Chapter 2

Non-Deterministic Finite Automata:-

NFA is a 5-tuple.

$N = (Q, \Sigma, \delta, q_0, F)$

$Q \rightarrow$ A Non empty finite set of states.

$\Sigma \rightarrow$ A finite Non-empty set of inputs.

$\delta \rightarrow$ It is a mapping (or) transition function

$$\boxed{\delta : Q \times \Sigma \longrightarrow 2^Q}$$

$q_0 \rightarrow q_0 \in Q$, is the initial state.

$F \rightarrow F \subseteq Q$, a set of final states.

NFA To DFA conversion:-

Let $N = (Q, \Sigma, \delta, q_0, F)$ is a NFA which accepts the language $L(N)$. There should be Equivalent DFA denoted by $M = (Q_D, \Sigma_D, \delta_D, q_{0D}, F_D)$.

The conversion steps:-

Step-1:- The start state (or) initial state of NFA, it will be the start state for DFA M. Hence, add q_0 of NFA to Q_D. Find the transitions from this start state.

Step-2:- For each state $[q_1, q_2, \ldots q_i]$ in Q_D, the transitions for each input symbol Σ can be obtained as. (i) $\delta_D([q_1, q_2, \ldots q_i], a) = \delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_3, a) \cup \ldots \cup \delta(q_i, a) = [q_1, q_2, \ldots q_i]$ may be some state.

13/12/1

Sol:-

(ii) Add The state $[q_1, q_2, \ldots, q_i]$ to DFA, if it is not already added in Q_D.

(iii) Then find The transitions for each input symbol from Σ for state $[q_1, q_2, \ldots, q_n]$ if we get some state $[q_1, q_2, \ldots, q_n]$ which is not in Q_D of DFA Then add This state to Q_D.

iv) If There is no new state generating Then stop The process after finding all The transitions.

Step-3:- ~~First take~~ For state $[q_1, q_2 \ldots q_n] \in Q_D$ of DFA if any one state q_i is a final state of NFA Then $[q_1, q_2, \ldots q_n]$ becomes a final state. Thus The set of all The final states belongs to F_i of DFA.

13/12/17 * convert The following NFA into DFA?

Σ/Q	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$

Sol:- Given data,

$\delta(q_0, 0) = \{q_0, q_1\}$

$\delta(q_0, 1) = \{q_1\}$

$\delta(q_1, 0) = \phi$

$\delta(q_1, 1) = \{q_0, q_1\}$.

Step-1:- The initial state q_0 Add to Q_D of DFA. Add transitions of q_0. added to Q_D states.

Step-2:- Find transitions for The newly generated states $[q_1]$ and $[q_0, q_1]$.

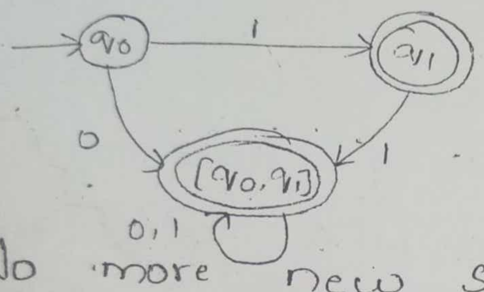
Σ/Q	0	1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_1]$
$[q_1]$	ϕ	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$

$\delta([q_1], 0) = \phi$

$\delta([q_1], 1) = [q_0, q_1]$

$\delta([q_0, q_1], 0) =$
$\delta([q_0], 0) \cup \delta([q_1], 0)$
$\{q_0, q_1\} \cup \{\phi\}$
$= [q_0, q_1]$

$\delta([q_0, q_1], 1) =$
$\delta([q_0], 1) \cup \delta([q_1], 1)$
$\{q_1\} \cup \{q_0, q_1\}$
$\{q_0, q_1\}$.



* No more new states.

So, Stop The construction of DFA.

Step-3:- In NFA q_1 is The final state, so, If q_1 is also a state of DFA, Then mark it as final state. And also if q_1 is an element of any state Then mark Those states as final states.

* convert The following NFA into DFA.

Σ/Q	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	q_1
q_2	q_3	q_3
q_3	ϕ	q_2

sol:- Given data,

$\delta(q_0, 0) = \{q_0, q_1\}$ $\quad$ $\delta(q_0, 1) = q_0$

$\delta(q_1, 0) = q_2$ $\quad$ $\delta(q_1, 1) = q_1$

$\delta(q_2, 0) = q_3$ $\quad$ $\delta(q_2, 1) = q_3$

$\delta(q_3, 0) = \phi$ $\quad$ $\delta(q_3, 1) = q_2$.

step-1:- The initial state q_0 of NFA is added to Q_D Add Q_D of DFA. Add transitions of q_0.

step-2:- Find transitions states for the newly generated states $[q_0, q_1]$ & q_0.

	0	1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1, q_2]$	$[q_0, q_1]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_3]$
$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_2, q_3]$
$[q_0, q_1, q_3]$	$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$

$\delta(q_0, q_1) =$
$\delta([q_0, 0) \cup \delta(q$
$\{q_0, q_1\} \cup q_2$
$= \{q_0, q_1, q_2$

$\delta(q_0, 1) \cup \delta(q_1, 1$
$\{q_0\} \cup \{q_1\}$
$= \{q_0, q_1\}$

$\delta[q_0, q_1, q_2]$
$\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta($
$\{q_0, q_1\} \cup \{q_2\} \cup \{q$
$\{q_0, q_1, q_2, q_3\}$

$\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta$
$\{q_0\} \cup \{q_1\} \cup \{q_3$

* No more new states so, stop the construction of DFA

$\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) \cup \delta(q_3, 1)$
$\{q_0\} \cup \{q_1\} \cup \{q_3\} \cup \{q_2\}$
$\{q_0, q_1, q_2, q_3\}$

$\delta(q_0, q_1, q_2, q_3)$
$\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)$
$\delta(q_3, 0)$
$\{q_0, q_1\} \cup \{q_2\} \cup \{q_3\} \cup \{\phi$
$= \{q_0, q_1, q_2, q_3\}$

$\delta(q_0, q_1, q_3)$

$\delta(q_0,0) \cup \delta(q_1,0) \cup \delta(q_3,0)$

$\{q_0, q_1\} \cup \{q_2\} \cup \{\phi\}$

$\{q_0, q_1, q_2\}$

$\delta(q_0,1) \cup \delta(q_1,1) \cup \delta(q_3,1)$

$\{q_0\} \cup \{q_1\} \cup \{q_2\}$

Step-3:- In NFA q_3 is The final state, if q_3 is also a state of DFA, Then mark it as final state And also if q_3 is an element of any state Then mark Those states as final states.

	0	1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1, q_2]$	$[q_0, q_1]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_3]$
$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_2, q_3]$
$[q_0, q_1, q_3]$	$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$.

* convert The following NFA to DFA.

	Σ / ϕ	0	1
$\rightarrow P$	$\rightarrow q_0$	$\{P, q\}$	P
q	q_1	r	r
&r	q_2	s	ϕ
s	q_3	s	s

$\delta(P, 0) = \{P, q\}$

$\delta(q, 0) = r$

$\delta(r, 0) = s$

$\delta(s, 0) = s$

$\delta(P, 1) = P$

$\delta(q, 1) = r$

$\delta(r, 1) = \phi$

$\delta(s, 1) = S$.

Step-1:- The initial state ~~Q~~ P of NFA, is added to Q_D Add to Q_D of DFA. Add transitions of P.

Step-2:- Find transition states for The newly generated states [P, q] and P

Σ/Q	0	1
→P	[P, q]	P
[P, q]	[P, q, r]	[P, r]
[P, r]	[P, q, S]	[P]
[P, q, r]	[P, q, r, S]	[P, r]
[P, q, S]	[P, q, S]	[P, r, S]
[P, r, S]	[P, q, S]	[P, S]
[P, q, r, S]	[P, q, r, S]	[P, r, S]
[P, S]	[P, q, S]	[P, S]

Step-3:- In NFA S is The final state if S is also a state of DFA, Then mark it as final state. And also S is an element of any state Then mark Those States as final states.

Σ/Q	0	1
→P	[P, q]	P
[P, q]	[P, q, r]	[P, r]
[P, r]	[P, q, S]	P
[P, q, r]	[P, q, r, S]	[P, r]
[P, q, S]	[P, q, S]	[P, r, S]
[P, r, S]	[P, q, S]	[P, S]
[P, q, r, S]	[P, q, r, S]	[P, r, S]
[P, S]	[P, q, S]	[P, S]

* Non-deterministic Finite Automata:-
* An Automata is an abstract Model of digital computer.
* An Automata has Mechanism to read input from input tape.
* Any language is recognised by Some Automata. Hence These Automata are basically language Acceptors (or) Language Recognizers.
* There are two types of finite Automata.
* Deterministic Finite Automata (DFA)
* Non-Deterministic Finite Automata (NFA).

Significance of NFA:-
* computers are basically deterministic Machines.
* That means on giving certain input, we get certain output either desirable (or) undesierable
* In Similar manner DFA are The Machines in which we get some predectiable state on certain input.
* But constructing such DFA's is very difficult.
* Hence There should Some ~~ma~~ way by which some easy to construct Machines can be built.
* In such a case, we construct a Non deterministic Machine which is called NFA (or) NDFA.
* This NFA can be easily converted to DFA.

14/12/1

from state P on ϵ - transitions such That

i) ϵ - closure (p) = P, where $p \in Q$

ii) If There exists ϵ - closure (P) = {q} and $\delta(q, \epsilon) = r$ Then ϵ - closure (p) = {q, r}.

Step-2:- Find δ' - transitions.

