

# HAOT: A Python package for hypersonic aero-optics analysis

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## Summary

Hypersonic flows present unique challenges due to the complex interplay of fluid dynamics, chemical reactions, and optical phenomena. As a signal from a Light Detection and Ranging (LiDAR) system travels through a hypersonic flow field, the beam is affected by the flow, potentially leading to errors in targeting and detection measurements.

HAOT is a Hypersonic Aerodynamics Optics Tools Python package developed to calculate different aerodynamic properties in a hypersonic medium. Its source code is available on [GitHub](#), the documentation is available on [Read the Docs](#) and an example on the usage of the package is given on the GitHub repo under the example folder.

## Statement of Need

Many techniques used to calculate optical properties are scattered across various papers, but there is no centralized repository containing all these calculations. Furthermore, some of these calculations require spectroscopy constants, which are often unclear or inconsistently presented in the literature. This package includes a constants module that provides and documents numerous spectroscopy constants for diatomic molecules.

This package was used by ([Liza et al., 2023](#)). In this work, they used results from Computational Fluid Dynamics (CFD) to calculate optical properties of interest.

## Algorithms

The HAOT package, contains five modules:

- Aerodynamics
- Optics
- Quantum Mechanics
- Constants
- Conversions

Each module can be imported independently. The [documentation](#) explains the functions in each module as well as their usage. Docstrings are included, so the function prototypes and usage can also be accessible in an interactive Python session. Results from these algorithms were compared with the literature, and a unit test was developed, which is located under the `unit_test` directory.

This section highlights some of the capabilities of the package but not all of them. For instance, the package can calculate various compressible flow properties, such as isentropic, normal shock, and oblique shock relations. Please refer to the [documentation](#) for a complete list of available functions.

## Results

Equation 1 was introduced by (Smith & Weintraub, n.d.), and it provides an approximation for the index of refraction as a function of atmospheric altitude.

$$n(h) \approx 1 + \frac{K_1}{T(h)} \left( p(h) + K_2 \frac{e(h)}{T(h)} \right) \quad (1)$$

Where:  $K_1$  and  $K_2$  are constants,  $T(h)$  is the temperature as a function of altitude,  $p(h)$  is pressure as a function of altitude, and  $e(h)$  is the partial pressure of water vapor.

Results for Equation 1 are provided in Figure 1. This approximation is a useful way of analyzing the region in which the index of refraction has the greatest impact on a seeker's performance. As expected, the critical region is between 20 [km] and 30 [km] above sea level, which is a region where a seeker's performance is particularly important.

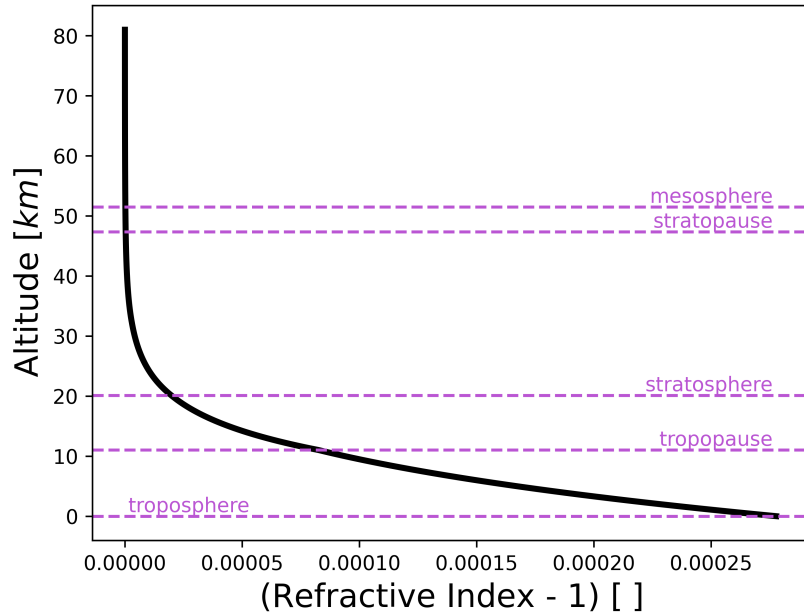


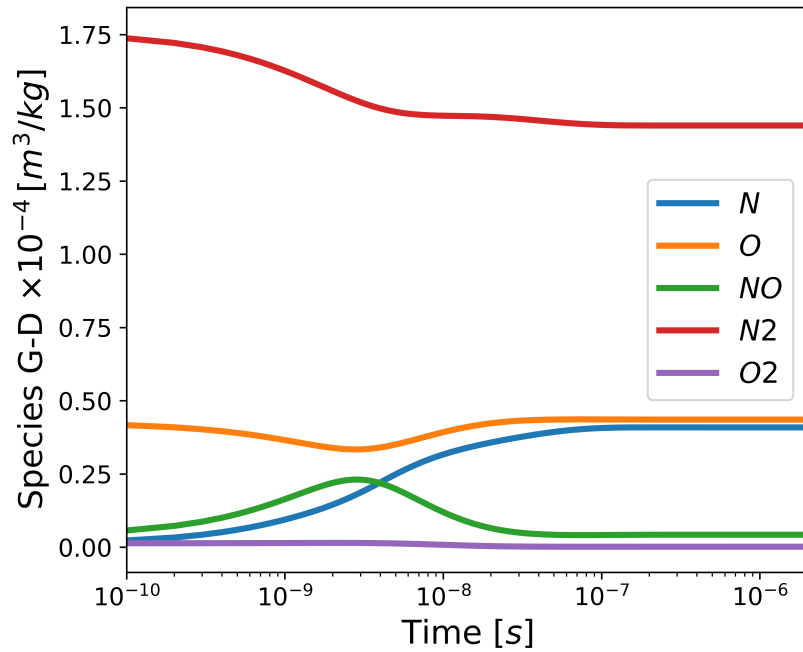
Figure 1: Atmospheric index of refraction for dry air.

