Illustrations

Oligopoly Model

We shall study the outcome of competition among the firms producing a product with their customers.

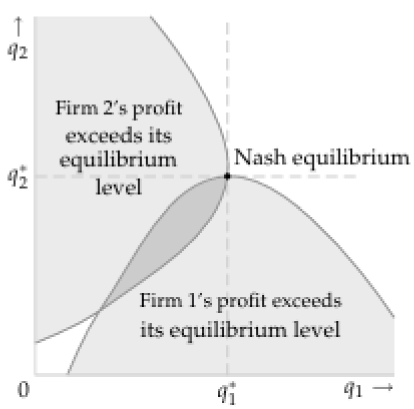
General Model

A single good is produced by n firm.

- $C_i(q_i)$: cost to firm i of producing a quantity of q_i units of the good (C_i is an increasing function)
- Q: total output of all firms (equivalently: $q_1 + q_2 + \cdots + q_n$)
- $P(Q)$: market price based on total quantity available (also called **inverse demand function**, P decreases when Q increases)
  A firm profit, or payoff function, is $\pi(q_1, \cdots, q_n) = q_i P(Q) - C_i(q_i)$, or gross revenue subtracted by cost.

Example

Let us consider the case where there're 2 firms and the price function is $P(Q) = \max(\alpha - Q, 0)$ for some fixed demand α. By finding the best response function for each firm given a fixed quantity, we can find that the equilibrium is unique and is the point $(\frac{1}{3}(\alpha - c), \frac{1}{3}(\alpha - c))$, where c is the cost to make each product by either company.
Note that there are outcomes where both firms can reduce their output and get better payoff, as illustrated in the figure



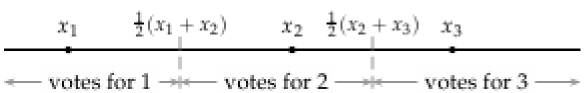
An interesting observation is that as the number of firms $n \to \infty$ and if they all have the same characteristics, then $Q \to \alpha$ and $P \to c$, which means the firms barely breakeven.

Electoral Competition

We shall study the a simplified model of an election. Suppose each candidate chooses a policy to go with, and voters cast their votes based on which policy they like the most. Winning is preferred to tie to losing.

General Model

We represent the policy as a number on the number line. A voter votes for the candidate who is closest to their preferred number.



This is like choosing number game where the one that's closer to the majority of numbers win the game

Example

Suppose there are 2 candidates. Let m be a value where there are exactly half the voters whose preferences is less than m and exactly half choose a policy greater than m. In other words, exactly half the numbers is on either side of m, the **median favorite position**.
We can easily show that (m_1, m_2) is a Nash equilibrium for any median positions m_1, m_2

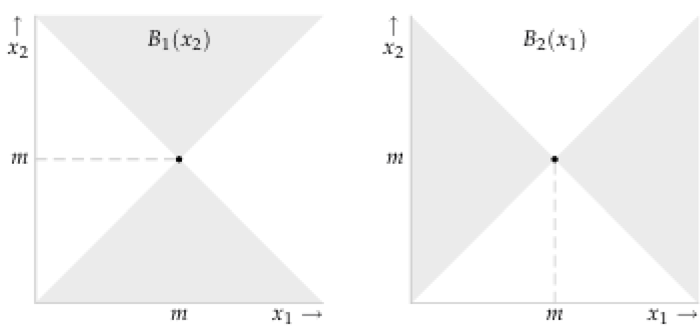


Figure 71.1 The candidates' best response functions in Hotelling's model of electoral competition with two candidates. Candidate 1's best response function is in the left panel; candidate 2's is in the right panel. (The edges of the shaded areas are excluded.)

The War of Attrition

This game serves as an extension of the game Hawk-Dove. An equivalent game would be betting on a $100 bill, where the winner gets the bill and both have to pay their bids.

General Model

We have two people who can either wait or concede for an item.

- t_i: time player i is waiting for
- v_i: how much player i values the reward item
  You get the item if you're the player who waited the longest. If you give up before any other players, you end up with nothing. If there's a tie, the reward is divided equally between the winners.

