Detection Method for Phase Synchronization in a Population of Spiking Neurons

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Abstract. Currently there are many methods to detect synchronization, each of them trying to extract some specific aspects or oriented to specific number or type of signals. In this paper, we present a new method to detect synchronization for multivariate signals, computationally light and not requiring a combinatorial number of operations on signals differences.The method is based on the Hilbert transform of the signals, which provides their instantaneous phases. The distribution of phases for all signals at a specific time is assimilated to a probability distribution. In this way, we obtain a sequence of probability distributions (one per time unit). Computing the entropy of the probability distributions we get finally a function of entropies along time. The average value of this final function provides a good estimate of the synchronization level of the multivariate signals ensemble, and the function itself can be used as a signature (descriptive function) of the whole multidimensional ensemble dynamics.

Keywords: synchronization detection, signal phase, multivariate signals.

1 Introduction

The study of synchronization is a main topic of current research in different fields: Computational neuroscience [1], Temporal synchronization of image sequence [10], Data mining in multivariate data, Information retrieval by content [9], Analysis of geologic and atmospheric data [4], Human movements, gaits and gesture recognition [5], Financial applications (trends, similarities,..) [8,2], Genetics (DNA,..) [3], etc ...

Synchronization is defined by Webster dictionary as: (1) happening, existing, or arising at precisely the same time, (2) recurring or operating at exactly the same periods.This apparently easy definition hides some troubles in its practical usage due to: (1) inherent signal noise, (2) imperfect signal coincidences, (3) alternatives in signal parameter used to establish signals similarity (instantaneous value, phase, frequency,), (4) establishing similarity between more than two signals, (5) similarity between two signals ensembles, (6) patterns recurrence (which pattern? spatio-temporal?), These difficulties imply more efforts dedicated to provide new and better methods for synchronization detection mechanisms.

J.M. Ferrández et al. (Eds.): IWINAC 2013, Part I, LNCS 7930, pp. 421–431, 2013.

These efforts are justified due to the importance of synchronization in the above mentioned fields. Particularly, in neuroscience, synchronization is been studied as one of the main elements that the brain uses in the integration of dissimilar information to form a coherent perception (binding problem) [6] and as an important mechanism for information coding and learning [7].

Considering all these aspects we propose in this paper a new method to detect synchronization for multivariate signals. The method is computationally low demanding as it is not based on a combinatorial number of differences between signals, but, in a time-dependent entropy function calculated on the signals probability phase distributions. Therefore, the number of main-step computations is made linear with the total number of time units of the signals: $O(T)$, where T is the total time length of the signals .

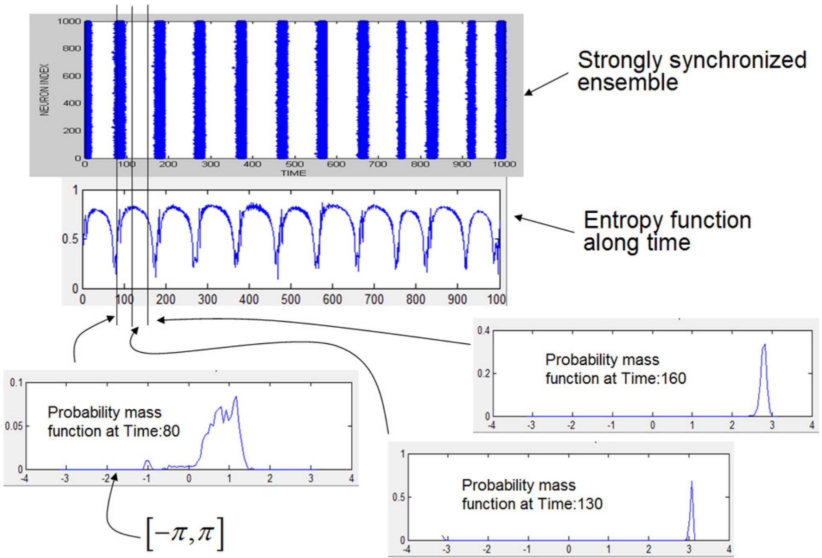

Fig. 1. Relationship between signal ensemble, entropy sequence function $\gamma_S(t)$ and probability mass functions p_t for different times

2 Multivariate Synchronization Detection Method

In this section is presented the detection method. First it is presented the Hilbert transform which allows getting the instantaneous signal phase, following with a detailed description of the proposed synchronization detection method.

