Pouring Tea (or Coffee)

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Abstract

There is a subtle shift in the quality of the sound produced by pouring tea as its temperature is allowed to vary. A modification of a model explaining the sound produced by the similar phenomenon of a single round object plunging into a body of water is proposed to describe this shift, and an analysis is presented. Timbral data obtained via auditory analysis software at a range of temperatures are compared as a means of verification, and an additional comparison with coffee is made.

1 Introduction

While pouring out a cup of tea to enjoy, any self-identifying tea drinker should have the opportunity to hear a slight distinction between sounds made by teas that are served at different temperatures[1]. If this tea drinker listens carefully, they might notice that while the *pitch* of the sound produced does not vary with temperature, the *timbre* of this sound—a related, yet distinct aspect of how it is auditorily percieved—does vary with temperature.

The starting point here for an explanation is a simple and effective model for what determines the pitch of the sound made by a stone dropping into a pond, detailed later in section 3. According to this model, the pitch depends on the physical dimensions of the stone being dropped, so for a simplified conceptualization in which a stream of tea being poured is understood merely as a rapid succession of equally-dimensioned (and perhaps equally spaced) drops, it is not hard to see how its characteristic sound is similarly produced.

[1]For a perhaps more detectable distinction, compare hot and cold water [1].

To see how this model may be modified to arrive at an explanation, it helps to consider in more detail the aspects of sound that influence timbre, as well as how physical characteristics of liquid water differ with temperature.

2 Background

2.1 Psychoacoustic perception

Loudness and pitch can both be physically defined relatively easily, namely in terms of the amplitude and frequency of a given waveform, respectively [2, 3]. For pure sinusoid tones with well-defined amplitudes and frequencies, these definitions coincide exactly. For arbitrary waveforms, a number of slightly looser interpretations are available: for example, a frequency-*like* parameter interpretable as pitch can be obtained 'pictographically' by counting the number of times a waveform crosses 'zero' per unit time. There are various other ways of obtaining pitch and loudness parameters for arbitrary waveforms, and this task of *pitch detection* is one of many aspects of the larger field of audio signal processing.

Timbre, on the other hand, is more elusive. According to Wikipedia [4], it can be described in simple terms as the combination of aspects of sound that allow humans to distinguish between two different types of instruments, e.g. a cello and a flute, playing at the same loudness and pitch. It is associated physically with a waveform's spectrum and envelope; a popular quote from McAdams and Bregman characterizes it well [5].

> "Timbre tends to be the psychoacoustician's multidimensional waste-basket category for everything that cannot be labeled pitch or loudness, including short-term spectral changes such as onset transients, long-term spectra, those dynamic qualities which a musician would term 'texture,' and so on."

There is therefore a variety of physical attributes that contribute to the perception of timbre. Despite this, as with pitch and loudness, a multlitude of techniques have been developed to obtain parameters that may be associated with the perception of the timbre of a sound, such as e.g. its zero-crossing rate, spectral centroid, spectral spread, spectral skewness, spectral kurtosis, roll-off, spectral entropy, spectral flatness, roughness, irregularity, inharmonicity, and spectral flux, just to list the features typically computed by analysis software. To this end, it seems at first that nearly anything might be able to contribute to the timbre of the sound produced by pouring tea, in that it seems nearly anything could contribute to a slightly different spectro-temporal signature that is then percieved as having a different timbre. However, as tea is really just water in flavorful and aromatic disguise, it is restricted by water's physical properties.

2.2 Water in the drinkable regime

Water and many other fluids exhibit physical properties that tend to vary with temperature. Those that could influence how it behaves acoustically are density, viscosity, and surface tension. The density of water does not vary too much with temperature within the drinkable regime[2], and neither does the surface tension [6]. Figs 1 and 2 below indicate how (in)significant those variations are.

Subject to abrupt changes in phase at certain temperature thresholds,

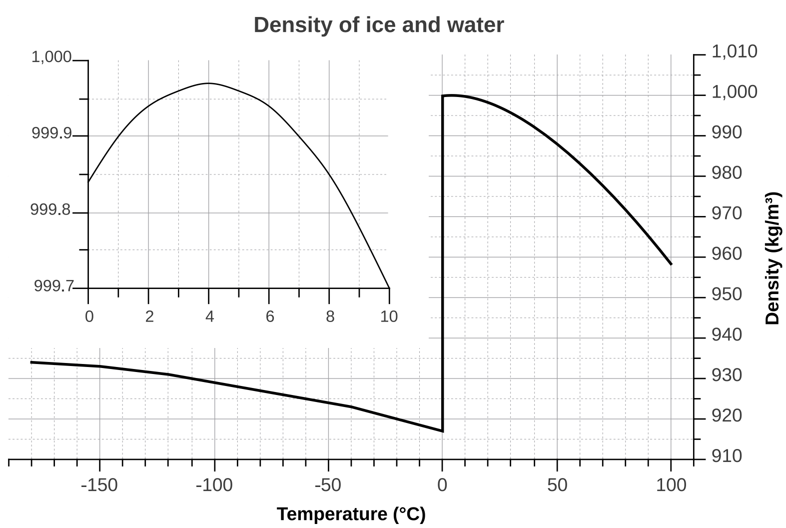

Figure 1: Density of water as it changes with temperature. Note the fractional change of less than 5% within the drinkable regime.

liquid water does not at first appear to vary considerably in its physical

[2] "Water near [its] boiling point is about 96% as dense as water at 4 °C" (at which it attains its maximum density), a difference which Wikipedia claims to be significant.