The δ' - transitions means an ϵ - closure on δ - moves.

$$\boxed{\delta'(q,a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q,\epsilon),a)) \quad \text{where } \hat{\delta}(q,\epsilon) = \epsilon\text{-closure}(q).}$$

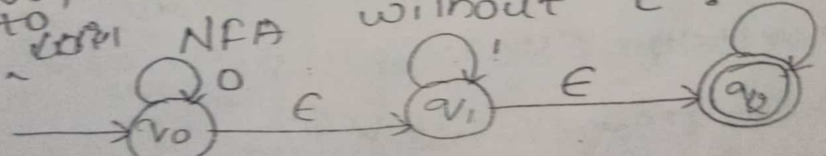
δ - transitions with ϵ

δ' - transitions without ϵ

Step-3:- Step - 2 is repeated for each input symbol and for each state of given NFA.

Step-4:- using The Resultant states The transitions table for equivalent NFA without Null can be built.

14M Imp **** Example:- Convert The given NFA with ϵ to NFA without ϵ ?



sol:- Given data,

Σ/Q	0	1	2	ϵ
→ q_0	q_0	—	—	{q_1, q_2}
q_1	—	q_1	—	{q_2}
(q_2)	—	—	q_2	-

conversion:-

Step-1:- Finding of ϵ-closure

ϵ-closure $(q_0) = \{q_0, q_1, q_2\}$

$\delta(q_0, \epsilon) = q_1$

$\delta(q_1, \epsilon) = q_2$

$\delta(q_2, \epsilon) = \phi$.

ϵ-closure $(q_1) = \{q_1, q_2\}$

$\delta(q_1, \epsilon) = q_2$

$\delta(q_2, \epsilon) = \phi$

ϵ-closure $(q_2) = \{q_2\}$

$\delta(q_2, \epsilon) = \phi$.

Step-2:- Finding of δ'

$$\boxed{\begin{array}{l} \delta'(q, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q, \epsilon), a)) \\ \text{where } \hat{\delta}(q, \epsilon) = \epsilon\text{-closure}(q). \end{array}}$$

Step-3:-

1) $\delta'(q_0, 0) = \epsilon$- closure $(\delta(\hat{\delta}(q_0, \epsilon), 0))$

$= \epsilon$-closure$(\delta(\epsilon\text{-closure}(q_0), 0))$

$= \epsilon$-closure$(\delta(\{q_0, q_1, q_2\}), 0)$

$= \epsilon$-closure$(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$= \epsilon$-closure$(q_0 \cup \phi \cup \phi)$

$= \epsilon$-closure(q_0)

$= \{q_0, q_1, q_2\}$

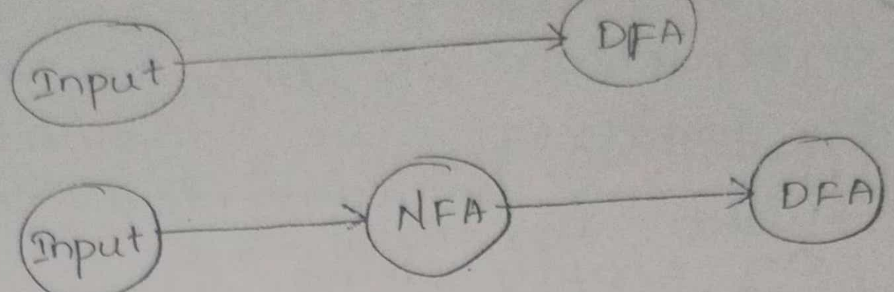
2) $\delta'(q_0, 1) = \epsilon$-closure$(\delta(\hat{\delta}(q_0, \epsilon), 1))$

$= \epsilon$-closure$(\delta(\epsilon\text{-closure}(q_0), 1))$

$= \epsilon$-closure$(\delta(\{q_0, q_1, q_2\}), 1)$

$= \epsilon$-closure$(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$

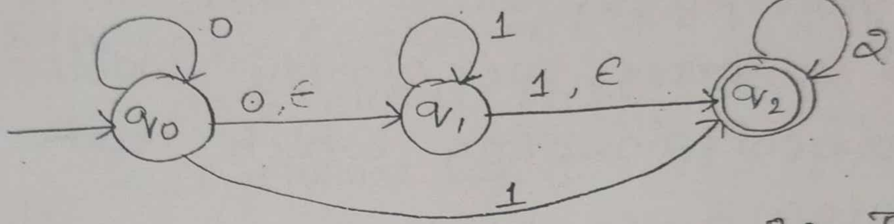
$= \epsilon$-closure$(\phi \cup q_1 \cup \phi)$



Imp**** **NFA with ε-Transitions:-**

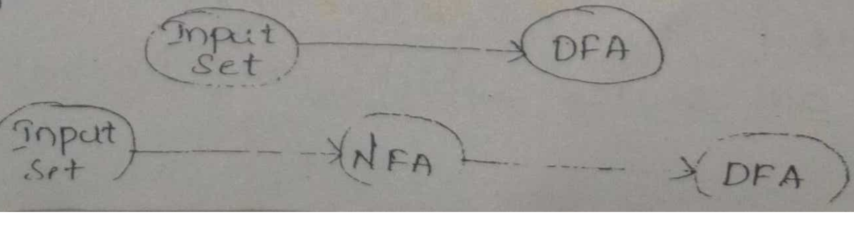
* The ε-Transitions in NFA are
given inorder to move from one state
to another without having any input
symbol from input set Σ.

Eg:-



14/12/17 * In This NFA with ε - q_0 is The start
state with input '0' ~~ei~~ can be either
state q_0 (or) q_1.
* If we get The input symbol '1' at
initial state Then with ε-move we can
change from state q_0 to q_1. And Then with
input we can be in state q_1. On The
Other hand on input ~~one~~ 1 from initial
state q_0, we can reach to state q_2. Thus
It is non-deterministic on input 1
whether we will be in state q_1 (or) q_2.
Hence, it is called NFA with Epsilon (ε).

Significance of NFA with ε:-



input set → NFA with ε → NFA → DFA

* As construction of DFA is very diffi... for certain input set, we construct NFA.
* This NFA can be converted to DFA.
* ε is an empty string.
* The ε-transitions are used simply to change from one state to other, without any input symbol from Σ.

P * * Sometimes to reach to final state we do not get proper state from initial state. In such a case, we simply wants to reach to certain state which leads to final state.
* Such a transition to that specific state should be without any input symbol.
* Hence, we require ε-moves by which a proper state can be obtained for reaching to final state.
* Thus, ε-moves play an important role in NFA.

NFA with ε to DFA conversion:-

Procedure for

1) NFA with ε-moves to NFA:-

Step-1:- Find ε-closure, for all states in Q.

ε-closure:- The ε-closure (P) is a set of all states which are reachable

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}.$

3) $\delta'(q_0, 2) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 2))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), 2))$

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 2)$

$= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2,$

$= \epsilon\text{-closure}(\phi \cup \phi \cup q_2)$

$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}.$

4) $\delta'(q_1, 0) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), 0))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 0))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 0)$

$= \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0)$

$= \epsilon\text{-closure}(\phi \cup \phi)$

$= \epsilon\text{-closure}(\phi)$

$= \phi.$

5) $\delta'(q_1, 1) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), 1))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1)), 1))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}), 1)$

$= \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1))$

$= \epsilon\text{-closure}(q_1 \cup \phi\}$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$

6) $\delta'(q_1, 2) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), 2))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), 2))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}), 2)$

$= \epsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2))$

$= \epsilon\text{-closure}(\phi \cup q_2)$

$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}.$

7) $\delta'(q_2, 0) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), 0))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 0))$

$= \epsilon\text{-closure}(\delta(\{q_2\}, 0)$

$= \epsilon\text{-closure}(\delta(q_2, 0))$

$= \epsilon\text{-closure}(\phi)$

$= \phi$

8) $\delta'(q_2, 1) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), 1))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 1))$

$= \epsilon\text{ closure}(\delta(\{q_2\}, 1)$

$= \epsilon\text{-closure}(\delta(q_2, 1)$

$= \epsilon\text{-closure}(\phi)$

$= \phi.$

9) $\delta'(q_2, 2) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), 2))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 2))$

$= \epsilon\text{-closure}(\delta(\{q_2\}, 2)$

$= \epsilon\text{-closure}(\delta(q_2, 2)$

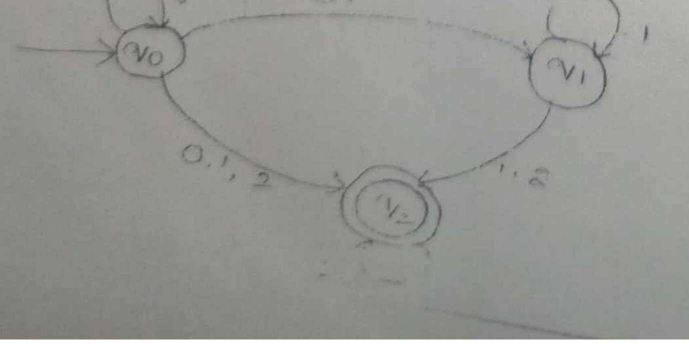
$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}.$

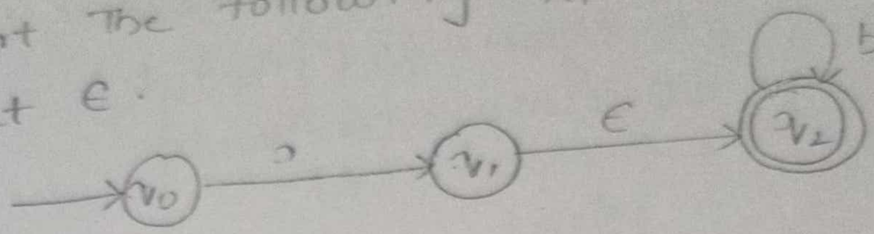
Step-4:- Transition table for NFA.

	0	1	2
$\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	—	$\{q_1, q_2\}$	$\{q_2\}$
q_2	—	—	$\{q_2\}$

* Its equivalent transition diagram is



* convert the following NFA with ε to N
without ε.



sol:- Given data,

	a	b	ε
$\rightarrow q_0$	q_1	-	-
q_1	-	$\{q_2\}$	$\{q_2\}$
q_2	-	q_2	-

Conversion:-

Step-1:- Finding of ε-closure.

