

# Network Games of Imperfect Competition: An Empirical Framework

Marco Loseto\*

March 2024

Please click [here](#) for the latest draft

## Abstract

This paper studies how product differentiation affects substitution patterns and firms' markups in oligopolistic markets where products are differentiated over multiple attributes and consumers have linear-quadratic preferences. Under these assumptions, oligopolistic competition in either prices or quantities is framed as a network game where each product identifies a node, and the vector of attributes pins down a product's location in the network. In equilibrium, firms' pricing power is summarized by a measure of network centrality, function of product characteristics. A more central product faces competitors with similar characteristics and charges lower margins. Under the assumption that any unobserved product characteristic enters consumers' utility only through the linear component of preferences, a simple linear IV strategy can be implemented to estimate the demand parameters and recover the equilibrium price-cost margins.

KEYWORDS: Oligopoly, Networks, Market Power, Markups.

JEL Classification: D2, D43, L1, L13, L2

---

\*Kenneth C. Griffin, Department of Economics, University of Chicago, mloseto@uchicago.edu. I am grateful to Lars Peter Hansen, Ali Hortaçsu, Scott Nelson, Michael Dinerstein, Ben Golub, Hugo Lopez, Federico Mainardi, Pauline Mourot, Bruno Pellegrino, Francesco Ruggieri, Xavier Vives and seminar participants at the UChicago Economic Dynamics and IO working groups for their feedback and comments. Financial support from the Bradley fellowship, Stevanovich fellowship, Neubauer PhD fellowship and the Bank of Italy's Bonaldo Stringher scholarship is gratefully acknowledged.

# 1 Introduction

Product differentiation plays a crucial role in allowing firms to price above costs. When products are homogeneous, standard models of price competition predict that firms should price at costs even in markets with only two competitors.<sup>1</sup> The most common approach to model product differentiation is to define a product in terms of a bundle of characteristics. Although models of oligopolistic competition may differ in the way they are specified, they typically carry the same intuition about how product differentiation affects products' substitution patterns and producers' price-cost margins. Products that have similar characteristics should be more substitutable to each other, and firms selling more differentiated products should be able to charge higher margins. Although the intuition is clear, in models where the characteristic space is multidimensional, it is often hard to characterize analytically how product differentiation enables firms to price above costs.<sup>2</sup>

This paper considers a framework in which product differentiation enters substitution patterns and firms' price-cost margins in an way that is analytically tractable and easy to interpret. At the same time, it allows for products to be differentiated over multiple attributes and for consumers to have heterogeneous preferences over these attributes. In modelling consumer preferences, I make two assumptions. First, consumers choose how much to consume of each product. With this first assumption, I depart from the unit demand framework commonly used in empirical applications.<sup>3</sup> Second, product characteristics enter consumers' preferences in a linear-quadratic fashion. The linear component of preferences mimics the linearity of consumers' indirect utility, often assumed under discrete choice demand. The quadratic term instead enters preferences with a negative sign, implying that consumers have a taste for variety. This preference specification induces an hedonic demand system similar to the generalized hedonic-linear (GHL) demand system recently developed in [Pellegrino \(2023\)](#).<sup>4</sup>

Under the previous assumptions, I show that the cross-price elasticity between any two products is proportional to a weighted inner product of their corresponding vector of attributes. This weighting has two properties. First, it is such that, regardless of the units with which characteristics are measured, the resulting inner product always lies in between  $-1$  and  $1$  and thus can be interpreted as a correlation between the

---

<sup>1</sup>Also assuming there are no capacity constraints.

<sup>2</sup>This is not the case in more stylized models, such as the Hotelling and Salop models, where product are differentiated along one dimension.

<sup>3</sup>However, if consumers were constrained to purchase at most one unit of a single product, the model would boil down to a standard discrete choice demand framework. The linear-quadratic model nests the discrete choice framework. The only difference would be that the inner-product of the vector of attributes of, say, product  $j$ , would enter the indirect utility  $u_{ij}$  as an additional product characteristic. For more details see [Appendix F](#).

<sup>4</sup>In studying the demand and supply of differentiated products under perfect competition, [Epple \(1987\)](#) also leverages these type of preferences to study the identification of hedonic demand systems of the type considered in the seminal work by [Rosen \(1974\)](#)

characteristics of the two products. Two products are substitutes if their characteristics are positively correlated and complements otherwise. Second, the weight on a given characteristic is inversely related to how much that characteristic varies across all products in the market. Thus, characteristics that are more homogeneous across all products available matter more in determining substitution patterns between products.

Next, I turn to the supply side to study how product attributes affect firms' pricing power. Although I will primarily focus on Bertrand competition because it is the workhorse model used in empirical applications, I also consider quantity competition à la Cournot. In both cases, I leverage the linear-quadratic structure of consumer preferences to frame the oligopolistic game as a network game in the spirit of [Ballester, Calvó-Armenagol and Zenou \(2006\)](#). Products are the network nodes, and the weighted inner product between their vector of attributes determines the strength of the links between nodes. The implied competitive network is weighted, undirected and such that the more substitutable two products are, the stronger their link. Within this network framing, I show that firms' equilibrium price-cost margins can be decomposed additively into two components: a monopolistic component and a product differentiation component. The monopolistic component captures the price-cost margins a monopolist would charge. The product differentiation component instead summarizes how differentiated a product is relative to its competitors and is proportional to the Bonacich network centrality of that product. This centrality measure enters firms' price-cost margins with a negative sign because a more central product or, equivalently, a less differentiated product faces more competition and, in turn, charges lower markups.

In the second part of the paper, I estimate the Bertrand Network model using price-quantity data on the US automobile industry. I show that the demand parameters can be identified and estimated with a simple linear IV strategy, assuming that any unobserved product characteristic enters consumers' utility only through the linear component of preferences. With the estimated demand parameters, margins and costs can be recovered from the supply equation. Although the model is not based on discrete choice demand, it delivers reasonable price elasticities and price-cost margins. When I decompose the estimated margins, I find that the network structure implied by the observed product characteristics is competitive. The Bonacich product centrality, on average, accounts for more than 90% of the monopolistic margins. By differentiating their products, car producers are able to capture from 2% to 7% of the potential monopolistic margins.

Lastly, to quantify how much product differentiation matters in determining price-cost margins, I compare the estimated margins with the ones firms would have charged if their products were to be homogeneous. The estimated margins can be as high as three times the homogeneous margins, suggesting that product differentiation plays an important role in allowing firms to price above costs.

The rest of the paper proceeds as follows. Section 2 describes how this paper fits the literature, Section 3 develops the demand side of the model, Section 4 focuses on the supply side and looks at both Bertrand and Cournot competition, Section 5 extends the supply to the case of multiproduct firms, Section 6 estimates the model with market level data on the US automobile industry and Section 7 concludes.

## 2 Contributions to the Literature

This paper contributes to the empirical industrial organization literature that estimates oligopolistic models of product differentiation using market-level data. Most of this literature models demand as a discrete choice problem and supply as a game of imperfect competition with differentiated products where, in most cases, firms are assumed to choose prices simultaneously (i.e., a la Bertrand).<sup>5</sup> Bresnahan (1987) was among the first to estimate a discrete choice model of oligopolistic competition with products that are vertically differentiated along one dimension (i.e., a la Hotelling). A few years later, motivated by the theoretical advancements in the modelling of product differentiation, Feenstra and Levinsohn (1995) extended the Bresnahan (1987) model to accommodate for product differentiation along multiple dimensions and showed how to recover price-cost margins in this more general context. Importantly, in their model, substitution patterns and markups are determined by the distance in the characteristic space to neighbouring products.

In parallel, Berry (1994) and Berry, Levinsohn and Pakes (1995) developed methodologies to estimate oligopolistic models in which products are horizontally differentiated. These methodologies can accommodate the presence of unobserved product characteristics, which is one of the reasons that made them the leading approaches to estimating oligopolistic models of product differentiation.<sup>6</sup> An important caveat with these models is that to obtain reasonable substitution patterns, in the sense that products with similar characteristics are more substitute to each other, one needs to introduce random coefficients on product characteristics and estimate the model via a non-convex optimization. In addition, while in most cases, the estimates obtained from these models match the common intuition that similar products are more substitutable, it is not immediate to confirm such intuition analytically from the model equations.

In the framework I consider, substitution patterns and price-cost margins are related to the distance between product characteristics as in Feenstra and Levinsohn (1995) but with the advantage that the characterization is analytical instead of being defined implicitly. More precisely, by framing oligopolistic competition as a network

---

<sup>5</sup>For an exception with continuous demand see Dubois, Griffith and Nevo (2014).

<sup>6</sup>Perhaps even more importantly, these methodologies enable researchers and practitioners to test firms' conduct on the the supply side (Nevo (2001)) and quantify the welfare effects of mergers.

game, I show that the equilibrium price-cost margins can be expressed as a function of a product’s Bonacich centrality, which captures how close that product is to its competitors and represents a summary statistic for the ability of firms to price above marginal costs. I also show that the cross-price elasticity between any two goods is determined by a weighted inner product of their vector of characteristics which, differently from the standard discrete-choice models, can accommodate the presence of complementary goods.<sup>7</sup> At the same time, the model allows for unobserved product characteristics, delivers reasonable substitution patterns and can be estimated with a simple linear IV strategy.

Product proximity in terms of characteristics also matters for more practical aspects of model estimation. In the context of the standard discrete choice framework developed by [Berry \(1994\)](#) and [Berry, Levinsohn and Pakes \(1995\)](#), a recent contribution by [Gandhi and Houde \(2023\)](#) shows that, under the common assumption that product characteristics are exogenous, relevant price instruments should reflect the degree of differentiation of a product relative to others available in the market. In particular, they show that the residual function of the model, which depends on endogenous prices, is a function of the distances between observed product characteristics. In the model considered here, the intuition of this result emerges clearly from the analytical solution of the equilibrium prices (i.e., [Proposition 3](#)) which are a function of the proximity of products in the characteristic space as measured by their Bonacich network centrality.

Also recently, [Magnolfi, McClure and Sorensen \(2022\)](#) showed how to incorporate online survey data on the product space into demand estimation. The data capture product distances in the form of “product A is closer to B than it is to C” and can be used to construct a low-dimensional representation of the latent product space (i.e., an embedding). The authors show that incorporating these embeddings into conventional random coefficient logit models delivers elasticity estimates that are similar to those from a model that uses observable characteristics, suggesting that what matters are not attributes per se but rather how a product’s attributes differ relative to its competitors. Similarly, in the network model developed here, substitution patterns depend only on how similar product characteristics are. Any set of characteristics that preserves the same network structure would generate the same substitution patterns.

This paper also contributes to a recent literature that applies results from network theory to study oligopolistic competition and market power. This literature builds on the seminal contribution by [Ballester, Calvó-Armenagol and Zenou \(2006\)](#) to frame

---

<sup>7</sup>A notable exception is [Gentzkow \(2007\)](#) who develops a discrete choice model with complementary goods. Departing from discrete choice, [Thomassen, Smith, Seiler and Schiraldi \(2017\)](#) develop and estimate a demand model of complementary product categories, where consumers have quadratic preferences. Similarly, [Lee and Allenby \(2009\)](#) study the role of complementarity across product categories. In their demand model, consumers’ utility is a composite function with linear sub-utility within each product category, and a quadratic specification across categories.

games of imperfect competition as network games with product differentiation. In a recent contribution, [Ushchev and Zenou \(2018\)](#) develop a model of price competition in which product varieties are differentiated over a network and show that a unique Bertrand-Nash equilibrium exists and is proportional to a sign-alternating version of the Bonacich centrality. [Galeotti, Golub, Goyal, Talamàs and Tamuz \(2022\)](#) also frame oligopolistic price competition as a network game and exploit properties of the singular value decomposition (SVD) of the Slutsky matrix to characterize optimal tax-subsidy designs in terms of properties of the underlying network structure. [Pellegrino \(2023\)](#) instead considers a general equilibrium Cournot oligopoly in which products are differentiated over multiple attributes and the network structure is pinned down by the characteristics of the products.<sup>8</sup> He decomposes Cournot markups into a (quality-adjusted) productivity component and a product centrality component and calibrates the model using data on firms' financials from Compustat while measuring the adjacency matrix of product characteristics using the data on product cosine similarities constructed in [Hoberg and Phillips \(2016\)](#).

This paper contributes to this network literature along several dimensions. First, as in [Pellegrino \(2023\)](#), but differently from [Ushchev and Zenou \(2018\)](#) and [Galeotti, Golub, Goyal, Talamàs and Tamuz \(2022\)](#), I assume that products are differentiated over multiple attributes, which imply that the characteristics of the products determine the network structure. Second, differently from [Pellegrino \(2023\)](#), I focus on Bertrand competition, the standard conduct assumption made in most empirical applications. Importantly, I show that in both Cournot and Bertrand games, equilibrium price-cost margins can be decomposed additively into a monopolistic component that captures the margin a monopolist would charge and into a product differentiation component which captures how differentiated a given product is relative to its competitors.<sup>9</sup> I show that, in both Cournot and Bertrand, this centrality component coincides with the standard Bonacich network centrality as defined in [Bonacich \(1987\)](#) and [Jackson \(2008\)](#) and, in both cases, the relevant adjacency matrix is a simple (possibly weighted) inner-product between the matrix of product characteristics  $X$ .<sup>10</sup> Moreover, similarly

---

<sup>8</sup>See also [Ederer and Pellegrino \(2022\)](#) which extends the Cournot network model of [Pellegrino \(2023\)](#) to account for firms' institutional ownership structure.

<sup>9</sup>The decomposition I obtain is remindful of the one obtained by [Chen, Zenou and Zhou \(2018\)](#) and [Chen, Zenou and Zhou \(2022\)](#) in a model of price competition with perfect price discrimination where firms sell products that generate network externalities between connected consumers. In that setting, the price charged to a given consumer decomposes additively into the monopoly price and the consumer network centrality. The latter enters negatively suggesting that firms' offer price discounts to more connected consumers. In the model considered here, firms cannot price discriminate and there are no network externalities. The network structure arises because products are differentiated over multiple attributes and more central products must charge lower prices because they are not differentiated enough.

<sup>10</sup>The decomposition I propose here is for the dollar price-cost margins  $p - c$ . For the Cournot case, [Pellegrino \(2023\)](#) instead develops a decomposition for markups defined as  $\mu = p/c$ . In Appendix C, I show that the two decompositions are related. In particular, I show that the measure of centrality defined in [Pellegrino \(2023\)](#) is an affine transformation of the Bonacich product centrality that enters

to [Galeotti, Golub, Goyal, Talamàs and Tamuz \(2022\)](#), I exploit the SVD of the Slutsky matrix to characterize and interpret own and cross-price elasticities (Proposition 2).

Overall, this paper provides a unified framework to model imperfect competition in quantities or prices as a network game in which products are differentiated over multiple attributes, and shows how to estimate the model with market-level data (e.g., prices, quantities and characteristics) on a given industry using a simple linear IV strategy.

Finally, this paper contributes to the theoretical industrial organization literature that compares equilibrium outcomes across price and quantity competition. In general, this literature finds that under strategic complementarity in prices, Cournot equilibrium prices are higher than Bertrand prices, thus confirming the intuition that Bertrand competition is more intense. The seminal contribution by [Singh and Vives \(1984\)](#) develops the argument for the case of a differentiated duopoly which was then extended to an  $N$  firms oligopoly in [Vives \(1985\)](#).<sup>11</sup> More recently, [Magnolfi, Quint, Sullivan and Waldfogel \(2022\)](#) pointed out that, under the assumption that prices are strategic complements, imposing Cournot competition would always lead the researcher to estimate higher markups. In Proposition 5 I show that these results also hold when comparing the Cournot network game with the Bertrand network game. In particular, I show that, under strategic complementarity in prices, the centrality of each product is higher under Bertrand competition.

## 3 Demand

This section sets up the demand side of the model. I start by describing the set of products available, consumers' preferences and by deriving individual demand functions. Then, I turn to aggregate demand and describe how product attributes affect own and cross price elasticities.

