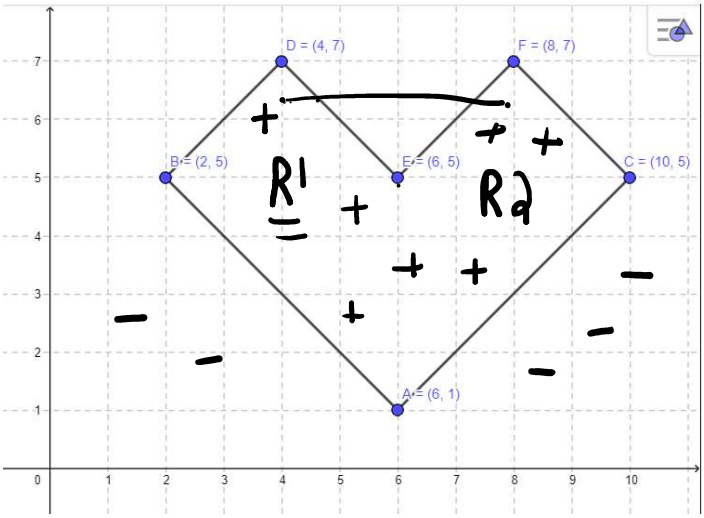
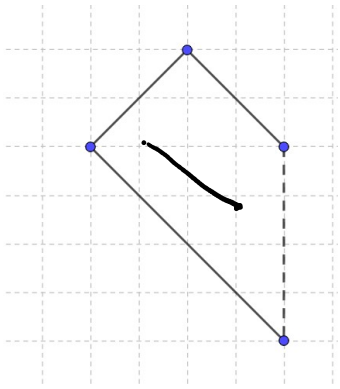
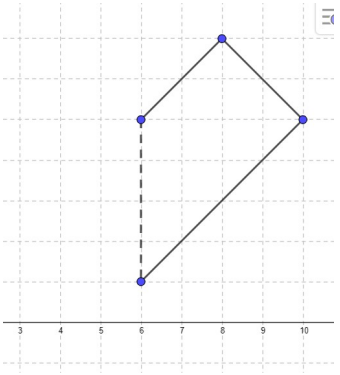
# Multi-Layer Neural network for a Convex Decision Boundary



Region1



region2

The equation of line through $A(6,1)$ & $B(2,5)$

$$\frac{x_2-5}{x_1-2} = \frac{1-5}{6-2} \Rightarrow 4(x_2-5) = -4(x_1-2)$$

$$\Rightarrow -x_1+2-x_2+5=0 \Rightarrow \boxed{x_1+x_2-7=0}$$

The equation of line through $B(2,5)$ & $D(4,7)$

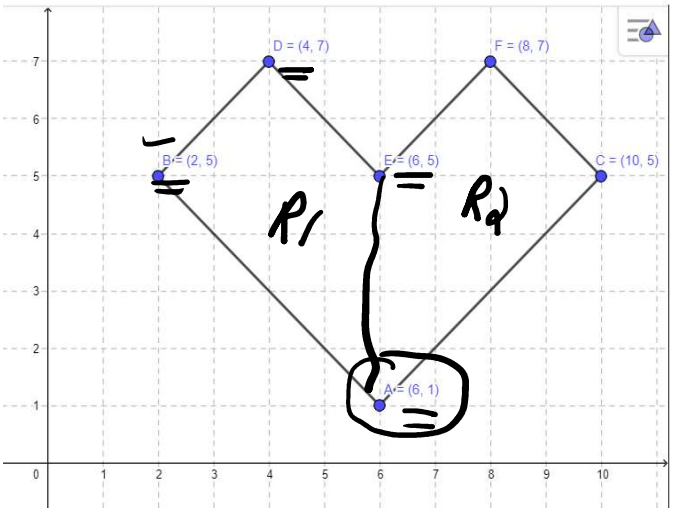
$$\frac{x_2-7}{x_1-4} = \frac{5-7}{2-4} \Rightarrow -2(x_2-7) = -2(x_1-4)$$