$\epsilon\text{-closure}(q_0) = \{q_0\}$

$\delta(q_0, \epsilon) = \phi$

$\epsilon\text{-closure}(q_1) = \{q_1, q_2\}$

$\delta(q_1, \epsilon) = q_2$

$\delta(q_2, \epsilon) = \phi$

$\epsilon\text{-closure}(q_2) = \{q_2\}$

$\delta(q_2, \epsilon) = \phi$.

Step-2:- Finding of δ'

$\delta'(q, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q, \epsilon), a))$

where $\hat{\delta}(q, \epsilon) = \epsilon\text{-closure}(q)$

Step-3:-

1) $\delta'(q_0, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), a))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), a))$

$= \epsilon\text{-closure}(\delta(\{q_0\}, a))$

$= \epsilon\text{-closure}(\delta(q_0, a))$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$.

2) $\delta'(q_0, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), b))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_0), b))$

$= \epsilon\text{-closure}(\delta(\{q_0\}, b)$

$= \epsilon\text{-closure}(\delta(q_0, b)$

$= \epsilon\text{-closure}(\phi)$

$= \phi.$

3) $\delta'(q_1, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), a))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), a))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}), a))$

$= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a))$

$= \epsilon\text{-closure}(\phi \cup \phi)$

$= \epsilon\text{-closure}(\phi)$

$= \phi.$

4) $\delta'(q_1, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), b))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), b))$

$= \epsilon\text{-closure}(\delta(q_1, q_2), b))$

$= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b))$

$= \epsilon\text{-closure}(\phi \cup q_2)$

$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}$

5) $\delta'(q_2, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), a))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), a))$

$= \epsilon\text{-closure}(\delta(\{q_2\}, a))$

$= \epsilon\text{-closure}(\delta(q_2, a)$

$= \epsilon\text{-closure}(\phi)$

$= \phi$

6) $\delta'(q_2, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), b))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), b))$

$= \epsilon\text{-closure}(\delta(\{q_2\}, b))$

$= \epsilon\text{-closure}(\delta(q_2, b)$

$= \epsilon\text{-closure}(q_2)$

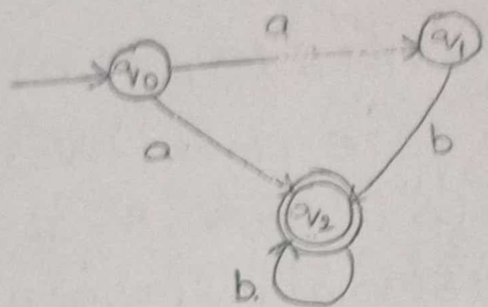
$= \{q_2\}.$

Sol

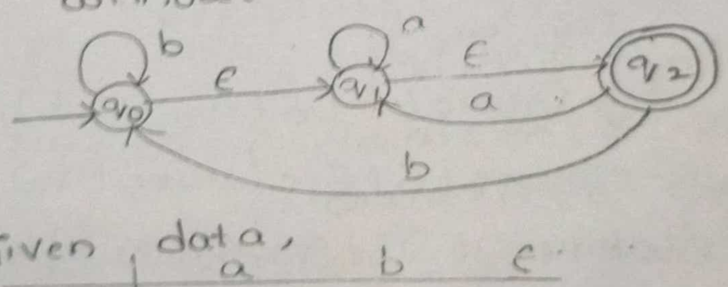
Step-4:- Transition table for NFA.

Σ/Q	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	-
q_1	-	$\{q_2\}$
q_2	-	$\{q_2\}$

* Its Equivalent transition diagram is



* convert the following NFA with ε to NFA without ε ?



Sol:- Given data,

	a	b	ε
$\rightarrow q_0$	-	q_0	$\{q_1, q_2\}$
q_1	q_1	-	$\{q_2\}$
q_2	q_1	q_0	-

Conversion:-

Step-1:- Finding of ε-closure

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

$\delta(q_0, \epsilon) = q_1$

$\delta(q_1, \epsilon) = q_2$

$\delta(q_2, \epsilon) = \phi$

ϵ-closure $(q_1) = \{q_1, q_2\}$

$\delta(q_1, \epsilon) = q_2$

$\delta(q_2, \epsilon) = \phi$

ϵ-closure $(q_2) = \{q_2\}$

$\delta(q_2, \epsilon) = \phi$.

Step-2:- Finding of δ'

$\delta'(q, a) = \epsilon$-closure $(\delta(\hat{\delta}(q, \epsilon), a))$

where $\hat{\delta}(q, \epsilon) = \epsilon$-closure (q).

Step-3:-

1) $\delta'(q_0, a) = \epsilon$-closure $(\delta(\hat{\delta}(q_0, \epsilon), a))$

$= \epsilon$-closure $(\delta(\epsilon$-closure $(q_0), a))$

$= \epsilon$-closure $(\delta(\{q_0, q_1, q_2\}, a))$

$= \epsilon$-closure $(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$

$= \epsilon$-closure $(\phi \cup q_1 \cup q_1)$

$= \epsilon$-closure $(\{q_1\})$

$= \{q_1, q_2\}$.

2) $\delta'(q_0, b) = \epsilon$-closure $(\delta(\hat{\delta}(q_0, \epsilon), b))$

$= \epsilon$-closure $(\delta(\epsilon$-closure $(q_0), b))$

$= \epsilon$-closure $(\delta(\{q_0, q_1, q_2\}, b))$

$= \epsilon$-closure $(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b))$

$= \epsilon$-closure $(q_0 \cup \phi \cup q_0)$

$= \epsilon$-closure (q_0)

$= \{q_0, q_1, q_2\}$

3) $\delta'(q_1, a) = \epsilon$-closure $(\delta(\hat{\delta}(q_1, \epsilon), a))$

$= \epsilon$-closure $(\delta(\epsilon$-closure $(q_1), a))$

$= \epsilon$-closure $(\delta(\{q_1, q_2\}), a))$

$= \epsilon$-closure $(\delta(q_1, a) \cup \delta(q_2, a))$

$= \epsilon$-closure $(q_1 \cup q_1)$

$= \epsilon$-closure (q_1)

$= \{q_1, q_2\}$

4) $\delta'(q_1, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), b))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1), b))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}), b))$

$= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b))$

$= \epsilon\text{-closure}(\emptyset \cup q_0)$

$= \epsilon\text{-closure}($

$= \epsilon\text{-closure}(q_0)$

$= \{q_0, q_1, q_2\}$.

5) $\delta'(q_2, a) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), a))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), a))$

$= \epsilon\text{-closure}(\delta(\{q_2\}), a)$

$= \epsilon\text{-closure}(\delta(q_2, a))$

$= \epsilon\text{-closure}(q_1)$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$

6) $\delta'(q_2, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_2, \epsilon), b))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), b))$

$= \epsilon\text{-closure}(\delta(\{q_2\}), b)$

$= \epsilon\text{-closure}(\delta(q_2, b)$

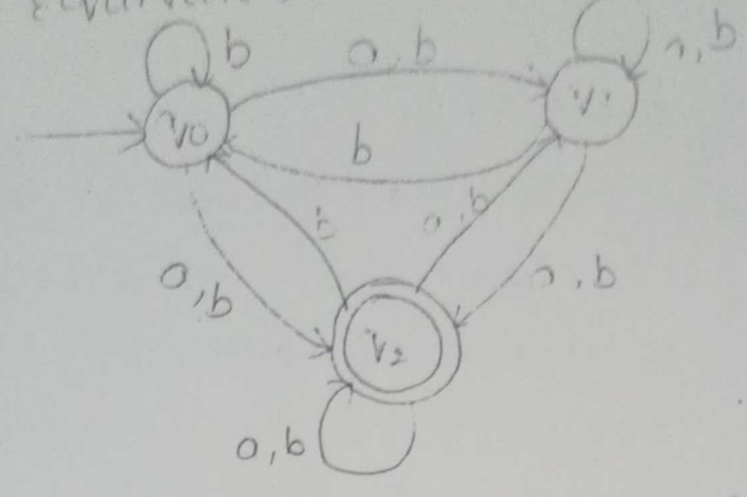
$= \epsilon\text{-closure}(q_0)$

$= \{q_0, q_1, q_2\}$.

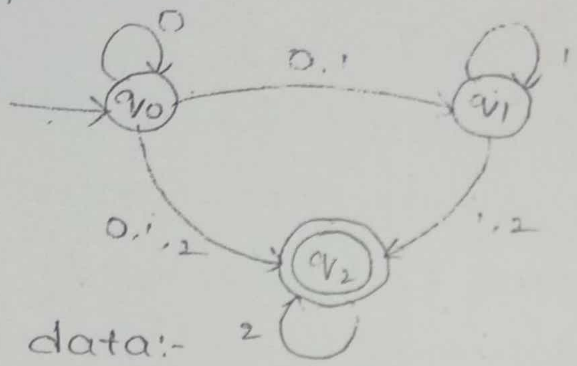
Step - 4 :- Transition table for NFA.

	a	b
$\rightarrow q_0$	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
q_1	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
q_2	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$.

* Its equivalent transition diagram is



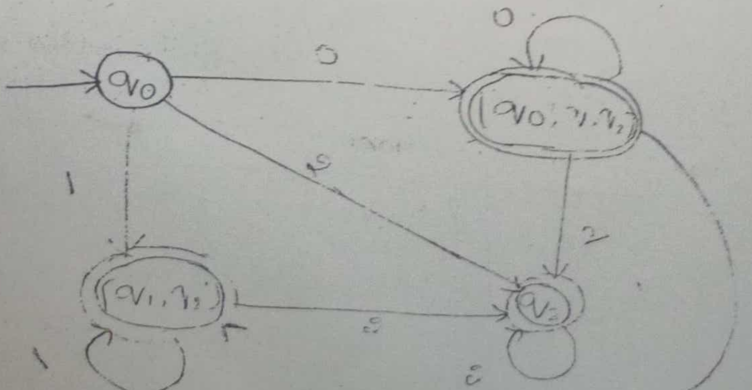
) Convert the following NFA with ε-move to DFA.



