<u>Circulation</u>

We are given a directed graph $G=(V,E)$
w/ capacities on the edges
Associated w/ each node $v \in V$ is a demand d_v.

if $d_v > 0$, node v has a demand of d_v for flow (Sink)

if $d_v < 0$, ~~the~~ node v has a supply of $|d_v|$ for flow (Source)

if $d_v = 0$ neither a sink nor a source

Def. A circulation with demands $\{d_v\}$ is a function f that assigns non-negative real numbers to each edge and satisfies:

1) Cap condition

for each edge $e \in E$, $0 \leq f(e) \leq C_e$

2) Demand Condition.

for each $v \in V$, $f^{in}(v) - f^{out}(v) = d_v$

FACT: If there is a feasible circulation w/ demands $\{d_v\}$ then

$$\sum_v d_v = 0$$

Proof $f^{in}(v) - f^{out}(v) = d_v$

$$\sum_v d_v = \sum_v f^{in}(v) - f^{out}(v) = 0$$

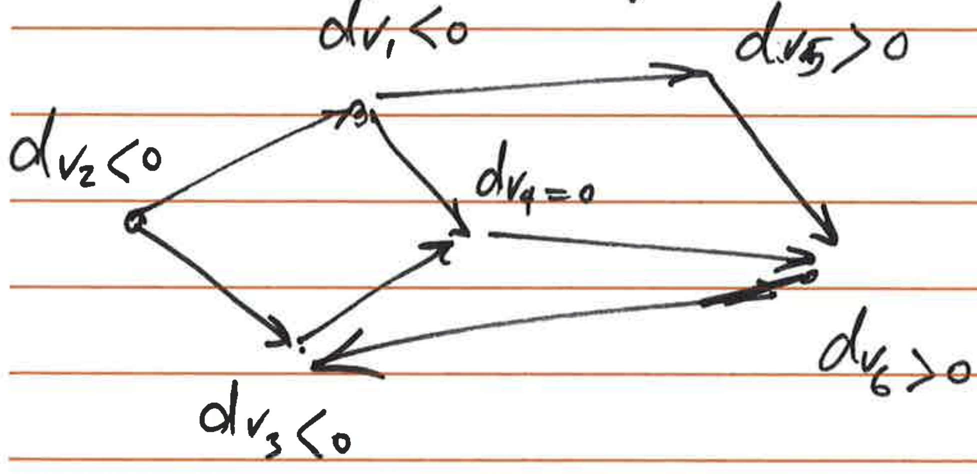
$$v \xrightarrow[f(e)]{e} u$$

$$\sum_v d_v = 0$$

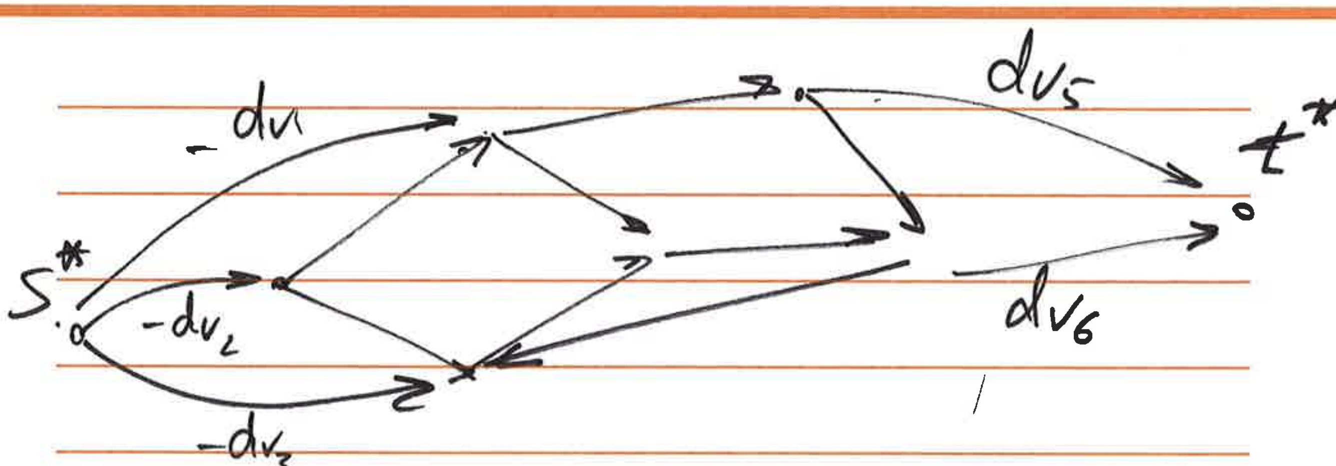
$$\sum_{v:d_v>0} d_v = \sum_{v:d_v<0} -d_v = D$$

total demand value.

