

Carteras de inversión.

Introducción al diseño y análisis.

Dr. Martín Lozano <https://mlozanoqf.github.io/>

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| | Fundamental | Intermedio | Especializado |
|-------------|-------------|------------|---------------|
| Finanzas | × | ✓ | × |
| Estadística | ✓ | × | × |
| R | × | ✓ | × |

1 Introducción.

- Conceptos fundamentales del análisis riesgo–rendimiento.
- Explicar la frontera media–varianza, destacando cómo se obtiene mediante métodos analíticos y numéricos.
- Construir y evaluar portafolios ingenuos, de mínima varianza, de 2 y 10 activos usando datos reales.
- Análisis cuando se permiten o restringen las ventas en corto.

2 Diez empresas públicas de Estados Unidos.

| Ticker | Nombre de la empresa | Industria |
|--------|-----------------------|---------------------------------|
| AMD | Advanced Micro Dev. | Computación de alto rendimiento |
| CNC | Centene Corp. | Servicios de salud |
| GIS | General Mills, Inc. | Productos de consumo envasados |
| LMT | Lockheed Martin Corp. | Aeroespacial y defensa |
| LRCX | Lam Research Corp. | Semiconductores |
| NEM | Newmont Corp. | Minería de metales preciosos |
| SNPS | Synopsys, Inc. | Software de diseño |
| SYF | Synchrony Financial | Servicios financieros |
| TRMB | Trimble Inc. | Tecnología geoespacial |
| TTD | The Trade Desk, Inc. | Tecnología publicitaria |

3 Paquetes.

```
1 library(tidyquant)
2 library(tidyverse)
3 library(lubridate)
4 library(scales)
5 library(quadprog)
```

4 Inicialización, parte 1.

```
1 # Empresas y rango temporal
2
3 tickers <- c("AMD", "CNC", "GIS", "LMT", "LRCX", "NEM", "SNPS", "SYF", "TRMB", "TTD")
4 n_months <- 60L
5 price_start <- ymd("2020-09-01")
6 price_end <- price_start %m+% months(n_months + 1) - days(1)
7 returns_start <- price_start %m+% months(1)
8 returns_end <- returns_start %m+% months(n_months) - months(1)
9
10 # Descarga de precios históricos de las acciones
11
12 prices <- tq_get(tickers, from = price_start, to = price_end, get = "stock.prices")
13
14 # Cálculo de retornos mensuales por activo
15
16 monthly_returns <- prices |>
17   arrange(symbol, date) |>
18   group_by(symbol) |>
19   tq_transmute(select = adjusted, mutate_fun = periodReturn,
20                 period = "monthly", type = "arithmetic",
21                 col_rename = "monthly_return") |>
22   filter(between(date, returns_start, returns_end)) |>
23   slice_head(n = n_months) |>
24   ungroup()
25
26 stopifnot(n_distinct(monthly_returns$symbol) == length(tickers))
27
28 # Función: estadísticas básicas por activo
29
30 asset_stats <- function(data) {
31   data |>
32     group_by(symbol) |>
33     summarise(ER = mean(monthly_return, na.rm = TRUE),
34               SD = sd(monthly_return, na.rm = TRUE),
35               SR = ER / SD, .groups = "drop")}
36
37 # Función: construcción de rendimiento acumulado por activo
38
39 cum_index <- function(data, start_date) {
40   base <- tibble(date = start_date, symbol = unique(data$symbol), index_level = 1)
41   data |>
42     arrange(symbol, date) |>
43     group_by(symbol) |>
44     mutate(index_level = cumprod(1 + monthly_return)) |>
45     ungroup() |>
46     select(date, symbol, index_level) |>
47     bind_rows(base) |>
48     arrange(symbol, date)}
```

5 Inicialización, parte 2.

```
1  # Función: construcción de rendimiento acumulado por cartera
2
3  portfolio_series <- function(data, weights, label, start_date) {
4    data |>
5      filter(symbol %in% names(weights)) |>
6      group_by(date) |>
7      summarise(portfolio_return = sum(monthly_return * weights[symbol]), .groups = "drop") |>
8      arrange(date) |>
9      mutate(index_level = cumprod(1 + portfolio_return), symbol = label) |>
10     select(date, symbol, index_level) |>
11     bind_rows(tibble(date = start_date, symbol = label, index_level = 1))}
12
13  # Función: estadísticas de un portafolio
14
15  portfolio_stats <- function(data, weights, label) {
16    pr <- data |>
17      filter(symbol %in% names(weights)) |>
18      group_by(date) |>
19      summarise(portfolio_return = sum(monthly_return * weights[symbol]), .groups = "drop")
20
21    tibble(symbol = label, ER = mean(pr$portfolio_return, na.rm = TRUE),
22          SD = sd(pr$portfolio_return, na.rm = TRUE), SR = ER / SD)}
23
24  # Conversión de retornos a formato ancho y matriz
25
26  returns_wide <- monthly_returns |>
27    select(date, symbol, monthly_return) |>
28    pivot_wider(names_from = symbol, values_from = monthly_return) |>
29    arrange(date)
30
31  returns_mat_all <- returns_wide |>
32    select(-date) |>
33    as.matrix()
34
35  # Cálculo de medias, covarianzas y estadísticas
36
37  mu_vec_all <- colMeans(returns_mat_all, na.rm = TRUE)
38  cov_mat_all <- cov(returns_mat_all, use = "pairwise.complete.obs")
39  n_assets <- length(mu_vec_all)
40
41  stats_all <- tibble(symbol = names(mu_vec_all), ER = mu_vec_all,
42                    SD = apply(returns_mat_all, 2, sd, na.rm = TRUE))
43
44  color_values_all <- setNames(scales::hue_pal()(length(mu_vec_all)),
45                             names(mu_vec_all))
```

6 Matriz de correlaciones.

```
1 corr_mat <- returns_wide |>
2   select(-date) |>
3   cor(use = "pairwise.complete.obs")
4
5 print(round(corr_mat, 2), quote = FALSE, right = TRUE)
```

| ## | | AMD | CNC | GIS | LMT | LRCX | NEM | SNPS | SYF | TRMB | TTD |
|----|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| ## | AMD | 1.00 | -0.10 | -0.16 | -0.12 | 0.71 | -0.06 | 0.70 | 0.35 | 0.50 | 0.51 |
| ## | CNC | -0.10 | 1.00 | 0.37 | 0.37 | 0.14 | 0.20 | -0.16 | 0.12 | 0.01 | -0.18 |
| ## | GIS | -0.16 | 0.37 | 1.00 | 0.46 | -0.20 | 0.25 | -0.27 | -0.15 | -0.20 | -0.20 |
| ## | LMT | -0.12 | 0.37 | 0.46 | 1.00 | 0.00 | 0.32 | -0.22 | 0.20 | 0.01 | -0.10 |
| ## | LRCX | 0.71 | 0.14 | -0.20 | 0.00 | 1.00 | 0.04 | 0.56 | 0.59 | 0.61 | 0.38 |
| ## | NEM | -0.06 | 0.20 | 0.25 | 0.32 | 0.04 | 1.00 | -0.04 | -0.08 | -0.05 | -0.26 |
| ## | SNPS | 0.70 | -0.16 | -0.27 | -0.22 | 0.56 | -0.04 | 1.00 | 0.33 | 0.48 | 0.31 |
| ## | SYF | 0.35 | 0.12 | -0.15 | 0.20 | 0.59 | -0.08 | 0.33 | 1.00 | 0.67 | 0.29 |
| ## | TRMB | 0.50 | 0.01 | -0.20 | 0.01 | 0.61 | -0.05 | 0.48 | 0.67 | 1.00 | 0.44 |
| ## | TTD | 0.51 | -0.18 | -0.20 | -0.10 | 0.38 | -0.26 | 0.31 | 0.29 | 0.44 | 1.00 |