2.1 Hilbert Transform

Assuming a discrete-time, continuous-valued function: $x = \{x_1, x_2, ..x_T\}$; $x_i \in \mathbf{R}$ the analytic function of $x(t)$ is defined as [11,3]:

$$\xi(t) = x(t) + x_H(t) = A(t)exp[i\phi(t)], \tag{1}$$

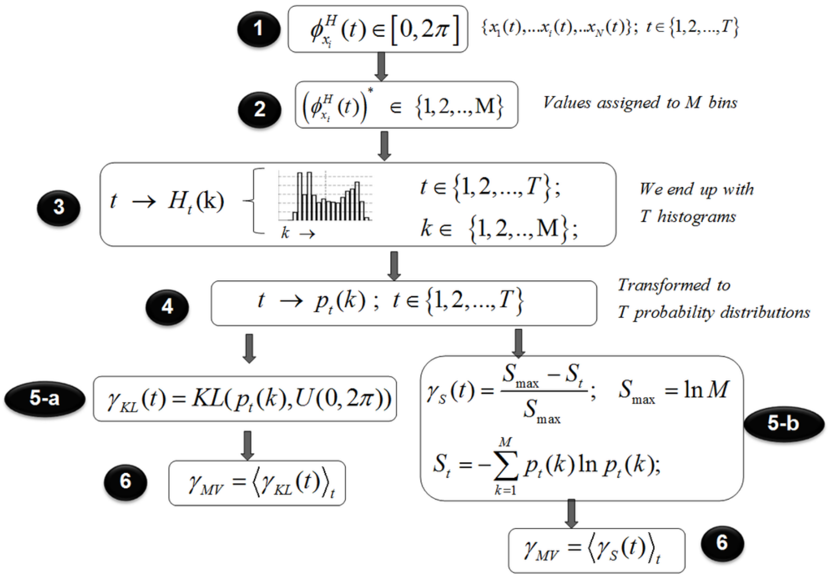

Fig. 2. Synopsis of method computation

$$\phi(t) = \tan^{-1}\left[\frac{x_H(t)}{x(t)}\right],\ x_H(t) = \frac{1}{\pi}P.V.\int_{-\infty}^{+\infty}\frac{x(\tau)}{t-\tau}\,d\tau, \tag{2}$$

where $x(t)$ is the signal and $x_H(t)$ is the Hilbert transform of it, and P.V. indicates that the integral is taken in the sense of the Cauchy principal value. In Eq. 1, $\xi(t)$ is the analytic function and the functions $A(t)$ and $\phi(t)$ define the instantaneous phase and instantaneous amplitude of function $x(t)$.

2.2 Method Description

To start with we assume having a set of N interconnected nodes (neurons). The activity of the N nodes produce N signals which we consider to be sampled at regular time intervals producing T samples per signal. Ending up with an activity matrix $\mathbf{X}$ of dimension $N \times T$ with the sampled values of all signals:

$$\mathbf{X} = \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^T \\ \dots & . & . & \dots \\ x_N^1 & x_N^2 & \dots & x_N^T \end{bmatrix} = \left(\boldsymbol{x}^1, \boldsymbol{x}^2, \dots, \boldsymbol{x}^T\right) \tag{3}$$

Activity matrix $\mathbf{X}$ can be also represented as an array of column vectors $\boldsymbol{x}^i$, each column vector having the activity of all nodes at time i .

Computing the Hilbert transform of all signals in matrix $\mathbf{X}$ will provide matrix $\mathbf{X_H}$ (Hilbert transform along rows of $\mathbf{X}$), from which we can compute the instantaneous matrix of signals phases $\boldsymbol{\Phi}$:

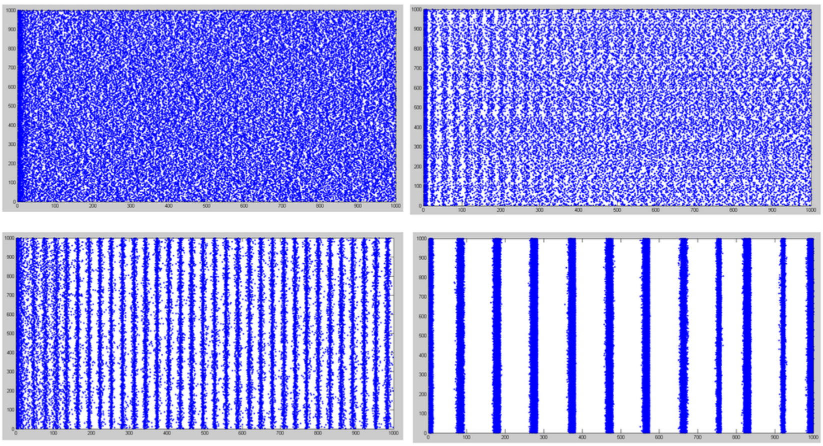

Fig. 3. Regular synchronous signal. We show four ensembles used for RS synchronization, from left to right and up to bottom: random (no synchronization), weakly, moderately and strongly synchronized ensembles.

$$\mathbf{X_H} = \begin{bmatrix} x_1^1{}_H & x_1^2{}_H & \dots & x_1^T{}_H \\ \dots & . & . & \dots \\ x_N^1{}_H & x_N^2{}_H & \dots & x_N^T{}_H \end{bmatrix} \tag{4}$$

$$\mathbf{\Phi} = \begin{bmatrix} \tan^{-1}\left(\frac{x_1^1{}_H}{x_1^1}\right) & \tan^{-1}\left(\frac{x_1^2{}_H}{x_1^2}\right) & \dots & \tan^{-1}\left(\frac{x_1^T{}_H}{x_1^T}\right) \\ \dots & . & . & \dots \\ \tan^{-1}\left(\frac{x_N^1{}_H}{x_N^1}\right) & \tan^{-1}\left(\frac{x_N^2{}_H}{x_N^2}\right) & \dots & \tan^{-1}\left(\frac{x_N^T{}_H}{x_N^T}\right) \end{bmatrix} \tag{5}$$

$$= \begin{bmatrix} \phi_1^1 & \phi_1^2 & \dots & \phi_1^T \\ \dots & . & . & \dots \\ \phi_N^1 & \phi_N^2 & \dots & \phi_N^T \end{bmatrix} = \left(\boldsymbol{\phi}^1, \boldsymbol{\phi}^2, \dots, \boldsymbol{\phi}^T\right) \tag{6}$$

Again, matrix $\mathbf{\Phi}$ can be represented as an array of column vectors $\boldsymbol{\phi}^i$, each column vector having the instantaneous phases of all nodes at time i .

Considering the phase values for each column vector $\boldsymbol{\phi}^i$: we build its histogram, assigning the range of phase values $[0, 2\pi]$ into M bins.

Finally we will have T histograms (one per column vector $\boldsymbol{\phi}^i$) where in bin k of histogram i one will have the total number of times that any signal phase at time i has a value between $2\pi\frac{(k-1)}{M}$ and $2\pi\frac{k}{M}$ radians.

Normalizing the histogram to have a total area of 1, we end up with T probability mass functions: p_t , one per time increment:

$$t \rightarrow p_t(k),\ t = \{1, 2, \dots, T\},\ k = \{1, 2, \dots, M\}. \tag{7}$$

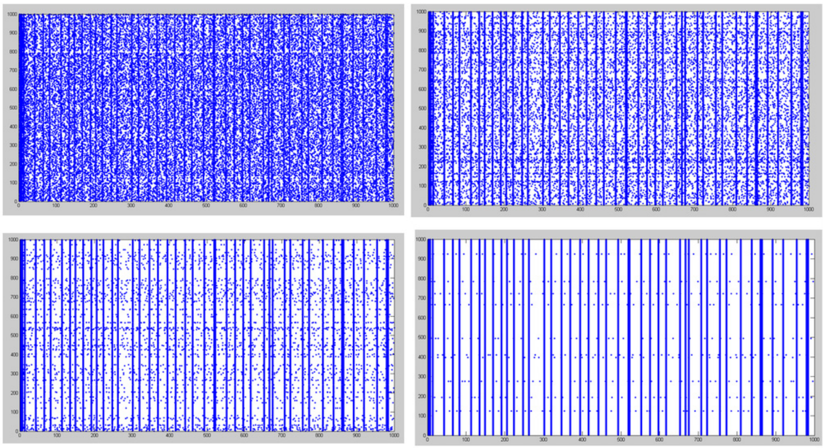

Fig. 4. Irregular synchronous signals with 30%, 70%, 90%, and 99% as percentage of identical signals in the ensemble (from left to right and up to bottom)

If the distribution of the probability mass functions is similar to a uniform distribution implies that the signals are not synchronized and the more the distribution is departed from the uniform distribution implies that the signals are more synchronized, being the limit a delta Dirac distribution with a single phase value for all signals.

