were insensitive to the Reynolds number, whereas distortions by the separated shear layers were sensitive to it.

2) Aero-optical effects in non-equilibrium air [2]

- In a flow environment, which may induce real gas effects and chemistry, the refractive index depends on the air density, composition and internal state.
- In many cases, flows behind shocks and in boundary layers are not in thermal equilibrium and thus may have associate with them thermal aero-optical effects significantly different from those associated with thermal equilibrium.
- The refractive index of air is proportional to the gas density, with the constant of proportionality known as the Gladstone-Dale constant. $R_g = \frac{\alpha_{(\omega)}}{2\epsilon_0} N$, where: $\alpha_{(\omega)}$ is the polarization constant, ϵ_0 is the permittivity in vacuum and N is the molecular density.
- The Gladstone-Dale constant might not be constant at high speed flows because particles are tend to be found in their excited state.
- $n = 1 + R_g N$, where: n is the index of refraction and N is the density of the gas in molecules per cubic meter.
- In thermal equilibrium the fractional population and polarization obeys the Boltzmann distribution; however, for a non-equilibrium, the fractional population and polarization does not necessary obeys the Boltzmann distribution.
- The total energy of a state is given by the summation of electronic, vibrational and rotational energies; which are determined by the molecular vibrational and rotational constants.
- A gas with a higher temperature will exhibit a larger refractive index.
- For nitrogen and oxygen, vibrational excitation has a significantly larger effect on the polarization compared with the rotational excitation.

3) Aero-Optic Distortion in Transonic and Hypersonic Turbulent Boundary Layers [3]

- In a compressible boundary layer, the density, temperature, pressure and velocity vary in space and time. However, only the density fluctuations are relevant to optical distortion.
- The density fluctuations can be related to the velocity fluctuations using the Strong Reynolds Analogy (SRA).
- Accepting that the pressure fluctuations are small, the ideal gas law for small fluctuations leads to: $\frac{\rho'}{\bar{\rho}} = (\gamma - 1) M^2 \frac{u'}{\bar{U}}$
- From previous equation, it is expected that aero-optical distortions increase at higher Mach numbers and lower altitudes.

- The linking equation assumes that the density fluctuations are random and normally distributed an that the integration length L is much greater than the integral scale Λ.
- $OPD^2_{rms} = 2R^2_g \int_0^L \bar{\rho'^2_{(y)}} \Lambda_{(y)} dy$ Linking equation.
- In the inner region of a turbulent boundary layer, the structures tend to organize into long, streamwise streaks; the inner region extends to approximately $10 - 15\%$ of the boundary layer thickness.
- In the outer layer, of the boundary layer is dominated by structures sometimes termed large scale motions.
- Because the phase distortion is integrated through the boundary layer, the contributions from the larger scale structures in the outer region wold be expected to be significant. However, the linking equation shows that the variation of relative density fluctuations with height above the wall must also be taken into consideration. The density fluctuations are stronger in the inner half of the boundary layer, but in a compressible flow the local density decreases sharply close to the wall especially in high Mach number flows.
- There are two different ways used to describe the degree of aberration in a wave front: the root-mean-square variation across the wave front, and the SR, defined as the ratio between the measure of irradiance in the far field to the diffraction limited theoretical maximum.
- Two ways of calculating SR. SR is calculated at a fixed point (centerline); others calculate the SR at the point of maximum irradiance, regardless of its location.

4) HyperCode:A framework for high-order accurate turbulent non-equilibrium hypersonic flow simulation [4]

- Coupled, multi-scale, multi-physics interactions not only have a significant role in signal propagation and sensing in extreme environments but also affect crucial decisions regarding thermal management, path planning and control.
- **HyperCode:** code that has unique non-equilibrium prediction capabilities for multicomponent hypersonic flows.
- **OTF:** Optical Transfer Function.
- **PSF:** Point Spread Function.
- For temporal behavior one should first integrate OTFs or PSFs over an exposure window before calculating spatial parameters. This procedure yields primarily information for image or beam quality, since wave-ront and phase errors are readily available.

5) Dome and mirror seeing estimates from the Thirty Meter Telescope [5]

- **Mirror Seeing:** degradation of the telescope image due to variations in the index of refraction of the air.
- **Dome Seeing:** degradation of telescope image quality due to variations in the index of refraction of the air by the dome.

- **PSSn:** Point Source Sensitivity normalized. This metric is representative of the relative integration time of a background limited point source observation normalized to the atmospheric seeing.
- Its assumed that the diffraction image is computed by summing the OPD error through the system, applying this to the exit pupil, and propagating from the exit pupil using the Fourier transform of the exit pupil ignoring the propagation effects within the system.
- Near field diffraction effects are ignored due to the large optical paths involved in the telescope and the large thermal mass of the mirror.
- Matlab script was used to perform calculations.

