Oligopolistic Competition, Fund Proliferation and Asset Prices*

Marco Loseto[†] Federico Mainardi [‡]

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Abstract

We develop and estimate a dynamic oligopoly model of the passive mutual fund industry in which multiproduct asset management firms act as fund initiators and decide how many funds to launch in a given investment sector. Both mutual funds and management companies compete a la Cournot and take into account the demand for asset management services from a representative household investor. In the first part of the paper, we provide sufficient conditions for the existence and uniqueness of a steady-state equilibrium in which each management firm operates a constant number of funds and the equilibrium index price is constant. In the second part of the paper, we develop a nested fixed-point algorithm to estimate fund initiation costs separately for the five biggest management companies in the US passive equity industry by matching fund proliferation patterns observed in the data. We find that the top five companies are substantially more efficient and enjoy large scale economies relatively to the rest of the market. In a series of counterfactual exercises, we show that removing the largest management companies from the market would reduce investors' welfare by as much as 25%. Lastly, we characterize analytically the steady-state multiplier of household wealth on the equity index price in terms of the technology primitives of the industry. Our estimates imply that a 1% increase in household wealth increases the valuation of the equity index by 5.5%, consistent with other estimates in the literature.

KEYWORDS: Asset Management, Dynamic Oligopoly, Asset Pricing

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†Kenneth C. Griffin Department of Economics, University of Chicago, mloseto@uchicago.edu.

‡Kenneth C. Griffin Department of Economics, University of Chicago and Booth School of Business, fmainard@chicagobooth.edu.

1 Introduction

The US mutual fund industry has witnessed a tremendous shift from active to passive investing in the past two decades. The share of assets under management (AUM) held in passive equity funds increased from about 20% at the beginning of 2000 to 50% in the first quarter of 2020 (Figure 9), making the passive industry the dominant one in the US equity market. While this shift from active to passive is a well-known fact, less is known about the competitive dynamics within the passive industry and their implications for asset prices and investors' welfare.

We start this paper by highlighting two facts about the structure of the passive mutual fund industry. First, the industry is concentrated in a handful of large fund families. Figure 13 plots the market share of the five biggest families for three different cap-based categories within the US passive equity industry: Large Cap, Mid Cap and Small Cap. In each category, the market share of the top five families has been roughly steady over time and amounts to more than 80% of the market.

Second, perhaps not surprisingly, the number of passive investment vehicles such as index funds and ETFs have been increasing in the past 20 years. More interestingly, Figure 14 shows how the average number of passive funds per fund family evolved over time, separately for the five biggest families (blue line) and the rest of them (red line). The pattern is striking: not only do the five biggest families manage more assets, but they do so by deploying many more funds relative to their competitors. In each investment category, this gap in the number of funds has been increasing over time, suggesting that fund proliferation might be a key mechanism through which these investment firms compete. To maintain their market shares, the largest families keep introducing new funds to capture household investors' demand.

To rationalize these patterns, we develop and estimate a dynamic oligopoly model of the mutual fund industry. A key component of our model is the distinction between individual mutual funds and fund families which we also refer to as management companies. While there exists an extensive literature that studies the mutual fund industry, most of it focuses on funds and disregards the role of management companies.[1] In practice, management companies are responsible for relevant decisions that shape the competitive environment in which funds operate. They decide if and when to introduce new funds, choose the investment sector in which to operate their new funds and establish the investment style of the new funds they create (e.g., the investment mandates). To accommodate the presence of both management companies and funds, to emphasize the key role of fund initiation and to analyze the competitive forces that drive the dynamics of this industry, our model builds on two layers of Cournot com-

[1] Some exceptions are Massa (2003), Gaspar, Massa and Matos (2006), Bhattacharya, Lee and Pool (2013), Berk, Binsbergen and Liu (2017), Betermier, Schumacher and Shahrad (2022) and Ørpetveit (2021).

petition. In the inner layer, mutual funds compete by choosing quantities and make a profit from fees (as a percentage of their AUM) determined, in equilibrium, by the investment service demand of a representative household investor. In the outer Cournot layer, an oligopoly of heterogeneous management companies compete with each other by deciding how many funds to operate.

On top of allowing the number of funds to emerge endogenously in equilibrium, the model links the technological primitives of the asset management industry, such as marginal costs and scale economies, to the elasticity of asset demand. We close the model and derive the equilibrium price that clears the asset market under the assumptions of strict mandates and a fixed supply of shares. After setting up the model, in Proposition (1), we analytically prove the existence and uniqueness of a steady state equilibrium in which the number of funds and the index price are constant. While we characterize the steady-state equilibrium analytically, we further solve the full model numerically. To do so, we develop a nested fixed point numerical routine that, for given parameter values, solves for the optimal equilibrium path to a given terminal condition. At each point in time, the path for the number of funds created is a pure strategy Markov Perfect Nash equilibrium of the dynamic game between management companies, and the path for asset prices clears the asset market in every period.

Secondly, we push forward the recent asset pricing literature that studies the role of institutional investors in determining asset market movements.[2] Contrary to the traditional assumptions that investors are atomistic and that their demand shocks are uncorrelated, this literature documents how asset demand is far from perfectly elastic and how demand shocks affect equilibrium asset prices. Intuitively, the large size of these investors and the presence of specific investment mandates contribute to generating correlated demand shocks, which will inevitably impact asset prices. Our model links the mutual fund industry technology fundamentals to equilibrium asset prices: the price impact of large institutional investors is micro-founded from technology primitives such as fund initiation costs and scale economies. A reduction in initiation costs pushes companies to introduce more products, which will lower the equilibrium fees and, in turn, attract more demand from households. Then, under fixed supply, asset prices will increase to clear the excess demand triggered by the initial reduction in initiation costs.

The second part of the paper turns to the estimation of the model using data on US passive equity funds. We do so by matching two types of data moments, the average per-period fund initiation rate and the average rate at which this fund initiation rate grows. In our model, these moments inform the two parameters that

[2]See for instance, Petajisto (2009) Basak and Pavlova (2013), Koijen and Yogo (2019), Haddad, Huebner and Loualiche (2022) and Pavlova and Sikorskaya (2022).

characterize the cost of introducing new funds for management company j, the linear cost parameter c_j and the adjustment cost parameter δ_j. In the data, we compute these moments separately for each of the five biggest management companies and the remaining companies pooled together.[3] For each of the five biggest companies, we then obtain an estimate of their fund initiation costs, and we show how our model can match the pattern of fund proliferation observed (and targeted) in our data. As a validation check, we also show how our dynamic model's time series of equilibrium fees closely follows the observed (but untargeted) time series of expense ratios.

Next, with our estimated model, we perform counterfactual exercises to study the welfare effects of removing the largest management companies from the market. We do so for each of the top 5 largest companies Blackrock, Charles Schwab, Fidelity, State Street and Vanguard, one at the time. The welfare effects of removing any one of those companies are large and heterogeneous. We estimate that removing Blackrock or Vanguard from the market would reduce household welfare by 25% and 9.2%, respectively. Such a large welfare loss is a consequence of the fact that although Blackrock and Vanguard are the largest asset managers, they are also the most efficient ones. To further corroborate our finding, in a second set of counterfactuals, we replace Blackrock with two different management companies with a cost structure analogous to Charles Schwab, which we estimate to be less efficient. The resulting welfare loss is slightly smaller than before and amounts to a 20% reduction in household welfare. Overall, our counterfactual exercises suggest that restricting the largest management companies to favor competition might ultimately hurt investors' welfare if those companies are also the most efficient ones.

Finally, in Proposition (2), we provide a closed-form expression of the equilibrium asset price multiplier with respect to investors' wealth. Our estimates imply that a 1% increase in household wealth increases the valuation of the equity index by 5.5%, which is in line with what the recent asset pricing literature has found. In the context of our model, the inelasticity of asset demand is driven by the structure of the asset management industry. The presence of large and multiproduct management companies exacerbates the price impact of a demand or cost shock. In Section 6.4, we perform a comparative static exercise and show that a reduction in fund initiation costs pushes management companies to introduce more funds which in turn reduces fees and further increases household demand for the index. Under fixed supply, asset prices will need to be higher to clear the asset market. Overall, our model rationalizes the high price multiplier often found in the empirical asset pricing literature through the competitive dynamics of large, heterogeneous and multiproduct management companies.

[3]Our model only focuses on passive funds, and for the estimation of the model, we will pool all the non-top five companies into one entity which we refer to as the outside company, so that the overall number of companies used in estimation will be 6. As shown in figure (10), more than 80% of the market is controlled by the top 5 companies, so the outside group consists of a set of small companies.

The rest of the paper proceeds as follows. Section 2 reviews the literature. Section 3 describes the model. Section 4 proves uniqueness and existence of the symmetric steady-state. Section 6 calibrates and estimates the model. Section 7 concludes.

2 Related Literature

Our paper contributes to the broad literature that studies theoretically and empirically the industrial organization of the asset management industry. Dermine, Neven and Thisse (1991) is one of the first contributions that considers strategic competition between funds. In their model, funds face demand from mean-variance investors with heterogeneous risk-aversion and choose where to locate on the mean-variance frontier. The equilibrium displays an Hotelling like property in which only two types of funds are supplied, one fully invested in the risk-free and the other fully invested in the market. Nanda, Narayanan and Warther (2000) propose a model of the mutual fund industry in which investors' heterogeneity in terms of liquidity needs implies that different load fee structures arise in equilibrium. Hortaçsu and Syverson (2004) show that presence of investors' search costs play a crucial role in rationalizing why homogeneous S&P 500 index funds with different fees are supplied. In our model we abstract from investors' heterogeneity but allow for multi-product management firms with heterogeneous production technologies who compete in a two-layers Cournot oligopoly.

Within this literature, a few papers highlight the importance of management companies in shaping the market structure and the proliferation of products in the asset management industry. Massa (2003) argues that fund families are incentivized to offer a broad menu of funds because investors value the possibility to switch across different funds belonging to the same family at no cost. Khorana and Servaes (1999) empirically analyze the determinants of mutual fund starts and show that scale and scope economies are among the factors that induce fund families to launch new funds. More recently, Betermier, Schumacher and Shahrad (2022) provide empirical evidence that incumbent families set up a large number of new funds in order to deter entry. The role of fund families and product proliferation are also crucial elements in our dynamic model.[4] In each period fund families decide how many new funds to introduce taking into account that operating more funds will generate scale economies next period but will increase competition and reduce profits of existing funds.

Our work also contributes to the recent asset pricing literature that highlights

[4]The importance of multi-product management companies is not limited to shaping the market structure of the industry. Gaspar, Massa and Matos (2006) provide empirical evidence of how fund families transfer performance across member funds to maximize family profits. Bhattacharya, Lee and Pool (2013) show that large families offer mutual funds that only invest in other funds in the family and how these type of funds provide insurance against liquidity shocks. Berk, Binsbergen and Liu (2017) argue that fund families exploit their private information about their managers skill and create value by reallocating capital efficiently among managers. Ørpetveit (2021) shows empirically that management companies improve the quality of their existing funds in response to higher competition.

the role of institutional investors in determining asset prices movements. In a static mean-variance framework Petajisto (2009) shows how the presence of demand for asset management services is enough to generate downward sloping demand curves for stocks. Starting from a simple portfolio choice problem Koijen and Yogo (2019) develop an equilibrium asset pricing model in which portfolio weights are function of stock characteristics. The model is estimated to match investors holdings and used in several applications to highlight the role of institutions in determining asset market movements. More recently, Haddad, Huebner and Loualiche (2022) extended the Koijen and Yogo (2019)'s characteristics-based framework by allowing investors to compete between each other in setting their trading strategies. They show that this type of strategic competition reduces the competitiveness of the stock market leading to inelastic demand curves.[5] In our model equilibrium asset prices are also related to the competitive behavior of large multi-product institutional investors. In particular, we show how technology primitives such us scale economies in the production of asset management services impact equilibrium asset prices.

Recently, the importance of large institutional investors has been also shown to be relevant for the efficiency of asset prices. In a recent paper, Kacperczyk, Nosal and Sundaresan (2022) consider an asset market with an oligopoly of large investors of exogenous sizes and study how market concentration affects price informativeness.[6] Although we abstract from the role price informativeness, our model endogenizes flows and market concentration leaving heterogeneity in production technologies to be the fundamental model primitive.

Finally, motivated by the increasing regulatory scrutiny toward the growth of index investing,[7] Schmalz and Zame (2023) propose a static equilibrium model with heterogeneous investors and show that the presence of an index fund might hurt investors' welfare if one takes into account the general equilibrium effect on asset prices. When

[5]Many other papers have included institutional investors in asset pricing models among which He and Krishnamurthy (2013) and Basak and Pavlova (2013). The former paper develops a dynamic model in which institutional investors are constrained in their portfolio choice and study the impact on risk premia in bad times. The latter studies the asset pricing effects of delegated portfolio managers who care about their performance relative to a benchmark in a dynamic economy. More recently, Pavlova and Sikorskaya (2022) develop an empirical measure of benchmarking intensity and provide evidence of inelastic demand of active managers for stocks in their benchmarks.

[6]With the rise of passive investing the literature that studies how the presence of large passive investors affects the information embedded in asset prices is growing. See for instance: Bai, Philippon and Savov (2016), Baruch and Zhang (2022), Bond and Garcia (2022), Farboodi, Matray, Veldkamp and Venkateswaran (2021), Coles, Heath and Ringgenberg (2022), Malikov (2021) and Sammon (2022).

[7]Regulators and antitrust legal scholars are investigating the consequences of the rise of passive investing on various economic outcomes. The trigger of many of the regulatory concerns has been a recent and growing literature that studies the anticompetitive effects of common ownership (Azar, Schmalz and Tecu (2018)), Posner, Scott Morton and Weyl (2017), Anton, Ederer, Gine and Schmalz (2017), Azar and Vives (2021), Backus, Conlon and Sinkinson (2021)). This literature asks whether product firms that share common owners, which in most cases are large passive asset managers, have less incentive to compete.

the index fund enters the market or lowers its fee investors increase their stock holdings relative to their bond holdings, which leads to higher asset prices and, in turn, to lower asset returns. Although our model is different in several respects, we also look at how investors' welfare changes when the structure of the asset management industry changes, while taking into account the effects on asset prices. Our counterfactual analysis in section (6.3) suggests that restricting the largest passive asset managers to favor competition might reduce investors' surplus. This happens because the largest management companies are far more efficient than the rest of market and thus the efficiency loss that results from removing them hurts investors despite asset prices increase.