We define a synchronization index, based in applying a measure to each probability distribution: $p_t(k)$, to finally have a single number that represents how close is the distribution to a uniform distribution. In this paper, we have used the Entropy.

Applying the Entropy operator to each probability mass function p_t, we transform a sequence of probability distributions $\mathbf{P} = [p_1, p_2, \ldots, p_T]$ into a sequence of scalars $\mathbf{S} = [S_1, S_2, \ldots, S_T]$, where:

$$S_t = -\sum_{k=1}^{M} p_t(k) \ln p_t(k), \;\; t = \{1, 2, \ldots, T\}. \tag{8}$$

Normalizing the entropies, with respect to the maximum entropy which is given by the uniform distribution ($\ln M$), we transform the entropies S_t into normalized entropies $\gamma_S(t)$ which range is $[0, 1]$:

$$\gamma_S(t) = \frac{S_{max} - S_t}{S_{max}}; \; S_{max} = \ln M \tag{9}$$

The final result is that the ensemble activity matrix $\mathbf{X}$ in Eq. 1 has been transformed into a sequence of scalars $[\gamma_S(1), \gamma_S(2), \ldots, \gamma_S(T)]$ of length T, which we call the **entropy sequence function**. This sequence of scalars actually represents a function of time of the normalized entropies associated to per-time-sliced phase values distributions along all nodes in the ensemble.

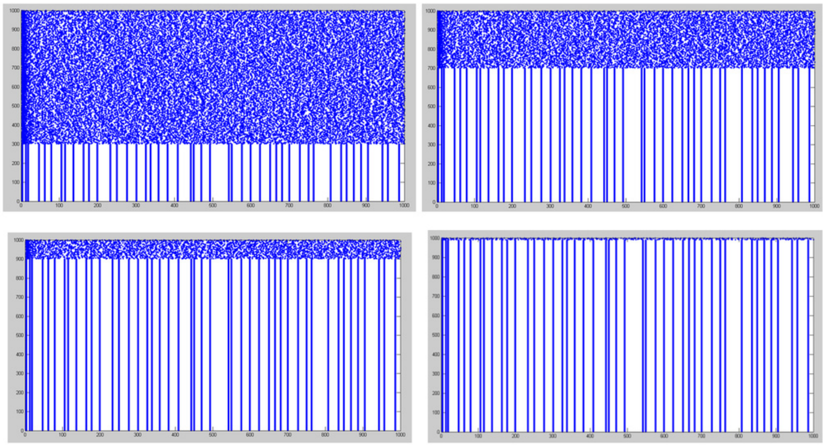

Fig. 5. Irregular synchronous signals forming clusters of contiguous identical signals. We show from left to right and up to bottom four ensembles with 30%, 70%, 90%, and 99% as percentage of identical signals.

From the entropy sequence function we can provide a summary value (a single scalar) which can represent the complete activity of the ensemble. Several *aggregation* functions can be taken, but we have used the simple average function along time:

$$\gamma_{MV} = \langle \gamma_S(t) \rangle_t \tag{10}$$

This final value is the **multivariate synchronization index** presented in this paper.

In Fig 1 is presented the relationship between the signal ensemble, the entropy sequence function: $\gamma_S(t)$ and probability mass functions: p_t . We can see that the points in time of maximum entropy corresponds with quasi Dirac deltas in the probability distributions.

In Fig 2 is presented a synopsis of the computations performed: In step 1, the Hilbert transform of each signal is calculated. In step 2 we discretize the phase values to M possible values (M bins). In step 3 we build a histogram per time increment, the histogram gives the relative frequencies of each possible value: $\{1, 2, \ldots, M\}$, along all the nodes. Normalizing the histograms we construct the corresponding probability mass functions in step 4. From there, we can use the Entropy to translate probability distributions to scalar values, which is done in steps 5. Note that we could use any alternative operator in order to translate probability distributions to scalar values, like for instance the Kullback-Leibler(KL) [12] divergence. In step 6 we provide an aggregate summary of the Entropies or KL divergence, which are functions of time; to do so we use the average along time.