6) Role of High Fidelity Nonequilibrium Modeling in Laminar and Turbulent Flows for High Speed ISR Missions [6]

- **DNS:** Direct Numerical Simulations.
- DNS of turbulence are performed for low Mach number compressible channel ideal gas flow.
- **STS:** State to State kinetics.
- **LLTR:** Lumped Landau Teller Relaxation.
- Laminar OPD's exhibit a tilt aberration with a positive gradient in the flow direction while in case of turbulent flows the integral length scale of the flow begins to dominate the OPD spatial behavior resulting in higher order aberrations.
- The complexity of hypersonic turbulent nonequilibrium boundary layers still merits the development of models that will retain both the spatial and temporal aero-optical behavior, that can lead to incorporating aberration mitigation strategies into sensor design.
- Hypersonic flow solvers currently employ simple algebraic or two equation turbulence models coupled with thermochemical nonequilibrium models which involve when needed macroscopic approximations and may therefore not be suitable or ISR studies due to the simplifications that prevent the proper resolution of the space and time scales that may affect the signal.
- Many current off the shelf (COTS) software technologies in this context lack key mechanisms that describe the interplay between turbulence and nonequilibrium distributions of molecules over vibrational, rotational and/or translational states. This leads to high uncertainty levels in the results and thus to potentially significant errors in signals analysis; other affected predictions included thrust, drag, aerodynamic heating, surface temperatures, chemical compositions, boundary layer transition and ablation.
- Nonequilibrium effect due to Internal energy exchanges between modes of vibration, rotation, translation, dissociation, recombination and radiation are calculated by two types of models.
- The LLTR model allows to calculate the coupling of vibrational-translational and dissociation reaction mechanisms on the flow physics, under the assumption that the molecule behaves as a harmonic oscillator.
- The STS model allows for prediction of nonequilibrium molecular distributions of energy among various internal energy modes, as well as state specific transport coefficient.

- Detailed representation of molecular distributions over internal energy states may be necessary for proper predictions of surface heat transfer and signal propagation errors.
- In conducting an aero-optics analysis o the signal propagation in high speed ISR missions, one would have to study teh propagation of E&M waves through a nonequilibrium flow field.
- The refractive index N of air is given by: $N - 1 = \rho\left(A + \dfrac{B}{\lambda^2}\right)$, where ρ is the mixture density, λ is the signal's wavelength, and A and B are constants depending on the air species.
- The OPL is the distance light travels in vacuum in the same time it travels a distance L in a medium.
- The difference between two OPL's at two locations is called OPD.
- An OPD map, also called wave-front, is the two-dimensional spatial distribution over an aperture of the deviation of the OPL of the signal from a reference distance.
- The length scale over which the phase error due to passage through a given column of medium equals to 1 [$radian$] is called the coherence length r_0; which is directly tied to the turbulence correlation and integral length scales.
- The coherence time is defined by the ratio between the coherence length and the average medium velocity relative to the sensor window; it sets the lower limit for exposure time before serious image degradation.
- An optical mesh of resolution comparable to that of the computational mesh and aligned with the signal propagation direction is defined.
- For a single flow field, either a steady state simulation converged solution or at a corresponding time of an unsteady solution, the process can be as follow: calculate the OPL for each cell o the optical mesh, then integrate the paths to obtain the total OPL for each point on the aperture. Subtracting the reference OPL, yields the OPD map. The OPD map can be further post processed for various optical parameters; such as SR, OTF, PSF.

7) Aero-thermal simulations of the TMT Laser Guide Star Facility [7]

- The time-step used in the initial simulation was 300 [s]. This is too coarse for beam jitter estimates but it is sufficient for temperature calculations.
- The grid of each region has been generated in such a way that each interface facet is common to the two cells of the two region involved. Convection, conduction and radiation heat transfer between regions is taking place through all interfaces.

8) Analytical Approach for Aero-Optical & Atmospheric Effects in Supersonic Flow Fields [8]

- The refractive index of a medium, such as air, governs the angular shift in the path of an optical signal. For a fluid the refractive index is a function of the thermodynamic state.
- The mix of dissociation and recombination reactions at high-supersonic and hypersonic speeds is the cause of communication blackout.
- At these speeds radio waves are unable to penetrate through the charged plasma's layer. Hence, RF communication is no longer effective.

- When an optical signal passes through a turbulent flow field, it undergoes deviation (due to refractive index changes) and distortion (due to refractive index fluctuations).
- Density ratio in supersonic flow increases with increasing Mach number and so would the downstream density.
- When a signal travels through a shock wave, the signal will deviate (due do the density gradient from the upstream and downstream regions in the flow field) at each intermediate interface of the shock layer.
- A shock wave in a supersonic flow can be consider as an interface between two medias of different refractive indices (think about Snell's law derivation).
- When refraction angle is the same as the shock angle, the signal propagates straight down.
- Shock layer thickness is relative small at high Mach numbers as the shock waves is closer to the body.

9) Optical distortion caused by propagation through turbulent shear layer [9]

- The fluctuations of the index of refraction are calculated using Wye's model.
- Methods use to investigate aero-optics problems: (1) Measuring the deviation of the optical radiation while passing through a flow field. (2) Finding an expression for the variations in the index of refraction (calculating fluid density).
- It is well known that the relation between the index of refraction and the density is given by the Gladstone Dale formula, which is valid for moderate high Mach number airflow.
- A shear narrow shear layer perpendicular to the laser beam's path, with a width of 0.03% of the path length causes a strong reduction in resolution.
- The reduction in resolution is dependent on the distance of the shear layer from the exit aperture of the beam.
- The turbulent limited resolution is inversely proportional to the Fried's coherence length.

10) Modeling Radio Communication Blackout and Blackout Mitigation in Hypersonic Vehicles [10]

- Weakly ionized plasma generated around the surface of a hypersonic reentry vehicle is simulated using full Navier-Stokes equations in multi-species single fluid form.
- The electromagnetic wave's interaction with the plasma layer is modeled using multi-fluid equations for fluid transport and full Maxwell's equations for the electromagnetic fields.
- Shock waves convert the kinetic energy to internal energy, an thereby increases the fluid temperature significantly.
- The electrons in the plasma layer may interrupt the propagation of RF waves when the plasma oscillation frequency is higher than the RF frequency.

