CDF and PDF of the Doppler Shift for Fixed v and f_c.

Let $\Theta \sim \text{Uniform}([-\pi, \pi])$, then the PDF of Θ is

$$F_\Theta(\theta) = \begin{cases} 0 & \text{if } \theta \in [-\infty, -\pi) \\ \dfrac{\theta - (-\pi)}{\pi - (-\pi)} = \dfrac{\theta}{2\pi} + \dfrac{1}{2} & \text{if } \theta \in [-\pi, \pi] \\ 1 & \text{if } \theta \in (\pi, \infty). \end{cases}$$

Let $Y = f_m \cdot \cos(\Theta)$ be a random variable, where $f_m = \dfrac{v \cdot f_c}{3 \times 10^8}$ is a constant since v and fc are fixed. For $y \in [-f_m, f_m]$, we have

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P(f_m \cos(\Theta) \leq y) \\ &= P(\Theta \leq -\cos^{-1}(\frac{y}{f_m})) + P(\Theta \geq \cos^{-1}(\frac{y}{f_m})) \\ &= F_\Theta(-\cos^{-1}(\frac{y}{f_m})) + (1 - F_\Theta(\cos^{-1}(\frac{y}{f_m}))) \\ &= \Big(\frac{-\cos^{-1}(\frac{y}{f_m})}{2\pi} + \frac{1}{2}\Big) + \Big(1 - (\frac{\cos^{-1}(\frac{y}{f_m})}{2\pi} + \frac{1}{2})\Big) \\ &= 1 - \frac{\cos^{-1}(\frac{y}{f_m})}{\pi} \end{aligned}$$

Thus, we have the CDF of Y

$$F_Y(y) = \begin{cases} 0 & \text{if } y \in [-\infty, -f_m) \\ 1 - \dfrac{\cos^{-1}(\frac{y}{f_m})}{\pi} & \text{if } y \in [-f_m, f_m] \\ 1 & \text{if } y \in (f_m, \infty). \end{cases}$$

Differentiate $F_Y(y)$ w.r.t y and we obtain the PDF of Y

$$f_Y(y) = \begin{cases} \dfrac{1}{\pi f_m} \cdot \dfrac{1}{\sqrt{1 - (\frac{y}{f_m})^2}} & \text{if } y \in (-f_m, f_m) \\ 0 & \text{otherwise.} \end{cases}$$