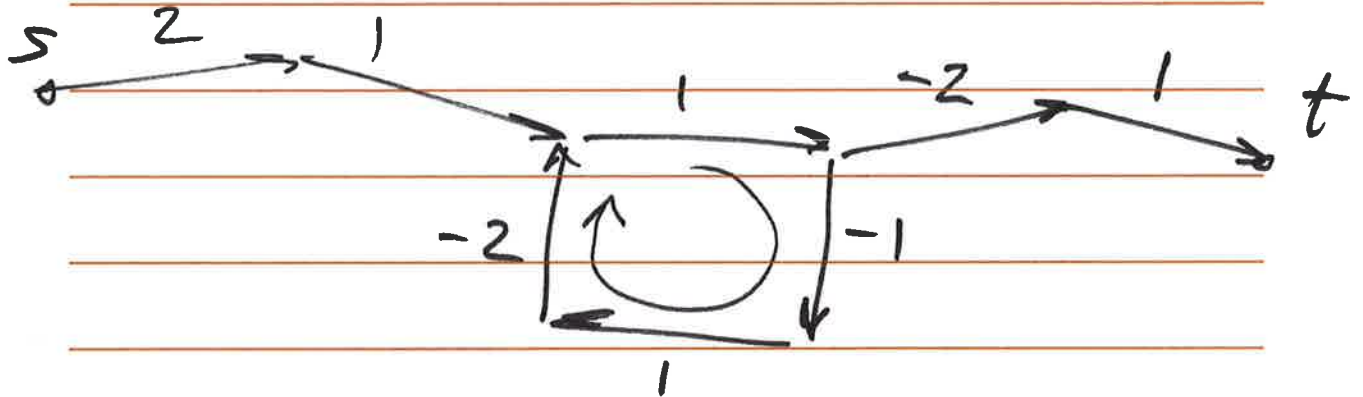


Shortest Path Problem

Dynamic Programming

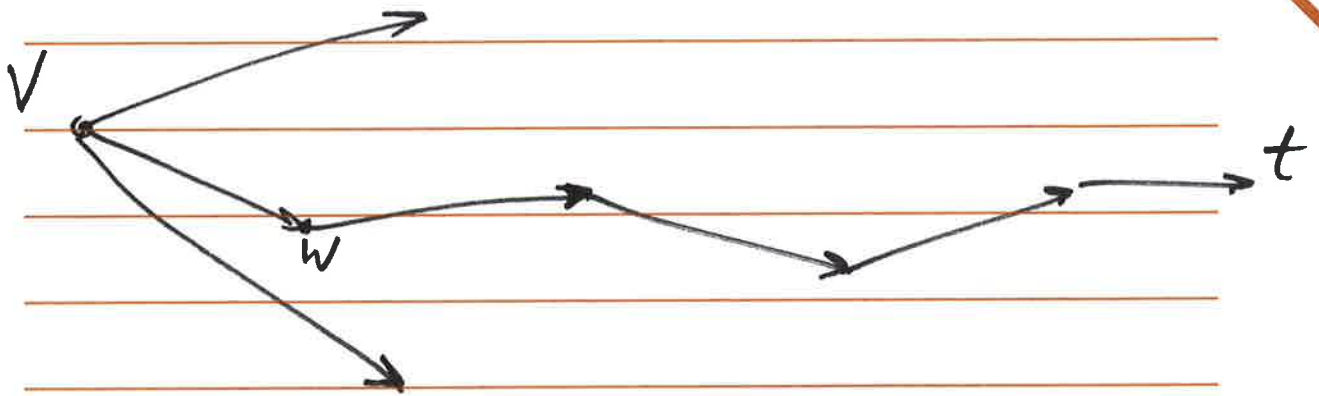
## Shortest path in a graph



If  $G$  has no negative cycles, then there is a shortest path from  $s$  to  $t$  that is simple and hence has at most  $n-1$  edges.

$OPT(i, v)$  denotes the min cost of a  $v \rightarrow t$  path using at most  $i$  edges

we want to compute  $OPT(n-1, s)$



either  $P$  uses at most  $n-1$  edges

$$OPT(n, v) = C_{vw} + OPT(n-2, w)$$

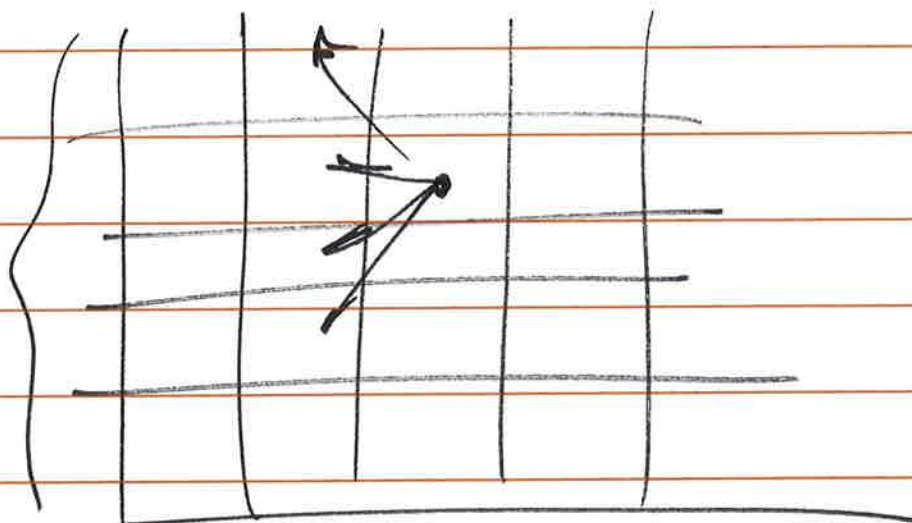
$$OPT(n-1, v) = \min_{w \in Adj(v)} (C_{vw} + OPT(n-2, w))$$

$$OPT(\cancel{n-1}, v) = OPT(n-2, v)$$

$$OPT(n-1, v) = \min (OPT(n-2, v),$$

$$\min_{w \in Adj(v)} (C_{vw} + OPT(n-2, w)))$$

V



n-1