$$\Rightarrow x_1-4-x_2+7=0 \Rightarrow \boxed{x_1-x_2+3=0}$$

The equation of line through $D(4,7)$ & $E(6,5)$

$$\frac{x_2-5}{x_1-6} = \frac{7-5}{4-6} \Rightarrow -2(x_2-5) = 2(x_1-6)$$

$$\Rightarrow x_1-6+x_2-5=0 \Rightarrow \boxed{x_1+x_2-11=0}$$ ✓




The equation of line through $E(6,5)$ and $A(6,1)$ is $\boxed{x_1+0x_2-6=0}$

The equation of line through $E(6,5)$ & $F(8,7)$

$$\frac{x_2-7}{x_1-8} = \frac{5-7}{6-8} \Rightarrow -2(x_2-7) = -2(x_1-8)$$

$$\Rightarrow x_1 - x_2 - 8 + 7 = 0 \Rightarrow \boxed{x_1 - x_2 - 1 = 0}$$ ✓

The equation of line through $F(8,7)$ & $C(10,5)$

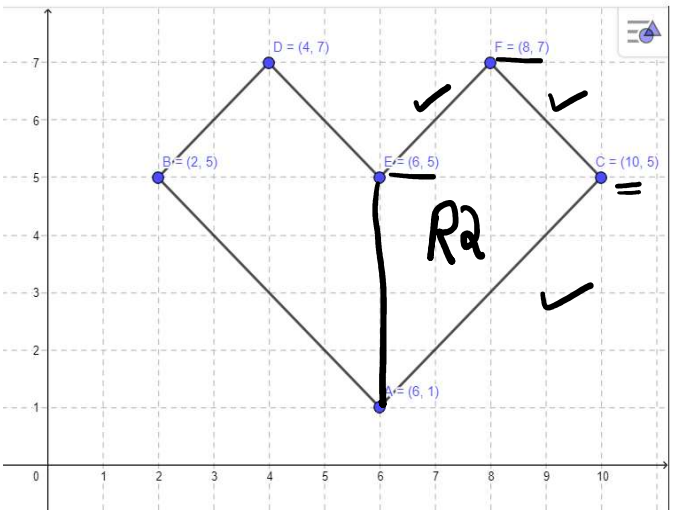
$$\frac{x_2-5}{x_1-10} = \frac{7-5}{8-10} \Rightarrow -2(x_2-5) = 2(x_1-10)$$

$$\Rightarrow x_1 - 10 + x_2 - 5 \Rightarrow \boxed{x_1 + x_2 - 15 = 0}$$ ✓

The equation of line through $C(10,5)$ & $A(6,1)$

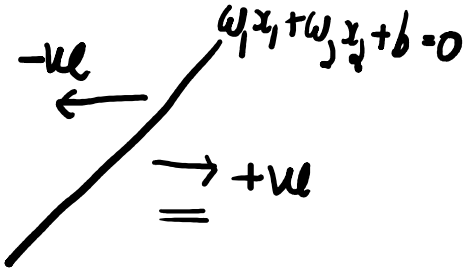
$$\frac{x_2-5}{x_1-10} = \frac{1-5}{6-10} \Rightarrow -4(x_2-5) = -4(x_1-10)$$