7 Pares de correlaciones mínimas.

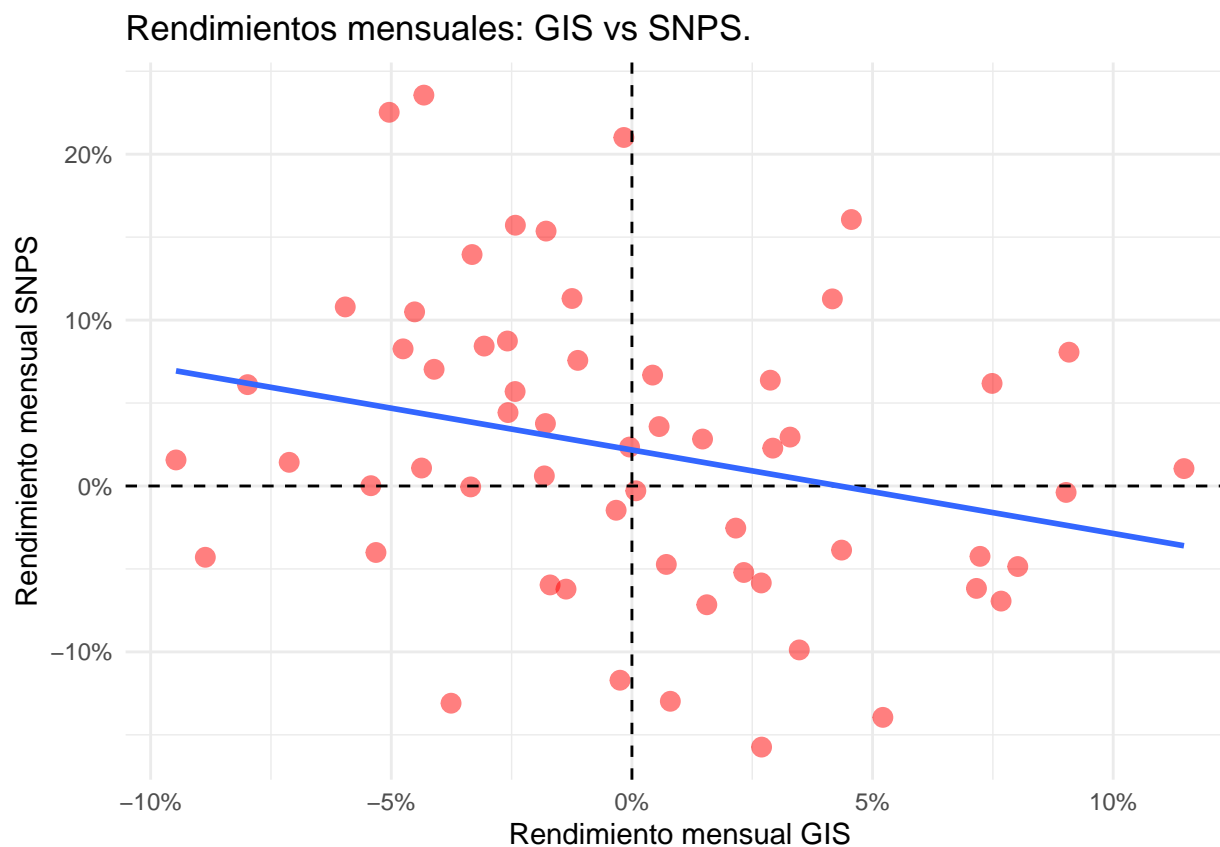
$$\sigma_{port}^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{A,B}.$$

```
1 lowest_pairs <- corr_mat |>
2   as_tibble(rownames = "symbol_i") |>
3   pivot_longer(-symbol_i, names_to = "symbol_j", values_to = "corr") |>
4   filter(symbol_i < symbol_j) |>
5   arrange(corr) |>
6   slice_head(n = 10)
7
8 print(lowest_pairs)
```

```
## # A tibble: 10 x 3
##   symbol_i symbol_j   corr
##   <chr>    <chr>    <dbl>
## 1 GIS      SNPS      -0.266
## 2 NEM      TTD       -0.263
## 3 LMT      SNPS      -0.217
## 4 GIS      TTD       -0.201
## 5 GIS      LRCX      -0.201
## 6 GIS      TRMB      -0.198
## 7 CNC      TTD       -0.180
## 8 AMD      GIS       -0.162
## 9 CNC      SNPS      -0.156
## 10 GIS     SYF       -0.148
```


8 Ilustración de la correlación.

```
1 # Pasar a formato ancho con una columna para GIS y otra para SNPS
2 gis_snps_wide <- monthly_returns |>
3   filter(symbol %in% c("GIS", "SNPS")) |>
4   select(date, symbol, monthly_return) |>
5   pivot_wider(names_from = symbol, values_from = monthly_return) |>
6   drop_na()
7
8 # Scatter GIS (eje X) vs SNPS (eje Y)
9 ggplot(gis_snps_wide, aes(x = GIS, y = SNPS)) +
10   geom_point(size = 3, alpha = 0.5, col = "red") +
11   geom_hline(yintercept = 0, linetype = "dashed") +
12   geom_vline(xintercept = 0, linetype = "dashed") +
13   scale_x_continuous(labels = percent_format(accuracy = 1)) +
14   scale_y_continuous(labels = percent_format(accuracy = 1)) +
15   labs(title = "Rendimientos mensuales: GIS vs SNPS.",
16        x = "Rendimiento mensual GIS", y = "Rendimiento mensual SNPS") +
17   geom_smooth(method = "lm", se = FALSE) +
18   theme_minimal()
```



9 Riesgo-rendimiento: 10 activos individuales.

```
1 stats <- asset_stats(monthly_returns) |>
2   arrange(-SR) |>
3   print()
```

