Network Games of Imperfect Competition: An Empirical Framework

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Abstract

This paper studies how product differentiation affects substitution patterns and firms' markups in oligopolistic markets where products are differentiated over multiple attributes and consumers have linear-quadratic preferences. Under these assumptions, the cross-price elasticity between any two goods is determined by a weighted inner product of their corresponding vector of attributes. The weight on a given attribute is proportional to how homogeneous that attribute is across all products available in the market. On the supply side, oligopolistic competition in either prices or quantities is framed as a network game in the spirit of Ballester, Calvó-Armenagol and Zenou (2006). Each product identifies a node, and the vector of attributes pins down a product's location in the competitive network. In Bertrand and Cournot games, product differentiation affects equilibrium price-cost margins only through a product's Bonacich centrality. Therefore, a product's centrality summarizes how product differentiation affects the ability of firms to charge positive markups. Using market-level data on the US automobile industry, the second part of the paper shows how to identify and estimate the model parameters. Under the assumption that any unobserved characteristic enters consumers' utility only through the linear component of preferences, a simple linear IV strategy can be implemented to estimate the demand parameters and recover equilibrium price-cost margins.

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1 Introduction

Product differentiation plays a crucial role in allowing firms to price above costs. Indeed, it is well-known that when firms compete on prices, product differentiation is necessary for firms to charge positive margins. If products were to be homogeneous, even in a duopoly, firms would price at costs leading to a perfectly competitive outcome.[1] There are many ways to model product differentiation, but defining a product in terms of a vector of attributes is the most common. Within this characteristic-based approach, models of demand differ in how consumers evaluate these attributes. At one extreme, when all consumers value attributes in the same way, products are vertically differentiated and can be ranked uniquely. Conversely, when consumers value attributes differently, products are horizontally differentiated, and there is no unique way to rank them across consumers.

While models of demand for differentiated products might differ in how consumer preferences are specified, they all carry the same intuition about how product differentiation affects substitution patterns and producers' price-cost margins. Products that are more similar in terms of characteristics should be more substitutable between each other, and firms selling a more differentiated product should be able to charge higher markups. Although the intuition is clear, in most models, it is hard to fully grasp how product attributes affect substitution patterns and firms' markups. The reason is that perhaps except for the most stylized models,[2] this relationship is often hidden behind implicit equations involving integration over a multidimensional space.

This paper considers a framework in which product differentiation enters substitution patterns and firms' price-cost margins in an way that is easy to interpret and that matches the common intuition about how product differentiation enables firms to price above marginal costs. At the same time, I allow for products to be differentiated over multiple attributes and for consumers to have heterogeneous preferences over these attributes. In modelling consumer preferences, I make two assumptions. First, product characteristics enter preferences in a linear-quadratic fashion, and second, consumers choose how much to consume of each product. With this second assumption, I depart from the unit demand framework commonly used in empirical applications. However, if consumers were constrained to consume at most one unit of a single product, the model would boil down to a standard discrete choice demand framework. Thus, the linear-quadratic model nests the discrete choice framework.[3] Turning to the first assumption, the utility's linear part mimics the indirect utility's linearity, often assumed under discrete choice demand. In contrast, the quadratic term enters consumer

[1] Also assuming there are no capacity constraints.

[2] E.g., in models of vertical product differentiation along a single attribute such as the Hotelling and Salop models.

[3] The only difference would be that the inner-product of the vector of attributes of, say, product j, would enter the indirect utility u_{ij} as an additional product characteristic.

preferences with a negative sign, implying that consumers value variety.

Under these two assumptions, after deriving the aggregate demand system, I show that the cross-price elasticity between any two products is proportional to a weighted inner product of their corresponding vector of attributes. This weighting has two essential properties. First, it is such that, regardless of the units with which characteristics are measured, the resulting inner product always lies in between $(-1, 1)$ and thus can be interpreted as a correlation between the characteristics of the two products. Moreover, the model allows for complementary goods because cross-price elasticities can be negative. Second, the weight on a given characteristic is inversely related to how much that characteristic varies across all products in the market. Thus, more homogeneous characteristics across all goods available matter more in determining substitution patterns between products.

Next, I turn to the supply side to study how product attributes affect firms' price-cost margins. Although I will primarily focus on Bertrand competition because it is the workhorse model used in empirical applications, I also consider quantity competition a la Cournot. In both cases, I leverage the linear-quadratic structure of consumer preferences to frame the oligopolistic game as a network game in the spirit of Ballester, Calvó-Armenagol and Zenou (2006). Products are the network nodes, and the weighted inner product between their vector of attributes determines the strength of the links between nodes. The implied competitive network is weighted, undirected and such that the more substitutable two products are, the stronger their link. Within this network framing, I show that firms' equilibrium price-cost margins can be decomposed additively into two components: a monopolistic component and a product differentiation component. The monopolistic component captures how much value a product generates for society; if the producer were to be a monopolist, it would charge a higher margin the higher the value its product generates for consumers. The product differentiation component instead summarizes how differentiated a product is relative to its competitors and is proportional to the Bonacich network centrality of that product. This centrality measure enters firms' price-cost margins with a negative sign because a more central product or, equivalently, a less differentiated product faces more competition and, in turn, charges lower markups.

In the second part of the paper, I estimate the Bertrand Network model using market-level data on the US automobile industry. I show that the demand parameters can be identified and estimated with a simple linear IV strategy, assuming that any unobserved product characteristic enters consumers' utility only through the linear component of preferences. With the estimated demand parameters, margins and costs can be recovered from the supply equation. Although the model is not based on discrete choice demand, it delivers reasonable price elasticities and price-cost margins. When I decompose the estimated margins, I find that the network structure based on the available characteristics seems quite competitive. The Bonacich product centrality, on average, kills more than 90% of the monopolistic margins. By differentiating their products, car producers are only able to capture from 2% to 7% of the potential monopolistic margins.

Lastly, to quantify how much product differentiation matters in determining price-cost margins, I compare the estimated margins with the ones firms would have charged if their products were to be homogeneous. The estimated margins can be as high as three times the homogeneous margins, suggesting that product differentiation is critical in allowing firms to price above costs.

The rest of the paper proceeds as follows. Section 2 describes how this paper fits the literature, Section 3 develops the demand side of the model, Section 4 focuses on the supply side and looks at both Bertrand and Cournot competition, Section 5 extends the supply to the case of multiproduct firms, Section 6 estimates the model with market level data on the US automobile industry and Section 7 concludes.

2 Contributions to the Literature

This paper contributes to the empirical industrial organization literature that estimates oligopolistic models of product differentiation using market-level data. Most of this literature models demand as a discrete choice problem and supply as a game of imperfect competition with differentiated products where, in most cases, firms are assumed to choose prices simultaneously (i.e., a la Bertrand).[4] Bresnahan (1987) was among the first to estimate a discrete choice model of oligopolistic competition with products that are vertically differentiated along one dimension (i.e., a la Hotelling). A few years later, motivated by the theoretical advancements in the modelling of product differentiation, Feenstra and Levinsohn (1995) extended the Bresnahan (1987) model to accommodate for product differentiation along multiple dimensions and showed how to recover price-cost margins in this more general context. Importantly, in their model, substitution patterns and markups are determined by the distance in the characteristic space to neighbouring products.

In parallel, Berry (1994) and Berry, Levinsohn and Pakes (1995) developed methodologies to estimate oligopolistic models in which products are horizontally differentiated. These methodologies can accommodate the presence of unobserved product characteristics, which is one of the reasons that made them the leading approaches to estimating oligopolistic models of product differentiation. One drawback with these models is that to obtain reasonable substitution patterns, in the sense that products with similar characteristics are more substitute to each other, one needs to introduce random coefficients on product characteristics and estimate the model via a nested-fixed-point optimization algorithm. In addition, while in most cases, the estimates

[4] For an exception with continuous demand see Dubois, Griffith and Nevo (2014).

obtained from these models match the common intuition that similar products are more substitutable, it is not immediate to confirm such intuition analytically from the model equations.

The model I developed in this paper allows for unobserved product characteristics and simultaneously delivers reasonable substitution patterns without requiring any sophisticated estimation algorithm. In particular, I show that the model can be consistently estimated using a simple linear IV strategy. Furthermore, in the model I consider, substitution patterns and price-cost margins are related to the distance between product characteristics as in Feenstra and Levinsohn (1995) but with the advantage that the characterization is analytical instead of being defined implicitly. More precisely, by framing oligopolistic competition as a network game, I show that the equilibrium price-cost margins can be expressed as a function of a product's Bonacich centrality, which captures how close that product is to its competitors and represents a summary statistic for the ability of firms to price above marginal costs. Moreover, I show that the cross-price elasticity between any two goods is determined by a weighted inner product of their vector of characteristics which, differently from the standard discrete-choice models, can accommodate the presence of complementary goods.

Product proximity in terms of characteristics also matters for more practical aspects of model estimation. In the context of the standard discrete choice framework developed by Berry (1994) and Berry, Levinsohn and Pakes (1995), a recent contribution by Gandhi and Houde (2023) shows that, under the common assumption that product characteristics are exogenous, relevant price instruments should reflect the degree of differentiation of a product relative to others available in the market. In particular, they show that the residual function of the model, which depends on endogenous prices, is a function of the distances between observed product characteristics. In the model considered here, the intuition of this result emerges clearly from the analytical solution of the equilibrium prices (i.e., Proposition 3) which are a function of the proximity of products in the characteristic space as measured by their Bonacich network centrality.

Also recently, Magnolfi, McClure and Sorensen (2022) showed how to incorporate online survey data on the product space into demand estimation. The data capture product distances in the form of "product A is closer to B than it is to C" and can be used to construct a low-dimensional representation of the latent product space (i.e., an embedding). The authors show that incorporating these embeddings into conventional random coefficient logit models delivers elasticity estimates that are similar to those from a model that uses observable characteristics, suggesting that what matters are not attributes per se but rather how a product's attributes differ relative to its competitors. Similarly, in the network model developed here, substitution patterns depend only on how similar product characteristics are. Any set of characteristics that preserves the same network structure would generate the same substitution patterns.

This paper also contributes to recent literature that applies results from network theory to study oligopolistic competition and market power. This literature builds on the seminal contribution by Ballester, Calvó-Armenagol and Zenou (2006) to frame games of imperfect competition as network games with product differentiation. In a recent contribution, Ushchev and Zenou (2018) develop a model of price competition in which product varieties are differentiated over a network and show that a unique Bertrand-Nash equilibrium exists and is proportional to a sign-alternating version of the Bonacich centrality. Galeotti, Golub, Goyal, Talamàs and Tamuz (2022) also frame oligopolistic price competition as a network game and exploit properties of the singular value decomposition (SVD) of the Slustky matrix to characterize optimal tax-subsidy designs in terms of properties of the underlying network structure. Pellegrino (2023) instead considers a general equilibrium Cournot oligopoly in which products are differentiated over multiple attributes and the network structure is pinned down by the characteristics of the products.[5] He decomposes Cournot markups into a (quality-adjusted) productivity component and a product centrality component and calibrates the model using data on firms' financials from Compustat while measuring the adjacency matrix of product characteristics using the data on product cosine similarities constructed in Hoberg and Phillips (2016).

This paper contributes to this network literature in several dimensions. First, as in Pellegrino (2023), but differently from Ushchev and Zenou (2018) and Galeotti, Golub, Goyal, Talamàs and Tamuz (2022), I assume that products are differentiated over multiple attributes, which imply that the characteristics of the products determine the network structure. Second, differently from Pellegrino (2023), I focus on Bertrand competition, the standard conduct assumption made in empirical applications. Importantly, I show that in both Cournot and Bertrand games, equilibrium price-cost margins can be decomposed additively into a monopolistic component that captures the margin a monopolist would charge and into a product differentiation component which captures how differentiated a given product is relative to its competitors. I show that, in both Cournot and Bertrand, this centrality component coincides with the standard Bonacich network centrality as defined in Bonacich (1987) and Jackson (2008) and, in both cases, the relevant adjacency matrix is a simple (possibly weighted) inner-product between the matrix of product characteristics X.[6] Moreover, similarly to Galeotti, Golub, Goyal, Talamàs and Tamuz (2022), I exploit the SVD of the Slusky matrix to characterize and interpret own and cross-price elasticities (Proposition 2). This setting provides a unified framework to model imperfect competition in quan-

[5]See also Ederer and Pellegrino (2022) which extends the Cournot network model of Pellegrino (2023) to account for firms' institutional ownership structure.

[6]The decomposition I propose here is for the dollar price-cost margins $p-c$. For the Cournot case, Pellegrino (2023) instead develops a decomposition for markups defined as $\mu = p/c$. In Appendix B, I show that the two decompositions are related. In particular, I show that the measure of centrality defined in Pellegrino (2023) is an affine transformation of the Bonacich product centrality that enters Cournot price-cost margins.

tities or prices as a network game in which products are differentiated over multiple attributes. Last, I show how to estimate the model with market-level data (e.g., prices, quantities and characteristics) on a given industry using a simple linear IV strategy.

Finally, this paper contributes to the theoretical industrial organization literature that compares equilibrium outcomes across price and quantity competition. In general, this literature finds that under strategic complementarity in prices, Cournot equilibrium prices are higher than Bertrand prices, thus confirming the intuition that Bertrand competition is more intense. The seminal contribution by Singh and Vives (1984) develops the argument for the case of a differentiated duopoly which was then extended to an N firms oligopoly in Vives (1985).[7] More recently, Magnolfi, Quint, Sullivan and Waldfogel (2022) pointed out that, again, under the assumption that prices are strategic complements, imposing Cournot competition would always lead the researcher to estimate higher markups. In Proposition 5 I show that these results also hold when comparing the Cournot network game with the Bertrand network game. In particular, I show that, under strategic complementarity in prices, the centrality of each product is higher under Bertrand competition.