How to find a feasible Circulation



G



G'

$v(f) = X$

$X > D$? not possible

$X < D$? no feasible flow

$X = D$ ✓ feasible flow!

Proof: A- ~~∃~~ If there is a feasible circulation f w demand values $\{d_v\}$ in G, we can find a Max flow ~~f~~ in G' of value D!

B- If there is a Max flow in G' of value D, we can find a feasible circulation in G.

<u>circulation w/ demands & lower bounds.</u>

Conditions

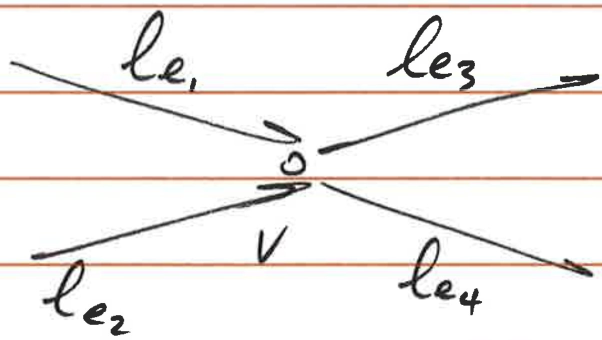
1) Cap. cond.

for each $e \in E$, $\ell_e \leq f(e) \leq c_e$

2) Demand Cond.

for every v, $f^{in}(v) - f^{out}(v) = d_v$

f_0 is a flow where $f(e) = l_e$ for all $e \in E$



L_v = imbalance at node $\underline{v}$ due to f_0

$$f_0^{in}(v) - f_0^{out}(v) = \sum_{e \text{ into } v} l_e - \sum_{e \text{ out of } v} l_e$$

$$= L_v$$

flow imbalance at $\underline{v}$.

1- push flow f_0 thru G where $f_0(e) = l_e$

2 - Construct G'

where $c'_e = c_e - l_e$

& $d'_v = d_v - L_v$

3 - find feasible circulation in G'

call this f_1

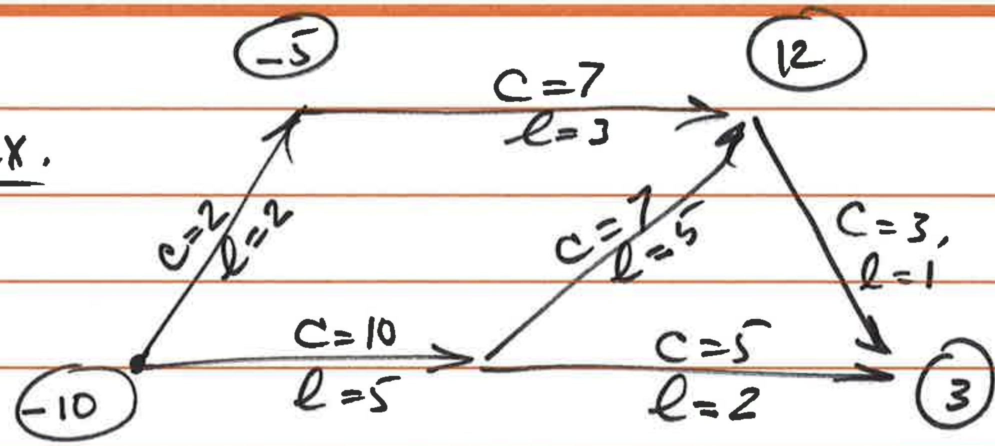
4 - if there is no feasible circ in G'