An important secondary result is the possible use of the function $\gamma_S(t)$ as a signature (single representative function) of the whole multidimensional

	IS Signals
Random	0.28745
IS-30%	0.39439
IS-70%	0.67452
IS-90%	0.87018
IS-99%	0.98441
IS-100%	1

	IS Signals in clusters
Random	0.28745
IS-30%	0.39423
IS-70%	0.67411
IS-90%	0.87006
IS-99%	0.98431
IS-100%	1

	RS Signals
Random	0.28745
RS-Weakly Synchronized	0.42574
RS-Moderately Synchronized	0.46936
RS-Strongly Synchronized	0.68369

Fig. 6. Multivariate synchronization index (γ_{MV}) results for different types of signals: RS, IS and IS-C (Section 3). At the bottom of each data set is presented a function representation of values (from top to bottom).

ensemble dynamics. So, these functions can be used to compare the similarity of two complete ensembles of signals.

In the following sections we will provide evidence of the suitability of the method to detect synchronization.

3 Data Test Signals

To test the method we have used three types of signals with increasing level of synchrony in each group.

3.1 Regular Synchronous (RS)

These signals have been produced with a specifically developed Matlab toolbox starting with a non-connected set of 1000 neurons with a random input signal for each of the neurons to produce a random signals ensemble. Then it has been incrementally increased the coupling weight and degree of connectivity of the network, producing three additional signals ensembles with: weak, medium and strong synchronization.

To create the simulation we have used the Izhikevich neuron model [13,14]. This model is a phenomenological spiking model as computationally efficient as the integrate-and-fire model and that depending only on four parameters can reproduce the spiking and bursting behavior of known types of cortical neurons. For this specific simulation we have created 1000 excitatory integrator neurons with a random connectivity network.

In Fig 3 are presented the four ensembles used for RS synchronization, from left to right and up to bottom: random (no synchronization), weak, medium and strong synchronization ensembles.

In all cases the signals present a periodic or quasi-periodic recurrence activity (except the first which is random). The signal coming from each neuron is different to all other signals but the global activity forms a periodic dynamic.

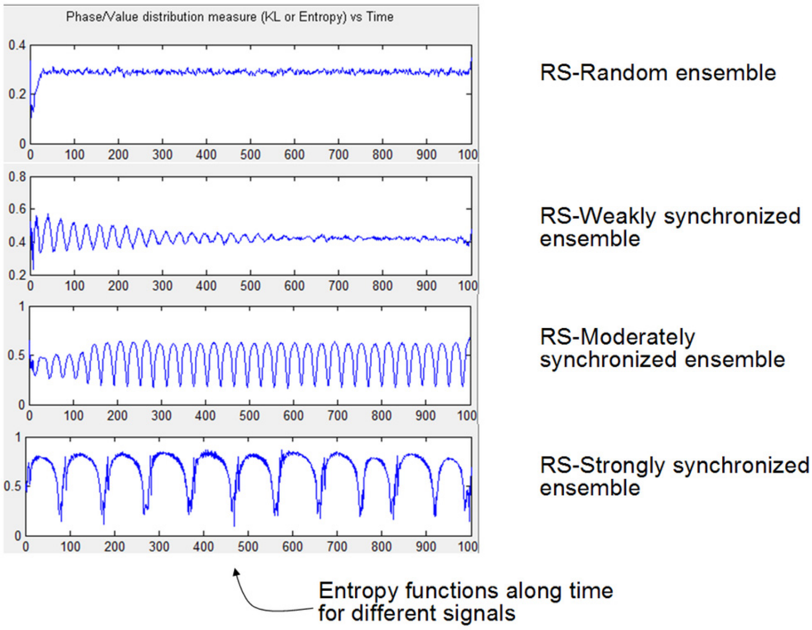

Fig. 7. Entropy sequence functions for the four RS signals. The x-axis is time.

3.2 Irregular Synchronous (IS)

These signals have been produced starting from the same random ensemble of 1000 neurons used in the RS case. From this ensemble, we have randomly selected a neuron signal and replaced a percentage of other neurons signals with the chosen one. In this way, it has been produced several ensembles with a different percentage of repetition of the one single signal chosen for substitution. There are ensembles with 30%, 70%, 90%, 99% and 100% as percentage of identical signals in the ensemble. For example, the 100% repetition case is an ensemble with all neurons signals been identical.