### 3.1 Products

In the market I consider, there are  $j \in \{1, \dots, J\}$  products available. Each product  $j$  is characterized by a set of  $K$  attributes whose values are collected in the  $K$  dimensional real-valued vector  $x_j = (x_{jk})_{k=1}^K$  where  $x_{jk}$  is measured in units of quantity consumed. Characteristic  $x_{jk}$  tells you how much of attribute  $k$  you would get if you consume one unit of product  $j$ .

---

Cournot price-cost margins.

<sup>11</sup>Similar results can be found in [Cheng \(1985\)](#), [Okuguchi \(1987\)](#) and [Amir and Jin \(2001\)](#).



### 3.2 Utility

Consumers are indexed by  $i \in I$ , take product prices  $p = (p_j)_{j=1}^J$  as given, and choose how much to consume of each product available. I denote by  $q_i = (q_{ij})_{j=1}^J$  the consumption vector of consumer  $i$ .

I define consumer  $i$  preferences as follows:<sup>12</sup>

$$\begin{aligned} u_i(q_i, X) &= q_{i0} + \left( q'_i \alpha_i^q - \frac{\beta_i}{2} q'_i q_i \right) + \eta \left( q'_i X \alpha_i^x - \frac{\beta_i}{2} q'_i X X' q_i \right) \\ &= q_{i0} + q'_i \alpha_i - \frac{\beta_i}{2} q'_i (I_J + \eta X X') q_i \end{aligned} \quad (1)$$

where  $q_{i0}$  is an outside good,  $X$  the  $J \times K$  matrix of products attributes,  $\eta > 0$  governs the extent with which product differentiation in terms of attributes matters for consumers,  $\alpha_i^q > 0$  and  $\alpha_i^x > 0$  are respectively a  $J$ -vector and  $K$ -vector of utility parameters. Both vectors of parameters affect the marginal utility that comes from the linear term in (1),  $\alpha_i \equiv \eta X \alpha_i^x + \alpha_i^q$ . Finally,  $\beta_i$  captures  $i$ 's love for varieties.

### 3.3 Individual Demand

Consumer  $i$  takes prices  $p$  as given and maximizes (1) subject to

$$q_{i0} + q'_i p \leq y_i \quad (2)$$

where  $y_i$  is consumer  $i$  income. After substituting for the budget constraint in (2), consumer  $i$ 's demand function is given by

$$q_i(p) = \frac{1}{\beta_i} (I_J + \eta X X')^{-1} (\alpha_i - p) \quad (3)$$

which is always well defined because  $(I_J + \eta X X')$  is positive definite and therefore non-singular.

### 3.4 Aggregate Demand and Price Elasticities

I assume there is a mass  $M$  of consumers that are heterogeneous in terms of  $(\alpha_i, \beta_i)$ . Further, I assume that all moments involving the random variables  $\alpha_i$  and  $\beta_i$  are well-defined. The next proposition characterizes the aggregate demand function.

**Proposition 1** *Under the above assumptions, the aggregate demand of product  $j$  as a function of own and competitors prices is given by*

$$q_j(p_j, p_{-j}) = a_j(\alpha, \beta, (\theta_{jl})_{l=1}^J) - \frac{1}{\beta} (1 - \theta_{jj}) p_j + \frac{1}{\beta} \sum_{l \neq j} \theta_{jl} p_l \quad (4)$$

---

<sup>12</sup>The same type of preferences are considered in [Pellegrino \(2023\)](#).



where  $\beta \equiv M \left( \int \frac{1}{\beta_i} di \right)^{-1}$ ,  $\theta_{jl} \equiv x'_j \Omega^{-1} x_l$  where  $\Omega \equiv \left( \frac{1}{\eta} I_K + X'X \right)$  is a positive definite weighting matrix independent of  $(j, l)$  and  $a_j$  is a  $j$ -specific demand intercept that depends on  $\alpha \equiv \int \frac{\alpha_i / \beta_i di}{1 / \beta_i di}$ . Moreover, for any  $(j, l)$ ,  $\theta_{jl} \in (-1, 1)$  if  $j \neq l$  whereas  $\theta_{jj} \in (0, 1)$  if  $j = l$ .

Equation (4) defines the aggregate demand for a given product  $j$ . Because consumer preferences are quadratic, aggregate demand is linear in prices. What is more interesting is how own and cross price elasticities relate to products characteristics. This relationship is enclosed in the elements  $(\theta_{jl})_{j,l}$  which determine the substitution patterns across products. In fact, the  $(j, l)$  element of the Slutsky matrix is given by

$$\frac{\partial q_j}{\partial p_l} = \begin{cases} -\frac{1}{\beta}(1 - \theta_{jj}) & \text{if } j = l \\ \frac{1}{\beta}\theta_{jl} & \text{if } j \neq l \end{cases} \quad (5)$$

so that products  $j$  and  $l$  are substitutes or complements whenever  $\theta_{jl}$  is respectively positive or negative. But Proposition 1 tells us more; the element  $\theta_{jl}$  is a weighted inner-product between  $j$  and  $l$  vectors of attributes. In other words, the closer  $j$  and  $l$  are in this inner-product space the more substitutable they are. The next result characterizes the weighting of the inner-product between  $x_j$  and  $x_l$  in terms of the principal components of the matrix of characteristics  $X$ .

**Proposition 2** *Let  $U$  be the  $K \times K$  matrix of principal components directions of  $X$  and for any  $j$  let  $\tilde{x}_j = U'x_j$  be the projection of  $x_j$  onto these principal components. Then for any  $(j, l)$*

$$\theta_{jl} = \sum_{k=1}^K \left( \frac{1}{1 + \lambda_k^{x'x}} \right) \tilde{x}_{jk} \tilde{x}_{lk} \quad (6)$$

where  $\lambda_k^{x'x}$  is the  $k$ -th eigenvalue of  $X'X$ .

Equation (6) has two main insights. First, the substitution between any two products  $j$  and  $l$  can be expressed as a weighted inner-product between the vectors of projected attributes  $\tilde{x}_j$  and  $\tilde{x}_l$ . Because  $U$  is an orthogonal matrix, this projection is innocuous in the sense that  $\tilde{X}\tilde{X}' = XU U'X' = XX'$  and we can replace  $XX'$  with  $\tilde{X}\tilde{X}'$  without affecting consumer preferences defined in (1), individual demand defined in (3) and aggregate demand defined in (4).

Second, the weighting of these (projected) product characteristics depends on how much variety in terms of each characteristic is available in the whole market. From expression (6) we can see that characteristics with smaller  $\lambda_k^{x'x}$  are weighted more. But what does a small  $\lambda_k^{x'x}$  mean in practice? It means that higher weight is assigned to (projected) characteristics that do not vary too much across products as measured by  $\tilde{X}'\tilde{X}$ . To see this more formally, let  $u_k$  be  $k$ -th principal component of  $X$ ,  $\tilde{x}_k$  the  $k$ -th

column of  $\tilde{X}$  and note that,

$$\tilde{x}'_k \tilde{x}_k = u'_k X' X u_k = u'_k U \Lambda^{x'x} U' u_k = \lambda_k^{x'x}. \quad (7)$$

where the first equality comes from the definition of  $\tilde{x}_j$  and  $\tilde{x}_k$ , the second from the eigen-decomposition of  $X'X$  and the last one from the fact that  $U$  is an orthogonal matrix. Overall, equation (7) tells us that characteristics that vary more across all products available in the market will have a higher  $\lambda_k^{x'x}$  and thus will matter less when computing substitution patterns between any two products.

To sum up, equation (6) highlights that the elasticity of substitution between two products is affected by not only how similar are their vector of characteristics, but also by how similar are characteristics across the whole market. The substitutability between any two products will be higher if their characteristics are similar but even more so if the characteristics in which they are similar are the ones that are more homogeneous across all products available.

## 4 Oligopolistic Competition

In this section, I turn to the analysis of the supply side. I will mainly focus on Bertrand competition because it is the workhorse model used in empirical applications. After solving the Bertrand game, I will turn to Cournot competition. In both cases, I show that the effect of product differentiation on equilibrium markups is summarized by a measure of how central a product is in the competitive network. I conclude the section by comparing the two cases and by showing that, under strategic complementarity, Cournot competition always leads to higher price-cost margins.

To start with, I assume that  $J$  single-product firms produce the  $J$  products with constant marginal costs. Then, section 5 deals with the case in which firms are multi-product. Moreover, throughout the following analysis, I will assume that an interior Nash equilibrium exists.<sup>13</sup>

### 4.1 Bertrand Competition

Firm  $j$  takes the vector of competitor prices  $p_{-j}$  as given and solves

$$\max_{p_j} (p_j - c_j) q_j(p_j, p_{-j}) \quad (8)$$

$$\text{s.t. } q_j(p_j, p_{-j}) = a_j - \frac{1}{\beta}(1 - \theta_{jj})p_j + \frac{1}{\beta} \sum_{l \neq j} \theta_{jl} p_l \quad (9)$$

---

<sup>13</sup>In Appendix B, I provide conditions for the existence and uniqueness of an Nash equilibrium where  $p_j \in [c_j, \alpha_j]$  for all  $j$ .

which is equivalent to

$$\max_{p_j} \left( a_j + \frac{c_j}{\beta} (1 - \theta_{jj}) \right) p_j - \frac{1}{\beta} (1 - \theta_{jj}) p_j^2 + \frac{1}{\beta} \sum_{l \neq j} \theta_{jl} p_j p_l. \quad (10)$$

The payoff function in equation (10) is analogous to the linear-quadratic utility functions considered in [Ballester, Calvó-Armenagol and Zenou \(2006\)](#) and, as such, defines a linear-quadratic network game in which each product is a node and the  $J \times J$  matrix

$$A(\Theta) \equiv \Theta - \text{diag}(\Theta) \quad (11)$$

is the weighted and undirected adjacency matrix of the network.

**Network game interpretation.** The adjacency matrix defined in (11) shows that network connections and products' substitution patterns are isomorphic to each other. From the previous section, we know that an off-diagonal element  $\theta_{jl}$  of the matrix  $\Theta$  captures the degree of substitution between product  $j$  and product  $l$  as measured by a weighted inner-product of their vector of characteristics  $x_j$  and  $x_l$  respectively. From equation (11), we know that we can interpret the inner-product  $\theta_{jl}$  as a weighted link between product  $j$  and product  $l$  and therefore, we can think of the product differentiation space as being a network whose nodes are the products and whose links tell us how close, or equivalently how substitutable, any two products are.

The natural question at this point is, why embedding product differentiation into a competitive network is relevant? Framing the product differentiation space as a network is important because it enables us to learn how product differentiation affects equilibrium outcomes by studying the topological properties of the network. In the next proposition, I show that the equilibrium Bertrand price-cost margins depend negatively on a product's Bonacich centrality, which, following [Jackson \(2008\)](#), I define as

**Definition 1** *Let  $(A, J)$  be a network with  $J$  nodes and adjacency matrix  $A$ . The  $J$ -vector of (weighted) Bonacich centralities  $\mathbf{b}(A, \delta, u)$  is given by*

$$\mathbf{b}(A, \delta, u) \equiv (I_J - \delta A)^{-1} \delta A u = \sum_{k=1}^{\infty} \delta^k A^k u, \quad (12)$$

where  $\delta > 0$  is a scalar and  $u > 0$  is  $J$ -vector.

The  $j$ -th element of  $\mathbf{b}(A, \delta, u)$  summarizes how central node  $j$  is in the network. This measure of centrality is widely used in social networks because it captures a node's importance in terms of how close/connected this node is to others and how close/connected the nodes it is connected to. According to the definition of Bonacich centrality, a node's importance is a weighted sum of the walks that emanate from it.

Moreover, if  $\delta \in (0, 1)$ , walks of shorter length are weighted more.<sup>14</sup>

**Proposition 3** *Assume that  $\theta_{jj} = \theta$  for all  $j \in \{1, \dots, J\}$ . If an interior equilibrium  $p^* = (p_j^*)_{j=1}^J$  of the Bertrand pricing game exists, then it is unique and is such that*

$$p^* - c = \frac{\alpha - c}{2} - \mathbf{b} \left( A(\Theta), \frac{1}{2(1 - \theta)}, \frac{\alpha - c}{2} \right), \quad (13)$$

*provided  $\theta < 1 - \frac{1}{2} \max_j |\lambda_j(A)|$  where  $\lambda_j(A)$  is the  $j$ -th eigenvalue of the adjacency matrix  $A(\Theta)$ .*

The key insight of Proposition 3 is that the more central a product is in the competitive network, the lower its equilibrium price-cost margins.<sup>15</sup> What does this mean in practice? From Definition 1, we can see that the higher any of the entries of the  $j$ -th row of  $A$ , the more central node  $j$  is. In our settings, product  $j$  is more central the higher its substitutability with any other product (i.e., the higher the elements  $(\theta_{jl})_{l \neq j}$  of the  $j$ -th row of  $\Theta$ ). Overall, the expression for the Bertrand price-cost margins in (13) tells us two things. First, a less central or, equivalently, more differentiated product will be able to charge higher markups. Second, a product's Bonacich centrality is a sufficient statistic to measure how product differentiation allows firms to price above marginal costs.

**Comparison with Ballester, Calvó-Armenagol and Zenou (2006).** In their seminal paper, Ballester et al. show that in a general network game with quadratic payoffs, the Nash equilibrium action of any player is increasing in their Bonacich centrality. For the Bertrand competition game I study, the opposite holds; Nash equilibrium prices decrease with a player's centrality. This mismatch in the results is a consequence of the fact that in Ballester et al., the coefficient on the linear component of players' utility does not depend on the network structure, whereas, in the Bertrand case, the linear term of the quadratic profit in (10) depends on  $\Theta$ .<sup>16</sup>

What is the economic interpretation of this result? The linear term in the Bertrand game corresponds to the marginal benefit of the very first unit for the average consumer and (assuming  $c_j = 0$ ) is given by

$$a_j = (1 - \theta_{jj})\alpha_j - \sum_{l \neq j} \theta_{jl}\alpha_l. \quad (14)$$

From expression (14), it is immediate to see that a more central product faces a lower residual demand. This *residual-demand* effect has to be contrasted with the *peer-effect*

---

<sup>14</sup>This interpretation is motivated by the fact that when  $A$  is a binary  $\{0, 1\}$  its  $k$ -th power  $A^k$  counts how many walks of length  $k$  are between any two nodes.

<sup>15</sup>In Appendix E I perform a simple simulation exercise to visualize and summarize the properties of the Bertrand network model.

<sup>16</sup>As I discuss in subsection 4.2, this is not the case with quantity competition à la Cournot. Consequently, the relationship between the Cournot-Nash equilibrium and the Bonacich network centrality follows immediately from the main result in Ballester, Calvó-Armenagol and Zenou (2006).

benefit of being more connected, which affects firms' payoffs through the interaction terms in

$$\sum_{l \neq j} \theta_{jl} p_l p_j. \quad (15)$$

From expression (15), we can see that holding everything else constant, being more central or having more substitutes increases profits. In the class of network games studied by Ballester et al., only this second effect is present, and it is the force that pushes players to increase their equilibrium action proportionally to their network centrality. In the Bertrand network game, not only the *residual-demand* effect in (14) is present but, as I show in Proposition 3, it dominates the *peer-effect*. As a result, the equilibrium actions (i.e., firms' prices) decrease with a product's centrality.

**Some remarks.** A couple of remarks are in order before turning to the Cournot case. First, the assumption that  $\theta_{jj}$  are homogeneous across  $j$  is made only for expositional purposes, and the proof presented in Appendix A is provided for the general heterogeneous case. Similarly, Section 5 generalizes the result to the case in which firms are multiproduct.

Second, the condition on the largest eigenvalue of  $A(\Theta)$  ensures that the Bonacich centrality can be rewritten as a convergent infinite sum. If the condition is not satisfied, expression (13) would still be well-defined but not expressible as an infinite series.