Sol:- Given data:-

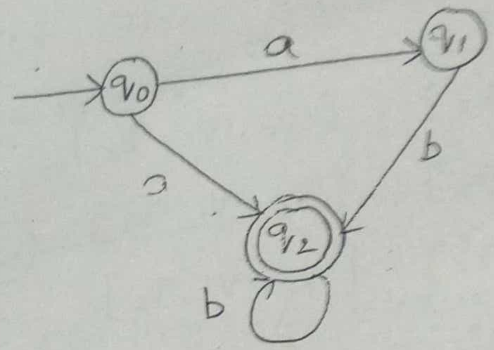
	0	1
$[q_0]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$
$[q_1, q_2]$	ϕ	$[q_1]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$
$[q_1]$	ϕ	$[q_1, q_2]$

No more new states. so we stop the construction.



	0	1	
$\rightarrow [q_0]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$	$[q_2]$
$[q_2]$	ϕ	ϕ	$[q_2]$
$[q_1, q_2]$	ϕ	$[q_1, q_2]$	$[q_2]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$	$[q_2]$

2) convert The following NFA with ϵ- move to DFA.



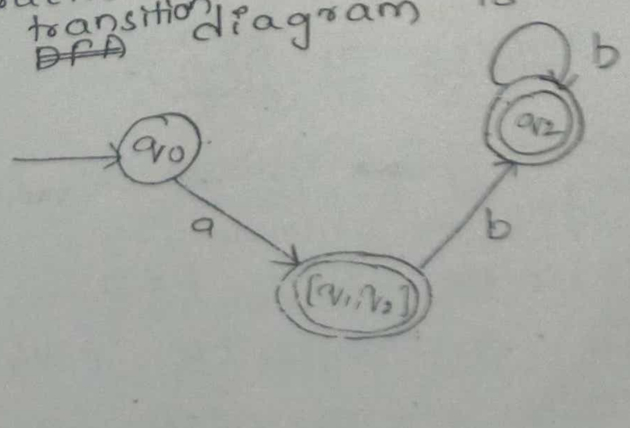
Sol: Given, data:-

	a	b
$\rightarrow [q_0]$	$[q_1, q_2]$	ϕ
$[q_1, q_2]$	ϕ	$[q_2]$
$[q_2]$	ϕ	$[q_2]$

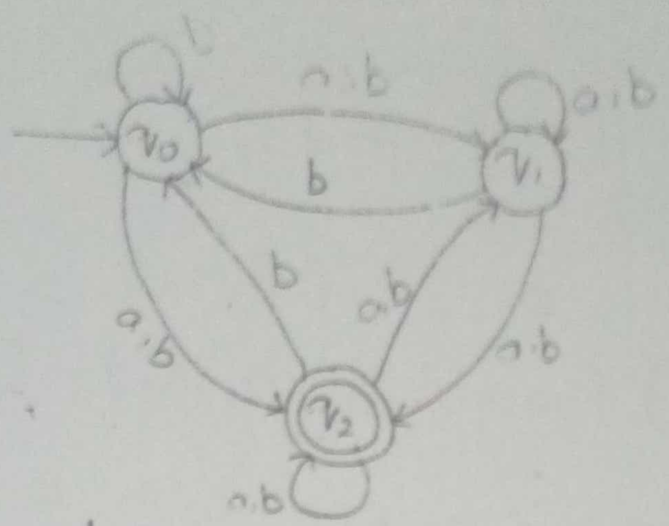
$(\delta(q_1, q_2) a)$
$\delta(q_1, a) \cup \delta(q_2$
$\delta(\phi \cup \phi)$

No more new states. so we stop The construction.

The ~~DFA~~ transition diagram is



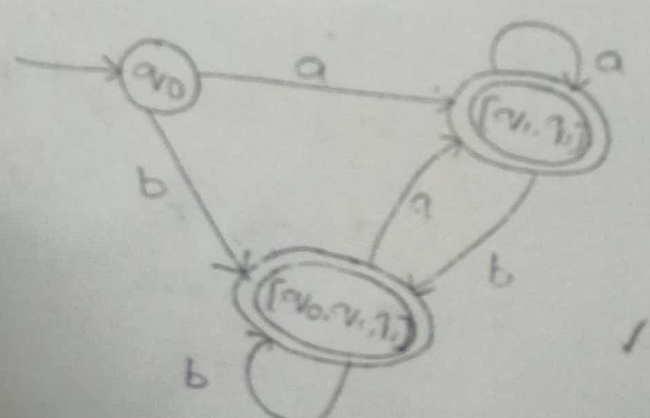
3) convert the following NFA with ε-move to DFA



Sol:- Given data:-

	a	b
→[q_0]	[q_1, q_2]	[q_0, q_1, q_2]
[q_1, q_2]	[q_1, q_2]	[q_0, q_1, q_2]
[q_0, q_1, q_2]	[q_1, q_2]	[q_0, q_1, q_2]

So No more transition states. So we stop The construction.



117 **NFA with ε- moves to DFA:-**

Procedure:-

Step-1:- Finding of ε-closure

ε-closure (P) = P, where P ∈ Q

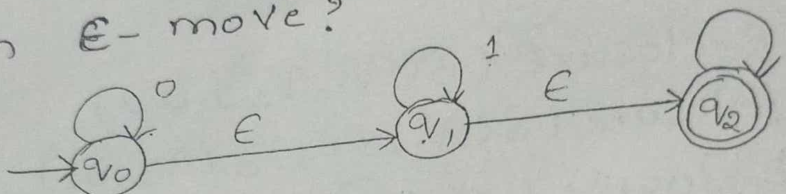
If There exists ε- closure (P) = {q} and

δ(q, ε) = r Then ε-closure (P) = {q, r}

Step-2:- Finding of δ'.
if ϵ-closure $(p) = \{P_1, P_2, \text{------} P_n\}$ Then
add $[P_1, P_2, \text{------} P_n]$ state to DFA.
$(\delta'[P_1, P_2, \text{------} P_n], a)$ is equal to
ϵ-closure $(\delta(P_1, a) \cup \delta(P_2, a) \cup \delta(P_3, a) \text{------} \cup \delta(P_n,$

Step-3:- construct its equivalent transitic table.

* construct The equivalent DFA from NFA with ϵ- move?



Step-1:- Finding ϵ- closure.

ϵ-closure $(q_0) = \{q_0, q_1, q_2\} = A$
ϵ-closure $(q_1) = \{q_1, q_2\} = B$.
ϵ-closure $(q_2) = \{q_2\} = C$.

	0	1	ϵ
q_0	$\{q_0\}$		$\{q_1, q_2\}$
q_1	ϕ	q_1	$\{q_2$
q_2	ϕ	ϕ	$\{$

Step-2:- Find δ'

$\delta'(A, 0) = \epsilon\text{-closure}(\delta([q_0, q_1, q_2], 0))$
$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$
$= \epsilon\text{-closure}(q_0 \cup \phi \cup \phi)$
$= \epsilon\text{-closure}(q_0)$
$= \{q_0, q_1, q_2\}$.

$\delta'(A, 1) = \epsilon\text{-closure}(\delta([q_0, q_1, q_2], 1))$
$= \epsilon\text{-closure}(\delta(q_0, 1), \cup \delta(q_1, 1) \cup \delta(q_2, 1))$
$= \epsilon\text{-closure}(\phi \cup q_1 \cup \phi)$
$= \epsilon\text{-closure}(q_1)$
$= \{q_1, q_2\}$

$\delta'(A, 2) = \epsilon\text{-closure}(\delta([q_0, q_1, q_2], 2))$
$= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$

$\delta'(B,0) \quad \epsilon\text{-closure}(\delta[q_1,q_2],0))$

$= \epsilon\text{-closure}(\delta(q_1,0) \cup \delta(q_2,0))$

$= \epsilon\text{-closure}(\phi) \times$

$= \epsilon\text{-closure}(\phi \cup \phi\, q_2)$

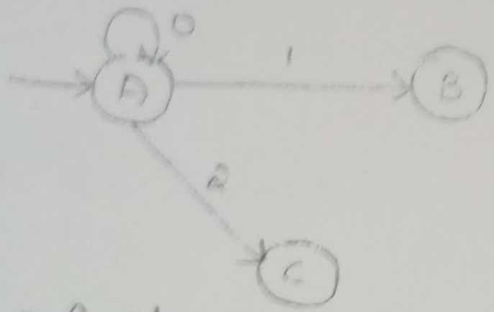
$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}$

$\delta'(A,0) = A$

$\delta'(A,1) = B$

$\delta'(A,2) = C$



$\delta'(B,0) = \epsilon\text{-closure}(\delta(q_1,q_2],0))$

$= \epsilon\text{-closure}(\delta(q_1,0) \cup \delta(q_2,0))$

$= \epsilon\text{-closure}(\phi \cup \phi)$

$= \cancel{\phi}\ \epsilon\text{-closure}(\phi).$

$= \phi.$

$\delta'(B,1) = \epsilon\text{-closure}(\delta(q_1,q_2],1))$

$= \epsilon\text{-closure}(\delta(q_1,1) \cup \delta(q_2,1))$

$= \epsilon\text{-closure}(q_1 \cup \phi)$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$ ✓

$\delta'(B,2) = \epsilon\text{-closure}(\delta[q_1,q_2],2))$

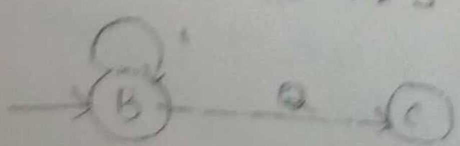
$= \epsilon\text{-closure}(\delta(q_1,2) \cup \delta(q_2,2))$

