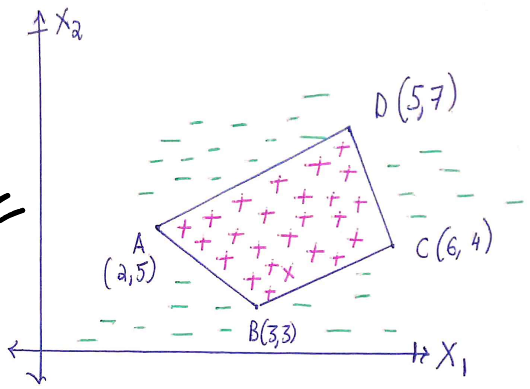
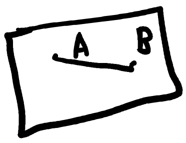
Multi-Layer Neural network for a Convex Decision Boundary





AB BC CD AD

M(x_1, y_1) N(x_2, y_2) M —— N

$$\frac{y - y_2}{x - x_2} = \frac{y_1 - y_2}{x_1 - x_2}$$

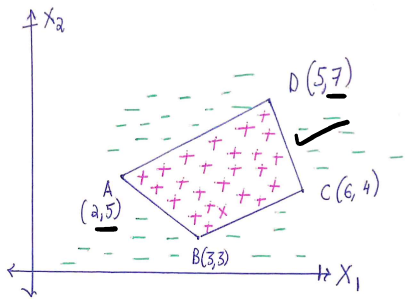
We have AD whose equation is given by.

$$\frac{x_2 - 5}{x_1 - 2} = \frac{7-5}{5-2}$$

$$\Rightarrow 3(x_2 - 5) = 2(x_1 - 2)$$

$$\Rightarrow 2x_1 - 3x_2 - 4 + 15 = 0$$

$$\Rightarrow \boxed{2x_1 - 3x_2 + 11 = 0}$$



We have DC whose equation is given by.

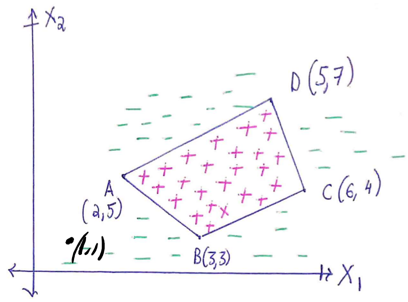
$$\frac{x_2 - 7}{x_1 - 5} = \frac{4-7}{6-5}$$

$$\Rightarrow 1(x_2 - 7) = -3(x_1 - 5)$$

$$\Rightarrow -3x_1 + 15 - x_2 + 7 = 0$$

$$\Rightarrow -3x_1 - x_2 + 22 = 0$$

$$\Rightarrow \boxed{3x_1 + x_2 - 22 = 0}$$



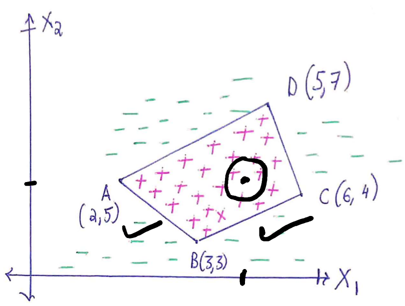
We have equation of CB as below.

$$\frac{x_2 - 4}{x_1 - 6} = \frac{3-4}{3-6}$$

$$\Rightarrow -3(x_2 - 4) = -1(x_1 - 6)$$

$$\Rightarrow -x_1 + 6 + 3x_2 - 12 = 0$$

$$\Rightarrow \boxed{x_1 - 3x_2 + 6 = 0}$$



The equation of AB is given as

$$x_2 - 3 = \frac{5-3}{}$$

The equation of AB is given as

$$\frac{x_2-3}{x_1-3} = \frac{5-3}{2-3}$$

$\Rightarrow -1(x_2-3) = 2(x_1-3)$

$\Rightarrow 2x_1 - 6 + x_2 - 3 = 0$

$\Rightarrow \boxed{2x_1 + x_2 - 9 = 0}$

$$\left.\begin{array}{ll} AD \rightarrow & 2x_1 + 3x_2 + 11 = 0 \\ DC \rightarrow & 3x_1 + x_2 - 22 = 0 \\ CB \rightarrow & x_1 - 3x_2 + 6 = 0 \\ AB \rightarrow & 2x_1 + x_2 - 9 = 0 \end{array}\right\}$$

$\underline{w_1x_1 + w_2x_2 + b = 0}$

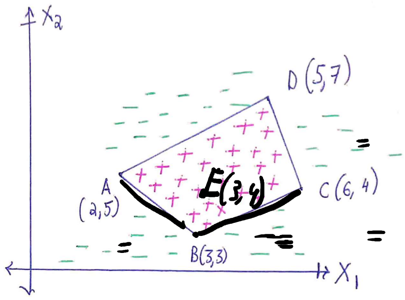
$\underline{CB}$ $w_1 = 1$, $w_2 = -3$, $b = 6$

DECISION RULE →

$$\begin{cases} +1 & \text{if } w_1x_1 + w_2x_2 + b \geq 0 \\ -1 & \text{else } (w_1x_1 + w_2x_2 + b < 0) \end{cases}$$

$w_1x_1 + w_2x_2 + b = \underline{\underline{0}}$

-w. ← → +w.