Third, Proposition 3 offers a two-terms decomposition of price-cost margins, a monopolistic component and a product differentiation component summarized in terms of the network centrality. Interestingly, this latter component matters only to the extent that firms are competing with each other. As I show in Section 5, when there is a multiproduct monopolist, the product differentiation component converges to zero, and monopolistic price-cost margins are charged to each of the  $J$  product varieties.

## 4.2 Cournot Competition

In this section, I focus on quantity competition a la Cournot. To define a firm's objective, I need the inverse aggregate demand instead of the aggregate demand derived in Proposition 1. Compared to Bertrand, deriving the aggregate demand for the Cournot case is almost immediate. Starting from (3), and defining  $\alpha$  and  $\beta$  as in Proposition 1 the aggregate inverse demand for product  $j$  is

$$p_j(q_j, q_{-j}) = \alpha - \beta(1 + \eta(x'_j x_j))q_j - \eta\beta \sum_{l \neq j} (x'_j x_l)q_l. \quad (16)$$

Similar to the Bertrand case, substitution patterns in Cournot are driven by an inner product between product attributes. In this case, the inner product is unweighted, and the substitutability between products  $j$  and  $l$  is given by  $x'_j x_l$ . One drawback of this

is that, without a normalization on the scale of characteristics, the implied product adjacency matrix can have weights larger than one in absolute value, which makes it less interpretable. In the Bertrand case, as shown in Proposition (3), no normalization of the product characteristics is required.

Firm  $j$  takes competitors' quantities as given and solves

$$\max_{q_j} (\alpha_j - c_j)q_j - \beta(1 + \eta(x'_j x_j))q_j^2 - \eta\beta \sum_{l \neq j} (x'_j x_l)q_l q_j \quad (17)$$

The Cournot objective in (17) also defines a network game with quadratic payoffs. In this case, the relevant adjacency matrix will be constructed from the matrix  $\Theta^- \equiv -XX'$ . Why this is important will be clear in the following Proposition, where I derive the Cournot price-cost margins as a function of the vector of Bonacich centralities.

**Proposition 4** *Let  $\Theta^- \equiv -XX'$  and assume that  $\theta_{jj}^- = \theta^-$  for all  $j \in \{1, \dots, J\}$ . If an interior equilibrium  $q^* = (q_j^*)_{j=1}^J$  of the Cournot game exists, then it is unique and is such that*

$$p^* - c = \frac{\alpha - c}{2} + \mathbf{b} \left( A(\Theta^-), \frac{\eta}{2(1 - \eta\theta^-)}, \frac{\alpha - c}{2} \right) \quad (18)$$

*provided  $\theta^- < \frac{1}{\eta} - \frac{1}{2} \max_j |\lambda_j(A(\Theta^-))|$  where  $\lambda_j(A)$  is the  $j$ -th eigenvalue of the adjacency matrix  $A(\Theta^-)$ .*

The expression for Cournot price-cost margins resembles very closely the one for Bertrand described in (13). The most apparent difference is that now price-cost margins seem to increase in the product's centrality, but this is not the case. To see this, note that the relevant adjacency matrix here is  $\Theta = -XX'$ , which, because of the minus sign, assigns higher centrality whenever a product becomes less substitutable with any other product or equivalently more differentiated. As expected, Cournot and Bertrand's price-cost margins are higher for more differentiated products.

In the context of Cournot competition, Pellegrino (2023) also develops a decomposition of firms' markups, defined as  $p_j/c_j$ , into a productivity component  $\alpha_j/c_j$  and a product centrality component, denoted by  $1 - \chi_j$  in the paper. Although different, the two decompositions are related to each other. More precisely, in Appendix C, I show that the product centrality defined in Pellegrino (2023) is an affine transformation of the Bonacich product centrality that enters the Cournot price-cost margins in equation (18).

**Comparison with Ballester, Calvó-Armenagol and Zenou (2006).** As I did for the Bertrand case, it is interesting to compare the result in Proposition 4 to the more general result provided in Ballester et al. Differently from Bertrand, the linear component of the quadratic payoff in (17) does not depend on the network structure, which

makes the Cournot game analogous to the network game considered in Ballester et al., provided one defines the adjacency matrix as  $-XX'$ .

### 4.3 Bertrand vs Cournot

In this subsection, I compare the Bertrand and Cournot equilibrium outcomes. I start by describing the main differences between the two in terms of the product characteristics space. Then, I study how the network structure depends on the nature of competition and show that when there is strategic complementarity in prices, Bertrand competition leads to a network in which each product is more central and where price-cost margins are lower.

**Product characteristics.** In both models, product characteristics affect substitution patterns through an inner product matrix. In Cournot, this matrix is simply the inner product between each product's vector of attributes and is given by  $XX'$ . In Bertrand, as shown in Proposition 1, this inner-product matrix is instead  $X\Omega^{-1}X'$  where  $\Omega$  is a  $K \times K$  matrix that reweights each vector of characteristics. At first glance, it might seem that product characteristics are different in the two models, but because  $\Omega$  is positive definite, this turns out not to be the case.

To see this more formally, note that  $\Omega$  is diagonalizable through an orthonormal basis of eigenvectors  $S$  such that  $\Omega^{-1} = S\Lambda S'$  where  $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_K)$  is a diagonal matrix that collects the eigenvalues of  $\Omega^{-1}$ . Then, by letting  $\tilde{X} = XS$ , we can redefine product characteristics without changing their inner-product  $\tilde{X}\tilde{X}' = XX'$ . Thus comparing  $XX'$  to  $X\Omega^{-1}X'$  is equivalent to comparing  $\tilde{X}\tilde{X}'$  to  $\tilde{X}\Lambda\tilde{X}'$ . From this second comparison, it is easy to see that for any characteristic  $k$ , if  $\tilde{x}_{jk} \geq \tilde{x}_{lk}$ , then  $\sqrt{\lambda_k}\tilde{x}_{jk} \geq \sqrt{\lambda_k}\tilde{x}_{lk}$ , where the latter inequality holds because  $\Omega$  is positive definite and thus  $\lambda_k > 0$  for all  $k$ . In practice, this means that, under Bertrand, each product characteristic is just rescaled by a positive number  $\lambda_k$ . Thus, whether product  $j$  offers more characteristic  $k$  than product  $l$  is independent of the type of competition model, as one would expect.

Another important difference is that the competitive network implied by Bertrand does not depend on the units with which product characteristics are measured. From Proposition 3, we know that  $\theta_{jl}$ , or equivalently  $x_j'\Omega^{-1}x_l$ , or equivalently  $\tilde{x}_j'\Lambda\tilde{x}_l$ , always lie in between  $-1$  and  $1$  regardless of the units with which each characteristic is measured, which is an appealing property in an empirical context.

**Competition and network structure.** The following proposition shows how the nature of competition influences the structure of the competitive network by comparing the network centralities implied by price and quantity competition, respectively.

**Proposition 5** *Assume that  $\theta_{jk} \geq 0$  if  $j \neq k$ . Then each node centrality in the network implied by Bertrand competition is higher than the one under Cournot competi-*



tion, i.e.,  $\tilde{\mathbf{b}}_b \geq -\tilde{\mathbf{b}}_c$  where  $\tilde{\mathbf{b}}_b \equiv (I_J - \text{diag}(\Theta))^{-1/2} \mathbf{b}_b$  and  $-\tilde{\mathbf{b}}_c \equiv (I_J - \text{diag}(\Theta^-))^{1/2} (-\mathbf{b}_c)$  are the vectors of Bertrand and Cournot centralities respectively.

As described in Section 2, several papers in the industrial organization literature have studied how equilibrium outcomes compare across the two types of competition models. In general, under strategic complementarity in prices, it is known that Bertrand competition with differentiated products is more efficient than Cournot competition and leads to lower prices and higher consumer surplus.<sup>17</sup> The same is true in this network setting where the vector of Bonacich centralities captures the intensity of competition. Proposition 5 shows that each node has a higher centrality under Bertrand and thus faces more intense competition. Moreover, looking back at the results in Propositions 3 and 4, one can see that the vector of Bonacich centralities is the only determinant of the wedge between Cournot and Bertrand prices. Hence, the effect of more intense competition on equilibrium prices must be entirely captured by differences in centrality.

**Remarks.** Before concluding this section, a couple of remarks are in order. First, note that the centrality implied by the Cournot model enters the inequality in Proposition 5 with a negative sign. This is because the centrality in Cournot is based on the matrix  $-XX'$ , which assigns a lower centrality to a more substitutable product. The measure  $\mathbf{b}_b$  implied by Bertrand instead assigns higher centrality to more substitutable products and should be compared to  $-\mathbf{b}_c$ . To see this mathematically, suppose we knew that Cournot equilibrium prices  $p_c$  were to be higher than Bertrand prices  $p_b$  then by combining equations (13) and (18), the inequality in Proposition 5 would follow immediately.

Second, the Bonancich centralities are scaled by two positive diagonal matrices. This scaling appears because I am not imposing homogeneity across  $j$  of the  $\theta_{jj}$  and  $\theta_{jj}^-$  and the vectors of Bonacich centralities enter equilibrium price-cost margin after this positive rescaling.<sup>18</sup> Furthermore, because the scaling is positive, the effect of product differentiation on equilibrium price-cost margins remains unaffected; higher centrality implies lower markups.<sup>19</sup>

<sup>17</sup>With homogenous products, the efficiency of Bertrand competition is maximized; even with only two firms, the perfectly competitive outcome obtains.

<sup>18</sup>In Proposition 3 and 4 this homogeneity assumption was made only for expositional purposes. For more details, refer to the proofs of Propositions 3, 4 and 5, which are presented for the non-homogeneous case.

<sup>19</sup>A similar type of rescaling also appears in the more general setting considered by Ballester, Calvó-Armenagol and Zenou (2006). If we allow for heterogeneity in the curvature ( $\sigma_{ii}$  in their notation) of own marginal returns, Remark 2 in their paper suggests scaling all elements of the adjacency matrix by its diagonal elements  $\sigma_{ii}$ . My scaling is similar, but it preserves the symmetry of the adjacency matrix.

## 5 Multiproduct Firms

In this section, I allow for the possibility that the same firm owns multiple products. I index firms by  $f$  and denote by  $J_f$  the set of products offered by firm  $f$ . Assuming there are  $F < J$  firms, the  $F \times J$  matrix  $R$  keeps track of which product belongs to which firm, i.e.,  $r_{fj} = \mathbf{1}\{j \in J_f\}$  and the  $J \times J$  matrix  $H = R'R$  denotes the ownership matrix. Moreover, as typically assumed in the context of multiproduct firms, I will assume that one product can only be owned by one firm or equivalently that the sets  $(J_f)_{f=1}^F$  forms a partition of the set  $\{1, \dots, J\}$ . The latter implies that the ownership matrix  $H$  will be a block-diagonal matrix with  $F$  blocks where the matrix of ones  $\mathbf{1}_{|J_f|}\mathbf{1}'_{|J_f|}$  corresponds to  $f$ -th block.

In what follows, I will focus on Bertrand competition, but everything can be restated in terms of Cournot competition. I start by defining the profit maximization problem for a generic firm  $f$  and frame it as a Network game. Next, I show that in the case of multiproduct firms, the implied adjacency matrix only keeps track of the competitive links between products across different firms but not between products within the same firm. Finally, I extend the result in Proposition 3 and derive equilibrium price-cost margins in terms of the vector of Bonacich centralities. The result points to an intuitive comparative static for the conduct parameter  $H$ : when moving toward a more collusive industry structure (e.g., as  $H \rightarrow \mathbf{1}_J\mathbf{1}'_J$ ), product differentiation (or equivalently product centrality) matters less and less for the equilibrium price-cost margins.

### 5.1 Multiproduct Firm Problem

Under Bertrand competition, firm  $f$  chooses prices  $(p_j)_{j \in J_f}$  taking prices of competitors firms as given to maximize

$$\begin{aligned} \max_{(p_j)_{j \in J_f}} \quad & \sum_{j=1}^J r_{fj}(p_j - c_j)q_j(p_j, p_{-j}) \\ \text{s.t.} \quad & q_j(p_j, p_{-j}) = \left( a_j(1 - \theta_{jj}) - \sum_{l \neq j} \theta_{jl}a_l \right) - (1 - \theta_{jj})p_j + \sum_{l \neq j} \theta_{jl}p_l \end{aligned} \quad (19)$$

Problem (19) is a quadratic problem in the vector of prices chosen by firm  $f$ ,  $(p_j)_{j \in J_f}$  and as such can be framed as a network game. The main difference from the single product problem in (10) is that now firm  $f$  will price its products jointly and will internalize how increasing a given price impacts the market shares of each of its products, i.e., the so-called portfolio effect.

How does the portfolio effect due to multiproduct pricing affect the equilibrium outcomes? When interpreting oligopolistic competition as a network game, the answer is fully captured by the adjacency matrix of the competitive network, which, as I show

formally in Proposition 6, is a function of the ownership structure  $H$

$$A_H(\Theta) \equiv \Theta - H \odot \Theta \quad (20)$$

where  $\odot$  is the Hadamard matrix product. The matrix  $\Theta$  still captures the competitive links between products. However, in the presence of multiproduct firms, the links between products within the same firms are set to zero by subtracting the block diagonal matrix  $H \odot \Theta$ . The resulting adjacency matrix  $A_H(\Theta)$  keeps track of the competitive links only between products owned by different firms.

## 5.2 Equilibrium price-cost Margins for Multiproduct Firms

In the next Proposition, I show that the result in Proposition 3 extends to the case in which firms sell multiple products and price them jointly. The result is presented for the case where own-price elasticities are heterogeneous across products, e.g.,  $\theta_{jj}$  varies across  $j$ .

**Proposition 6** *For a given ownership structure  $H$ , if an interior equilibrium  $(p_j^*)_{j=1}^J$  of the Bertrand pricing game exists, then it is unique and is such that*

$$p^* - c = \frac{\alpha - c}{2} - (I - H \odot \Theta)^{-1/2} \mathbf{b} \left( G_H(\Theta), \frac{1}{2}, (I - H \odot \Theta)^{1/2} \frac{\alpha - c}{2} \right) \quad (21)$$

*provided  $\max_j |\lambda_j(G)| \leq 2$  and where  $G_H(\Theta) \equiv (I - H \odot \Theta)^{-1/2} A_H(\Theta) (I - H \odot \Theta)^{-1/2}$  is a weighted adjacency matrix with elements in  $(-1, 1)$ .*

Similarly to the single-product case, in the multiproduct case, price-cost margins can be decomposed into a monopolistic component  $(\alpha - c)/2$  and into a product differentiation component proportional to the vector of Bonacich centralities  $\mathbf{b}$ . The fact that some subsets of products are priced jointly influences the network structure and, in turn, the centrality of each node. In particular, it affects the adjacency matrix by setting all the competitive links between products owned by the same firm to zero, i.e., any  $(j, l)$  element of the matrix  $A_H(\Theta)$  is zero whenever  $i, j \in J_f$  for some firm  $f$  and the same is true for the matrix  $G_H(\Theta)$ .

One slight difference between the single and multiproduct cases is that the matrix  $G(\Theta)$  that enters the Bonacich centrality is not necessarily symmetric, although its elements still belong to  $(-1, 1)$ . Luckily, this is not too much of a problem because one advantage of the Bonacich centrality compared to other measures is that it can be applied regardless of whether the network is directed or undirected.<sup>20</sup> This would not be the case if we were to use a measure of degree centrality, which, in the case of directed networks, needs to consider the direction of the link.

<sup>20</sup>See also Remark 3 in [Ballester, Calvó-Armenagol and Zenou \(2006\)](#).