$= \epsilon\text{-closure}(\phi \cup q_2)$

$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}.$

*



$\delta'(B,0) = \phi$

$\delta'(B,1) = B$

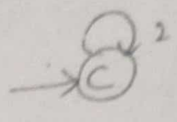
$\delta'(B,2) = C.$

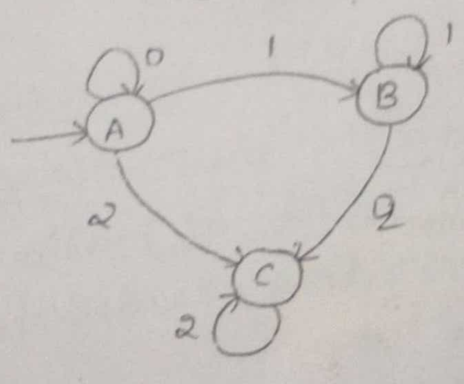
$\delta'(C,0) = \epsilon\text{-closure}(\delta[q_2],0))$
$= \epsilon\text{-closure}(\delta(q_2,0))$
$= \epsilon\text{-closure}(\phi)$
$= \phi.$

$\delta'(C,1) = \epsilon\text{-closure}(\delta[q_2],1))$
$= \epsilon\text{-closure}(\delta(q_2,1))$
$= \epsilon\text{-closure}(\phi)$
$= \phi$

$\delta'(C,2) = \epsilon\text{-closure}(\delta[q_2],2))$
$= \epsilon\text{-closure}(\delta(q_2,2))$
$= \epsilon\text{-closure}(q_2)$
$= \{q_2\}$

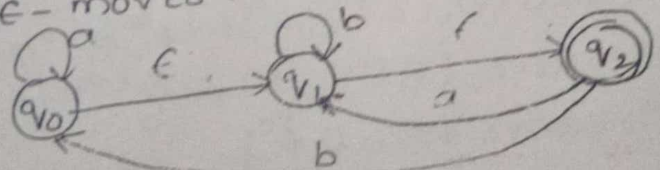
$\delta'(C,0) = \phi$
$\delta'(C,1) = \phi$
$\delta'(C,2) = C.$





	0	1	2
→A	A	B	C
B	ϕ	B	C
(C)	ϕ	ϕ	C

* Convert the Following NFA from DFA with ϵ-moves.



Step-1:- Finding ϵ-closure

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\} = A$

$\epsilon\text{-closure}(q_1) = \{q_1, q_2\} = B$

$\epsilon\text{-closure}(q_2) = \{q_2\} = C.$

Step-2:- Find δ'

$\delta'(A, a) = \epsilon\text{-closure}(\delta[q_0, q_1, q_2], a)$

$= \epsilon\text{-closure}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$

$= \epsilon\text{-closure}(q_0 \cup \phi \cup q_1)$

$= \epsilon\text{-closure}\{q_0, q_1\}$

$= \epsilon\text{-closure}[q_0] \cup \epsilon\text{-closure}[q_1]$

$= \epsilon\text{-closure}\ \{q_0, q_1, q_2\} \cup \{q_1, q_2\}$

$= \{q_0, q_1, q_2\} = A$

$\delta'(A, b) = \epsilon\text{-closure}(\delta[q_0, q_1, q_2], b)$

$= \epsilon\text{-closure}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b))$

$= \epsilon\text{-closure}(\phi \cup q_1 \cup q_0)$

$= \epsilon\text{-closure}(q_0, q_1)$

$= \epsilon\text{-closure}(q_0) \cup \epsilon\text{-closure}(q_1)$

$\{q_0, q_1, q_2\} \cup \{q_1, q_2\}$

$= \{q_0, q_1, q_2\} = A.$

$\delta'(A, a) = A$

$\delta'(A, b) = A.$

$\delta'(B, a) = \epsilon\text{-closure}(\delta[q_1, q_2], a))$

$= \epsilon\text{-closure}(\delta(q_1, a) \cup \delta(q_2, a))$

$= \epsilon\text{-closure}(\phi \cup q_1)$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\} = B$ ✓

$\delta'(B, b) = \epsilon\text{-closure}(\delta[q_1, q_2], b))$

$= \epsilon\text{-closure}(\delta(q_1, b), \cup \delta(q_2, b))$

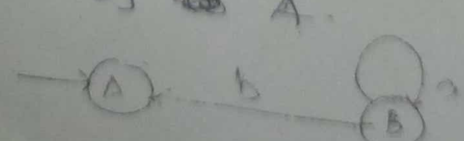
$= \epsilon\text{-closure}(q_1 \cup \cancel{\phi}\ q_0)$

$= \epsilon\text{-closure}(q_1) \cup \epsilon\text{-closure}(q_0)$

$= \{q_0, q_1, q_2\}$ A.

$\delta'(B, a) = B$

$\delta'(B, b) = A$

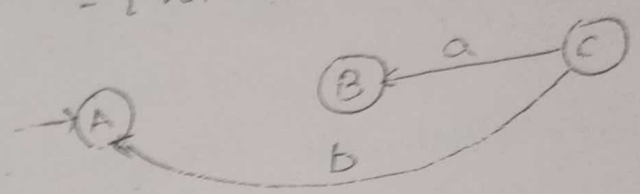


20/11

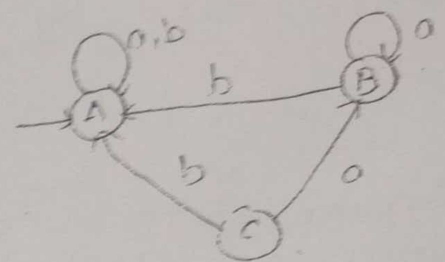
$\delta'(c,a) = \epsilon\text{-closure}(\delta[q_2],a))$
$= \epsilon\text{-closure}(\delta(q_2,a))$
$= \epsilon\text{-closure}(q_1)$
$= \{q_1, q_2\} = B$

$\delta'(c,b) = \epsilon\text{-closure}(\delta[q_2],b))$
$= \epsilon\text{-closure}(\delta[q_2,b))$
$= \epsilon\text{-closure}(q_0)$
$= \{q_0, q_1, q_2\} = A.$

$\delta'(c,a) = B$
$\delta'(c,b) = A$



Step-3:-



Minimisation of DFA:-

20/12/17 Equivalence:- Two states q_1 and q_2 are k-equivalent if both $\delta(q_1,a)$ and $\delta(q_2,$ are final states (or) non-final states.

Procedure:-

Step-1:- we will create a set Π_0 as $\Pi_0 = \{Q_1^0, Q_2^0\}$ where Q_1^0 is a set of final states. and
$Q_2^0 = Q - Q_1^0$, a set of non-final states
where Q is set of states.

Step-2:- construction of Π_K from Π_{K+1}.

* let Q_i^K be any subset in Π_K.
* if q_1 and q_2 are in Q_i^K. They are $(K+1)$ Equivalent provided $\delta(q_1,a)$ and $\delta(q_2,a)$

are residing in same Equivalence class π_K.

* Then it is said That q_1 and q_2 are $(K+1)$-Equivalent.

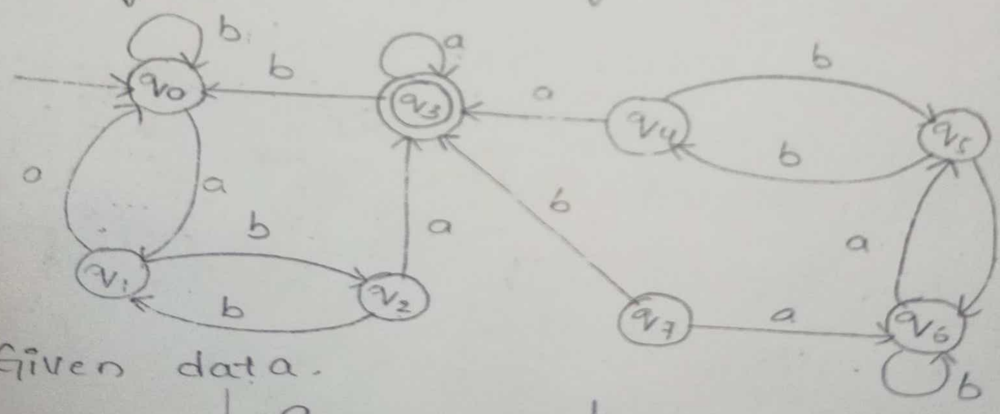
* Thus Q_i^K is further divided into $(K+1)$-Equivalence classes.

* Repeat step-2 for every Q_i^K in π_K and obtain all The elements of π_{K+1}.

Step-3:- construct π_n for $n = 1, 2, \text{----}$ until $\pi_n = \pi_{n+1}$.

Step-4:- Replace all The Equivalent states in one Equivalence class by representative state.

Problem:-
construct The minimum state Automata for The following transition diagram?



Sol:- Given data.

	a	b
$\rightarrow q_0$	q_1	q_0
q_1	q_0	q_2
q_2	q_3	q_1
q_3	q_3	q_0
q_4	q_3	q_5
q_5	q_6	q_4

q_6	q_5	q_6
q_7	q_6	q_3

step-1 :- construction of Π_0

$$\Pi_0 = \{ Q_1^0, Q_2^0 \}$$

$$\Pi_0 = \{ \{q_3\}, \{q_0, q_1, q_2, q_4, q_5, q_6, q_7\} \}$$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

Q_1^0 — final states $\{q_3\}$

Q_2^0 — Non-final states $\{q_0, q_1, q_2, q_4, q_5, q_6, q_7\}$

step-2 :- construction of Π_1 from Π_0.

$$\Pi_0 = \{ Q_1^0, Q_2^0 \}.$$

where $Q_1^0 = \{q_3\}$, $Q_2^0 = \{q_0, q_1, q_2, q_4, q_5, q_6, q_7\}$ so, it There is only one state in Q_1^0. is not further divided. find the Equivalent states consider Q_2^0,

	a	b
q_0	q_1	q_0
q_1	q_0	q_2

	a	b
q_0	q_1	q_0
q_2	q_3	q_1

	a	b
q_0	q_1	q_0
q_4	q_3	q_5

	a	b
q_0	q_1	q_0
q_5	q_6	q_4

	a	b
q_0	q_1	q_0
q_6	q_5	q_6

	a	b
q_0	q_1	q_0
q_7	q_6	q_3

	a	b
q_1	q_0	q_2
q_2	q_3	q_1

	a	b
q_1	q_0	q_2
q_4	q_3	q_5

The Equivalent states of q_0 are q_1, q_5, q_6.

	a	b
q_1	q_0	q_2
q_5	q_6	q_4

	a	b
q_1	q_0	q_2
q_6	q_5	q_6

	a	b
q_1	q_0	q_2
q_7	q_6	q_3

The Equivalent states of q_1 are q_5, q_6.

	a	b
q_2	q_3	q_1
q_4	q_3	q_5

	a	b
q_2	q_3	q_1
q_5	q_6	q_4

	a	b
q_2	q_3	q_1
q_6	q_5	q_6

	a	b
q_2	q_3	q_1
q_7	q_6	q_3

The Equivalent state for q_2 is q_4

There no Equivalent state for

	a	b
q_4	q_3	q_5
q_5	q_6	q_4

	a	b
q_4	q_3	q_5
q_6	q_5	q_6

	a	b
q_4	q_3	q_5
q_7	q_6	q_3

	a	b
q_5	q_6	q_4
q_6	q_5	q_6

	a	b
q_5	q_6	q_4
q_7	q_6	q_3

	a	b
q_6	q_5	q_6
q_7	q_6	q_3

There are no Equivalent states for q_4

The Equivalent state for q_5 is q_6

There are no Equivalent state for q_6.