3 Model

Time is discrete and indexed by $t \in \{1, 2, ...\}$. We consider an economy populated by three types of agents: a representative household , mutual funds and management companies. The representative household allocates wealth between the mutual fund sector and a risk-free asset to finance consumption and takes expected return, variance and fees as given. The mutual fund sector is populated by a discrete number of identical funds that internalize household demand and that optimally choose their size to maximize dollar revenues. Each fund invests in the same underlying index and takes the total number of operational funds as given. Finally, each management company is responsible for fund initiation. Specifically, at each time t, each management company controls a number of pre-existing funds that carries from previous periods and chooses the number of new funds to create. We close the model and derive equilibrium market prices by assuming that mutual funds have a strict mandate to invest in the underlying index and that the index is available in fixed supply. The major contribution of the model is to describe the competitive dynamics of the mutual fund industry assuming that not only mutual funds but also management companies simultaneously and dynamically compete a la Cournot. Despite the complications created by the two layers of Cournot competition, we provide sufficient conditions under which the model admits a unique steady-state equilibrium.

We now proceed to describe in details the problem solved by each agent in the model.

3.1 Household

In each period, a representative household with log utility over consumption decides how much of its current wealth A_t to consume and how much to invest in the financial market. The investment opportunity set consists of two broad asset classes, namely a risk-free asset with return normalized to zero and a mutual fund sector. The mutual fund sector is populated by a discrete number of identical funds that invest in the same underlying index and that charge fee f_t at time t.[8] Because all mutual funds are identical, each household is indifferent between investing in any specific fund and it will only choose the fraction of wealth to invest in the aggregate mutual fund sector. The size of each individual fund will then be determined via Cournot competition.

We assume that the index tracked by each mutual fund pays a constant dollar dividend D and we denote by P_t the index price at time t. Next, we define the net of fee index return at time $t+1$ as

$$1 + R_{t+1} = \frac{P_{t+1} + D}{P_t} - f_t. \tag{1}$$

Our representative household knows D and f_t but is not able to foresee the equilibrium path of asset prices. In other words, the household is not able to anticipate the effect that actions of mutual funds and management companies have on equilibrium asset prices. Instead, he perceives the index log net returns to evolve as a Gaussian stationary process

$$r_{t+1} \equiv \log\left(1 + R_{t+1}\right) = \rho_t - f_t + \sigma_t \varepsilon_{t+1}$$

where $\varepsilon_{t+1} \sim \mathcal{N}\left(0, 1\right)$.[9] Letting w_t denote the portfolio weight on the mutual fund sector, the problem solved by the representative household at time t is

$$V(A_t) = \max_{C_t, w_t} \mathbb{E}_t \left[\sum_{s=t}^{\infty} \beta^{s-t} \log(C_s)\right] \tag{2}$$

$$\text{s.t. } A_{t+1} = (1 + w_t R_{t+1})(A_t - C_t) \tag{3}$$

with associated Euler equation given by

$$1 = \mathbb{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1} (1 + R_{t+1})\right] \tag{4}$$

Under standard arguments, which we detail in Appendix A, the household optimally consumes $C_t = (1-\beta)A_t$ and invests βA_t in the financial market. Moreover, household optimal portfolio allocation is given by

$$w_t = \frac{\mu_t - f_t}{\sigma_t^2}. \tag{5}$$

where $\mu_t \equiv \rho_t + \sigma_t^2/2$.

[8] We will discuss this product homogeneity assumption in Section 3.5.

[9] When simple net returns are sufficiently small $\log(1+R_{t+1}) \approx R_{t+1} = \frac{P_{t+1}+D}{P_t} - f_t - 1$ so that ρ_t can be interpreted has the household subjective belief about the next period capital gain and dividend yield.

3.2 Mutual Funds

In any period t, each mutual fund takes the total number of funds in the market n_t as given and chooses its market share to maximize dollar profits. Given the optimal market share chosen by each fund, the equilibrium fee at time t is pinned down by the demand of the representative household in (5). In other words, we are assuming that, at each time t, mutual funds compete simultaneously and repeatedly a la Cournot. Each mutual fund internalizes that a higher individual market share leads to a higher market share of the aggregate mutual fund industry and to a lower fee that the household is willing to pay.

Let w_{it} denote the weight on mutual fund i in the portfolio of the representative household, and consider to rewrite household demand in (5) as

$$f_t = \mu_t - w_t \sigma_t^2 \tag{6}$$

Then fund i at time t solves

$$\max_{w_{it}} \; f_t w_{it} \beta A_t$$

subject to

$$f_t = \mu_t - w_t \sigma_t^2$$
$$w_t = \sum_{i=1}^{n_t} w_{it}$$

Taking the first order condition with respect to w_{it} we obtain fund i best response

$$w_{it} = \frac{\mu_t}{\sigma_t^2} - w_t. \tag{7}$$

Summing across funds yields the Cournot total quantity

$$w_t = \frac{\mu_t n_t}{\sigma_t^2 (n_t + 1)}, \tag{8}$$

and by replacing (8) in (6) we recover the equilibrium fee

$$f_t = \frac{\mu_t}{n_t + 1}. \tag{9}$$

Finally, we solve for the symmetric equilibrium w_{it} by replacing (8) in (7)

$$w_{it} = \frac{\mu_t}{\sigma_t^2 (n_t + 1)}. \tag{10}$$

In equilibrium at time t and conditional on n_t, each mutual fund i realizes dollar profits

$$\pi_t(n_t) = f_t w_{it} \beta A_t = \frac{\mu_t^2}{\sigma_t^2 (n_t+1)^2} \beta A_t.$$

To save notation, we will rewrite dollar profits gained by each fund as

$$\pi_t(n_t) = \frac{\pi_t}{(n_t+1)^2} \tag{11}$$

where $\pi_t \equiv \frac{\mu_t^2}{\sigma_t^2} \beta A_t$.

3.3 Management Companies

Consider an oligopoly of M multi-product management companies indexed by $j \in \{1, 2, \ldots, M\}$. Each management company j enters time t with n_{jt-1} pre-existing funds and chooses the number of funds n_{jt} to operate in the current period with the objective of maximizing the present discounted value of dollar profits. Equivalently, the decision of management company j to operate n_{jt} funds at time t requires the creation or deletion of $n'_{jt} = n_{jt} - n_{jt-1}$ funds.

While pre-existing funds do not carry any cost for the controlling management company, opening a new fund is costly. We allow the initiation cost to depend on the size of management companies and we parameterize the cost of creating a new fund for management company j at time t as

$$C_j \left(n_{jt}, n_{jt-1}; c_j, \delta_j\right) = c_j n'_{jt} + \delta_j \left(\frac{n'_{jt}}{n_{jt-1}}\right)^2 n_{jt-1} \tag{12}$$

where $c_j > 0$ is the linear component of the initiation cost and δ_j captures an additional cost of adjusting the menu of funds, which we assume decreasing in the size of the management company, i.e. in the number of pre-existing funds n_{jt-1}. The suggested functional form for $C_j(\cdot)$ implies that management companies with a higher number of pre-existing funds face a lower initiation cost and it is motivated by the newly documented evidence that largest management companies are responsible for most of fund creation. We interpret this evidence as suggesting that largest management companies face a lower cost of initiating a new fund and we incorporate this empirical fact in the model. Holding n_{jt-1} constant, the parameter δ_j captures how costly is for company j to adjust its menu of funds. A lower δ_j and a higher n_{jt-1} both reduce j's cost of launching a new fund.

What is the trade-off that management companies face when initiating a new fund? First, in the current period, there is an ambiguous effect on profits. On one hand, profits increase because the company operates one additional fund thereby increasing its market share. On the other hand, creating one additional fund increases competition in the mutual fund sector, leading to a decrease in fee f_t and profit $\pi_t(n_t)$. Second, expanding the current menu of funds carries the additional benefit of reducing the initiation cost in future periods.

Overall, management company j at time t solves the following dynamic problem

$$V\left(n_{jt-1}\right) = \max_{n_{jt}} \; n_{jt}\frac{\pi_t}{(n_t+1)^2} - C_j\left(n_{jt}, n_{jt-1}; c_j, \delta_j\right) + \beta V\left(n_{jt}\right) \tag{13}$$

subject to

$$n'_{jt} = n_{jt} - n_{jt-1}$$
$$n_t = \sum_{j=1}^{M} n_{jt}$$

Effectively, we are considering a dynamic game in which management companies compete simultaneously a la Cournot. For each management company j, the optimal strategies $n_{-jt} = (n_{j't})_{j'\neq j}$ chosen by the other enter management companies $j' \neq j$ enter the problem only through the total number of funds $n_t = n_{jt} + \sum_{j'\neq j} n_{j't}$.

3.4 Financial Market

The last aspect of the model that still has to be addressed is how the price of the index in which mutual funds are invested will be pinned down in equilibrium. To this end, we assume that mutual funds have a strict mandate to invest in the underlying index and that the index is available in fixed supply $\bar{Q}$. Letting Q_{it} denote the number of index shares demanded by mutual fund i at time t, then the assumption of strict mandate requires

$$Q_{it}P_t = w_{it}\beta A_t \qquad \forall i, t \tag{14}$$

Equation (14) simply states that, if mutual fund i has a strict mandate to invest in the index, then, at any time t, the dollar investment in the index (left hand-side) has to equal the total assets under management of mutual fund i (right hand-side). Summing (14) across funds and imposing market clearing yields

$$P_t = \frac{w_t(\beta A_t)}{\bar{Q}} = \frac{\mu_t n_t}{\sigma_t^2(n_t+1)}\frac{\beta A_t}{\bar{Q}} \tag{15}$$

where in the second equality we used equation (8). In equilibrium, the wealth A_t of the representative household will evolve according to the following law of motion:

$$A_{t+1} = \beta A_t \left[1 + w_t \left(\frac{P_{t+1} + D}{P_t} - f_t - 1\right)\right] \tag{16}$$

$$= \beta A_t \left[1 + \left(\frac{\mu_t n_t}{\sigma_t^2 (n_t + 1)}\right) \left(\frac{P_{t+1} + D}{P_t} - f_t - 1\right)\right] \tag{17}$$

$$= \beta A_t + \bar{Q}\left(D + \Delta P_{t+1} - f_t P_t\right) \tag{18}$$

where the second equality substitutes for the equilibrium portfolio weight w_t in (8) and the third equality uses expression (15). We stress that, because the household is not able to internalize the effect on asset prices coming from the actions of mutual funds and management companies, then the law of motion of wealth derived in (18) will not in general be equivalent to the budget constraint used in (3).

3.5 Discussion of model assumptions

Before turning to the definition of equilibrium and proving existence and uniqueness of a steady state, we now discuss in detail some of our modelling assumptions. Although in some cases restrictive, all of the assumptions are needed to balance the model tractability and its ability to capture what we believe are the most relevant dynamics of the industry.

Myopic portfolio choice. In our model the optimal portfolio choice is myopic because our representative household has log preferences over consumption. Unless the belief process is independent and identically distributed over time, relaxing this assumption would affect our household portfolio choice in a way that would prevent us from obtaining a closed form solution for the portfolio weight w_t. In the case in which the belief process is time-varying the optimal portfolio choice would need to account for the incentives to hedge intertemporally. The resulting asset demand function would only be defined implicitly, making it hard to set up the supply side oligopolistic game in a tractable way. Overall, although the incentives to hedge intertemporally are important, our static demand framework allows us to derive a simple demand for asset management services in each period and to enrich the dynamics on the supply side without losing tractability.

Product homogeneity. In our model all funds are identical and in equilibrium will charge the same fee and manage the same amount of AUM. In practice though the funds offered by a management company are never perfectly identical. Even within the same investment category, funds in a company's menu might have slightly different holdings, different managers, different fee structures, tax benefits and so on. Allowing for this type of product heterogeneity would require extending the model in two directions: on the supply side, we would need to introduce some dimension of horizontal differentiation and characterize a fund with a vector of both portfolio (e.g., type of holdings, factor exposures, etc.) and non-portfolio (e.g., management tenure, advertising, fund age etc.) characteristics. On the demand side, we would need to modify households' preferences for all these product characteristics to be valuable.

Product differentiation is without any doubt an important dimension through which investment firms compete to attract investors with heterogeneous preferences.[10] However, it should also be clear that introducing horizontal differentiation in our equilibrium framework would make it intractable and that is why we abstract from it. To further back up our homogeneity assumption, Tables 7 and 8, present some characteristics of the top 30 passive funds supplied in the Large Cap and Mid Cap sectors in 2018. The exposures to the 4 Carhart factors, the alphas and the gross-returns are similar across all funds especially within but also across the two sectors. Also, note that our model can be easily extended to include multiple index sectors with sector specific investors that do not substitute across sectors.[11]

Overall, we believe that this homogeneity assumption could be a good modelling compromise that would still allow us to study the competitive and asset pricing implications of fund proliferation while keeping the model tractable.

Investor learning. A large body of the literature on mutual funds has studied the so called flow-performance relationship.[12] According to the literature, past performance attracts new inflows regardless of whether performance persists or not. Building on this empirical findings, theoretical models studying the flow-performance relationship typically feature a learning component in which investors learn about unobserved managerial skills from past performance.[13] In our model we do not have investor learning because we are focusing on passive investment vehicles that track an underlying index. Therefore, we decided to not include investor learning in our dynamic model.

[10] Kostovetsky and Warner (2020) develop a textual measure of product differentiation and show that more differentiated/unique funds are able to attract higher inflows at least the first few years upon introduction. Abis and Lines (2022) use a k-mean mean clustering algorithm based on a textual analysis of fund prospectuses and show that funds are differentiated in groups and that investors withdraw money if funds tend to diverge from their prospectus strategy. Ben-David, Franzoni, Kim and Moussawi (2022) provide evidence that specialized ETFs that track niche portfolios are supplied to cater investors heterogeneous beliefs.

[11] The case in which investors substitute across sectors, say because of diversification motives, would substantially complicate the oligopolistic game between funds and management companies. The reason is that funds are now also competing across sectors with products that have different characteristics.

[12] See for instance the two seminal contributions Chevalier and Ellison (1997) and Sirri and Tufano (1998).

[13] The seminal contribution here is Berk and Green (2004) which rationalizes the flow-performance relationship in a model with rational investors who learn about managers' alphas. More recently, Roussanov, Ruan and Wei (2021) extends the Berk and Green (2004) to allow for search friction as in Hortaçsu and Syverson (2004).