The Gladstone-Dale constant is an important constant used to calculate the index of refraction (equation 2). For a dilute gas, the index of refraction can be approximated as:

$$n - 1 = \sum_{s=1}^N K_s \rho_s \quad (2)$$

Where:  $n$  is the index of refraction,  $K_s$  is the species' Gladstone-Dale constants, and  $\rho_s$  is the species density.

Figure 2 shows the Gladstone-Dale constants for a five-species gas. The species density was calculated using the CFD tool, SU2 (W. T. Maier et al., 2021), (W. Maier et al., 2023). These densities were then loaded into the HAOT tool to calculate Figure 2.



**Figure 2:** Species Gladstone-Dale constants for a five-species gas.

Another way of calculating the index of refraction is using the polarizability, as shown in equation 3.

$$n - 1 = \frac{1}{2\epsilon_0} \sum_{s=1} \alpha_s N_s \quad (3)$$

Where:  $\epsilon_0$  is the dielectric constant in vacuum,  $\alpha_s$  is the species' polarizability constant, and  $N_s$  is the partial species mass fraction.

Figure 3 uses the extrapolation (equation 4) developed by (Kerl et al., 1992), based on the work of (Hohm & Kerl, 1986).

$$\alpha(\omega, T) = \frac{\alpha(0, 0)}{1 - \left(\frac{\omega}{\omega_0}\right)^2} (1 + bT + cT^2) \quad (4)$$

Where:  $\alpha$  is the polarizability as a function of the laser's frequency  $\omega$  and temperature  $T$ .  $b$  and  $c$  are extrapolation constants, all of which are implemented in the HAOT package, and  $\omega_0$  is the oscillation frequency of the diatomic molecule.

Figure 3 shows the change of polarizability as a function of temperature.

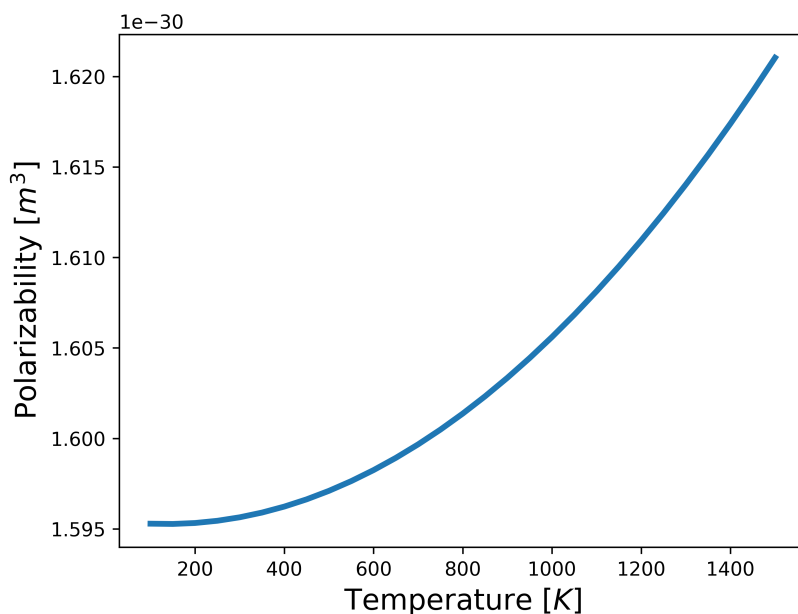


Figure 3: Polarizability of  $O_2$  using Kerl's method for a laser of  $633 \text{ nm}$ .

64 The results presented so far highlight some of the capabilities of the aerodynamics and optics  
 65 modules. Figure 4 shows the results of the Boltzmann Distribution for  $N_2$  (in the quantum  
 66 mechanics module). Some aero-optics calculations (Buldakov et al., 2000), (Tropina et al.,  
 67 2018) required a Boltzmann Distribution to calculate the polarizability and index of refraction  
 68 for diatomic molecules.

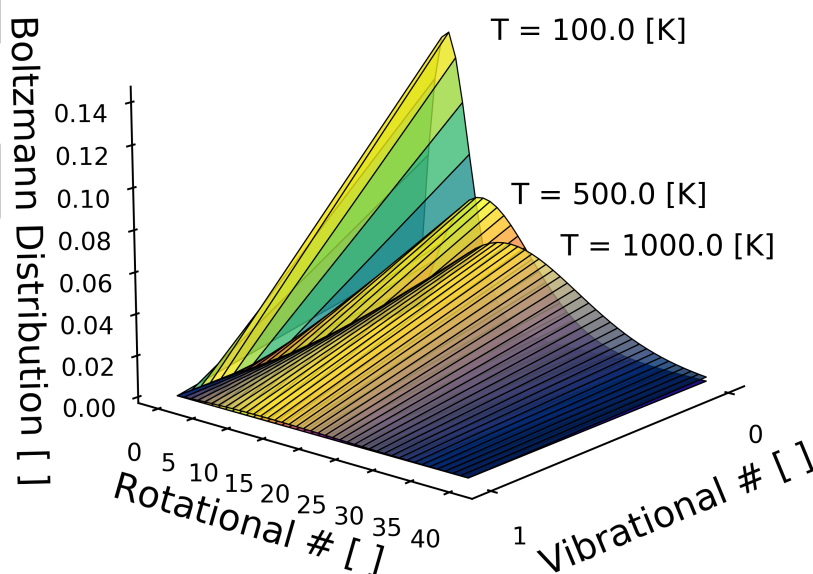


Figure 4: Boltzmann Distribution for  $N_2$ .

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