**Comparative static with respect to  $H$ .** The expression in equation (21) points to an insightful comparative static in terms of the ownership structure matrix or, equivalently, the conduct parameter  $H$ . Consider first the extreme cases in which either firms are single-products (equivalently no collusion), or there is a single monopolist (equivalently perfect collusion). In the non-collusive case, we can see that expression (21) collapses to the outcome described in Proposition 3 because  $H = I_J$  and  $H \odot \Theta = \text{diag}(\Theta)$ . Conversely, under perfect collusion,  $H = 1_J 1_J'$ ,  $H \odot \Theta = \Theta$  and price-cost margins collapse to the monopolistic price-cost margins given by  $(\alpha - c)/2$ . The reason is that when all products are priced jointly, all the competitive links in the adjacency matrix  $A_{1_J 1_J'}(\Theta) = \Theta - \Theta = O$  are set to zero, and product differentiation does not matter for price-cost margins.

To gain more intuition, suppose that  $A(\Theta)$  is non-negative or equivalently that prices are strategic complements. Then, as the market becomes more collusive (i.e.,  $H \rightarrow 1_J 1_J'$ ), the centrality of each product decreases because  $A_H(\Theta) \rightarrow O$  element-wise and the importance of the monopolistic component (i.e., portfolio effect) in determining price-cost margins increases relative to the product differentiation component.

### 5.3 Market Definition and Mergers

Framing oligopolistic competition with product differentiation as a network game is insightful for at least two reasons.

First, from Proposition 6, we know that changes in the ownership structure will affect equilibrium price-cost margins only through the product differentiation component. Therefore, assuming there are no cost synergies, the only thing needed to assess the effect of mergers on equilibrium prices is the vector of product centralities under the merger and no merger scenarios. In practice, this requires taking a stand on the relevant product characteristics for consumers. Depending on the industry under consideration, this might be more or less difficult. However, if one is willing to assume Bertrand competition, no normalization on the levels or the scale of product attributes is needed because, as I showed in Propositions 1 and 3, characteristics affect substitution patterns and price-cost margins only through a normalized inner-product.

A second advantage of modelling product differentiation as a competitive network is that, as already pointed out in Hoberg and Phillips (2016), there is no need to put too much thought into defining the relevant market. To the extent that the vector of attributes adequately describes substitution patterns across products, the implied network structure defines the market; products that are more substitutable to each other will share a stronger link in the network. Thus, even a broad definition of the market, such as an entire industry, would not affect the merger analysis as long as the product characteristics considered capture the true substitution patterns across products.

## 6 Application: The US Automobile Industry

In this section, I estimate the model using data on the US automobile industry. The same data have been used extensively in the empirical industrial organization literature starting from the seminal contribution in [Berry, Levinsohn and Pakes \(1995\)](#).

Before going into the estimation details, I want to remark that, although the car industry is important, it does not represent the most ideal application for the current setting. The reason is that consumers in the model have a taste for variety and will consume more than one good. In the context of car choice, this is not the most realistic assumption because households typically own no more than two cars and, in most cases, would buy only one car at the time of purchase. Nonetheless, as I show later, the model produces reasonable substitution patterns and price-cost margins.

In what follows, I describe the data sources and the empirical model. Then, I recover own-cross price elasticities and price-cost margins. Lastly, I decompose price-cost margins and quantify how much of those are attributable to product differentiation.

### 6.1 Data

I obtain data on the US automobile industry from two different sources. First, I downloaded the data in [Berry, Levinsohn and Pakes \(1995\)](#) from the replication package accompanying a recent paper by [Andrews, Gentzkow and Shapiro \(2017\)](#). These data include the quantity sold by each car brand which I need because aggregate demand is derived in terms of quantities and not market shares. The second source of data is included in the recent Python package developed in [Conlon and Gortmaker \(2020\)](#), which again contains the very same automobile data but also includes the set of demand and supply instruments used in [Berry, Levinsohn and Pakes \(1995\)](#), which I will use to estimate demand in my model. Lastly, I collect data on the number of US households from the FRED website.

Overall, the data contains information on prices, quantities, market shares and characteristics of several car models sold in the US from 1971 to 1990. Table 1 reports sales-weighted averages of some relevant variables for each year. Average quantities sold are in units of 1000, average prices are in \$1000 units, and the number of households is in millions. The last five columns are sales-weighted averages of product characteristics: HP/WT is the ratio of horsepower to weight, Air is a dummy for whether air conditioning is standard, Size captures the space of the car, MP\$ measures the number of ten-mile increments one could drive for \$1 worth of gasoline, and lastly MPG measures the number of ten-mile increments one could drive with one gallon of gasoline.

Several interesting patterns emerge from Table 1. The number of competing firms has been roughly constant, ranging between 17 and 22 multiproduct car manufactur-

Year	Firms	Models	Own models	Competitors	Quantity	Price	Households	HP/WT	Air	Size	MP\$	MPG
1971	18	92	5.11	86.89	86.89	7.87	64.78	0.49	0.00	1.50	1.85	1.66
1972	19	89	4.68	84.32	98.62	7.98	66.68	0.39	0.01	1.51	1.87	1.62
1973	17	86	5.06	80.94	92.79	7.53	68.25	0.36	0.02	1.53	1.82	1.59
1974	17	72	4.24	67.76	105.12	7.51	69.86	0.35	0.03	1.51	1.45	1.57
1975	19	93	4.89	88.11	84.77	7.82	71.12	0.34	0.05	1.48	1.50	1.58
1976	21	99	4.71	94.29	93.38	7.79	72.87	0.34	0.06	1.51	1.70	1.76
1977	18	95	5.28	89.72	97.73	7.65	74.14	0.34	0.03	1.47	1.83	1.95
1978	18	95	5.28	89.72	99.44	7.64	76.03	0.35	0.03	1.40	1.93	1.98
1979	18	102	5.67	96.33	82.74	7.60	77.33	0.35	0.05	1.34	1.66	2.06
1980	19	103	5.42	97.58	71.57	7.72	80.78	0.35	0.08	1.30	1.47	2.21
1981	19	116	6.11	109.89	62.03	8.35	82.37	0.35	0.09	1.29	1.56	2.36
1982	19	110	5.79	104.21	61.89	8.83	83.53	0.35	0.13	1.28	1.82	2.44
1983	18	115	6.39	108.61	67.88	8.82	83.92	0.35	0.13	1.28	2.09	2.60
1984	20	113	5.65	107.35	85.93	8.87	85.41	0.36	0.13	1.29	2.12	2.47
1985	20	136	6.80	129.20	78.14	8.94	86.79	0.37	0.14	1.26	2.02	2.26
1986	22	130	5.91	124.09	83.76	9.38	88.46	0.38	0.18	1.25	2.86	2.42
1987	21	143	6.81	136.19	67.67	9.97	89.48	0.39	0.23	1.25	2.79	2.33
1988	20	150	7.50	142.50	67.08	10.07	91.07	0.40	0.24	1.25	2.92	2.33
1989	21	147	7.00	140.00	62.91	10.32	92.83	0.41	0.29	1.26	2.81	2.31
1990	20	131	6.55	124.45	66.38	10.34	93.35	0.42	0.31	1.27	2.85	2.27

Table 1: Sales-weighted averages.

ers. The same is true for the average number of car models produced by a single manufacturer, which increased slightly from 5 to 6.5. On the other hand, the total number of available car models increased more than 40%, from 92 in 1971 to 131 in 1990. At the same time, the average number of models produced by competitor firms also increased by 40%, from 87 models in 1971 to 124 in 1990, suggesting that competition in the number of products increased for the average firm. Next, looking at the average quantity and prices, the former decreased with cyclical ups and downs over time. In contrast, sales-weighted prices increased throughout the '80s while being roughly constant in the '70s. Product characteristics have also changed. For instance, air-conditioning becomes a more common feature over time, and both tens of miles per dollar (MP\$) and tens of miles per gallon (MPG) increased, suggesting that cars have become more efficient over time. At the same time, car size seems to have decreased, whereas horsepower has been roughly constant.

Before turning to the estimation of the model and recovering products' price-cost margins, it is helpful to get a sense of how prices correlate with product characteristics in the raw data without imposing any modelling structure. To this end, Figure 1 presents a binscatter of car prices against an unweighted measure of product centrality which summarizes how differentiated a product is relative to its competitors. The measure of centrality on the x-axis is motivated by the result I provided in Proposition 3 but differs in several respects because some terms are unobserved. The intuition described in equation (13) is simple: the margin over marginal cost a firm can charge depends negatively on how central its product is in the competitive network. Empirically though, whether or not this relationship between margins and centrality holds

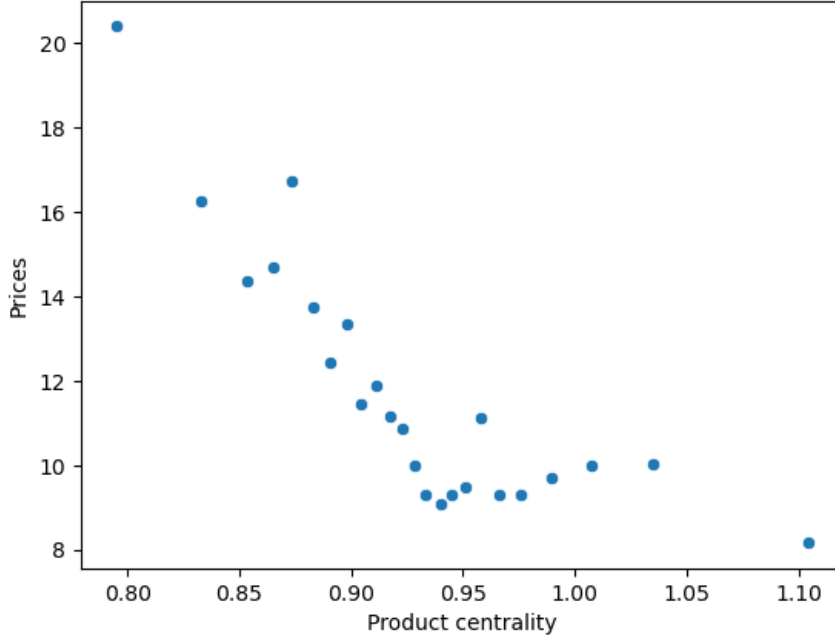


Figure 1: Binscatter of car prices against (unweighted) product Bonacich centrality after partialling out time fixed effects.

cannot be tested directly because marginal costs ( $c$ ) and consumer preferences ( $\alpha$ ) are unobserved. Nonetheless, Figure 1 attempts to do so naively by proxying margins (i.e., the left-hand side of equation (13)) with observed prices and the Bonacich centrality  $\mathbf{b}(A(\Theta), 1/2, (\alpha - c)/2)$  (i.e., the second term in (13)) with a similar centrality measure where  $((\alpha - c)/2)$  is replaced with a vector of ones  $1_J$ . The resulting measure is an unweighted Bonacich centrality given by

$$\mathbf{b}(A(\Theta), 1/2, 1_J) = \left( I_J - \frac{1}{2}A(\Theta) \right)^{-1} \frac{1}{2}A(\Theta)1_J, \quad (22)$$

which I can compute directly from the raw data. To compute  $\Theta$ , I use the same product characteristics that enter the indirect utility in Berry, Levinsohn and Pakes (1995) namely horsepower (HP/WT), air conditioning, size and miles-per-dollar (MP\$), and I calibrate  $\eta$  to 0.136.<sup>21</sup>

Returning to Figure 1, we can see that products with higher unweighted Bonacich centrality tend to charge lower prices. This decreasing relationship looks stronger for lower values of product centrality and seems to flatten as centrality increases. Although prices and centrality are negatively correlated as predicted by the network Bertrand model, from Figure 1, we cannot conclude that less central firms can charge higher margins and thus have more market power.<sup>22</sup> The reason is that the relationship

<sup>21</sup>The parameter  $\eta$  corresponds to the  $\frac{\alpha}{1-\alpha}$  in Pellegrino (2023) which is calibrated to  $\alpha$  to 0.12.

<sup>22</sup>Similarly, if prices and centrality were positively correlated or uncorrelated in the data, we could



can be driven by unobserved costs or heterogeneity in consumer preferences, which we are not accounting for. To uncover these unobservable components, we need a structural model of competition that allows us to identify consumer preferences and marginal costs. In the next section, I leverage the structure of the Bertrand network model to identify and estimate both the linear and quadratic components of consumer preferences. After estimating these demand parameters, I recover firms' marginal costs from the Nash-Bertrand equilibrium conditions.

## 6.2 The Empirical Nash-Bertrand Network Model

In this section, I make the Bertrand network model described in Section 4.1 empirically operational and adapt it to the context of car consumption.

An implicit assumption of the demand model introduced in Section 3 is the absence of income effects due to the quasi-linearity of consumer preferences in the outside good. In the context of car purchases, this assumption might be unrealistic. For this reason, I will now extend the demand model parsimoniously to accommodate income effects. To this end, I follow [Berry, Levinsohn and Pakes \(1995\)](#), and model consumer preferences for the inside and outside goods in a Cobb-Douglas fashion

$$U_i(y_i - p'q_i, q_i; X) = (y_i - p'q_i)^\gamma [G_i(q_i, X)]^\phi \quad (23)$$

where the first term in  $U$  already substitutes for the budget constraint and

$$G(q_i, X) \equiv \exp \left\{ q_i' \alpha_i - \frac{\beta_i}{2} q_i' (I_J + \eta X' X) q_i \right\}.$$

Next, substituting  $G$  into (23) and taking logs, consumer  $i$ 's utility can be written as

$$u_i(q_i, X) \equiv \log(U_i) = \gamma \log(y_i - p'q_i) + q_i' \alpha_i - \frac{\beta_i}{2} q_i' (I_J + \eta X' X) q_i \quad (24)$$

$$\approx q_i' \left( \alpha_i - \frac{p}{y_i} \right) - \frac{\beta_i}{2} q_i' (I_J + \eta X' X) q_i \quad (25)$$

where I normalize  $\phi = 1$  and  $\gamma = 1$  because they cannot be separately identified from  $\alpha_i$  and  $\beta_i$ , and I use a first order Taylor expansion to approximate  $\log(y_i - p'q_i)$ .<sup>23</sup> The preferences in (25) are identical to the ones described in Section 3 except for the fact that prices are now measured relative to income. Consumer  $i$ 's demand system can be derived as before

$$q_i(p) = \frac{1}{\beta_i} (I_J + \eta X X')^{-1} \left( \alpha_i - \frac{p}{y_i} \right). \quad (26)$$

---

not conclude that centrality does not influence price-cost margins as predicted by the model.

<sup>23</sup>The same approximation has been used in [Berry, Levinshon and Pakes \(1999\)](#).

To derive the aggregate demand for product  $j$  we need to integrate over the distribution of consumer preferences  $(\alpha_i, \beta_i, y_i)$ . Assuming that  $y_i$  is independent of  $(\alpha_i, \beta_i)$ ,<sup>24</sup> aggregate demand for product  $j$  is given by:

$$q_j(p_j, p_{-j}) = a_j - \frac{1}{\beta}(1 - \theta_{jj})\frac{p_j}{y} + \frac{1}{\beta} \sum_{k \neq j} \theta_{jk} \frac{p_k}{y} \quad (27)$$

where  $\alpha = \int \frac{\alpha_i/\beta_i}{1/\beta_i} di$ ,  $\beta = M \left[ \int \frac{1}{\beta_i} di \right]^{-1}$ ,  $y = M \left[ \int \frac{1}{y_i} di \right]^{-1}$  and  $\theta_{jk} = x'_j \Omega^{-1} x_k$ . Overall, the demand functions are identical to the ones presented in Section 3 except that income effects are now present.