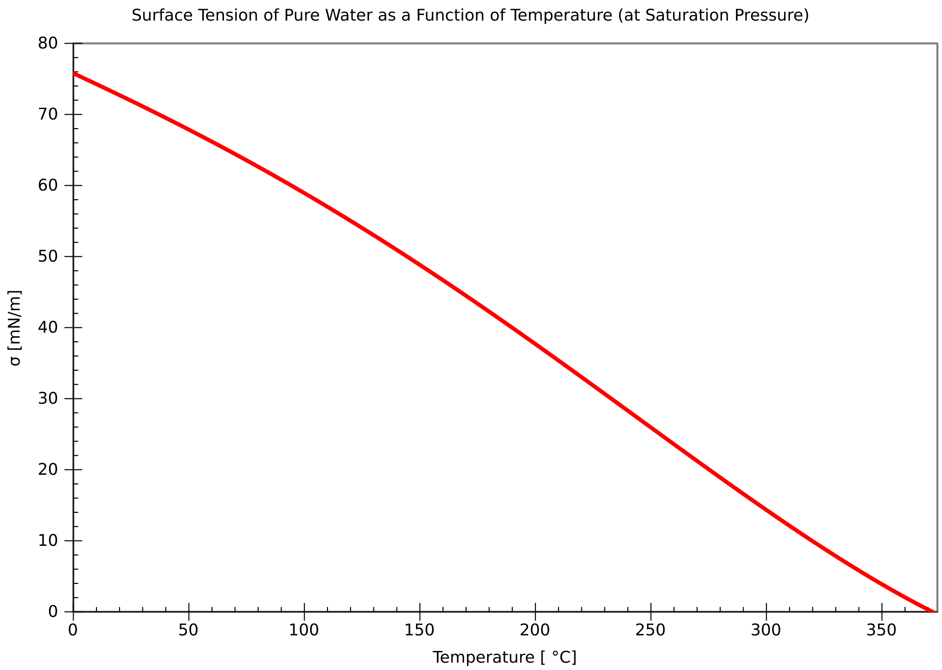

Figure 2: Surface tension of water as it changes with temperature. Note the miniscule scale of variation within the drinkable regime.

properties *between* those thresholds, especially compared to e.g. glass or honey, which transition more or less continuously between liquid and solid phases. However, the viscosity of water actually does change significantly between room temperature and boiling [7], as can be seen in fig. 3. Viscosity is also a property associated with drag forces [8], which have the effect of damping motion, which may affect the envelope and spectrum of the sound each pour makes.

The considerations above suggest that the effects of viscosity would need to be accounted for within any model that explained a temperature-dependent variation in timbre between tea pouring sounds. The next step is doing so while keeping things relatively simple.