3 Demand

This section sets up the demand side of the model. I start by describing the set of products available, consumers' preferences and by deriving individual demand functions. Then, I turn to aggregate demand and describe how product attributes affect own and cross price elasticities.

3.1 Products

In the market I consider, there are $j \in \{1, ..., J\}$ products available. Each product j is characterized by a set of K attributes whose values are collected in the K dimensional real-valued vector $x_j = (x_{jk})_{k=1}^{K}$ where x_{jk} is measured in units of quantity consumed. Characteristic x_{jk} tells you how much of attribute k you would get if you consume one unit of product j.

3.2 Utility

Consumers are indexed by $i \in I$, take product prices $p = (p_j)_{j=1}^{J}$ as given, and choose how much to consume of each product available. I denote by $q_i = (q_{ij})_{j=1}^{J}$ the consumption vector of consumer i.

[7] Similar results can be found in Cheng (1985), Okuguchi (1987) and Amir and Jin (2001).

I define consumer i preferences as follows:[8]

$$
\begin{aligned}
u_i(q_i, X) &= q_{i0} + \left(q_i'\alpha_i^q - \frac{\beta_i}{2}q'q\right) + \eta\left(q'X\alpha_i^x - \frac{\beta_i}{2}q'XX'q\right) \\
&= q_{i0} + q_i'\alpha_i - \frac{\beta_i}{2}q_i'\left(I_J + \eta XX'\right)q_i
\end{aligned} \tag{1}
$$

where q_{i0} is an outside good, X the $J \times K$ matrix of products attributes, $\eta > 0$ governs the extent with which product differentiation in terms of attributes matters for consumers, $\alpha_i^q > 0$ and $\alpha_i^x > 0$ are respectively a J-vector and K-vector of utility parameters. Both vectors of parameters affect the marginal utility that comes from the linear term in (1), $\alpha_i \equiv \eta X\alpha_i^x + \alpha_i^q$. Finally, β_i captures i's love for varieties.

3.3 Individual Demand

Consumer i takes prices p as given and maximizes (1) subject to

$$
q_{i0} + q_i'p \leq y_i \tag{2}
$$

where y_i is consumer i income. After substituting for the budget constraint in (2), consumer i's demand function is given by

$$
q_i(p) = \frac{1}{\beta_i}\left(I_J + \eta XX'\right)^{-1}(\alpha_i - p) \tag{3}
$$

which is always well defined because $(I_J + \eta XX')$ is positive definite and therefore non-singular.

3.4 Aggregate Demand and Price Elasticities

I assume there is a mass M of consumers that are heterogeneous in terms of (α_i, β_i). Further, I shall assume that all moments involving the random variables α_i and β_i are well-defined. The next proposition characterizes the aggregate demand function.

Proposition 1 *Under the above assumptions, the aggregate demand of product j as a function of own and competitors prices is given by*

$$
q_j(p_j, p_{-j}) = a_j(\alpha, \beta, (\theta_{jl})_{l=1}^J) - \frac{1}{\beta}(1 - \theta_{jj})p_j + \frac{1}{\beta}\sum_{l \neq j}\theta_{jl}p_l \tag{4}
$$

where $\beta \equiv M\left(\int \frac{1}{b_i}di\right)^{-1}$, $\theta_{jl} = x_j'\Omega^{-1}x_l$ where $\Omega \equiv \left(\frac{1}{\eta}I_K + X'X\right)$ is a positive definite weighting matrix independent of (j, l) and a_j is a j-specific demand intercept that depends on $\alpha \equiv \int \frac{\alpha_i/\beta_i di}{\int 1/\beta_i di}$. Moreover, for any (j, l), $\theta_{jl} \in (-1, 1)$ if $j \neq l$ whereas $\theta_{jj} \in (0, 1)$ if $j = l$.

[8] The same type of preferences are considered in Pellegrino (2023).

Equation (4) defines the aggregate demand for a given product j. Because consumer preferences are quadratic it is not surprising that aggregate demand is linear in prices. What is more interesting is how own and cross price elasticities relate to products characteristics. This relationship is enclosed in the elements $(\theta_{jl})_{j,l}$ which determine the substitution patterns across products. In fact, the (j,l)th element of the Slutsky matrix is given by

$$\frac{\partial q_j}{\partial p_l} = \begin{cases} -\frac{1}{\beta}(1-\theta_{jj}) & \text{if } j = l \\ \frac{1}{\beta}\theta_{jl} & \text{if } j \neq l \end{cases} \tag{5}$$

so that products j and l are substitutes or complements whenever θ_{jl} is respectively positive or negative. But Proposition 1 tells us more; the element θ_{jl} is a weighted inner-product between j and l vectors of attributes. In other words, the closer j and l are in this inner-product space the more substitutable they are. The next result characterizes the weighting of the inner-product between x_j and x_l in terms of the principal components of the matrix of characteristics X.

Proposition 2 *Let U be the $K \times K$ matrix of principal components directions of X and for any j let $\tilde{x}_j = U'x_j$ be the projection of x_j onto these principal components. Then for any (j,l)*

$$\theta_{jl} = \sum_{k=1}^{K} \left(\frac{1}{1+\lambda_k^{x'x}} \right) \tilde{x}_{jk}\tilde{x}_{lk} \tag{6}$$

where $\lambda_k^{x'x}$ is the k-th eigenvalue of $X'X$.

Equation (6) has two main insights. First, the substitution between any two products j and l can be expressed as a weighted inner-product between the vectors of projected attributes $\tilde{x}_j$ and $\tilde{x}_l$. Because U is an orthogonal matrix, this projection is innocuous in the sense that $\tilde{X}\tilde{X}' = XUU'X' = XX'$ and we can replace XX' with $\tilde{X}\tilde{X}'$ without affecting consumer preferences defined in (1), individual demand defined in (3) and aggregate demand defined in (4).

Second, the weighting of these (projected) product characteristics depends on how much variety in terms of each characteristic is available in the whole market. From expression (6) we can see that characteristics with smaller $\lambda_k^{x'x}$ are weighted more. But what does a small $\lambda_k^{x'x}$ mean in practice? It means that higher weight is assigned to (projected) characteristics that do not vary too much across products as measured by $\tilde{X}'\tilde{X}$. To see this more formally, let u_k be k-th principal component of X, $\tilde{x}_k$ the k-th column of $\tilde{X}$ and note that,

$$\tilde{x}_k'\tilde{x}_k = u_k'X'Xu_k = u_k'U\Lambda^{x'x}U'u_k = \lambda_k^{x'x}. \tag{7}$$

where the first equality comes from the definition of $\tilde{x}_j$ and $\tilde{x}_k$, the second from the eigen-decomposition of $X'X$ and the last one from the fact that U is an orthogonal matrix. Overall, equation (7) tells us that characteristics that vary more across all products available in the market will have a higher $\lambda_k^{x'x}$ and thus, from equation (6), will matter less when computing substitution patterns between any two products.

To sum up, Proposition (6) highlights that the elasticity of substitution between two products is affected by not only how similar are their vector of characteristics, but also by how similar are characteristics across the whole market. The substitutability between any two products will be higher if their characteristics are similar but even more so if the characteristics in which they are similar are the ones that are more homogeneous across all products available.

4 Oligopolistic Competition

In this section I turn to the analysis of the supply side. I will mainly focus on Bertrand competition because it is the workhorse model used in empirical applications. After, solving the Bertrand game, I will turn to Cournot competition. In both cases I show that the effect of product differentiation on equilibrium markups is summarized by a measure of how central is a product in the competitive network. I conclude the section by comparing the two cases and by showing that, under strategic complementarity, Cournot competition always leads to higher price cost margins.

To start with, I assume that the J products are produced by J single-product firms with constant marginal costs. Section 5 deals with the case in which firms are multi-product. Moreover, throughout the analysis that follows, I will assume that an interior Nash equilibrium exists.

4.1 Bertrand Competition

Firm j takes the vector of competitor prices p_{-j} as given and solves

$$\max_{p_j} \quad (p_j - c_j) q_j(p_j, p_{-j}) \tag{8}$$

$$\text{s.t.} \quad q_j(p_j, p_{-j}) = a_j - \frac{1}{\beta}(1-\theta_{jj})p_j + \frac{1}{\beta}\sum_{l \neq j} \theta_{jl} p_l \tag{9}$$

which is equivalent to

$$\max_{p_j} \left(a_j + \frac{c_j}{\beta}(1-\theta_{jj}) \right) p_j - \frac{1}{\beta}\left(1-\theta_{jj}\right) p_j^2 + \frac{1}{\beta}\sum_{l \neq j} \theta_{jl} p_j p_l. \tag{10}$$

The payoff function in equation (10) is analogous to the linear-quadratic utility functions considered in Ballester, Calvó-Armenagol and Zenou (2006) and, as such, defines a linear-quadratic network game in which each product is a node and the $J \times J$ matrix

$$A(\Theta) \equiv \Theta - diag(\Theta) \tag{11}$$

is the weighed and undirected adjacency matrix of the network.

Network game interpretation. The adjacency matrix defined in (11) shows that network connections and products' substitution patterns are isomorphic to each other. From the previous section, we know that an off-diagonal element θ_{jl} of the matrix Θ captures the degree of substitution between product j and product l as measured by a weighted inner-product of their vector of characteristics x_j and x_l respectively. From equation (11), we know that we can interpret the inner-product θ_{jl} as a weighted link between product j and product l and therefore we can think of the product differentiation space as being a network whose nodes are the products and whose links tell us how close, or equivalently how substitutable, are any two products.

A natural question is then, why embedding product differentiation into a competitive network is relevant? Framing the product differentiation space as a network is important because it enables us to learn about how product differentiation affects equilibrium outcomes by studying the topological properties of the competitive network. In the next proposition, I show that equilibrium Bertrand price-cost margins depend negatively on a product's Bonacich centrality which, following Jackson (2008), I define as

Definition 1 *Let (A, J) be a network with J nodes and adjacency matrix A. The J-vector of (weighted) Bonacich centralities $\mathbf{b}(A, \delta, u)$ is given by*

$$\mathbf{b}(A, \delta, u) \equiv (I_J - \delta A)^{-1}\delta A u = \sum_{k=1}^{\infty} \delta^k A^k u, \tag{12}$$

where $\delta > 0$ is a scalar and $u > 0$ is J-vector.

The j-th element of $\mathbf{b}(A, \delta, u)$ summarizes how central node j is in the network. This measure of centrality is widely used in social networks because it captures a node's importance in terms not only of how close/connected this node is to others but also on how close/connected are the nodes it is connected to. According to the definition of Bonacich centrality, a node's importance is a weighted sum of the walks that emanate from it. Moreover, if $\delta \in (0, 1)$, walks of shorter length are weighted more.[9]

Proposition 3 *Assume that $\theta_{jj} = \theta$ for all $j \in \{1, ..., J\}$. If an interior equilibrium $p^* = (p_j^*)_{j=1}^J$ of the Bertrand pricing game exists then it is unique and is such that*

$$p^* - c = \frac{\alpha - c}{2} - \mathbf{b}\left(A(\Theta), \frac{1}{2(1-\theta)}, \frac{\alpha - c}{2}\right), \tag{13}$$

[9] This interpretation is motivated by the fact that when A is a binary $\{0, 1\}$ it k-th power A^k counts how many walks of length k are between any two nodes.

provided $\theta < 1 - \frac{1}{2}\max_j |\lambda_j(A)|$ *where* $\lambda_j(A)$ *is the* j*-th eigenvalue of the adjacency matrix* $A(\Theta)$*.*

The key insight of Proposition 3 is that the more central a product is in the competitive network the lower its equilibrium price-cost margins.[10] What does this mean in practice? From Definition 1 we can see that the higher any of the entries of the j-th row of A the more central node j is. In our settings, product j is more central the higher its substitutability with any other product (i.e., the higher the elements $(\theta_{jl})_{l\neq j}$ of the j-th row of Θ). Overall, the expression for the Bertrand price-cost margins in (13) tells us two things. First, a less central or, equivalently, more differentiated product, will be able to charge higher markups. Second, a product's Bonacich centrality is a sufficient statistics to measure how product differentiation allows firms to price above marginal costs.

Comparison with Ballester, Calvó-Armenagol and Zenou (2006). In their seminal paper, Ballester et.al. show that in a general network game with quadratic payoffs the Nash equilibrium action of any player is increasing in their Bonancich centrality. For the Bertrand competition game I study the opposite holds; Nash equilibrium prices decrease with a player's centrality. This mismatch in the results is a consequence of the fact that in Ballester et.al. the coefficient on the linear component of players utility does not depend on the network structure whereas, in the Bertrand case, the linear term of the quadratic profit in (10) depends on Θ.[11]

What is the economic interpretation of this result? The linear term in the Bertrand game corresponds to the marginal benefit of the very first unit for the average consumer and (assuming $c_j = 0$) is given by,

$$a_j = (1 - \theta_{jj})\alpha_j - \sum_{l\neq j} \theta_{jl}\alpha_l. \tag{14}$$

From expression (14) it is immediate to see that a more central product faces a lower residual demand. This *residual-demand* effect has to be contrasted with the *peer-effect* benefit of being more connected which affects firms' payoffs through the interaction terms in

$$\sum_{l\neq j} \theta_{jl} p_l p_j. \tag{15}$$

From expression (15) we can see that, holding everything else constant, being more central or, equivalently, having more substitutes increases profits. In the class of

[10] In Appendix D I perform a simple simulation exercise to visualize and summarize the properties of the Bertrand network model.

