Discussion 1

CSCI 570

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DISCUSSION 1

Outline

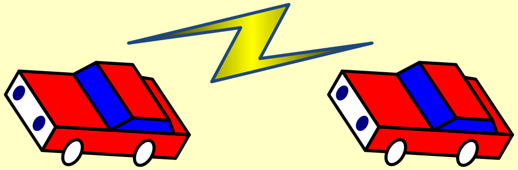
- Introduction
- Problems with Solutions

Background

- Graduated with a BS in CECS, MS in CS, and Ph.D. in CS from USC
- Taught at Cal State LA for 5 years during grad school
- Assistant/Associate Professor at University of Alaska for 6.5 years
- Back at USC as an Associate Professor of Engineering Practices in 2014
- Worked part-time and full-time as a system administrator, junior programmer, intermediate programmer, senior programmer, technical lead, chief architect, director of engineering, and founder of a company
- Still do consulting work for all types of applications and companies

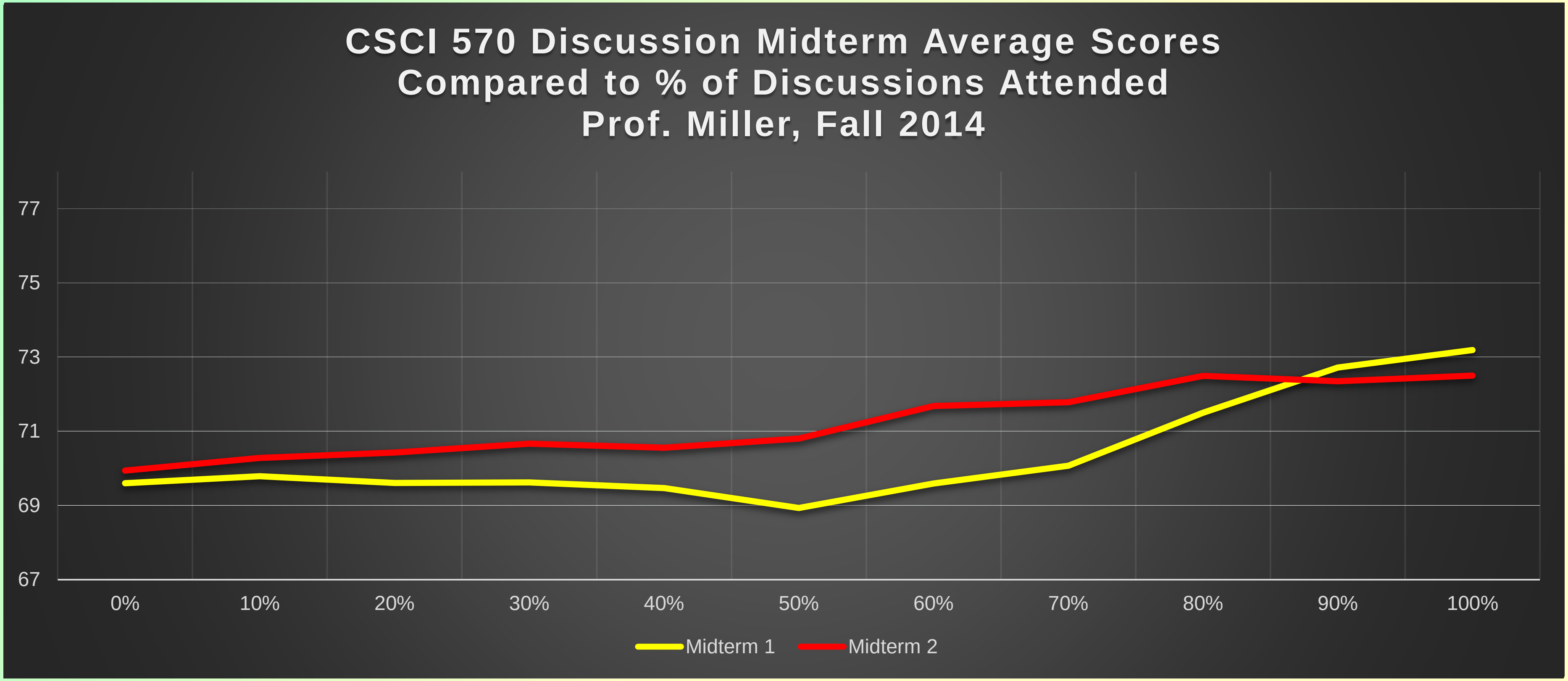
Research Interests

- Computer science education
  - Undergraduate
  - Graduate
  - K12 Science, Technology, Engineering, Mathematics (STEM) education
- Intelligent Transportation Systems (ITS)
  - Routing algorithms
  - Dynamic graph algorithms
  - Data gathering and mining
- Vehicular Networking
  - Vehicle-to-Vehicle (V2V)
  - Vehicle-to-Infrastructure (V2I)
  - Vehicle-to-Vehicle-to-Infrastructure (V2V2I)