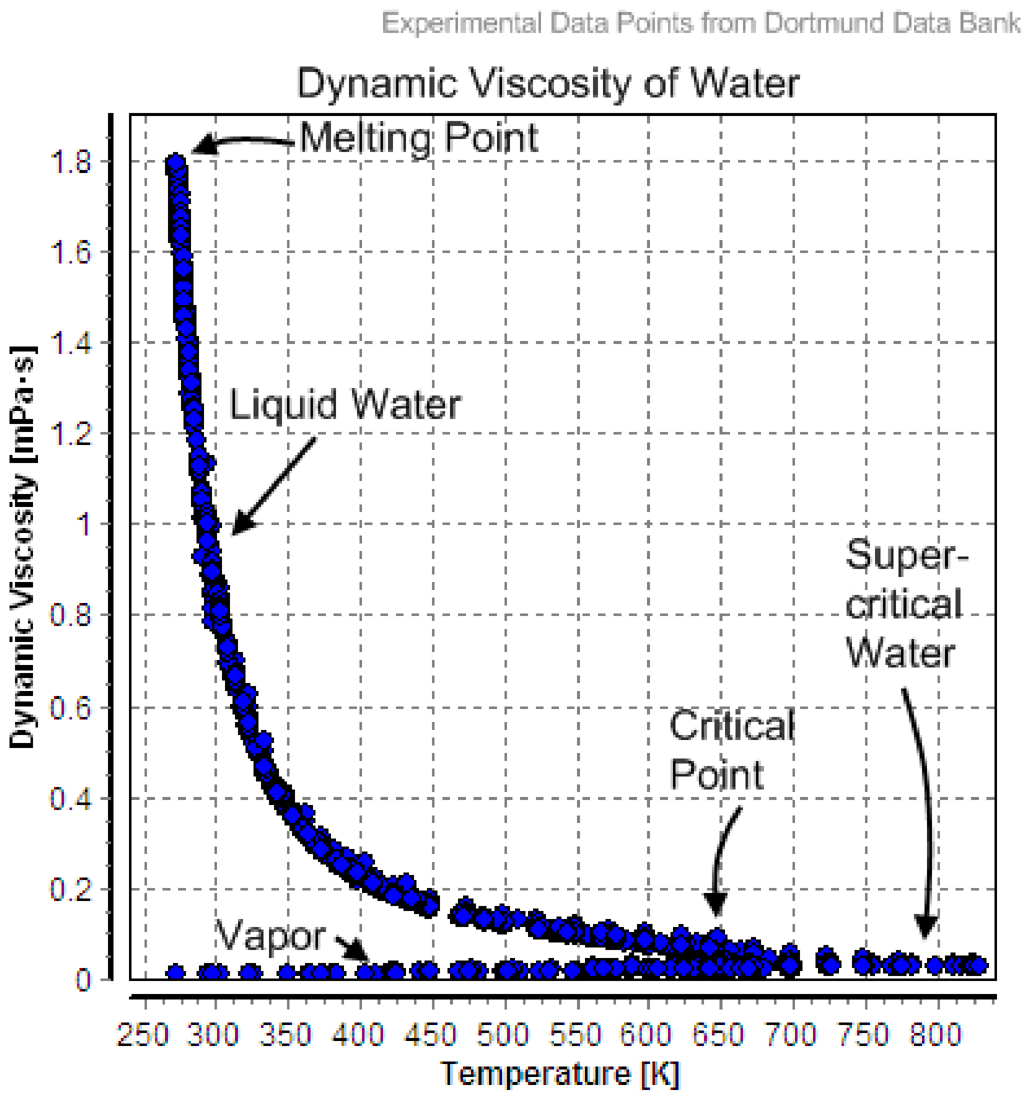

Figure 3: Empirical plot of the variation of the dynamic viscosity of water with temperature. Note the fractional change within the drinkable regime of ∼89%—for this it deserves its own page.

3 Model

3.1 Stepping stone: stone in a pond

The stone-in-a-pond model [9] approximates the physical configuration of a stone plunging into a pond as a simple harmonic oscillator consisting of:

- the air-filled cavity that the stone produces under the water's surface
- the tube-like 'neck' of this cavity extending up past the water's surface
- the column of air filling this tube-like neck

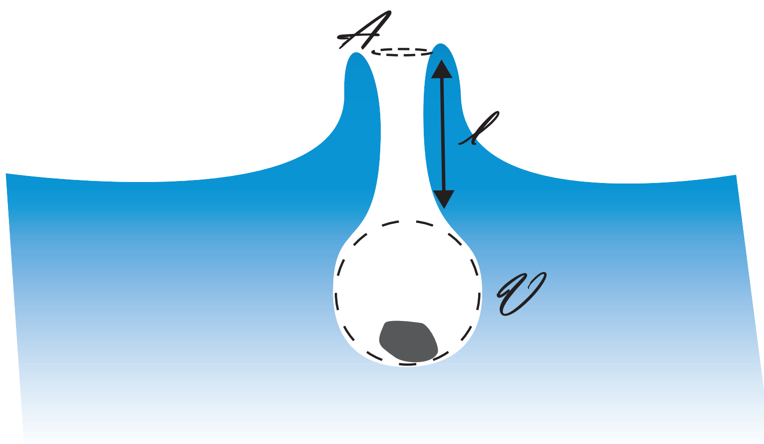

Figure 4: A stone plunging into a pond. The volume of the air cavity is V, the neck of the cavity has cross-sectional area A, and it extends a length l above the surface.

An illustration is provided with physical dimensions labeled in fig. 4. The natural frequency of such a harmonic oscillator would then be evident from its equation of motion

$$\begin{aligned}\text{mass} \times \text{acceleration} &= \text{force on air column}\\ (\rho A l) \times \frac{d^2}{dt^2}(\Delta z) &= A \times \Delta P = A\Delta P \cdot \frac{\Delta\rho}{\Delta\rho} \cdot \frac{\rho}{\rho}\\ &= A\rho \cdot \frac{\Delta P}{\Delta\rho} \cdot \frac{\Delta\rho}{\rho}\\ &= A\rho \cdot c^2 \cdot \left(-\frac{A\Delta z}{V}\right)\end{aligned}$$

which, once put in a more familiar form, is

$$\frac{d^2}{dt^2}(\Delta z) + \left(c^2 \frac{A}{Vl}\right)(\Delta z) = 0 \tag{1}$$

none other than the equation of motion for a simple harmonic oscillator with natural frequency $\omega_0^2 = c^2 A/Vl$. This suggests that the pitch of the noise produced as a function of the parameters is something like

$$\nu = \frac{\omega_0}{2\pi} \approx \frac{c}{2\pi}\sqrt{\frac{A}{Vl}} \tag{2}$$

which comes very close to observed pitches with reasonable[3] choices of relationships amongst A, V, and l.