[11] As I discuss in subsection 4.2, this is not the case with quantity competition a la Cournot. As a consequence, the relationship between the Cournot-Nash equilibrium and the Bonacich network centrality follows immediately from the main result in Ballester, Calvó-Armenagol and Zenou (2006).

network games studied in Ballester et.al. only this second effect is present and is the force that pushes players to increase their equilibrium action proportionally to their network centrality. In the Bertrand network game not only the *residual-demand* effect in (14) is present but, as I show in Proposition 3, it dominates the *peer-effect*. As a result, the equilibrium actions (i.e., firms' prices) decrease with a product's centrality.

Some remarks. Before turning to the Cournot case, a couple of remarks are in order. First, the assumption that θ_{jj} are homogeneous across j is made only for expositional purposes and the proof presented in Appendix A is provided for the general heterogeneous case. Similarly, Section 5 generalizes the result to the case in which firms are multi-product.

Second, the condition on the largest eigenvalue of $A(\Theta)$ ensures that the Bonacich centrality can be rewritten as a convergent infinite sum. If the condition is not satisfied, expression (13) would still be well-defined but not expressable as an infinite series.

Third, Proposition (3) offers a two-terms decomposition of price-cost margins, a monopolistic component and a product differentiation component summarized in terms of network centrality. Interestingly, this latter component matters only to the extent that firms are competing between each other. As I show in Section 5, when there is a multi-product monopolist the product differentiation component converges to zero and monopolistic price cost margins are charged to each of the J product varieties.

4.2 Cournot Competition

In this section I focus on quantity competition a la Cournot. In this case, to define a firm's objective I need the inverse aggregate demand instead of the aggregate demand derived in Proposition (1). Compared to Bertrand, deriving the aggregate demand for the Cournot case is almost immediate. Starting from (3), and defining α and β as in Proposition (1) the aggregate inverse demand for product j is

$$p_j(q_j, q_{-j}) = \alpha - \beta(1 + \eta(x_j' x_j))q_j - \eta\beta \sum_{l \neq j}(x_j' x_l)q_l. \tag{16}$$

Similar to the Bertrand case, substitution patterns in Cournot are driven by an inner-product between products attributes. In this case, the inner-product is unweighted and the substitutability between products j and l is simply given by $x_j' x_l$. One drawback of this is that, without a normalization on the scale of characteristics, the implied product adjacency matrix can have weights larger than one in absolute value which makes it less interpretable. In the Bertrand case, as shown in Proposition (3), no normalization of the product characteristics is required.

Firm j takes competitors quantities as given and solves

$$\max_{q_j} \quad (\alpha_j - c_j)q_j - \beta(1 + \eta(x_j' x_j))q_j^2 - \eta\beta \sum_{l \neq j}(x_j' x_l)q_l q_j \tag{17}$$

The Cournot objective in (17) also defines a network game with quadratic payoffs. In this case though, the relevant adjacency matrix will be constructed from the matrix $\Theta^- \equiv -XX'$. Why this is important will be clear in the next Proposition where I derive the Cournot price cost margins as a function of the vector of Bonacich centralities.

Proposition 4 *Let $\Theta^- \equiv -XX'$ and assume that $\theta_{jj}^- = \theta^-$ for all $j \in \{1, ..., J\}$. If an interior equilibrium $q^* = (q_j^*)_{j=1}^J$ of the Cournot game exists then it is unique and is such that*

$$p^* - c = \frac{\alpha - c}{2} + \mathbf{b}\left(A(\Theta^-), \frac{\eta}{2(1 - \eta\theta^-)}, \frac{\alpha - c}{2}\right) \tag{18}$$

provided $\theta^- < \frac{1}{\eta} - \frac{1}{2}\max_j |\lambda_j(A(\Theta^-))|$ where $\lambda_j(A)$ is the j-th eigenvalue of the adjacency matrix $A(\Theta^-)$.

The expression for Cournot price-cost margins resembles very closely the one for Bertrand described in (13). The most apparent difference is that now price-cost margins seem to be increasing a product's centrality, but this is not case. To see this, note that the relevant adjacency matrix here is $\Theta = -XX'$ which, because of the minus sign, assigns higher centrality whenever a product becomes less substitutable with any other product or equivalently more differentiated. Overall, as one would expect, both Cournot and Bertrand price-cost margins are higher for more differentiated products.

In the context of Cournot competition, Pellegrino (2023) also develops a decomposition of firms' markups, defined as p_j/c_j, into a productivity component, defined as α_j/c_j and into a product centrality component, denoted by $1 - \chi_j$ in the paper. Although different, the two decompositions are related to each other. More precisely, in Appendix B, I show that the product centrality defined in Pellegrino (2023) is an affine transformation of the Bonacich product centrality that enters the Cournot price-cost margins in equation (18).

Comparison with Ballester, Calvó-Armenagol and Zenou (2006). As I did for the Bertrand case, it is interesting to compare the result in Proposition (4) to the more general result provided in Ballester et.al. Differently from Bertrand, the linear component of the quadratic payoff in (17) does not depend on the network structure which makes the Cournot game analogous to the network game considered in Ballester et.al., provided one defines the adjacency matrix as $-XX'$.

4.3 Bertrand vs Cournot

In this subsection, I compare the Bertrand and Cournot equilibrium outcomes. I start by describing the main differences between the two in terms of the product characteristics space. Then, I study how the network structure depends on the nature of competition and show that, when there is strategic complementarity in prices, Bertrand competition leads to a network in which each node is more central and where price cost margins are lower.

Product characteristics. In both models product characteristics affect substitution patterns through an inner-product product matrix. In Cournot this matrix is simply the inner product between each product's vector of attributes and is given by XX'. In Bertrand, as shown in Proposition 1, this inner-product matrix is instead $X\Omega^{-1}X'$ where Ω is a $K \times K$ matrix that reweights each vector of characteristics. At first glance, it might seem that product characteristics are different in the two models but, because Ω is positive definite, this turns out not to be the case.

To see this more formally, note that Ω is diagonalizable through an orthonormal basis of eigenvectors S such that $\Omega^{-1} = S'\Lambda S$ where $\Lambda = diag(\lambda_1, ..., \lambda_K)$ is a diagonal matrix that collects the eigenvalues of Ω^{-1}. Then, by letting $\tilde{X} = XS$, we can redefine product characteristics without changing their inner-product $\tilde{X}\tilde{X}' = XX'$. Thus comparing XX' to $X\Omega^{-1}X'$ is equivalent to comparing $\tilde{X}\tilde{X}'$ to $\tilde{X}\Lambda\tilde{X}'$. From this second comparison it is easy to see that for any characteristic k, if $\tilde{x}_{jk} \geq \tilde{x}_{lk}$, then $\sqrt{\lambda_k}\tilde{x}_{jk} \geq \sqrt{\lambda_k}\tilde{x}_{jk}$, where the latter inequality holds because Ω is positive definite and thus $\lambda_k > 0$ for all k. In practice this means that, under Bertrand, each product characteristic is just rescaled by a positive number λ_k and thus whether product j offers more of characteristic k than product l is independent of the type of competition model, as one would expect.

Another important difference, is that the competitive network implied by Bertrand does not depend on the units with which product characteristics are measured. From Proposition 3, we know that θ_{jl}, or equivalently $x_j'\Omega^{-1}x_l$, or equivalently $\tilde{x}_j'\Lambda\tilde{x}_l$, always lie in $(-1, 1)$ regardless of the units with which each characteristic is measured. This is an appealing property in an empirical context.

Competition and network structure. The next proposition shows how the nature of competition influences the structure of the competitive network by comparing the network centralities implied by price and quantity competition respectively.

Proposition 5 *Assume that $\theta_{jk} \geq 0$ if $j \neq k$. Then each node centrality in the network implied by Bertrand competition is higher than the one under Cournot competition i.e., $\tilde{\mathbf{b}}_b \geq -\tilde{\mathbf{b}}_c$ where $\tilde{\mathbf{b}}_b \equiv (I_J - diag(\Theta))^{-1/2}\mathbf{b}_b$ and $-\tilde{\mathbf{b}}_c \equiv (I_J - diag(\Theta^-))^{1/2}(-\mathbf{b}_c)$ are the vectors of Bertrand and Cournot centralities respectively.*

As described in Section 2 several papers in the industrial organization literature have studied how equilibrium outcomes compare across the two types of model of competition. In general, under strategic complementarity in prices, it is known that Bertrand competition with differentiated products is more efficient than Cournot competition and leads to lower prices and higher consumer surplus.[12] The same is true in this network setting where the intensity of competition is captured by the vector of Bonacich centralities. Proposition (5) shows that under Bertrand each node has an higher centrality and thus faces more intense competition. Moreover, looking back at the results in Propositions (3) and (4), one can see that the vector of Bonacich centralities is the only determinant of the wedge between Cournot and Bertrand prices. Hence, the effect of having a more intense competition on equilibrium prices must be entirely captured by differences in centrality.

A couple of remarks. Before concluding this section, a couple of remarks are in order. First, note that the centrality implied by the Cournot model enters the inequality in Proposition (5) with a negative sign. This is because the centrality in Cournot is based on the matrix $-XX'$ which assigns a lower centrality to a more substitutable product. The measure $\mathbf{b}_b$ implied by Bertrand instead assigns higher centrality to more substitutable products and should be compared to $-\mathbf{b}_c$. To see this mathematically, suppose we knew that Cournot equilirbium prices p_c were to be higher than Bertrand prices p_b then by combining equations (13) and (18) the inequality in Proposition (5) would follow immediately.

Second, the Bonancich centralities are scaled by two positive diagonal matrices respectively. This scaling appears because I am not imposing homogeneity across j of the θ_{jj} and θ_{jj}^{-} and the vectors of Bonacich centralities enter equilibrium price cost margin after this positive re-scaling.[13] Furthermore, because the scaling is positive, the effect of product differentiation on equilibrium price cost margins remains unaffected; higher centrality implies lower markups.[14]

5 Multiproduct Firms

In this section I allow for the possibility that the same firm owns multiple products. I index firms by f and denote by J_f the set of products offered by firm f. Assuming there are $F < J$ firms, the $F \times J$ matrix R keeps track of which product belongs to which firm i.e., $r_{fj} = \mathbf{1}\{j \in J_f\}$ and the $F \times F$ matrix $H = R'R$ denotes the ownership

[12] With homogenous products the efficiency of Bertrand competition is maximized; even with only two firms the perfectly competitive outcome obtains.

[13] In Proposition 3 and 4 this homogeneity assumption was made only for expositional purposes. For more details refer to the proofs of Propositions 3, 4 and 5 which are presented for the homogeneous case.

[14] A similar type of re-scaling also appears in the more general setting considered by Ballester, Calvó-Armenagol and Zenou (2006). If we allow for heterogeneity in the curvature (σ_{ii} in their notation) of own marginal returns Remark 2 in their paper suggests to scale all elements of the adjacency matrix by its diagonal elements σ_{ii}. My scaling is similar but it preserves the symmetry of the adjacency matrix.

matrix. Moreover, as typically assumed in the context of multi-product firms, I will assume that one product can only be owned by one firm or equivalently that the sets $(J_f)_{f=1}^F$ forms a partition of the set $\{1, ..., J\}$. The latter implies that the ownership matrix H will be a block-diagonal matrix with F blocks where the f-th block is given by the matrix of ones $1_{|J_f|}1'_{|J_f|}$.

In what follows I will focus on Bertrand competition but everything can be restated in terms of Cournot competition. I start by defining the profit maximization problem for a generic firm f and frame it as a Network game. I show that in the case of multiproduct firms the implied adjacency matrix only keeps track of the competitive links between products across different firms but not between products within the same firm. Finally, I extend the result in Proposition 13 and derive equilibrium price cost margins in terms of the vector of Bonacich centralities. The result points to an intuitive comparative static in terms of the conduct parameter H: when moving toward a more collusive industry structure (e.g., as $H \to 1_J 1'_J$) product differentiation (or equivalently product centrality) matters less and less for the equilibrium price cost margins.

5.1 Multiproduct Firm Problem

Under Bertrand competition, firm f chooses prices $(p_j)_{j\in J_f}$ taking prices of competitors firms as given to maximize

$$\max_{(p_j)_{j\in J_f}} \sum_{j=1}^{J} r_{fj}(p_j - c_j)q_j(p_j, p_{-j}) \tag{19}$$
$$\text{s.t.} \quad q_j(p_j, p_{-j}) = \left(a_j(1-\theta_{jj}) - \sum_{l\neq j}\theta_{jl}a_l\right) - (1-\theta_{jj})p_j + \sum_{l\neq j}\theta_{jl}p_l$$

Problem (19) is a quadratic problem in the vector of prices chosen by firm f, $(p_j)_{j\in J_f}$ and as such can be framed as network game. The main difference from the single product problem in (10) is that now firm f will price its products jointly and will internalize how increasing a given price impacts the market shares of each of its products i.e., the so called portfolio effect.

How does the portfolio effect due to multi-product pricing affect the equilibrium outcomes? When interpreting oligopolistic competition as a network game the answer is fully captured by the adjacency matrix of the competitive network which, as I show formally in Proposition 6, is a function of the ownership structure H

$$A_H(\Theta) \equiv \Theta - H \odot \Theta \tag{20}$$

where $\odot$ is the Hadamard matrix product. The matrix Θ still captures the competitive links between products but in the presence of multi-product firms the links between products within the same firms are set to zero by subtracting the block diagonal matrix $H \odot \Theta$. The resulting adjacency matrix $A_H(\Theta)$ keeps track of the competitive links only between products owned by different firms.

5.2 Equilibrium Price Cost Margins for Multiproduct Firms

In the next proposition I show that the result in Proposition (3) extends to the case in which firms sell multiple products and price them jointly. The result is presented for the case in which own-price elasticities are heterogeneous across products e.g., θ_{jj} varies across j.