* The one - Equivalent are :-

$Q_1' = \{q_3\}$

$Q_2' = \{q_0, q_1, q_5, q_6\}$

$Q_3' = \{q_2, q_4, q_7\}$

2/17

* Construction of π_2 from π_1.

consider The subsets of π_1 $Q_1' = \{q_3\}$, $Q_2' = \{q_0, q_1, q_5, q_6\}$, $Q_3' = \{q_2, q_4, q_7\}$

* Q_1' has only one state which cannot be further divided.

	a	b
q_0	q_1	$q_0 \in Q_2'$
q_1	$q_0 \in Q_2'$	$q_2 \in Q_3'$

	a	b
q_0	q_1	q_0
q_5	q_6	q_4

	a	b
q_0	q_1	q_0
q_6	q_5	q_6

There is only state Equivalent to q_0 is q_6.

	a	b
q_1	q_0	q_2
q_5	q_6	q_4

	a	b
q_1	q_0	q_2
q_6	q_5	q_6

	a	b
q_5	q_6	q_4
q_6	q_5	q_6

~~(There is only)~~

* Q_2' is partioned into $Q_2' = \{q_0, q_1, q_5, q_6\}$.

Q_2' - Equivalence :-

* There is only one state equivalent to q_1 is q_5 (or) q_1 & q_5 are Equivalent.

* There are no Equivalent states for q_5.

* The two - equivalent ~~are~~

$Q_2^1 = \{q_0, q_1, q_5, q_6\}$

Q_2^2 $\{q_0, q_6\}$ $\quad$ Q_3^2 $\{q_1, q_5\}$.

* consider $Q_3^1 = \{q_2, q_4, q_7\}$

q_2 - equivalence :-

	a	b
q_2	q_3	q_1
q_4	q_3	q_5

	a	b
q_2	q_3	q_1
q_7	q_6	q_3

q_2 and q_4 are equivalent.

Q_3^1

Q_4^2 $\{q_2, q_4\}$ $\quad$ Q_5^2 $\{q_7\}$.

q_4 - equivalence :-

	a	b
q_4	q_3	q_5
q_7	q_6 x	q_3 x

There are no equivalent states.

* $\pi_2 = \{Q_1^2, Q_2^2, Q_3^2, Q_4^2, Q_5^2\}$

$\boxed{\pi_2 = \{\{q_3\}, \{q_0, q_6\}, \{q_1, q_5\}\ \{q_2, q_4\}, \{q_7\}\}}$

* Construction of π_3 from π_2. $\quad Q_1^2 = \{q_3\}$

consider $Q_2^2 = \{q_0, q_6\}$

	a	b
q_0	q_1	q_0
q_6	q_5	q_6

q_0 and q_6 are equivalent.

consider $Q_3^2 = \{q_1, q_5\}$

	a	b
q_1	q_0	q_2
q_5	q_6	q_4

q_1 and q_5 are equivalent.

consider $Q_4^2 = \{q_2, q_4\}$

	a	b
q_2	q_3	q_1
q_4	q_3	q_5

q_2 and q_4 are equivalent.

$Q_5^2 = \{q_7\}$

$\Pi_3 = \{\{q_3\}, \{q_0, q_6\}, \{q_1, q_5\}, \{q_2, q_4\}, \{q_7\}\}$

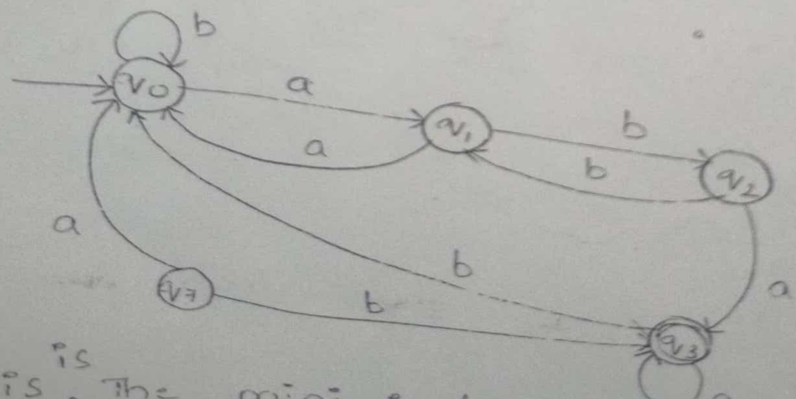
$\therefore \Pi_3 = \Pi_2$

Stop the construction.

Step-4:- Replace all the equivalent states.

		a	b
q_0q_6	$\rightarrow q_0$	q_1	q_0
q_1q_5	q_1	q_0	q_2
q_2q_4	q_2	q_3	q_1
q_3	q_3	q_3	q_0
q_7	q_7	q_0	q_3

The transition diagram is.



This is the minimised Finite Automata.

Equivalence between two finite Automatas:- The two finite Automata are said to be equivalent if both The Automata accept The same set of strings over Input set Σ.

Procedure:-

* Let M and M' be two finite Automatas and Σ is set of Input strings.

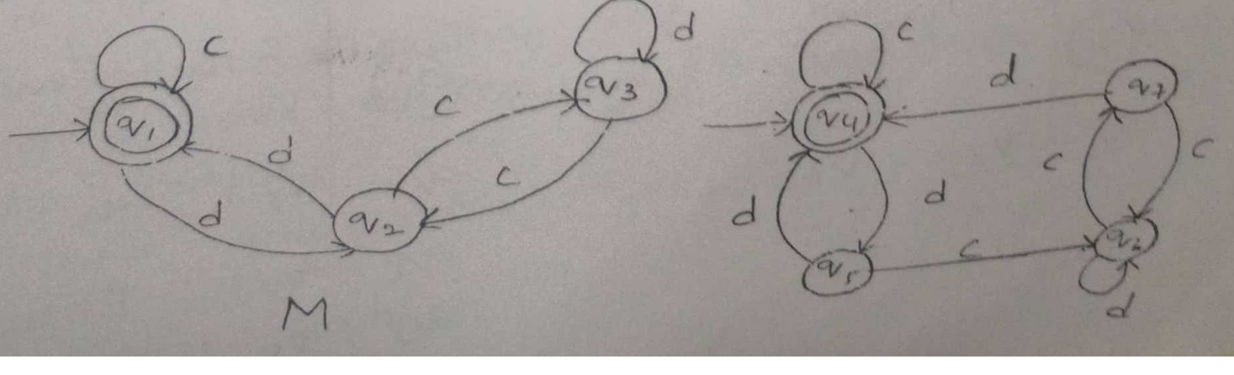
Step-1:- construct a transition table have Pair wise entries (q, q') where $q \in M$ and $q' \in M'$ for each input symbol.

Step-2:- If we get a pair as one final state and other as a non-final state, Then we terminate The construction of transition table and we declare That two finite Automatas are not equivalent.

Step-3:- The construction of Transition table gets terminated when There is no new pair appearing in The transition table.

Problem:-

* consider The Two DFA's and find The equivalence between Them?



Given data:

M

	c	d
→ q_1	q_1	q_2
q_2	q_3	q_1
q_3	q_2	q_3

M'

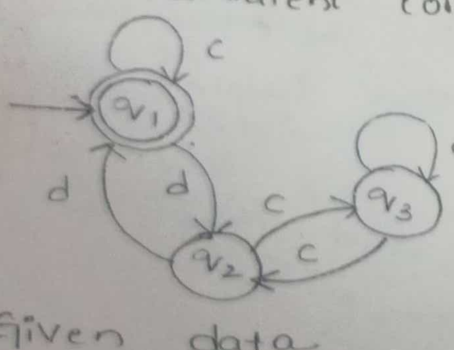
	c	d
→ q_4	q_4	q_5
q_5	q_6	q_4
q_6	q_7	q_6
q_7	q_6	q_4

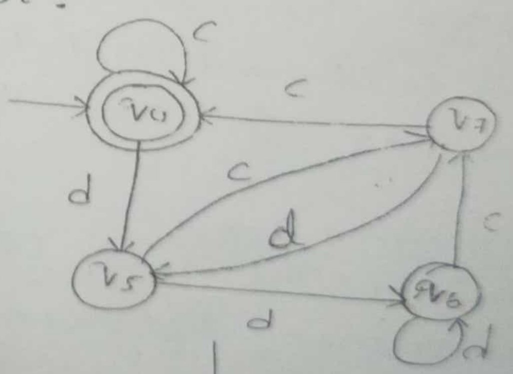
Similar.

	c	d
(q_1, q_4)	(q_1, q_4) $\in F$ $\in F$	(q_2, q_5) $\notin F$ $\notin F$
(q_2, q_5)	(q_3, q_6) $\notin F$ $\notin F$	(q_1, q_4) $\in F$ $\in F$
(q_3, q_6)	(q_2, q_7) $\notin F$ $\notin F$	(q_3, q_6) $\notin F$ $\notin F$
(q_2, q_7)	(q_3, q_6) $\notin F$ $\notin F$	(q_1, q_4) $\in F$ $\in F$

* No more new pairs, Hence the two Automata's are Equivalent.

