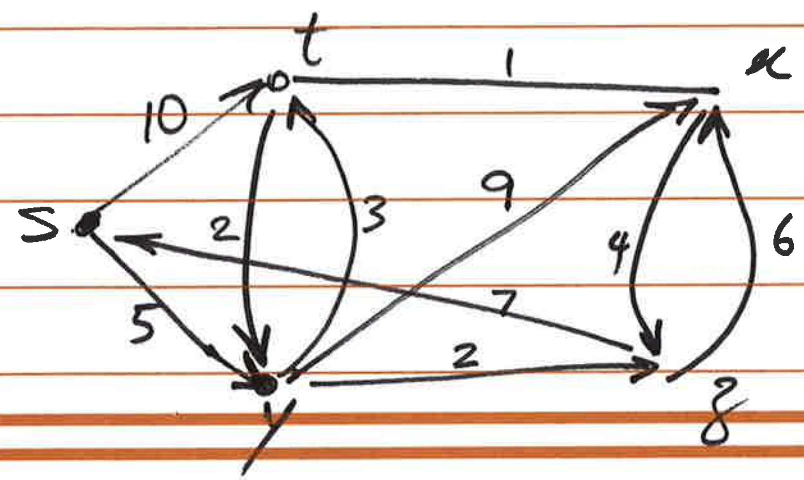
Greedy Part $\underline{2}$

Shortest Path

Shortest path

Given $G = (V, E)$ w/ $w(u,v) \geq 0$ for each edge $(u,v) \in E$ find the shortest path from $s \in V$ to $V - \{s\}$

