We have 
$$X = (x_1, x_3, x_3)$$
 as  $x_4, x_5, x_6, \dots, x_m$  an infinite data of singe  $IX = (x_1, x_3, x_3)$  as  $x_4, x_5, x_6, \dots, x_m$  as  $x_5 = x_5$  and  $x_5$ 

Similarly, we can calculate  $3_d$ ,  $3_3$ ,  $3_4$ ,  $3_m - 2_m - 2_m$ With  $3_m = k_1^2 x_m + 3k_1 k_2 x_{m+1} + (k_2^2 + 3k_1 k_3) x_{m+2} + 3k_2 k_3 x_{m+3} + k_3^2 x_{m+4}$ 

X->Y->2

(1) JUL LA JK.K. AK.K. K.A)

## Question

Consider a  $1 \times 3$  convolution (as per Deep Learning terminology) kernel  $\frac{1}{6}$  =[0, 0, 0] = [-12-1]. Output is generated by stacking 4 such operators on the 1-D input data (i.e., the operator is applied four times on the input). What is the width of the equivalent operator O that will give the same output, when applied only once on the input data?

IX3

ble have an input data X as below

$$X = (x_1, x_2, \dots, x_m)$$
We have kernel  $K = (-1, 2, -1)$ . We have to apply it 4 times.

L's try to wouldy unduration of the spirations.

The 4 convolution A will look like below-

$$\frac{V_{1}}{=} = \frac{k_{1} u_{1} + k_{2} u_{3} + k_{3} u_{3}}{k_{1} u_{2} + k_{3} u_{3}} \qquad (2)$$
Here  $u_{1} = \frac{k_{1} u_{3} + k_{2} u_{3} + k_{3} u_{3}}{k_{2} u_{3} + k_{3} u_{3} + k_{3} u_{3}} \qquad (2)$ 

$$u_{3} = k_{1} 3_{3} + k_{3} 3_{3} + k_{3} 3_{4} \times J$$

$$u_{3} = k_{1} 3_{3} + k_{4} 3_{4} + k_{3} 3_{5}$$

$$U_{3} = k_{1} 3_{3} + k_{4} 3_{4} + k_{3} 3_{5}$$

$$3_{1} = k_{1} y_{1} + k_{3} y_{3} + k_{3} y_{3} + k_{3} y_{4}$$

$$3_{2} = k_{1} y_{2} + k_{3} y_{3} + k_{3} y_{4} + k_{3} y_{5}$$

How, 
$$y_1 = k_1 x_1 + k_2 x_2 + k_3 x_3$$
  
 $y_3 = k_1 x_2 + k_3 x_3 + k_4 x_4$   
 $y_3 = k_1 x_3 + k_3 x_4 + k_4 x_5$ 

34 = K, Y4 + K2 45 + K3 46

35 = K, ys + K, y6 + K, y7 /

$$33 = k, y_3 + k_3 y_3 + k_3 y_4$$
 $33 = k, y_4 + k_5 y_5 + k_3 y_4$ 
 $33 = k, y_4 + k_5 y_5 + k_5 y_6$ 

(in midd value  $y_1, y_3, \dots y_7$ )

$$\chi = (x_1, x_2, x_3, \dots)$$

For calculating  $V_1$ , we will need  $(x_1, x_2, \dots x_q)$ . Henre we will need an operator of will g that will be applied on X to give V u,  $X \xrightarrow{k'} V$  (where K' is IX9)

Video3 Page 4

- A. Consider a 1x3 convolution (as per Deep Learning terminology) operator o =[01, 02, 03]=[-12 -1]. Output is generated by stacking two such operators on the 1-D input data (i.e., the operator is applied twice on the input).
  - a) What is the size of the equivalent operator O that will give the same output, when applied only once on the input data?

Height of 
$$O = 1$$
 Width of  $O = 2*3-1 = 5$ 

b) Show the elements O(i,j) of equivalent operator O. O can be obtained by cross-correlation of o with o.

$$0 = [1 - 4 6 - 4 1] = V$$

$$O(1, j) = \sum_{k=1}^{j} o_k o_{4-k}$$

$$\begin{array}{c} X \xrightarrow{k} Y \xrightarrow{k} Z \\ X \xrightarrow{k'} Z \end{array}$$

We have imput data as  $\frac{X}{k} = (x_1, x_2, \dots, x_m)$ 

we apply the found Kon X twee to get  $X \xrightarrow{K} Y \xrightarrow{K} Z$ 

for 31 € Z we have 3, = K, y, + k, y, + k, y, + k, y, = [ We need y, y, y, y, for]

New 
$$y_1 = k_1 x_1 + k_2 x_3 + k_3 x_3$$

$$y_2 = k_1 x_2 + k_3 x_3 + k_3 x_4$$

$$y_3 = k_1 x_3 + k_3 x_4 + k_3 x_5$$

$$y_4 = k_1 x_3 + k_3 x_4 + k_3 x_5$$

$$y_5 = k_1 x_3 + k_3 x_4 + k_3 x_5$$

$$y_7 = k_1 x_3 + k_3 x_4 + k_3 x_5$$

$$y_8 = k_1 x_8 + k_3 x_4 + k_3 x_5$$

$$y_9 = k_1 x_9 + k_3 x_4 + k_3 x_5$$

$$y_9 = k_1 x_9 + k_3 x_9 + k_3 x_9 + k_3 x_9$$

$$y_9 = k_1 x_9 + k_3 x_9 +$$

Honce we must have I weter K' of order 1X5 as a kernel to

reach Z from X.