To make the model empirically operational, let  $t$  denote a market (i.e., a year in our empirical context) and note that equation (27) can be rearranged in vector form as

$$\tilde{q}_t \equiv (I_{J_t} + \eta X_t X'_t) q_t = \frac{1}{\beta} \left( \alpha_t - \frac{p_t}{y_t} \right) \quad (28)$$

where  $J_t$  is the number of car models available in market  $t$ ,  $X_t$  is a  $J_t \times K$  matrix of product characteristics and the linear preference parameter vector  $\alpha$  is allowed to vary over time. Next, consider the  $j$ th equation of the above system

$$\tilde{q}_{jt} = \frac{1}{\beta} \left( \alpha_{jt} - \frac{p_{jt}}{y_t} \right) \quad (29)$$

and note that, upon calibrating  $\eta$  and assuming that the matrix  $X_t$  contains only observable characteristics, the left-hand side in (27), denoted by  $\tilde{q}_{jt}$ , is directly measurable. Conversely, on the right-hand side of (27), only  $p_t$  and  $y_t$  are observable.<sup>25</sup>

More generally, equation (29) suggests that we can estimate demand using the following linear specification

$$\tilde{q}_{jt} = -\frac{1}{\beta} \frac{p_{jt}}{y_t} + w'_{jt} \zeta + \xi_{jt} \quad (30)$$

where  $w_{jt}$  is a vector of observable product and demographic characteristics which, in this context, includes both  $x_{jt}$  and  $y_t$ . On the other hand,  $\xi_{jt}$  includes characteristics that are unobservable to the econometrician but known by the agents.

To summarise, two assumptions allow us to estimate demand from (30). First, all the characteristics that enter consumer preferences in the quadratic term are ob-

<sup>24</sup>In empirical IO it is commonly assumed that preferences parameters are idiosyncratic and independent from demographics.

<sup>25</sup>To measure income in a given year, I use the simulated draws that come with the pyBLP package developed in [Conlon and Gortmaker \(2020\)](#), and I average them using the weights provided. The draws come from a log-normal distribution of income whose location and scale parameters are estimated from the Current Population Survey (CPS) each year as described in [Berry, Levinsohn and Pakes \(1995\)](#).

servable. Second, any unobserved characteristic ( $\xi_{jt}$ ) enters consumer preferences only through the linear parameter vector  $\alpha$ , i.e.,<sup>26</sup>

$$\alpha_{jt} = \beta(w'_{jt}\zeta + \xi_{jt}) \quad (31)$$

To consistently estimate (30), we need to instrument  $p_{jt}/y_t$  because prices will be correlated with the unobservable component  $\xi_{jt}$ . The reason is that firms internalize  $\xi_{jt}$  before setting prices simultaneously. To see this formally, recall that Nash equilibrium prices are given by

$$p_{jt} = c_{jt} + \frac{y_t \alpha_{jt} - c_{jt}}{2} + \mathbf{b}_{jt} \quad (32)$$

and note that those prices are a function of  $\alpha_{jt}$  (and in turn function of  $\xi_{jt}$ ) both directly and indirectly through product  $j$ 's Bonacich centrality  $\mathbf{b}_{jt}$ .<sup>27</sup> Under this setting, demand can be estimated with a simple linear instrumental variable strategy which I describe next.

### 6.3 Estimation Results

I start by estimating demand from the linear specification in (30). To do so, I instrument the term  $p_{jt}/y_t$  using the set of demand instruments  $z_{jt}$  constructed in [Berry, Levinsohn and Pakes \(1995\)](#) and available in the pyBLP Python package developed by [Conlon and Gortmaker \(2020\)](#). These instruments are a function of product characteristics and, as a consequence, are correlated with prices because, per the supply equation (32), characteristics affect firms' pricing decisions through the centrality term  $\mathbf{b}_{jt}$  and the observable part of  $\alpha_{jt}$ . The fact that our instruments are correlated with prices is not enough, and we also need to ensure that demand remains constant while instruments shift the supply. The identifying assumption relies on the idea that firms choose characteristics before observing any demand shock  $\xi_{jt}$  which formally boils down to requiring that  $\mathbb{E}[\xi_{jt}|z_{jt}] = 0$ .

Table 2 reports both OLS and 2SLS demand estimates for the linear specification in equation (30). In both cases, the vector of characteristics  $w_{jt}$  includes a dummy for air conditioning, miles per dollar (MP\$), horsepower (HP/WT), space and the interaction between all of those with income. The coefficients on characteristics are similar across the two specifications, and, in both cases, only horsepower and miles per dollar are significantly different from zero. Both coefficients are positive, suggesting that the average consumer prefers cars with more horsepower and cars that are more efficient in gasoline consumption. The estimated price-to-income coefficient is the most evident

<sup>26</sup>I am also assuming that the parameter  $\beta$  is constant across markets. This assumption can be partially relaxed by interacting prices with any market level observable in equation (30).

<sup>27</sup>The derivation of (30) is analogous to the one presented in the proof of Proposition (3) with the exception that the income term now appears.

	OLS	IV-2SLS
Constant	-0.1023 (0.0043)	-0.1005 (0.0044)
Air (dummy)	-0.0179 (0.0145)	0.0039 (0.0183)
MP\$	0.0331 (0.0104)	0.0368 (0.0104)
HP/WT	0.1432 (0.0448)	0.1040 (0.0476)
Space	-0.0138 (0.0168)	-0.0115 (0.0166)
price/income	-0.0043 (0.0037)	-0.0327 (0.0121)
Fstat (Excluded)	-	92.1276
R2	0.8704	0.6257
Observations	2,217	2,217

Table 2: Demand estimates. Both specifications include interactions between characteristics and income, not reported here but available in Appendix D. Standard errors are clustered at the car model level.

difference between the OLS and 2SLS specifications. With OLS, the coefficient is not statistically different from zero. In contrast, when we instrument for prices, the coefficient becomes negative and significant, and its magnitude increases almost ten folds in absolute value. The discrepancy between OLS and IV estimates is quite common in contexts where prices and quantities are determined simultaneously in equilibrium. OLS estimates often imply inelastic demand curves because the observed variation in quantity and prices is also due to shifts in demand. However, after instrumenting for prices, the resulting estimates recover demand curves that are much more elastic.

After estimating the demand parameters, one would recover firms' marginal costs from equation (32). One issue in our context is that we do not observe  $\alpha_{jt}$ , and we need to estimate it before being able to back out  $c_{jt}$ . Luckily, we can obtain an estimate of  $\hat{\alpha}_{jt}$  for each product  $j$  and market  $t$  by simply plugging our demand estimates ( $\hat{\beta}, \hat{\zeta}$ ) and the estimated regression residuals ( $\hat{\xi}_{jt}$ ) into equation (31),

$$\hat{\alpha}_{jt} = \hat{\beta} (w'_{jt} \hat{\zeta} + \hat{\xi}_{jt}). \quad (33)$$

From the pricing equation in (32), with an estimate of  $\alpha_{jt}$ , we can then recover marginal costs  $c_{jt}$ , price-cost margins  $p_{jt} - c_{jt}$  and decompose the latter into monopolistic price-cost margins  $(\alpha_{jt} - c_{jt})/2$  and network centrality  $\mathbf{b}_{jt}$ .

Figures 2 and 3 show the distribution of both marginal costs and price-cost margins

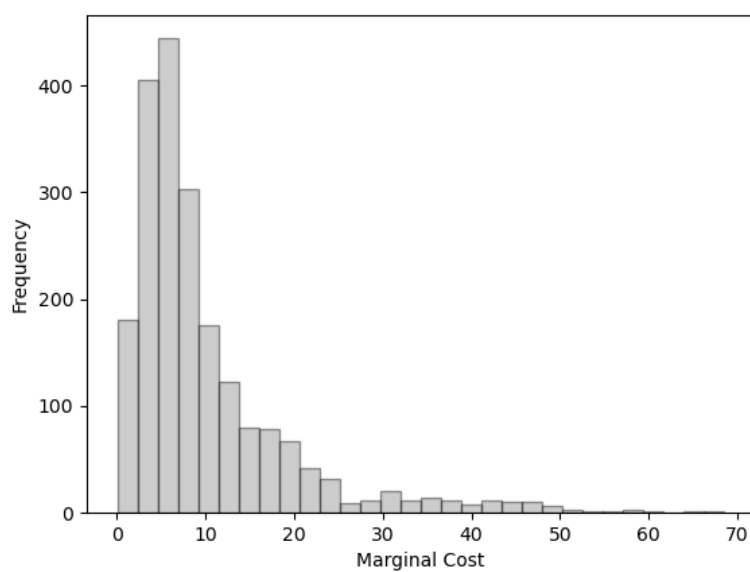


Figure 2: Distribution of marginal costs.

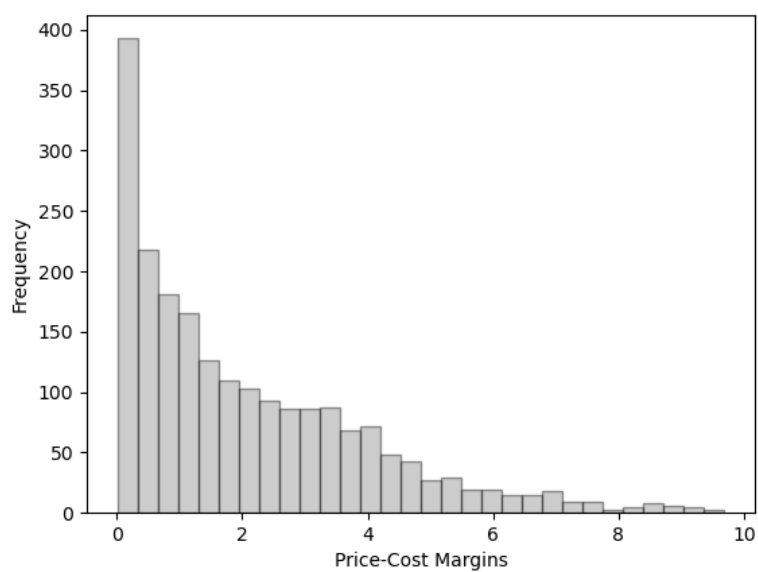


Figure 3: Distribution of price-cost margins.

in \$1000.<sup>28</sup> The median marginal cost across models and years is around \$6,900, and 3/4 of car models have a marginal cost lower than \$12,500. The distribution of estimated price-cost margin is more spread out, with a median margin around \$1,500 and 75% of car models charging a margin lower than \$3,200. Overall, although I started from linear-quadratic preferences rather than discrete choice, the estimated price-cost margins seem reasonably close to the ones recovered in [Berry, Levinsohn and Pakes \(1995\)](#).

Next, I decompose price-cost margins into a monopolistic component and a product centrality component which measures how differentiated a given product is relative to its competitors. Table 3 reports sales-weighted averages of such decomposition for each year available. The second column shows that the average marginal cost (MC) was stable throughout the '70s and then increased in the '80s. The third column reports average price-cost margins (PCM) in \$1000. Following equation (13), the fourth and fifth columns decompose PCMs into monopolistic price-cost margins (MPCM) and a Bonacich centrality component (BC).<sup>29</sup> The monopolistic margins, given by  $(\alpha_{jt} - c_{jt})/2$  vary considerably across years ranging from around \$50,000 up to almost \$170,000 and suggesting that the linear component of consumer preferences  $\alpha_{jt}$  varies substantially over time. This time variation in preferences can capture changes in economic conditions, changes in regulation such as the import restriction on Japanese cars imposed throughout the '80s and changes in the type and number of car models available. The product centrality component  $\mathbf{b}_{jt}$  also shows the same type of time variation and is, in magnitude, quite close to the monopolistic margins.

Figure 4 takes a closer look by plotting the monopolistic margins (MPCM) together with the product centrality (BC) on the left y-axis and the price-cost margins on the right y-axis over time. Monopolistic margins and product centrality follow a similar cyclical behaviour suggesting that the main driver could be time variation in consumer preferences  $\alpha_{jt}$ , which affects monopolistic margins but also the Bonacich product centrality through the weights.<sup>30</sup> Price-cost margins slightly fluctuate over time and seem to follow the cyclical behaviour of the other two variables. However, because the scale of the magnitude is way smaller, it is safe to conclude that PCMs have been basically constant over time, ranging on average between \$3000 and \$5000 overall but mostly in between \$3000 and \$4000 from 1975 onward.

The last two columns of Table 3 show what percentage of the monopolistic margins are captured by car manufacturers (PCM/MPCM) and what percentage of these margins is lost to competition between products (BC/MPCM). Not surprisingly, given the magnitudes observed in columns four and three, the last column shows that more

<sup>28</sup>I removed roughly 6% of observations estimated to have negative marginal costs.

<sup>29</sup>The decomposition used in Table 3 takes into account the fact that products have heterogeneous own-price elasticities and that firms are multiproduct.

<sup>30</sup>See equation (13) and Definition 1.

Year	MC (\$1000)	PMC (\$1000)	MPCM (\$1000)	BC (\$1000)	PMC/MPCM (%)	BC/MPCM (%)
1971	4.435	4.668	92.937	88.269	4.99	95.01
1972	4.290	4.825	101.971	97.146	4.79	95.21
1973	4.096	4.695	89.996	85.301	5.26	94.74
1974	4.256	4.295	65.251	60.956	6.51	93.49
1975	4.462	3.803	66.107	62.304	5.70	94.30
1976	4.247	3.990	88.535	84.545	4.53	95.47
1977	4.299	4.014	93.829	89.815	4.33	95.67
1978	4.198	4.091	101.548	97.457	4.08	95.92
1979	4.541	3.708	70.299	66.592	5.24	94.76
1980	4.503	3.514	49.657	46.143	7.05	92.95
1981	5.627	3.151	49.460	46.309	6.36	93.64
1982	6.270	2.848	55.414	52.566	5.13	94.87
1983	5.906	3.281	75.745	72.464	4.39	95.61
1984	4.917	4.259	98.949	94.690	4.36	95.64
1985	5.767	3.614	101.260	97.645	3.62	96.38
1986	5.556	4.133	175.756	171.623	2.42	97.58
1987	6.578	3.807	150.768	146.960	2.56	97.44
1988	6.782	3.834	167.875	164.042	2.28	97.72
1989	7.288	3.600	146.416	142.816	2.46	97.54
1990	7.417	3.326	135.846	132.520	2.48	97.52

Table 3: Sales-weighted averages of the decomposition of price-cost margins (PMC).

than 90% of monopolistic price-cost margins are lost to competition, suggesting that the competitive network is dense and most of the products are close substitutes to each other. The reason for this could be that, although products are differentiated, a firm faces more than 85 competitors each year (i.e., Table 1 column four) on average, and each product likely has a close substitute in terms of the observable characteristics we are considering.<sup>31</sup> The second to last column shows the other side of the medal; firms only capture from 2% to 7% of the margins they could potentially charge in a monopolistic market.

Our estimates suggest that car manufacturers capture only a small fraction of the monopolistic margins they could charge. However, are those margins as tiny as they look? To answer this question, I compare the estimated PCMs (as a percentage of the MPCMs) with the margin a firm would charge in a Cournot game with homogeneous products. In a homogenous Cournot model with  $N$  symmetric firms and linear demand, the equilibrium (dollar) price-cost margins are given by

$$p^{\text{hom. cournot}} - c = \frac{\alpha - c}{N + 1} \quad (34)$$

where  $\alpha$  is the demand intercept and  $c$  is the marginal cost. As a fraction of the monopolistic price-cost margin, the homogenous Cournot margins are only a function

<sup>31</sup>Recall that we are assuming that all characteristics that determine a product's centrality are observable to the econometrician.