3.2 A viscous pond

Revisiting the force balance for the cavity-tube-air column system in the stone-in-a-pond model, a first choice for including the effects of water's viscosity might be in a velocity-dependent damping term in the ODE describing the system's motion. For simplicity, the drag force on the air column as it is forced through the water by the pressure oscillations in the cavity will be approximated as the viscous drag experienced by a comparably-sized 'ball of air' moving through the water. The magnitude of this quantity is given by Stokes' Law [10]:

$$F_{\text{drag}} = 6\pi\mu RV \tag{3}$$

[3] e.g. $A = l^2$, and $v = (2l)^3$ for a round-ish stone of diameter l.

where μ is the fluid's dynamic viscosity, R is the radius of the ball, and V is the magnitude of the velocity of the ball. Working this into the force balance (and allowing 6π to equal 20 as well as R to equal $l/2$), it becomes:

$$\begin{aligned}(\rho A l)\frac{d^2}{dt^2}(\Delta z) = A\Delta P - F_{\text{drag}} &= A\Delta P - 20\mu\frac{l}{2}\cdot\frac{d}{dt}(\Delta z) \\ &= A\rho\cdot c^2\cdot\left(-\frac{Az}{V}\right) - 10\mu l\cdot\frac{d}{dt}(\Delta z)\end{aligned}$$

giving

$$\frac{d^2}{dt^2}(\Delta z) + \frac{10\nu}{A}\cdot\frac{d}{dt}(\Delta z) + \left(c^2\frac{A}{Vl}\right)\Delta z = 0 \tag{4}$$

where $\nu = \mu/\rho$ is the kinematic viscosity of water. This is describes a damped oscillator system with damping term

$$\begin{aligned}2\zeta\omega_0 &= \frac{10\nu}{A} \\ \zeta &= \frac{5\nu}{A\omega_0} = \frac{5\nu}{c}\sqrt{\frac{Vl}{A^3}}\end{aligned} \tag{5}$$

Using the same reasonable estimates as in the previous section for A, l, and V, this results in an approximate $\zeta \approx 0.41/(\frac{m^2}{s}$ of kinematic viscosity) relationship between viscosity and damping term, placing the system in the underdamped regime. The *ringing frequency* of such a system is then

$$\nu = \frac{\omega}{2\pi} = \frac{\omega_0}{2\pi}\sqrt{1-\zeta^2} \tag{6}$$

where it is emphasized on Wikipedia that this frequency does not change in time. The solution for Δz in this case can be written

$$\Delta z = e^{-\zeta\omega_0 t}(A\cos\omega_0 t + B\sin\omega_0 t)$$

for some Fourier coefficients A and B that are of little importance here. The important part is the factor $e^{-\zeta\omega_0 t}$ out front, which leads to the more useful

$$|\Delta z| \sim e^{-\zeta\omega_0 t} \tag{7}$$

which captures the decay in amplitude of each mode. Upon brief consideration of equations (6) and (7), we may arrive at the following conclusions:

1. The difference in pitch between two teas being poured at different temperatures (read: different viscosities) is exceedingly[4] small, in that the same fundamental modes should be present, and viscosity does not end up shifting any mode's frequency significantly.

2. However, the difference in the *rate of decay of the amplitude*, i.e. the die-off or envelope (a consequence of the proportionality $|\Delta z| \sim e^{-\zeta\omega_0 t}$ in this model) of each mode present should, in comparison, vary more notably with viscosity.

This comparison is obtained by observing that the resulting difference in decay rate depends on the *relative* difference between damping ratios (e.g. ζ_1/ζ_2) while the difference in pitch depends on the *absolute* difference as measured on some fixed scale (e.g. $\sqrt{1-\zeta_1^2}/\sqrt{1-\zeta_2^2}$). The relative difference is independent of the scale of ζ, but this absolute difference depends much more on the size of ζ relative to 1. In the drinkable regime, water's viscosity results in damping ratios between 0.12 and 0.73, so even a 'significant' relative difference (e.g. $\zeta_1/\zeta_2 = 3$) would not lead to a detectable difference in pitch, but should lead to a noticable change in envelope.

4 Evidence

4.1 On the quantification of timbre

As previously mentioned, timbre is determined by a combination of spectral and temporal characteristics of a waveform. MIRToolbox, a popular MATLAB software, provides a few tools [11] for timbral analysis of an audio sample. In addition to pitch information, it can estimate features like envelope, as well as collect a great deal of information about a sample spectrum and said spectrum's time-evolution. For the results presented here, the `mirenvelope()` function was used to compare the envelope shapes between each tea pour. The `'Log'` option was used for easier comparison of their decay rates, with a sampling rate of 3000 Hz.

[4] In the sense that one should not expect the average human to reliably discern by ear alone.

