3SAT vs Indep set

Reduction using <u>gadget</u>

- Given $\underline{n}$ Boolean variables $x_1 \ldots x_n$
<u>a clause</u> is a disjunction of
terms $t_1 \vee t_2 \vee \ldots \vee t_\ell$
where $t_i \in \{x_1, \ldots x_n, \bar{x}_1, \ldots \bar{x}_n\}$

- A ~~tract~~ <u>Truth assignment</u> for X is an
assignment of values $\underline{0}$ or 1
to each x_i

- An assignment satisfies a clause C
if it causes C to evaluate to $\underline{1}$.

- An assignment satisfies a collection
of clauses if

$$C_1 \wedge C_2 \wedge \ldots \wedge C_k$$
~~eh~~ evaluates to 1.

ex. $(x_1 \vee \bar{x}_2), (\bar{x}_1 \vee \bar{x}_3), (x_2 \vee \bar{x}_3)$ ✓

$x_1 = 1$, $x_2 = 1$, $x_3 = 1$ ✗

$x_1 = 0$, $x_2 = 0$, $x_3 = 0$ ✓

Prob. statement: Given a set of clauses $C_1 \ldots C_k$ over a set of variables $X = \{x_1 \ldots x_n\}$ does there exist a satisfying truth assignment?

Satisfiability Problem (SAT)

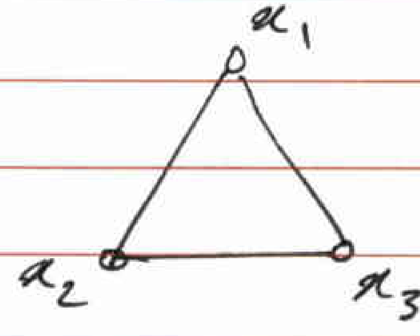
Prob. statement Given a set of clauses $C_1 \ldots C_k$ each of length $\underline{3}$ over a set of variables $X = \{x_1 \ldots x_n\}$ does there exist a satisfying truth assignment?

(3SAT)

$3SAT \leq_p$ Indep Set

Plan: Given an instance of 3SAT w/ k clauses we will construct a graph G such that G has an indep set of size at least k iff there is a satisfying truth assignment in the 3SAT problem.

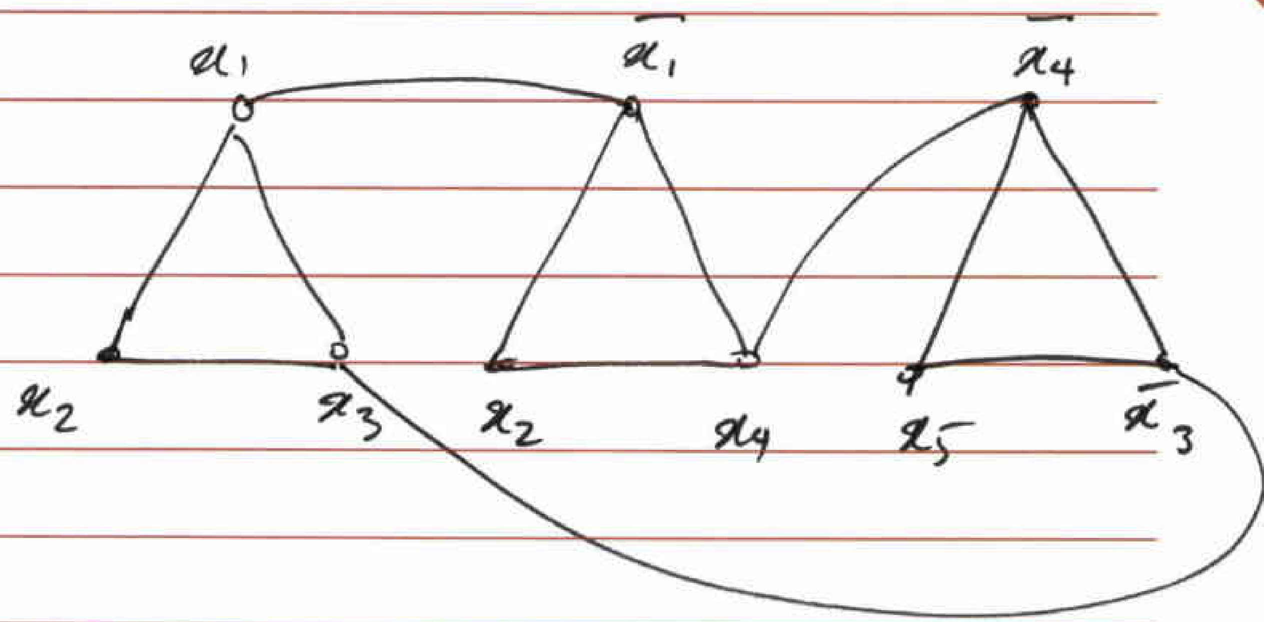
$(x_1 \vee x_2 \vee x_3)$



$C_1 = (x_1 \vee x_2 \vee x_3)$

$C_2 = (\bar{x}_1 \vee x_2 \vee x_4)$

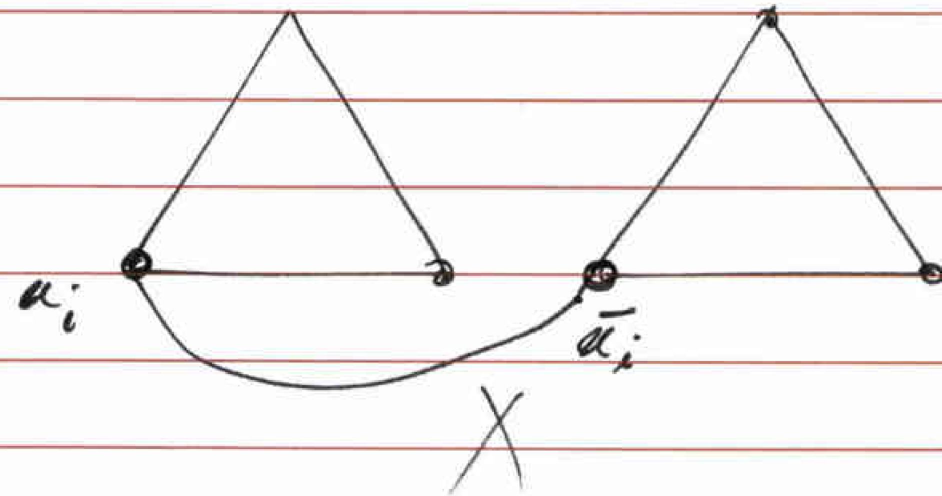
$C_3 = (\bar{x}_4 \vee x_5 \vee \bar{x}_3)$



claim: The original 3-SAT instance is satisfiable iff the graph G has an indep set of size $\underline{k}$.

<u>Proof</u> A) If the 3-SAT instance is satisfiable there is at least one node per triangle that evaluates to 1.

Let S be ~~an~~ a set containing one such node from each triangle



$\Longrightarrow$ S is an indep set

B) Suppose G has an indep set S of size at least $\underline{k}$.

If x_i appears as a node in S, then $x_i = 1$

If $\bar{x}_i$ appears as a node in S, then $x_i = 0$

If neither x_i nor $\bar{x}_i$ appear as a node in S, then $x_i = \underline{1}$ or $\underline{0}$