4 Equilibrium

4.1 Equilibrium definition

We are now ready to define the equilibrium of our dynamic game. Following the industrial organization literature on dynamic oligopolies, we restrict our attention to Markovian strategies i.e., strategies that are function of payoff relevant state variables.[14] From problem (13), we can see that the payoff relevant state-variables for management company j are its menu of funds active in the previous period n_{jt-1} as well as competitors' menu of funds active in the previous period $(n_{j't-1})_{j'\neq j}$. Indeed, management companies $j' \neq j$ choose an optimal strategy $(n_{j't})_{j'\neq j}$ which is a function of $(n_{j't-1})_{j'\neq j}$. Because $(n_{j't})_{j'\neq j}$ enter company j's problem through n_t, then the optimal strategy of each management company depends on its own menu of pre-existing funds as well as the menu of pre-existing funds of all its competitors. To preserve computational tractability, for each management company j, we restrict attention to strategies that are function only of company j's own state (in our case, n_{jt-1}) and denote the policy function by $\alpha_j : [0, \infty) \to [0, \infty)$.[15]

Definition 1 *An equilibrium of our dynamic model consists in a profile of strategies $\alpha^* = (\alpha^*_j)_{j=1}^M$ with $\alpha^*_j : [0,\infty) \to [0,\infty)$, a path of asset prices $(P_t)_{t=1}^{\infty}$ and wealth $(A_t)_{t=1}^{\infty}$ such that:*
(1) in any period t, $\alpha^* = (\alpha^*_j)_{j=1}^M$ is a pure strategy Markov perfect equilibrium such that for all j

$$\alpha^*_j(n_{jt-1}) = \arg\max_{n_{jt}} \left\{ n_{jt}\frac{\pi_t}{(1+n_t)^2} - c_j(n_{jt} - n_{jt-1}) - \delta_j \left(\frac{n_{jt} - n_{jt-1}}{n_{jt-1}}\right)^2 n_{jt-1} + \beta V(n_{jt}) \right\}$$

where n_{jt} denotes the number of company j's funds active in the current period, n_{jt-1} the the number of company j's funds active in the previous period, $n_t = n_{jt} + \sum_{j'\neq j} n_{j't}$ with $n_{j't} = \alpha^*_{j'}(n_{j't-1})$ and $\pi_t = \frac{\mu_t^2}{\sigma^2}\beta A_t$.
(2) in any period t the asset market clears,

$$P_t = \frac{\mu_t n_t}{\sigma_t^2(1+n_t)}\frac{\beta A_t}{\bar{Q}},$$

and the path of wealth solves,

$$A_{t+1} = \beta A_t + \bar{Q}\left(D + \Delta P_{t+1} - f_t P_t\right).$$

[14] See for instance, Maskin and Tirole (1988), Ericson and Pakes (1995) and for a self-contained review Aguirregabiria, Collard-Wexler and Ryan (2021).

[15] Weintraub, Benkard and Van Roy (2008) show that this restriction is appropriate in oligopolies with many firms and that as the number of firms increases the equilibrium converges to the unrestricted Markov perfect equilibrium.

Before discussing existence and uniqueness of our equilibrium a few remarks are in order. First, our restricted Markovian strategies allow us to treat the Bellman of each of the M management companies as an independent single-agent dynamic problem. In other words, the strategy of each player does not depend on other players' states and thus, we can compute the envelope condition as in the standard single-agent frameworks. Second, we will assume that, when adjusting their menu of funds, management companies do not internalize the price impact generated by their actions. The profit earned by each management company indeed depends on expected returns and asset prices through the term π_t and asset prices in turn depend on the total number of funds n_t. Nonetheless we assume throughout that management companies take π_t as given.

4.2 Steady state definition and existence

While the computational complexity of the general model will require a numerical solution, we are able to formally characterize a steady-state equilibrium of our model characterized by

- $n_{j,t} = n_j > 0$ for any management company j and time t;
- $P_t = P > 0$ for any time t;
- $A_t = A > 0$ for any time t.

In other words, we are able to formally characterize a steady-state with constant index price, constant household wealth, and in which the dynamic game between management companies resolves with each company having incentive to keep the same number of funds over time.

We now turn to provide sufficient conditions for the existence and uniqueness of such a steady state equilibrium. We start by assuming that household subjective beliefs are constant over time, that is $\mu_t = \mu$ and $\sigma_t^2 = \sigma$ for all t. We maintain this assumption from here throughout the paper. It follows that, in steady state, the term π_t in (13) is also constant over time and equal to

$$\pi_t \equiv \pi = \frac{\mu^2}{\sigma^2}\beta A.$$

Proposition (1) provides sufficient conditions under which such steady state exists and is unique.

Proposition 1 *Let* $\tilde{\pi} \equiv \frac{D}{1-\beta}\frac{\beta\mu^2}{\sigma^2}$, *assume* $M\tilde{\pi} > (1-\beta)c$ *with* $c = \sum_j c_j$ *and, without loss of generality, let* $\bar{Q} = 1$. *Then, there exists a unique steady-state* $\{(n_j)_{j=1}^M, P, A\}$ *such that:*

(1) for any management company j and any period t, $n_{jt} = n_j = \alpha_j^*(n_j)$ satisfies

$$n_j = \frac{1+n}{2} - \frac{(1-\beta)}{2\pi} c_j (1+n)^3 \tag{19}$$

where $n = \sum_{j=1}^{M} n_j$ and $\pi = \frac{\mu^2}{\sigma^2}\beta A$;

(2) the market clearing price $P_t = P$, the wealth $A_t = A$ and the total number of funds n solve simultaneously

$$A = \left(\frac{1}{1+\zeta(n)}\right)\frac{D}{1-\beta} \tag{20}$$

$$P = \frac{\mu}{\sigma^2}\frac{n}{1+n}\left(\frac{1}{1+\zeta(n)}\right)\frac{\beta}{1-\beta}D \tag{21}$$

$$\tilde{\pi}(M + n(M-2)) = (1-\beta)c(1+n)^3\,(1+\zeta(n)) \tag{22}$$

where

$$\zeta(n) \equiv \frac{\mu^2}{\sigma^2}\frac{\beta}{1-\beta}\frac{n}{(1+n)^2}. \tag{23}$$

Moreover, $n_j > 0$ and company j remains active if $\frac{\pi}{(1+n)^2} > (1-\beta)c_j$.

Proof: See Appendix A.

Equations (20) and (21) describe the equilibrium wealth and asset prices as functions of the equilibrium number of funds. Equation (20) suggests that the steady-state wealth A is proportional to the present discounted value of future dollar dividend D where the constant of proportionality depends on n, i.e. on the competitive outcome among management companies. In particular, it is easy to notice that $\zeta(n) > 0$ and $\zeta'(n) < 0$ for $n > 1$. Thus, when competitive forces push companies to initiate a higher number of funds n, then $\zeta(n)$ declines and the steady-state wealth A increases. In the limit for $n \to \infty$, then $\zeta(n) = 0$ and $A = D/(1-\beta)$, i.e. the steady state wealth converges to the present discounted value of the dollar dividend D.

Similarly, according to equation (21), higher steady-state n leads to a higher index price P. In the limit for $n \to \infty$, we now have $P = \frac{\mu}{\sigma^2}\frac{\beta}{1-\beta}D$. More generally, equation (21) relates the equilibrium index price to the marginal cost of initiating new funds and thus microfounds the price impact of institutional investors in terms of the technological primitives of the asset management industry. In section 6.4, we will use equations (20), (21) and (22) to characterize the steady-state index price multiplier with respect to household wealth and we will show that this suggested measure of price impact crucially depends on the competitive outcome in the mutual fund sector. We will further perform a comparative static exercise to explore how the steady-state equilibrium, including the suggested measure of elasticity, vary with the dollar dividend D and the total fund initiation cost c.

While the result in Proposition (1) guarantees existence and uniqueness of a steady state in which all companies have no incentives to create additional funds and the index asset price is constant, we know less about the path $\{(n_{jt})_{j=1}^{M}, P_t\}_{t=1}^{T}$ that leads to such steady state. In the next section we provide a numerical algorithm that, for a given initial condition on the number of active funds $(n_{j0})_{j=1}^{M}$ and a given terminal date T, finds the optimal path if such path exists. The algorithm can be used to solve the model numerically and thus to derive the optimal path that, for given initial conditions, leads to the steady-state characterized in this section. For the purpose of this paper, we will use the algorithm to solve the model numerically and estimate the unobserved parameters in the cost function of management companies.

4.3 Numerical solution for the equilibrium path

The complexity of the problem prevents us from deriving formal properties of the equilibrium path out of the steady state. While formalizing its existence, convergence and stability properties is beyond the scope of the paper, the goal of our model is still quantitative and, as we will see in the next sections, we will estimate the model using data from the US mutual fund industry. With such goal in mind, in this section we propose a numerical procedure that, given proper initial and terminal conditions, allows to derive the equilibrium path if such path exists.

Our algorithm amounts to solving two fixed points, one nested into the other, that for a given set of initial conditions and parameter values, finds the optimal path of fund initiation, index price and household wealth. The numerical procedure can be summarized in the following steps:

Step 0. Set exogenous parameters to be kept constant throughout the algorithm:

- Fix exogenous parameters $\left\{\sigma,\ D,\ \mu,\ M,\ (c_j)_{j=1}^{M},\ (\delta_j)_{j=1}^{M},\ \bar{Q},\ \beta\right\}$.
- Fix the initial household wealth A_0.
- Fix the initial number of funds managed by each company j, $(n_{j0})_{j=1}^{M}$.
- Fix a terminal date T and the terminal number of funds managed by each company j, $(n_{jT})_{j=1}^{M}$.

Step 1. Solve the inner loop for a given path of asset prices $(P_t)_{t=1}^{T}$ and household wealth $(A_t)_{t=1}^{T}$ as follows:

- Construct the path for $(\pi_t)_{t=1}^{T}$.
- Guess a path for the number of funds managed by each company j: $\left(\left(n_{jt}^{(k)}\right)_{t=1}^{T}\right)_{j=1}^{M}$.

- Use euler equation (35) to find a new path $\left(\left(\tilde{n}_{jt}^{(k)}\right)_{t=1}^{T}\right)_{j=1}^{M}$.

- Update the path of funds using

$$n_{jt}^{(k+1)} = n_{jt}^{(k)} + \chi_n \left(\tilde{n}_{jt}^{(k)} - n_{jt}^{(k)}\right) \quad \forall j, t. \tag{24}$$

- Repeat until convergence.

Step 2. Run the outer loop to find the equilibrium path of index price and household wealth:

- Guess a path of prices $\left(P_t^{(q)}\right)_{t=1}^{T}$ and household wealth $\left(A_t^{(q)}\right)_{t=1}^{T}$.

- Run inner loop as in Step 1 to obtain $\left(\left(n_{jt}^{(q)}\right)_{t=1}^{T}\right)_{j=1}^{M}$.

- Use market clearing in (15) and the law of motion of wealth in (18) to find new paths $\left(\tilde{P}_t^{(q)}\right)_{t=1}^{T}$ and $\left(\tilde{A}_t^{(q)}\right)_{t=1}^{T}$.

- Update price and wealth using

$$P_t^{(q+1)} = P_t^{(q)} + \chi_p \left(\tilde{P}_t^{(q)} - P_t^{(q)}\right) \tag{25}$$

$$A_t^{(q+1)} = A_t^{(q)} + \chi_a \left(\tilde{A}_t^{(q)} - A_t^{(q)}\right) \tag{26}$$

- Repeat until the maximum of $||P^{(q+1)} - P^{(q)}||_\infty$ and $||A^{(q+1)} - A^{(q)}||_\infty$ is below some tolerance

To sum up, for a given set of parameter values, the routine just described starts in Step 2 with a guess for the equilibrium index price and wealth. It then moves to the inner loop (Step 1) and solves for the equilibrium number of funds taking the path of index price and wealth as given. Finally, it returns to Step 2 to update the equilibrium price and wealth. This routine is repeated until convergence. It is a nested procedure because the fixed point that solves for the Markov perfect Nash equilibrium is solved within each iteration of the fixed point that solves for the market clearing price and wealth evolution.

5 Data

Before turning to the estimation of our model we overview our data sources and how we constructed our estimation dataset.

5.1 Data sources

We obtained data on US mutual funds from the Center for Research in Security Prices (CRSP) which we accessed through the Wharton Research Data Services (WRDS). The data provide detailed information on US mutual funds at monthly frequency starting from 1961 but we restrict the sample from year 2000 to 2020 for the reasons we describe in the following subsection.

The data is at the share class level but we collapse everything at the fund-by-year level. Moreover, we focus on US domestic equity funds that, according to the CRSP investment objective classification, belong to the Large Cap, Mid Cap and Small Cap sectors. Among those, we identify passive funds as either index funds or ETFs as classified by CRSP. The resulting sample contains around 16,500 fund-by-year observations of which around 3,700 are passive investment vehicles.

Table (5) presents some summary statistics of our data. The average amount of asset under management at the end of year is around 2 billions but the distribution is quite skewed due to the presence of extremely large funds. The average monthly gross return in a given year is around 0.9%, with an average monthly alpha of 0.04% and an average market beta of 0.97. These latter are estimated for each year and each fund from a monthly regression of gross returns on the 3 Fama-French factors plus momentum including observations from the previous 3 years. Finally, the average market share at the management company level is around 1.7%, although also in this case the distribution is very skewed because, for most years, more than 50% of the market is captured by the five biggest management companies.

Table (6) replicates table (5) restricting the sample to passive funds only. As expected passive funds tend to be cheaper with an average expense ratio of 0.5% and larger, managing an average of 5.7 billions of assets. On average passive funds also seem to deploy more funds with an average of 4.5 funds per management company.

5.2 Data construction

We now discuss in detail the way we constructed our final dataset which we will use for estimating the model in the next section.

Filling missing of fund and company identifiers. Information about US mutual funds collected by CRSP is provided at the share class level. Data on returns and asset under management are at the monthly frequency whereas information on fund characteristics are provided quarterly. The first thing we do is to aggregate all share classes of the same fund in one single observation so that the resulting dataset is at the fund level. To this end, we exploit a grouping variable constructed by CRSP (denoted by *crsp_cl_grp*) that contains an unique code for all share classes that belong to the same fund. This variable is available starting from 1999 which is the main reason for why we restrict our sample to start from 2000. To identify funds of the same share class when *crsp_cl_grp* is not available we rely on the WFICN identifiers and on fund names. Fund names in CRSP are useful because contain three types of information: the name of the management company, followed by the name of the fund, followed by the type of share class. The former two are separated by a colon while the latter by a semicolon. Following this rule we parse each fund name in each month in three parts and then assign the same *crsp_cl_grp* to funds with the same fund name (i.e., the same second part of the name) in the same quarter. This procedure leaves us with 625 share class by quarter observations with a missing *crsp_cl_grp* out of more than 2 millions share class by quarter observations.

Key to our analysis is the role of management companies as fund initiators. In the data, we identify the management company that offers each fund using a unique management company identifier *mgmt_cd*, provided by CRSP, which is available starting from December 1999. Roughly 11% of share class by quarter observations have a missing *mgmt_cd* which we refill again exploiting the information available in the fund name. The first part of each fund name corresponds to the name of the management company; whenever missing we assign the same *mgmt_cd* to funds that feature the same management company name in the same quarter. This procedure fills around 60% of the missing *mgmt_cd*. Whenever this procedures fails because of mistakes in fund name spellings we refill *mgmt_cd* manually.[16] Overall, we were not able to identify the controlling management company for less than 1% of share class by quarter observations.