4.2 The tea procedure

The kettle in the lobby of ICME[5] has a nice selection of six temperatures to suit any self-identifying tea drinker's brewing needs, summarized in table 1.

Using these temperatures as reference by recording water being poured as soon as the kettle beeped, a series of audio samples were taken using a mobile phone. An additional sample was also taken at room temperature. Each sample lasted about 15 seconds, obtained by pouring from a height of about 10 cm from the top of the mug. The pouring was performed so that the stream of tea had a diameter of about 1 cm at the moment of surface contact. Auditory analysis software was used to extract the timbral attributes of interest[6]. An overplot of the log of the envelopes of each temperature setting can be found in fig. 5, while a comparison of room temperature water and boiling water is in fig. 6.

setting	DELICATE	GREEN	WHITE	OOLONG	FRENCH PRESS	BLACK
temperature (°F)	160	175	185	190	200	212

Table 1: How the kettle suggests you brew your beverages.

These envelope data are qualitatively consistent with each pour experiencing a viscosity-driven decay of its fundamental frequency, where the viscosity has an inverse correlation with temperature. That is, the harmonics produced by pouring cooler teas decay more rapidly in general, resulting in steeper envelopes as expected.

[5] Institute for Computational and Mathematical Engineering, Stanford University

[6] The samples can be found at:
`soundcloud.com/mll-284056533/sets/pouring-tea-and-coffee`.

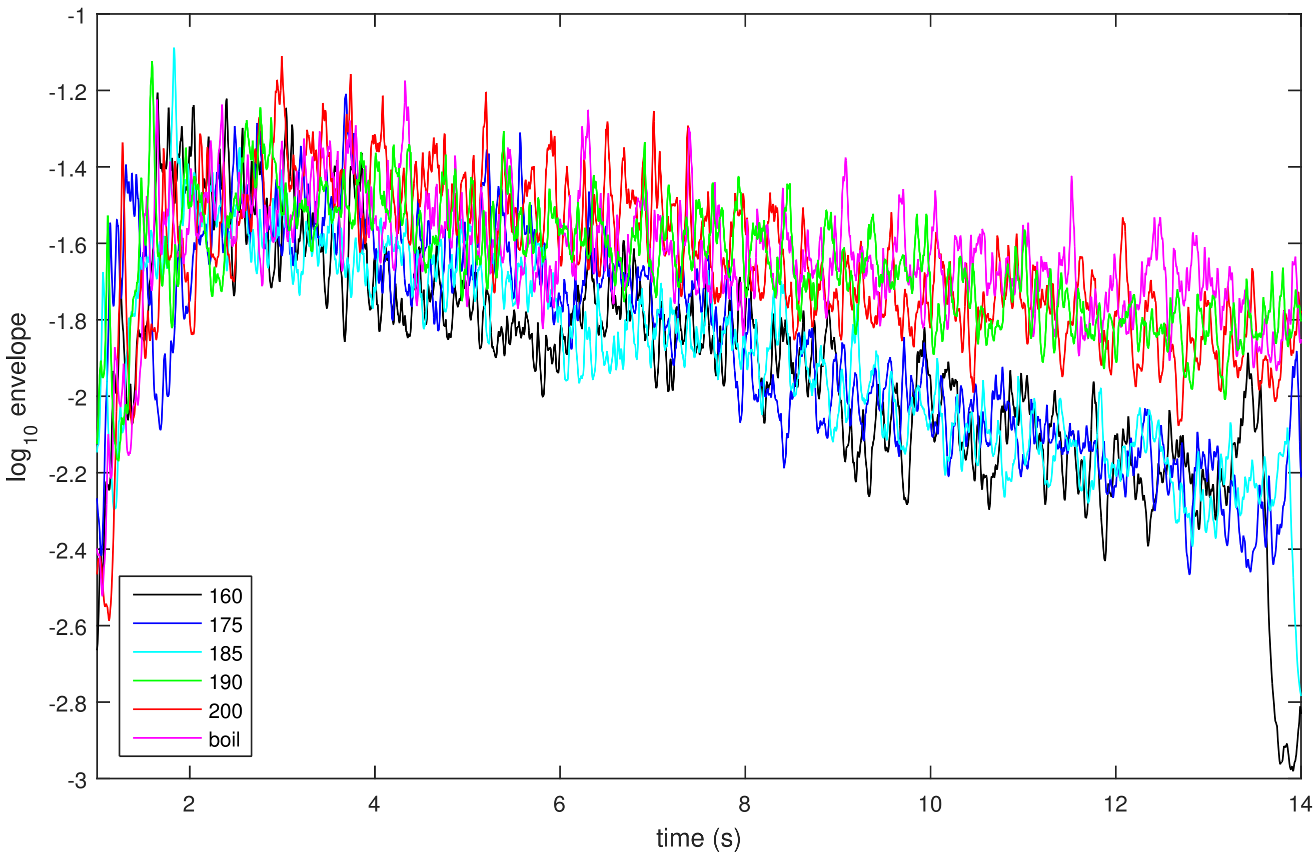

Figure 5: Log of the envelopes of each pour's waveform. Note the increase in steepness as the temperature of the tea declines.