$$\Rightarrow x_1 - 10 - x_2 + 5 = 0 \Rightarrow \boxed{x_1 - x_2 - 5 = 0}$$ ✓




# Decision Boundary.

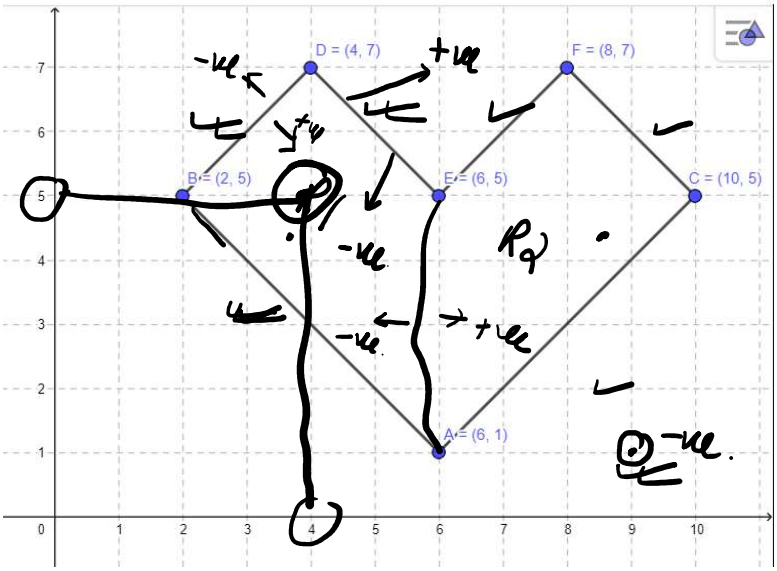
$$\begin{cases} +1, & w_1 x_1 + w_2 x_2 + b \geq 0 \\ -1, & w_1 x_1 + w_2 x_2 + b < 0 \end{cases}$$