- Steps: (1) modeling and simulation of the multi-species hypersonic flow over the vehicle to obtain plasma density distribution, (2) validation of the plasma density distribution with the results from literature, (3) modeling of RF waves propagation into the plasma and validation with the dispersion relations, (4) the propagation of a plane E&M wave on to the vehicle's surface through the plasma layer using a magnetic window and the whistler wave conversion.
- The properties' viscosity, thermal conductivity (mole fraction averaging), and specific heat (mass fraction average) of the individual species are obtained from the kinetic theory of gases. Gas constant is computing using the mole fraction averaged molecular weight.
- The interaction between RF waves and plasma are calculated by a MHD model without the assumption of quasi neutrality, without the assumption that the light wave is infinitely fast and by using full Ohm's law (including electron inertia) in the MHD system.
- The plasma frequency timescale is much smaller than the advection, diffusion and collision timescales; ergo these terms can be neglected.
- Wall temperature is lower than the boundary layer temperature.
- Inclusion of radiation looses from the plasma and the diffusion of electrons in the simulation could decrease the density to some extent.
- The dispersion relation is derived from the two fluid electromagnetic plasma model and written in terms of plasma parameters and the speed of light.
- The plasma E&M wave does not propagate below the plasma frequency; this is the reason for radio communication blackout.

11) Assessment of Hypersonic Flow Physics on Aero-Optics [11]

- The computations are performed with the aid of finite rate chemistry and the Landau-Teller vibrational energy relaxation model.
- High kinetic energy of the oncoming flow causes the molecules in the flow to be thermally excited, leading to dissociation.
- To accurately assess signal propagation through hypersonic flow field, high-enthalpy-flow physics need to be appropriately accounted for in optical distortion predictions.
- When flow passes through region of high gradient enthalpy, it takes a finite amount of collisions for the internal energies to adjust to a new thermodynamic state, often resulting in regions of nonequilibrium and relaxation.
- Vibrational and chemical energy exchange processes are often relatively slow compared with translational and rotational energy exchange processes.
- In nonequilibrium regions, chemical dissociation often takes place at freestream velocities greater than Mach 6.
- High enthalpy flow effects usually manifest as a relatively thin boundary later with steep gradient in density.

12) Influence of vibrational non-equilibrium on the polarizability and refraction index in air: computational study [12]

- Knowledge of variations of refractive index in air can ensure the reliable operation of optical on-board instrumentation such as optical air-data systems, airborne LIDAR, and directed energy systems.
- Polarizability in general, is represented by the polarizability tensor, reflecting a fact that an applied electric field can induce different polarization components. In its turn the scalar polarizability is an average of the diagonal elements of the polarizability tensor.
- Commonly tabulated values of R_g and α are given only for atoms and molecules in their ground state.
- Atomic species are not often found in appreciable quantities in their excited electronic states in hypersonic flows, it is well-known fact that their polarizabilities can be significantly larger than the ground state.
- From the perspective of molecular dynamics theory, polarizability dependence on temperature is caused by centrifugal stretching and anharmonicity of the intramolecular potential.
- Increase of the vibrational temperature leads to an increase of the polarizability due to overpopulation of higher lying vibrational states and, depending on the level of vibrational nonequilibrium and gas temperature, leads to the polarizability increase on the order of $1\% - 9.5\%$.

13) Table of Gladstone-Dale constants on the ground state

$\times 10^{-3}\ \left[\frac{m^3}{kg}\right]$	N_2	N	O_2	O	NO	Ref.
R_{GD}	2.38	3.01	1.90	1.82	2.21	[13]
			1.93	2.04		[14]
		3.10		1.80		[15]
	2.38		1.89	1.73		[16]

14) Atmospheric Propagation Vs. Aero-Optics [17]

- In the case of atmospheric propagation, the index of refraction variations are due to temperature fluctuations caused by very large scale (relative to the beam aperture) temperature gradients in a region of the atmosphere that itself is n shear and has turbulence scales that cascade from the largest scales down to sub aperture scales.
- In the case of aero-optics, the index of refraction variations for weakly-compressible flows are caused by density fluctuations that form in and around the coherent structures in the turbulent flow.
- At the heart of the atmospheric propagation, a single parameter C_n^2. This parameter describes not only the scale size of the aberrating index of refraction fluctuations, but also the magnitude of the aberration with them.
- In aero-optics the C_n^2 parameter is irrelevant, because of the characterization of aero-optic turbulence since the range of scales over which aero-optic turbulence can be consider Kolmogorov places them below the scale sizes that are relevant to the optical problem.

- The Kolmogorov model of turbulence; is a statistical, incompressible model; which based largely on the idea of energy being transferred between various length scales in the flow. At the largest scales, persisting eddies act as the energy supply for the turbulent flow. At the small scales, viscous forces become dominant, and the kinetic energy in the flow is dissipated as heat.

- Kolmogorov proposed that in fully developed turbulence flow, the properties in the inertial range were only dependent upon the flux of energy though those scales. Note, Kolmogorov' turbulence model is assumed to be an incompressible flow.

- Variations in temperature and pressure of a gas or collection of gases will produce variations in density.

- In air, for wavelengths in or near the visible range, the index of refraction can be approximated to be: $dn = 7.77 \times 10^{-7} \frac{P}{T} \left(\frac{dP}{P} - \frac{dT}{T} \right)$

- In the free atmosphere, pressure gradients severe enough to produce significant density variation on the length scale of most optical path diameters tend to disperse at sonic speeds and the small pressure fluctuations that are coupled to velocity variations in atmospheric turbulence tend to have a low order optical impact compared to the effect of temperature fluctuations.