Line	Equation	w_1	w_2	b	Decision Rule $\begin{cases}+1 & \text{if } w_1x_1+w_2x_2+b\geq 0\\ -1 & \text{else}\end{cases}$	outcome in Region
✓ AD	$2x_1-3x_2+11=0$	2	-3	11	+1 if $2x_1-3x_2+11 \geq 0$; -1 else	+1 ✓
X DC	$3x_1+x_2-22=0$	3	1	-22	+1 if $3x_1+x_2-22 \geq 0$; -1 else	-1
X CB	$x_1-3x_2+6=0$	1	-3	6	+1 if $x_1-3x_2+6 \geq 0$; -1 if $x_1-3x_2+6<0$; $w_1x_1+w_2x_2+b \geq 0$	-1
✓ AB	$2x_1+x_2-9=0$	2	1	-9	+1 if $2x_1+x_2-9 \geq 0$; -1 if $2x_1+x_2-9<0$	+1

✓ ✓

$(3, 4)$, $3 \rightarrow x_1$, $4 \rightarrow x_2$

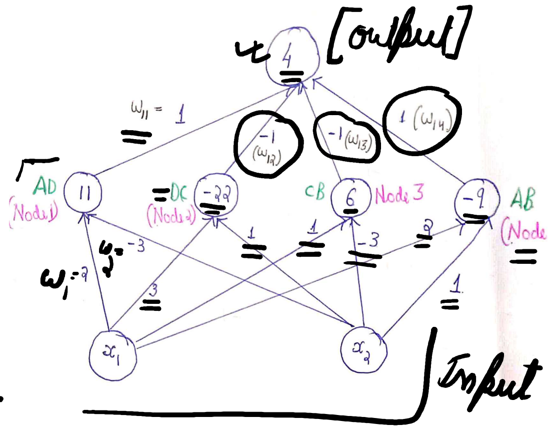
$2(3)-3(4)+11 = 5 \geq 0$

$3(3)+4-22<0$

$3-3(4)+6 = -3<0$

$2(3)+4-9 = 1 \geq 0$

Line	Equation	w_1	w_2	b	Decision Rule $\begin{cases}+1 & \text{if } w_1x_1+w_2x_2+b\geq 0\\ -1 & \text{else}\end{cases}$	outcome in Region
AD	$2x_1-3x_2+11=0$	2	-3	11	$+1$ if $2x_1-3x_2+11\geq 0$; -1 else	+1 ✓
DC	$3x_1+x_2-22=0$	3	1	-22	$+1$ if $3x_1+x_2-22\geq 0$; -1 else	-1 ✗
CB	$x_1-3x_2+6=0$	1	-3	6	$+1$ if $x_1-3x_2+6\geq 0$; -1 if $x_1-3x_2+6<0$	-1 ✗
AB	$2x_1+x_2-9=0$	2	1	-9	$+1$ if $2x_1+x_2-9\geq 0$; -1 if $2x_1+x_2-9<0$	+1 ✓



$$+1(+1)-1(-1)-1(-1)+1(+1)=4$$

Let's consider a point (4, 5)

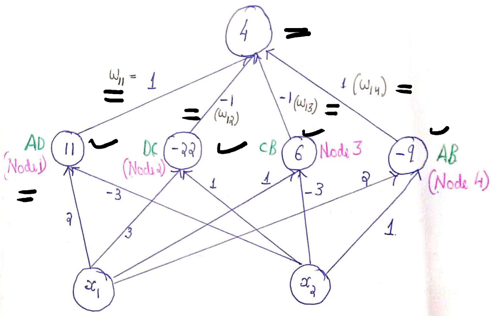
$$2(4)-3(5)+11=4$$
$$3(4)+1(5)-22=-5$$
$$1(4)-3(5)+6=-5$$
$$2(4)+5(1)-9=4$$

outcome at Node 1 = 1 = z_1

Outcome at Node 2 = -1 = z_2

Outcome at Node 3 = -1 = z_3

outcome at Node 4 = 1 = z_4



Final decision rule = $\begin{cases}+1 & \text{if } w_{11}z_1+w_{12}z_2+w_{13}z_3+w_{14}z_4-4\geq 0\\ -1 & \text{else}\end{cases}$

Now $w_{11}z_1+w_{12}z_2+w_{13}z_3+w_{14}z_4 = 1(1)+(-1)(-1)+(-1)(-1)+1(1)$

$= 4$

The final result is +1.

Let us consider (1, 1)

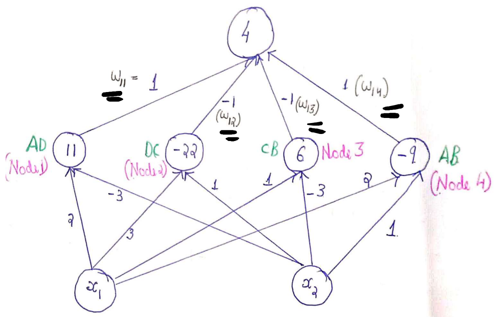
$$2(1)-3(1)+11=10\geq 0$$
$$-3(1)+1(1)-22<0$$
$$1(1)-3(1)+6>0$$
$$2(1)+1(1)-9<0$$

outcome at Node 1 (z_1) = +1

Node 2 (z_2) = -1

Node 3 (z_3) = 1

Node 4 (z_4) = -1



Now $w_{11}z_1+w_{12}z_2+w_{13}z_3+w_{14}z_4-4$

$= 1(1)+(-1)(-1)+1(-1)+1(-1)-4$

$= -4 < 0$

As $-4<0 \Rightarrow$ final result is -1 $\quad$ -ve (1, 1)