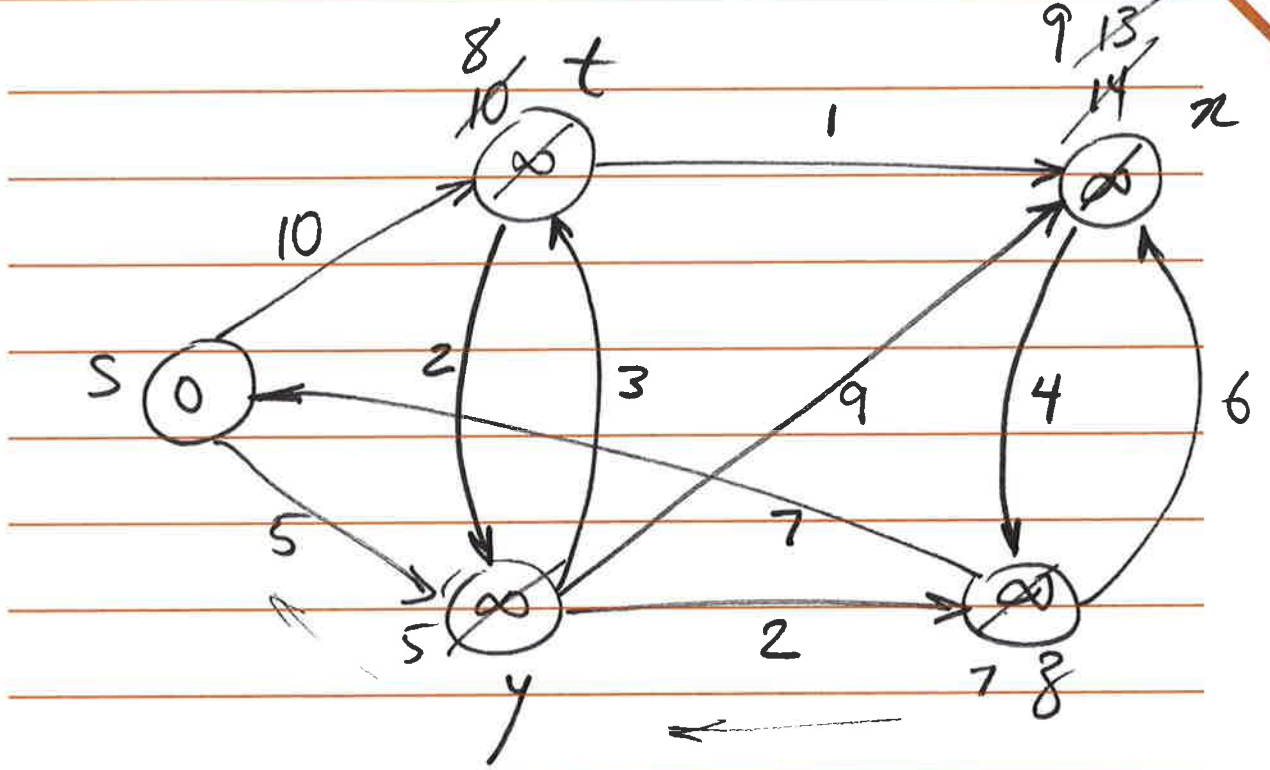


Solution:

1- start w/ a set $\underline{S}$ of vertices whose final shortest path we ~~already~~ know already!

2- At each step find a vertex $u \in V - S$ with shortest distance from s.

3- Add u to S, repeat.



1 $S = \{s\}$

2 $S = \{s, y\}$

3 $S = \{s, y, z\}$ 4 $S = \{s, y, z, t\}$

5 $S = \{s, y, z, t, x\}$

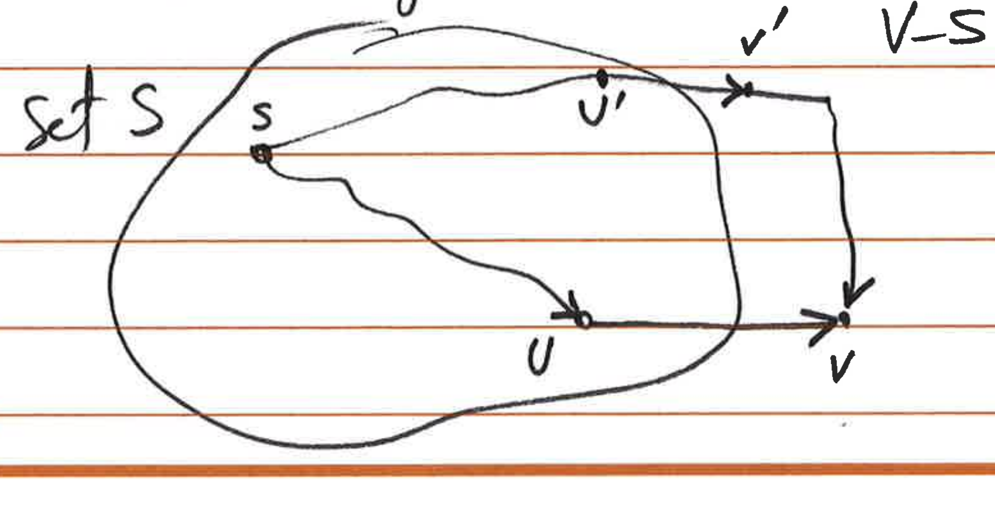
<u>Dijkstra's</u> shortest path algorithm.

→ At each step Dijkstra's alg. finds the shortest path to a new node in the graph.

Proof by math. induction

Case $|S| = 1$, $S = \{s\}$ and $d(s) = 0$ ✓

Suppose claim holds when $|S| = k$ for some $k \geq 1$. We now grow S to size $k+1$ by adding the node v. Let (u, v) be the final edge on our s-v path.



Implementation of Dijkstra's

Initially $S = \{s\}$ and $d(s) = 0$
for all other nodes $d(u) = \infty$
while $S \neq V$
select a node $v \notin S$ with at least one edge from S for which $d(v) = \min_{e(u,v):u \in S} (d(u) + l_e)$
Add v to S
endwhile

$S = \text{Null}$
Initialize priority queue Q with all nodes V, w/ $d(v)$ as their key value (all $d(v)$'s are ∞ except for s where $d(s) = 0$)
while $S \neq V$
  $v = \text{Extract\_Min}(Q)$
  $S = S \cup \{v\}$
  for each vertex $u \in Adj(v)$
    if $d(u) > d(v) + l_e$: Decrease-key $(Q, u, d(v) + l_e)$
  end for
endwhile