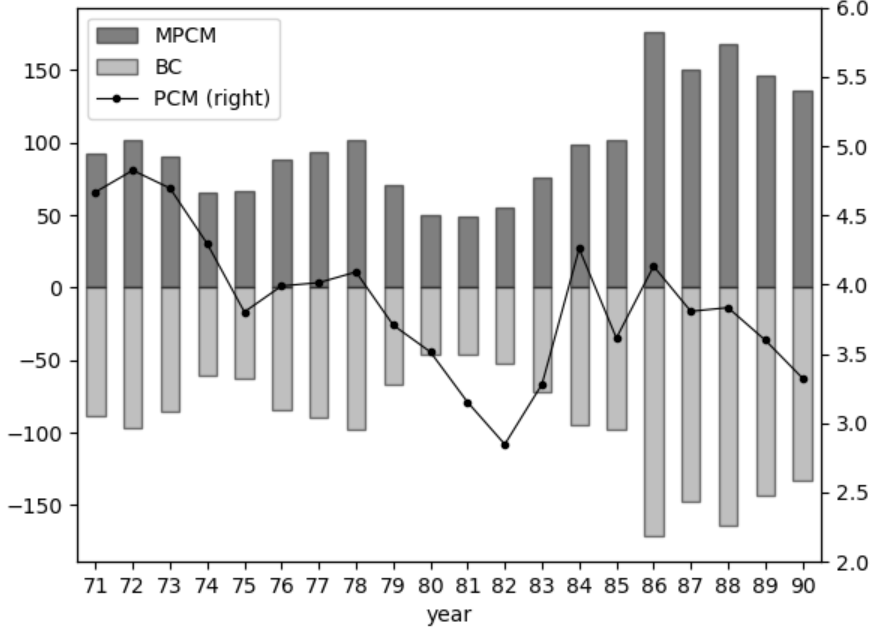


Figure 4: Decomposition of price-cost margins. All variables are measured in \$1000

of the number of firms  $N$

$$\frac{p^{\text{hom. cournot}} - c}{p^{\text{monopolist}} - c} = \frac{\alpha - c}{N + 1} \cdot \frac{2}{\alpha - c} = \frac{2}{N + 1}. \quad (35)$$

In Figure 5, I plot the estimated Bertrand PCM/MPCM reported in Table 3, together with the PCM/MPCM for the homogeneous Cournot model derived in equation (35) where I set the number of firms  $N$  equal to the average number of competitors a given car manufacturer faces in a given year (i.e., the number reported in Table 1) plus one. The pattern is clear; the margins firms charge in the Network Bertrand game are consistently above the margins under the homogeneous Cournot. Depending on the year, Bertrand margins can be more than three times higher than Cournot.<sup>32</sup> More generally, because the average number of competitors is the same across the two models, the difference between the two curves can be interpreted as the increase in margins that oligopolistic firms can capture when they offer differentiated products. Lastly, note that the difference between the Network Bertrand margins and the homogeneous Cournot margins represents a conservative estimate of the ability of firms to increase markups when products are differentiated. If we used a homogeneous Bertrand as benchmark, the increase would be more significant because firms, under homogeneous Bertrand, would be pricing at cost and charge zero margins.

<sup>32</sup>Interestingly, Bertrand margins peak throughout the '80s when a voluntary export restraint was placed on exports of automobiles from Japan to the United States, thereby reducing the supply of car models available.

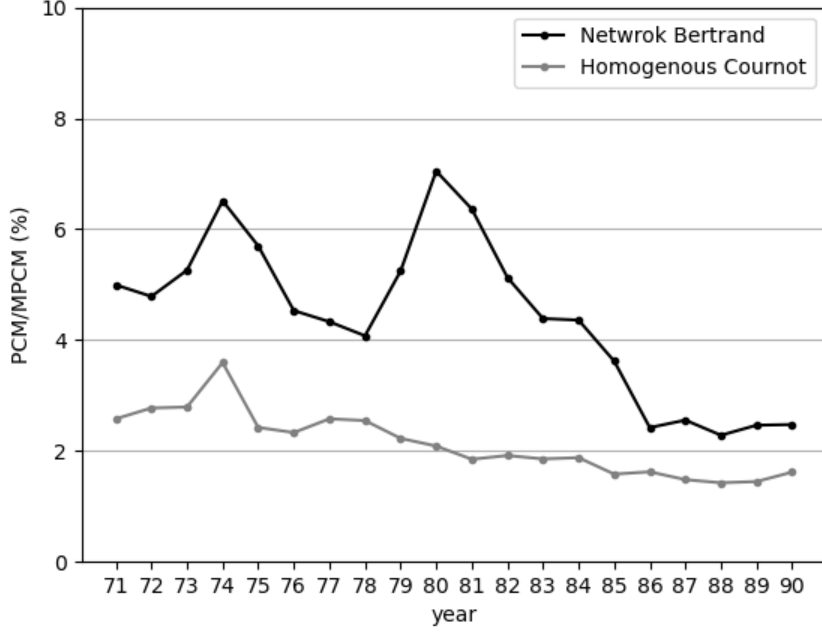


Figure 5: PCM/MPCM in Network Bertrand

I conclude the section by recovering own and cross-price elasticities implied by the demand estimates reported in Table 2. Recall from Section 3 that the  $(j, l)$  element of the elasticity matrix is given by

$$\epsilon_{jl} = \begin{cases} -\frac{1}{\beta}(1 - \theta_{jj}) & \text{if } j = l \\ \frac{1}{\beta}\theta_{jl} & \text{if } j \neq l \end{cases} \quad (36)$$

Because  $\theta_{jl}$  is a function of observable product characteristics, we only need an estimate of  $1/\beta$  to recover both own and cross-price elasticities.

Figure 6 plots a heatmap of the elasticity matrix for the year 1990. In that year, there were 131 car models available (Table 1), and Figure 6 shows the estimated own and cross-price elasticities for those models with an estimated own price elasticity larger than median.<sup>33</sup> Two remarks are in order. First, the magnitudes of own and cross-price elasticities are reasonable, with the own-price elasticities estimated to be negative and larger than the cross-price elasticities. Second, almost all cross-price elasticities are estimated to be small and positive. The latter would not be surprising in a model based on logit demand because that model restricts substitution patterns in a way that makes all products substitutes. However, this is not true in the linear-quadratic demand model I developed in Section 2. This type of quadratic preference does not impose any restrictions on product substitution patterns. However, the esti-

<sup>33</sup>This is done for visibility purposes. Estimates remain reasonable if we look at all models with own-price elasticity above the 25th percentile, reported in Appendix D Figure 7

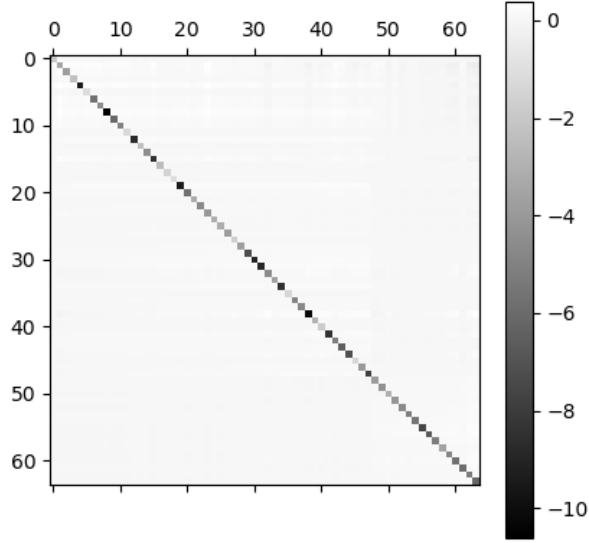


Figure 6: Matrix of estimated own and cross price elasticities for the year 1990. The products included are the ones with an estimated own price elasticity above the median.

mated cross-price elasticities are positive, suggesting that most car models are indeed substitutes for each other. Moreover, note that the fact that cross-price elasticities are positive is not a consequence of the fact that all product characteristics are positive numbers. More formally, having that  $x_{jk} > 0$  for any product  $j$  and characteristic  $k$  does not imply that all products are substitutes or, equivalently, that  $\theta_{jl}$  is always positive.

## 7 Conclusion

This paper studies how product differentiation affects substitution patterns, and firms' price-cost margins in oligopolistic markets where products are differentiated over multiple attributes and consumers have linear-quadratic preferences. Under these assumptions, oligopolistic competition in either prices or quantities can be framed as a network game where a product location in the network is determined by its vector of attributes, and the network links between products capture the extent with which two products compete. Products with similar characteristics will be closer to each other and will compete more intensely. On the other hand, products with more unique characteristics will have a more peripheral location and enjoy more market power.

More precisely, I show that firms' price-cost margins can be decomposed additively into a monopolistic component and a product differentiation component. The first component coincides with the margins a monopolist would charge. In contrast, the

second component is proportional to how central a product is in the network as measured by its Bonacich centrality. Products with higher centrality must give up a more significant fraction of the monopolistic margins because they face more competition. Furthermore, I showed that this decomposition holds for both Bertrand and Cournot price-cost margins and can be extended to allow the presence of multiproduct firms.

In the second part of the paper, I show how to estimate the model using market-level data on a given industry. Under the assumption that unobserved product characteristics enter consumer preferences only through the linear component of the utility, a simple linear IV strategy identifies the demand parameters. Then, marginal costs and margins can be recovered from the Nash equilibrium pricing equations. In addition, using the decomposition of price-cost margins, one can quantify what fraction of the potential monopolistic margins a firm can capture by differentiating its product from its competitors.

In the last part of the paper, I estimate the model using data on the US automobile industry from 1971 to 1990. I find that it delivers substitution patterns and price-cost margins comparable to the ones estimated in the literature that models automobile demand starting from individual discrete choice problems. Interestingly, although the linear-quadratic demand model does not restrict substitution patterns in any way, the estimated cross-price elasticities are almost always positive, suggesting that cars are substitutes, something the discrete choice demand framework instead imposes a priori. Finally, I decompose firms' price-cost margins and find that car manufacturers capture from 2% to 7% of the monopolistic margins depending on the year. These margins can be as high as three times the margins firms would be able to charge if their products were to be homogeneous.

## References

- Amir, R. and Jin, J. (2001), ‘Cournot and bertrand equilibria compared: substitutability, complementarity and concavity’, *International Journal of Industrial Organization* 19, 303–317.
- Andrews, I., Gentzkow, M. and Shapiro, J. (2017), ‘Measuring the sensitivity of parameter estimates to estimation moments’, *Quarterly Journal of Economics* 132(4), 1533–1592.
- Ballester, C., Calvó-Armenagol, A. and Zenou, Y. (2006), ‘Who’s who in networks. wanted: The key player’, *Econometrica* 74(5), 1403–1417.
- Berry, S. (1994), ‘Estimating discrete-choice models of product differentiation’, *The RAND Journal of Economics* 25(2), 242–262.
- Berry, S., Levinshon, J. and Pakes, A. (1999), ‘Voluntary export restraints on automobiles: Evaluating a trade policy’, *American Economic Review* 89(3), 400–430.
- Berry, S., Levinsohn, J. and Pakes, A. (1995), ‘Automobile prices in market equilibrium’, *Econometrica* 63(4), 841–890.
- Bonacich, P. (1987), ‘Power and centrality: A family of measures’, *American Journal of Sociology* 92(5), 1170–1182.
- Bresnahan, T. (1987), ‘Competition and collusion in the american automobile industry: The 1955 price war’, *The Journal of Industrial Organization* 35(4), 457–481.
- Chen, Y.-J., Zenou, Y. and Zhou, J. (2022), ‘The impact of network topology and market structure on pricing’, *Journal of Economic Theory* 204, 105491.
- Chen, Y., Zenou, Y. and Zhou, J. (2018), ‘Competitive pricing strategies in social networks’, *The RAND Journal of Economics* 49(3), 672–705.
- Cheng, L. (1985), ‘Comparing bertrand and cournot equilibria: A geometric approach’, *RAND Journal of Economics* 16(1), 146–152.
- Conlon, C. and Gortmaker, J. (2020), ‘Best practices for differentiated products demand estimation with pyblp’, *RAND Journal of Economics* 51(4), 1108–1161.
- Dubois, P., Griffith, R. and Nevo, A. (2014), ‘Do prices and attributes explain international differences in food purchases?’, *American Economic Review* 104(3), 832–867.
- Ederer, F. and Pellegrino, B. (2022), ‘A tale of two networks: Common ownership and product market rivalry’, SSRN working paper.

- Epplé, D. (1987), ‘Hedonic prices and implicit markets: Estimating demand and supply functions for differentiated products’, *Journal of Political Economy* 95(1), 59–80.
- Feenstra, R. and Levinsohn, J. (1995), ‘Estimating markups and market conduct with multidimensional product attributes’, *Review of Economic Studies* 62(1), 19–52.
- Galeotti, A., Golub, B., Goyal, S., Talamàs, E. and Tamuz, O. (2022), ‘Taxes and market power: A principal components approach’, arXiv preprint arXiv:2112.08153v2.
- Gandhi, A. and Houde, J. (2023), ‘Measuring substitution patterns in differentiated-products industries’, NBER working paper.
- Gentzkow, M. (2007), ‘Valuing new goods in a model with complementarity: Online newspapers’, *American Economic Review* 97(3), 713–744.
- Hoberg, G. and Phillips, G. (2016), ‘Text-based network industries and endogenous product differentiation’, *Journal of Political Economy* 124(5), 1423–1465.
- Jackson, M. (2008), *Social and Economic Networks*, Princeton University Press.
- Lee, S. and Allenby, G. (2009), ‘A direct utility model for market basket data’, *working paper*.
- Magnolfi, L., McClure, J. and Sorensen, A. (2022), ‘Triplet embeddings for demand estimation’, SSRN working paper.
- Magnolfi, L., Quint, D., Sullivan, C. and Waldfogel, S. (2022), ‘Differentiated-products cournot attributes higher markups than bertrand–nash’, *Economics Letters* 219, 166–175.
- Nevo, A. (2001), ‘Measuring market power in the ready-to-eat cereal industry’, *Econometrica* 69(2), 307–342.
- Okuguchi, K. (1987), ‘Equilibrium prices in the bertrand and cournot oligopolies’, *Journal of Economic Theory* 42, 128–139.
- Pellegrino, B. (2023), ‘Product differentiation and oligopoly: A network approach’, SSRN working paper.
- Rosen, S. (1974), ‘Hedonic prices and implicit markets: Product differentiation in pure competition’, *Journal of Political Economy* 82(1), 34–55.
- Singh, N. and Vives, X. (1984), ‘Price and quantity competition in a differentiated duopoly’, *RAND Journal of Economics* 15(4), 546–554.
- Thomassen, , Smith, H., Seiler, S. and Schiraldi, P. (2017), ‘Multi-category competition and market power: A model of supermarket pricing’, *American Economic Review* 107(8), 2308–2351.

- Ushchev, P. and Zenou, Y. (2018), ‘Price competition in product variety networks’, *Games and Economic Behavior* 110(2), 226–247.
- Vives, X. (1985), ‘On the efficiency of bertrand and cournot equilibria with product differentiation’, *Journal of Economic Theory* 36, 166–175.



## A Proofs

**Proof of Proposition 1.** Taking the horizontal sum of individual demands in (3) it is easy to check that the aggregate demand system is given by

$$q(p) = \int q_i(p) \frac{di}{M} \quad (37)$$

$$= \frac{1}{\eta\beta} \left( \frac{1}{\eta} I_J + X X' \right)^{-1} (\alpha - p) \quad (38)$$

where  $\beta \equiv \int \frac{1}{\beta_i} \frac{di}{M}$  and  $\alpha \equiv \int \frac{\alpha_i/\beta_i di}{\int 1/\beta_i di}$ . Next, note that

$$q(p) = \frac{1}{\eta\beta} \left( \frac{1}{\eta} I_J + X X' \right)^{-1} (\alpha - p) \quad (39)$$

$$= \frac{\alpha - p}{\beta} - X \Omega^{-1} X' \frac{(\alpha - p)}{\beta} \quad (40)$$

where  $\Omega \equiv \frac{1}{\eta} I_K + X' X$ . Then let  $a \equiv \frac{\alpha}{\beta} - X \Omega^{-1} X' \frac{\alpha}{\beta}$  and  $\Theta \equiv X \Omega^{-1} X'$  to obtain the expression in the main text. To complete the proof, we are left to show that  $\theta_{jj} \in (0, 1)$  and that  $\theta_{jl} \in (-1, 1)$  for any  $j \neq l$ . To this end, first note that by construction both  $\Theta$  and  $I_J - \Theta$  are positive definite matrices. Then, letting  $e_j$  be the  $j$ -th unit vector, we have that  $\theta_{jj} = e_j' \Theta e_j > 0$  and similarly  $1 - \theta_{jj} = e_j' (I - \Theta) e_j > 0$ . Next, take any  $(j, l)$  pair with  $j \neq l$  and note that

$$\theta_{jj} + \theta_{ll} - 2\theta_{jl} = (e_j - e_l)' \Theta (e_j - e_l) > 0 \quad (41)$$

where the first equality exploits the fact that  $\Theta$  is symmetric. From (41) and the fact that  $\theta_{jj} < 1$  for all  $j$ , we can conclude that  $\theta_{jl} < 1$ . To show that  $\theta_{jl} > -1$  it is enough to repeat the previous argument using  $\theta_{jj} + \theta_{ll} + 2\theta_{jl}$ .