Example

Suppose there are two players. If you wait and the other player gives up, the payoff is $v_i - t_j$, the time the other give up at. If you give up before the other, the payoff is $-t_i$ for the lost time, and if there's a tie, the payoff is $\frac{1}{2}v_i - t_i$

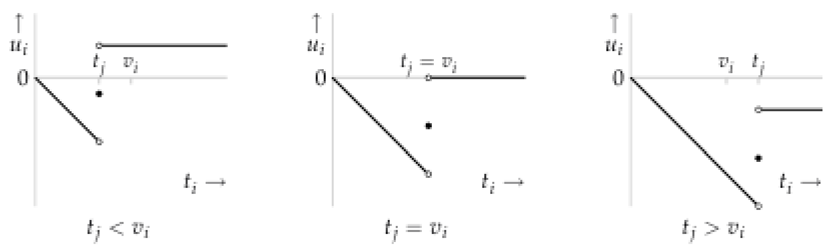


Figure 76.1 Three cross-sections of player i's payoff function in the *War of Attrition*.

Solving this problem, we can see that in every equilibrium, either player 1 concedes immediately and player 2 concedes at time v_1 or later (player 2 values the item more than the wasted time) or vice versa.

Auctions

Let us consider a second-price sealed-bid auctions, where the highest bidder wins, but pays the price according to the second highest bidder.

General Model

- n: number of bidders
- v_i: how much someone values the item being bid. Suppose $v_i > v_j$ for any $i < j$
- b_i: how much bidder i is bidding
  The payoff of any player i is therefore $v_i - b_j$ where b_j is at least the second highest bid. If there's a tie, the player with smallest index i wins.

Example

A second-price auction has many Nash equilibria, including one where everyone bids there maximum evaluation $b_i = v_i$, in which every player's action weakly dominates all their other actions. An example where player 1 with highest evaluation doesn't win and is an equilibrium is $b_1 = v_2, b_2 = v_1$ and all other players don't bid.

Accident Law

We wish to make a law so that both the injurer and victim have the necessary precaution. In turns out that suppose we determine the threshold that each player needs to prepare (supposed it's a socially acceptable outcome), then we can model a game with a single Nash equilibrium.

General Model

Suppose player 1 is the injurer and player 2 is the victim

- a_i: amount of care player i takes (measured in monetary terms)
- $L(a_1, a_2)$: the expected loss suffered by the victim (it's not happening with certainty, so we model this as a probability)
  Assume that $L > 0$ for all values of a_1, a_2 and it is moving in the opposite direction as a_i, assuming the other a_j is fixed. In other words, if more care is taken, there's less loss.
  Player 1 payoff is therefore $-a_1 - L(a_1, a_2)$ if $a_1 < X_1$ and $a_2 \geq X_2$ (injurer is not careful enough and victim is more cautious than needed) or $-a_1$ otherwise (doesn't need to pay the victim). Player 2 payoff is similar to this.

Example

We would want to maximize the sum of both players' payoffs (or minimize social loss), which is maximizing $-a_1 - a_2 - L(a_1, a_2)$. Suppose we've determined a socially acceptable pair $(\hat{a}_1, \hat{a}_2)$ that maximizes our payoff. We can show that this is a Nash equivalent and it is unique (equivalent to (X_1, X_2)).

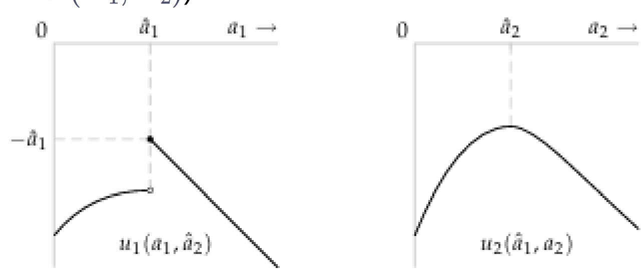


Figure 92.1 Left panel: the injurer's payoff as a function of her level of care a_1 when the victim's level of care is $a_2 = \hat{a}_2$ (see (91.1)). Right panel: the victim's payoff as a function of her level of care a_2 when the injurer's level of care is $a_1 = \hat{a}_1$ (see (92.1)).

In other words, both side is expected to take the minimal required care towards their action. (How does this hold for car insurance?)