Proposition 6 *For a given ownership structure H, if an interior equilibrium $(p_j^*)_{j=1}^J$ of the Bertrand pricing game exists then it is unique and is such that*

$$p^* - c = \frac{\alpha - c}{2} - (I - H \odot \Theta)^{-1/2}\mathbf{b}\left(G_H(\Theta), \frac{1}{2}, (I - H \odot \Theta)^{1/2}\,\frac{\alpha - c}{2}\right) \qquad (21)$$

provided $\max_j |\lambda_j(G)| \leq 2$ *and where* $G_H(\Theta) \equiv (I - H \odot \Theta)^{-1/2}\, A_H(\Theta)\, (I - H \odot \Theta)^{-1/2}$ *is a weighted adjacency matrix with elements in* $(-1, 1)$.

Similarly to the single-product case, in the multi-product case price-cost margins can be decomposed into a monopolistic component $(\alpha - c)/2$ and into a product differentiation component proportional to the vector of Bonacich centralities $\mathbf{b}$. The fact that some subsets of products are priced jointly influences the network structure and, in turn, the centrality of each node. In particular, it affects the adjacency matrix by setting all the competitive links between products owned by the same firm to zero i.e., any (j, l) element of the matrix $A_H(\Theta)$ is zero whenever $i, j \in J_f$ for some firm f and the same is true for the matrix $G_H(\Theta)$.

One small difference between the single and multiproduct cases is that the matrix $G(\Theta)$ that enters the Bonacich centrality is not necessarily symmetric although its elements still belong to $(-1, 1)$. Luckily, this is not too much of a problem because one advantage of the Bonacich centrality compared to other measures is that it can be applied regardless of whether the network is directed or undirected.[15] This would not be the case if we were to use a measure of degree centrality which, in the case of directed networks needs to take into account the direction of the link.

Comparative static with respect to H. The expression in equation (21) points out to an insightful comparative statics in terms of the ownership structure matrix or, equivalently, the conduct parameter H. Consider first the extreme cases in which either firms are single-products (equivalently no collusion) or there is a single monopolist (equivalently perfect collusion). In the non-collusive case we can see that expression (21) collapses to the outcome described in Proposition (3) because $H = I_J$ and $H \odot \Theta =$

[15] See also Remark 3 in Ballester, Calvó-Armenagol and Zenou (2006).

$diag(\Theta)$. One the other extreme, under perfect collusion, $H = 1_J 1_J'$, $H \odot \Theta = \Theta$ and price-cost margins collapse to the monopolistic price-cost margins given by $\frac{a-c}{2}$. The reason is that, when all products are priced jointly all the competitive links in the adjacency matrix $A_{1_J 1_J'}(\Theta) = \Theta - \Theta = O$ are set to zero and product differentiation does not matter for price cost margins.

To gain more intuition, suppose that $A(\Theta)$ is non-negative or equivalently that prices are strategic complements, then as the market becomes more collusive (i.e., $H \to 1_J 1_J'$) the centrality of each product decreases because $A_H(\Theta) \to O$ element-wise and the importance of the monopolistic component (i.e., portfolio effect) in determining price-cost margins increases relative to the product differentiation component.

5.3 Market Definition and Mergers

Framing oligopolistic competition with product differentiation as a network game is insightful for at least two reasons.

First, from Proposition 6, we know that changes in the ownership structure will affect equilibrium price-cost margins only through the product differentiation component. Therefore, assuming there are no cost synergies, the only thing one needs to assess the effect of mergers on equilibrium prices is the vector of product centralities under the merger and no merger scenarios. In practice, this requires taking a stand on the relevant product characteristics for consumers. Depending on the industry under consideration, this might be more or less difficult. However, if one is willing to assume Bertrand competition, no normalization on the levels or the scale of product attributes is needed because, as I showed in Propositions 1 and 3, characteristics affect substitution patterns and price cost margins only through a normalized inner-product.

A second advantage of modelling product differentiation as a competitive network is that, as already pointed out in Hoberg and Phillips (2016), there is no need to put too much thought into defining the relevant market. To the extent that the vector of attributes adequately describes substitution patterns across products, the implied network structure defines the market; products that are more substitutable to each other will share a stronger link in the network. Thus, even a broad definition of the market, such as an entire industry, would not affect the merger analysis as long as the product characteristics considered capture the true substitution patterns across products.

6 Application: The US Automobile Industry

In this section, I estimate the model using data on the US automobile industry. The same data have been used extensively in the empirical industrial organization literature starting from the seminal contribution in Berry, Levinsohn and Pakes (1995).

Before going into the estimation details, I want to remark that although the car industry is important, it does not represent the ideal application for the current setting. The reason is that consumers in the model have a taste for variety and will consume more than one good. In the context of car choice, this is not the most realistic assumption because households typically own no more than two cars and, in most cases, would buy only one car at the time of purchase. Nonetheless, as I show later, the model produces reasonable substitution patterns and price-cost margins. To make the assumption about taste for variety empirically more appealing, in future iterations of the paper, I will estimate the model using data from industries offering products that are consumed more often, have a lower expenditure share and are differentiated more broadly.

In what follows, I describe the data sources and the empirical model. Then, I recover own-cross price elasticities and price cost margins. Lastly, I decompose price-cost margins and quantify how much of those are attributable to product differentiation.

6.1 Data

I obtain data on the US automobile industry from two different sources. First, I downloaded the data in Berry, Levinsohn and Pakes (1995) from the replication package accompanying a recent paper by Andrews, Gentzkow and Shapiro (2017). These data include the quantity sold by each car brand which I need because aggregate demand is derived in terms of quantities and not market shares. The second source of data is included in the recent Python package developed in Conlon and Gortmaker (2020), which again contains the very same automobile data but also includes the set of demand and supply instruments used in Berry, Levinsohn and Pakes (1995), which I will use to estimate demand in my model. Lastly, I collect data on the number of US households from the FRED website.

Overall, the data contains information on prices, quantities, market shares and characteristics of several car models sold in the US from 1971 to 1990. Table 1 reports sales-weighted averages of some relevant variables for each year. Average quantities sold are in units of 1000, average prices are in $1000 units, and the number of households is in millions. The last five columns are sales-weighted averages of product characteristics: HP/WT is the ratio of horsepower to weight, Air is a dummy for whether air conditioning is standard, Size captures the space of the car, MP$ measures the number of ten-mile increments one could drive for $1 worth of gasoline, and lastly MPG measures the number of ten-mile increments one could drive with one gallon of gasoline.

Several interesting patterns emerge from Table 1. The number of competing firms has been roughly constant, ranging between 17 and 22 multiproduct car manufacturers. The same is true for the average number of car models produced by a single

Year	Firms	Models	Own models	Competitors	Quantity	Price	Households	HP/WT	Air	Size	MP$	MPG
1971	18	92	5.11	86.89	86.89	7.87	64.78	0.49	0.00	1.50	1.85	1.66
1972	19	89	4.68	84.32	98.62	7.98	66.68	0.39	0.01	1.51	1.87	1.62
1973	17	86	5.06	80.94	92.79	7.53	68.25	0.36	0.02	1.53	1.82	1.59
1974	17	72	4.24	67.76	105.12	7.51	69.86	0.35	0.03	1.51	1.45	1.57
1975	19	93	4.89	88.11	84.77	7.82	71.12	0.34	0.05	1.48	1.50	1.58
1976	21	99	4.71	94.29	93.38	7.79	72.87	0.34	0.06	1.51	1.70	1.76
1977	18	95	5.28	89.72	97.73	7.65	74.14	0.34	0.03	1.47	1.83	1.95
1978	18	95	5.28	89.72	99.44	7.64	76.03	0.35	0.03	1.40	1.93	1.98
1979	18	102	5.67	96.33	82.74	7.60	77.33	0.35	0.05	1.34	1.66	2.06
1980	19	103	5.42	97.58	71.57	7.72	80.78	0.35	0.08	1.30	1.47	2.21
1981	19	116	6.11	109.89	62.03	8.35	82.37	0.35	0.09	1.29	1.56	2.36
1982	19	110	5.79	104.21	61.89	8.83	83.53	0.35	0.13	1.28	1.82	2.44
1983	18	115	6.39	108.61	67.88	8.82	83.92	0.35	0.13	1.28	2.09	2.60
1984	20	113	5.65	107.35	85.93	8.87	85.41	0.36	0.13	1.29	2.12	2.47
1985	20	136	6.80	129.20	78.14	8.94	86.79	0.37	0.14	1.26	2.02	2.26
1986	22	130	5.91	124.09	83.76	9.38	88.46	0.38	0.18	1.25	2.86	2.42
1987	21	143	6.81	136.19	67.67	9.97	89.48	0.39	0.23	1.25	2.79	2.33
1988	20	150	7.50	142.50	67.08	10.07	91.07	0.40	0.24	1.25	2.92	2.33
1989	21	147	7.00	140.00	62.91	10.32	92.83	0.41	0.29	1.26	2.81	2.31
1990	20	131	6.55	124.45	66.38	10.34	93.35	0.42	0.31	1.27	2.85	2.27

Table 1: Sales-weighted averages.

manufacturer, which increased slightly from 5 to 6.5. On the other hand, the total number of available car models increased more than 40%, from 92 in 1971 to 131 in 1990. At the same time, the average number of models produced by competitor firms also increased by 40%, from 87 models in 1971 to 124 in 1990, suggesting that competition in the number of products increased for the average firm. Next, looking at the average quantity and prices, the former decreased with cyclical ups and downs over time. In contrast, sales-weighted prices increased throughout the '80s while being roughly constant in the '70s. Product characteristics have also changed. For instance, air-conditioning becomes a more common feature over time, and both tens of miles per dollar (MP$) and tens of miles per gallon (MPG) increased, suggesting that cars have become more efficient over time. At the same time, car size seems to have decreased, whereas horsepower has been roughly constant.

Before turning to the estimation of the model and recovering products' price cost margins, it is helpful to get a sense of how prices correlate with product characteristics in the raw data without imposing any modelling structure. To this end, Figure 1 presents a binscatter of car prices against an unweighted measure of product centrality which summarizes how differentiated a product is relative to its competitors. The measure of centrality on the x-axis is motivated by the result I provided in Proposition (3) but differs in several respects because some terms are unobserved. The intuition described in equation (13) is simple: the margin over marginal cost a firm can charge depends negatively on how central its product is in the competitive network. Empirically though, whether or not this relationship between margins and centrality holds cannot be tested directly because marginal costs (c) and consumer preferences (α) are

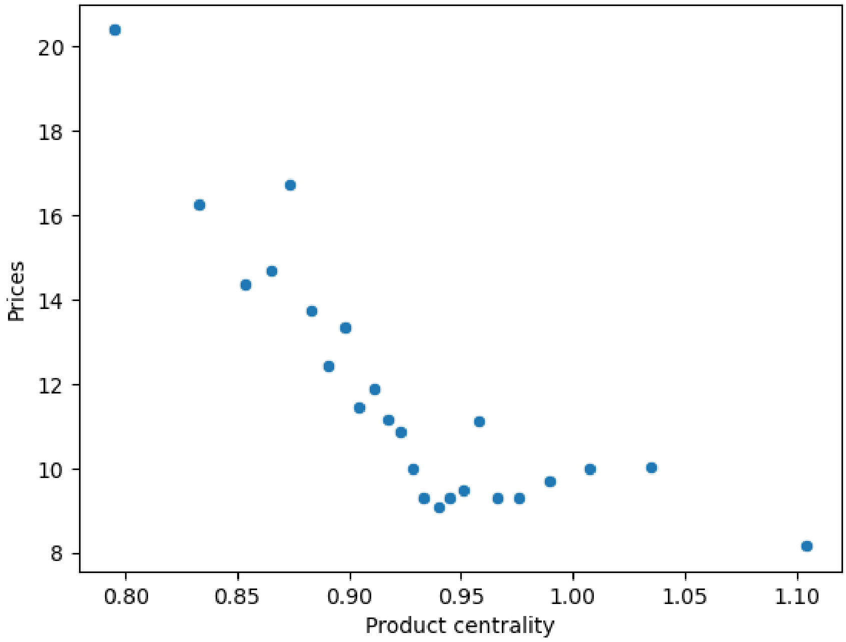

Figure 1: Binscatter of car prices against (unweighted) product Bonacich centrality after partialling out time fixed effects.

unobserved. Nonetheless, Figure 1 attempts to do so naively by proxying margins (i.e., the left-hand side of equation (13)) with observed prices and the Bonacich centrality $\mathbf{b}(A(\Theta), 1/2, (\alpha - c)/2)$ (i.e., the second term in (13)) with a similar centrality measure where $((\alpha - c)/2)$ is replaced with a vector of ones 1_J. The resulting measure is an unweighted Bonacich centrality given by

$$\mathbf{b}\left(A(\Theta), 1/2, 1_J\right) = \left(I_J - \frac{1}{2}A(\Theta)\right)^{-1} \frac{1}{2}A(\Theta)1_J, \tag{22}$$

which I can compute directly from the raw data. To compute Θ, I use the same product characteristics that enter the indirect utility in Berry, Levinsohn and Pakes (1995) namely horsepower (HP/WT), air conditioning, size and miles-per-dollar (MP$), and I calibrate η to 0.136.[16]

Returning to Figure 1, we can see that products with higher unweighted Bonacich centrality tend to charge lower prices. This decreasing relationship looks stronger for lower values of product centrality and seems to flatten as centrality increases. Although prices and centrality are negatively correlated as predicted by the network Bertrand model, from Figure 1, we cannot conclude that less central firms can charge higher margins and thus have more market power.[17] The reason is that the relationship

[16] The parameter η corresponds to the $\frac{\alpha}{1-\alpha}$ in Pellegrino (2023) which is calibrated to α to 0.12.