Aggregation of share classes and further cleaning. After the refilling procedure, we merge the quarterly level data on funds' characteristics (which include the *crsp_cl_grp* and *mgmt_cd* identifiers) with the monthly data on returns and AUM. Then, for each month we aggregate share classes of the same fund into one observation based on the *crsp_cl_grp* identifier. To do so we sum the end of month AUM of all share classes and take averages of other relevant variables, such as monthly returns and expense ratios, weighting by the AUM at the end of the previous month. Finally we only keep domestic equity funds and, to remove incubation bias, we drop funds that we observe for less than 12 months and whose AUM are less than 15 millions.[17] The resulting dataset contains around 650,000 fund by month observations.

[16] In some cases, mergers and acquisitions between companies create mismatches between fund names and the *mgmt_cd* code provided by CRSP which we manually correct whenever possible. For instance, after Blackrock acquired the iShare business from Barclays in June 2009 the *mgmt_cd* has not been updated accordingly. In this case there were two *mgmt_cd* codes "BZW" and "BLK" for Blackrock but we replaced "BZW" with "BLK" after 2009. Similarly, we replaced "PDR", the *mgmt_cd* for PDR services LLC owned by the American Stock Exchange, with "SSB" the *mgmt_cd* for State Street Bank which acquired the SPDR ETF license from PDR services LLC in 2005.

[17] To identify domestic equity funds we exploit the variable *crsp_obj_cd* which classifies funds based on their investment style. The variable is constructed by CRSP building on Strategic Insights, Wiesenberger, and Lipper objective codes

Dataset for model estimation. Our model focuses on homogenous passive investment vehicles that track an underlying index. In the data we identify passive funds using the variables *et_flag* and *index_fund_flag* and consider as passive both index funds and ETFs. Moreover, we restrict ourselves only to the Large Cap, Mid Cap and Small Cap sectors as identified by the *crsp_cl_grp* variable constructed by CRSP.[18] The reason is that more than half of pure index funds belong to these sectors and, as shown in Table 8 these products seem to be sufficiently homogeneous in terms of the risk-return profile they offer. Finally, we collapse everything at the year level and we obtain a dataset of 16,500 fund by year observations of which 3,700 are passive.

6 Model estimation

Using the numerical algorithm discussed in section 4, we now turn to estimate the model and discuss the results. Specifically, in section 6.1, we provide details of the estimation procedure. In section 6.2, we comment on the results, including the ability of the model to match targeted as well as untargeted moments. We then turn in section 6.3 to perform a series of counterfactuals devoted to assessing the contribution of each management companies to fund proliferation, fee and household surplus. Finally, section 6.4 shows that the model is suitable to speak to a growing literature that focuses on the asset pricing implications of inelastic demand for financial assets. Despite being completely untargeted, we estimate that a 1% increase in household wealth increases the steady-state valuation of the equity index by 5.5%. The resulting 5.5 steady-state multiplier is not only aligned with what the previous literature has documented, but it also provides an alternative microfoundation of inelastic markets, namely the competition among oligopolistic investment management companies.

6.1 Estimation procedure

The estimation procedure relies on calibrating a subset of the parameters while inferring a second subset of parameters from data. Because our model abstracts from product differentiation, we estimate the model to match features of mutual funds classified as either Large Cap or Mid Cap in CRSP. In other words, we limit ourselves to passive funds that track reasonably mature firms and exclude instead mutual funds that track growing or developing companies. Table 1 summarizes the calibrated inputs.

From our dataset, we estimate a dividend yield on the Russell 2000 equal to 2.14%. We then calibrate the dollar dividend D to match a dividend yield equal to 2.14% at time $t = 0$. We set household expected return μ to match the average return on the Russell 2000 index which we estimate equal to 6.12%. Similarly, we set the return

[18]Funds classified to belong to these sectors determine their holdings primarily on market capitalization considerations.

Parameter	Description	Value
$\frac{D}{P_0}$	Dividend yield	2.14%
σ	Volatility	25.09%
μ	Expected return	6.12%
β	Discount factor	0.98
M	Number of management companies	6
A_0	Initial wealth	1.00
$\bar{Q}$	Supply of shares	1.00
T	Terminal date (years)	20

Table 1: Calibrated inputs

volatility σ to match the standard deviation of the Russell 2000, equal to 25.09%. Since our attention is focused on Large Cap and Mid Cap funds, we use the Russell 2000 rather than the S&P500 as the counterpart of the equity index in our model. We set the number of management companies equal to 6. This choice is motivated by the newly documented evidence that the top five management companies behave very differently compared to other management companies and are responsible for most of mutual fund proliferation. For this reason, we directly model competition among the top five firms and classify all other management companies in one residual group (from here on, we will refer to this residual group as the outside management company, indexed by $j = 0$). We identify the top management companies as the five firms with the highest average annual market share throughout our sample.[19] We further normalize both household initial wealth A_0 and the supply of index shares $\bar{Q}$ to one. Finally, we set the terminal date $T = 20$ to match the length of our dataset which ranges between 2000 and 2020. For the purpose of our solution algorithm, we then use as terminal condition $(n_{jT})_{j=0}^{5}$ the number of funds that we observe in our dataset for each management company in 2020.

While all the parameters discussed so far can be easily obtained from data or can be reasonably linked to observables, the same is not true for the parameters $(c_j)_{j=0}^{5}$ and $(\delta_j)_{j=0}^{5}$ that characterize the cost function of the management companies in our model. For this reason, we estimate both set of parameters directly from data using the following estimation procedure.

Let $\theta = (c_j, \delta_j)_{j=0}^{5}$ denote the set of parameters to be estimated. We estimate θ by

[19]The market share of a given management company in a given year is simply computed as the sum of net assets across all funds operated by the management company, rescaled by the sum of net assets across all funds that appear in our dataset in a given year.

solving

$$\min_{\theta} \sum_{s=1}^{S} \sum_{j=0}^{5} \left(\Lambda_{sj}(\theta) - \Lambda_{sj}\right)^2 . \tag{27}$$

where $\Lambda_{sj}(\theta)$ denotes the s^{th} moment for management company j implied by the model and expressed as a function of the unknown parameters in θ. On the other hand, we denote by Λ_{sj} the empirical analogue of $\Lambda_{sj}(\theta)$ observed in the data.

From the problem solved by a generic management company j in our model, it can be noticed that c_j governs the linear trend in its menu of funds. Thus, for given initial condition n_{j0} on the number of funds that management company j operates at time $t = 0$, c_j is tightly linked to the average number of funds that management company j originates in each period. Differently from c_j, δ_j governs how costly is for management company j to adjust and restructure its menu of funds. Specifically, given two management companies j and j' at time t with $n_{jt-1} = n_{j't-1}$, if $\delta_j < \delta_{j'}$ then adjusting the menu of funds is less costly for company j. In other words, management company j can more easily adapt its supply of funds without incurring in large adjustment costs. In mathematical terms, δ_j is directly related to the curvature across time in the equilibrium number of funds offered by company j, with lower δ_j translating into higher curvature in the equilibrium path.

Informed by the above discussion, we select the following two moments for a given management company j

$$\Lambda_{1j} = \sum_{t=1}^{T} \frac{\Delta n_{jt}}{T} \quad \forall j \tag{28}$$

$$\Lambda_{2j} = \sum_{t=1}^{T} \frac{\Delta\left(\Delta n_{jt}\right)}{T} \quad \forall j \tag{29}$$

In words, Λ_{1j} captures the average creation rate in absolute terms of management company j, which allows to pin down c_j. On the contrary, Λ_{2j} captures the concavity/convexity of management company j creation rate over time, and it allows to pin down δ_j. For each of the top five management companies, both moments are computed from the time-series of the number of funds operated by the management company between 2000 and 2020. For the outside management company, both moments are computed from the time-series of the average number of funds operated by non-top five management companies over the same time interval. Finally, notice that the estimation problem involves $6 \times 2 = 12$ moments and $6 \times 2 = 12$ unknowns, so that it is exactly identified.

Concretely, we employ the following steps to obtain an estimate of θ:

- At the end of each iteration in the nested fixed loop described in section 4.3, we compute $\Lambda_{1j}(\theta)$ and $\Lambda_{2j}(\theta)$ for any management company j.

- Given Λ_{1j} and Λ_{2j} from data, we form the objective function in equation (27).
- We iterate over θ until the objective is minimized

The minimization works robustly and it takes around three seconds to solve one iteration on a standard portable computer.

We conclude this section by reporting in table 2 summary statistics for the six management companies used in our estimation procedure. In reporting the last row, we first construct the outside management company by averaging in each year across all non-top five management companies and by subsequently averaging in the time-series.[20]

Management company	Share	Num. of funds	n_{j0}	n_{jT}	Λ_{1j}	Λ_{2j}
Vanguard	46.73%	6.95	4	9	0.25	0.00
State Street	17.25%	6.24	1	10	0.45	-0.05
Blackrock	9.89%	11.90	2	13	0.55	-0.36
Fidelity	9.61%	3.10	2	8	0.30	0.05
Charles Schwab	2.28%	2.62	2	4	0.10	0.00
Outside MC	0.17%	1.78	1.33	1.97	0.03	-0.01

Table 2: Summary statistics and estimated inputs

The top five management companies have reported, on average, a cumulative market share of 85.76%. In other words, differentiating the top five management companies and regrouping all other firms allow modeling directly more than 80% of the market on average. A second relevant feature of the data is a clear positive relation between the average market share and the average number of controlled funds. This feature is consistent with the mechanisms in our model where, given the absence of fund differentiation, a management company can increase its market share only by increasing the number of funds it operates. The last three columns in table 2 further provide three set of parameters that directly enter the estimation procedure. Columns (4) and (5) report the number of funds that each management company used to operate in year 2000 and 2020 respectively, which we employ as initial and terminal conditions in the model estimation. Columns (6) and (7) provide the empirical analogue of the moments we use to estimate the model. Blackrock and State Street are characterized by the highest absolute rate of fund creation Λ_{1j} but also by the most concave transition

[20]This practice has the shortcoming that average market shares do not generally sum to one, but it has the advantage that the outside management company can be interpreted as a representative "small" management company.

pattern Λ_{2j}. These features already point to the presence of both a low linear cost c_j and low adjustment cost δ_j. Both the rate of fund creation, as well as the concavity of the transition pattern, are lower for Vanguard, Fidelity and Charles Schwab, but well above the corresponding moments reported for the outside management company. Interestingly, Fidelity is the only management company with positive Λ_{2j}, determined by the fact that Fidelity started engaging in fund creation only in recent years, after 2015. These features of data are confirmed by figure 1, which reports the time-series of the number of funds controlled by the top 5 management companies.

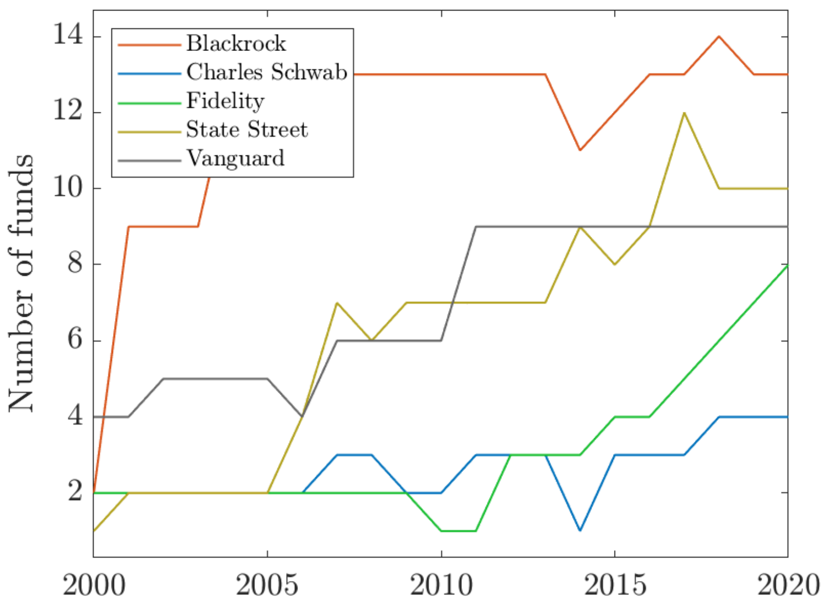

Figure 1: Number of funds operated by each of the top five management companies over time.

6.2 Results

We use the procedure as well as the moments discussed in section 3 to estimate the model. We start by reporting in table 3 the estimated vector of parameters θ across the six management companies considered in the estimation. For a direct comparison between estimated parameters and target moments, we reinclude n_{j0}, Λ_{1j} and Λ_{2j} in table 3 as well.

With the lowest linear cost c_j and the highest speed of adjustment δ_j, Blackrock is the most efficient management company.[21] Such efficiency allowed Blackrock to in-

[21] In the estimation we considered Blackrock and Barclays an unique entity even before their merger in 2009 when Blackrock acquired the Barclays' iShare business. Before the acquisition Blackrock market share was small whereas Barclays was one of the top 5, the opposite happens after the acquisition. Another way to interpret this is thinking to iShare to be itself a multi-product company and according to our estimate the most efficient one. In practice, although the owner of the iShare business changed, iShare has always been one of the market leader since the early 2000.

Management company	n_{j0}	Λ_{1j}	Λ_{2j}	c_j	δ_j
Vanguard	4	0.25	0.00	0.0099	7.9003
State Street	1	0.45	-0.05	0.0007	3.0815
Blackrock	2	0.55	-0.36	0.0004	0.0001
Fidelity	2	0.30	0.05	0.0871	5.7127
Charles Schwab	2	0.10	0.00	0.0584	7.6989
Outside MC	1.33	0.03	-0.01	0.0238	5.0298

Table 3: Estimated parameters

crease massively the number of controlled funds from 2 to 13 throughout the sample, with 0.55 funds created on average in each year. Compared to the beginning of our sample, Blackrock managed to become an absolute industry leader by 2020. The second most efficient firm is State Street. While in 2000 State Street was controlling only one fund (less than the average number of funds controlled by the outside management companies), it managed to create 0.45 funds per year on average, concluding the 2020 with 10 funds, second only to Blackrock. In 2000, Vanguard controlled more funds than any other management companies. The rate of fund creation, however, has been lower for Vanguard than for Blackrock and State Street. Finally, Fidelity and Charles Schwab appear as the least efficient firms among the top five management companies, although Fidelity experienced a significant bounce up that started in 2015.

