Discussion 3

CSCI 570

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DISCUSSION 3

Outline

- Review Terms
- Problems with Solutions

Review Terms

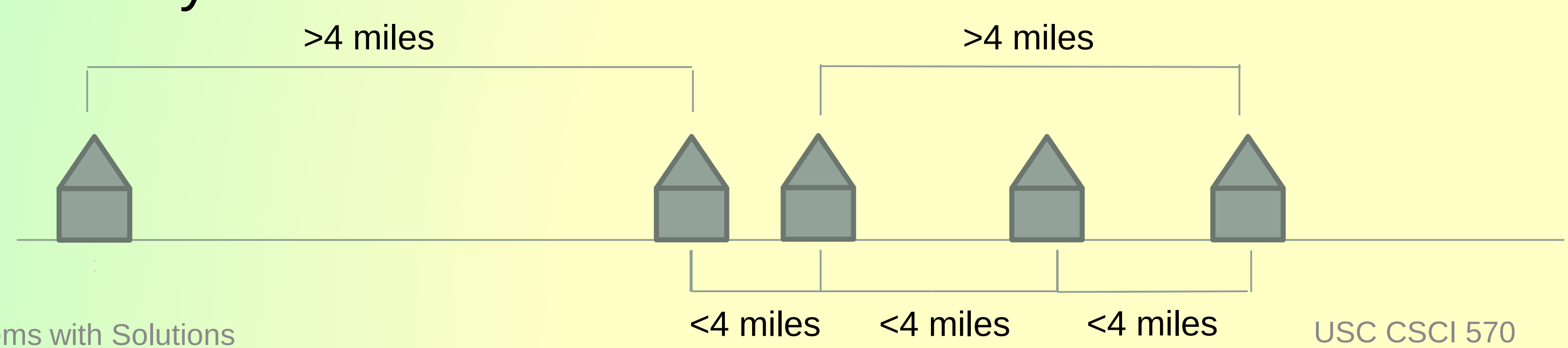
- Binary Tree
- Full Binary Tree
- Complete Binary Tree
- Max Heap
- Min Heap
- Binary Search Tree

Review Terms

- Binary Tree
  - Tree where each node has at most 2 children
- Full Binary Tree
  - Binary tree of depth k with exactly $2^k – 1$ nodes
- Complete Binary Tree
  - Full binary tree through depth k-1 with the nodes at depth k filled from left to right
- Max Heap
  - Complete binary tree where the value of the parent is greater than the values of the children
- Min Heap
  - Complete binary tree where the value of the parent is less than the values of the children
- Binary Search Tree
  - Binary tree where the value of the parent is greater than the value of the left child and less than the value of the right child
  - Equality can be implemented with the left or right child

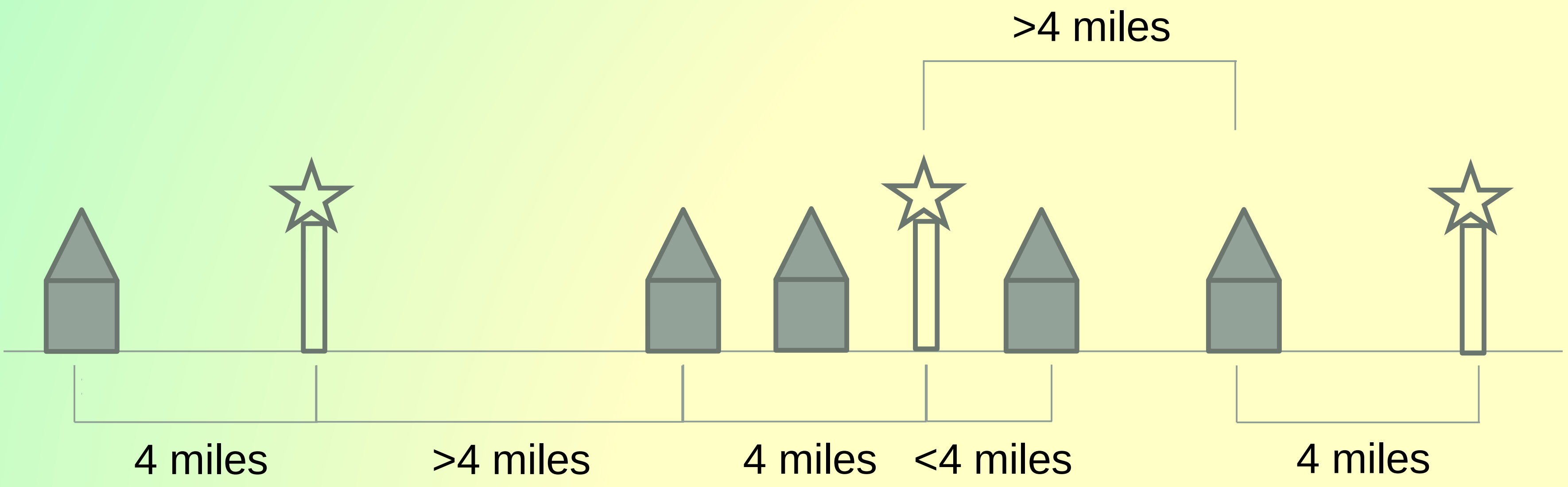
Problem 1

- Let's consider a long, quiet country road with houses scattered very sparsely along it. We can picture the road as a long line segment, with an eastern endpoint and a western endpoint. Further, let's suppose that, despite the rural setting, the residents of all these houses are avid cell phone users. You want to place cell phone base stations at certain points along the road so that every house is within four miles of one of the base stations.
- Give an efficient algorithm that achieves this goal and uses as few base stations as possible. Prove that your algorithm correctly minimizes the number of base stations.