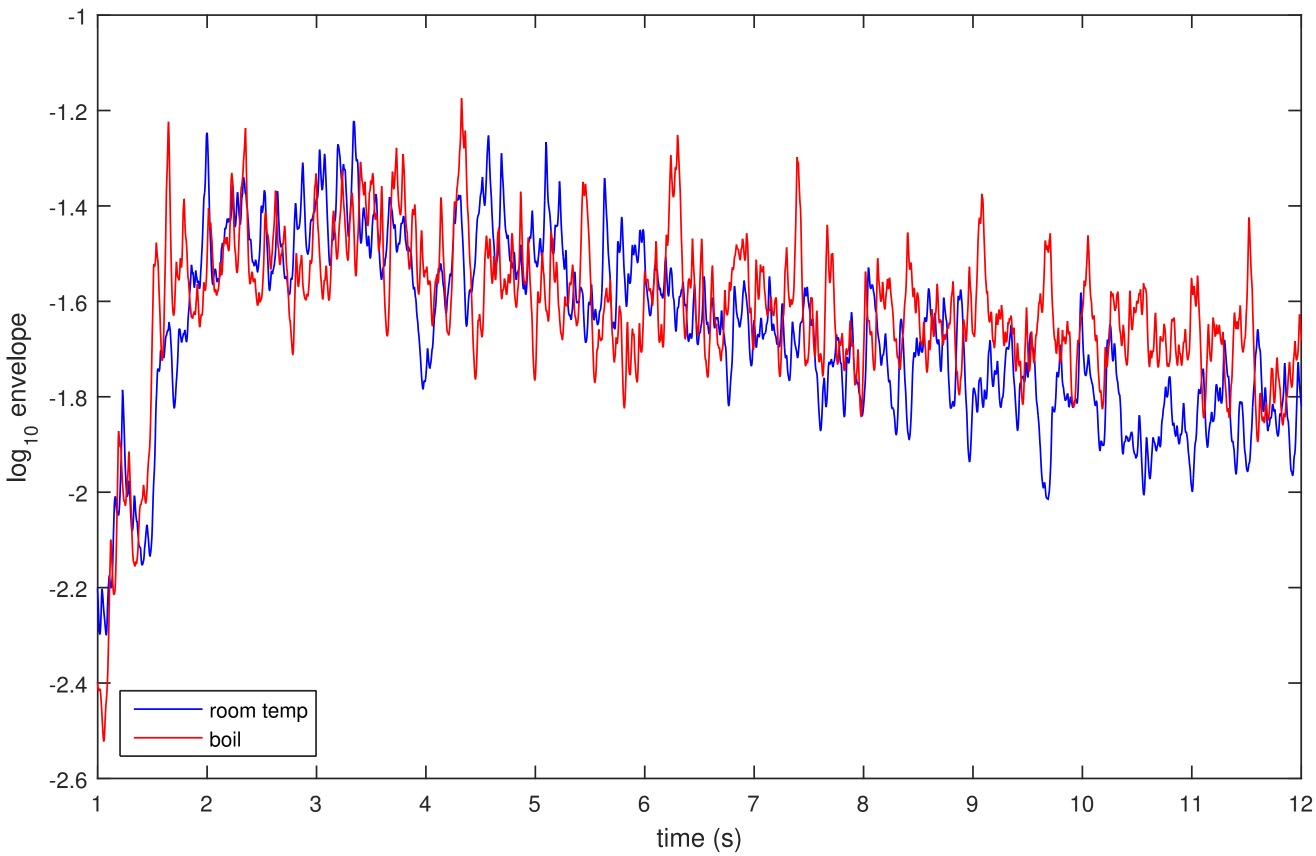

Figure 6: Comparison between envelopes of lowest and highest temperatures. That of the boiling water is flatter.

4.3 Coffee

A sample was also recorded by the same procedure for coffee. The motivation is that the oils and solids present in coffee that qualitatively distinguish it from tea should also affect its viscosity and, in turn, the apparent timbre of its pouring sound. Fig. 7 includes a comparison between tea and coffee pour envelopes.

The plot indicates that coffee indeed does make a slightly different sound/envelope