We next turn to validate our estimated parameters. Using the estimated vector of parameters θ, we reconstruct the time-series of n_{jt} for each management company $j \in \{0, ..., 5\}$ as implied by the model solution. We further average n_{jt} across the top five management companies $j \in \{1, .., 5\}$ in each period. Figure 2 compares the time-series of n_{0t} and $\sum_{j=1}^{5} \frac{n_{jt}}{5}$ in model vs data.

The model is able to exactly match the high creation rate observed for the top management companies as well as the low rate of fund creation observed for other management companies. Despite our model is estimated from a set of exactly identified equations, we believe these estimates provide a useful quantitative benchmark to explain dynamics in this industry. This is confirmed by the fact that the estimated model is able to match extremely closely also untargeted moments and, in particular, the secular decline in average fee charged by Mid and Large Cap passive funds. Figure 3 compares the value-weighted fee observed in data against the equilibrium fee implied by the model and estimated using equation (9).

Lastly, we compare the revenues gained by top five management companies with the revenues gained by the outside management company in the estimated model. We report the output in figure 4.

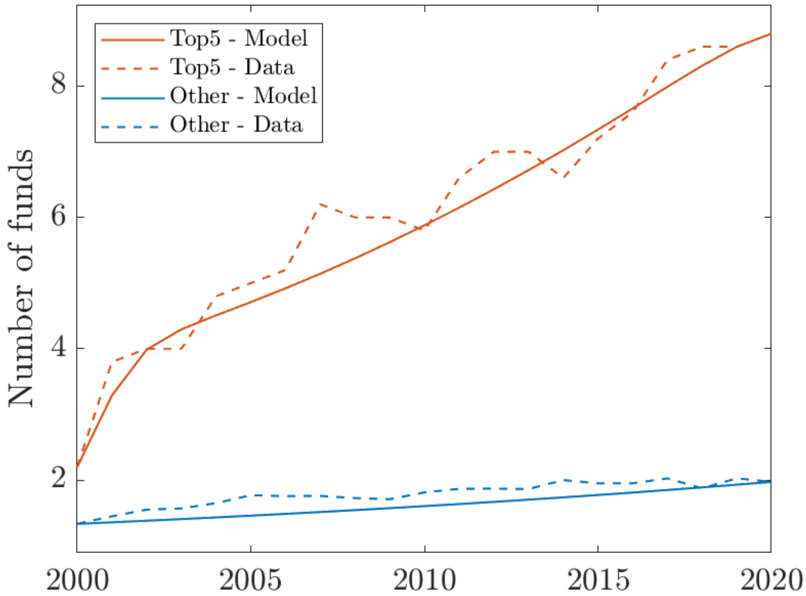

Figure 2: Time-series of the average number of funds operated by the top five management companies as well as the number of funds operated by the outside management company in model vs data. In data, the time-series of the number of funds operated by the outside management company is computed as simple average of the number of funds operated by all non-top five management companies

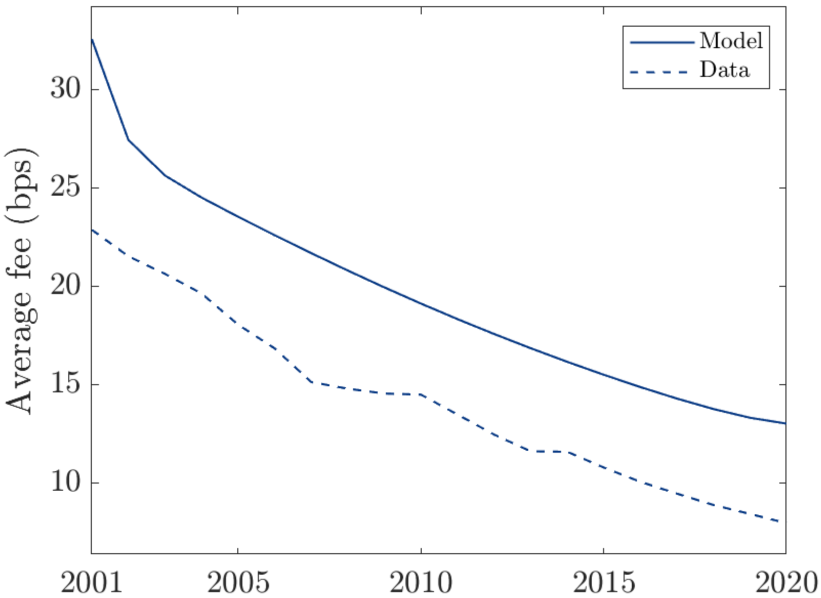

Figure 3: Time-series of value-weighted fee from data and equilibrium fee estimated from the model. The value-weighted fee from data is estimated, for each year, by averaging the expense ratio reported by CRSP for each fund with weights proportional to lagged total net assets.

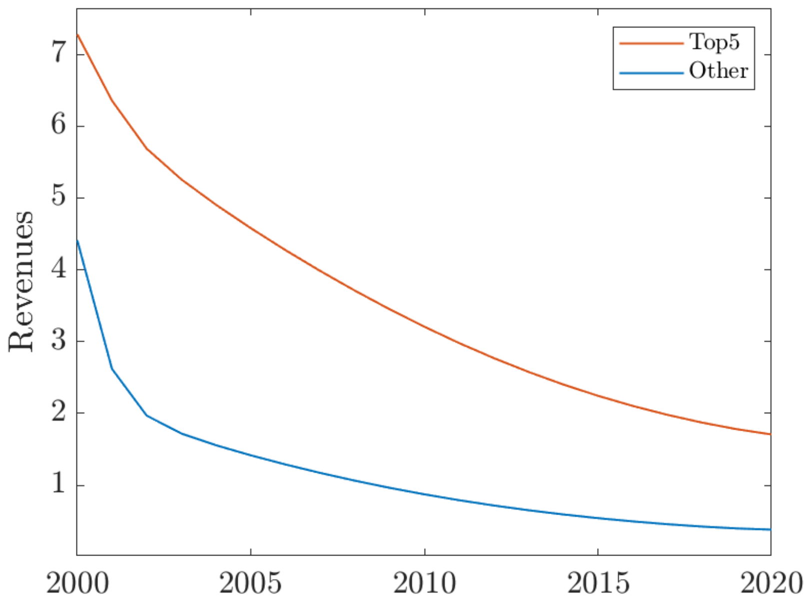

Figure 4: Estimated time-series of average revenues gained by the top five management companies and by the outside management company in the estimated model.

Management companies compete with each other over time and try to gain market share by creating new funds. As the number of funds in the market increase, the equilibrium fee declines and revenues decline as well both for top five management companies and for the outside management company. However, because the top five management companies are more efficient than the outside management company, they can efficiently use fund initiation as a tool to saturate the market, leading the outside management company to earn close to zero revenues by the end of the sample.

6.3 Counterfactuals and welfare analysis

We now turn to the key section of the paper. Using the estimated model, we perform a series of counterfactuals devoted to understand the contribution of each management company to the secular decline in fees and to consumer surplus.

For each management company $j \in \{0, .., 5\}$ we start by fixing the initial number of funds n_{j0} at the level observed in 2000, the beginning of our sample. We further fix c_j and δ_j to the estimates obtained and discussed in section 6.2. Using the calibrated parameters in Table 1, we numerically solve for the model equilibrium over a long horizon which we set equal to $T = 100$ years. The model solution allows us to derive the equilibrium path for the number of funds n_{jt} held by each management company, the total number of funds n_t, the fee f_t and the revenues earned by each management company j. We further construct consumer surplus as

$$S = \sum_{t=0}^{99} \beta^t \log(C_t) + \beta^{100} \frac{\log(C_{100})}{1-\beta}$$

The equation for S implicitly assumes that household consumption remains constant at C_{100} for all $t \geq 100$. While this assumption is not an equilibrium outcome, setting the terminal date $T = 100$ so far away in the future implies that the terminal value $\beta^{100} \frac{\log(C_{100})}{1-\beta}$ only accounts for 2.74% of household overall surplus S.

Given the model solution, we perform a series of counterfactuals devoted to understanding the impact of each management company on equilibrium outcomes. Specifically, we solve the model after removing each management company j, one at a time. For each of the remaining management companies $j' \neq j$, we keep the same initial condition $n_{j'0}$ and the same estimates c_j' and δ_j'. This procedure allows us to construct the counterfactual equilibrium path for number of funds, fee, revenue and consumer surplus that would have prevailed if management company j had not been operational.

Figure 5 provides the results of our counterfactual analysis. Each bar is labelled after the name of the management company that is excluded in the counterfactual of interest.

We start from discussing the top-left panel, which reports the percentage change in average number of funds in each counterfactual compared to the model solution. Removing Vanguard would lead to the largest decline in the number of funds operating in the market and equal to 31.95%. Removing Blackrock and State Street would also lead to a significant decline in the number of funds, equal to 21.00% and 19.20% respectively. Excluding Charles Schwab would decrease the number of operating funds by 14.10% while, interestingly, excluding Fidelity would leave the number of funds basically unaltered.

Turning to fees, the pattern is symmetric compared to the one seen for the number of funds. Removing Vanguard and Blackrock would significantly decrease competition in the mutual fund industry, leading to a 45.88% and 33.71% increase in the average fee respectively. The counterfactual increase in fee would be lower but still significant after removing State Street or Charles Schwab, equal to 21.32% and 16.12% respectively. The large increase in fees after removing Vanguard would significantly boost management companies revenues by 57.75%. The increase in average revenue would instead lie between 28.02% and 39.25% if any other management company is removed from the market.

Finally, we turn to household surplus. Removing Blackrock would lead to the largest decline in household surplus, equal to 24.64%. This is expected since Blackrock is by far the most efficient firm based on the estimates discussed in section 6.2. The second largest decline in household surplus is equal to 9.19% and is observed after

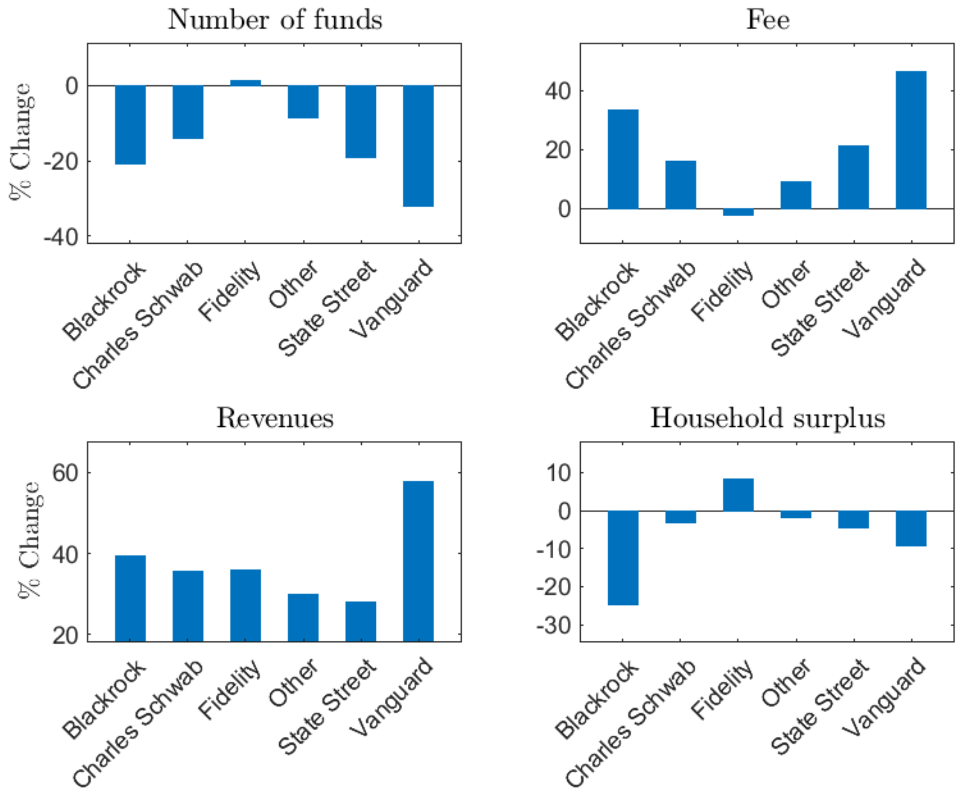

Figure 5: Percentage change in average number of funds, average fee, revenues and consumer surplus in each counterfactual compared to the model solution. For number of funds, we report the percentage change in average number of funds, where the average is computed across time. For the fee, we report the percentage change in the average fee, where the average is computed across time. For revenues, we report the percentage change in average revenues, where the average is computed across time and management companies.

removing Vanguard. Removing other management companies would lead to a change in household surplus between −4.57% and 8.02%.

Overall, the results so far suggest that large and efficient management companies play a crucial role in shaping the competition in the mutual fund sector. At the same time, the decline in household surplus may appear mechanical since removing any management company from the market would trivially lead to lower competition in the mutual fund sector. Lower competition would in turn increases equilibrium fees thereby reducing household welfare. We show that this is not the case. Specifically, we conduct a counterfactual where Blackrock is replaced by two additional management companies equal to Charles Schwab. This implies that the number of management companies competing in the industry increases from 6 to 7. We report in figure 6 the percentage change in the number of funds, fee, revenues and household surplus between this counterfactual and the model solution.

Replacing Blackrock with two less efficient firms would lead to a 6.23% increase in

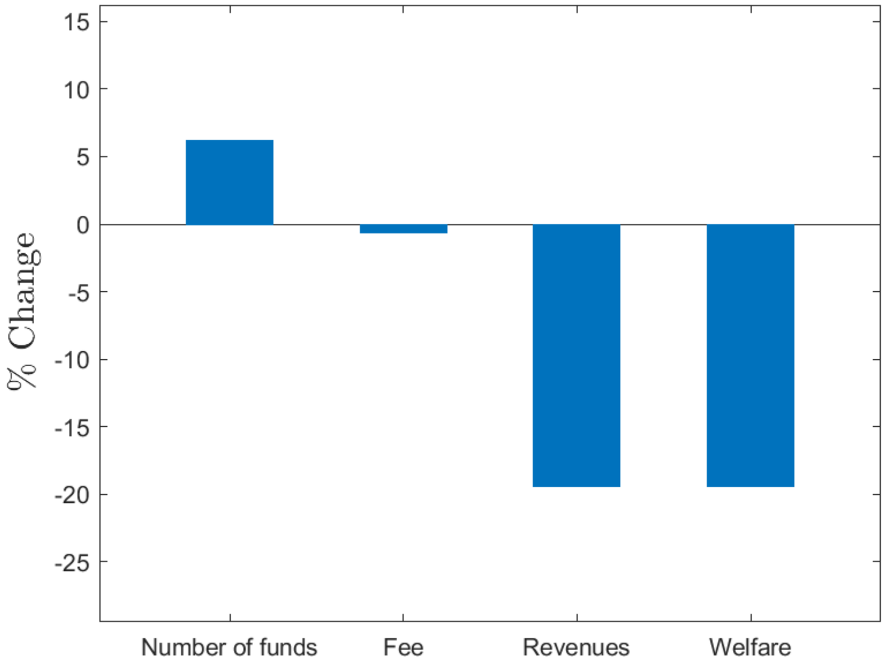

Figure 6: Percentage change in average number of funds, average fee, revenues and consumer surplus in the counterfactual where Blackrock is replaced by two firms identical to Charles Schwab. For number of funds, we report the percentage change in average number of funds, where the average is computed across time. For the fee, we report the percentage change in the average fee, where the average is computed across time. For revenues, we report the percentage change in average revenues, where the average is computed across time and management companies.

the average number of funds while leaving the average fee basically unaltered. While relatively more inefficient management companies can eventually substitute Blackrock funds, they can do so only over the long run while smoothing fund creation over time. This gradual substitution leads to a large a decrease in household surplus, equal to 19.46%. Thus, constraining efficient management companies in favor of competition is not necessarily the optimal solution for a policy maker interested in maximizing household welfare.