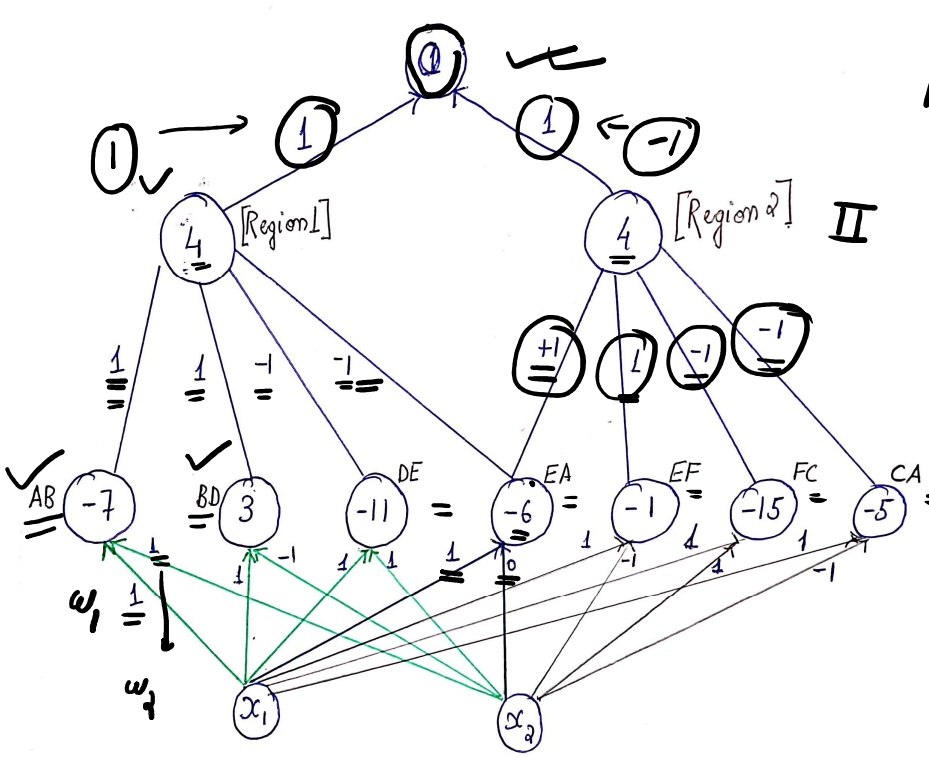



| Line | equation | $w_1$ | $w_2$ | $b$ | Decision Boundary $\begin{cases} +1 & \text{if } w_1x_1+w_2x_2+b \geq 0 \\ -1 & \text{else.} \end{cases}$ | |
|---|---|---|---|---|---|---|
| AB | $x_1+x_2-7=0$ | 1 | 1 | -7 | $\begin{cases} +1 & \text{if } x_1+x_2-7 \geq 0 \\ -1 & \text{else.} \end{cases}$ | |
| BD | $x_1-x_2+3=0$ | 1 | -1 | 3 | $\begin{cases} +1 & \text{if } x_1-x_2+3 \geq 0 \\ -1 & \text{else.} \end{cases}$ | R1 |
| DE | $x_1+x_2-11=0$ | 1 | 1 | -11 | $\begin{cases} +1 & \text{if } x_1+x_2-11 \geq 0 \\ -1 & \text{else.} \end{cases}$ | R1 |
| EA | $x_1+0x_2-6=0$ | 1 | 0 | -6 | $\begin{cases} +1 & \text{if } x_1-6 \geq 0 \\ -1 & \text{else.} \end{cases}$ | R2 |
| EF | $x_1-x_2-1=0$ | 1 | -1 | -1 | $\begin{cases} +1 & \text{if } x_1-x_2-1 \geq 0 \\ -1 & \text{else.} \end{cases}$ | R2 |
| FC | $x_1+x_2-15=0$ | 1 | 1 | -15 | $\begin{cases} +1 & \text{if } x_1+x_2-15 \geq 0 \\ -1 & \text{else.} \end{cases}$ | R2 |
| CA | $x_1-x_2-5=0$ | 1 | -1 | -5 | $\begin{cases} +1 & \text{if } x_1-x_2-5 \geq 0 \\ -1 & \text{else.} \end{cases}$ | |

$$w_1x_1 + w_2x_2 + b = 0$$

| Line | outcome in region 1 | outcome in region 2 |
|---|---|---|
| AB | 1 | 0 |
| BD | 1 | 0 |
| DE | -1 | 0 |
| EA | -1 | +1 |
| EF | 0 | +1 |
| FC | 0 | -1 |
| CA | 0 | -1 |