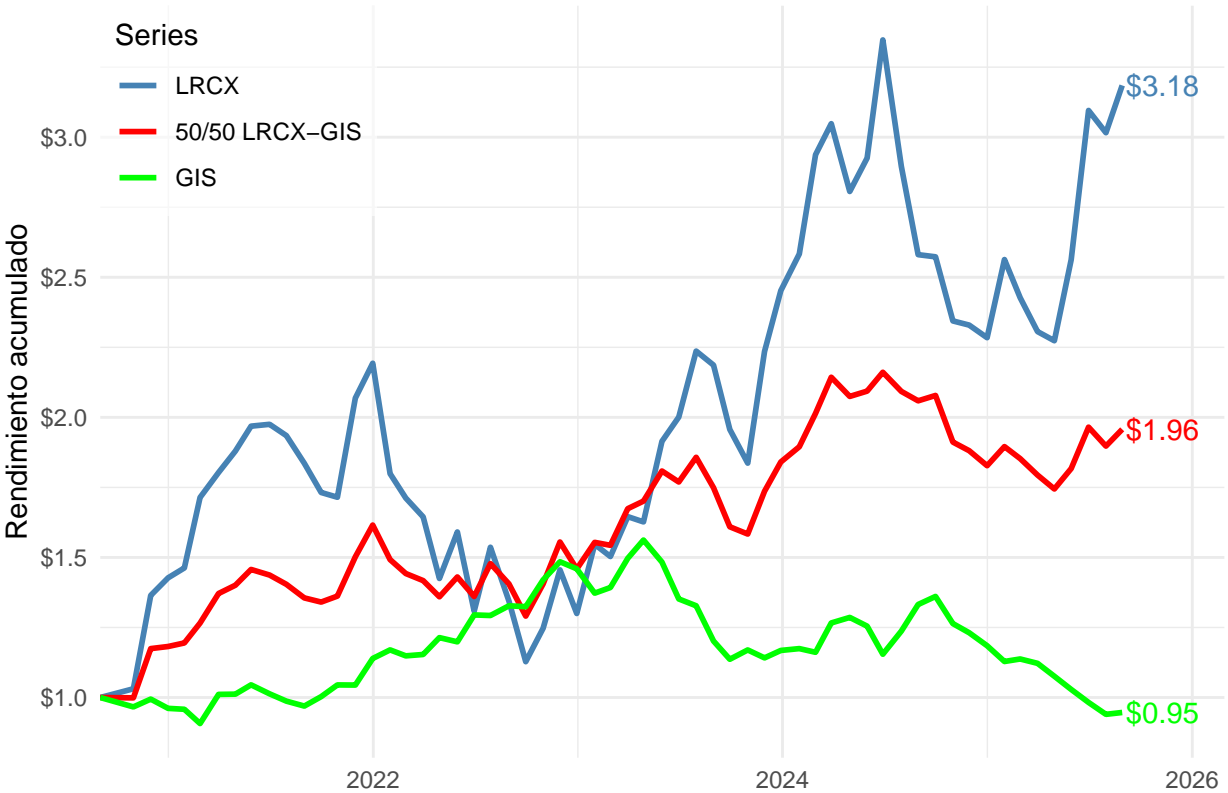
```
## # A tibble: 10 x 4
##   symbol      ER      SD      SR
##   <chr>    <dbl> <dbl>   <dbl>
## 1 SYF      0.0259  0.106  0.244
## 2 SNPS     0.0216  0.0902  0.240
## 3 LRCX     0.0257  0.111  0.231
## 4 AMD      0.0223  0.150  0.149
## 5 TRMB     0.0132  0.0970  0.136
## 6 LMT      0.00709 0.0647  0.110
## 7 NEM      0.0107  0.103  0.103
## 8 TTD      0.0170  0.184  0.0926
## 9 GIS      0.000182 0.0477  0.00382
## 10 CNC     -0.00542 0.102 -0.0532
```

10 Rendimiento acumulado, cartera ingenua.

$$R_{port} = 0.5\mu_{LRCX} + 0.5\mu_{GIS}.$$

```
1 focus <- c("LRCX", "GIS")
2 color_values <- c("LRCX"="steelblue", "GIS"="green", "50/50 LRCX-GIS"="red")
3
4 cum_focus <- monthly_returns |>
5   filter(symbol %in% focus) |>
6   cum_index(price_start)
7
8 portfolio_50_50 <- portfolio_series(monthly_returns, c(LRCX = 0.5, GIS = 0.5),
9                                     "50/50 LRCX-GIS", price_start)
10
11 plot_data <- bind_rows(cum_focus, portfolio_50_50)
12
13 last_vals <- plot_data |>
14   group_by(symbol) |>
15   summarise(final_value = last(index_level), .groups = "drop") |>
16   arrange(desc(final_value))
17
18 label_data <- plot_data |>
19   group_by(symbol) |>
20   filter(date == max(date)) |>
21   mutate(label = paste0("$", formatC(index_level, format="f", digits=2))) |>
22   ungroup()
23
24 ggplot(plot_data, aes(date, index_level,
25                       color = factor(symbol, levels = last_vals$symbol))) +
26   geom_line(linewidth = 1) +
27   geom_text(data = label_data, aes(label = label),
28             hjust = -0.05, vjust = 0.5, show.legend = FALSE) +
29   scale_color_manual(values = color_values, limits = last_vals$symbol) +
30   scale_x_date(expand = expansion(mult = c(0, 0.1))) +
31   scale_y_continuous(labels = dollar_format(prefix = "$", accuracy = 0.1)) +
32   labs(title = "Rendimiento acumulado, 2 activos y una cartera ingenua.",
33        y = "Rendimiento acumulado",
34        x = NULL, color = "Series") +
35   theme_minimal() +
36   theme(legend.position = c(0,1), legend.justification = c(0,1),
37         legend.background = element_rect(fill = alpha("white", 0.6), color = NA))
```

Rendimiento acumulado, 2 activos y una cartera ingenua.



11 Riesgo-rendimiento: cartera ingenua y componentes.

$$R_{port} = 0.5\mu_{LRCX} + 0.5\mu_{GIS}.$$

```
1 weights_50_50 <- c(LRCX = 0.5, GIS = 0.5)
2
3 stats_lrcx_gis <- stats |>
4   filter(symbol %in% names(weights_50_50))
5
6 portfolio_stats_50_50 <- portfolio_stats(monthly_returns, weights_50_50, "50/50 LRCX-GIS")
7
8 final_table <- bind_rows(stats_lrcx_gis, portfolio_stats_50_50) |>
9   arrange(-SR)
10
11 print(final_table)
```

```
## # A tibble: 3 x 4
##   symbol          ER      SD      SR
##   <chr>        <dbl>  <dbl>  <dbl>
## 1 50/50 LRCX-GIS 0.0130  0.0560 0.231
## 2 LRCX          0.0257  0.111  0.231
## 3 GIS           0.000182 0.0477 0.00382
```