6.4 Asset pricing implications

In this section, we go back to the steady-state equilibrium that we are able to characterize analytically and, within this equilibrium, we use the estimates from section 6.2 to connect the competitive response of management companies to the evidence of inelastic demand for financial assets that the previous literature has documented. In particular, we estimate that a reduction in initiation costs c that induces a 1% increase in the steady-state wealth A increases the valuation of the equity index by 5.5%. In other words, the steady-state of our estimated model implies a multiplier ξ

Parameter	Description	Value
c	Estimated initiation cost	0.18
P	Steady-state index price	0.54
A	Steady-state financial wealth	0.66
$\frac{D}{P}$	Steady-state dividend yield	0.03
n	Steady-state total number of funds	5.8
ξ	Steady-state multiplier	5.5

Table 4: Estimated multiplier

of household wealth on the equity index price equal to 5.5.

We start with the following proposition that provides a closed-form expression for the multiplier ξ in the steady-state of the model.

Proposition 2 *Under the conditions detailed in section 4.2, the steady-state multiplier ξ is given by*

$$\xi \equiv \frac{dP}{dA}\frac{A}{P} = \left(1 - \frac{1}{n(1+n)}\frac{1+\zeta(n)}{\zeta'(n)}\right) \tag{30}$$

with $\zeta(n) > 0$ and $\zeta'(n) < 0$ for $n > 1$.

Proof: See Appendix A.

We use the estimates for $\{c_j\}_{j=0}^{5}$ derived and discussed in section 6.2 to compute $c = \sum_{j=0}^{5} c_j$. Moreover, we use equations (20), (21) and (22) to solve for the steady-state wealth A, index price P and number of funds n. Thus, we have all the inputs needed to produce an estimate for the steady-state multiplier ξ using equation (30). Details about the inputs used to estimate ξ are provided in Table 4. For completeness and to ease comparison, we reinclude in table 4 also parameters that have been already introduced but that enters the expression of ξ.

Our estimated steady-state multiplier of 5.5 was untargeted in the estimation, yet very aligned to estimates that the previous literature has reported. Among others, Gabaix and Koijen (2021) estimates a macro equity multiplier equal to 5 and shows that previous estimates range approximately between 1.5 and 5.5. Our estimate is thus consistent with previous work. Yet, to our knowledge, no previous work has microfounded the macro equity multiplier starting from the competitive dynamics of passive mutual funds and dominant management companies.

We conclude this section by performing a comparative static exercise in the steady-state of our model, where we vary the dollar dividend D and fund initiation costs c around the values reported in Table 4.

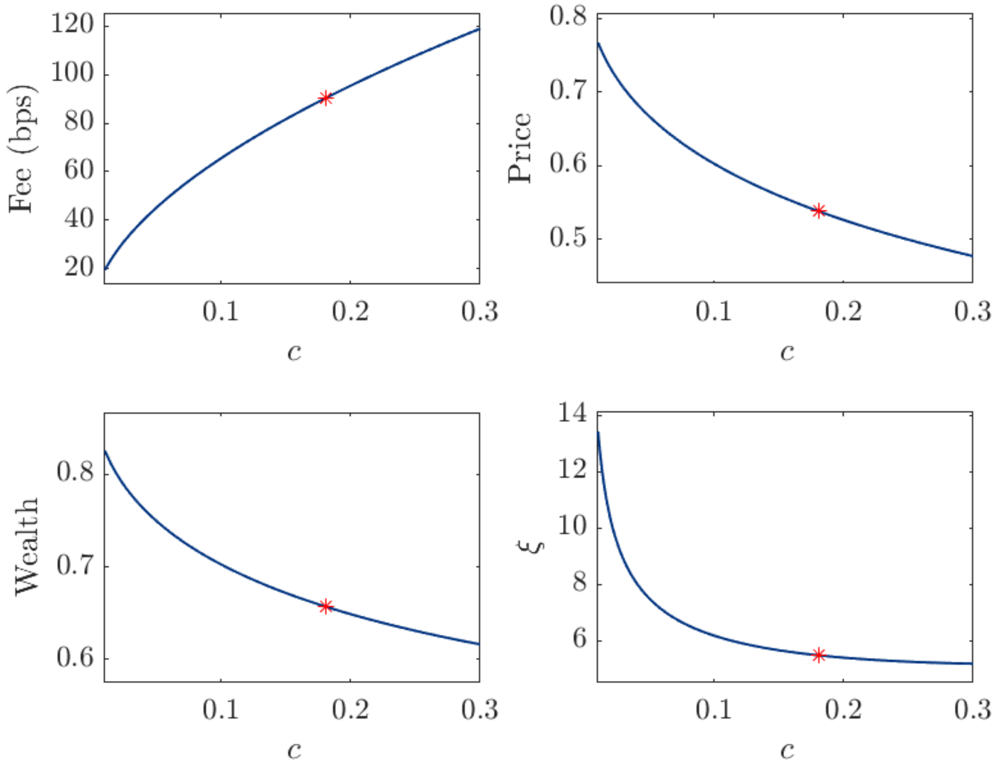

Figure 7: Equilibrium comparative static with respect to initiation costs c.

We start from varying c in Figure 7 and we flag in red the estimates we obtain in our model. The top left panel shows the steady state fee as function of c. Not surprisingly, the equilibrium fee increases with the initiation costs. From the perspective of our model, higher costs will push management companies to supply less funds. Lower competition in the mutual fund sector would then endogenously lead to higher fees. The top right panel looks at the equilibrium index price P and shows that as initiation costs rise, the equilibrium index price decreases. From the top left panel we know that higher initiation costs are passed-through investors via higher fees which in turn reduce household demand for the equity index. Finally, via market-clearing, lower demand for the equity index leads to a lower equilibrium price. Higher initiation costs also lead to lower equilibrium wealth A as shown in the bottom left panel. Once again, the mechanism for this outcome is driven by the competitive incentives in the mutual fund sector. Higher costs lead to lower fund creation and higher fees resulting in redistribution of wealth from household to mutual funds and management companies.

Lastly, the bottom right panel shows how the multiplier ξ varies with initiation costs. Increasing initiation costs from 0.01 to 0.3 decreases the steady-state multiplier from 13 to 4. To understand this result, consider first the case when c is small. As A increases, management companies find it optimal to create additional funds to collect part of the increase in wealth. Because c is small, management companies are able

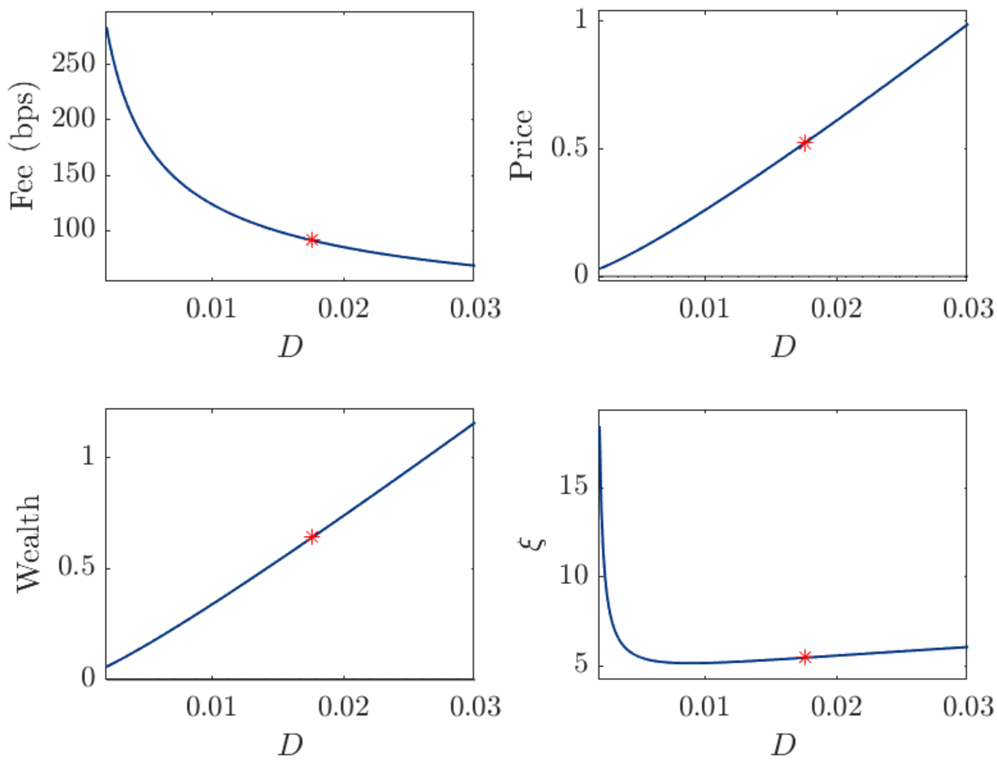

Figure 8: Equilibrium comparative static with respect to initiation costs c.

to initiate a large number of funds thereby significantly increasing competition in the mutual fund sector. As the equilibrium fee declines, household optimally invests a larger fraction of its wealth in the stock market, leading to an increase in the equity index price by market clearing. However, when c is higher, the creation of new funds becomes more and more costly for management companies. Thus, when A increases, management companies abstain from creating a large number of new funds. The result is that the level of competition in the mutual fund sector stays low, fees decline less and household increases less its demand for the equity index. The ultimate result is that, by market clearing, P increases but it increases less compared to the case of lower c.

Next, in Figure 8 we consider the comparative static of the same variables with respect to the dollar dividend D. As before, the top left panel shows the comparative static for the equilibrium fees. In this case, a higher dollar dividend leads to a decline in fees because a higher D increases the rate at which wealth accumulates. To accommodate the increase in asset demand, management companies create more funds. The stronger competition in the mutual fund sector ultimately leads to lower fees. Turning to the comparative static for P (top right panel) and A (bottom left), we notice that, differently from initiation costs, D affects the equilibrium price and wealth both directly and indirectly through the equilibrium number of funds n. Starting from the top right panel, we see that the index price increases with D. Indeed, a higher dividend increases the rate at which household wealth accumulates which in turn increases household demand for the equity index. Moreover, management companies accommodate the increase in demand by creating additional funds, leading to a decrease in fees and to a further increase in household demand. Both the direct effect on wealth as well as the indirect effect through n contribute to increasing household demand, ultimately leading to an increase in the index price through market clearing.

Turning to the bottom left panel, we can see that the equilibrium wealth increases with the dollar dividend D. The dividend affects the equilibrium wealth directly because it mechanically increases the rate at which wealth accumulates and indirectly through fund initiation. In other words, higher D directly increases wealth accumulation rate and indirectly prompts management companies to increase the number of funds, given the increase in demand. Stronger competition in the mutual fund sector leads to a decline in fees which further and indirectly accelerate wealth accumulation. This indirect effect is summarized by the term $\frac{1}{1+\zeta(n)}$ in equation (20). Because $\frac{1}{1+\zeta(n)}$ is an increasing function of n, it contributes to amplify the initial and direct increase in D.

Finally, the bottom right panel describes how the steady-state multiplier varies with D. Notice that the steady-state multiplier depends on D only indirectly, through n. Consider first the case of small D. In this case, household demand for the equity index is relatively low with the consequence that management companies are constrained to manage a relatively limited menu of funds. It follows however that any increase in household wealth is particularly attractive for management companies and they respond by creating to an increase in A by creating a higher number of funds compared to the case of high D. The larger response of management companies in turn leads to a larger decline in fee, a larger increase in household demand and, ultimately, to a larger increase in the equity index price via market clearing.

7 Conclusions

In this paper, we propose a model where the competitive dynamics in the mutual fund industry are driven by the decisions of heterogeneous and multi-product management companies to initiate new funds. We provide sufficient conditions that guarantee existence and uniqueness of a steady state equilibrium characterized by a constant number of funds operated by each management company and a constant index price. In addition, we develop a numerical algorithm that solves for the equilibrium path of the number of funds created by each management company and the equilibrium market clearing asset price.

In the second part of the paper, we estimate the model using data on US passive equity funds that operate in the Large and Mid Cap sectors. For each of the five biggest management companies, we estimate their cost of initiating new funds and match the fund proliferation patterns observed in the data with the ones implied by our model. Moreover, to further validate the model, we show how the model implied time series of equilibrium fees closely follows the observed time series of average expense ratio.

With our estimated model parameters, we study several counterfactuals to understand the contribution of each management company to the secular decline in fees and the surplus of our household investors. In the first set of counterfactuals, we remove, one at the time, each of the top 5 management companies from the market. In all cases, investor surplus decreases substantially, although the magnitude is heterogeneous and depends on how efficient the removed company is. Removing the most efficient management company, Blackrock, reduces household welfare by 25%. In a second set of counterfactuals, we perform a similar exercise. However, instead of simply removing the most efficient company from the market, we replace it with two less efficient companies. Interestingly, investor surplus still goes down with a reduction of about 20%. The key insight is that restricting efficient management companies to favor competition might ultimately hurt investor welfare.

Finally, we turn to the asset pricing implications of our model. Modelling competition across management companies in an asset market equilibrium framework allows us to microfound the price impact of large institutional investors through their technological primitives. A reduction in initiation costs pushes companies to create more funds, reducing equilibrium fees and increasing household asset demand for the equity index. The index price will then need to increase to clear this excess demand. Lastly, we derive a closed-form expression for the steady-state multiplier of the equity index price with respect to household wealth. Using our initiation cost estimates, we find that a 1% increase in household wealth implies a 5.5% increase in the steady-state index price. While this estimate is aligned with previous results in the literature, we microfound the equity multiplier through the competitive forces among large, heterogeneous and multiproduct investment companies.