How 
$$k_1 y_1 + k_3 y_3 + k_3 y_3$$
  
=  $k_1 \left( k_1 x_1 + k_3 x_3 + k_3 x_3 \right) + k_2 \left[ k_1 x_4 + k_4 x_3 + k_3 x_4 \right] + k_3 \left[ k_1 x_3 + k_3 x_4 + k_3 x_5 \right]$ 

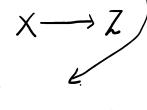
$$= \kappa_{1}^{3}x_{1} + 2\kappa_{1}\kappa_{2}^{3}x_{3} + (3\kappa_{1}\kappa_{3} + \kappa_{3}^{3})x_{3} + 2\kappa_{3}\kappa_{3}^{3}x_{4} + \kappa_{3}^{3}x_{5}$$

$$+ \kappa_{3}^{3}x_{5}$$

$$= \left(\begin{array}{ccccc} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & \alpha_5 \end{array}\right) \left(\begin{array}{cccccc} k_1^2 & & & \\ \gamma k_1 & k_2 & & \\ k_2^2 + \gamma k_1 & k_3 & & \\ \gamma k_2 & k_3 & & \\ k_3^2 & k_2^3 & & \end{array}\right)$$

$$K = \begin{bmatrix} -1, 2, -1 \end{bmatrix}$$

$$\times \longrightarrow Z$$



$$U_{2} \text{ get } K' = /(-1)^{\frac{3}{4}}.$$

$$2(-1)(2)$$

$$2^{\frac{3}{4}} + 2(-1)(-1)$$

$$2(2)(-1)$$

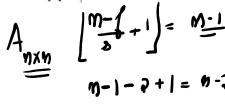
$$(-1)^{\frac{3}{4}}.$$

## Question

$$A \xrightarrow{k} B \xrightarrow{k} (m-1) \times (m-1) \times (m-1) \times (m-1)$$

Consider a 2 X 2 Convolution (as per Deep Learning Terminology) operator O

-1	-1	_ k
1	1	= 5 N



Output is generated by stacking 2 such operators on the 2-D input data (i.e. the operator is applied twice on the input)

What is the size of the equivalent Operator "V" when applied only once on input data?

Show the elements of equivalent operator V

$$A_{3x3} \rightarrow B_{3x3} \rightarrow C_{1x1}$$

We have 
$$K = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix}$$
 as a kernel of  $AXA$  mige.

Lit us consider a 3x3 imput matrix A as below.

$$A = \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_q \end{pmatrix}$$

the apply the knowl k on A time to get  $A \xrightarrow{K} B \xrightarrow{K} C$ 

We have B of size 
$$\left\lfloor \frac{m-l}{2} + 1 \right\rfloor \times \left\lfloor \frac{m-l}{2} + 1 \right\rfloor = \left\lfloor \frac{3-3}{l} + 1 \right\rfloor \times \left\lfloor \frac{3-3}{l} + 1 \right\rfloor$$

$$= 2 \times 3$$

we get 
$$B = \begin{pmatrix} \beta & q \\ g_1 & g_2 \end{pmatrix}$$
 on applying  $K$  on  $B$  we get
$$C = \langle G \rangle \quad \text{where} \quad C = \langle K, \beta + K, q + k_3 g_1 + k_4 g_2 \rangle$$
[re,  $C$  is a matrix of rige  $|X|$ ]

a, K, + 9, K, + 94 K, +95 K4

$$k = \begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} a_1 & a_1 & a_3 \\ a_4 & a_5 & a_4 \end{pmatrix}$$

$$Q = -a_3 - a_3 + a_5 + a_6 = a_3 k_1 + a_3 k_2 + a_5 k_5 + a_5 k_4 + a_5 k_5 + a_5 k$$

New 
$$L = -\int_{-q}^{-q} - q + y_1 + y_2$$

$$= -\left(-a_1 - a_3 + a_5 + a_4\right) - \left(-a_3 - a_3 + a_5 + a_6\right) + \frac{a_3 - a_4 + a_5 + a_6}{2}$$

$$= -(-a_1 - a_3 + a_5 + a_4) - (-a_3 - a_6 + a_8 + a_9)$$

$$(-a_4 - a_5 + a_7 + a_8) + (-a_5 - a_6 + a_8 + a_9)$$

lile con use a kvonel

$$\frac{K'}{2} = \begin{pmatrix} 1 & 3 & 1 \\ -3 & -4 & -3 \end{pmatrix}$$
 dually on A to get  $c$  attained about

This means we apply a 3x3 knowned on 3x3 matrix to get result c. The required 3x3 formed is K'.