12 Frontera media-varianza: 2 activos.

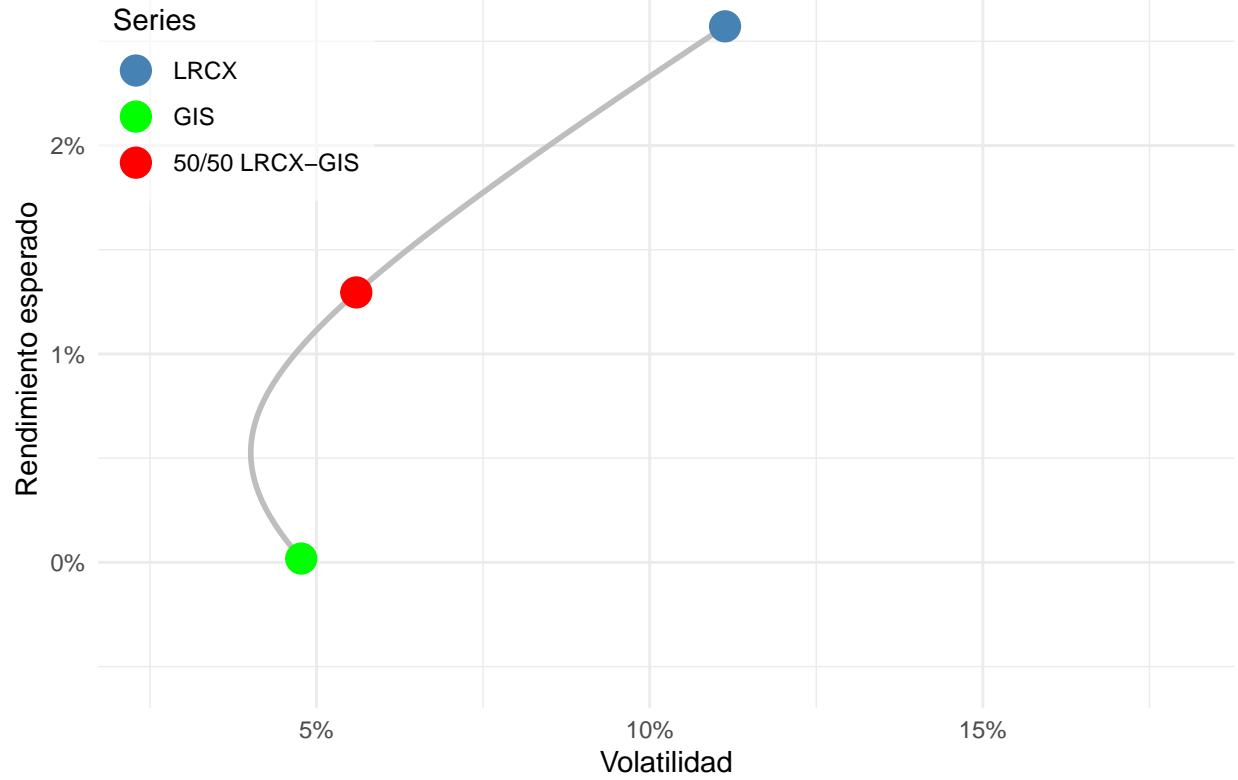
$$ER_{port} = w_A \mu_A + w_B \mu_B.$$

$$\sigma_{port}^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_{A,B}.$$

$$\sigma_{port}^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{A,B}.$$

```
1 returns_matrix_2 <- returns_wide |>
2   select(all_of(focus))
3
4 cov_mat_2 <- cov(returns_matrix_2, use = "pairwise.complete.obs")
5 mu_vec_2 <- colMeans(returns_matrix_2, na.rm = TRUE)
6
7 frontier_data <- tibble(weight_lrcx = seq(0, 1, length.out = 200)) |>
8   mutate(weight_gis = 1 - weight_lrcx,
9          ER = weight_lrcx * mu_vec_2["LRCX"] + weight_gis * mu_vec_2["GIS"],
10         variance = weight_lrcx^2 * var(returns_matrix_2$LRCX, na.rm = TRUE) +
11         weight_gis^2 * var(returns_matrix_2$GIS, na.rm = TRUE) +
12         2 * weight_lrcx * weight_gis * cov_mat_2["LRCX", "GIS"],
13         SD = sqrt(variance))
14
15 mean_var_points <- final_table |>
16   mutate(symbol = factor(symbol, levels = c("LRCX", "GIS", "50/50 LRCX-GIS"))) |>
17   arrange(symbol)
18
19 ggplot() +
20   geom_path(data = frontier_data, aes(SD, ER), color = "grey", linewidth = 1) +
21   geom_point(data = mean_var_points, aes(SD, ER, color = symbol), size = 5) +
22   scale_color_manual(values = color_values, limits = levels(mean_var_points$symbol)) +
23   scale_x_continuous(limits = c(0.025, 0.18), labels = percent_format(accuracy = 1)) +
24   scale_y_continuous(limits = c(min(mu_vec_all), max(mu_vec_all)),
25                     labels = percent_format(accuracy = 1)) +
26   labs(title = "Frontera media-varianza, 2 activos.",
27        x = "Volatilidad", y = "Rendimiento esperado", color = "Series") +
28   theme_minimal() +
29   theme(legend.position = c(0,1), legend.justification = c(0,1),
30         legend.background = element_rect(fill = alpha("white", 0.6), color = NA))
```

Frontera media-varianza, 2 activos.



13 Cartera de mínima varianza: método numérico de optimización.

$\min_{\mathbf{w}} \mathbf{w}^\top \Sigma \mathbf{w}$ sujeto a: $\mathbf{1}^\top \mathbf{w} = 1$.

```
1 minvar2_label <- "Min-Var 2-Asset (LRCX-GIS)"
2
3 assets_2 <- focus
4 qp_2 <- solve.QP(Dmat = 2 * cov_mat_2,
5                 dvec = rep(0, length(assets_2)),
6                 Amat = matrix(1, nrow = length(assets_2), ncol = 1),
7                 bvec = 1, meq = 1)
8
9 weights_minvar2_qp <- setNames(qp_2$solution, assets_2)
10
11 print(weights_minvar2_qp)
```

```
##           LRCX           GIS
## 0.1990058 0.8009942
```


14 Resumen de componentes y carteras.

```
1 weights_50_50 <- c(LRCX = 0.5, GIS = 0.5)
2
3 stats_lrcx_gis <- stats |>
4   filter(symbol %in% names(weights_50_50))
5 stats_minvar2_qp <- stats |>
6   filter(symbol %in% names(weights_minvar2_qp))
7
8 portfolio_stats_50_50 <- portfolio_stats(monthly_returns, weights_50_50, "50/50 LRCX-GIS")
9 portfolio_stats_minvar2_qp <-
10   portfolio_stats(monthly_returns, weights_minvar2_qp, "MV LRCX-GIS")
11
12 final_table <- bind_rows(stats_lrcx_gis, portfolio_stats_50_50,
13                           portfolio_stats_minvar2_qp) |>
14   arrange(-SR)
15
16 print(final_table)
```

```
## # A tibble: 4 x 4
##   symbol          ER      SD      SR
##   <chr>        <dbl>  <dbl>  <dbl>
## 1 50/50 LRCX-GIS 0.0130  0.0560 0.231
## 2 LRCX          0.0257  0.111  0.231
## 3 MV LRCX-GIS   0.00526 0.0401 0.131
## 4 GIS           0.000182 0.0477 0.00382
```

15 Cartera de mínima varianza: método analítico algebraico para 2 activos.

$$w_1^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}, \quad w_2^* = 1 - w_1^*.$$

```
1 Sigma_2 <- cov_mat_2[assets_2, assets_2]
2 var1 <- Sigma_2[1, 1]; var2 <- Sigma_2[2, 2]; cov12 <- Sigma_2[1, 2]
3 w1 <- (var2 - cov12) / (var1 + var2 - 2 * cov12)
4 weights_minvar2_closed <- setNames(c(w1, 1 - w1), assets_2)
5 print(weights_minvar2_closed)
```

```
##          LRCX          GIS
## 0.1990058 0.8009942
```