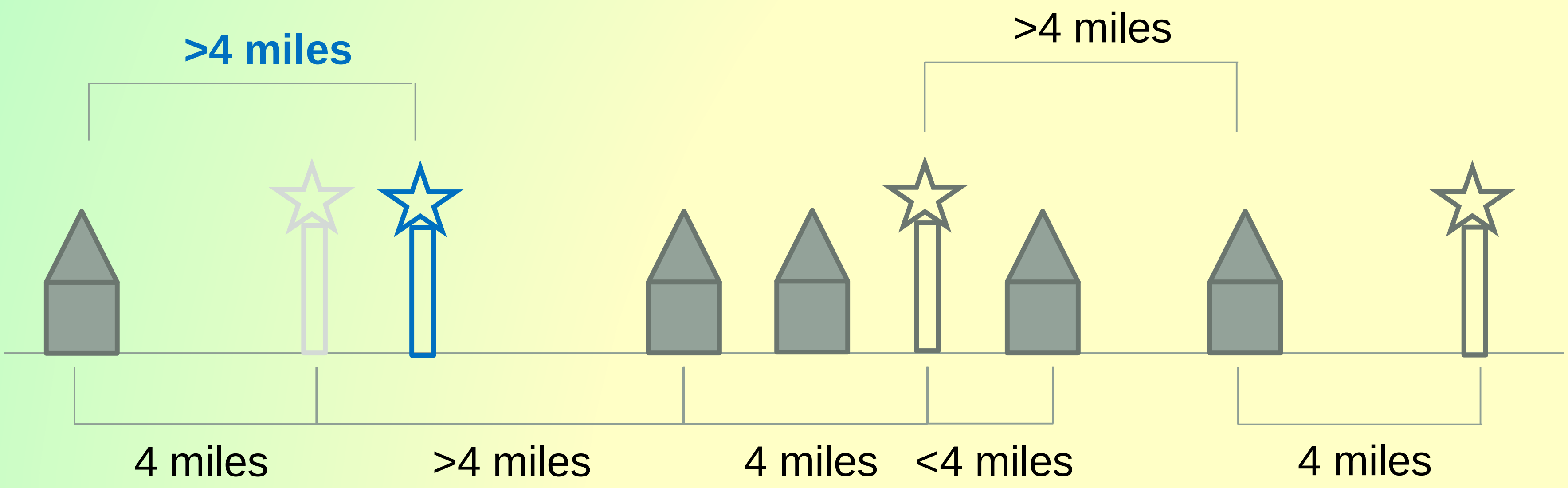
Solution 1

- Place the first cell tower four miles east of the west-most uncovered house. Mark all houses within four miles of that cell tower as “covered.” Repeat.



