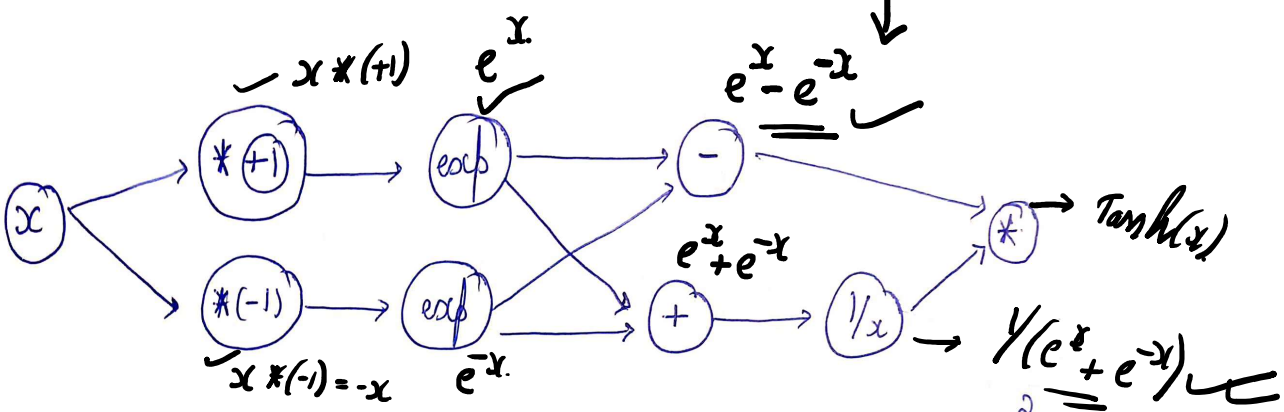
Computational Graphs (Basics)

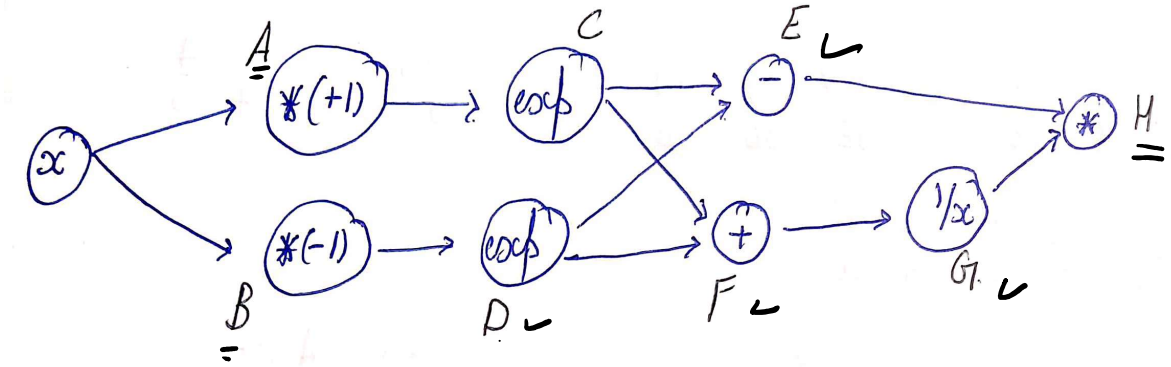
$$f(x) = \text{Tanh}(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$x,\ -x,\ e^x,\ e^{-x}$

$e^x - e^{-x},\quad e^x + e^{-x}$



$$f(x) = \text{Tanh}(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \Rightarrow f'(x) = 1 - [\text{Tanh}(x)]^2$$