16 Cartera de mínima varianza: método analítico matricial para 2 activos.

$$w^* = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^\top \Sigma^{-1}\mathbf{1}}.$$

```
1 Sigma_2 <- cov_mat_2[assets_2, assets_2, drop = FALSE]
2 ones <- rep(1, length(assets_2))
3 Sigma_inv <- solve(Sigma_2)
4 num <- Sigma_inv %*% ones
5 den <- as.numeric(t(ones) %*% Sigma_inv %*% ones)
6 w_star <- as.numeric(num / den)
7 names(w_star) <- assets_2
8 print(w_star)
```

```
##          LRCX          GIS
## 0.1990058 0.8009942
```

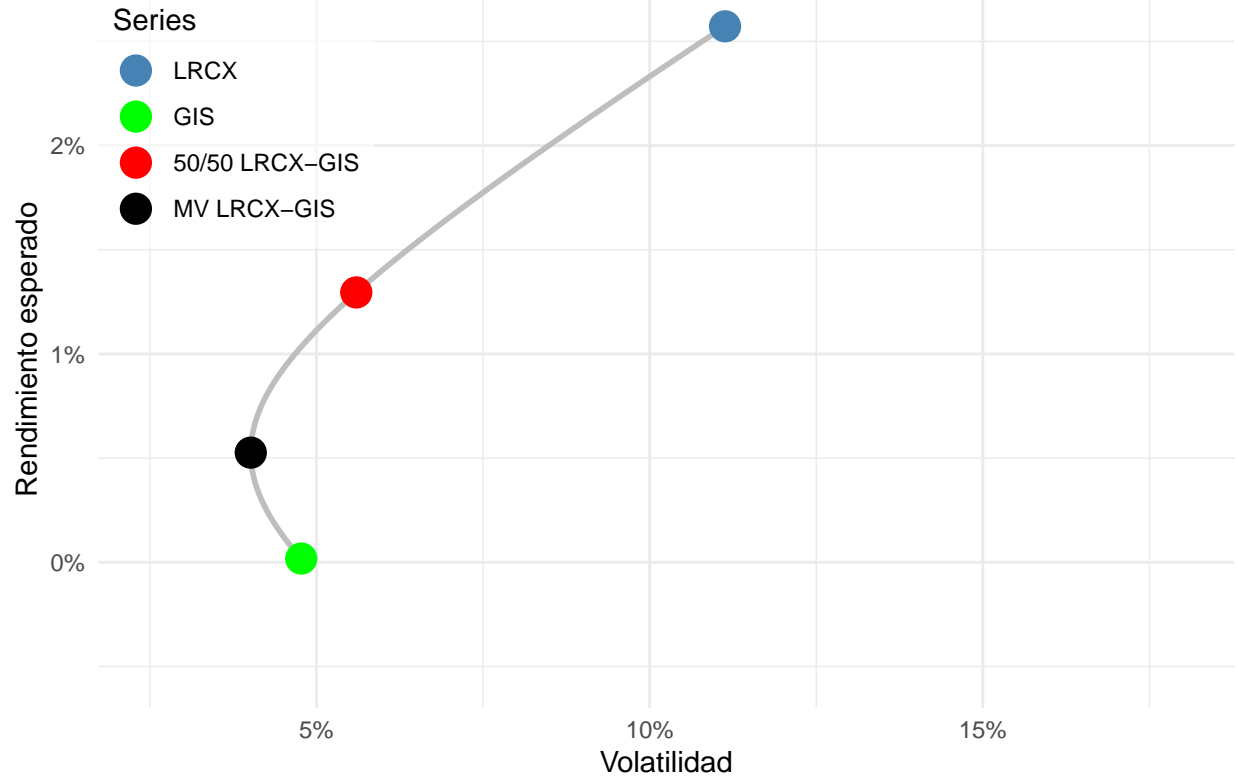
17 Frontera media-varianza: 2 activos, cartera MV.

$$ER_{port} = \sum_{i=1}^n w_i \mu_i.$$

$$\sigma_{port} = \sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}.$$

```
1 minvar2_label <- "MV LRCX-GIS"
2 minvar_point_2 <- tibble(symbol = minvar2_label,
3   ER = sum(weights_minvar2_qp * mu_vec_2),
4   SD = sqrt(as.numeric(t(weights_minvar2_qp) %*% cov_mat_2 %*% weights_minvar2_qp)))
5
6 mean_var_points_ext <- bind_rows(stats_lrcx_gis, portfolio_stats_50_50, minvar_point_2) |>
7   mutate(symbol = factor(symbol,
8     levels = c("LRCX", "GIS", "50/50 LRCX-GIS", minvar2_label))) |>
9   arrange(symbol)
10
11 color_values_ext <- c(color_values, setNames("black", minvar2_label))
12
13 ggplot() +
14   geom_path(data = frontier_data, aes(SD, ER), color = "grey", linewidth = 1) +
15   geom_point(data = mean_var_points_ext, aes(SD, ER, color = symbol), size = 5) +
16   scale_color_manual(values = color_values_ext, limits = levels(mean_var_points_ext$symbol)) +
17   scale_x_continuous(limits = c(0.025, 0.18), labels = percent_format(accuracy = 1)) +
18   scale_y_continuous(limits = c(min(mu_vec_all), max(mu_vec_all)),
19     labels = percent_format(accuracy = 1)) +
20   labs(title = "Frontera media-varianza, 2 activos y 2 carteras.",
21     x = "Volatilidad", y = "Rendimiento esperado", color = "Series") +
22   theme_minimal() +
23   theme(legend.position = c(0, 1), legend.justification = c(0, 1),
24     legend.background = element_rect(fill = alpha("white", 0.6), color = NA))
```

Frontera media-varianza, 2 activos y 2 carteras.



18 Cartera de mínima varianza: método numérico para 10 activos.

$\min_{\mathbf{w}} \mathbf{w}^\top \Sigma \mathbf{w}$ sujeto a: $\mathbf{1}^\top \mathbf{w} = 1, \mathbf{w} \geq \mathbf{0}$.

```
1 minvar10_ns_label <- "MV 10-Asset (No Shorts)"
2
3 weights_minvar10_ns <- solve.QP(2 * cov_mat_all, rep(0, length(mu_vec_all)),
4   cbind(rep(1, length(mu_vec_all)), diag(length(mu_vec_all))),
5   c(1, rep(0, length(mu_vec_all))), meq = 1)$solution
6
7 names(weights_minvar10_ns) <- names(mu_vec_all)
8
9 minvar_point_10_ns <- tibble(symbol = minvar10_ns_label,
10   ER = sum(weights_minvar10_ns * mu_vec_all),
11   SD = sqrt(as.numeric(t(weights_minvar10_ns) %*% cov_mat_all %*% weights_minvar10_ns)),
12   SR = ER/SD)
13
14 minvar_point_10_ns
```

```
## # A tibble: 1 x 4
```

```
##   symbol          ER      SD      SR
##   <chr>        <dbl>  <dbl> <dbl>
## 1 MV 10-Asset (No Shorts) 0.00717 0.0337 0.213
```

