Divide & Conquer

Dense Matrix Multiplication

$$\underbrace{\left.\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}\right\}}_{n} n \quad \underbrace{\begin{bmatrix} & & \\ & B & \\ & & \end{bmatrix}}_{n} = \underbrace{\begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix}}_{n}\left.\vphantom{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}\right\} n$$

Brute force method gives you $O(n^3)$

$$\left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right] = \left[\begin{array}{c|c} C_{11} & C_{12} \\ \hline C_{21} & C_{22} \end{array}\right]$$

$$C_{11} = \underline{A_{11} \cdot B_{11}} \; \underline{+} \; \underline{A_{12} \cdot B_{21}}$$
$$C_{12} = \underline{A_{11} \cdot B_{12}} \; \underline{+} \; \overline{A_{12} \cdot B_{22}}$$
$$C_{21} = A_{21} \cdot B_{11} \; \underline{+} \; A_{22} \cdot B_{21}$$
$$C_{22} = A_{21} \cdot B_{12} \; \underline{+} \; A_{22} \cdot B_{22}$$

$D(n) = O(n)$
$C(n) = O(n^2)$

$a = 8$
$b = 2$

$f(n) = O(n^2)$ $\quad n^{\log_b a} = n^{\log_2 8} = n^3$

Case 1 $\Rightarrow$ $O(n^3)$

Strassen's Alg.

$P = (A_{11} + A_{22})(B_{11} + B_{22})$
$Q = (A_{21} + A_{22})\, B_{11}$
$R = A_{11}\, (B_{12} - B_{22})$
$S = A_{22}\, (B_{21} - B_{11})$
$T = (A_{11} + A_{12})\, B_{22}$
$U = (A_{21} - A_{11})(B_{11} + B_{12})$
$V = (A_{12} - A_{22})(B_{21} + B_{22})$

$C_{11} = P + S - T + V$

$C_{12} = R + T$

$C_{21} = Q + S$

$C_{22} = P + R - Q + U$

$a = 7$

$b = 2$

$n^{\log_b a} = n^{\log_2 7} = n^{2.81}$

$f(n) = O(n^2)$

$\Rightarrow \Theta(n^{2.81})$