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A Derivations and Proofs

Derivation of HH portfolio allocation. Under the assumption of log utility, it is easy to verify that consuming a constant fraction of wealth is optimal for HH. In particular, from the Euler equation (4) and the budget constraint, one can verify that $C_t = (1-\beta)A_t$ is the optimal consumption in each period.

To derive the optimal portfolio allocation w_t, denote the log consumption and log wealth by $c_t \equiv \log(C_t)$ and $a_t \equiv \log(A_t)$ respectively so that $c_t = \log(1-\beta) + a_t$. The budget constraint in logs is then

$$\begin{aligned} \Delta a_{t+1} &= \log(1 + w_t R_{t+1}) + \log(1-\beta) && (31) \\ &\approx w_t r_{t+1} + \frac{1}{2} w_t (1-w_t)\sigma_t^2 + \log(1-\beta) && (32) \end{aligned}$$

where $\Delta a_{t+1} \equiv a_{t+1} - a_t$, $r_{t+1} \equiv \log(1+R_{t+1})$ and the second line follows the log-linear approximation of log portfolio returns in Campbell and Viceira (2002). Next, note that under the assumption that r_{t+1} is a Gaussian stationary process, we can take logs on both sides of (4) to obtain

$$\mathbb{E}_t[\Delta c_{t+1}] = \log(\beta) + \rho_t - f_t + \frac{1}{2}\sigma_t^2 + \frac{1}{2}\mathbb{V}_t[\Delta c_{t+1}] - Cov_t[\Delta c_{t+1}, r_{t+1}].$$

Moreover, because we normalized the return on the risk-free to zero, the above expression boils down to

$$\mu_t - f_t + \frac{1}{2}\sigma_t^2 = Cov_t[\Delta c_{t+1}, r_{t+1}]. \tag{33}$$

Lastly, approximation (32) and the constant consumption-wealth ratio imply that we can solve for w_t in (33) to obtain

$$w_t = \frac{\rho_t + \sigma_t^2/2 - f_t}{\sigma_t^2}. \tag{34}$$

Proof of Proposition 1. For given π_t, company j's Euler equation implied by problem (13) is given by

$$\begin{aligned} &\frac{\pi_t}{(1+n_t)^2} + \delta_j \beta \left(\frac{n_{jt+1} - n_{jt}}{n_{jt}} \right) \left[\frac{n_{jt+1}}{n_{jt}} + 1 \right] = \\ &\frac{2\pi_t n_{jt}}{(1+n_t)^3} + (1-\beta)c_j + \delta_j \left(\frac{n_{jt} - n_{jt-1}}{n_{jt-1}} \right) && (35) \end{aligned}$$

If a steady $\{(n_j)_{j=1}^M, P, A\}$ exists, then for given P and A, n_j must satisfy (35) which boils down to

$$n_j = \frac{1+n}{2} - \frac{(1-\beta)}{2\pi} c_j (1+n)^3 \tag{36}$$

where $\pi = \beta A \frac{\mu^2}{\sigma^2}$. Summing across j, the steady state total number of funds n in the market solves

$$\beta \frac{\mu^2}{\sigma^2} A(M + n(M-2)) = (1-\beta)(1+n)^3 c \tag{37}$$

Moreover, given the steady state fee

$$f = \frac{\mu}{n+1} \tag{38}$$

we can rewrite the equations that pin down the steady state P and A as

$$P = \frac{\mu n}{\sigma^2(1+n)} \beta A \tag{39}$$

$$A = \beta A + D - \frac{\mu}{1+n} P \tag{40}$$

where without loss of generality we normalized $\bar{Q} = 1$. From (39) and (40) we can solve for A and P as function of n and other parameters

$$P = \left(\frac{\frac{\mu}{\sigma^2} \frac{\beta}{1-\beta} \frac{n}{1+n}}{1+\zeta(n)} \right) D \tag{41}$$

$$A = \left(\frac{1}{1+\zeta(n)} \right) \frac{D}{1-\beta} \tag{42}$$

where

$$\zeta(n) \equiv \frac{\mu^2}{\sigma^2} \frac{\beta}{1-\beta} \frac{n}{(1+n)^2}. \tag{43}$$

The steady-state n can then be found by substituting (42) into (37)

$$\tilde{\pi} \left(\frac{1}{1+\zeta(n)} \right) (M + n(M-2)) = (1-\beta)(1+n)^3 c \tag{44}$$

which can be rearranged more conveniently as

$$\tilde{\pi} \left(M + n(M-2) \right) = (1-\beta) c \left[(1+n)^3 + \frac{\mu^2}{\sigma^2} \frac{\beta}{1-\beta} n(1+n) \right] \tag{45}$$

with $\tilde{\pi} \equiv \frac{D}{1-\beta} \frac{\beta \mu^2}{\sigma^2}$.

To show existence and uniqueness, note that at $n = 0$, the LHS of (45) is greater than its RHS provided $\tilde{\pi}M > (1-\beta)c$. Next, note that the LHS increases in n at a constant rate, whereas the RHS increases in n at an increasing rate. Thus, there will be one and only one $n > 0$ at which (45) is satisfied ■

Proof of Proposition 2. Consider an increase in fund initiation costs c and note that the only way this change in costs affects the equilibrium wealth A and asset prices P is through the effect on n. Differentiating (41) and (39) with respect to n gives

$$\frac{dA}{dn} = -\frac{\zeta'(n)}{1+\zeta(n)}\frac{D}{1-\beta}$$
$$\frac{dP}{dn} = \frac{\mu}{\sigma^2}\frac{n}{1+n}\beta\frac{dA}{dn} + \frac{1}{(1+n)^2}\frac{\mu}{\sigma^2}\left(\frac{1}{1+\zeta(n)}\right)\frac{\beta}{1-\beta}D$$

Next, take the ratio of the two expressions above and note that

$$\frac{P}{A} = \frac{\mu}{\sigma^2}\beta\frac{n}{1+n} \tag{46}$$

we obtain

$$\frac{dP}{dA} = \frac{P}{A}\left(1 - \frac{1}{n(1+n)}\frac{1+\xi(n)}{\xi'(n)}\right) \tag{47}$$

with

$$\zeta'(n) = \frac{\mu^2}{\sigma^2}\frac{\beta}{1-\beta}\frac{1-n}{(1+n)^3} \tag{48}$$

which is negative for $n > 1$ ■

B Figures

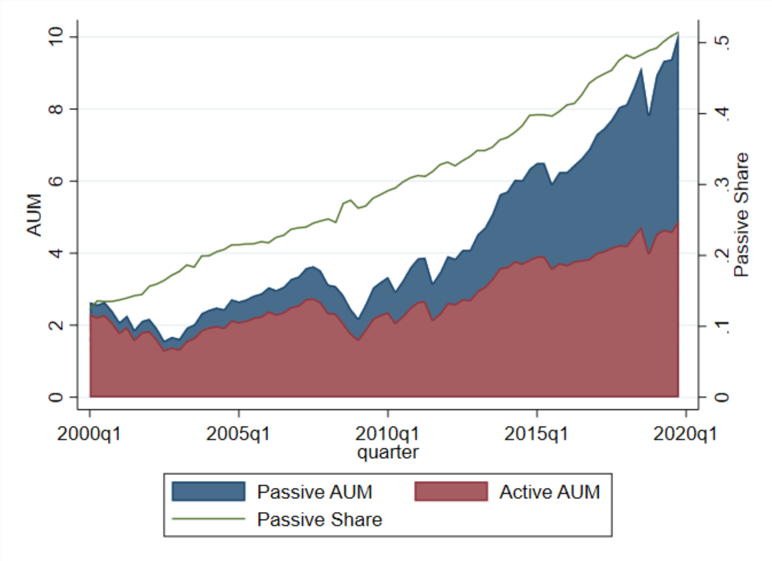

Figure 9: Left Axis: AUM in trillions of $ for both passive and active equity industry. Right Axis: Share of AUM held in the passive industry.

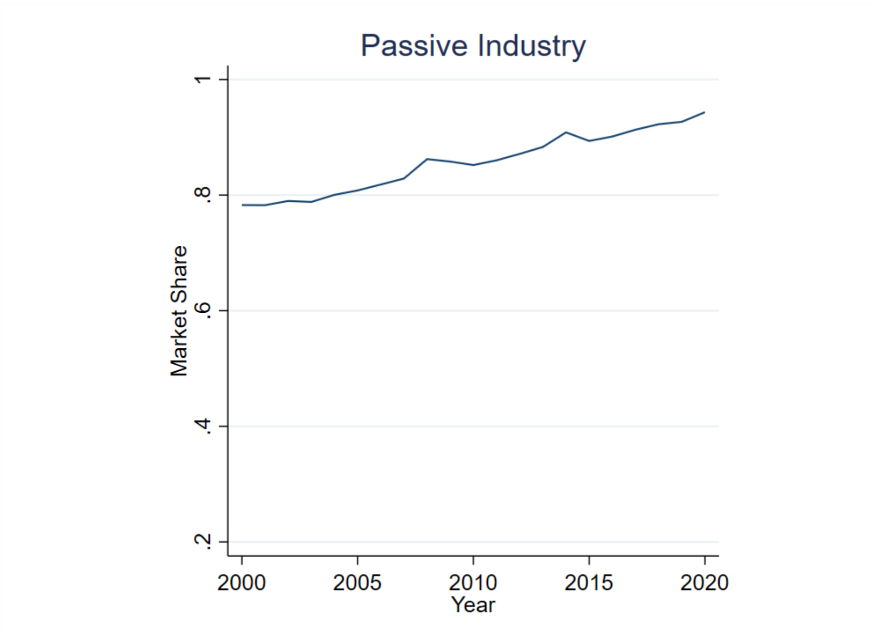

Figure 10: Market share of the five biggest investment companies in the passive industry. Market shares are in terms of end-of-year assets under management (AUM).

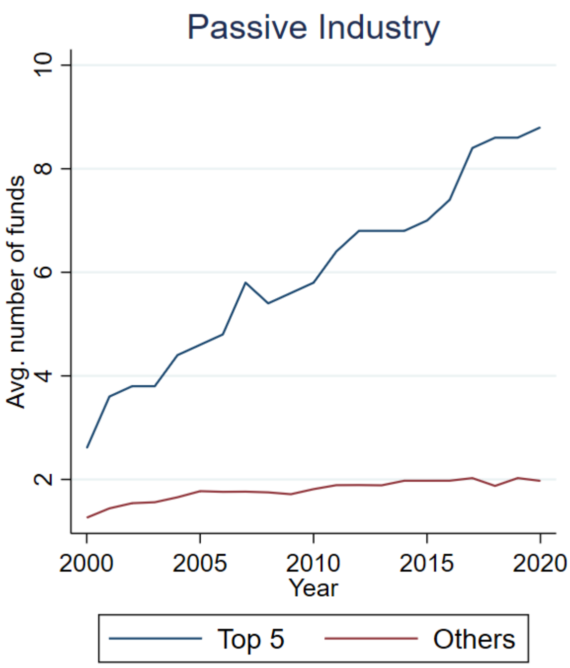

Figure 11: Average number of passive funds per management company. Funds with different share classes count as a single fund.

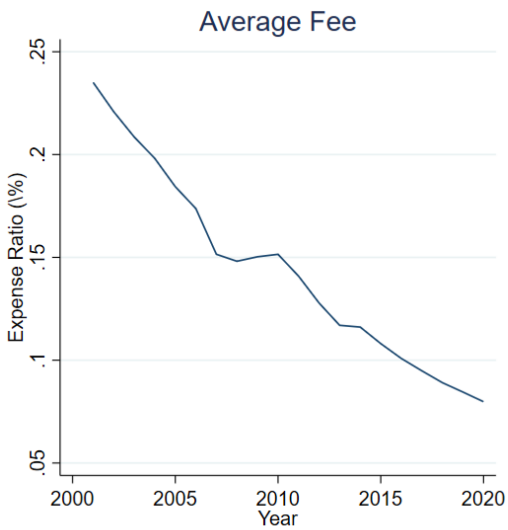

Figure 12: Average asset-weighted fee across passive funds. Funds with different share classes count as a single fund.

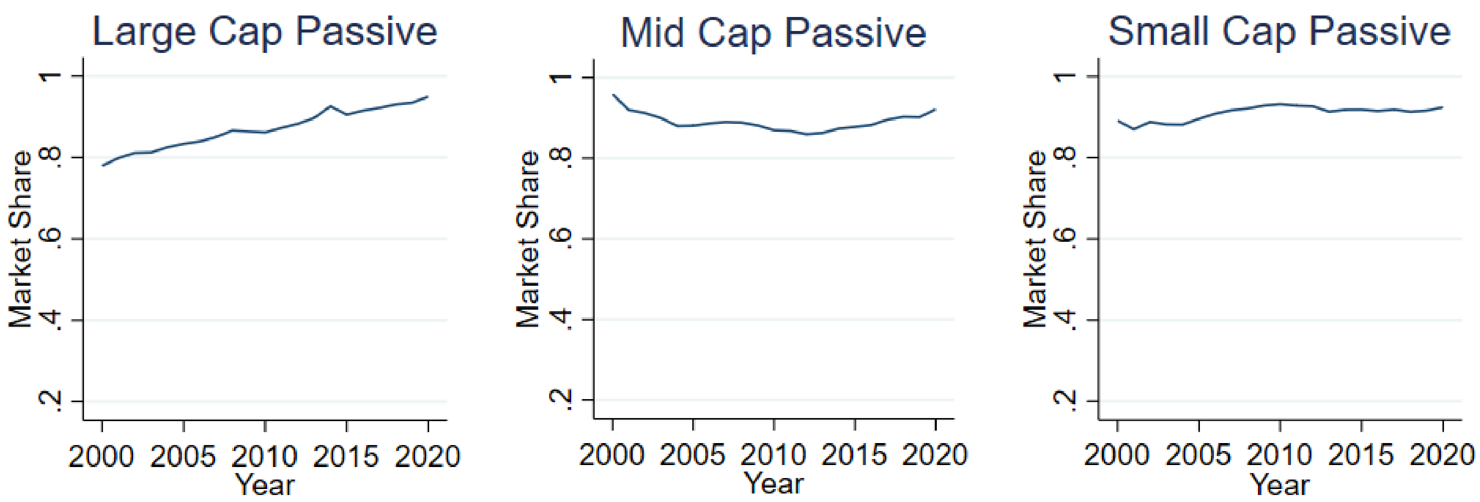

Figure 13: Market share of the five biggest investment companies by investment strategy. Market shares are in terms of end-of-year assets under management (AUM).