- As light travels slower in areas with a higher index of refraction, the same absolute path length becomes effectively longer or shorter from an optical standpoint in regions of greater or lesser index of refraction.

- The OPD can be scaled by the wavelength of light to indicate differences in phase over the recieving plane. These differences in phase produce differences in amplitude and intensity as a wave propagates.

- Fried, assumed that if one was attempting to recover information from light received through an aperture, then there would be some signal modulated onto a carrier wave, such that the amplitude of the carrier would be much greater than the amplitude of the signal.

- Having an aperture larger than the Friend's parameter (r_0), will not significantly improve the signal to noise ratio of a signal or resolution of an image.

- The Friend's parameter is physically defined as the size of an aperture over which the root-mean square phase variance is approximately 1 $[radian]$.

- The Greenwood frequency is commonly used as a guideline for the bandwidth required of a corrective system intended to deal with optical turbulence.

- The assumptions of isotropy and homogeneity of the flow don't apply in aero-optics.

- Experimental studies of transonic flow have shown that resulting flow structures are a source of optical aberrations.

- Studies have shown that in transonic shear layers, there are rolling vortices that make up flows containing significant wells of low pressure. These low pressure regions can produce significant optical distortions not predicted by models based solely on temperature differences.

- Shear layers are of particular interest in aero-optics as they may form over cavities or in any instance of separated flow.

- Aero-optic flows do not follow atmospheric flows' rules of scale, because they tend to have a characteristic length and associated frequency. This is because these flows are not in equilibrium state as is it assumed for Kolmogorov turbulence's model.

- A possible corrective filter with a transfer function that remove optical distortions can be derived from the Greenwood frequency.

15) Study on the Effect of Compressibility and Knudsen number on Aero Optics in Supersonic/Hypersonic Flows [18]

- **Kn:** Knudsen number; is a dimensionless number defines as the ratio of the molecular mean free path length (λ) to a representative physical length (L). This ratio helps to determine where statistical mechanics or continuum mechanics formulation of fluid dynamics should be used to model a situation. If Kn is ≥ 1, then $\lambda \approx L$ and the continuum assumption of fluid mechanics is no longer a good approximation; hence, statistical methods should be use. $Kn = \dfrac{\lambda}{L}$
- A smaller Knudsen number would lead to a larger optical distortion because the media was denser and larger strength per unit which will lead to a larger distortion. $OPD_{RMS} \approx Kn^{-1}$.
- Compressibility played an important role in optical distortion. For supersonic flows, optical distortion was lower at higher Mach number $\left(OPD_{RMS} \approx Ma^{-1}\right)$; and for hypersonic flows, optical distortion became more severe as Mach number increased ($OPD_{RMS} \approx Ma$).
- It was confirmed that shock wave regions contributed most to the distortion. Near wall flow regions contributed less than 10% to the total distortion in supersonic flows; and in hypersonic flows the near wall flow regions contributed 40% to the total distortion.
- The blurring due to a hypersonic flow, can lead to a decrease in the signal to noise ration and image degradation, consequently affecting the target's location. (Cohesive Reference Point CRP)
- Navier-Stokes equations might not be capable to resolve the physical process at order of magnitudes less than nano-meters/seconds and/or micro-meters/seconds. Thereby, a numerical method for Boltzmann equation becomes necessary to capture the physical phenomenon in the small scale.
- Flow fields were computing using Direct Simulation Monte Carlo (DSMC) and a new ray tracing technique was proposed to computer the phase difference from the index of refraction.
- The aero-optical analysis, a new ray tracing technique based on CFD meshes was proposed, and it was assumed that the grid resolution from the CFD simulation was considered to be an accurate representation of the aero-optical effects.
- OPD is also called the relative phase (ϕ).
- The optically-active region in supersonic flows was larger than in hypersonic flows, because of larger compressibility.
- The aero-optical effects caused by boundary layers in hypersonic flows was more severe than in supersonic flows.

16) Molecular Polarizability - an Introduction

- When a beam of light is incident on a transparent material medium of refractive index different from that of its surroundings, the medium gets polarized.

- If a light wave of electric intensity $\vec{E}$ goes through a molecule in the medium, it induces an optic moment in the molecule. The molecule is said to be polarized.
- Theoretically; therefore, polarizability should be a function of the incident light frequency.
- α can be regarded as a constant; unless, when measuring n.
- $\boldsymbol{P} = \alpha \boldsymbol{E}$, $\boldsymbol{P}$ is the dipole moment, $\boldsymbol{E}$ is the electric field, α is the polarizability of the molecule.
- In isotropic materials α can be treated as a scalar of constant magnitude.
- If the material is anisotropic, the polarizability acquires directional property.
- For optically inactive molecules, α is a symmetrical tensor.
- An optically active molecule, rotates the plane of polarization after it goes through the molecule. Dextorotatory component rotates the light clockwise, and Levorotatory components rotates the light counterclockwise.
- The mean molecular polarizability is given by: $\alpha = \frac{2}{3}\left(b_L + b_T + b_V\right)$. Where b_L is the longitudinal link polarizability, b_T is the link polarizability in the plane of the molecule or group containing the link and at right angles to it and b_V is the one normal to the plane.
- Individual chemical bonds can be associated with polarizability components along their lengths.