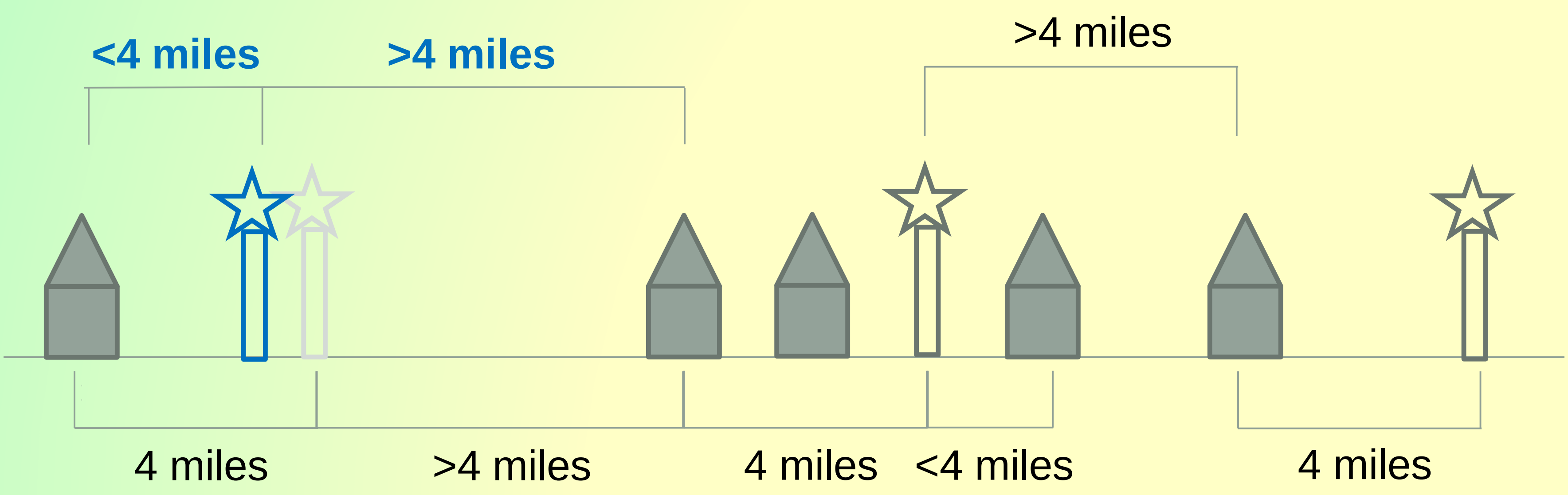
Solution 1

- Proof of Correctness
  - Let’s look at the first location of our cell tower where another solution disagrees with ours
  - Clearly the other solution can’t have a cell tower east of ours or it wouldn’t be covering the west-most house that our solution is covering



Solution 1

- Proof of Correctness
  - Since the cell tower can't be east of ours, it must be west of ours
  - This means that our cell tower covers no fewer houses than theirs
    - The houses covered to the west are exactly the same as ours
    - The houses to the east are no more than ours since there is a region to the east that we are covering and they aren't



Problem 2

- Your friend is working as a camp counselor, and he is in charge of organizing activities for a set of campers. One of his plans is the following mini-triathlon exercise: each contestant must swim 20 laps of a pool, then bike 10 miles, then run 3 miles. The plan is to send the contestants out in a staggered fashion, via the following rule:
  - **The contestants must use the pool one at a time.**
- In other words, first one contestant swims the 20 laps, gets out, and starts biking. As soon as this first person is out of the pool, a second contestant begins swimming the 20 laps; as soon as he or she is out and starts biking, a third contestant begins swimming, and so on.
- Each contestant `i` has a projected *swimming time* (S_i), a projected *biking time* (B_i), and a projected *running time* (R_i).
- Your friend wants to decide on a *schedule* for the triathlon: an order in which to sequence the starts of the contestants. Let's say that the *completion time* of a schedule is the earliest time at which **all** contestants will be finished with all three legs of the triathlon, assuming the time projections are accurate.
- What is the best order for sending people out, if one wants the whole competition to be over as soon as possible? More precisely, give an efficient algorithm that produces a schedule whose completion time is as small as possible. Prove that your algorithm achieves this.

Solution 2

- As an example, take the following times for different athletes i

i	S_i	B_i	R_i	Ordering
1	30	30	30	
2	80	10	80	
3	10	20	30	
4	70	90	60	

Solution 2

- Sort by non-increasing non-swim time (B_i + R_i) since multiple people could be on the land at the same time
  - The restriction we had is that only one person could be swimming at a time
- The total time will be *at least* the sum of all of the swim times Si plus the B_i + R_i of the last contestant to start, which will be the minimum B_i + R_i of the contestants
- As an example, take the following table

i	S_i	B_i	R_i	$B_i + R_i$	Ordering
1	30	30	30	60	3
2	80	10	80	90	2
3	10	20	30	50	4
4	70	90	60	150	1

  - S_4=70, B_4+R_4=150 220
  - S_4=70 + S_2=80, B_2+R_2=90 240
  - S_4=70 + S_2=80 + S_1=30, B_1+R_1=60 240
  - S_4=70 + S_2=80 + S_1=30 + S_3=10, B_3+R_3=50 240
- The maximum is 240, which is the overall completion time

Solution 2

- If there is a better solution, we should be able to swap some of our contestants to produce a lower completion time
  - If we swap the start time of contestants i and i+1, contestant i+1 will finish earlier than he finished in the original ordering
  - Contestant i is now finishing later than in the original ordering
  - We know from our definition that, after the swap, $B_i + R_i <= B_{i+1} + R_{i+1}$
  - Then $S_{i+1} + S_i + B_i + R_i <= S_i + S_{i+1} + B_{i+1} + R_{i+1}$
  - Therefore swapping produced a result that is potentially larger than the original result we found

i	S_i	B_i	R_i	$B_i + R_i$	Ordering
1	30	30	30	60	3
2	80	10	80	90	~~2~~ **1**
3	10	20	30	50	4
4	70	90	60	150	~~1~~ **2**

  - $S_2=80$, $B_2+R_2=90$ 170
  - $S_2=80 + S_4=70$, $B_4+R_4=150$ 300
  - $S_2=80 + S_4=70 + S_1=30$, $B_1+R_1=60$ 240
  - $S_2=80 + S_4=70 + S_1=30 + S_3=10$, $B_3+R_3=50$ 240
- The maximum is 300, which is the overall completion time