* check whether the following Automata's are Equivalent (or) not?





Sol:- Given data

	c	d
→ q_1	q_1	q_2
q_2	q_3	q_1
q_3	q_2	q_3

	c	d
→ q_4	q_4	q_5
q_5	q_7	q_6
q_6	q_7	q_6
q_7	q_4	q_5

	c	d
(q_1, q_4)	(q_1, q_4) $\in F$ $\in F$	(q_2, q_5) $\notin F$ $\notin F$
(q_2, q_5)	(q_3, q_7) $\notin F$ $\notin F$	(q_1, q_6) $\in F$ $\notin F$
(q_3, q_7)	(q_2, q_4)	(q_3, q_5)
(q_1, q_6)		

∴ The two finite Automata's are not equivalent.

Finite Automata with output :-

* Here we will discuss two different models of FA's with output capabilities.
* These were created by G.H. Mealy in ~~195~~ 1955 and E.F. Moore in 1956.
* Generally, The Finite Automata is a collection of $(Q, \Sigma, \delta, q_0, F)$. In FA after reading The input string, if we get final state Then The string is said to be acceptable.
* If we do not get final state. Then The string is said to be Rejected.
* The Accept (or) Reject acts as yes (or) no, But if There is a need for specifying The output other Than yes or no. Then In such a case we require finite-Automata with output.
* There are two types of FA with output.
  * Moore Machine
  * Mealy Machine.

Moore Machine:- It is the FA in which the **output** is associated with **state**.

* Moore Machine is a 6-tuple.

$(Q, \Sigma, \Delta, \delta, \lambda, q_0)$.

where,

Q - is the set of states.

Σ - finite set of Input symbols

Δ - is an output alphabet.

δ - is an transition function $\boxed{\delta : Q \times \Sigma \rightarrow Q}$

λ - is an output function

$$\boxed{\lambda : Q \longrightarrow \Delta}$$

q_0 - is initial state.

Representation of Moore Machine:-

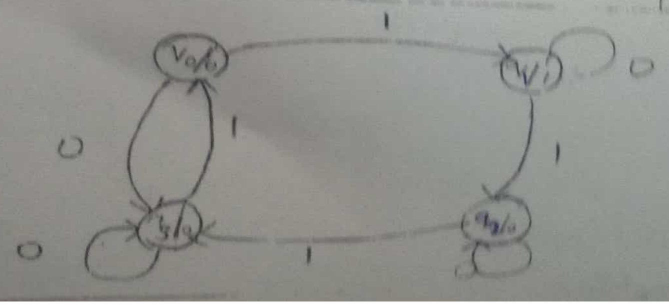
Moore Machine can be represented by

* Transition table and
* Transition diagram.

Example:-

Present State	Next State $a=0$	Next State $a=1$	output
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

*



Mealy Machine:-

A Mealy Machine is a 6-tup

$M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$.

where,

Q - a finite set of states.

Σ - a finite set of input symbols.

Δ - a finite set of outputs.

q_0 - initial state.

δ - Transition function.

$$\boxed{\delta : Q \times \Sigma \rightarrow Q}$$

λ - is an output function.

$$\boxed{\lambda : Q \times \Sigma \rightarrow \Delta}$$

* In Mealy Machine The output depends on both state and input.

Mealy Machine Representation:-

A Mealy Machine can be Represer by

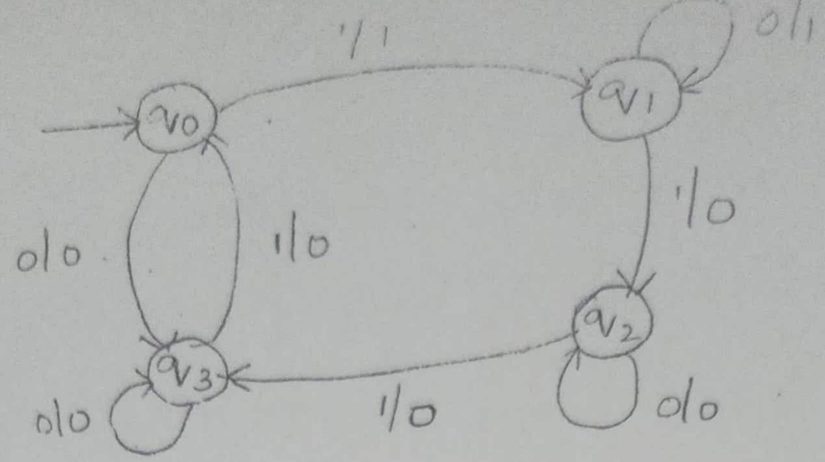
* Transition table and
* Transition diagram.

Example:-

Present state	a=0		a=1	
	Ns	o/p	Ns	o/p
$\rightarrow q_0$	q_3	0	q_1	1
q_1	q_1	1	q_2	0
q_2	q_2	0	q_3	0
q_3	q_3	0	q_0	0

Ns - Next state

*



Conversion:-

Moore to Mealy:-

* Let $M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ be a Moore Machine. The Equivalent Mealy Machine can be represented by $M' = (Q, \Sigma, \Delta, \delta, \lambda', q_0)$ where $\boxed{\lambda'(q, a) = \lambda(\delta(q, a))}$

Example:-

Construct the Mealy Machine from the following Moore Machine? (Residue Modulo-3)?

Present State	a=0	a=1	o/p.
$\rightarrow q_0$	q_0	q_1	0
q_1	q_2	q_0	1
q_2	q_1	q_2	2

28/2/F

* The output function of Mealy Machine λ' can be obtained using the following rule $\boxed{\lambda'(q, a) = \lambda(\delta(q, a))}$

1) $\lambda'(q_0, 0) = \lambda(\delta(q_0, 0))$
$= \lambda(q_0)$
$= 0.$

2) $\lambda'(q_0, 1) = \lambda(\delta(q_0, 1))$
$= \lambda(q_1) \Rightarrow 1.$

3) $\lambda'(q_1, 0) = \lambda(\delta(q_1, 0))$
$= \lambda(q_2)$
$= 2$

4) $\lambda'(q_1, 1) = \lambda(\delta(q_1, 1))$
$= \lambda(q_0)$
$= 0$

5) $\lambda'(q_2, 0) = \lambda(\delta(q_2, 0))$
$= \lambda(q_1)$
$= 1$

6) $\lambda'(q_2, 1) = \lambda(\delta(q_2, 1))$
$= \lambda(q_2)$
$= 2.$

* The transition table for Mealy Machine is.

Present state	a = 0 NS	o/p	a = 1 NS	o/p
$\rightarrow q_0$	q_0	0	q_1	1
q_1	q_2	2	q_0	0
q_2	q_1	1	q_2	2

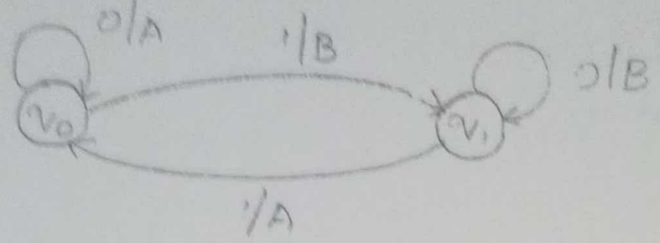
28|04|17 * **Mealy to Moore** :-

* By converting The Mealy Machine to Moore Machine we will create a separate path for every new output symbol. Let $M = (Q, \Sigma, \Delta, \delta, \lambda, q_0)$ be a Mealy Machine and There exists $M' = (Q \times \Delta, \Sigma, \Delta, \delta', \lambda', [q_0, b_0])$ where b_0 is an output selected from Δ.

we will calculate δ', λ' as follows :-

* $\boxed{\delta'([q, b], a) = (\delta(q, a), \lambda(q, a))}$

Example:
convert the following Mealy Machine into Equivalent Moore Machine?



1:- Given data:-

present state	a = 0		a = 1	
	Ns	o/p	Ns	o/p
q_0	q_0	A	q_ϕ	B
q_1	q_1	B	q_0	A

12 $\delta'([q, b], a) = (\delta(q, a), \lambda(q, a))$

where
b = is output
a - is input.

i) $\delta'([q_0, A], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$

$= [q_0, A]$

$\delta(q_0, 0) = q_0$
$\lambda(q_0, 0) = A.$

$(\delta'([q_1, A], 0) = [\delta(q_1, 0), \lambda) \times$

$\lambda'(q_0, A) = A.$

ii) $\delta'([q_1, A], 0) = [\delta(q_1, 0), \lambda(q_1, 0)]$

$= [q_1, B]$

$\lambda'(q_1, A) = A.$

iii) $\delta'([q_1, A], 1) = [\delta(q_1, 1), \lambda(q_1, 1)]$

$= [q_0, A)$

$\lambda'(q_1, A) = A.$

iv) $\delta'([q_1, B], 0) = [\delta(q_1, 0), \lambda(q_1, 0)$

$[q_1, B].$

$\lambda'(q_1, B) = B$

v) $\delta'([q_1, B], 1) = [\delta(q_1, 1), \lambda(q_1, 1)]$
$= [q_0, A]$.
$\lambda'(q_1, B) = B$.

vi) $\delta'([q_0, A], 1) = [\delta(q_0, 1), \lambda(q_0, 1)]$
$= [q_1, B]$
$\lambda'(q_0, A) = A$.

vii) $\delta'([q_0, B], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$
$= [q_0, A)$
$\lambda'(q_0, B) = B$.

viii) $\delta'([q_0, B], 1) = [\delta(q_0, 1) ; \lambda(q_0, 1)]$
$= [q_1, B]$
$\lambda'(q_0, B) = B$.