17) Physics and Measurement of Aero-Optical Effects: Past and Present [19]

- The United States had two periods of funding, both associated with the development of high-speed (transonic) airborne laser systems.
- Optical turbulence is defined as density fluctuations in the air due to atmospheric turbulence and temperature gradients.
- In order to study aero-optical effects, one needs to measure the aberrated wave fronts imposed on a otherwise planar wave front for a large-aperture laser projected through aero-optical turbulence.
- The measurement of the aberrations is a direct result of the air's unsteady density field in the flow given that the air's index of refraction is directly linked to the fluctuating density through the Gladstone-Dale relationship.
- The basic theory depended on the pressure fluctuations to be negligible. However, this is not the case or some flows.
- The OPD_{rms} is given by: $\overline{\langle OPD^2 \rangle} = R_{GD}^2 \int_0^{z_1} \int_0^{z_1} Cov_{\rho'}\left(z, z'\right) dz' dz$
- The covariance function is defined as $Cov_{\rho'}\left(z, z'\right) = \overline{\left[\rho_{(z)} - \bar{\rho}_{(z')}\right]\left[\rho_{(z')} - \bar{\rho}_{(z')}\right]}$; note the prime represents the fluctuations in the medium.
- The covariance function is usually modeled by either an exponential or Gaussian functional form based on the length scale (Λ), and the square of the fluctuation density $\left(\rho'^2_{rms}\right)$
- The Linking Equation is given by: $\overline{\langle OPD^2 \rangle} = \alpha R_{GD}^2 \int_0^{z_1} \rho'^2_{rms}\left(z\right) \Lambda\left(z\right) dz$, where α depends on the form of the covariance function equal to 2 if exponential and $\sqrt{\pi}$ is Gaussian.

18) Resolution requirements for aero-optical simulations [20]

- In the high Reynolds number limit, for a given geometry and Mach number, the spatial resolution required to capture aero-optics within a pre-specified error margin does not scale with Reynolds number.
- In typical aero-optical applications this resolution requirement is much lower than the resolution required for direct numerical simulation, and therefore, a typical large-eddy simulation can capture aero-optical effects.
- Such initial distortions reduce the coherence o the beams, so that by the time they arrive at the receivers, they have much lower intensities and are more dispersed compared to undistorted beams.
- An optical beam under such conditions experiences two regimes of propagation: first, propagation through the turbulence near the optical device, where densities fluctuations are significant and of relatively high frequency/wavenumber and second, propagation through the atmosphere.
- In the first region, the distance is very short and the optical wavelength is much smaller than the flow structures, hence the ray optics assumptions are valid. As a result, scattering and change in the optical wave amplitude can be ignore in this region.
- The problem of distortion due to atmospheric turbulence can generally be corrected using adaptive optics, because the refractive index field involves larger spatio-temporal scales.
- Aero-optical distortions usually take place at spatial and temporal frequencies which are order of magnitudes higher than the capabilities of current correction technologies.
- Aero-optic distortions are highly dependent on fluctuations in the turbulent density field, an accurate representation of aero-optical distortions with a RANS simulation alone is not possible.
- LES is of major recent interest for aero-optical computations; because is capable of capturing a range of flow structures, from large-scale motions down to the scale comparable to the mesh spacing.
- In previous studies it has been implicitly assumed that sub-grid scales and artificially damped resolved small-scale features of the flow are optically unimportant. Although, these simulations may computers low order flow statistics, this does not necessarily guarantee that the aero-optical effects of the flow can be provided with the same resolution.
- $P - P_0 = \nu_0^2 (\rho - \rho_0)$, where 0 is the free stream conditions and ν is the speed of sound.
- Spectral information regarding the pressure field can be linked to the spectrum index field, because the equation above and the Gladstone-Dale relation are linear.
- Fourier optics methods can compute the wave propagation after the turbulent medium accurate if and only if, the error beam has much less energy than the computed beam.
- The accuracy of the aero-optical computations can be based on this energy ratio. If the ration is smaller than a threshold (ex. 5%), then it can be assumed that the resolution of the flow simulation is adequate to capture the aero-optical effects.
- The grid resolution (l_c) is given by: $l_c = \dfrac{2\pi}{k_c}$, where k_c is the cut-off wave number.
- The cut-off wave-number is usually in the inertial range and is smaller than the wave-number associated with Kolmogorov length scale $\left(k_\eta = \dfrac{2\pi}{\eta}\right)$.

- $\dfrac{12\pi^3 B_p}{7\lambda^2}\left[\dfrac{(n_0-1)^2\,\epsilon^{4/3}}{\nu^4}\right]\left(\dfrac{l_c}{2\pi}\right)^{7/3}\Delta z < \zeta$ review paper to understand equation, this is the result from the paper.

- For a flow with a characteristic length (l) and a characteristic velocity, the l_c can be approximated to be: $l_c \approx \dfrac{\zeta^{3/7}}{M^{12/7}\,(n_0-1)^{6/7}}\left(\dfrac{\lambda^{6/7} l^{4/7}}{\Delta z^{3/7}}\right)$

- The importance flow parameters are the turbulence length scale and the turbulent Mach number; the important optical parameters are the characteristic refractive index and the optical wavelength; the important geometric parameter is the depth of the turbulence field.

- In a typical aero-optical setup, one can have $l \approx 1\ [m]\,, \Delta z \approx 1\ [m]\,, \lambda \approx 1\ [\mu m]\,, M \approx 0.4\ [\,]\,, (n-1) \approx 10^{-4}$, and $\zeta \approx 5\%$. Based on these parameters and assuming $B_p = 8.5$, the value of $l_c = 1.2\ [cm]$, with a grid spacing of 6 $[mm]$ (based on Nyquist criterion). The required number of grid points in each direction would be approximately 170.