**Proof of Proposition 2** Because  $U$  is the matrix of principal component directions of  $X$ , it is also the matrix whose columns correspond to eigenvectors of  $X' X$ . Then we have, assuming without loss that  $\eta = 1$ ,

$$\Theta = X(I_K + X' X)^{-1} X' \quad (42)$$

$$= X(I_K + U \Lambda^{x'x} U')^{-1} X' \quad (43)$$

$$= X U (I_K + \Lambda^{x'x})^{-1} U' X' \quad (44)$$

$$= \tilde{X} (I_K + \Lambda^{x'x})^{-1} \tilde{X}' \quad (45)$$

where  $\Lambda^{x'x}$  is the diagonal matrix that collects the eigenvalues of  $X' X$ . For  $\eta \neq 1$ , it is enough to redefine  $X = \sqrt{\eta} X$  to obtain the same result.

**Proof of Proposition 3** The proof presented is for the more general case in which  $\theta_{jj}$

are heterogeneous across  $j$ . The first order condition of (10) with respect to  $p_j$  reads:

$$\alpha_j(1 - \theta_{jj}) - \sum_{l \neq j} \theta_{jl} \alpha_l + c_j(1 - \theta_{jj}) - 2(1 - \theta_{jj})p_j + \sum_{l \neq j} \theta_{jl} p_l = 0 \quad (46)$$

after rearranging in vector form and solving for  $p^*$  one obtains

$$\begin{aligned} p^* - c &= \frac{1}{2} \left( I_J - \text{diag}(\Theta) - \frac{A(\Theta)}{2} \right)^{-1} (I - \Theta)(\alpha - c) \\ &= \frac{1}{2} \left( I_J - \left( I_J - \text{diag}(\Theta) - \frac{A(\Theta)}{2} \right)^{-1} \frac{A(\Theta)}{2} \right) (\alpha - c) \\ &= \frac{\alpha - c}{2} - (I_J - \text{diag}(\Theta))^{-1/2} \left( I_J - \frac{G(\Theta)}{2} \right)^{-1} \frac{G(\Theta)}{2} (I - \text{diag}(\Theta))^{1/2} \frac{(\alpha - c)}{2} \\ &= \frac{\alpha - c}{2} - (I_J - \text{diag}(\Theta))^{-1/2} \mathbf{b} \left( G(\Theta), \frac{1}{2}, (I - \text{diag}(\Theta))^{1/2} \frac{\alpha - c}{2} \right) \end{aligned} \quad (47)$$

where

$$G(\Theta) \equiv (I - \text{diag}(\Theta))^{-1/2} A(\Theta) (I - \text{diag}(\Theta))^{-1/2} \quad (48)$$

and the last equality is well-defined provided  $\max_j |\lambda_j(G)| < 2$ . Expression (13) then obtains when imposing  $\theta_{jj} \equiv \theta$ . To complete the proof for the more general case, I need to show that  $G(\Theta)$  preserves the properties of  $A(\Theta)$ . It is immediate to see that  $G(\Theta)$  is 0-diagonal and symmetric. I am left to show that all its off-diagonal elements  $g_{ij}$  lie in  $(-1, 1)$ . To see this recall that  $I_J - \Theta$  is positive definite and note that

$$(I_J - \text{diag}(\Theta))^{1/2} (I_J - G(\Theta)) (I_J - \text{diag}(\Theta))^{1/2} = I_J - \Theta \quad (49)$$

which implies that

$$1 - g_{ij}^2 = (g_{ij}e_i + e_j)' (I_J - G(\Theta)) (g_{ij}e_i + e_j) > 0 \quad (50)$$

which completes the proof.

**Proof of Proposition 4** The proof presented is for the more general case in which  $\theta_{jj}^-$  are heterogeneous across  $j$ . To find Cournot price-cost margins we first need to find the Cournot equilibrium quantities which are given by

$$q^* = \left( 2I_J - \eta \text{diag}(\Theta^-) - \eta \Theta^- \right)^{-1} \frac{\alpha - c}{\beta} \quad (51)$$

where  $\Theta^- \equiv -XX'$ . Plugging (51) into the aggregate inverse demand one obtains

$$\begin{aligned}
p^* - c &= \alpha - c - \beta (I_J - \eta \Theta^-) q^* \\
&= \frac{\alpha - c}{2} + \eta \frac{A(\Theta^-)}{2} \left( I_J - \eta \text{diag}(\Theta^-) - \eta \frac{A(\Theta^-)}{2} \right)^{-1} \frac{\alpha - c}{2} \\
&= \frac{\alpha - c}{2} + \left( I_J - \eta \text{diag}(\Theta^-) \right)^{1/2} \left( \eta \frac{G(\Theta^-)}{2} \right) \left( I_J - \eta \frac{G(\Theta^-)}{2} \right)^{-1} \times \\
&\quad \times \left( I_J - \eta \text{diag}(\Theta^-) \right)^{-1/2} \frac{\alpha - c}{2} \\
&= \frac{\alpha - c}{2} + \left( I_J - \eta \text{diag}(\Theta^-) \right)^{1/2} \times \\
&\quad \times \mathbf{b} \left( G(\Theta^-), \frac{\eta}{2}, \left( I_J - \eta \text{diag}(\Theta^-) \right)^{-1/2} \frac{\alpha - c}{2} \right)
\end{aligned} \tag{52}$$

where

$$G(\Theta^-) \equiv \left( I_J - \text{diag}(\Theta^-) \right)^{-1/2} A(\Theta^-) \left( I_J - \text{diag}(\Theta^-) \right)^{-1/2} \tag{53}$$

where the last equality is well-defined provided  $\eta < \max_j |\lambda(G)|/2$ . Expression (18) in the main text then obtains by replacing  $\theta_{jj}^-$  with  $\theta^-$  for any  $j$  and thus completes the proof. Note that, as mentioned in the main text, without any normalization the elements of the matrix  $\Theta^-$  can take any value possibly outside  $(-1, 1)$ . While this is not too much of a problem mathematically, it is unappealing conceptually because it implies that the adjacency matrix  $A(\Theta^-)$  can have weights greater than 1 in absolute value.

**Proof of Proposition 5** Let  $f$  and  $g$  denote the inverse demand and demand systems respectively:

$$p = f(q) = \alpha - \beta (I_J + \eta XX') q \tag{54}$$

$$q = g(p) = (I_J - \Theta) \frac{(\alpha - p)}{\beta} \tag{55}$$

where  $\Theta = X\Omega^{-1}X'$  and  $\Omega = \frac{1}{\eta}I_K + X'X$ . Firm  $j$ 's first order condition of the Bertrand problem can be written as

$$g_j(p) + (f_j(g(p)) - c) \frac{\partial g_j(p)}{\partial p_j} \tag{56}$$

$$= g_j(p) \sum_{l \neq j} \frac{\partial f_j}{\partial g_k} \frac{\partial g_k}{\partial p_j} + \left( f_j(g(p)) - c + \frac{\partial f_j}{\partial g_j} \right) \frac{\partial g_j(p)}{\partial p_j} \tag{57}$$

where the second equality differentiates the identity  $p_j \equiv f_j(g(p))$  with respect to  $p_j$ . Next, denote the Cournot equilibrium price  $p^c$  and note that the above Bertrand FOC

evaluated at  $p^c$  reduces to

$$g_j(p^c) \sum_{l \neq j} \frac{\partial f_j}{\partial g_k} \frac{\partial g_k}{\partial p_j} = g_j(p^c) \sum_{l \neq j} (-\beta \eta x'_j x_k) \frac{\theta_{kj}}{\beta} < 0 \quad (58)$$

where the last inequality holds because  $\theta_{kj} \geq 0$  implies that  $x'_j x_k \geq 0$ . To see this note that both  $I_J - \Theta$  and  $\Theta$  are positive definite matrices. But this implies that all eigenvalues of  $\Theta$  must lie in  $(0, 1)$  and, consequently

$$(I_J + \eta X X') = \quad (59)$$

$$= \left( I_J - X \left( \frac{1}{\eta} I_K + X' X \right)^{-1} X' \right)^{-1} \quad (60)$$

$$= (I_J - \Theta)^{-1} = \sum_{s=1}^{\infty} \Theta^s \geq 0, \quad (61)$$

where the last equality is well defined because  $\max_j |\lambda_j(\Theta)| < 1$ . Thus, because  $\Theta$  is non-negative by assumption we can conclude that  $x'_j x_k \geq 0$  if  $j \neq k$ . Because equation (58) holds for all  $j$  we can consider the vector of Bertrand FOCs evaluated at the Cournot price  $p_c$

$$(I_J - \Theta)\alpha + (I_J - \text{diag}(\Theta))c - 2 \left( I_J - \frac{\text{diag}(\Theta)}{2} - \frac{\Theta}{2} \right) p_c < 0 \quad (62)$$

$$\Leftrightarrow p_b = \frac{1}{2} \left( I_J - \frac{\text{diag}(\Theta)}{2} - \frac{\Theta}{2} \right)^{-1} ((I_J - \Theta)\alpha + (I_J - \text{diag}(\Theta))c) < p_c \quad (63)$$

where  $p_b$  denotes the Bertrand equilibrium price vector. Then, from Propositions 3 and 4 we can conclude that Bertrand centralities are higher than the ones implied by Cournot:

$$\tilde{\mathbf{b}}_b \equiv (I_J - \text{diag}(\Theta))^{-1/2} \mathbf{b}_b \geq \left( I_J - \eta \text{diag}(\Theta^-) \right)^{1/2} (-\mathbf{b}_c) \equiv -\tilde{\mathbf{b}}_c. \quad (64)$$

**Proof of Proposition 6** The proof is identical to the proof of Proposition 3 with the difference that the matrix  $\text{diag}(\Theta)$  must be replaced by  $H \odot \Theta$ . The first order condition of (19) with respect to  $p_j$  is given by

$$a_j + c_j - \sum_{l \in J_f} \theta_{lj} c_l - 2(1 - \theta_{jj}) p_j + \sum_{l \in J_f / \{j\}} \theta_{lj} p_l + \sum_{l \neq j} \theta_{jl} p_l = 0 \quad (65)$$

and after rearranging in vector form and solving for  $p^*$  one obtains

$$p^* - c = \frac{1}{2} \left( I_J - H \odot \Theta - \frac{A_H(\Theta)}{2} \right)^{-1} (I_J - \Theta)(\alpha - c) \quad (66)$$

$$= \frac{1}{2} \left[ I_J - \left( I_J - H \odot \Theta - \frac{A_H(\Theta)}{2} \right)^{-1} \frac{A_H(\Theta)}{2} \right] (\alpha - c) \quad (67)$$

$$= \frac{\alpha - c}{2} - (I_J - H \odot \Theta)^{-1/2} \left( I_J - \frac{G_H(\Theta)}{2} \right)^{-1} \frac{G_H(\Theta)}{2} \times \quad (68)$$

$$\times (I_J - H \odot \Theta)^{1/2} \frac{(\alpha - c)}{2} \quad (69)$$

$$= \frac{\alpha - c}{2} - (I_J - H \odot \Theta)^{-1/2} b \left( G_H(\Theta), \frac{1}{2}, (I_J - H \odot \Theta)^{1/2} \frac{(\alpha - c)}{2} \right) \quad (70)$$

where

$$G_H(\Theta) \equiv (I_J - H \odot \Theta)^{-1/2} A_H(\Theta) (I_J - H \odot \Theta)^{-1/2} \quad (71)$$

and the last equality is well-defined provided  $\max_j |\lambda_j(G)| < 2$ , where  $\lambda_j$  is the an eigenvalue of  $G_H(\Theta)$ . To complete the proof I need to show that  $G_H(\Theta)$  is a weighted adjacency matrix. First note that because  $A_H(\Theta) = \Theta - H \odot \Theta$  is a 0-block diagonal matrix the same holds for  $G_H(\Theta)$ . Next, I show that all its elements  $g_{ij}$  lie in  $(-1, 1)$ . To see this recall that  $I_J - \Theta$  is positive definite and note that

$$(I_J - H \odot \Theta)^{1/2} (I_J - G_H(\Theta)) (I_J - H \odot \Theta)^{1/2} = I_J - \Theta \quad (72)$$

which implies that  $(I_J - G_H(\Theta))$  is also positive definite, but then

$$1 - g_{ij}^2 = (g_{ij}e_i + e_j)' (I_J - G_H(\Theta)) (g_{ij}e_i + e_j) > 0. \quad (73)$$

So far we have used extensively  $(I_J - H \odot \Theta)^{1/2}$ , to make sure that this is well defined I need to show that  $I_J - H \odot \Theta$  is positive definite. To see this, note that  $I_J - \Theta > 0$  implies that all its principal submatrices are positive definite, but then all blocks in the block-diagonal matrix  $H \odot (I_J - \Theta)$  are also positive definite which makes  $H \odot (I_J - \Theta)$  positive definite as well.

## B Equilibrium existence

Throughout the analysis I have assumed that an interior Nash equilibrium exists. In this appendix, I provide conditions for the existence and uniqueness of the equilibrium. Specifically, I show that the following dominance-diagonal type of condition,

$$(1 - \theta_{jj})(\alpha_j - c_j) \geq \sum_{k \neq j} |\theta_{jk}|(\alpha_k - c_k) \quad \text{all } j \quad (74)$$

ensures that each firm  $j$  charges a price  $p_j$  that belongs to the interval  $[c_j, \alpha_j]$  when competing a la Bertrand.<sup>34</sup> Recall, that the system of FOCs is linear in the vector of prices  $p$  and given by

$$(I - \Theta)p + (I - \text{diag}(\Theta))p = (I - \Theta)\alpha + (I - \text{diag}(\Theta))c.$$

The above system has a well-defined solution as long as  $(I - \Theta)$  is non-singular, which is always the case because  $(I - \Theta)$  is a positive definite matrix. Moreover, linearity would also imply that such solution is unique. What we do not know is whether the equilibrium prices are non-negative and above firms' marginal costs. The above dominance diagonal condition ensures that the system of best replies is a self-map over the set  $\times_{j \in \{1, \dots, J\}} [c_j, \alpha_j]$ . To start with define the linear operator  $T : \mathbb{R}^J \rightarrow \mathbb{R}^J$  whose  $j$ th component is fund  $j$  best reply:

$$T_j(p) = \frac{1}{2} \left[ \alpha_j - c_j - \sum_{k \neq j} \frac{\theta_{jk}}{1 - \theta_{jj}} (\alpha_k - p_k) \right].$$

Next, we can show that whenever  $p \in \times_{j \in \{1, \dots, J\}} [c_j, \alpha_j]$ , condition (74) implies that  $T_j \in [c_j, \alpha_j]$ . To see this, take any  $p \in \times_{j \in \{1, \dots, J\}} [c_j, \alpha_j]$  and note that

$$\begin{aligned} c_j &\leq T_j(p) \leq \alpha_j \\ \Leftrightarrow \left| \sum_{k \neq j} \frac{\theta_{jk}}{1 - \theta_{jj}} \frac{\alpha_k - c_k}{\alpha_j - c_j} \frac{\alpha_k - p_k}{\alpha_k - c_k} \right| &\leq 1 \\ \Leftrightarrow \sum_{k \neq j} \frac{|\theta_{jk}|}{1 - \theta_{jj}} \frac{\alpha_k - c_k}{\alpha_j - c_j} \frac{\alpha_k - p_k}{\alpha_k - c_k} &\leq 1 \end{aligned}$$

is always satisfied when (74). Because  $T$  maps a closed and bounded set into itself, Bower fixed point theorem implies that there exists an  $p^* \in \times_{j \in \{1, \dots, J\}} [c_j, \mu_j]$  such that  $T(p^*) = p^*$ . Moreover, from (75) we know that such fixed point is unique, because  $T$  is linear.