19 Cartera de mínima varianza: método numérico para 10 activos, pesos.

$\min_{\mathbf{w}} \mathbf{w}^\top \Sigma \mathbf{w}$ sujeto a: $\mathbf{1}^\top \mathbf{w} = 1, \mathbf{w} \geq 0$.

```
1 minvar_weights_table_ns <- tibble(symbol = names(mu_vec_all),
2   weight = weights_minvar10_ns,
3   weight_pct = scales::percent(weights_minvar10_ns, accuracy = 0.01)) |>
4   arrange(desc(weight)) |>
5   bind_rows(tibble(symbol = "Total",
6     weight = sum(weights_minvar10_ns),
7     weight_pct = scales::percent(sum(weights_minvar10_ns),
8                                   accuracy = 0.01)))
9
10 print(minvar_weights_table_ns)
```

```
## # A tibble: 11 x 3
##   symbol    weight weight_pct
##   <chr>      <dbl> <chr>
## 1 GIS      5.40e- 1 54.01%
## 2 SNPS     1.83e- 1 18.34%
## 3 LMT      1.12e- 1 11.17%
## 4 TRMB     4.62e- 2 4.62%
## 5 NEM      4.52e- 2 4.52%
## 6 SYF      3.05e- 2 3.05%
## 7 TTD      3.00e- 2 3.00%
## 8 CNC      1.30e- 2 1.30%
## 9 AMD      1.31e-17 0.00%
## 10 LRCX     0      0.00%
## 11 Total   1      e+ 0 100.00%
```

20 Cartera de mínima varianza: método analítico matricial para 10 activos, pesos.

$$w^* = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^\top \Sigma^{-1}\mathbf{1}}. \text{ Markowitz, ventas en corto permitidas.}$$

```

1 minvar10_short_label <- "MV (shorts)"
2
3 Sigma_all <- cov_mat_all
4 ones_all <- rep(1, length(mu_vec_all))
5 inv_Sigma_all <- solve(Sigma_all)
6 A_all <- as.numeric(t(ones_all) %*% inv_Sigma_all %*% ones_all)
7
8 weights_minvar10_short <- as.numeric(inv_Sigma_all %*% ones_all / A_all)
9 names(weights_minvar10_short) <- names(mu_vec_all)
10
11 minvar_point_10_short <-
12   tibble(symbol = minvar10_short_label,
13     ER = sum(weights_minvar10_short * mu_vec_all),
14     SD = sqrt(as.numeric(t(weights_minvar10_short) %*% Sigma_all %*% weights_minvar10_short)))
15
16 weights_gmv_table_short <- tibble(symbol = names(mu_vec_all),
17   weight = weights_minvar10_short,
18   weight_pct = scales::percent(weights_minvar10_short, accuracy = 0.01)) |>
19   arrange(desc(weight)) |>
20   bind_rows(tibble(symbol = "Total",
21     weight = sum(weights_minvar10_short),
22     weight_pct = scales::percent(sum(weights_minvar10_short),
23       accuracy = 0.01)))
24
25 print(weights_gmv_table_short)

```

```

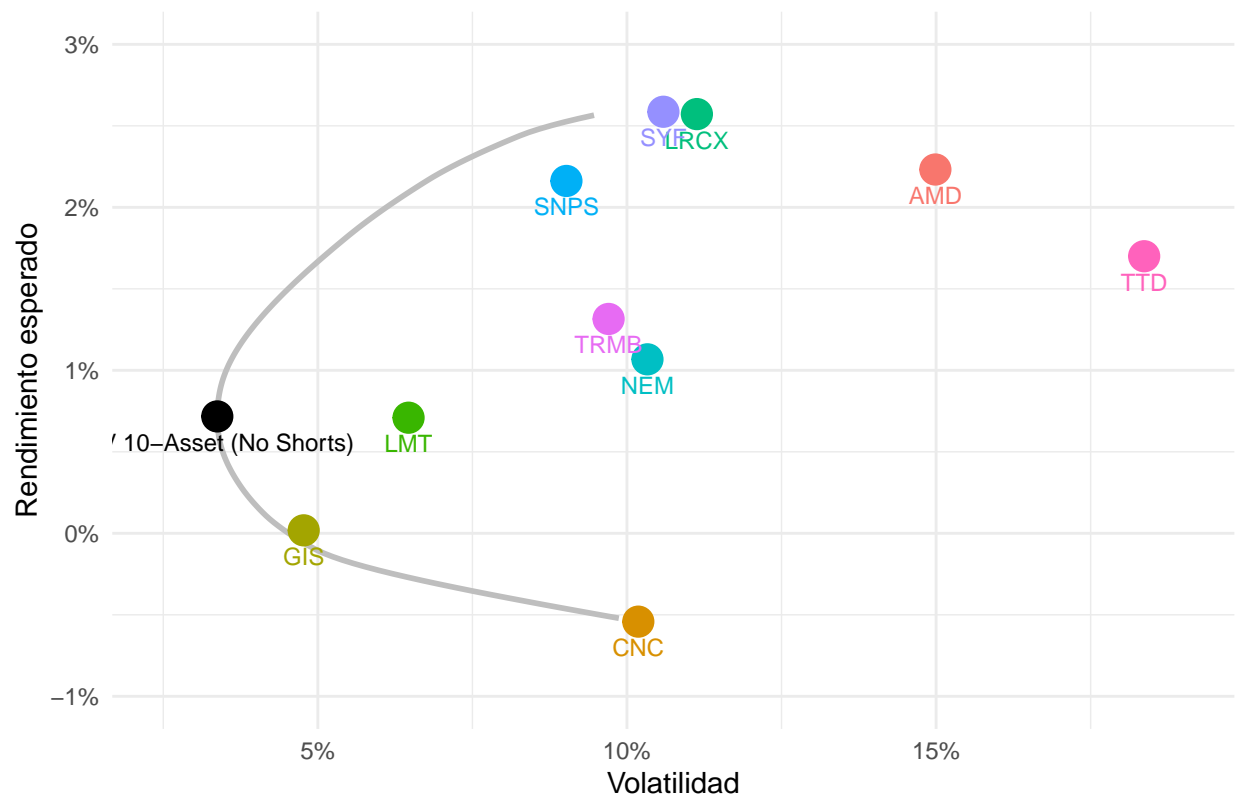
## # A tibble: 11 x 3
##   symbol    weight weight_pct
##   <chr>      <dbl> <chr>
## 1 GIS        0.539  53.93%
## 2 SNPS       0.240  24.05%
## 3 LMT        0.106  10.62%
## 4 TRMB       0.0490  4.90%
## 5 TTD        0.0469  4.69%
## 6 NEM        0.0419  4.19%
## 7 LRCX       0.0416  4.16%
## 8 SYF        0.0187  1.87%
## 9 CNC        0.00510 0.51%
## 10 AMD      -0.0893 -8.93%
## 11 Total     1      100.00%