- In summary, $l_c << l_{geometry}, l_{aperture}$.

- Optical wavelength, dept of the turbulent field, length scale of large flow structures, characteristic Mach number and index of refraction are the governing parameters in determining the required resolution. The required resolution does not scale with Reynolds number flow.

19) Theory and applications of atomic and ionic polarizabilities [21]

- The dielectric constant and the refractive index of any gas are examples of macroscopic properties that are largely determined by the dipole polarizability.

- The calculation o atomic polarizabilities is a demanding special case of the calculation of atomic structure.

- On this paper, only low-order polarizabilities are considered, high-order polarizabilities are not.

- The most widely used unit for theoretical atomic physics is atomic units (a.u.), in which, $q = m_e = 4\pi\epsilon_0 = \hbar = 1$.

20) Development of an Integrated Aero-Optics Modeling Capability: OVERFLOW-aeroOptics [22]

- OVERFLOW-aeroOptics is a specialized version of the OVERFLOW 2.1x overset CFD solver that has been developed to study aerodynamically-induced aberrations to optical wave forms as they are propagated through an unsteady, compressible and turbulent flow field.

- A reduced form of Maxwell's equations, the Paraxial Beam equation, is solved concurrently with the Navier-Stokes equations to propagate a given wave for through the CFD generated density field.

- Code validation is made by performing a mesh refinement study and comparing computed to exact solutions for the propagation of a Gaussian beam through a vacuum.

- Near-field propagation (aeo-optics) maintains some similarities to the far-field (atmospheric) propagation, but due to the interaction between turbulence length scales, beam wavelengths, aperture and distances, the two often require different approaches to tackle.

- Diffraction may be neglected when $\frac{\lambda L}{l^2} << 1$ (Fresnel condition), where λ is the wavelength, L is the propagation length and l is the turbulence length scale.
- OVERFLOW is an overset, finite-difference, compressible, Reynolds-Averaged Navier-Stokes (RANS) and Detached Eddy Simulation (DES) solver.
- OVERFLOW is written in Fortran90, the Paraxial solver has been developed as a F90 module, named aeroOpticsModule.
- Interpolation from a CFD optic grid to the spectral grid is made using an 8-noded isoparametric element interpolation that is simplified for these orthogonal and grids.
- The geometric OPD is computed on the optic grids using procedures invoked separately from the paraxial solver, and is stored on disk after each calculation.
- The paraxial beam solver is validated by comparing the computed to exact solutions for a Gaussian beam propagating through a vacuum, and by comparing computed values to those computed by the AOQ solver.
- A mesh refinement study is performed and the error norms are plotted against mesh size.

21) Numerical study of evaluating the optical quality of supersonic flow fields [23]

- A numerical method base on the uniform and hexahedral grids generated from computational fluid dynamics is presented for the analysis of aero-optical performance. A single grid is taken as a cell with isotropy and homogeneity inside, and it is assumed that the light rays transmit grit by grid.
- Aero-optics is a phenomenon of fluid-optic interactions.
- Compared with atmospheric optics, aero-optics is a near field optics. It typically involves the turbulent boundary layer, wake, and shear layer.
- If the grids are of fine resolution, we can assume that the gaseous medium inside a single grid is uniform and isotropic. If not, we should interpolate CFD grids so as to improve the resolution and gain approximately continuous flow data.
- For this paper, the grids generated from CFD are uniform and hexahedral, the size of which is equal to 1 $[mm]$. Each CFD hexahedral grid is considered as an index cell with uniform refractive index.
- The computational mesh has $64 \times 64 \times 80$ grid points, ranging from 69 to 132 in the $\hat{x}$-direction and from -31 to 32 in the $\hat{y}$-direction.
- If the temperature is very high, the index of refraction will be dependent mainly on the temperature and components of fluids.
- If $\left| n_j - n_i \right| s < \frac{\lambda}{4}$, wave-front errors can be neglected. Where, s is the path length.
- The OPD can be transform to wave front phase by: $\Delta\phi_{(\vec{r},t)} = k\,OPD$, where k is the wave number, $k = \frac{2\pi}{\lambda}$.
- The OPD directly reflects the variations of the wave-front phase errors.

- The wavefront variance σ^2 is given by: $\sigma^2 = 2R_{GD}^2 \int_0^L {\rho'}^2 l' \, dl$, where ρ' is the fluctuation density and l' is the turbulence length scale calculated from CFD.
- The wave-front variance is a measure of the dispersion about the mean for OPLs along a wavefront, and is crucial for modeling aero-optical parameters such as Strehl ratios and OTF for turbulent flows.
- Only if σ^2 are not too large, an estimate of the time-averaged Strehl ratio or a given wavefront phase variance is approximated by: $SR = e^{-(k\sigma)^2}$.
- In this paper, LES was used.