The repetition of equal signals in the ensemble is done in a random manner, meaning that the replacing signal occupies random positions in the signals matrix rows.

In Fig 4 are presented several ensembles (from left to right and up to bottom) for 30%, 70%, 90% and 99% repetition percentage. The figure presents the spikes activity of the signals, being time in horizontal and nodes in vertical.

These signals have been produced using a specifically developed Matlab code; they have not been generated by the Matlab toolbox as part of the simulation of a dynamical system, as for the RS signals.

3.3 Irregular Synchronous in Cluster (IS-C)

Similar to IS signals but with equal signals placed in contiguous positions. In Fig 5 are presented several ensembles (from left to right and up to bottom) for 30%, 70%, 90% and 99% repetition percentage of the IS signals in cluster.

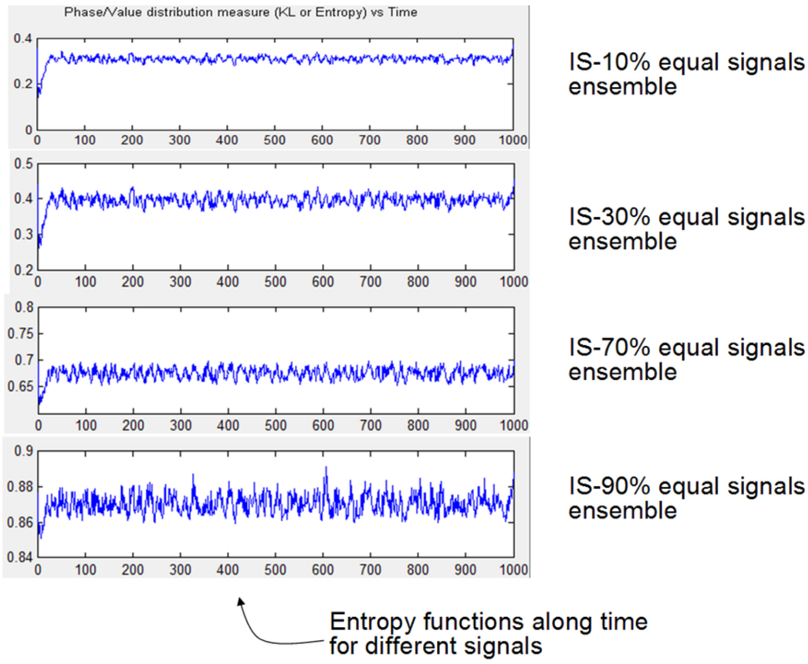

Fig. 8. Entropy sequence functions for the four IS signals. The x-axis is time.

4 Experimental Results

In this section we present the numerical results obtained after applying the multivariate index of section 2.2 to the signals in section 3. The results are presented in Fig 6. In this figure we can observe that the evolution of the phase index is monotonically increasing as synchronization increases in the ensembles, providing a convenient measure of synchronization between ensembles, even of different types (IS vs. RS). In Fig 7 are shown the entropy sequence functions for the four RS signals in section 3. The four diagrams correspond to (from up to bottom): random (no synchronization), weakly, moderately and strongly synchronized signals. In Fig 8 is presented a similar diagram but for the four IS signals of sections 3. We can see, observing these figures, how the differences between the IS and RS signals are expressed in the shape of the entropy functions ($\gamma_S(t)$). The IS signals show an almost constant entropy sequence function only modulated by noise, with an average value increasing as the signals ensemble increase in synchrony. Meanwhile, the RS signals present and oscillating shape, centered around an almost constant value but with the oscillation amplitudes increasing as the synchronization increases. Considering the above mentioned properties, the entropy function can be used as a one-dimensional signature function to represent the whole signals ensemble.

5 Conclusions

It has been presented a synchronization detection method for multivariate signals which is suitable to detect the synchronization level of a signals ensemble. The method grows linearly in computational demands with the increase in number of signals and sampling time steps. It is not based in a combinatorial growing number of signals differences.

The method is presented in detail and the results show its adequacy to detect synchronization for different types of signals.

The method gives two main results:

- **A scalar value (phase index)**: This provides a single aggregate of synchronization level.
- **A time function of entropies**: This provides a signature of the ensemble behavior good to capture the similarity signals level along time. This function can be used to compare two ensembles of signals.

Acknowledgements. This work was supported by the Spanish Government projects TIN2010-19607.

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