CSCI 570 Discussion 14

1: The *bin packing* problem is as follows. You have an infinite supply of bins, each of which can hold M maximum weight. You also have n objects, each of which has a (possibly distinct) weight w_i (any given w_i is at most M). Our goal is to partition the objects into bins, such that no bin holds more than M total weight, and that we use as few bins as possible. This problem in general is **NP**-hard.

Give a 2-approximation to the *bin packing* problem. That is, give an algorithm that will compute a valid partitioning of objects into bins, such that no bin holds more than M weight, and our algorithm uses at most twice as many bins as the optimal solution. Prove that the approximation ratio of your algorithm is two.

2: Suppose you are given a set of positive integers A: a_1 a_2 ... a_n and a positive integer B. A subset $S \subseteq A$ is called *feasible* if the sum of the numbers in S does not exceed B.

The sum of the numbers in S will be called the *total sum* of S. You would like to select a feasible subset S of A whose total sum is as large as possible.

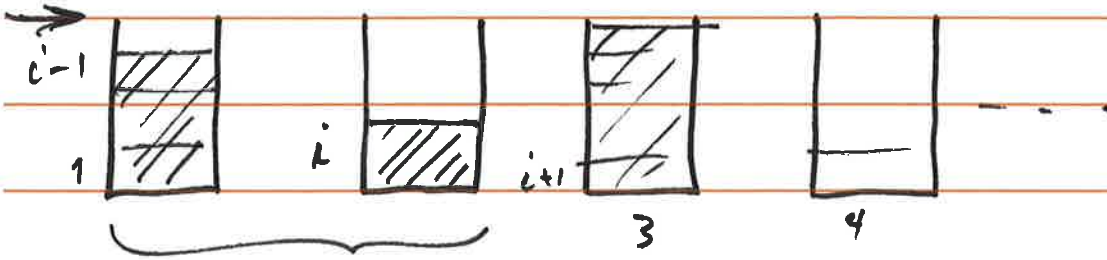
Example: If A = {8, 2, 4} and B = 11 then the optimal solution is the subset S = {8, 2}.

Give a linear-time algorithm for this problem that finds a feasible set $S \subseteq A$ whose total sum is at least half as large as the maximum total sum of any feasible set $S' \subseteq A$. Prove that your algorithm achieves this guarantee.
You may assume that each $a_i \leq B$.

3: A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, up to the maximum available limits given below.

- Material 1 has density 2 tons/cubic meter, maximum available amount 40 cubic meters, and revenue $1,000 per cubic meter.
- Material 2 has density 1 ton/cubic meter, maximum available amount 30 cubic meters, and revenue $1,200 per cubic meter.
- Material 3 has density 3 tons/cubic meter, maximum available amount 20 cubic meters, and revenue $12,000 per cubic meter.

Write a linear program that optimizes revenue within the constraints. You do not need to solve the linear program.



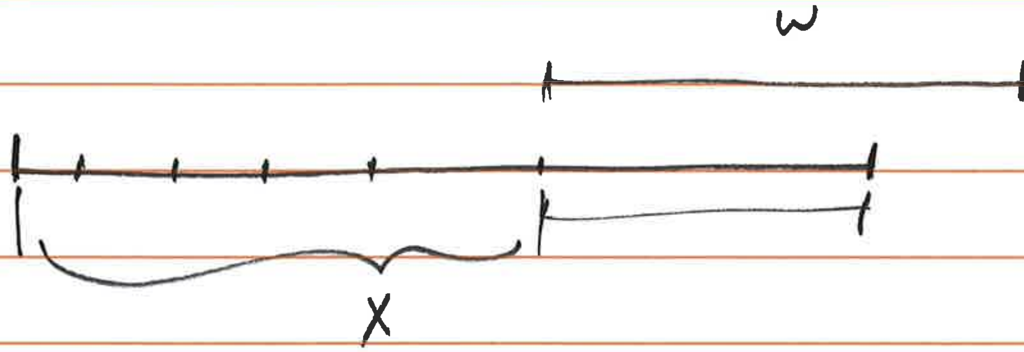
place as many objects in bin 1 as you can. If the next object causes the total weight of the bin to exceed M then place this object in bin 2.

Bound on how good the opt. sol. can be :

value of opt. sol. $\geq$ 1

our sol. has value 2

$\Rightarrow$ we are within a factor of 2 of the opt. sol.



either X or w must be greater than $B/2$. whichever that is bigger is our approx. sol. and that would give us a .5-approximation

Variables M1, M2, M3

representing volumns corresponding to material 1, 2 & 3.

Objective function :

Maximize $1000*M1 + 2000*M2 + 12000*M3$

Subject to :

$M1 + M2 + M3 \leq 60$

$2*M1 + 1*M2 + 3*M3 \leq 100$

$M1 \leq 40$

$M2 \leq 30$

$M3 \leq 20$

Solution Outlines.

1: The simple greedy algorithm achieves this: throw objects into the first bin until adding the next object would exceed the weight limit; close that bin and open a new one. Repeat until all objects are assigned.

To prove the approximation ratio, consider the first and second bins constructed. Their total weight must be at least M -- otherwise, bin two would never have been started. This is also true for all bins numbered $(2i-1, 2i)$ for any $i >= 1$. Thus, for each pair of bins we use, no solution can use fewer than one to store the same objects.

(the case where there are an odd number of bins adds one to both our usage and that of OPT).

2: Add each element from A in order to some set X, until adding some a_j would cause the total sum of X to exceed B. Now consider set X and set Y = $\{a_j\}$ and return the larger one.

Because the sum of elements in X + Y > B, the larger must be at least B/2, and no feasible subset can be more than twice as big.

3: [redacted]

Let M1, M2, and M3 denote the cubic meters of the three materials we're going to transport.

We want to maximize the profit: maximize 1000 * M1 + 2000 * M2 + 12000 * M3

Subject to these constraints:

M1 + M2 + M3 <= 60 // 60 cubic meters space available
2 * M1 + M2 + 3 * M3 <= 100 // 100 tons weight capacity
M1 <= 40
M2 <= 30 // maximum available for these three
M3 <= 20