Independent Set vs Vertex Cover

Reduce one to the other

FACT: Let $G = (V, E)$ be a graph, then S is an indep set iff its complement $(V - S)$ is a vertex cover.

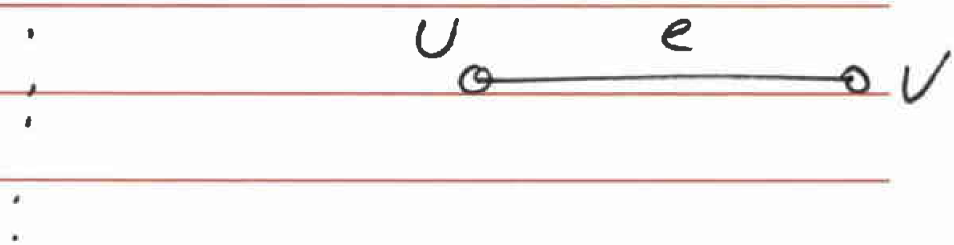
Proof A) First suppose that S is an indep set

1- U is in S & V is not
$\Rightarrow$ $V - S$ will have V and not U

2- V is in S & U is not
$\Rightarrow$ $V - S$ will have U and not V

3- Neither V nor U is in S
$\Rightarrow$ $V - S$ will have both U & V

B) Suppose $V-S$ is a vertex cover set
→ prove that S is an indep set.



claim: Indep set $\leq_p$ vertex cover

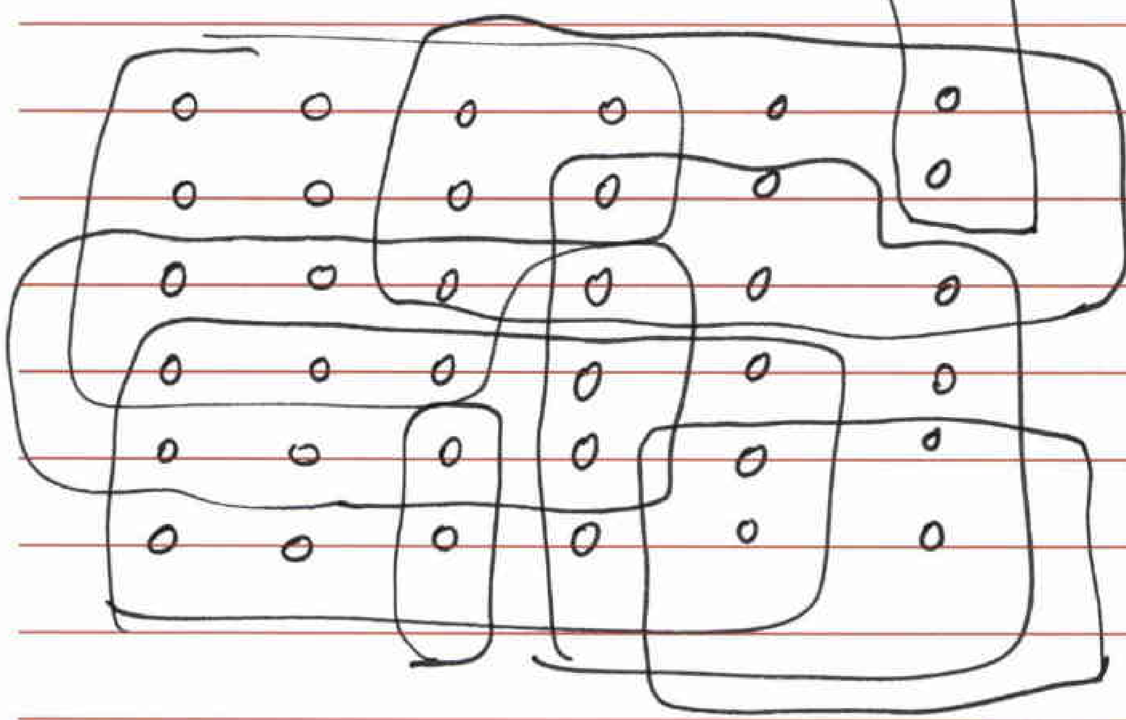
Proof: If we have a black box to solve vertex cover, we can

decide if G has an indep set of size at least k, by asking the blackbox if G has a vertex cover of size at most $n-k$.

claim: vertex cover $\leq_p$ Indep set

Proof: - - - - Similar

Set Cover Problem

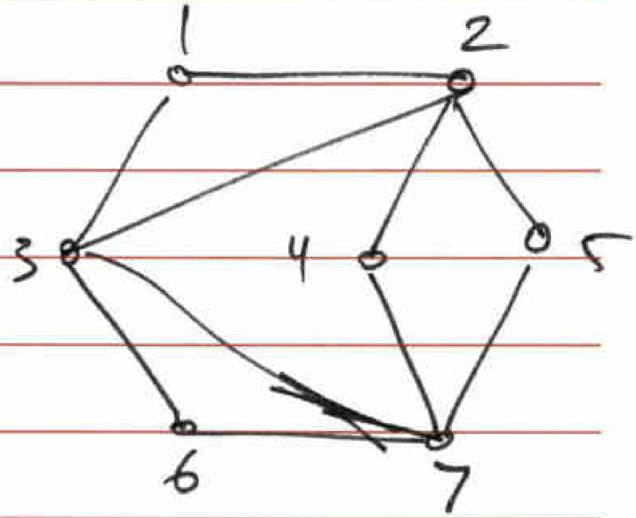


Set Cover Problem

Given a set U of $\underline{n}$ elements, a collection $S_1 \ldots S_m$ of subsets of U, and a no. $\underline{k}$, does there exist a collection of at most $\underline{k}$ of these sets whose union is equal to all of $\underline{U}$.

vertex cover $\leq_p$ set cover

$S_1 = \{(1,2),(1,3)\}$



$S_2 = \{(1,2),(2,3),$
$(2,4),(2,5)\}$

$\vdots$

$S_7 = \{ - \; — \; - \}$

Proof: A) If I have a vertex cover of size k in G, I can find a collection of $\underline{k}$ sets whose union is equal to all of $\underline{U}$.

B) If I have $\underline{k}$ sets whose union is equal to all of U I can find a vertex cover of size $\underline{k}$ in G.