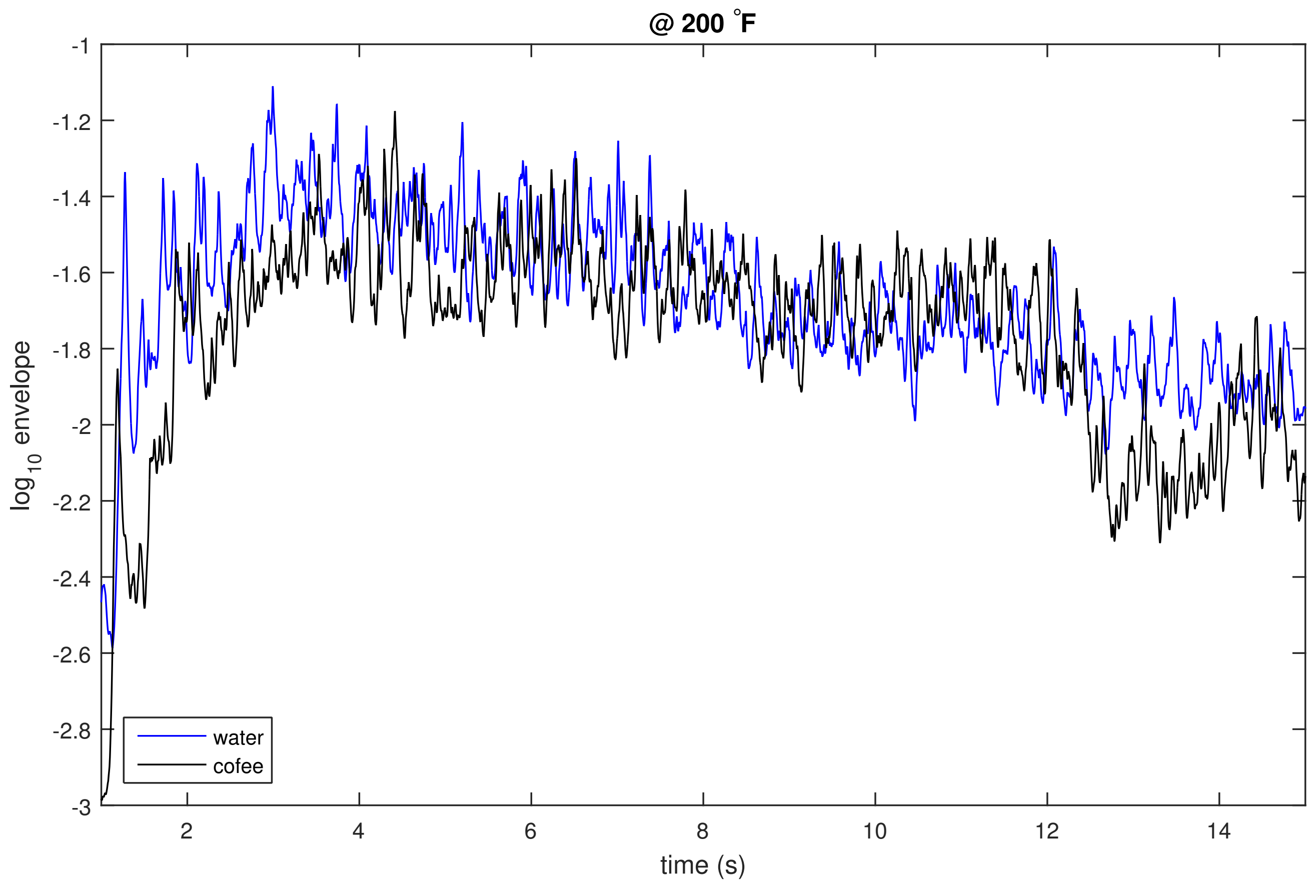

Figure 7: Respective envelopes of coffee and tea being poured at 200 °F. That of coffee is slightly steeper, indicating higher viscosity.

that of water at the same temperature. Coffee's difference in viscosity is great enough to change its acoustic properties noticeably[7] compared to those of water or tea poured at the same temperature, though it is hard to see from this comparison of their log envelopes.

4.4 Curve-fitting

To investigate this 'cooler $\leftrightarrow$ steeper' trend a bit more quantitatively, one simple check that is a linear fit to the log of the envelopes. Assuming the model is not too poor, the envelopes should possess the signature of exponential damping at a viscosity-dependent rate. When the log envelopes from each waveform are fit with linear functions (taking care to omit the first and last five seconds of audio), the corresponding slopes only follow the trend loosely (i.e. only for larger discrepancies in temperature): table 2 compares the magnitude of the slope of each fit. A visual comparison of the fitting curves can be found in fig. 8.
The plot in fig. 3 provides a possible explanation for the values fit to temperatures in the middle range: these temperatures of water are not very far from one another in terms of viscosity, since the viscosity curve loses most of its height before it reaches that range. It is likely, then, that for nearby values, the trend is too weak compared to the magnitude of error in the analysis to observe plainly.

temperature (°F)	parameter ($\times 10^{-5}$)
room temp	-1.47*
160	-2.32
175	-2.02
185	-2.00
190	-0.94
200	-1.20
coffee (200)	-1.00
boiling	-0.73

Table 2: Steepness parameters for linear fitting on each log envelope. For nearby temperatures (i.e. smaller-magnitude differences), the trend is likely obscured by error (* this value likely due to nonstandard pouring technique).

[7] Anecdotally (and in vague terms), the sound made by coffee seems 'smoother'/'cooler'/'lower.'