$A = x$ ✓

$B = -x$ ✓

$C = e^A$ ✓

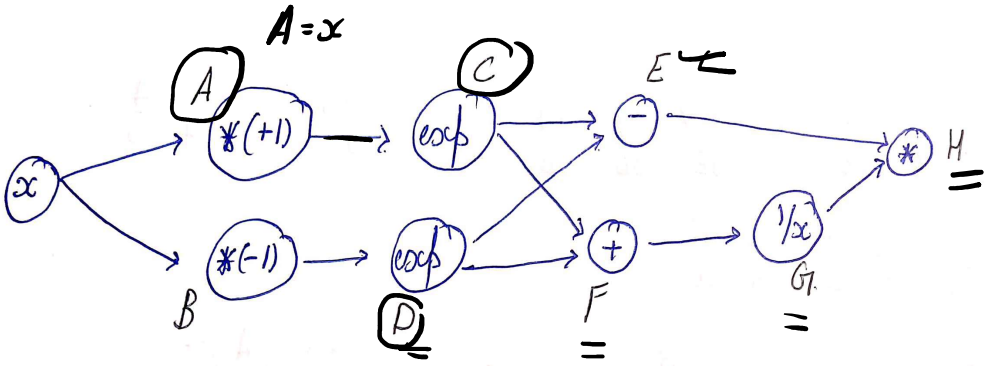
$D = e^B$ ✓

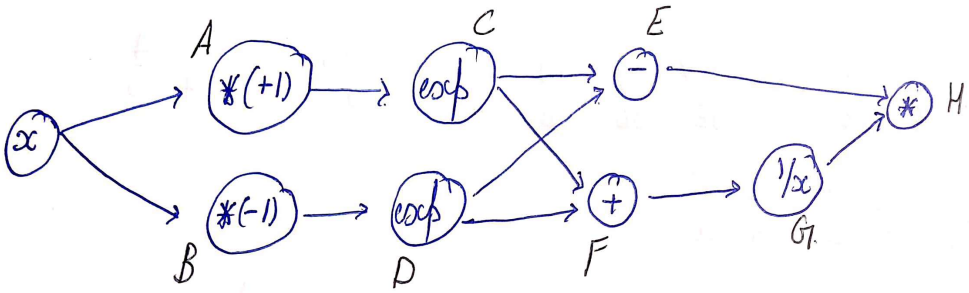
$E = C - D$ ✓

$F = C + D$ ✓

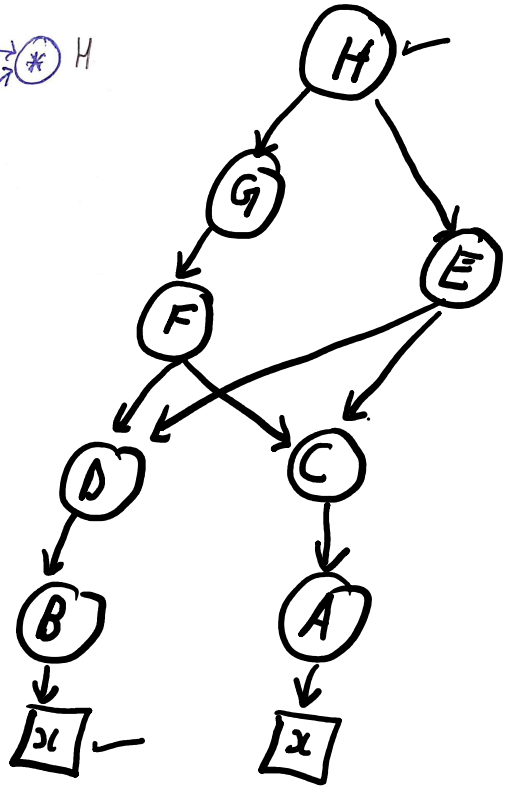
$G_1 = \frac{1}{F}$ ✓

$H = G_1 * E$ ✓





$f'(x) = \partial H / \partial x$



H G F D B x ✓

H G F C A x ✓ $\frac{\partial H}{\partial G_1}\frac{\partial G_1}{\partial F}\frac{\partial F}{\partial C}\frac{\partial C}{\partial A}\frac{\partial A}{\partial x}$

H E D B x ✓ $\frac{\partial H}{\partial E}\frac{\partial E}{\partial D}\frac{\partial D}{\partial B}\frac{\partial B}{\partial x}$

H E C A x ✓

$\frac{\partial H}{\partial E}\frac{\partial E}{\partial C}\frac{\partial C}{\partial A}\frac{\partial A}{\partial x}$

$$\frac{\partial H}{\partial x} = \underbrace{\frac{\partial H}{\partial E}\frac{\partial E}{\partial C}\frac{\partial C}{\partial A}\frac{\partial A}{\partial x}}_{HEC A} + \underbrace{\frac{\partial H}{\partial E}\frac{\partial E}{\partial D}\frac{\partial D}{\partial B}\frac{\partial B}{\partial x}}_{HEDB} + \underbrace{\frac{\partial H}{\partial G}\frac{\partial G}{\partial F}\frac{\partial F}{\partial C}\frac{\partial C}{\partial A}\frac{\partial A}{\partial x}}_{HGFCA} + \underbrace{\frac{\partial H}{\partial G}\frac{\partial G}{\partial F}\frac{\partial F}{\partial D}\frac{\partial D}{\partial B}\frac{\partial B}{\partial x}}_{HGFDB}$$

$$= G_1(1)\, e^A(1) + G_1(-1)\, e^B(-1) + E\left(\frac{-1}{F^2}\right)(1)\, e^A(1) + E(-1)(1)\, e^B(-1)$$

$A = x$

$B = -x$

$C = e^A$

B

$$= G_1(1)\,e^A(1) + G_1(-1)\,e^B(-1) + E\left(\frac{-1}{F^2}\right)(1)\,e^A(1) + E\left(\frac{-1}{F^2}\right)(1)\,e^B(-1)$$

$$\boxed{\frac{\partial H}{\partial x} = G e^A + G e^B - \frac{E}{F^2} e^A + \frac{E}{F^2} e^B}$$

$$= \left(\frac{1}{F} - \frac{E}{F^2}\right) e^A + \left(\frac{1}{F} + \frac{E}{F^2}\right) e^B \qquad [\because G_1 = 1/F]$$

$$= \left(\frac{F-E}{F^2}\right) e^A + \left(\frac{F+E}{F^2}\right) e^B \qquad \left[\begin{matrix} F = C+D \\ E = C-D \end{matrix}\right]$$

$$= \frac{2De^A}{F^2} + \frac{2Ce^B}{F^2} \qquad [\because C = e^A,\ D = e^B]$$

$$= \frac{2e^B e^A + 2e^A e^B}{F^2} = \frac{4e^{A+B}}{F^2} = \frac{4}{F^2} = \frac{4}{(e^x + e^{-x})^2}$$

$C = e^A$

$D = e^B$

$E = C - D$

$F = C + D$

$G_1 = \frac{1}{F}$

$H = G_1 * E$

$$\boxed{\frac{\partial H}{\partial x} = \frac{4}{(e^x + e^{-x})^2}}$$

$\frac{\partial}{\partial x}\,\text{Tanh}(x)$

Now $1 - [\tanh(x)]^2 = 1 - \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^2$

$$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2}$$

$$= \frac{e^{2x} + e^{-2x} + 2e^x e^{-x} - e^{2x} - e^{-2x} + 2e^x e^{-x}}{(e^x + e^{-x})^2}$$

$$= \frac{4}{(e^x + e^{-x})^2} = \frac{\partial H}{\partial x}$$

$$\Rightarrow \boxed{\frac{\partial H}{\partial x} = f'(x)}$$