[17] Similarly, if prices and centrality were positively correlated or uncorrelated in the data, we could not conclude that centrality does not influence price cost margins as predicted by the model.

can be driven by unobserved costs or heterogeneity in consumer preferences, which we are not accounting for. To uncover these unobservable components, we need a structural model of competition that allows us to identify consumer preferences and marginal costs. In the next section, I leverage the structure of the Bertrand network model to identify and estimate both the linear and quadratic components of consumer preferences. After estimating these demand parameters, I recover firms' marginal costs from the Nash-Bertrand equilibrium conditions.

6.2 The Empirical Nash-Bertrand Network Model

In this section, I make the Bertrand network model described in Section 4.1 empirically operational and adapt it to the context of car consumption.

An implicit assumption of the demand model introduced in Section 3 is the absence of income effects due to the quasi-linearity of consumer preferences in the outside good. In the context of car purchases, this assumption might be unrealistic. For this reason, I will now extend the demand model parsimoniously to accommodate income effects. To this end, I follow Berry, Levinsohn and Pakes (1995), and model consumer preferences for the inside and outside goods in a Cobb-Douglas fashion

$$U_i\left(y_i - p'q_i, q_i; X\right) = \left(y_i - p'q_i\right)^{\gamma}\left[G_i\left(q_i, X\right)\right]^{\phi} \tag{23}$$

where the first term in U already substitutes for the budget constraint and

$$G(q_i, X) \equiv \exp\left\{q_i'\alpha_i - \frac{\beta_i}{2}q_i'\left(I_J + \eta X'X\right)q_i\right\}.$$

Next, substituting G into (23) and taking logs, consumer i's utility can be written as

$$\begin{aligned} u_i(q_i, X) \equiv \log\left(U_i\right) &= \gamma\log(y_i - p'q_i) + q_i'\alpha_i - \frac{\beta_i}{2}q_i'\left(I_J + \eta X'X\right)q_i & (24)\\ &\approx q_i'\left(\alpha_i - \frac{p}{y_i}\right) - \frac{\beta_i}{2}q_i'\left(I_J + \eta X'X\right)q_i & (25) \end{aligned}$$

where I normalize $\phi = 1$ and $\gamma = 1$ because they cannot be separately identified from α_i and β_i, and I use a first order Taylor expansion to approximate $\log\left(y_i - p'q_i\right)$.[18] The preferences in (25) are identical to the ones described in Section 3 except for the fact that prices are now measured relative to income. Consumer i's demand system can be derived as before

$$q_i(p) = \frac{1}{\beta_i}\left(I_J + \eta XX'\right)^{-1}\left(\alpha_i - \frac{p}{y_i}\right). \tag{26}$$

To derive the aggregate demand for product j we need to integrate over the distribu-

[18]The same approximation has been used in Berry, Levinshon and Pakes (1999).

tion of consumer preferences (α_i, β_i, y_i). Assuming that y_i is independent of (α_i, β_i),[19] aggregate demand for product j is given by:

$$q_j(p_j, p_{-j}) = a_j - \frac{1}{\beta}(1-\theta_{jj})\frac{p_j}{y} + \frac{1}{\beta}\sum_{k\neq j}\theta_{jk}\frac{p_k}{y} \tag{27}$$

where $\alpha = \int \frac{\alpha_i/\beta_i}{1/\beta_i}di$, $\beta = M\left[\int \frac{1}{\beta_i}di\right]^{-1}$, $y = M\left[\int \frac{1}{y_i}di\right]^{-1}$ and $\theta_{jk} = x_j'\Omega^{-1}x_k$. Overall, the demand functions are identical to the ones presented in Section 3 except that income effects are now present.

To make the model empirically operational, let t denote a market (i.e., a year in our empirical context) and note that equation (27) can be rearranged in vector form as

$$\tilde{q}_t \equiv (I_{J_t} + \eta X_t X_t')q_t = \frac{1}{\beta}\left(\alpha_t - \frac{p_t}{y_t}\right) \tag{28}$$

where J_t is the number of car models available in market t, X_t is a $J_t \times K$ matrix of product characteristics and the linear preference parameter vector α is allowed to vary over time. Next, consider the jth equation of the above system

$$\tilde{q}_{jt} = \frac{1}{\beta}\left(\alpha_{jt} - \frac{p_{jt}}{y_t}\right) \tag{29}$$

and note that, upon calibrating η and assuming that the matrix X_t contains only observables characteristics, the left-hand side in (27), denoted by $\tilde{q}_{jt}$, is directly measurable. Conversely, on the right-hand side of (27), only p_t and y_t are observable.[20]

More generally, equation (29) suggests that we can estimate demand using the following linear specification

$$\tilde{q}_{jt} = -\frac{1}{\beta}\frac{p_{jt}}{y_t} + w_{jt}'\zeta + \xi_{jt} \tag{30}$$

where w_{jt} is a vector of observable product and demographic characteristics which, in this context, includes both x_{jt} and y_t. On the other hand, ξ_{jt} includes characteristics that are unobservable to the econometrician but known by the agents.

To summarise, two assumptions allow us to estimate demand from (30). First, all the characteristics that enter consumer preferences in the quadratic term are observable. Second, any unobserved characteristic (ξ_{jt}) enters consumer preferences only

[19] In empirical IO it commonly assumed that preferences parameters are idiosyncratic and independent from demographics.

[20] To measure income in a given year, I use the simulated draws that come with the pyBLP package developed in Conlon and Gortmaker (2020), and I average them using the weights provided. The draws come from a log-normal distribution of income whose location and scale parameters are estimated from the Current Population Survey (CPS) each year as described in Berry, Levinsohn and Pakes (1995).

through the linear parameter vector α, i.e.,[21]

$$\alpha_{jt} = \beta(w'_{jt}\zeta + \xi_{jt}) \tag{31}$$

To consistently estimate (30), we need to instrument p_{jt}/y_t because prices will be correlated with the unobservable component ξ_{jt}. The reason is that firms internalize ξ_{jt} before setting prices simultaneously. To see this formally, recall that Nash equilibrium prices are given by

$$p_{jt} = c_{jt} + \frac{y_t\alpha_{jt} - c_{jt}}{2} + \mathbf{b}_{jt} \tag{32}$$

and note that those prices are a function of α_{jt} (and in turn function of ξ_{jt}) both directly and indirectly through product j's Bonacich centrality $\mathbf{b}_{jt}$.[22] Under this setting, demand can be estimated with a simple linear instrumental variable strategy which I describe next.

6.3 Estimation Results

I start by estimating demand from the linear specification in (30). To do so, I instrument the term p_{jt}/y_t using the set of demand instruments z_{jt} constructed in Berry, Levinsohn and Pakes (1995) and available in the pyBLP Python package developed by Conlon and Gortmaker (2020). These instruments are a function of product characteristics and, as a consequence, are correlated with prices because, per the supply equation (32), characteristics affect firms' pricing decisions through the centrality term $\mathbf{b}_{jt}$ and the observable part of α_{jt}. The fact that our instruments are correlated with prices is not enough, and we also need to ensure that demand remains constant while instruments shift the supply. The identifying assumption relies on the idea that firms choose characteristics before observing any demand shock ξ_{jt} which formally boils down to requiring that $\mathbb{E}[\xi_{jt}|z_{jt}] = 0$.

Table 2 reports both OLS and 2SLS demand estimates for the linear specification in equation (30). In both cases, the vector of characteristics w_{jt} includes a dummy for air conditioning, miles per dollar (MP\$), horsepower (HP/WT), space and the interaction between all of those with income. The coefficients on characteristics are similar across the two specifications, and, in both cases, only horsepower and miles per dollar are significantly different from zero. Both coefficients are positive, suggesting that the average consumer prefers cars with more horsepower and cars that are more efficient in gasoline consumption. The estimated price-to-income coefficient is the most evident difference between the OLS and 2SLS specifications. With OLS, the coefficient is not

[21] I am also assuming that the parameter β is constant across markets. This assumption can be partially relaxed by interacting prices with any market level observable in equation (30).

[22] The derivation of (30) is analogous to the one presented in the proof of Proposition (3) with the exception that the income term now appears.

	OLS	IV-2SLS
Constant	-0.1023	-0.1005
	(0.0043)	(0.0044)
Air (dummy)	-0.0179	0.0039
	(0.0145)	(0.0183)
MP$	0.0331	0.0368
	(0.0104)	(0.0104)
HP/WT	0.1432	0.1040
	(0.0448)	(0.0476)
Space	-0.0138	-0.0115
	(0.0168)	(0.0166)
price/income	-0.0043	-0.0327
	(0.0037)	(0.0121)
Fstat (Excluded)	-	92.1276
R2	0.8704	0.8660
Observations	2,217	2,217

Table 2: Demand estimates. Both specifications include interactions between characteristics and income, not reported here but available in Appendix C. Standard errors are clustered at the car model level.

statistically different from zero. In contrast, when we instrument for prices, the coefficient becomes negative and significant, and its magnitude increases almost ten folds in absolute value. The discrepancy between OLS and IV estimates is quite common in contexts where prices and quantities are determined simultaneously in equilibrium. OLS estimates often imply inelastic demand curves because the observed variation in quantity and prices is also due to shifts in demand. However, after instrumenting for prices, the resulting estimates recover demand curves that are much more elastic.

After estimating the demand parameters, one would recover firms' marginal costs from equation (32). One issue in our context is that we do not observe α_{jt}, and we need to estimate it before being able to back out c_{jt}. Luckily, we can obtain an estimate of $\hat{\alpha}_{jt}$ for each product j and market t by simply plugging our demand estimates $(\hat{\beta}, \hat{\zeta})$ and the estimated regression residuals $(\hat{\xi}_{jt})$ into equation (31),

$$\hat{\alpha}_{jt} = \hat{\beta}\left(w'_{jt}\hat{\zeta} + \hat{\xi}_{jt}\right). \tag{33}$$

From the pricing equation in (32), with an estimate of α_{jt}, we can then recover marginal costs c_{jt}, price-cost margins $p_{jt} - c_{jt}$ and decompose the latter into monopolistic price cost margins $(\alpha_{jt} - c_{jt})/2$ and network centrality $\mathbf{b}_{jt}$.

Figures 2 and 3 show the distribution of both marginal costs and price-cost margins in \$1000.[23] The median marginal cost across models and years is around \$6,900,

[23] I removed roughly 6% of observations estimated to have negative marginal costs.

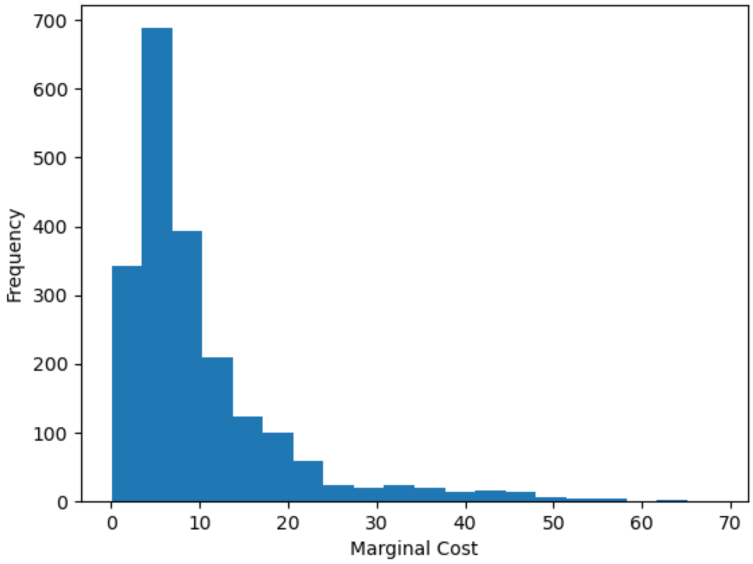

Figure 2: Distribution of marginal costs.

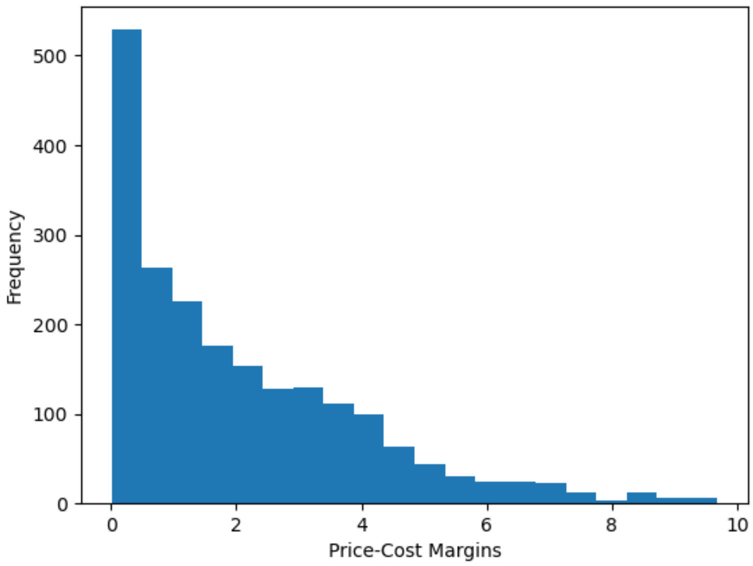

Figure 3: Distribution of price-cost margins.

and 3/4 of cars model have a marginal cost lower than \$12,500. The distribution of estimated price-cost margin is more spread out, with a median margin around \$1,500 and 75% of car models charging a margin lower than \$3,200. Overall, although I started from linear-quadratic preferences rather than discrete choice, the estimated price cost margins seem reasonably close to the ones recovered in Berry, Levinsohn and Pakes (1995).

