

CSCI 570 Spring 2015 Discussion 5

0. Solve the following recurrences using the Master Method:

- a. $A(n) = 3A(n/3) + 15$
- b. $B(n) = 4B(n/2) + n^3$
- c. $C(n) = 4C(n/2) + n^2$
- d. $D(n) = 4D(n/2) + n$

1. Suppose we have two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, along with T_1 which is a MST of G_1 and T_2 which is a MST of G_2 . Now consider a new graph $G = (V, E)$ such that $V = V_1 \cup V_2$ and $E = E_1 \cup E_2 \cup E_3$ where E_3 is a new set of edges that all cross the cut (V_1, V_2) .

Consider the following algorithm, which is intended to find a MST of G .

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Maybe-MST( $T_1, T_2, E_3$ )  
     $e_{\min}$  = a minimum weight edge in  $E_3$   
     $T = T_1 \cup T_2 \cup \{e_{\min}\}$   
    return  $T$ 
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Does this algorithm correctly find a MST of G ? Either prove it does or prove it does not.

2. You are given a graph representing the several career paths available in industry. Each node represents a position and there is an edge from node v to node u if and only if v is a prerequisite for u . Top positions are the ones which are not prerequisites for any positions. The cost of an edge (v, u) is the effort required to go from one position v to position u . Ivan wants to start a career and achieve a top position with minimum effort. Using the given graph can you provide an algorithm with the same run time complexity as Dijkstra's? You may assume the graph is a DAG.

3: (a): Suppose we are given an instance of the Minimum Spanning Tree problem on a graph G . Assume that all edge costs are distinct. Let T be a minimum spanning tree for this instance. Now suppose that we replace each edge cost c_e by its square, c_e^2 thereby creating a new instance of the problem with the same graph but different costs.

Prove or disprove: T is still a MST for this new instance.

(b): Consider an undirected graph $G = (V, E)$ with distinct nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G . Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$. Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

$$O. a \quad f(n) = O(1) \quad n^{\log_a b} = n^{\log_3 3} = n$$

$$\text{Case 1} \Rightarrow \Theta(n)$$

$$b \quad f(n) = n^3 \quad n^{\log_a b} = n^{\log_2 4} = n^2$$

$$\text{Case 3} \Rightarrow \Theta(n^3)$$

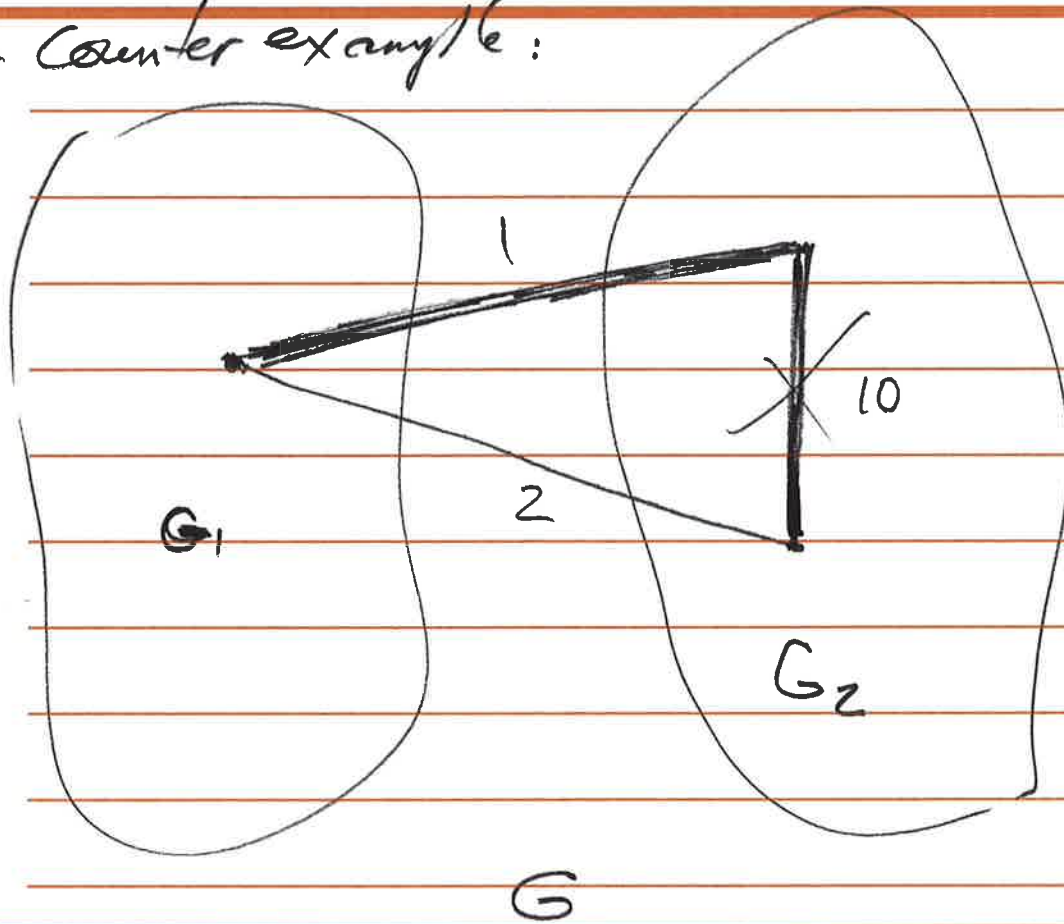
$$c \quad f(n) = n^2 \quad n^{\log_a b} = n^2$$