- Simulated Environments
  - Software
  - Hardware

CSCI 570 Study from Fall 2014



My Office Hours for Spring 2015

- Office: SAL 342
- Monday: 11:30a.m.-1:00p.m.
- Tuesday 9:30a.m.-10:30a.m.
- Wednesday 11:30a.m.-1:00p.m.
- Other days by appointment – email me

Outline

- Introduction
- Problems with Solutions

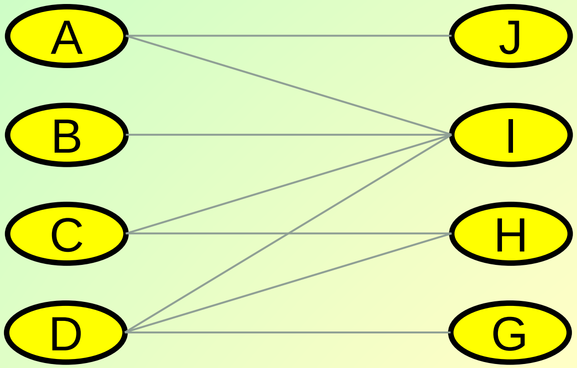
Problem 1

- Define each of the following terms
  - Tree
  - Forest
  - Cycle
  - Graph
  - Adjacency list
  - Adjacency matrix

Solution 1

- A tree is an undirected graph in which any two vertices are connected by exactly one simple path.
- A forest is a disjoint union of trees.
- A cycle is a sequence of vertices starting and ending at the same vertex with each two consecutive vertices in the sequence adjacent to each other in the graph.
- A simple cycle is a cycle with no repetitions of vertices other than the first and last.
- An adjacency list is a representation of a graph in which there is one set for each vertex in the graph.
- An adjacency matrix is a representation of a graph in which the vertices adjacent to other vertices is shown in a tabular format.

Solution 1



- Adjacency List
  - A->J, I
  - B->I
  - C->I, H
  - D->I, H, G
  - J->A
  - I->A, B, C, D
  - H->C, D
  - G->D

Adjacency Matrix

	A	**B**	**C**	**D**	**G**	**H**	**I**	**J**
A	0	0	0	0	0	0	1	1
B	0	0	0	0	0	0	1	0
C	0	0	0	0	0	1	1	0
D	0	0	0	0	1	1	1	0
G	0	0	0	1	0	0	0	0
H	0	0	1	1	0	0	0	0
I	1	1	1	1	0	0	0	0
J	1	0	0	0	0	0	0	0

Problem 2

- For this problem, we will explore the issue of *truthfulness* in the Stable Matching Problem and specifically in the Gale-Shapley algorithm. The basic question is: Can a man or a woman end up better off by lying about his or her preferences? More concretely, suppose each participant has a true preference order. Now consider a woman *w*. Suppose *w* prefers man *m* to *m'*, but both *m* and *m'* are low on her list of preferences. Can it be the case that by switching the order of *m* and *m'* on her list of preferences and running the algorithm with this false preference list, *w* will end up with a man *m"* that she truly prefers to both *m* and *m'* ?

- Resolve this question by doing one of the following two things:
  - Give a proof that, for any set of preference lists, switching the order of a pair on the list cannot improve a woman's partner in the Gale-Shapley algorithm; or
  - Give an example of a set of preference lists for which there is a switch that would improve the partner of a woman who switched preferences.

Solution 2

- Gale-Shapley Algorithm
  - Each unengaged man proposes to the woman he most prefers
  - Each woman replies “maybe” to the suitor she most prefers from the ones who have proposed and “no” to all other suitors. She is then *provisionally* engaged to the suitor who proposed to her that she most prefers
  - In each subsequent round, each unengaged man proposes to the most-preferred woman to whom he has not yet proposed (regardless of whether the woman is already engaged)
  - Each woman replies with “maybe” to her suitor she most prefers and rejects the rest again
  - This allows a woman to “trade up” even if she is already engaged
- This results in:
  - Everyone will be married
  - The marriages are stable – it is not possible for Alice and Bob to both prefer each other over their current partners

- Consider the following example with n=3 men and women. Let’s run the algorithm with w3’s true preference list then with her false (but stated) preference list. For the sake of this example, let’s assume that Gale-Shapley breaks ties by using the lowest-numbered unmatched man to ask.