22) Computation study of aero-optical distortions by a turbulent wake [24]

- The small-scales of the flow are found to play a significant role in causing optical aberrations, which places a more stringent grid-resolution requirement than that needed to capture the low order flow statistics.
- The depth of the aberrating flow field is usually smaller than or comparable to the projecting (or imaging) aperture.
- Jones and Bender used LES to study aero-optical distortions in a fuselage/turret configuration. They used ray tracing to obtain the wavefront error and represented it in terms of Zernike polynomials.
- Most of the previous work to aero-optics has assumed that the major contribution to optical distortion was from the large resolved scales of the flow, and the sub-grid scale effect were simply ignore.
- TVD or upwinding techniques are highly dissipative.
- For this paper, the Reynolds number is 3900 and the Mach number is 0.4, LES was used to compute the CFD part.
- The numerical scheme used in LES is based on a non-dissipative staggered-mesh formulation, which leads to more accurate representations of a wide range of optically important flow scales. The effects of flow resolution and small-scale turbulence on optical aberrations are investigated by subjecting the refractive-index field to a low-pass filtering operation. Furthermore, a statistical analysis is carried out for the optical far field and its dependence on wavelength and distance of propagation is revealed.
- The LES code, originally written for a C-mesh, has been modified for a generalized O-mesh to enhance grid-smoothness and hence numerical stability.
- The computational domain for the cylinder has a radius of about $20D$ (D is the cylinder's diameter) and a width of πD in the spanwise direction.
- The mesh size is $288 \times 200 \times 48$ in the wall normal, azimuthal and spanwise directions, respectively. A 16 grid point sponge later is applied at the outer boundary to make it non-reflecting.
- The total integration time was over 70 shedding cycles. From the last 14 shedding cycles, of the simulation, about 800 snapshots of the density field were saved for aero-optical study.
- The domain of beam propagation is divided into two parts: the small near field in which ray tracing can be applied accurately, and the far field in which the density is assumed spatially uniform and Fourier optics can be applied.

- The near field is taken to be within $10D$, while the far-field distance varies from $10^3 D$ to infinity. The optical beam has a diameter of $0.3D$ and us directed at 17° from the downstream axis.
- The optical wavelengths vary from $1.25 \times 10^{-6} D$ to $10^{-5} D$.

23) A simple adaptive mesh generator for $2D$ finite element calculations (optical-wave theory) [25]

- The adaptive mesh method consists of the use of a suitable defined 'density function'. The new mesh is built starting from a minimal number of triangular elements which are then in several sweeps, repeatedly refined according to the density function. The Delaunay algorithm is used in each stage to keep the shape of the triangles as equilateral as possible.
- Adaptive mesh refinement is a technique whereby a solution obtained with a rather coarse mesh is used to identify selected areas of the domain to refine.
- Adaptive remeshing is a method whereby a completely new mesh is generated adaptively, according to the results from the previous calculation.
- The remeshing proceeds as follows: once a result has been found for a certain value of the power, the power level is varied and the same mesh is used to obtain an approximate result. This is used in turn to generate the new mesh by providing the updated density function.
- Any suitable node-based function can be used as a 'density function'.
- The defining condition for a region in respect to the mesh generator is that no mesh elements will be allowed to cross the region boundaries.
- Stage one of the algorithm is to have specified the regions, materials and their boundaries, to subdivide each o them into a minimum number of triangles (the primary triangles).
- Step one of the algorithm is to found the local maximum density function in each region and join all vertices to that point. The information relative to the triangles so formed is stored in an array containing the material type, the nodal number of the three vertices, the element number of the three neighbours (triangle sharing a side) and a flag.
- The total 'weight' W of the problem is calculated by integrating the density function using either the primary triangulation (at the start of the process) or the current mesh. $W = \sum_{i=1}^{M} w_i = \sum_{i=1}^{M} \int f_{(x,y)} dxdy$, where M is the current number of elements. Assuming first order elements, the individual element weights are: $w_i = \frac{A}{3}(f_1 + f_2 + f_3)$. Where, f_i is the value of the density function at the node i and A is the area of the triangle.
- The mesh generator can be controlled by specifying a desired total number of elements in the mesh and monitoring the individual element weights.

24) CFD Mesh Generation: A Practical Guideline

- The quality of the mesh has significant implications on the convergence and stability of the numerical simulation and accuracy of the computational results.

- By definition, a mesh in itself consist of an arrangement of discrete number of points overlaying the whole domain geometry.
- Body fitted meshes in Cartesian coordinates, are tedious and time-consuming to be set up; these steps at the boundary may also introduce errors in computations of the wall stresses, heat fluxes, boundary-layer effects, etc.
- Inner mesh can be algebraically determined through interpolation from the boundary values, which is computationally inexpensive.
- Look at transfinite interpolation methods, matlab script.
- The method of transfinite interpolation according to Gordon and Thiel (1982), consists of generating the interior mesh from the boundary grid data using appropriate interpolation functions or 'blending' functions.
- Hermite interpolation, is a method of interpolating data as a polynomial function. Poisson equation can be use or this (read more).
- Are we using structured or unstructured mesh? Are we having matching or unmatching cell faces?

Bibliography

[1] Meng Wang, Ali Mani, and Stanislav Gordeyev. "Physics and Computation of Aero-Optics". In: *Annual Review of Fluid Mechanics* 44 (2012), pp. 299–321.

[2] Albina Tropina et al. "Aero-optical effects in non-equilibrium air". In: *2018 Plasmadynamics and Lasers Conference*. Atlanta, Georgia: AIAA 2018-3904, June 2018.

[3] Christopher M. Wyckham and Alexander J. Smits. "Aero-Optic Distortion in Transonic and Hypersonic Turbulent Boundary Layers". In: *AIAA Journal* 47.9 (Sept. 2009), pp. 2158–2168.

[4] Konstantinos Vogiatzis et al. "HyperCode: A framework for high-order accurate turbulent non-equilibrium hypersonic flow simulations". In: *AIAA Scitech 2020 Forum*. Orlando, FL: AIAA 2020-2192, Jan. 2020.