$\Rightarrow$ no feasible circ. in G

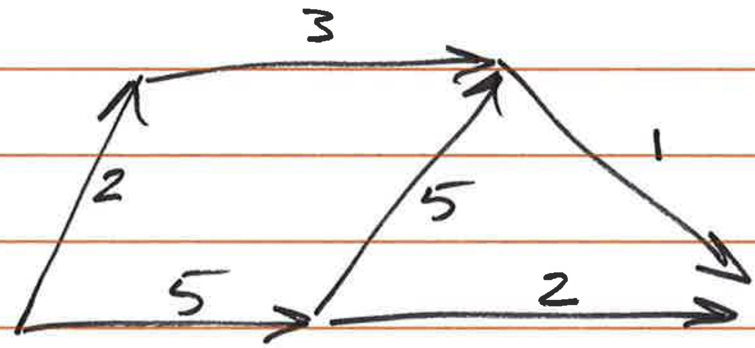
otherwise, feasible circ in $G =$

$f_0 + f_1$

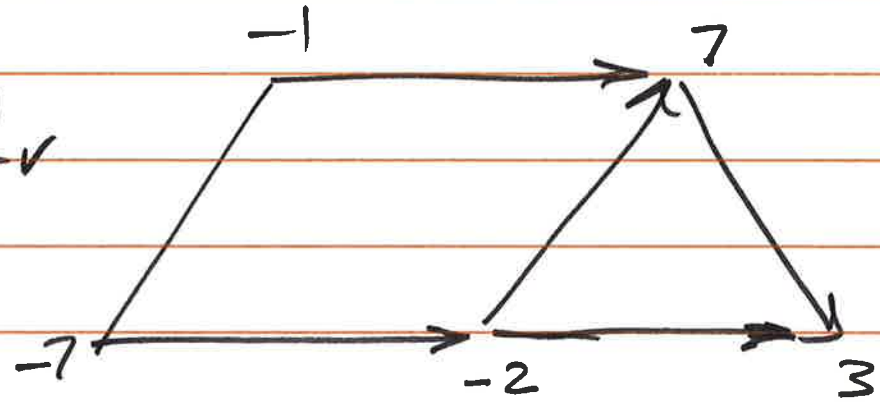
ex.



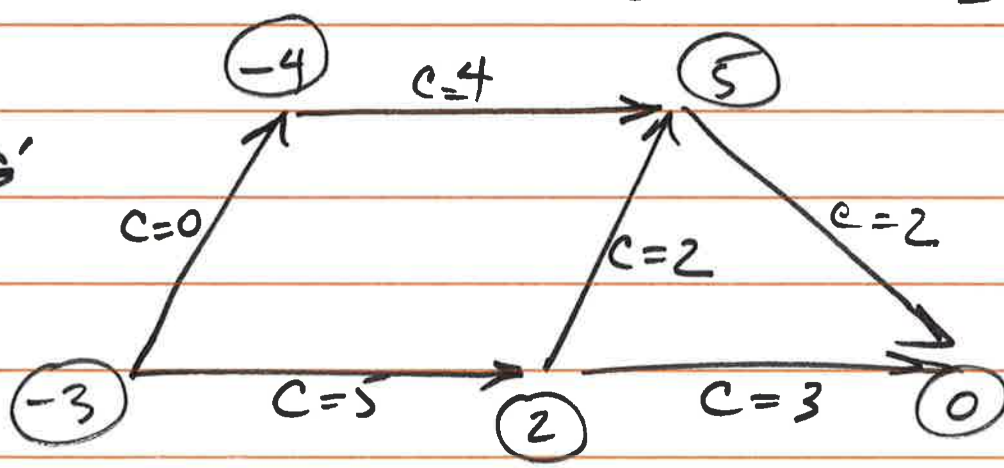
f_0



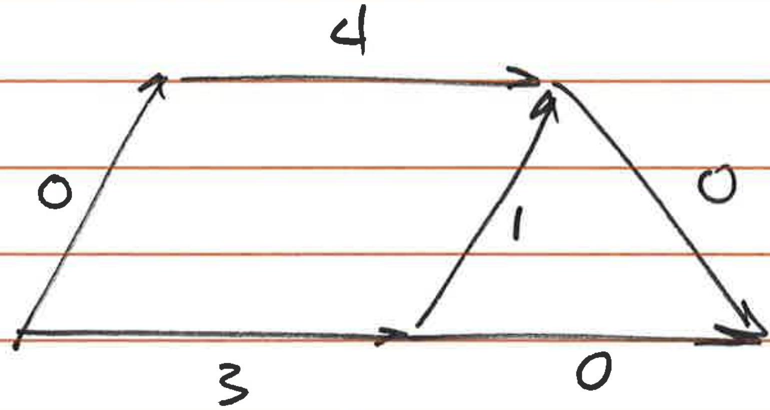
find L_v



G'



f_1'



$f = f_0 + f_1$