Solution 2

Person	m1	m2	m3	w1	w2	w3	w3’ (lying)
Rank #1	w3	w1	w3	m1	m1	m2	m2
Rank #2	w1	w3	w1	m2	m2	m1	m3
Rank #3	w2	w2	w2	m3	m3	m3	m1

<u>Round 1</u>
M1 proposes to W3
M2 proposes to W1
M3 proposes to W3
W3 accepts M1
W1 accepts M2

<u>Round 2</u>
M3 proposes to W1
W1 rejects M3

<u>Round 3</u>
M3 proposes to W2
W2 accepts M3

<u>Final Pairing</u>
M1, W3
M2, W1
M3, W2

All men and women are married, and there is no man and woman who would both prefer each other over the partner they have.
With w3’s false listing, here are the iterations

<u>Round 1</u>
M1 proposes to W3
M2 proposes to W1
M3 proposes to W3
W3 accepts M3
W1 accepts M2

<u>Round 2</u>
M1 proposes to W1
W1 accepts M1

<u>Round 3</u>
M2 proposes to W3
W3 accepts M2

<u>Round 4</u>
M3 proposes to W1
W1 rejects M3

<u>Round 5</u>
M3 proposes to W2
W2 accepts M3

<u>Final</u>
M1, W1
M2, W3
M3, W2

Initially, with the listed tie-breaker, Gale-Shapley will produce the pairs (m1,w3) (m2, w1) and (m3,w2). However, if w3’s false preference list is used (and the other five remain truthful), we are left with (m1,w1), (m2,w3), and (m3,w2) -- leaving w3 with her truly first choice.

Problem 3

- Given *n* men and *n* women along with their preference lists, a *consensus-optimal* stable matching is a matching which simultaneously pairs every man with his **best valid** partner and pairs every woman with her **best valid** partner. Recall that a valid partnership must be a matched pair in some stable matching solution. Give an algorithm to determine whether a consensus-optimal stable matching exists for a given set of preference lists.

Solution 3

- Run Gale-Shapley with the input and store the solution. Then run it again, but with the genders reversed (women ask men, or reverse the inputs). If the solutions are the same, we have a consensus-optimal stable matching. If they aren't, we don't.

Solution 3

Person	m1	m2	m3	w1	w2	w3	w3’ (lying)
Rank #1	w3	w1	w3	m1	m1	m2	m2
Rank #2	w1	w3	w1	m2	m2	m1	m3
Rank #3	w2	w2	w2	m3	m3	m3	m1

When men asked women with W3 being truthful, we ended up with (M1, W3), (M2, W1), and (M3, W2)

Round 1	Round 2	Round 3	Final Pairing
W1 proposes to M1	W2 proposes to M2	W2 proposes to M3	M1, W1
W2 proposes to M1	M2 rejects W2	M3 accepts W2	M2, W3
W3 proposes to M2			M3, W2
M1 accepts W1			
M2 accepts W3			

This is **not** a consensus-optimal stable matching.

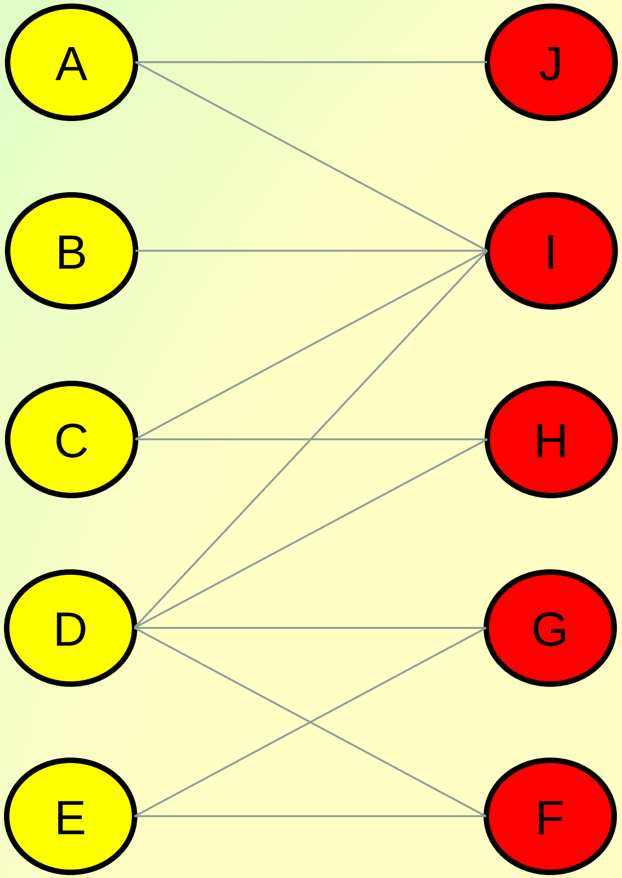
With w3’s false listing, we ended up with (M1, W1), (M2, W3), and (M3, W2)