```


21 Frontera media-varianza: 10 activos, sin ventas en corto.

```
1 target_returns <- seq(min(mu_vec_all), max(mu_vec_all), length.out = 150)
2 Amat_ns <- rbind(rep(1, n_assets), mu_vec_all, diag(n_assets))
3
4 frontier_points_ns <- map_dfr(target_returns, function(tr) {
5   bvec <- c(1, tr, rep(0, n_assets))
6   sol <- tryCatch(solve.QP(2 * cov_mat_all, rep(0, n_assets),
7     t(Amat_ns), bvec, meq = 2), error = function(e) NULL)
8   if (is.null(sol)) return(NULL)
9   tibble(ER = tr,
10     SD = sqrt(as.numeric(t(sol$solution) %*% cov_mat_all %*% sol$solution)))) |>
11   drop_na()
12
13 stats_all_ext_ns <- bind_rows(stats_all, minvar_point_10_ns)
14 color_values_ext_ns <- c(color_values_all, setNames("black", minvar10_ns_label))
15
16 ggplot() +
17   geom_path(data = frontier_points_ns, aes(SD, ER), color = "grey", linewidth = 1) +
18   geom_point(data = stats_all_ext_ns, aes(SD, ER, color = symbol), size = 5) +
19   geom_text(data = stats_all_ext_ns, aes(SD, ER, label = symbol, color = symbol),
20     vjust = 2, size = 3, show.legend = FALSE) +
21   scale_color_manual(values = color_values_ext_ns, limits = stats_all_ext_ns$symbol) +
22   scale_x_continuous(limits = c(0.025, 0.19), labels = percent_format(accuracy = 1)) +
23   scale_y_continuous(limits = c(-0.01, 0.03), labels = percent_format(accuracy = 1)) +
24   labs(title = "Frontera media-varianza, 10 activos sin ventas en corto.",
25     x = "Volatilidad", y = "Rendimiento esperado") +
26   guides(color = "none") +
27   theme_minimal()
```

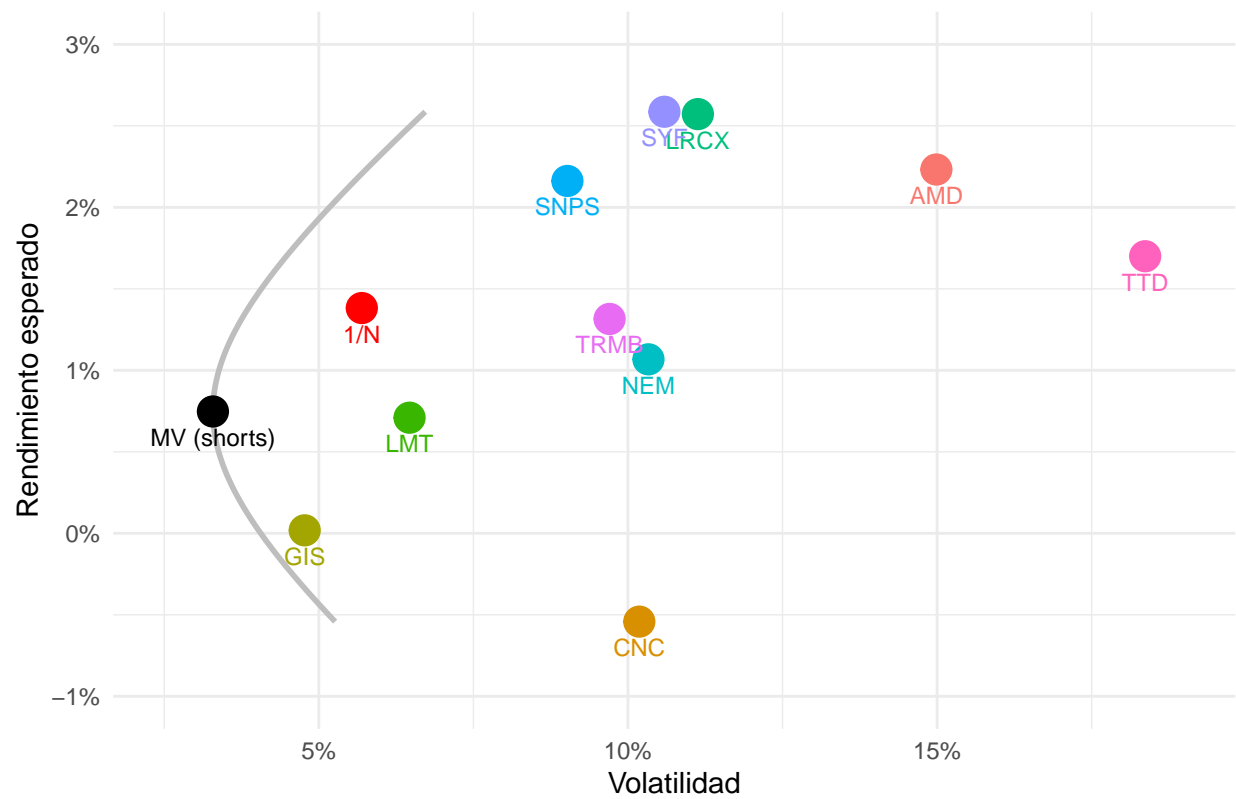
Frontera media–varianza, 10 activos sin ventas en corto.



22 Frontera media-varianza: 10 activos, con ventas en corto.

```
1 Amat_short <- rbind(rep(1, n_assets), mu_vec_all)
2
3 frontier_points_short <- map_dfr(target_returns, function(tr) {
4   bvec <- c(1, tr)
5   sol <- tryCatch(solve.QP(2 * cov_mat_all, rep(0, n_assets),
6     t(Amat_short), bvec, meq = 2), error = function(e) NULL)
7   if (is.null(sol)) return(NULL)
8   tibble(ER = tr,
9     SD = sqrt(as.numeric(t(sol$solution) %*% cov_mat_all %*% sol$solution)))) |>
10  drop_na()
11
12 # Punto 1/N (igual ponderación)
13 weights_1N <- setNames(rep(1 / n_assets, n_assets), names(mu_vec_all))
14 point_1N <- tibble(symbol = "1/N", ER = sum(weights_1N * mu_vec_all),
15   SD = sqrt(as.numeric(t(weights_1N) %*% cov_mat_all %*% weights_1N)))
16
17 stats_all_ext_short <- bind_rows(stats_all, minvar_point_10_short, point_1N)
18 color_values_ext_short <- c(color_values_all,
19   setNames("black", minvar10_short_label), "1/N" = "red")
20
21 ggplot() +
22   geom_path(data = frontier_points_short, aes(SD, ER), color = "grey", linewidth = 1) +
23   geom_point(data = stats_all_ext_short, aes(SD, ER, color = symbol), size = 5) +
24   geom_text(data = stats_all_ext_short, aes(SD, ER, label = symbol, color = symbol),
25     vjust = 2, size = 3, show.legend = FALSE) +
26   scale_color_manual(values = color_values_ext_short, limits = stats_all_ext_short$symbol) +
27   scale_x_continuous(limits = c(0.025, 0.19), labels = percent_format(accuracy = 1)) +
28   scale_y_continuous(limits = c(-0.01, 0.03), labels = percent_format(accuracy = 1)) +
29   labs(title = "Frontera media-varianza, 10 activos con ventas en corto y 2 carteras.",
30     x = "Volatilidad", y = "Rendimiento esperado") +
31   guides(color = "none") +
32   theme_minimal()
```

Frontera media–varianza, 10 activos con ventas en corto y 2 carteras.