Next, I decompose price cost margins into a monopolistic component and a product centrality component which measures how differentiated a given product is relative to its competitors. Table 3 reports sales-weighted averages of such decomposition for each year available. The second column shows that the average marginal cost (MC) was stable throughout the '70s and then increased in the '80s. The third column reports average price-cost margins (PCM) in \$1000. Following equation (13), the fourth and fifth columns decompose PCMs into monopolistic price-cost margins (MPCM) and a Bonacich centrality component (BC).[24] The monopolistic margins, given by $(\alpha_{jt} - c_{jt})/2$ vary considerably across years ranging from around \$50,000 up to almost \$170,000 and suggesting that the linear component of consumer preferences α_{jt} varies substantially over time. This time variation in preferences can capture changes in economic conditions, changes in regulation such as the import restriction on Japanese cars imposed throughout the '80s and changes in the type and number of car models available. The product centrality component $\mathbf{b}_{jt}$ also shows the same type of time variation and is, in magnitude, quite close to the monopolistic margins.

Figure 4 takes a closer look by plotting the monopolistic margins (MPCM) together with the product centrality (BC) on the left y-axis and the price cost margins on the right y-axis over time. Monopolistic margins and product centrality follow a similar cyclical behaviour suggesting that the main driver could be time variation in consumer preferences α_{jt}, which affects monopolistic margins but also the Bonacich product centrality through the weights.[25] Price-cost margins slightly fluctuate over time and seem to follow the cyclical behaviour of the other two variables. However, because the scale of the magnitude is way smaller, it is safe to conclude that PCMs have been basically constant over time, ranging on average between \$3000 and \$5000 overall but mostly in between \$3000 and \$4000 from 1975 onward.

The last two columns of Table 3 show what percentage of the monopolistic margins are captured by car manufacturers (PCM/MPCM) and what percentage of these margins is lost to competition between products (BC/MPCM). Not surprisingly, given the magnitudes observed in columns four and three, the last column shows that more than 90% of monopolistic price cost margins are eaten up by competition, suggesting that the competitive network is dense and most of the products are close substitutes to

[24]The decomposition used in Table 3 takes into account the fact that products have heterogeneous own-price elasticities and that firms are multiproduct.

[25]See equation (13) and Definition 1.

Year	MC ($1000)	PMC ($1000)	MPCM ($1000)	BC ($1000)	PMC/MPCM (%)	BC/MPCM (%)
1971	4.435	4.668	92.937	88.269	4.99	95.01
1972	4.290	4.825	101.971	97.146	4.79	95.21
1973	4.096	4.695	89.996	85.301	5.26	94.74
1974	4.256	4.295	65.251	60.956	6.51	93.49
1975	4.462	3.803	66.107	62.304	5.70	94.30
1976	4.247	3.990	88.535	84.545	4.53	95.47
1977	4.299	4.014	93.829	89.815	4.33	95.67
1978	4.198	4.091	101.548	97.457	4.08	95.92
1979	4.541	3.708	70.299	66.592	5.24	94.76
1980	4.503	3.514	49.657	46.143	7.05	92.95
1981	5.627	3.151	49.460	46.309	6.36	93.64
1982	6.270	2.848	55.414	52.566	5.13	94.87
1983	5.906	3.281	75.745	72.464	4.39	95.61
1984	4.917	4.259	98.949	94.690	4.36	95.64
1985	5.767	3.614	101.260	97.645	3.62	96.38
1986	5.556	4.133	175.756	171.623	2.42	97.58
1987	6.578	3.807	150.768	146.960	2.56	97.44
1988	6.782	3.834	167.875	164.042	2.28	97.72
1989	7.288	3.600	146.416	142.816	2.46	97.54
1990	7.417	3.326	135.846	132.520	2.48	97.52

Table 3: Sales-weighted averages of the decomposition of price-cost margins (PMC).

each other. The reason for this could be that, although products are differentiated, on average, a firm faces more than 85 competitors each year (i.e., Table 1 column four), and each product likely has a close substitute in terms of the observable characteristics we are considering.[26] The second to last column shows the other side of the medal; firms only capture from 2% to 7% of the margins they could potentially charge in a monopolistic market.

Our estimates suggest that car manufacturers capture only a small fraction of the monopolistic margins they could charge. However, are those margins as tiny as they look? To answer this question, I compare the estimated PCMs (as a percentage of the MPCMs) with the margin a firm would charge in a Cournot game with homogeneous products. In a homogenous Cournot model with N symmetric firms and linear demand, the equilibrium (dollar) price cost margins are given by

$$p^{\text{hom. cournot}} - c = \frac{\alpha - c}{N+1} \tag{34}$$

where α is the demand intercept and c is the marginal cost. As a fraction of the monopolistic price cost margin, the homogenous Cournot margins are only a function of the number of firms N

$$\frac{p^{\text{hom. cournot}} - c}{p^{\text{monopolist}} - c} = \frac{\alpha - c}{N+1} \cdot \frac{2}{\alpha - c} = \frac{2}{N+1}. \tag{35}$$

[26] Recall that we are assuming that all characteristics that determine how central/differentiated a given product is are observable.

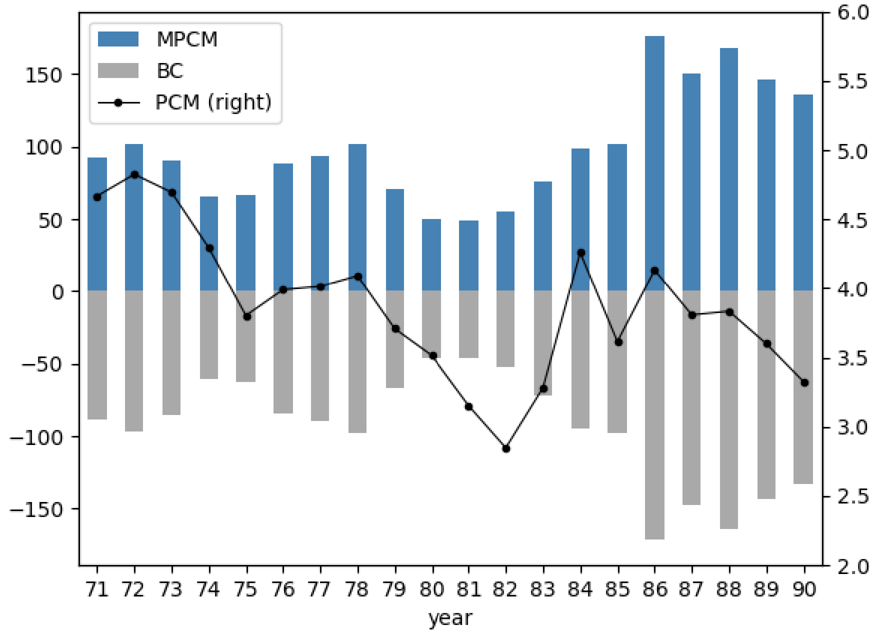

Figure 4: Decomposition of price-cost margins. All variables are measured in \$1000

In Figure 5, I plot the estimated Bertrand PCM/MPCM reported in Table 3, together with the PCM/MPCM for the homogeneous Cournot model derived in equation (35) where I set the number of firms N equal to the average number of competitors a given car manufacturer faces in a given year (i.e., the number reported in Table 1) plus one. The pattern is clear; the margins firms charge in the Network Bertrand game are consistently above the margins under the homogeneous Cournot. Depending on the year, Bertrand margins can be more than three times higher than Cournot. This happened throughout the '80s when a voluntary export restraint was placed on exports of automobiles from Japan to the United States. More generally, because the average number of competitors is the same across the two models, the difference between the two curves can be interpreted as the increase in margins that oligopolistic firms can capture when they offer differentiated products. Offering a differentiated product allows firms to increase their margins by as much as three folds. Lastly, note that the difference between the Network Bertrand margins and the homogeneous Cournot margins represents a conservative estimate of the ability of firms to increase markups when products are differentiated. The reason is that Cournot competition generates positive margins even if products are homogeneous. If we used a homogeneous Bertrand to compare, the markup increase would be more significant because firms would be pricing at cost and charge zero margins.

I conclude the section by recovering own and cross-price elasticities implied by the demand estimates reported in Table 2. Recall from Section 3 that the (j, l) element of

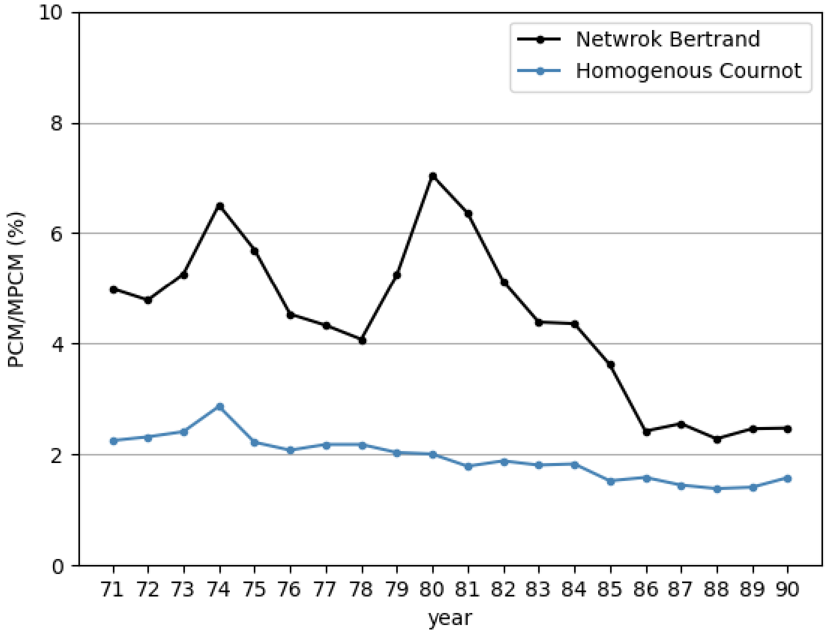

Figure 5: PCM/MPCM in Network Bertrand

the elasticity matrix is given by

$$\epsilon_{jl} = \begin{cases} -\frac{1}{\beta}(1-\theta_{jj}) & \text{if } j = l \\ \frac{1}{\beta}\theta_{jl} & \text{if } j \neq l \end{cases} \tag{36}$$

Because θ_{jl} is a function of observable product characteristics, we only need an estimate of $1/\beta$ to recover both own and cross-price elasticities.

Figure 6 plots a heatmap of the elasticity matrix for the year 1990. In that year, there were 131 car models available (Table 1), and Figure 6 shows the estimated own and cross-price elasticities for those models with an estimated own price elasticity larger than median.[27] Two remarks are in order. First, the magnitudes of own and cross-price elasticities are reasonable, with the own-price elasticities estimated to be negative and larger than the cross-price elasticities. Second, almost all cross-price elasticities are estimated to be small and positive. The latter would not be surprising in a model based on logit demand because that model restricts substitution patterns in a way that makes all products substitutes. However, this is not true in the linear-quadratic demand model I developed in Section 2. This type of quadratic preference does not impose any restrictions on product substitution patterns. However, the estimated cross-price elasticities are positive, suggesting that most car models are indeed substitutes for each other. Moreover, note that the fact that cross-price elasticities are

[27]This is done for visibility purposes. Estimates remain reasonable if we look at all models with own-price elasticity below the 75th percentile, reported in Appendix C Figure 7

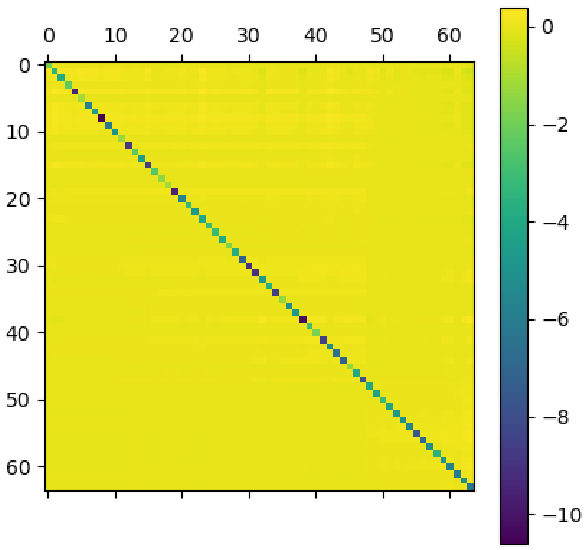

Figure 6: Matrix of estimated own and cross price elasticities for the year 1990. The products included are the ones with an estimated own price elasticity above the median.

positive is not a consequence of the fact that all product characteristics are positive numbers. More formally, having that $x_{jk} > 0$ for any product j and characteristic k does not imply that all products are substitutes or, equivalently, that θ_{jl} is always positive.

7 Conclusion

This paper studies how product differentiation affects substitution patterns, and firms' price cost margins in oligopolistic markets where products are differentiated over multiple attributes and consumers have linear-quadratic preferences. Under these assumptions, oligopolistic competition in either prices or quantities can be framed as a network game where a product location in the network is determined by its vector of attributes, and the network links between products capture the extent to which two products compete. Products with similar characteristics will be closer to each other and will compete more intensely. On the other hand, products with more unique characteristics will have a more peripheral location and enjoy more market power.

More precisely, I showed that firms' price cost margins can be decomposed additively into a monopolistic component and a product differentiation component. The first component coincides with the margins a monopolist would charge. In contrast, the second component is proportional to how central a product is in the network as measured by its Bonacich centrality. Products with higher centrality must give up a more significant fraction of the monopolistic margins because they face more competition. Furthermore, I showed that this decomposition holds for both Bertrand and Cournot price cost margins and can be extended to allow the presence of multiproduct firms.

In the second part of the paper, I show how to estimate the model using market-level data on a given industry. Under the assumption that unobserved product characteristics enter consumer preferences only through the linear component of the utility, a simple linear IV strategy identifies the demand parameters. Then, marginal costs and margins can be recovered from the Nash equilibrium pricing equations. In addition, using the decomposition of price-cost margins, one can quantify what fraction of the potential monopolistic margins a firm can capture by differentiating its product from its competitors.