Round 1	Round 2	Round 3	Final Pairing
W1 proposes to M1	W2 proposes to M2	W2 proposes to M3	M1, W1
W2 proposes to M1	M2 rejects W2	M3 accepts W2	M2, W3
W3 proposes to M2			M3, W2
M1 accepts W1			
M2 accepts W3			

This **is** a consensus-optimal stable matching

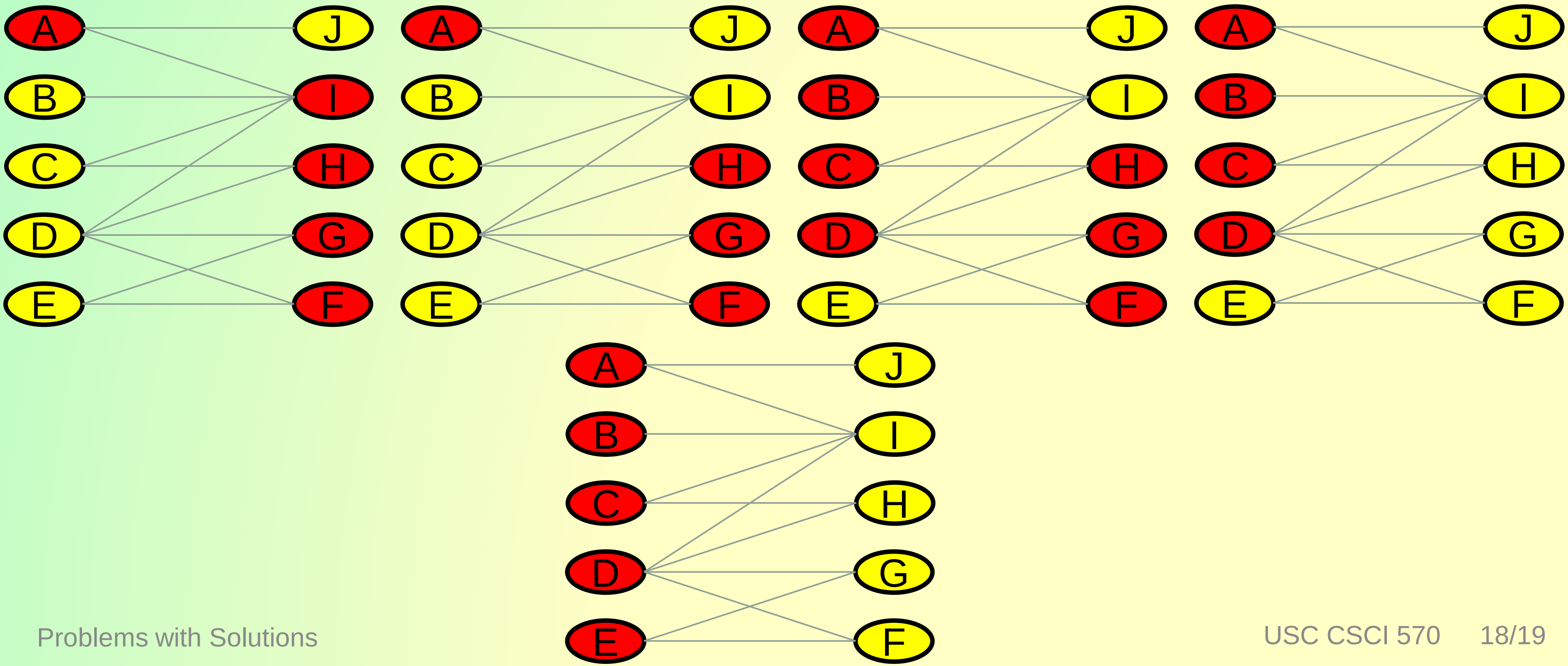
Problem 4

- In a connected bipartite graph, is the bipartition unique? Justify your answer.



Solution 4

- To get a different partition, you would need to switch the colors of two connected nodes
- This would have a trickle effect of requiring changing the colors of all the nodes to which one of the changed nodes is connected
- You would end up with the same partition, but the colors would be reversed



Solution 4

- Note that if the graph *isn't* connected, swapping colors in one or more connected components, and leaving one or more unchanged, *does* result in a different bipartition.