Shortest Path problem

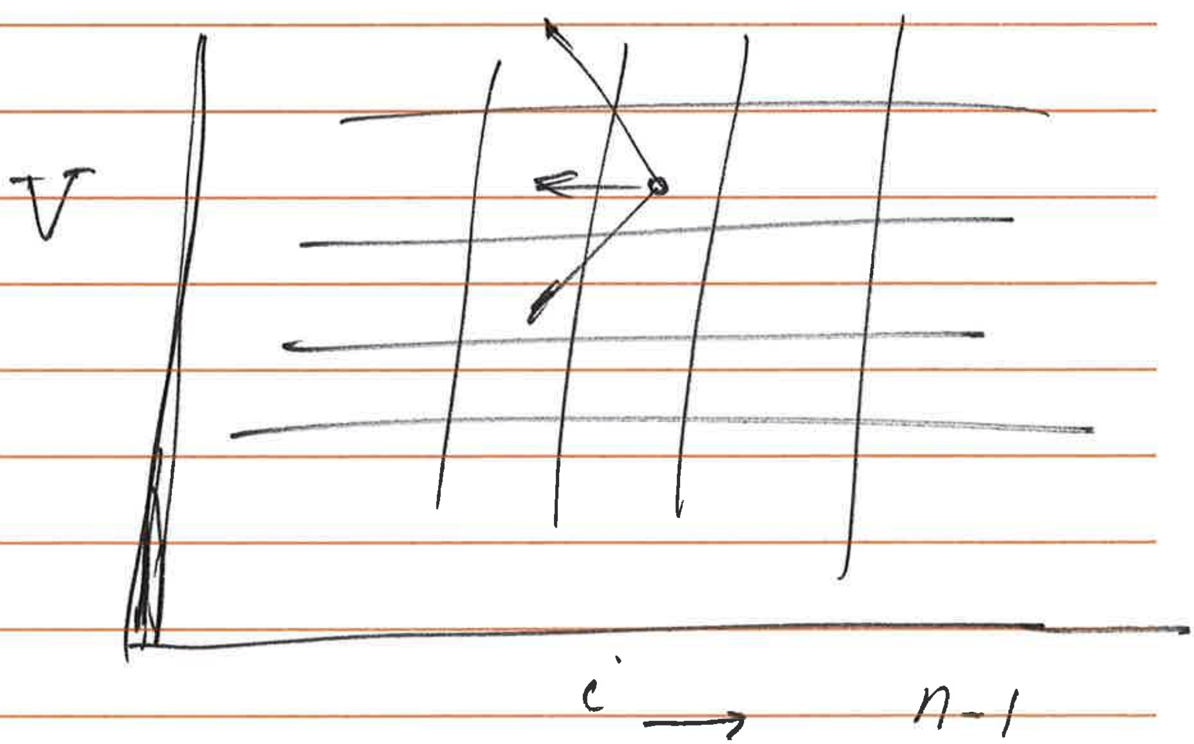
Bellman-Ford Alg.

Part II

recurrence formula:

$$OPT(i,v) = \min\Big(OPT(i-1, v),\ \min_{w \in Adj(v)} \big(OPT(i-1, w) + c_{vw} \big) \Big)$$

where $OPT(i,v)$ denotes the opt. distance from $\underline{v}$ to $\underline{t}$ using at most $\underline{i}$ edges.



Bellman-Ford Alg.

shortest path (G, s, t)

n = no of nodes in G

define $M[0, t] = 0$

$M[0, v] = \infty$

for $i = 1$ to $\underline{n-1}$ — $O(n)$

for $v \in V$ in any order — $O(n)$

$M[i, v] = \min\left(M[i-1, v], \min_{w \in Adj(v)}\left(M[i-1, w] + c_{vw}\right)\right)$ — $O(n)$