[5] John S. Pazder, Konstantinos Vogiatzis, and George Z. Angeli. "Dome and mirror seeing estimates for the Thirty Meter Telescope". In: *Modeling, Systems Engineering, and Project Management for Astronomy III*. Vol. 7017. Marseille, France: SPIE, July 2008, pp. 229–237.

[6] Konstantinos Vogiatzis, Eswar Josyula, and Prakash Vedula. "Role of High Fidelity Nonequilibrium Modeling in Laminar and Turbulent Flows for High Speed ISR Missions". In: *46th AIAA Thermophysics Conference*. Washington, D.C.: AIAA 2016-4317, June 2016.

[7] Konstantinos Vogiatzis et al. "Aero-thermal simulations of the TMT Laser Guide Star Facility". In: *Adaptive Optics Systems IV*. Ed. by Enrico Marchetti, Laird M. Close, and Jean-Pierre Véran. Vol. 9148. International Society for Optics and Photonics. Montreal, Quebec: SPIE, Aug. 2014, pp. 2024–2033.

[8] Anubhav Gupta and Brian Argrow. "Analytical Approach for Aero-Optical and Atmospheric Effects in Supersonic Flow Fields". In: *AIAA Scitech 2020 Forum*. Orlando, FL: AIAA 2020-0684, Jan. 2020.

[9] Offer Pade, Evgeny Frumker, and Paula Ines Rojt. "Optical distortions caused by propagation through turbulent shear layers". In: *Optics in Atmospheric Propagation and Adaptive Systems VI*. Ed. by John D. Gonglewski and Karin Stein. Vol. 5237. International Society for Optics and Photonics. Barcelona, Spain: SPIE, Feb. 2004, pp. 31–38.

[10] Madhusudhan Kundrapu et al. "Modeling Radio Communication Blackout and Blackout Mitigation in Hypersonic Vehicles". In: *Journal of Spacecraft and Rockets* 52.3 (May 2015), pp. 853–862.

[11] Lauren E. Mackey and Iain D. Boyd. "Assessment of Hypersonic Flow Physics on Aero-Optics". In: *AIAA Journal* 57.9 (Sept. 2019), pp. 3885–3897.

[12] Albina A Tropina et al. "Influence of vibrational non-equilibrium on the polarizability and refraction index in air: computational study". In: *Journal of Physics D: Applied Physics* 53.10 (Dec. 2019), p. 105201.

[13] Sebastian Karl, Jan Martinez Schramm, and Klaus Hannemann. "High Enthalpy Cylinder Flow in HEG: A Basis for CFD Validation". In: *33rd AIAA Fluid Dynamics Conference and Exhibit*. Orlando, FL: AIAA 2003-4252, June 2003.

[14] J. H. B. Anderson. "Experimental Determination of the Gladstone-Dale Constants for Dissociating Oxygen". In: *The Physics of Fluids* 12.5 (1969), pp. I-57-I–60.

[15] Ralph A. Alpher and Donald R. White. "Optical Refractivity of High-Temperature Gases. I. Effects Resulting from Dissociation of Diatomic Gases". In: *The Physics of Fluids* 2.2 (Mar. 1959), pp. 153–161.

[16] Xiao Qin et al. "Effect of varying composition on temperature reconstructions obtained from refractive index measurements in flames". In: *Combustion and Flame* 128.1 (2002), pp. 121–132.

[17] John Siegenthaler, Eric Jumper, and Stanislav Gordeyev. "Atmospheric Propagation vs. Aero-Optics". In: *46th AIAA Aerospace Sciences Meeting and Exhibit*. Reno, Nevada: AIAA 2008-1076, Jan. 2008.

[18] Wei Ren and Hong Liu. "Study on the Effect of Compressibility and Knudsen Number on Aero Optics in Supersonic/Hypersonic Flows". In: *42nd AIAA Fluid Dynamics Conference and Exhibit*. New Orleans, Louisiana: AIAA 2012-2988, June 2012.

[19] Eric J. Jumper and Stanislav Gordeyev. "Physics and Measurement of Aero-Optical Efects: Past and Present". In: *Annual Review of Fluid Mechanics* 49.1 (2017), pp. 419–441.

[20] Parviz Moin Ali Mani Meng Wang. "Resolution requirements for aero-optical simulations". In: *Journal of Computational Physics* 227.21 (Feb. 2008), pp. 9008–9020.

[21] M S Safronova J Mitroy and Charles W Clark. "Theory and applications of atomic and ionic polarizabilities". In: *Journal of Physics B: Atomic, Molecular and Optical Physics* 43.20 (Oct. 2010), p. 202001.

[22] William Coirier and Robert Tramel. "Development of An Integrated Aero-Optics Modeling Capability: OVERFLOW-AeroOptics". In: *48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*. Orlando, FL: AIAA Paper 2010-557, Jan. 2010.

[23] Tao Wang et al. "Numerical study of evaluating the optical quality of supersonic flow fields". In: *Applied Optics* 46.23wang-a (Aug. 2007), pp. 5545–5551.

[24] Parviz Moin Ali Mani Meng Wang. "Computational Study of Aero-Optical Distortion by Turbulent Wake". In: *36th AIAA Plasmadynamics and Lasers Conference*. Toronto, Canada, June 2005, AIAA Paper 2005–4655.

[25] F. A. Fernandez, Y. C. Yong, and R. D. Ettinger. "A simple adaptive mesh generator for 2-D finite element calculations (optical waveguide theory)". In: *IEEE Transactions on Magnetics* 29.2 (Mar. 1993), pp. 1882–1885.