- Define the problem

- Present a solution

- Prove the solution is correct

- Determine the complexity of the sol.

Stable Matching

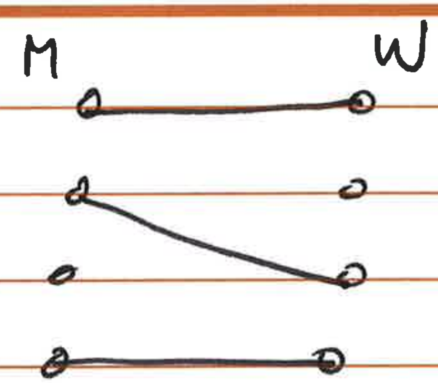
Problem we are interested in matching n men with n women so that they could end up happily married ever after.

we have a set of $\underline{n}$ men $M = \{m_1 \dots m_n\}$

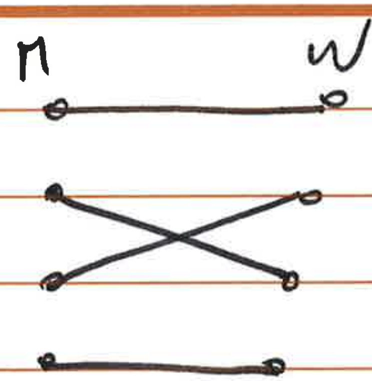
" " " " " women $W = \{w_1 \dots w_n\}$

Notation: an ordered pair m_i, w_j is shown as (m_i, w_j)

Def. A matching S is a set of ordered pairs



Def. A perfect matching S' is a matching with the property that each member of M and each member of W appears in exactly one pair in S'.



Add notion of preference

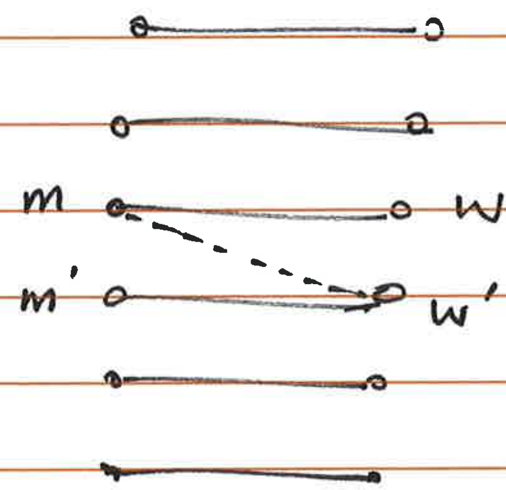
Each man $m \in M$ ranks all women

- m prefers w to w' if m ranks w higher than w'

  - ordered ranking of m is his preference list

$$P_{m_i} = \{ w_{i_1} . w_{i_2} . \ldots . w_{i_n} \}$$

Same for women



Def. Such a pair (m, w') is an instability WRT S.

(m prefers w' to w & w' prefers m to m')

Def. Matching S is stable if
1 - It is perfect
2 - There are no instabilities WRT S.

Input: preference lists for a set of
$\underline{n}$ men & $\underline{n}$ women.

Output: Set of marriages ~~w/ no instabilities~~
representing a stable matching

<u>Gale-shapley alg.</u>

<u>Proof of correctness:</u>

① From the w's perspective, she
she starts single and once she gets
engaged, she can only get into
better engagements

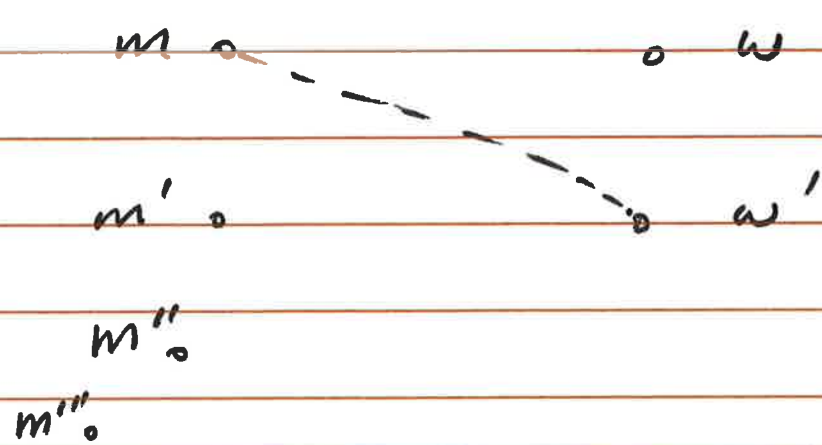
② From the m's perspetive, he starts single and gets engaged, and might be dropped repeatedly only to ~~see~~ settle for a lower ranking woman.

③ Alg terminates after n^2 iterations

④ Solution is a perfect matching

⑤ Solution is stable

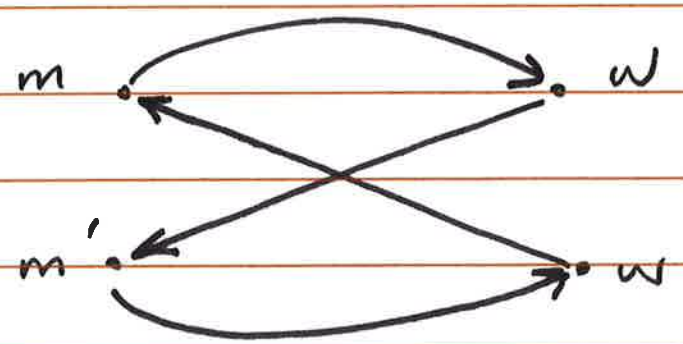
Assume instability with two pairs (m, w) and (m', w')



Question: Did m propose to w' at some point in the execution?

If no, then w must be higher than w' on his list $\Rightarrow$ contradiction!

If yes, he must have been rejected in favor of m'' and due to 1 either $m'' = m'$ or m' is better than m'' $\Rightarrow$ contradiction!



→ Men proposing: $(m, w), (m', w')$

→ Women proposing: $(m', w), (m, w')$

Complexity of Gale-Shapley

1 - Identify a free man

2 - For a man m, identify the highest ranked woman to whom he has not yet proposed.

3 - For a woman w, decide if w is engaged, if so to whom

4 - For a woman w and two men m & m' decide which man is prefered by w.

5 - place a man back in the list of free men