Lastly, I estimated the model using data on the US automobile industry from 1971 to 1990. I found that it delivers substitution patterns and price-cost margins comparable to the ones estimated in the literature that models automobile demand starting from individual discrete choice problems. Interestingly, although the linear-quadratic demand model does not restrict substitution patterns in any way, the estimated cross-price elasticities are almost always positive, suggesting that cars are substitutes, something the discrete choice demand framework instead imposes a priori. Finally, I decompose firms' price cost margins and find that car manufacturers capture from 2% to 7% of the monopolistic margins depending on the year. These margins can be as high as three times the margins firms would be able to charge if their products were to be homogeneous.

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A Proofs

Proof of Proposition 1. Taking the horizontal sum of individual demands in (3) it is easy to check that the aggregate demand system is given by

$$q(p) = \int q_i(p)\frac{di}{M} \tag{37}$$

$$= \frac{1}{\eta\beta}\left(\frac{1}{\eta}I_J + XX'\right)^{-1}(\alpha - p) \tag{38}$$

where $\beta \equiv \int \frac{1}{\beta_i}\frac{di}{M}$ and $\alpha \equiv \int \frac{\alpha_i/\beta_i di}{\int 1/\beta_i di}$. Next, note that

$$q(p) = \frac{1}{\eta\beta}\left(\frac{1}{\eta}I_J + XX'\right)^{-1}(\alpha - p) \tag{39}$$

$$= \frac{\alpha - p}{\beta} - X\Omega^{-1}X'\frac{(\alpha - p)}{\beta} \tag{40}$$

where $\Omega \equiv \frac{1}{\eta}I_K + X'X$. Then let $a \equiv \frac{\alpha}{\beta} - X\Omega^{-1}X'\frac{\alpha}{\beta}$ and $\Theta \equiv X\Omega^{-1}X'$ to obtain the expression in the main text. To complete the proof, we are left to show that $\theta_{jj} \in (0,1)$ and that $\theta_{jl} \in (-1,1)$ for any $j \neq l$. To this end, first note that by construction both Θ and $I_J - \Theta$ are positive definite matrices. Then, letting e_j be the j-th unit vector, we have that $\theta_{jj} = e_j'\Theta e_j > 0$ and similarly $1 - \theta_{jj} = e_j'(I - \Theta)e_j > 0$. Next, take any (j,l) pair with $j \neq l$ and note that

$$\theta_{jj} + \theta_{ll} - 2\theta_{jl} = (e_j - e_l)'\Theta(e_j - e_l) > 0 \tag{41}$$

where the first equality exploits the fact that Θ is symmetric. From (41) and the fact that $\theta_{jj} < 1$ for all j, we can conclude that $\theta_{jl} < 1$. To show that $\theta_{jl} > -1$ it is enough to repeat the previous argument using $\theta_{jj} + \theta_{ll} + 2\theta_{jl}$.

Proof of Proposition 2 Because U is the matrix of principal component directions of X, it is also the matrix of eigenvectors of $X'X$. Then we have, assuming without loss that $\eta = 1$,

$$\Theta = X(I_K + X'X)^{-1}X' \tag{42}$$

$$= X(I_K + U\Lambda^{x'x}U')^{-1}X' \tag{43}$$

$$= XU(I_K + \Lambda^{x'x})^{-1}U'X' \tag{44}$$

$$= \tilde{X}(I_K + \Lambda^{x'x})^{-1}\tilde{X}' \tag{45}$$

where $\Lambda^{x'x}$ is the diagonal matrix that collects the eigenvalues of $X'X$. For $\eta \neq 1$, it is enough to redefine $X = \sqrt{\eta}X$ to obtain the same result.

Proof of Proposition 3 The proof presented is for the more general case in which θ_{jj}

are heterogeneous across j. The first order condition of (10) with respect to p_j reads:

$$\alpha_j(1-\theta_{jj}) - \sum_{l\neq j}\theta_{jl}\alpha_l + c_j(1-\theta_{jj}) - 2(1-\theta_{jj})p_j + \sum_{l\neq j}\theta_{jl}p_l = 0 \tag{46}$$

after rearranging in vector form and solving for p^* one obtains

$$\begin{aligned}
p^* - c &= \frac{1}{2}\left(I_J - diag(\Theta) - \frac{A(\Theta)}{2}\right)^{-1}(I-\Theta)(\alpha - c) \\
&= \frac{1}{2}\left(I_J - \left(I_J - diag(\Theta) - \frac{A(\Theta)}{2}\right)^{-1}\frac{A(\Theta)}{2}\right)(\alpha - c) \\
&= \frac{\alpha - c}{2} - (I_J - diag(\Theta))^{-1/2}\left(I_J - \frac{G(\Theta)}{2}\right)^{-1}\frac{G(\Theta)}{2}(I - diag(\Theta))^{1/2}\frac{(\alpha - c)}{2} \\
&= \frac{\alpha - c}{2} - (I_J - diag(\Theta))^{-1/2}\,\mathbf{b}\left(G(\Theta), \frac{1}{2}, (I - diag(\Theta))^{1/2}\frac{\alpha - c}{2}\right)
\end{aligned} \tag{47}$$

where

$$G(\Theta) \equiv (I - diag(\Theta))^{-1/2}\,A(\Theta)\,(I - diag(\Theta))^{-1/2} \tag{48}$$

and the last equality is well-defined provided $\max_j |\lambda_j(G)| < 2$. Expression (13) then obtains when imposing $\theta_{jj} \equiv \theta$. To complete the proof for the more general case, I need to show that $G(\Theta)$ preserves the properties of $A(\Theta)$. It is immediate to see that $G(\Theta)$ is 0-diagonal and symmetric. I am left to show that all its off-diagonal elements g_{ij} lie in $(-1, 1)$. To see this recall that $I_J - \Theta$ is positive definite and note that

$$(I_J - diag(\Theta))^{1/2}(I_J - G(\Theta))^{-1}(I_J - diag(\Theta))^{1/2} = I_J - \Theta \tag{49}$$

which implies that $(I_J - G(\Theta))$ is also positive definite, but then

$$1 - g_{ij}^2 = (g_{ij}e_i + e_j)'(I_J - G(\Theta))(g_{ij}e_i + e_j) > 0 \tag{50}$$

which completes the proof.

Proof of Proposition 4 The proof presented is for the more general case in which θ_{jj}^- are heterogeneous across j. To find Cournot price-cost margins we first need to find the Cournot equilibrium quantities which are given by

$$q^* = \left(2I_J - \eta diag(\Theta^-) - \eta\Theta^-\right)^{-1}\frac{\alpha - c}{\beta} \tag{51}$$

where $\Theta^{-} \equiv -XX'$. Plugging (51) into the aggregate inverse demand one obtains

$$\begin{aligned}
p^{*}-c &= \alpha - c - \beta\left(I_J - \eta\Theta^{-}\right)q^{*} \\
&= \frac{\alpha-c}{2} + \eta\frac{A(\Theta^{-})}{2}\left(I_J - \eta diag(\Theta^{-}) - \eta\frac{A(\Theta^{-})}{2}\right)^{-1}\frac{\alpha-c}{2} \\
&= \frac{\alpha-c}{2} + \left(I_J - \eta diag(\Theta^{-})\right)^{1/2}\left(\eta\frac{G(\Theta^{-})}{2}\right)\left(I_J - \eta\frac{G(\Theta^{-})}{2}\right)^{-1} \times \\
&\times \left(I_J - \eta diag(\Theta^{-})\right)^{-1/2}\frac{\alpha-c}{2} \\
&= \frac{\alpha-c}{2} + \left(I_J - \eta diag(\Theta^{-})\right)^{1/2} \times \\
&\times \mathbf{b}\left(G(\Theta^{-}), \frac{\eta}{2}, \left(I_J - \eta diag(\Theta^{-})\right)^{-1/2}\frac{\alpha-c}{2}\right)
\end{aligned} \tag{52}$$

where

$$G(\Theta^{-}) \equiv \left(I_J - diag(\Theta^{-})\right)^{-1/2}A(\Theta^{-})\left(I_J - diag(\Theta^{-})\right)^{-1/2} \tag{53}$$

where the last equality is well-defined provided $\eta < \max_j |\lambda(G)|/2$. Expression (18) in the main text then obtains by replacing θ^{-}_{jj} with θ^{-} for any j and thus completes the proof. Note that, as mentioned in the main text, without any normalization the elements of the matrix Θ^{-} can take any value possibly outside $(-1,1)$. While this is not too much of a problem mathematically, it is unappealing conceptually because it implies that the adjacency matrix $A(\Theta^{-})$ can have weights greater than 1 in absolute value.

Proof of Proposition 5 Let f and g denote the inverse demand and demand systems respectively:

$$p = f(q) = \alpha - \beta\left(I_J + \eta XX'\right)q \tag{54}$$

$$q = g(p) = (I_J - \Theta)\frac{(\alpha - p)}{\beta} \tag{55}$$

where $\Theta = X\Omega^{-1}X'$ and $\Omega = \frac{1}{\eta}I_K + X'X$. Firm j's first order condition of the Bertrand problem can be written as

$$g_j(p) + (f_j(g(p)) - c)\frac{\partial g_j(p)}{\partial p_j} \tag{56}$$

$$= g_j(p)\sum_{l\neq j}\frac{\partial f_j}{\partial g_k}\frac{\partial g_k}{\partial p_j} + \left(f_j(g(p)) - c + \frac{\partial f_j}{\partial g_j}\right)\frac{\partial g_j(p)}{\partial p_j} \tag{57}$$

where the second equality differentiates the identity $p_j \equiv f_j(g(p))$ with respect to p_j. Next, denote the Cournot equilibrium price p^c and note that the above Bertrand FOC evaluated at p^c reduces to

$$g_j(p^c)\sum_{l\neq j}\frac{\partial f_j}{\partial g_k}\frac{\partial g_k}{\partial p_j} = g_j(p^c)\sum_{l\neq j}(-\beta\eta x_j' x_k)\frac{\theta_{kj}}{\beta} < 0 \tag{58}$$

where the last inequality holds because $\theta_{kj} \geq 0$ implies that $x_j' x_k \geq 0$. To see this note that both $I_J - \Theta$ and Θ are positive definite matrices. But this implies that all eigenvalues of Θ must lie in $(0, 1)$ and, consequently

$$(I_J + \eta X X') = \tag{59}$$

$$= \left(I_J - X\left(\frac{1}{\eta}I_K + X'X\right)^{-1} X'\right)^{-1} \tag{60}$$

$$= (I_J - \Theta)^{-1} = \sum_{s=1}^{\infty}\Theta^s \geq 0, \tag{61}$$

where the last inequality is well defined because $\max_j |\lambda_j(\Theta)| < 1$. Thus, because Θ is non-negative by assumption we can conclude that $x_j' x_k \geq 0$ if $j \neq k$. Because equation (58) holds for all j we can consider the vector of Bertrand FOCs evaluated at the Cournot price p_c

$$(I_J - \Theta)\alpha + (I_J - diag(\Theta))c - 2\left(I_J - \frac{diag(\Theta)}{2} - \frac{\Theta}{2}\right)p_c < 0 \tag{62}$$

$$\Leftrightarrow p_b = \frac{1}{2}\left(I_J - \frac{diag(\Theta)}{2} - \frac{\Theta}{2}\right)^{-1}((I_J - \Theta)\alpha + (I_J - diag(\Theta))c) < p_c \tag{63}$$

where p_b denotes the Bertrand equilibrium price vector. Then, from Propositions 3 and 4 we can conclude that Bertrand centralities are higher than the ones implied by Cournot:

$$\tilde{\mathbf{b}}_b \equiv (I_J - diag(\Theta))^{-1/2}\,\mathbf{b}_b \geq \left(I_J - \eta diag(\Theta^-)\right)^{1/2}(-\mathbf{b}_c) \equiv -\tilde{\mathbf{b}}_c. \tag{64}$$

Proof of Proposition 6 The proof is identical to the proof of Proposition 3 with the difference that the matrix $diag(\Theta)$ must be replaced by $H \odot \Theta$. The first order condition of (19) with respect to p_j is given by

$$a_j + c_j - \sum_{l\in J_f}\theta_{lj}c_l - 2(1-\theta_{jj})p_j + \sum_{l\in J_f/\{j\}}\theta_{lj}p_l + \sum_{l\neq j}\theta_{jl}p_l = 0 \tag{65}$$

and after rearranging in vector form and solving for p^* one obtains

$$p^* - c = \frac{1}{2}\left(I_J - H \odot \Theta - \frac{A_H(\Theta)}{2}\right)^{-1}(I_J - \Theta)(\alpha - c) \tag{66}$$

$$= \frac{1}{2}\left[I_J - \left(I_J - H \odot \Theta - \frac{A_H(\Theta)}{2}\right)^{-1}\frac{A_H(\Theta)}{2}\right](\alpha - c) \tag{67}$$

$$= \frac{\alpha - c}{2} - (I_J - H \odot \Theta)^{-1/2}\left(I_J - \frac{G_H(\Theta)}{2}\right)^{-1}\frac{G_H(\Theta)}{2} \times \tag{68}$$

$$\times (I_J - H \odot \Theta)^{1/2}\frac{(\alpha - c)}{2} \tag{69}$$

$$= \frac{\alpha - c}{2} - (I_J - H \odot \Theta)^{-1/2}\, b\left(G_H(\Theta), \frac{1}{2}, (I_J - H \odot \Theta)^{1/2}\frac{(\alpha - c)}{2}\right) \tag{70}$$

where

$$G_H(\Theta) \equiv (I_J - H \odot \Theta)^{-1/2} A_H(\Theta)\,(I_J - H \odot \Theta)^{-1/2} \tag{71}$$

and the last equality is well-defined provided $\max_j |\lambda_j(G)| < 2$, where λ_j is the an eigenvalue of $G_H(\Theta)$. To complete the proof I need to show that $G_H(\Theta)$ is a weighted adjacency matrix. First note that because $A_H(\Theta) = \Theta - H \odot \Theta$ is a 0-block diagonal matrix the same holds for $G_H(\Theta)$. Next, I show that all its elements g_{ij} lie in $(-1, 1)$. To see this recall that $I_J - \Theta$ is positive definite and note that

$$(I_J - H \odot \Theta)^{1/2}(I_J - G_H(\Theta))\,(I_J - H \odot \Theta)^{1/2} = I_J - \Theta \tag{72}$$

which implies that $(I_J - G_H(\Theta))$ is also positive definite, but then

$$1 - g_{ij}^2 = (g_{ij}e_i + e_j)'\,(I_J - G_H(\Theta))\,(g_{ij}e_i + e_j) > 0. \tag{73}$$

So far we have used extensively $(I_J - H \odot \Theta)^{1/2}$, to make sure that this is well defined I need to show that $I_J - H \odot \Theta$ is positive definite. To see this, note that $I_J - \Theta > 0$ implies that all its principal submatrices are positive definite, but then all blocks in the block-diagonal matrix $H \odot (I_J - \Theta)$ are also positive definite which makes $H \odot (I_J - \Theta)$ positive definite as well.