$1(1) + 1(-1) = 0$

$1(-1) + 1(1) = 0$

$1(1) + 1(1) + (-1)(-1) + (-1)(-1) = 4$

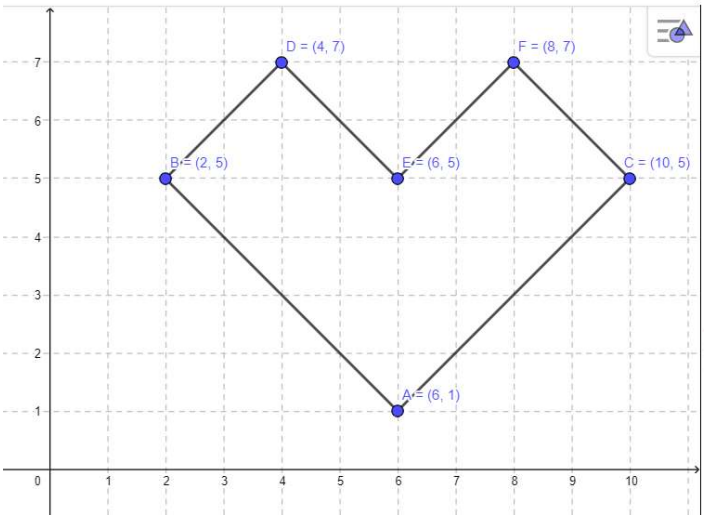
II

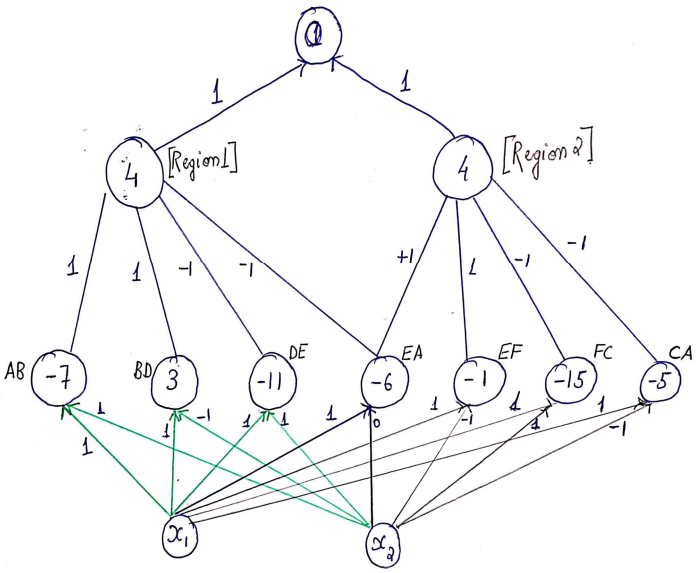
I

| Line | equation | $w_1$ | $w_2$ | b. | Decision Boundary $\begin{cases}+1 & \text{if } w_1x_1+w_2x_2+b \geq 0 \\ -1 & \text{else.}\end{cases}$ |
|---|---|---|---|---|---|
| AB | $x_1+x_2-7=0$ | 1 | 1 | -7 | $\begin{cases}+1 & \text{if } x_1+x_2-7\geq 0 \\ -1 & \text{else.}\end{cases}$ |
| BD | $x_1-x_2+3=0$ | 1 | -1 | 3. | $\begin{cases}+1 & \text{if } x_1-x_2+3\geq 0 \\ -1 & \text{else.}\end{cases}$ |
| DE | $x_1+x_2-11=0$ | 1 | 1 | -11 | $\begin{cases}+1 & \text{if } x_1+x_2-11\geq 0 \\ -1 & \text{else.}\end{cases}$ |
| EA | $x_1+0x_2-6=0$ | 1 | 0 | -6 | $\begin{cases}+1 & \text{if } x_1-6\geq 0 \\ -1 & \text{else.}\end{cases}$ |
| EF | $x_1-x_2-1=0$ | 1 | -1 | -1 | $\begin{cases}+1 & \text{if } x_1-x_2-1\geq 0 \\ -1 & \text{else.}\end{cases}$ |
| FC | $x_1+x_2-15=0$ | 1 | 1 | -15 | $\begin{cases}+1 & \text{if } x_1+x_2-15\geq 0 \\ -1 & \text{else.}\end{cases}$ |
| CA | $x_1-x_2-5=0$ | 1 | -1 | -5 | $\begin{cases}+1 & \text{if } x_1-x_2-5\geq 0 \\ -1 & \text{else.}\end{cases}$ |

| Value | Layer 1 | Layer 2 | Final result |
|---|---|---|---|
| (4, 5) | AB = 1<br>BD = 1<br>DE = -1<br>EA = -1 | Region 1 value<br>= 1(1) + 1(1) + (-1)(-1) + (-1)(-1) = 4<br>Decision = 1 [∵ 4 ≥ 4] | 1(1) + (-1)(1) = 0 ≥ 0<br>result = 1 |
| | EA = -1<br>EF = -1<br>FC = -1<br>CA = -1 | Region 2 value<br>= -1(1) + 1(-1) + (-1)(-1) + (-1)(-1) = 0<br>Decision = -1 [∵ 0 < 4] | |

| Value | Layer 1 | Layer 2. | Final result |
|---|---|---|---|
| (9, 1) | AB = 1<br>BD = 1<br>DE = -1<br>EA = 1 | Region 1 value<br>= 1(1) + 1(1) - 1(-1) - 1(1) = 2 < 4<br>Decision = -1 | -1(1) - 1(1) = -2 < 0<br>Decision = -1 |
| | EA = 1<br>EF = 1<br>FC = -1<br>CA = 1 | Region 2 value<br>1(1) + 1(1) - 1(-1) + (-1)(1) = 2 < 4<br>Decision = -1 | |


D = (4, 7)
F = (8, 7)
B = (2, 5)
E = (6, 5)
C = (10, 5)
A = (6, 1)


0
1
1
4
[Region1]
4
[Region2]
AB
-7
BD
3
DE
-11
EA
-6
EF
-1
FC
-15
CA
-5
$x_1$
$x_2$