endfor

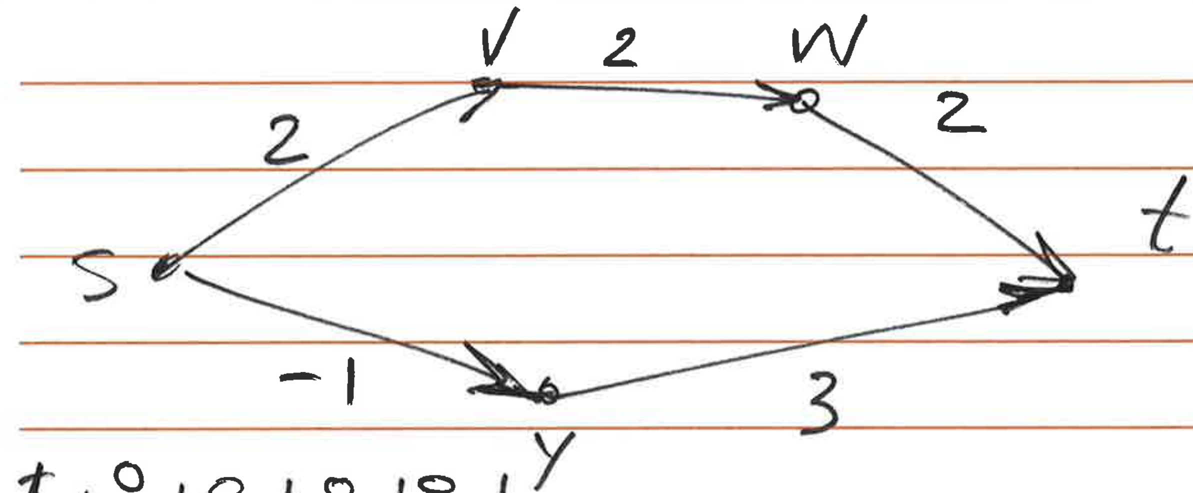
endfor

Return $M[n-1, s]$

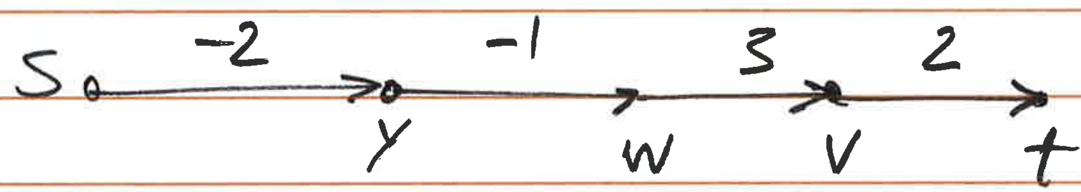
$\Rightarrow \quad O(n^3)$

$\downarrow$

$O(mn)$ is the complexity of Bellman-Ford.



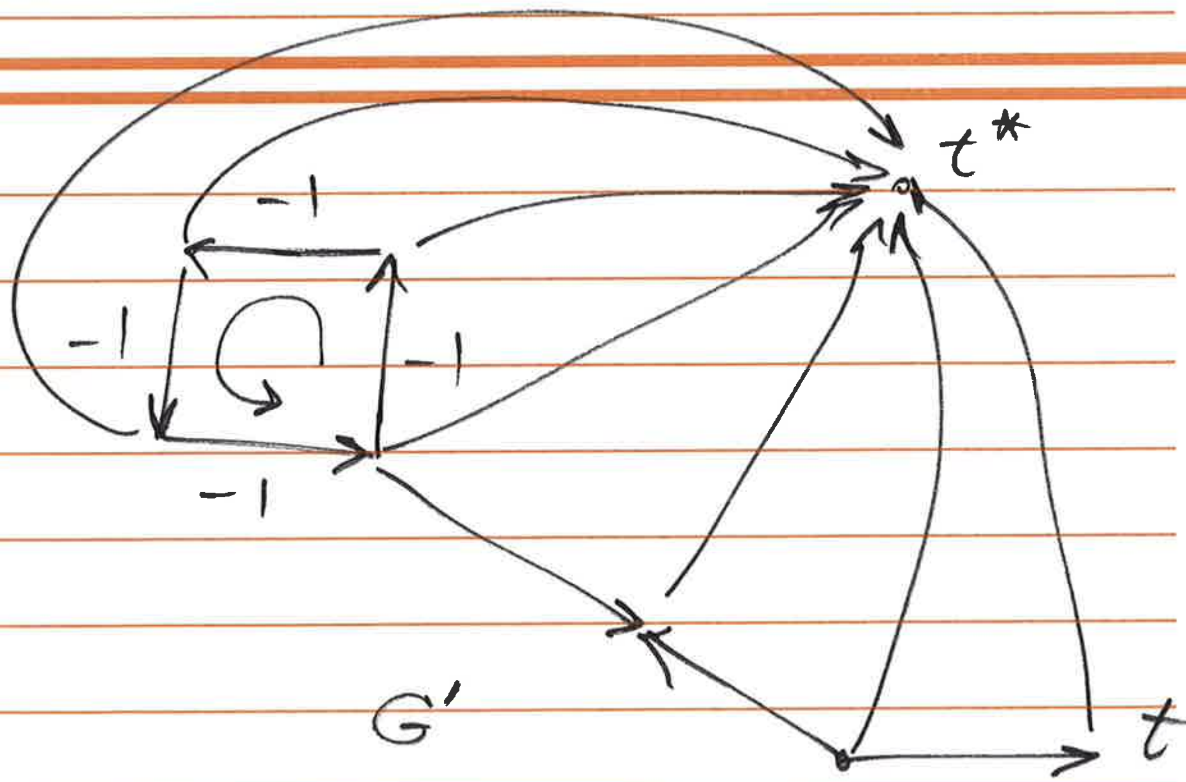
t	0	0	0	0	
y	∞	3	3	3	
w	∞	2	2	2	
v	∞	∞	4	4	
s	∞	∞	2	2	
	0	1	2	3	4



t	0	0	0
v	∞	2	2
w	∞	5	5
y	∞	4	4
s	∞	2	2

		1	2	3	4
t	0	0	0	0	0
v	∞	2	2	2	2
w	∞	∞	5	5	5
y	∞	∞	∞	4	4
s	∞	∞	∞	∞	2

$\underline{n-1}$



Bellman-Ford Dijkstra's

$O(mn)$ $O(m \lg n)$