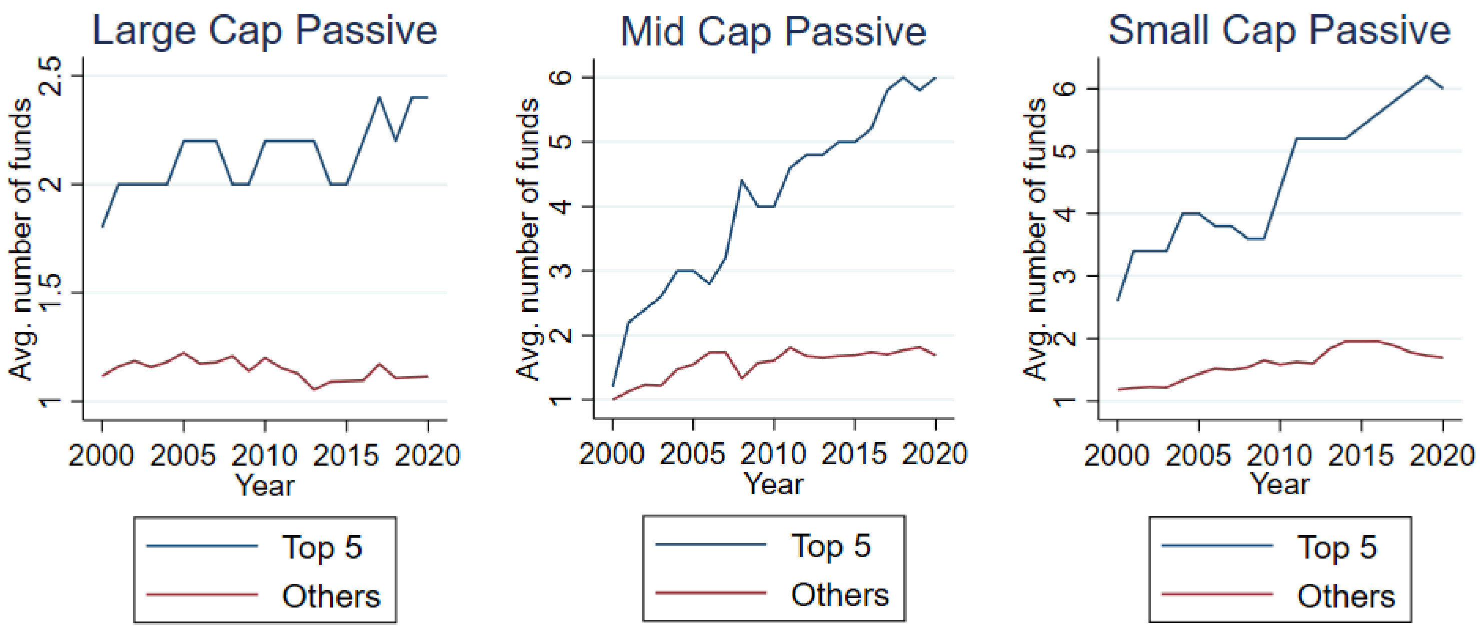

Figure 14: Average number of funds per management company. Funds with different share classes count as a single fund.

C Tables

	Obs.	Mean	Std. Dev	p5	p25	p50	p75	p95
AUM (bln.)	16552	2.00	13.41	0.02	0.08	0.28	0.96	5.76
Gross return (%)	16159	0.89	1.79	-2.52	-0.07	1.09	2.04	3.27
Expense Ratio (%)	16160	1.06	0.48	0.19	0.83	1.11	1.35	1.83
Passive	16552	0.22	0.42	0.00	0.00	0.00	0.00	1.00
Alpha (%)	13552	0.04	0.59	-0.47	-0.12	0.02	0.19	0.54
Market beta	13552	0.97	0.21	0.77	0.91	0.98	1.03	1.18
Market sharev(%)	16552	1.80	4.45	0.01	0.08	0.33	1.07	10.28
# of funds per company	16552	3.79	3.31	1.00	1.00	3.00	5.00	11.00

Table 5: Summary statistics of the full sample. All variables are winsorized at 1% and 99% levels. Returns and alpha are monthly. The expense ratio is annual.

	Obs.	Mean	Std. dev.	p5	p25	p50	p75	p95
AUM (bln.)	3697	5.70	27.72	0.02	0.09	0.42	1.89	18.10
Gross return (%)	3620	0.94	1.75	-2.12	-0.02	1.11	2.04	3.12
Expense Ratio	3621	0.48	0.42	0.08	0.20	0.35	0.60	1.57
Alpha (%)	3112	0.04	0.51	-0.31	-0.07	0.00	0.09	0.37
Market beta	3112	0.97	0.15	0.83	0.94	0.98	1.01	1.09
Market share (%)	3697	3.88	7.59	0.01	0.10	0.47	3.29	20.11
# of funds per company	3697	4.48	3.95	1.00	1.00	3.00	6.00	13.00

Table 6: Summary statistics for the passive sample. All variables are winsorized at 1% and 99% levels. Returns, alpha and expense ratios are monthly. The expense ratio is annual.

Company Code	Ticker	Fund Name	Beta	SMB	HML	MOM	Alpha	Gross Monthly Returns	Expense Ratio
VAN	VFIAX	Vanguard 500 Index Fund	0.9841	-0.1190	-0.0083	0.0002	-0.0001	-0.0028	0.0006
VAN	VINIX	Vanguard Institutional Index Fund	0.9864	-0.1196	-0.0082	0.0001	-0.0001	-0.0028	0.0005
SSB	SPY	SPDR S&P 500 ETF Trust	0.9863	-0.1222	-0.0057	0.0029	0.0000	-0.0028	0.0009
SSB	SSEYX	State Street Equity 500 Index II Portfolio	0.9830	-0.1251	-0.0141	0.0018	0.0003	-0.0028	0.0005
SSB	SVSPX	State Street S&P 500 Index Fund	0.9866	-0.1215	-0.0072	-0.0001	-0.0001	-0.0029	0.0016
SSB	SSSYX	State Street Equity 500 Index Fund	0.9889	-0.1157	-0.0066	0.0028	-0.0001	-0.0028	0.0013
BLK	IVV	iShares Core S&P 500 ETF	0.9864	-0.1200	-0.0081	0.0003	-0.0001	-0.0028	0.0005
BLK	WFSPX	iShares S&P 500 Index Fund	0.9858	-0.1191	-0.0079	-0.0003	-0.0001	-0.0028	0.0012
FID	FXAIX	Fidelity 500 Index Fund	0.9864	-0.1199	-0.0086	-0.0001	-0.0001	-0.0028	0.0005
CSW	SWPPX	Schwab S&P 500 Index Fund	0.9845	-0.1193	-0.0095	0.0003	-0.0001	-0.0028	0.0005
PRI	PREIX	T Rowe Price Equity Index 500 Fund	0.9855	-0.1194	-0.0083	-0.0002	-0.0001	-0.0028	0.0019
DFA	DFUSX	US Large Company Portfolio	0.9864	-0.1173	-0.0109	0.0000	-0.0001	-0.0028	0.0008
NTC	NOSIX	Stock Index Fund	0.9858	-0.1177	-0.0089	-0.0009	-0.0001	-0.0028	0.0010
USA	USPRX	S&P 500 Index Fund	0.9860	-0.1181	-0.0085	0.0000	-0.0001	-0.0028	0.0020
PGI	PLFIX	LargeCap S&P 500 Index Fund	0.9845	-0.1187	-0.0091	-0.0002	-0.0001	-0.0028	0.0027
DRY	DSPIX	Dreyfus Institutional S&P 500 Stock Index Fund	0.9857	-0.1206	-0.0063	0.0013	-0.0001	-0.0028	0.0028
DRY	PEOPX	Dreyfus S&P 500 Index Fund	0.9873	-0.1200	-0.0075	0.0009	-0.0001	-0.0028	0.0050
TIA	TISPX	S&P 500 Index Fund	0.9854	-0.1202	-0.0072	-0.0004	-0.0001	-0.0028	0.0011
SEI	SPINX	S&P 500 Index Fund	0.9853	-0.1167	-0.0055	0.0023	-0.0001	-0.0028	0.0005
SEI	SSPIX	S&P 500 Index Fund	0.9856	-0.1192	-0.0085	0.0003	-0.0001	-0.0028	0.0028
JPM	OGFAX	JPMorgan Equity Index Fund	0.9862	-0.1202	-0.0085	0.0005	-0.0001	-0.0028	0.0017
LBR	NINDX	Columbia Large Cap Index Fund	0.9887	-0.1206	-0.0081	0.0006	-0.0001	-0.0029	0.0026
GWC	MXVIX	Great-West S&P 500 Index Fund	1.0046	0.3668	0.2179	0.0117	0.0031	-0.0033	0.0040
MAS	MMIZX	MM S&P 500 Index Fund	0.9880	-0.1200	-0.0083	0.0001	-0.0001	-0.0028	0.0039
NFS	GRMIX	Nationwide S&P 500 Index Fund	0.9863	-0.1219	-0.0089	-0.0013	-0.0001	-0.0028	0.0030
DWS	SCPIX	DWS S&P 500 Index Fund	0.9792	-0.1191	-0.0122	-0.0001	0.0000	-0.0025	0.0045
DWS	BTIEX	DWS Equity 500 Index Fund	0.9790	-0.1204	-0.0109	0.0000	0.0000	-0.0025	0.0027
WFB	WFILX	Wells Fargo Index Fund	0.9864	-0.1196	-0.0085	0.0003	-0.0001	-0.0028	0.0038
AIM	SPIAX	Invesco S&P 500 Index Fund	0.9851	-0.1187	-0.0071	0.0008	-0.0001	-0.0028	0.0072
ABF	GEQYX	Equity Index Fund	0.9904	-0.1039	-0.0027	-0.0020	0.0000	-0.0026	0.0012

Table 7: Top 30 passive funds in the Large Cap sector.

Company Code	Ticker	Fund Name	Beta	SMB	HML	MOM	Alpha	Gross Monthly Returns	Expense Ratio
VAN	VIMAX	Vanguard Mid-Cap Index Fund	0.9266	0.1544	-0.1520	-0.0888	-0.0016	-0.0070	0.0005
VAN	VEXAX	Vanguard Extended Market Index Fund	0.9793	0.5387	-0.0685	0.0017	-0.0010	-0.0068	0.0006
VAN	VOE	Vanguard Mid-Cap Value Index Fund	0.9127	0.0971	0.0370	-0.1222	-0.0015	-0.0100	0.0007
VAN	VMGMX	Vanguard Mid-Cap Growth Index Fund	0.9390	0.2126	-0.3615	-0.0567	-0.0017	-0.0036	0.0007
VAN	VSPMX	Vanguard S&P Mid-Cap 400 Index Fund	0.9550	0.4006	0.0800	0.0367	-0.0006	-0.0085	0.0010
VAN	IVOG	Vanguard S&P Mid-Cap 400 Growth Index Fund	0.9667	0.3504	-0.0314	0.1663	-0.0008	-0.0077	0.0015
VAN	IVOV	Vanguard S&P Mid-Cap 400 Value Index Fund	0.9408	0.4493	0.1865	-0.1036	-0.0005	-0.0093	0.0015
BLK	IJH	iShares Core S&P Mid-Cap ETF	0.9623	0.3957	0.0776	0.0412	-0.0005	-0.0084	0.0008
BLK	IWR	iShares Russell Mid-Cap ETF	0.9243	0.1976	-0.0896	-0.0579	-0.0014	-0.0068	0.0019
BLK	IWS	iShares Russell Mid-Cap Value ETF	0.8752	0.1899	0.0787	-0.0871	-0.0016	-0.0099	0.0024
BLK	IWP	iShares Russell Mid-Cap Growth ETF	0.9805	0.2031	-0.2968	-0.0121	-0.0009	-0.0028	0.0024
BLK	IJK	iShares S&P Mid-Cap 400 Growth ETF	0.9685	0.3477	-0.0329	0.1651	-0.0010	-0.0079	0.0024
BLK	IJJ	iShares S&P Mid-Cap 400 Value ETF	0.9413	0.4490	0.1859	-0.1039	-0.0005	-0.0093	0.0025
BLK	BRMKX	iShares Russell Mid-Cap Index Fund	0.9267	0.1785	-0.0932	-0.1354	-0.0017	-0.0068	0.0015
BLK	JKG	iShares Morningstar Mid-Cap ETF	0.9659	0.1216	-0.0766	-0.0828	-0.0026	-0.0099	0.0025
BLK	JKI	iShares Morningstar Mid-Cap Value ETF	0.8690	0.1603	0.2226	-0.1096	-0.0012	-0.0101	0.0030
BLK	JKH	iShares Morningstar Mid-Cap Growth ETF	0.9811	0.2746	-0.3795	-0.0104	-0.0012	-0.0015	0.0030
BLK	BSMKX	iShares Russell Small/Mid-Cap Index Fund	1.0170	0.5240	0.0045	-0.0239	-0.0015	-0.0075	0.0013
BLK	SMMD	iShares Russell 2500 ETF	1.1511	0.0000	0.0000	0.0000	-0.0018	-0.0452	0.0006
FID	FSMAX	Fidelity Extended Market Index Fund	0.9792	0.5365	-0.0687	0.0034	-0.0010	-0.0069	0.0005
FID	FSMDX	Fidelity Mid Cap Index Fund	0.9233	0.1932	-0.0919	-0.0591	-0.0013	-0.0069	0.0005
FID	FZFLX	Fidelity SAI Small-Mid Cap 500 Index Fund	0.9554	0.3259	-0.0766	-0.0718	-0.0017	-0.0067	0.0014
SSB	MDY	SPDR S&P MidCap 400 ETF	0.9527	0.3999	0.0794	0.0363	-0.0006	-0.0085	0.0024
SSB	MDYG	SPDR S&P 400 Mid Cap Growth ETF	1.0514	0.4881	0.1740	0.0753	0.0000	-0.0077	0.0015
SSB	SPMD	SPDR Portfolio S&P 400 Mid Cap ETF	1.0051	0.5495	0.0771	0.0314	-0.0012	-0.0076	0.0006
SSB	MDYV	SPDR S&P 400 Mid Cap Value ETF	0.9637	0.6339	0.2154	-0.2250	0.0013	-0.0093	0.0015
SSB	SSMHX	State Street Small/Mid Cap Equity Index Portfolio	1.0035	0.5111	-0.0461	-0.0204	-0.0011	-0.0066	0.0005
SSB	SSMKX	State Street Small/Mid Cap Equity Index Fund	0.9645	0.5212	-0.0466	0.0150	-0.0012	-0.0066	0.0008
DFA	DFFVX	US Targeted Value Portfolio	1.0049	0.7027	0.3546	-0.0201	-0.0013	-0.0125	0.0037
CSW	SCHM	Schwab US Mid-Cap ETF	0.9576	0.3030	-0.0782	-0.0053	-0.0005	-0.0064	0.0005

Table 8: Top 30 passive funds in the Mid Cap sector.