---

<sup>34</sup>Clearly, there is an implicit assumption here that consumers willingness to pay for the very first unit  $\alpha_j$  (e.g., the demand intercept) is greater than the marginal cost  $c_j$ .

## C Decomposition of Cournot Markups

In this section, I show how to decompose the markups of a given product  $j$  in the case of Cournot competition and formally show that the product centrality measure defined in Pellegrino (2023), and denoted by  $1 - \chi_j$ , is an affine transformation of the Bonacich centrality measure  $\mathbf{b}_j$  I have been using throughout this paper.

To start with, I make a few notational adjustments to conform my notation to the one in Pellegrino (2023). I normalize the squared norm of product characteristics to one, i.e., for any product  $j$ , I assume that  $\|x_j\|_2^2 = x_j'x_j = 1$ . Following the notation I used in Proposition 4, this normalization is equivalent to set  $\theta^- = -1$ . Also, I set  $\beta_i \equiv 1$  for any consumer  $i$ , which further implies that in the aggregate demand derived in Proposition 1,  $\beta = 1$  and the aggregate demand intercept  $\alpha = M^{-1} \int \alpha_i di$ . In matrix form, the aggregate demand and its inverse are given by

$$q = (I + \eta XX')^{-1} (\alpha - p) \quad (75)$$

$$p = \alpha - (I + \eta XX')q. \quad (76)$$

Next, let  $\nu$  be such that  $\eta = \frac{\nu}{1-\nu}$  and note that

$$I + \eta XX' = \frac{1}{1-\nu}I + \frac{\nu}{1-\nu}(XX' - I) \quad (77)$$

where the  $\nu$  plays the role of  $\alpha$  in the notation of Pellegrino (2023). Next, assuming that  $(1 - \nu) > 0$ , I can re-scale consumer preferences in 1 by  $(1 - \nu)$  to obtain

$$v_i(q_i, X) \equiv (1 - \nu)u_i(q_i, X) = q_i'(\alpha_i - p) - \frac{1}{2}q_i'(I + \Sigma)q_i \quad (78)$$

where  $\Sigma \equiv \nu(XX' - I)$ , I redefined  $\alpha_i \equiv \nu X \alpha_i^x + (1 - \nu)\alpha_i^q$  and I already substituted in for the budget constraint.<sup>35</sup> The preferences in (78) are exactly the same ones used in Pellegrino (2023) and the associated aggregate demand and its inverse are

$$q = (I + \Sigma)^{-1} (\alpha - p) \quad (79)$$

$$p = \alpha - (I + \Sigma)q. \quad (80)$$

Next, I show how the product centrality measure defined in Pellegrino (2023) and denoted by  $1 - \chi_j$  is an affine transformation of product  $j$ 's Bonacich network centrality  $\mathbf{b}_j(-\Sigma, 1/2, (\alpha - c)/2)$ . The steps are similar to the ones in Proposition 4 with  $\Theta^- =$

---

<sup>35</sup>Note that it is not necessary to rescale the outside good  $q_{i0}$  by  $(1 - \nu)$  because I can always normalize its price to  $1/(1 - \nu)$  in the budget constraint.

$-\Sigma$ . To start with, note that the Cournot equilibrium quantities are given by

$$q^* = \left(1 - \frac{1}{2}(-\Sigma)\right)^{-1} \frac{\alpha - c}{2}. \quad (81)$$

Replacing (81) in the inverse demand, we get the price-cost margin decomposition obtained in Proposition (4)

$$p^* - c = \alpha - c + \left(I - \frac{1}{2}(-\Sigma)\right)^{-1} \frac{1}{2}(-\Sigma) \frac{\alpha - c}{2} \quad (82)$$

$$= \frac{\alpha - c}{2} + \mathbf{b} \left(-\Sigma, \frac{1}{2}, \frac{\alpha - c}{2}\right). \quad (83)$$

Next, define product  $j$  markups as

$$\mu_j \equiv \frac{p_j}{c_j} \quad (84)$$

and letting  $\mathbf{b}_j$  the  $j$ -th component of the vector of Bonacich centralities  $\mathbf{b}$ , from (83) we obtain

$$\mu_j = \frac{\alpha_j + c_j}{2c_j} + \frac{1}{c_j} \mathbf{b}_j = \bar{\mu}_j + \frac{1}{c_j} \mathbf{b}_j \quad (85)$$

where  $\bar{\mu}_j$  denotes the monopolistic markup.

Pellegrino (2023) shows that product  $j$  markup is given by

$$\mu_j = \chi_j + (1 - \chi_j) \bar{\mu}_j \quad (86)$$

where the product  $j$ 's centrality  $1 - \chi_j$  is defined as

$$1 - \chi_j \equiv \frac{1}{\alpha_j - c_j} \left[ \left( I + \frac{1}{2} \Sigma \right)^{-1} (\alpha - c) \right]_j \quad (87)$$

where for a generic vector  $y$ ,  $[y]_j$  denotes its  $j$ -th component. Combining (85) and (86) and solving for  $\chi_j$  one obtains

$$\chi_j = -\frac{2}{\alpha_j - c_j} \mathbf{b}_j \quad (88)$$

from which it is immediate to see that Pellegrino (2023)'s centrality measure is an affine transformation of the Bonacich centrality

$$1 - \chi_j = 1 + \frac{2}{\alpha_j - c_j} \mathbf{b}_j. \quad (89)$$

To complete the argument, I am left to show that from (89) I can back out the



definition of product centrality in (87). To see this, let  $\chi$  be the  $J$ -vector with  $j$ -th component  $\chi_j$  and building from (89)

$$1_J - \chi = [\text{diag}(\alpha - c)]^{-1} (\alpha - c + 2\mathbf{b}) \quad (90)$$

$$= [\text{diag}(\alpha - c)]^{-1} \left[ I - \left( I + \frac{1}{2}\Sigma \right)^{-1} \Sigma \frac{1}{2} \right] (\alpha - c) \quad (91)$$

$$= [\text{diag}(\alpha - c)]^{-1} \left( I + \frac{1}{2}\Sigma \right)^{-1} (\alpha - c) \quad (92)$$

where  $1_J$  is the  $J$ -vector of ones, the first equation is just (89) in vector forms, the second equation substitutes for the definition of Bonacich centrality and the last equation rearrange terms. Overall, the  $j$ -th equation in (92) coincides with the definition of product centrality for product  $j$  in (87), thus proving that the argument is consistent.

## D Figures and Tables

	OLS	IV-2SLS
Constant	-0.1023 (0.0043)	-0.1005 (0.0044)
Air (dummy)	-0.0179 (0.0145)	0.0039 (0.0183)
MP\$	0.0331 (0.0104)	0.0368 (0.0104)
HP/WT	0.1432 (0.0448)	0.1040 (0.0476)
Space	-0.0138 (0.0168)	-0.0115 (0.0166)
air $\times$ income	0.0001 0.0001	0.0000 0.0001
MP\$ $\times$ income	0.0001 0.0001	0.0001 0.0001
HP/WT $\times$ income	-0.0005 0.0002	-0.0003 0.0003
Space $\times$ income	0.0004 0.0001	0.0004 0.0001
price/income	-0.0043 (0.0037)	-0.0327 (0.0121)
Fstat (Excluded)	-	92.1276
R2	0.8704	0.6257
Observations	2,217	2,217

Table 4: Demand estimates. Standard errors are clustered at the model level.

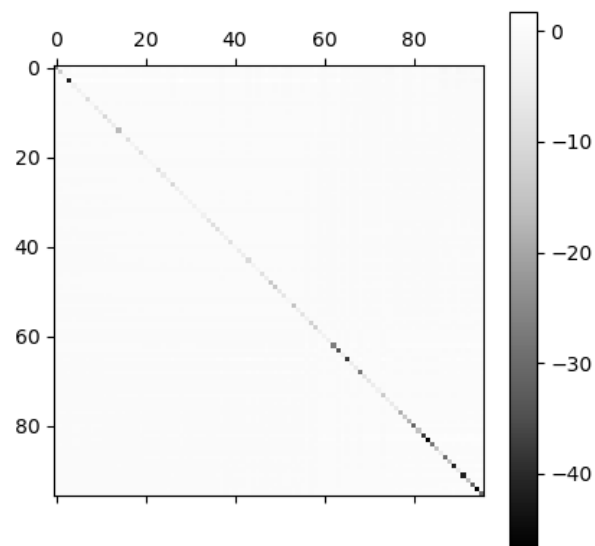


Figure 7: Matrix of estimated own and cross price elasticities for the year 1990. The products included are the ones with an estimated own price elasticity above the 25th percentile.

## E Simulation of the Bertrand Network game

In this section, I perform a simple simulation exercise to summarize and visualize how the Bertrand Network model works.

Table 5 describes the parameters used in the simulation. There is a single market with  $J = 30$  products and  $K = 7$  characteristics whose values are drawn from a uniform distribution in between  $[0, 1]$ . The demand intercept  $\alpha$  is the same across all products and set to 0.15 whereas marginal costs are heterogeneous across products and drawn from a  $[.01, .03]$  uniform distribution.

Parameter	Value
$J$	30
$K$	7
$\alpha$	0.15
$c$	U[0.01, 0.03]
$x_{jk}$	U[0, 1]

Table 5: Parameters for simulation of Bertrand network game

Given this parameters, Figure 8 plots the underlying Bertrand network. Each product is a node and the edges capture the degree of substitution between any two products/nodes; the longer the edge the less substitute are the two products. The location of dots and edges is exogenous and entirely determined by the realization of the draws of product characteristics. Conversely, the size of the dots is endogenous and it is proportional to the equilibrium price-cost margins. The plot shows that nodes that are more peripheral tend to have larger dot sizes whereas dots that are more central are smaller. The intuition for this result is the following: peripheral products are more unique or equivalently less central and, per equation (13) will charge higher margins in equilibrium. On the other hand, more central nodes face more intense competition and must lower their margins.

Figure 9 instead is a visualization of Proposition 3 and plots the equilibrium price-cost margins on the y-axis against the Bonacich product centrality on the x-axis. It should be clear by now why the relationship is decreasing; higher centrality implies lower equilibrium markups. The noise around the downward sloping relationship is due to the fact that marginal costs are heterogeneous. By increasing the variance of the distribution of costs, Figure 9 would start looking noisier and the resulting relationship between centrality and margins might not look as clear. This highlights how empirically it is important to control for the unobserved costs in order to recover the downward sloping relationship. The same would be true if we were to introduce heterogeneity in the demand intercept  $\alpha$ .

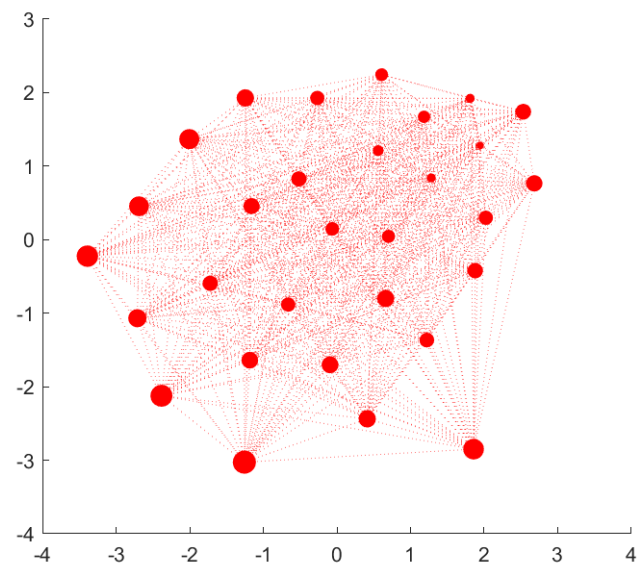


Figure 8: Simulated Network. Location is exogenous. Node size is proportional to markups.

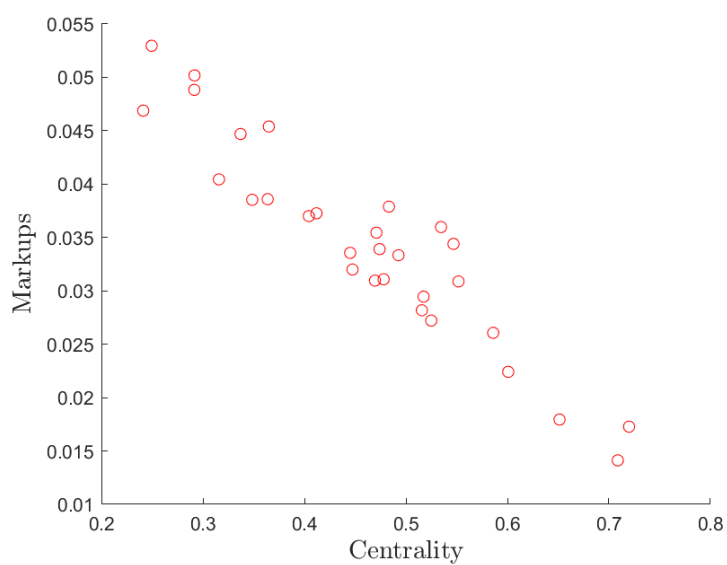


Figure 9: Simulated Network. Price-cost margins (y-axis) against Bonacich centrality (x-axis).

## F From linear-quadratic to discrete choice

This section shows how the linear-quadratic demand model nests the discrete choice demand framework. To see this suppose consumers can only purchase one unit of a product. Under this assumption consumer  $i$  problem can be formalized as

$$\max_{(q_{ij})_{j=1}^J} q'_i(\alpha_i - p) - \frac{\beta_i}{2} q'_i (I_J + \eta X X') q_i \quad (93)$$

$$\text{s.t. } q_{ij} \in \{0, 1\} \quad (94)$$

$$\sum_{j=1}^J q_{ij} \leq 1. \quad (95)$$

From the constraints it is immediate to see that the problem boils down to simply choosing the product that provides the highest utility

$$\max_{j \in \{1, \dots, J\}} \alpha_{ij} - p_j - \frac{\beta_i}{2} (1 + \eta x'_j x_j). \quad (96)$$

Next, recalling that  $\alpha_{ij} = \eta x'_j \alpha_i^x + \alpha_{ij}^q$ , redefining the vectors of product characteristics and preference parameters

$$\hat{x}_j \equiv (x'_j, (1 + \eta x'_j x_j)/2)' \quad (97)$$

$$\hat{\beta}_i \equiv (\eta \alpha_i, \beta_i)', \quad (98)$$

and, rewriting  $\alpha_{ij}^q = \hat{\xi}_j + \varepsilon_{ij}$  we can write (96) following the standard random utility notation used in empirical applications

$$\max_{j \in \{1, \dots, J\}} u_{ij} \equiv -p_j + \hat{x}'_j \beta_i + \hat{\xi}_j + \varepsilon_{ij} \quad (99)$$

where  $\hat{x}_j$  is the vector of product  $j$ 's characteristics observed by the econometrician,  $\hat{\xi}_j$  is a scalar that captures characteristics or demand shocks unobserved by the econometrician,  $\beta_i$  is a vector of preference parameters capturing how consumer  $i$  values different product attributes and, lastly,  $\varepsilon_{ij}$  is a random utility shock.<sup>36</sup>

By making specific assumptions on the distribution of  $\varepsilon_{ij}$  one can recover all standard models of discrete choice demand, such as logit, probit, nested-logit and random-coefficient logit.

---

<sup>36</sup>I am implicitly assuming that the econometrician knows or can calibrate  $\eta$ . If this is not the case then the quadratic term should be treated as unobserved and included in the  $\hat{\xi}_j$ .