B Decomposition of Cournot Markups

In this section I show how to decompose the markups of a given product j in the case of Cournot competition, and show formally that the product centrality measure defined in Pellegrino (2023), and denoted by $1-\chi_j$, is an affine transformation of the Bonacich centrality measure $\mathbf{b}_j$ I have been using throughout this paper.

To start with I make a few notational adjustments to conform my notation to the one in Pellegrino (2023). I normalize the squared norm of product characteristics to one i.e., for any product j I assume that $||x_j||_2^2 = x_j' x_j = 1$. Following the notation I used in Proposition 4 this normalization is equivalent to set $\theta^- = -1$. Also, I set $\beta_i \equiv 1$ for any consumer i, which further implies that, in the aggregate demand derived in Proposition 1, $\beta = 1$ and the aggregate demand intercept $\alpha = M^{-1} \int \alpha_i di$. In matrix form, the aggregate demand and its inverse are given by

$$q = (I + \eta X X')^{-1} (\alpha - p) \tag{74}$$

$$p = \alpha - (I + \eta X X') q. \tag{75}$$

Next, let ν be such that $\eta = \frac{\nu}{1-\nu}$ and note that

$$I + \eta X X' = \frac{1}{1-\nu} I + \frac{\nu}{1-\nu}(X X' - I) \tag{76}$$

where the ν plays the role of α in the notation of Pellegrino (2023). Next, assuming that $(1-\nu) > 0$, I can re-scale consumer preferences in 1 by $(1-\nu)$ to obtain

$$v_i(q_i, X) \equiv (1-\nu) u_i(q_i, X) = q_i'(\alpha_i - p) - \frac{1}{2} q_i' (I + \Sigma) q_i \tag{77}$$

where $\Sigma \equiv \nu(XX' - I)$, I redefined $\alpha_i \equiv \nu X \alpha_i^x + (1-\nu)\alpha_i^q$ and I already substituted in for the budget constraint.[28] The preferences in (77) are exactly the same ones used in Pellegrino (2023) and the associated aggregate demand and its inverse are

$$q = (I + \Sigma)^{-1} (\alpha - p) \tag{78}$$

$$p = \alpha - (I + \Sigma) q. \tag{79}$$

Next, I show how the product centrality measure defined in Pellegrino (2023) and denoted by $1-\chi_j$ is an affine transformation of product j's Bonacich network centrality $\mathbf{b}_j(-\Sigma, 1/2, (\alpha - c)/2)$. The steps are similar to the ones in Proposition 4 with $\Theta^- =$

[28] Note that it is not necessary to rescale the outside good q_{i0} by $(1-\nu)$ because I can always normalize its price to $1/(1-\nu)$ in the budget constraint.

$-\Sigma$. To start with note that the cournot equilibrium quantities are given by

$$q^* = \left(1 - \frac{1}{2}(-\Sigma)\right)^{-1} \frac{\alpha - c}{2}. \tag{80}$$

Replacing (80) in the inverse demand, we get the price-cost margin decomposition obtained in Proposition (4)

$$\begin{aligned} p^* - c &= \alpha - c + \left(I - \frac{1}{2}(-\Sigma)\right)^{-1} \frac{1}{2}(-\Sigma)\frac{\alpha - c}{2} && (81) \\ &= \frac{\alpha - c}{2} + \mathbf{b}\left(-\Sigma, \frac{1}{2}, \frac{\alpha - c}{2}\right). && (82) \end{aligned}$$

Next, define product j markups as

$$\mu_j \equiv \frac{p_j}{c_j} \tag{83}$$

and letting $\mathbf{b}_j$ the j-th component of the vector of Bonacich centralities $\mathbf{b}$, from 82 we obtain

$$\mu_j = \frac{\alpha_j + c_j}{2c_j} + \frac{1}{c_j}\mathbf{b}_j = \bar{\mu}_j + \frac{1}{c_j}\mathbf{b}_j \tag{84}$$

where $\bar{\mu}_j$ denotes the monopolistic markup.

Pellegrino (2023) shows that product j markup is given by

$$\mu_j = \chi_j + (1 - \chi_j)\bar{\mu}_j \tag{85}$$

where the product j's centrality $1 - \chi_j$ is defined as

$$1 - \chi_j \equiv \frac{1}{\alpha_j - c_j}\left[\left(I + \frac{1}{2}\Sigma\right)^{-1}(\alpha - c)\right]_j \tag{86}$$

where for a genric vector y, $[y]_j$ denotes its j-th component. Combining (84) and (85) and solving for χ_j one obtains

$$\chi_j = -\frac{2}{\alpha_j - c_j}\mathbf{b}_j \tag{87}$$

from which it is immediate to see that Pellegrino (2023)'s centrality measure is an affine trasformation of the Bonacich centrality

$$1 - \chi_j = 1 + \frac{2}{\alpha_j - c_j}\mathbf{b}_j. \tag{88}$$

To complete the argument, I am left to show that from (88) I can backout the definition of product centrality in (86). To see this, let χ be the J-vector with j-th component χ_j and bulding from (88)

$$1_J - \chi = [diag(\alpha - c)]^{-1} (\alpha - c + 2\mathbf{b}) \tag{89}$$

$$= [diag(\alpha - c)]^{-1} \left[I - \left(I + \frac{1}{2}\Sigma \right)^{-1} \Sigma \frac{1}{2} \right] (\alpha - c) \tag{90}$$

$$= [diag(\alpha - c)]^{-1} \left(I + \frac{1}{2}\Sigma \right)^{-1} (\alpha - c) \tag{91}$$

where 1_J is the J-vector of ones, the first equation is just (88) in vector forms, the second equation substitutes for the definition of Bonacich centrality and the last equation rearranges terms. Overall, the j-th equation in (91) coincides with the definition of product centrality for product j in (86) thus proving that the argument is consistent.

C Figures and Tables

	OLS	IV-2SLS
Constant	-0.1023	-0.1005
	(0.0043)	(0.0044)
Air (dummy)	-0.0179	0.0039
	(0.0145)	(0.0183)
MP$	0.0331	0.0368
	(0.0104)	(0.0104)
HP/WT	0.1432	0.1040
	(0.0448)	(0.0476)
Space	-0.0138	-0.0115
	(0.0168)	(0.0166)
air×income	0.0001	0.0000
	0.0001	0.0001
MP$×income	0.0001	0.0001
	0.0001	0.0001
HP/WT×income	-0.0005	-0.0003
	0.0002	0.0003
Space×income	0.0004	0.0004
	0.0001	0.0001
price/income	-0.0043	-0.0327
	(0.0037)	(0.0121)
Fstat (Excluded)	-	92.1276
R2	0.8704	0.8660
Observations	2,217	2,217

Table 4: Demand estimates. Standard errors are clustered at the model level.

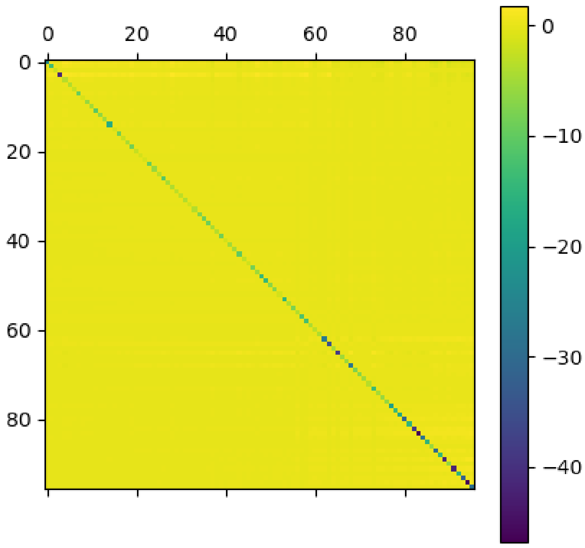

Figure 7: Matrix of estimated own and cross price elasticities for the year 1990. The products included are the ones with an estimated own price elasticity above the 75th percentile.

D Simulation of the Bertrand Network game

In this section, I perform a simple simulation exercise to summarize and visualize how the Bertrand Network model works.

Table 5 describes the parameters used in the simulation. There is a single market with $J = 30$ products and $K = 7$ characteristics whose values are drawn from a uniform distribution in between $[0, 1]$. The demand intercept α is the same across all products and set to 0.15 whereas marginal costs are heterogeneous across products and drawn from a $[.01, .03]$ uniform distribution.

Parameter	Value
J	30
K	7
α	0.15
c	U[0.01, 0.03]
x_{jk}	U[0, 1]

Table 5: Parameters for simulation of Bertrand network game

Given this parameters, Figure 8 plots the underlying Bertrand network. Each product is a node and the edges capture the degree of substitution between any two products/nodes; the longer the edge the less substitute are the two products. The location of dots and edges is exogenous and entirely determined by the realization of the draws of product characteristics. Conversely, the size of the dots is endogenous and it is proportional to the equilibrium price-cost margins. The plot shows that nodes that are more peripheral tend to have larger dot sizes whereas dots that are more central are smaller. The intuition for this result is the following: peripheral products are more unique or equivalently less central and, per equation (13) will charge higher margins in equilibrium. On the other hand, more central nodes face more intense competition and must lower their margins.

Figure 9 instead is a visualization of Proposition 3 and plots the equilibrium price-cost margins on the y-axis against the Bonacich product centrality on the x-axis. It should be clear by now why the relationship is decreasing; higher centrality implies lower equilibrium markups. The noise around the downward sloping relationship is due to the fact that marginal costs are heterogeneous. By increasing the variance of the distribution of costs, Figure 9 would start looking noisier and the resulting relationship between centrality and margins might not look as clear. This highlights how empirically it is important to control for the unobserved costs in order to recover the downward sloping relationship. The same would be true if we were to introduce heterogeneity in the demand intercept α.

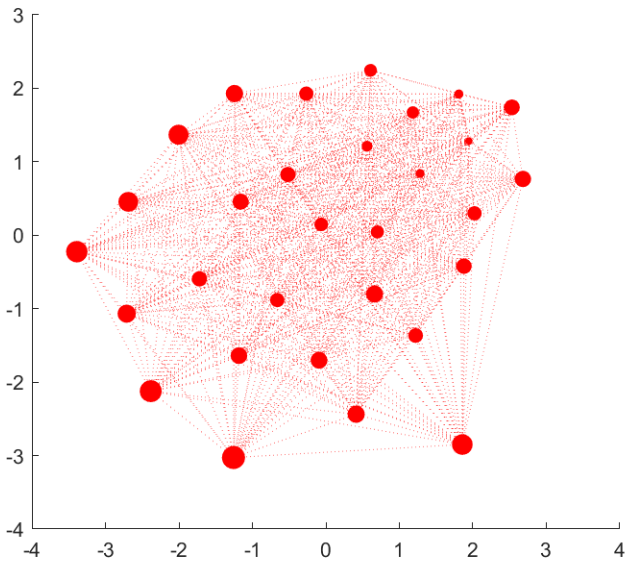

Figure 8: Simulated Network. Location is exogenous. Node size is proportional to markups.

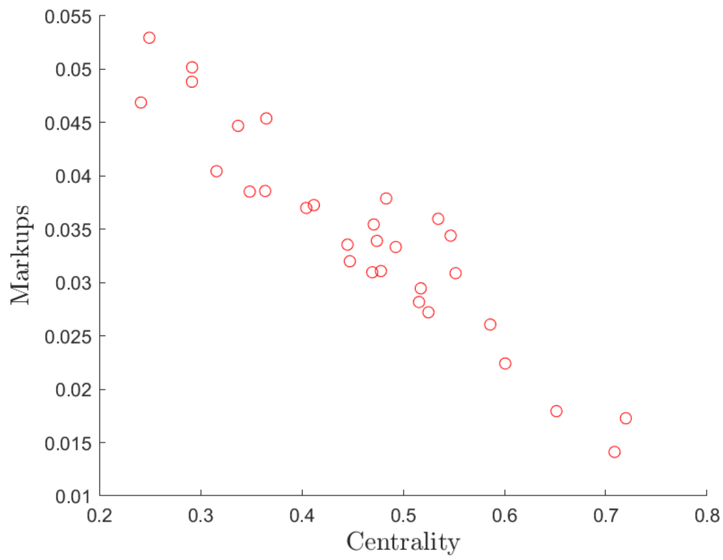

Figure 9: Simulated Network. Price-cost margins (y-axis) against Bonacich centrality (x-axis).