$$\text{Case 2} \Rightarrow \Theta(n^2 f(n))$$

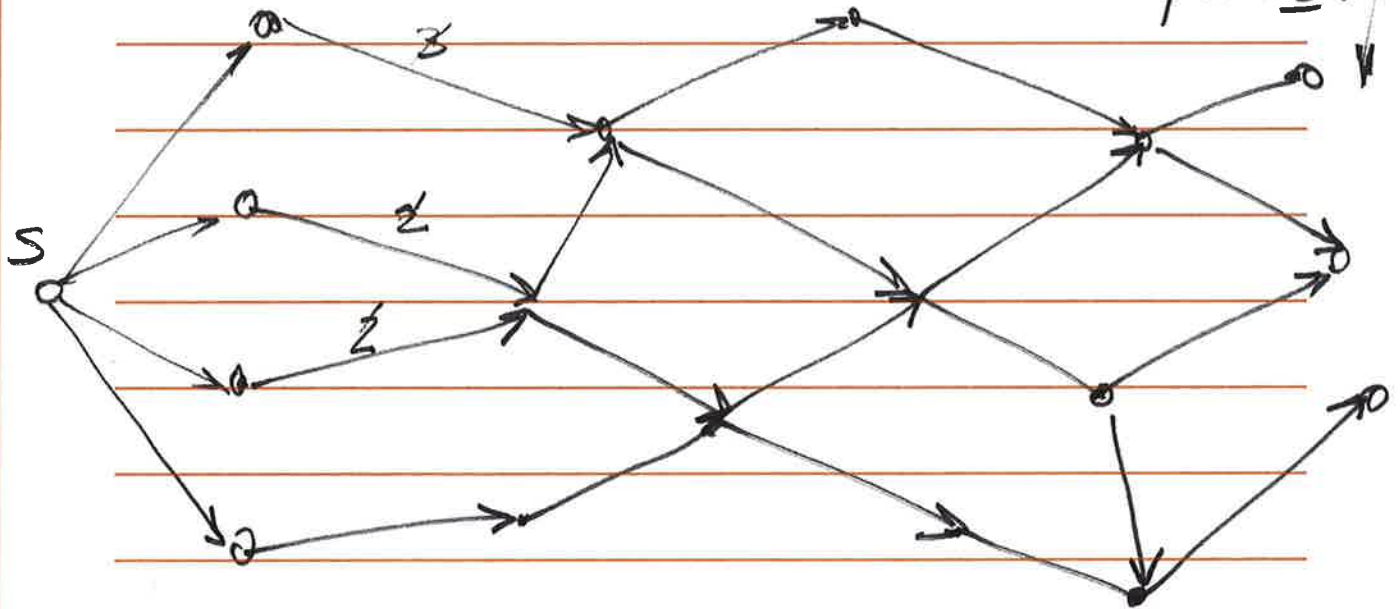
$$d \quad f(n) = n \quad \dots \dots \dots n^2$$

$$\text{Case 1} \Rightarrow \Theta(n^2)$$

1- Counter example:



2 - Add a node s and connect s to all starting nodes w/ edges of cost 0. Run Dijkstra's from s .



3. a

No.

because of possible
neg. cost edges

b

No.

No. won't change