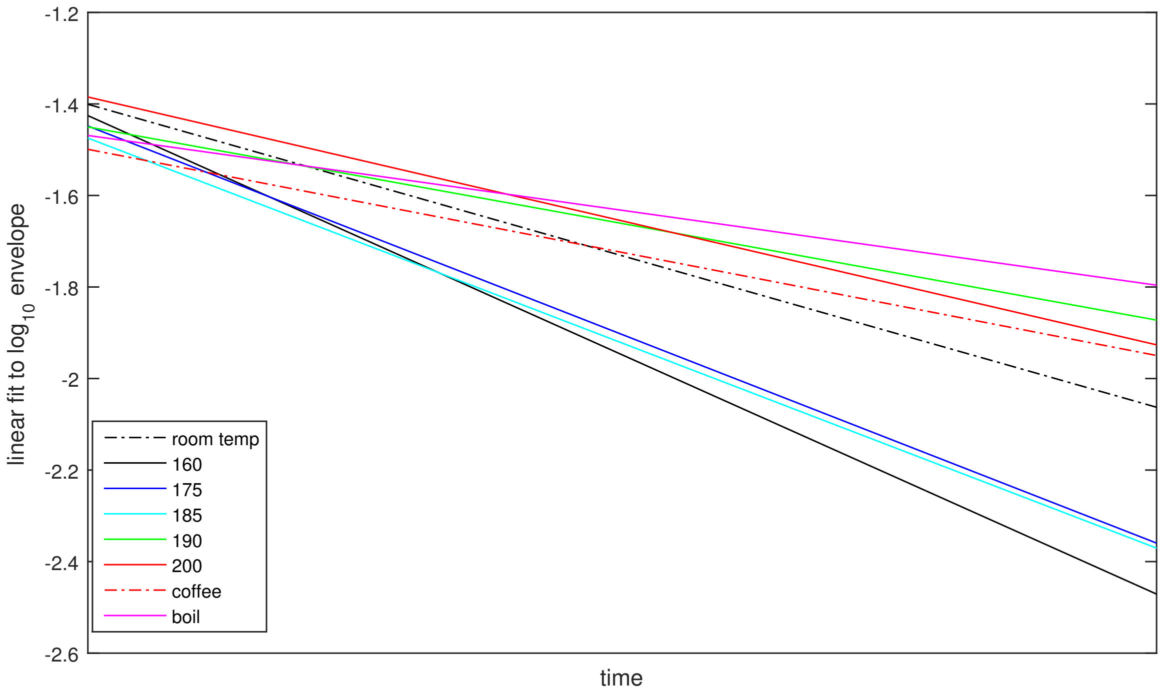

Figure 8: Slope comparison between linear fits on the log envelope curves of each pour. The fits for coffee and room temperature poors do not demonstrate the 'cooler $\leftrightarrow$ steeper' trend clearly, indicating that data collection could be improved.

5 Considerations

With regard to the experimental setup, there are a few details that introduce some challenges to obtaining 'clean' data:

- The kettle tends to be a bit loose in its interpretation of temperature, especially during repeated use within a short interval of time
- Each pour was performed by a human and therefore is subject to variations in height, total water poured, rate of pouring, duration, etc.
- The recording environment and tools were far from studio quality, so there is a great deal of noise—both figuratively and literally—in each sample that could be reduced by simply using fancier equipment.

Efforts to mitigate these sources of error include not using adjacent temperature settings on the kettle back-to-back, as well as time spent/care taken to improve the uniformity of the pouring technique. Fortunately, the trend is prominent enough (after all, it is detectable by human ears) to be observed 'more broadly' in the sense that the comparison is obvious between e.g. water at 160 °F and boiling water.

While the specific details of the model—namely that 'sphere of air'—are not the most precise depictions of what occurs physically, they are close enough on some basic level that this model is able to provide a sought-after explanation for perceptual differences in tea pouring sounds. There are certainly many ways to treat viscous effects in a fluid acoustic model that are more precise in terms of 'what's actually happening,' but it is also important to consider the relative lack of precision with which the phenomenon can be expressed. One one hand, it is easy for one to recognize that there is a distinction between each pouring sound produced, but on the other hand, it is difficult for one to provide a satisfactory characterization of this distinction in one's own words.

This degree of simplification, then, is an advantage of the viscous pond model. By restricting analysis to a vastly simplified physical picture, it avoids dealing with information that may lead to higher-order effects or more complicated predictions that are harder to interpret. No overly-complicated mathematical machinery is required to reach its conclusions. Simple predictions are easier to interpret (perhaps at some cost to precision), and they can still be accurate.

6 Conclusions

Water is pretty strange as far as fluids go; its physical characteristics may not initially seem to vary detectably, but upon closer inspection they do, although somewhat subtly. Also, the physics and perception of sound are intimately related, but the most interesting phenomena perceptually are often the most difficult to characterize physically. One may conclude that, insofar as each tea and coffee has some distinct combination of things that determine the flavor and aroma of its brew, this combination also determines its viscosity and therefore even its acoustic properties—yet another subtle distinctness about each cup that makes it special.

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