	0	1	output
$[q_0, A]$	$[q_0, A]$	$[q_1, B]$	A
$[q_0, B]$	$[q_0, A]$	$[q_1, B]$	B
$[q_1, A]$	$[q_1, B]$	$[q_0, A]$	A
$[q_1, B]$	$[q_1, B]$	$[q_0, A]$	B

Second procedure:-

Step-1:- Identify the states which are associated with different outputs.

Eg:-

P.S	N.S	o/p	N.S	o/p
$\rightarrow q_0$	q_1	[illegible]	q_2	0
q_1	q_2	0	q_1	0
q_2	q_0	0	q_2	0

* Here q_1 associated with ~~output~~ different outputs 0 and 1.

Step-2:- Partition The different output states into Number of partitions based on Number of different outputs.

Eg:- q_1 is associated with output 0 and 1 so partition The q_1 into q_{10} and q_{11}.

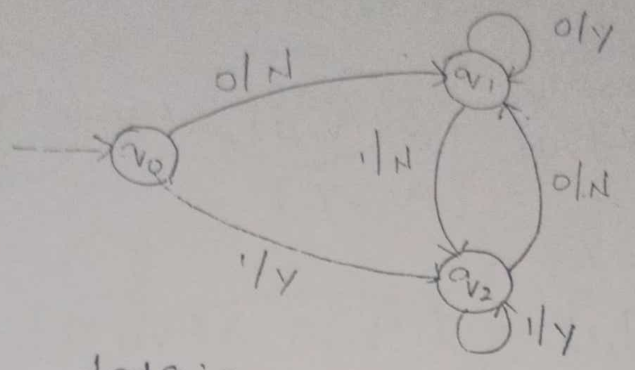
Step-3:- Reconstruct The table with partitioned states and Substitute The partioned states in place of original.

$\rightarrow q_0$	q_{11}	1	q_2	0
q_{10}	q_2	0	q_{10}	0
q_{11}	q_2	0	q_{10}	0
q_2	q_0	0	q_2	0

Step-4:- Again verify The states which are associated with different outputs. If No such states are exist, construct its Equivalent Moore Machine.

	0	1	Output
$\rightarrow q_0$	q_{11}	q_2	0
q_{10}	q_2	q_{10}	0
q_{11}	q_2	q_{10}	1
q_2	q_0	q_2	0

* convert The following Mealy Machine in Equivalent Moore Machine? ①



Sol:- Given data:-

Present state	a = 0		a = 1	
	Ns	o/p	Ns	o/p
$\rightarrow q_0$	q_1	N	q_2	Y
q_1	q_1	Y	q_2	N
q_2	q_1	N	q_2	Y

$\delta'([q, b], a) = [\delta(q, a), \lambda(q, a)]$

where b is output, a is input.

i) $\delta'([q_0, N], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$
$= [q_1, N]$

$\lambda'(q_0, N] = N.$

ii) $\delta'([q_0, N], 1) = [\delta(q_0, 1), \lambda(q_0, 1)]$
$= [q_2, Y].$

$\lambda'(q_0, N] = N.$

iii) $\delta'([q_0, Y], 0) = [\delta(q_0, 0), \lambda(q_0, 0)]$
$= [q_1, N]$

$\lambda'(q_0, Y) = Y.$

iv) $\delta'([q_0, Y], 1) = [\delta(q_0, 1), \lambda(q_0, 1)]$
$= [q_2, Y]$

$\lambda'(q_0, Y) = Y.$

v) $\delta'([q_1, N], 0) = [\delta(q_1, 0), \lambda(q_1, 0)]$
$= [q_1, Y]$

~~steps~~: $\lambda'[q_1, N] = N$.

vi) $\delta'([q_1, N], 1) = [\delta'(q_1, 1), \lambda(q_1, 1)]$
$= [q_2, N]$

$\lambda'(q_1, N) = N$

vii) $\delta'([q_1, Y], 0) = [\delta'(q_1, 0), \lambda(q_1, 0)]$
$= [q_1, Y]$

$\lambda'(q_1, Y) = Y$.

viii) $\delta'([q_1, Y], 1) = [\delta'(q_1, 1), \lambda(q_1, 1)]$
$= [q_2, N]$

$\lambda'(q_1, Y) = Y$.

ix) $\delta'([q_2, N], 0) = [\delta'(q_2, 0), \lambda(q_2, 0)]$
$= [q_1, N]$

$\lambda'(q_2, N) = N$

x) $\delta'([q_2, N], 1) = [\delta'(q_2, 1), \lambda(q_2, 1)]$
$= [q_2, Y]$

$\lambda'(q_2, N) = N$

xi) $\delta'([q_2, Y], 0) = [\delta'(q_2, 0), \lambda(q_2, 0)]$
$= [q_1, N]$

$\lambda'(q_2, Y) = Y$

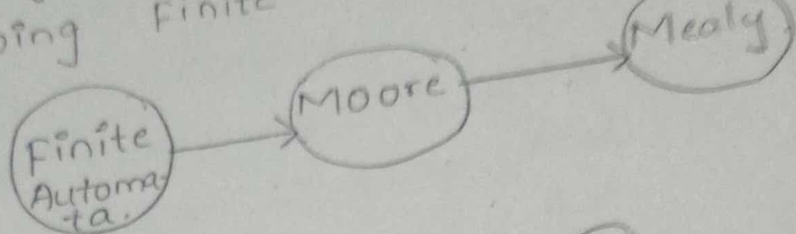
xii) $\delta'([q_2, Y], 1) = [\delta'(q_2, 1), \lambda(q_2, 1)]$
$= [q_2, Y]$

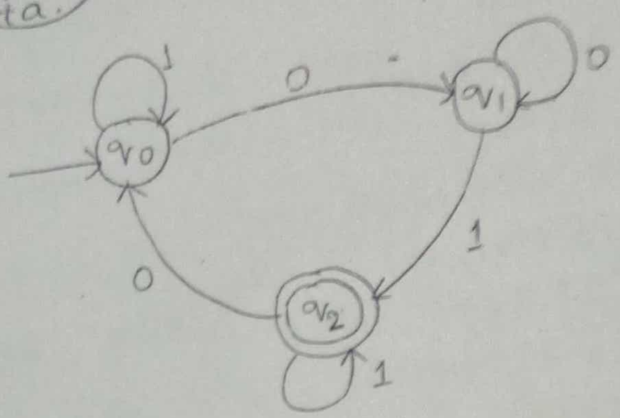
$\lambda'(q_2, Y) = Y$.

	0	1	output
$[q_0, N]$	$[q_1, N]$	$[q_2, Y]$	N
$[q_0, Y]$	$[q_1, N]$	$[q_2, Y]$	Y
$[q_1, N]$	$[q_1, Y]$	$[q_2, N]$	N
$[q_1, Y]$	$[q_1, Y]$	$[q_2, N]$	Y
$[q_2, N]$	$[q_1, N]$	$[q_2, Y]$	N
$[q_2, Y]$	$[q_1, N]$	$[q_2, Y]$	Y

* Design Mealy Machine from the following Finite Automata ?

Sol:





Given data:-

	0	1
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_2
q_2	q_0	q_2

Construction of Moore from Finite Automata:

$$\boxed{\begin{array}{l} \lambda(q) = 0 \ ; \ q \notin F \\ \lambda(q) = 1 \ ; \ q \in F. \end{array}}$$

* Now, The outputs are

$\lambda(q_0) = 0$

$\lambda(q_1) = 0$

$\lambda(q_2) = 1.$

* The Moore Machine is.

	0	1	Output
$\rightarrow q_0$	q_1	q_0	0
q_1	q_1	q_2	0
q_2	q_0	q_2	1

$$\lambda'(q,a) = \lambda(\delta(q,a))$$

1) $\lambda'(q_0, 0) = \lambda(\delta(q_0, 0))$
$= \lambda(q_1)$
$= 0$

2) $\lambda'(q_0, 1) = \lambda(\delta(q_0, 1))$
$= \lambda(q_0)$
$= 0.$

3) $\lambda'(q_1, 0) = \lambda(\delta(q_1, 0))$
$= \lambda(q_1)$
$= 0$

4) $\lambda'(q_1, 1) = \lambda(\delta(q_1, 1))$
$= \lambda(q_2)$
$= 1$

5) $\lambda'(q_2, 0) = \lambda(\delta(q_2, 0))$
$= \lambda(q_0)$
$= 0$

6) $\lambda'(q_2, 1) = \lambda(\delta(q_2, 1))$
$= \lambda(q_2)$
$= 1$

The transition table for Mealy Machine is.

Present state	a=0		a=1	
	NS	o/p	NS	o/p
$\rightarrow q_0$	q_1	0	q_0	0
q_1	q_1	0	q_2	1
q_2	q_0	0	q_2	1

Date
3/1/18

Unit - II
Regular ~~Gram~~ Languages

* Regular languages are accepted by finite Automata.
* A Regular language can be represented by
  * Regular sets and
  * Regular expressions.

Regular Set:- A Regular set consist of set of strings, all These strings are valid In The language.

Eg:- $R = \{ \epsilon, a, aa, aaa, aaaa, \text{-----} \}$

* This set Represents The any number of a
* $R = \{ \epsilon, aa, aaaa, aaaaaa, \text{-------} \}$
* This set Represents The evennumber of a's
* $R = \{ \epsilon, ab, aabb, aaabbb, \text{------} \}$.
* This set Represents no. of a's followed by same number of b's.
* $R = \{ a, aab, aaabb, aaaabbb, \text{-------} \}$
* This set Represents 'n' no. of a's followed by (n-1) no. of b's.
* $R = \{ \epsilon, ab, ba, abab, baba, abba, baab, \text{--}$
* This set Represents equal number of a's and b's.
* $R = \{ aa, aba, aaa, abba, abaa, aaba, aaaa \text{-------} \}$
* This set Represents begining with a and ending with a.
* $R = \{ a, baa, aaa, aab, bab, aabb, aaba, aaab \text{------} \}$