unsolvable
undecidable
problems

P

Plan: Explore the space of computationally hard problems to arrive at a mathematical characterization of a large class of them.

Technique: Compare relative difficulty of different problems.

Loose definition: If problem X is at least as hard as problem Y, it means that if we could solve X, we could also solve Y.

$Y \leq_p X$ (Y is polynomial time reducible to X)

if Y can be solved using a polyn. no. of std computational steps + a polyn. no. of calls to a blackbox that solves X.

FACT: Suppose $Y \leq_p X$, if X can be solved in polyn. time, then Y can be solved in polyn. time.

FACT: Suppose $Y \leq_p X$, if Y cannot be solved in polyn. time, Then X cannot be solved in polyn. time.

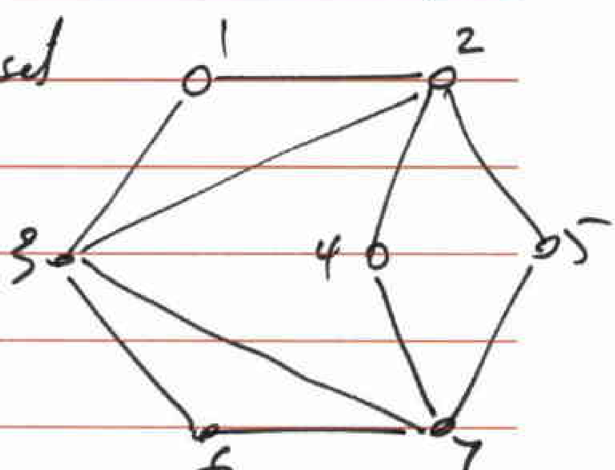
<u>Independent set</u>

Df. In a graph $G = (V, E)$, we say a set of nodes $S \subseteq V$ is "independent" if no two nodes in S are joined by an edge.

$\{1, 4, 5, 6\}$ ← largest indep set

$\{1\}$

$\{1, 7\}$



Independent set problem

- Find the largest indep set in graph G. (optimization version)

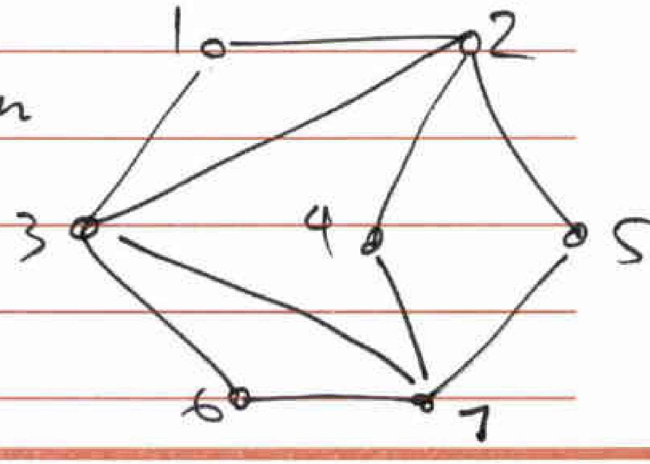
- Given a graph G and a no. k, does G contain an indep set of size at least k? (decision version)

Vertex Cover

Def. Given a graph G=(V,E) we say that a set of nodes $S \subseteq V$ is a vertex cover if every edge in $e \in E$ has at least one end in S.

$\{2,3,7\}$ smallest vertex cover

$\{1,2,3,4,5,6,7\}$



Vertex Cover problem

Find the smallest vertex cover set in G

(opt. version)

Given a graph G and a no. $\underline{k}$,
does G contain a vertex cover set
of size at most k?

(decision version)