CSCI 570 Fall 2014: Discussion 1 Problems

1.8 For this problem, we will explore the issue of *truthfulness* in the Stable Matching Problem and specifically in the Gale-Shapley algorithm. The basic question is: Can a man or a woman end up better off by lying about his or her preferences? More concretely, we suppose each participant has a true preference order. Now consider a woman *w*. Suppose *w* prefers man *m* to *m'*, but both *m* and *m'* are low on her list of preferences. Can it be the case that by switching the order of *m* and *m'* on her list of preferences and running the algorithm with this false preference list, *w* will end up with a man *m"* that she truly prefers to both *m* and *m'*?

Resolve this question by doing one of the following two things:

* Give a proof that, for any set of preference lists, switching the order of a pair on the list cannot improve a woman's partner in the Gale-Shaply algorithm; or
* Give an example of a set of preference lists for which there is a switch that would improve the partner of a woman who switched preferences.

2. Given *n* men and *n* women along with their preference lists, a *consensus-optimal* stable matching is a matching which is simultaneously pairs every man with his best **valid** partner and pairs every woman with her best **valid** partner. Recall that a valid partnership must be a matched pair in some stable matching solution. Give a polynomial-time algorithm to determine whether a consensus-optimal stable matching exists for a given set of preference lists.

3. In a connected bipartite graph, is the bipartition unique? Justify your answer.

Solutions

1.8 Yes, it's possible. Consider the following instance with n=3 men and women. Woman w3 will lie in this instance; the first six columns are true preference lists, and the final one is w3's false, but stated, preference list. For the sake of this example, let's assume that G-S breaks ties by using the lowest-numbered unmatched man to ask; similar examples exist for other tiebreakers.

m1	m2	m3	w1	w2	w3	w3'
w3	w1	w3	m1	m1	m2	m2
w1	w3	w1	m2	m2	m1	**m3**
w2	w2	w2	m3	m3	m3	**m1**

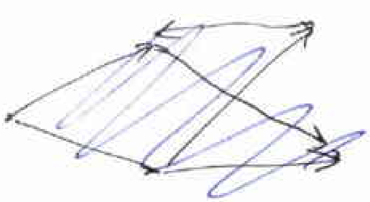
Initially, with the listed tie-breaker, Gale-Shapley will produce the pairs (m1,w3) (m2, w1) and (m3,w2). However, if w3's false preference list is used (and the other five remain truthful), we are left with (m1,w1), (m2,w3), and (m3,w2) -- leaving w3 with her truly first choice.

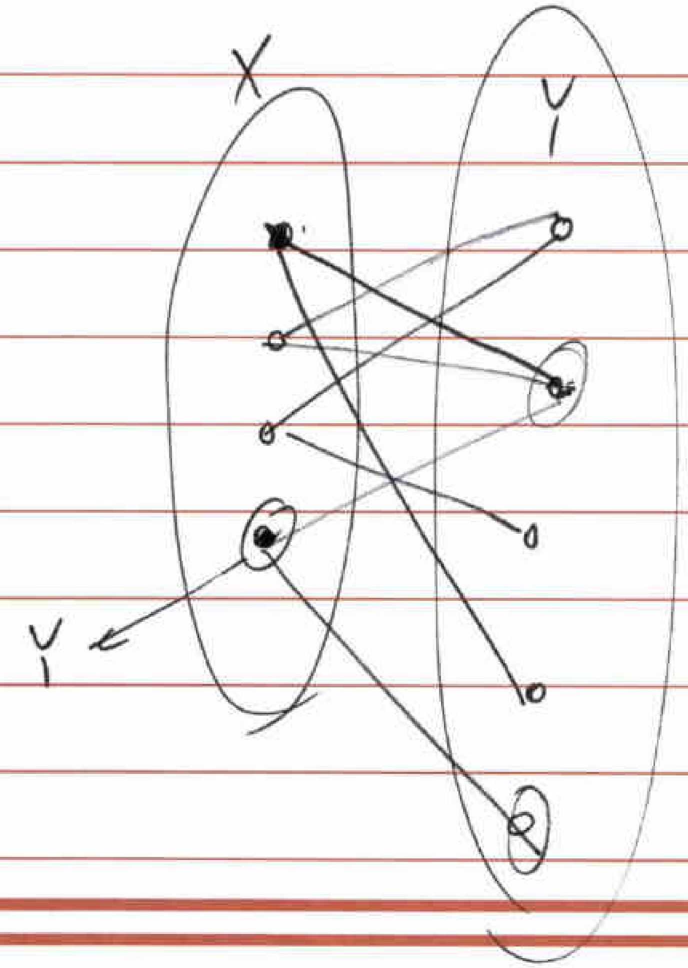
2. Recall that Gale-Shapley matches each man with his top-preference among valid partners, and each woman with her bottom-preference. For a consensus-optimal matching to exist, each woman would also have to be matched with her top-preference. As such, the stable matching must be unique (since, for each woman, her top valid choice and bottom valid choice must be the same).

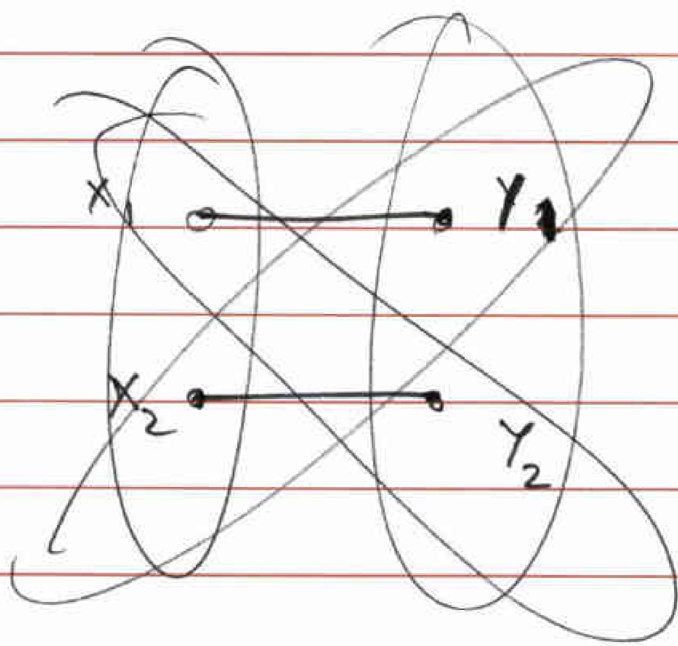
Run Gale-Shapley with the input and store the solution. Then run it again, but with the genders reversed (women ask men, or reverse the inputs). If the solutions are the same, we have a consensus-optimal stable matching. If they aren't, we don't.

3. Yes, unless you consider switching the two colors to be a different bipartition (it shouldn't be). Because the graph is connected, changing any one vertex's color would require each of its neighbors to be changed, and so on in this fashion, until we end up with just a pallet swap.

Note that if the graph *isn't* connected, swapping colors in one or more connected components, and leaving one or more unchanged, *does* result in a different bipartition.




X
Y
Y


X1
Y1
X2
Y2

m_1	m_2	m_3	w_1	w_2	w_3	w_3'
w_3	w_1	w_3	m_1	m_1	m_2	m_2
w_1	w_3	w_1	m_2	m_2	m_1	m_3
w_2	w_2	w_2	m_3	m_3	m_3	m_1

$(m_1, w_3), (m_2, w_1), (m_3, w_2)$

$(m_3, w_2), (m_1, w_1), \cancel{(m_2, w_2)}$ (M_2, w_3)