23 Riesgo-rendimiento: 10 activos individuales y carteras.

```

1 weights_50_50 <- c(LRCX = 0.5, GIS = 0.5)
2 custom_weights <- c(LRCX = 0.1990055, GIS = 0.8009945)
3 portfolio_label <- "19.90% LRCX / 80.10% GIS"
4
5 weights_1N <- setNames(rep(1 / n_assets, n_assets), names(mu_vec_all))
6
7 weights_specs <- list("50/50 LRCX-GIS" = weights_50_50,
8                       "19.90% LRCX / 80.10% GIS" = custom_weights,
9                       "MV 10-Asset (No Shorts)" = weights_minvar10_ns,
10                      "MV 10-Asset (Shorts Allowed)" = weights_minvar10_short,
11                      "1/N" = weights_1N)
12
13 portfolio_stats_ext <- imap_dfr(weights_specs, ~ portfolio_stats(monthly_returns, .x, .y))
14
15 final_table <- bind_rows(asset_stats(monthly_returns), portfolio_stats_ext) |>
16   mutate(symbol = factor(symbol, levels = c(tickers, names(weights_specs)))) |>
17   arrange(symbol)
18
19 print(final_table)

```

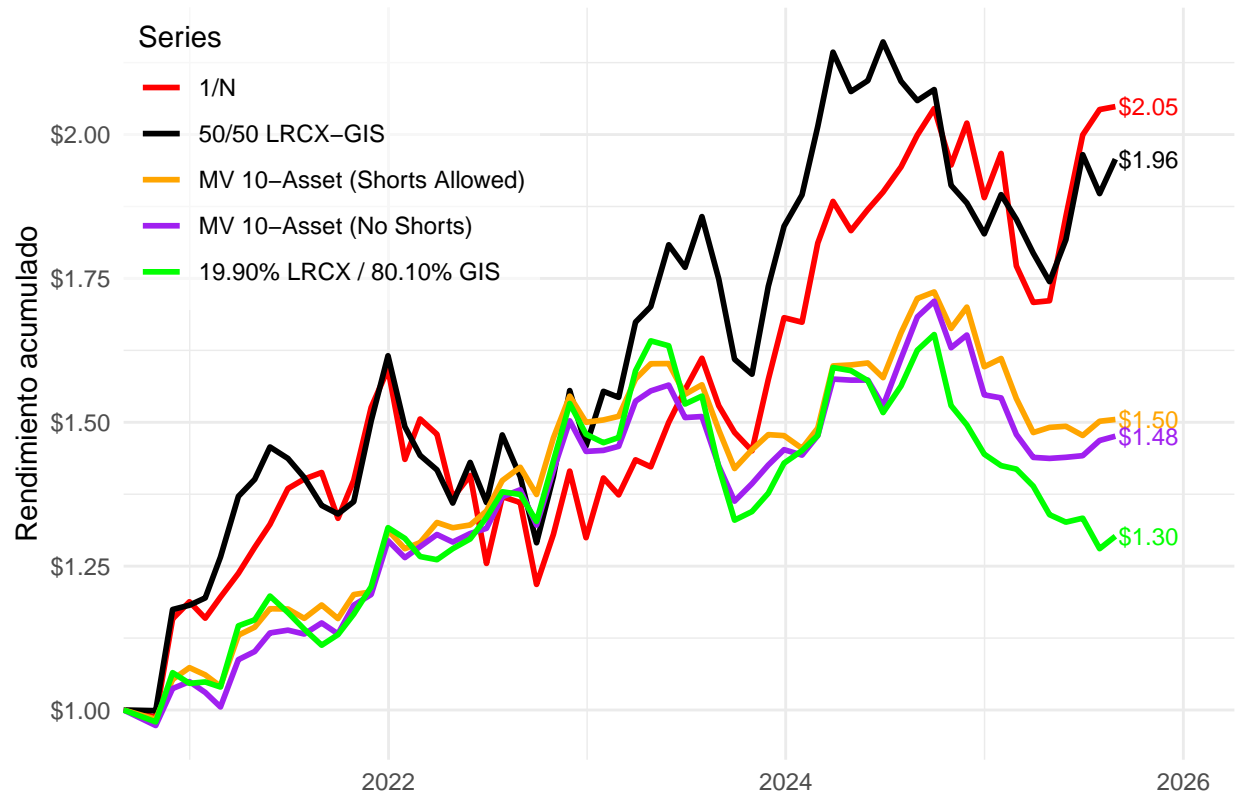
A tibble: 15 x 4

| ## | symbol | ER | SD | SR |
|----|---------------------------------|----------|--------|---------|
| ## | <fct> | <dbl> | <dbl> | <dbl> |
| ## | 1 AMD | 0.0223 | 0.150 | 0.149 |
| ## | 2 CNC | -0.00542 | 0.102 | -0.0532 |
| ## | 3 GIS | 0.000182 | 0.0477 | 0.00382 |
| ## | 4 LMT | 0.00709 | 0.0647 | 0.110 |
| ## | 5 LRCX | 0.0257 | 0.111 | 0.231 |
| ## | 6 NEM | 0.0107 | 0.103 | 0.103 |
| ## | 7 SNPS | 0.0216 | 0.0902 | 0.240 |
| ## | 8 SYF | 0.0259 | 0.106 | 0.244 |
| ## | 9 TRMB | 0.0132 | 0.0970 | 0.136 |
| ## | 10 TTD | 0.0170 | 0.184 | 0.0926 |
| ## | 11 50/50 LRCX-GIS | 0.0130 | 0.0560 | 0.231 |
| ## | 12 19.90% LRCX / 80.10% GIS | 0.00526 | 0.0401 | 0.131 |
| ## | 13 MV 10-Asset (No Shorts) | 0.00717 | 0.0337 | 0.213 |
| ## | 14 MV 10-Asset (Shorts Allowed) | 0.00747 | 0.0329 | 0.227 |
| ## | 15 1/N | 0.0138 | 0.0569 | 0.243 |

24 Rendimiento acumulado, todas las carteras.

```
1 portfolio_cum <- imap_dfr(weights_specs,
2                             ~ portfolio_series(monthly_returns, .x, .y, price_start))
3
4 # Último valor real de cada serie (usa la fecha máxima, no la última fila)
5 last_values_port <- portfolio_cum |>
6   group_by(symbol) |>
7   slice_max(order_by = date, n = 1, with_ties = FALSE) |>
8   select(symbol, final_value = index_level) |>
9   arrange(desc(final_value))
10
11 # Orden de series según rendimiento final (mayor a menor)
12 series_levels <- last_values_port$symbol
13
14 palette_base <- c("50/50 LRCX-GIS" = "black",
15                  "19.90% LRCX / 80.10% GIS" = "green",
16                  "MV 10-Asset (No Shorts)" = "purple",
17                  "MV 10-Asset (Shorts Allowed)" = "orange",
18                  "1/N" = "red")
19
20 portfolio_colors <- palette_base[series_levels]
21
22 portfolio_cum <- portfolio_cum |>
23   mutate(symbol = factor(symbol, levels = series_levels))
24
25 label_data_port <- portfolio_cum |>
26   group_by(symbol) |>
27   slice_max(order_by = date, n = 1, with_ties = FALSE) |>
28   mutate(label = paste0("$", formatC(index_level, format = "f", digits = 2))) |>
29   ungroup()
30
31 ggplot(portfolio_cum, aes(date, index_level, color = symbol)) +
32   geom_line(linewidth = 1) +
33   geom_text(data = label_data_port, aes(label = label),
34             hjust = -0.05, vjust = 0.5, size = 3.2, show.legend = FALSE) +
35   scale_color_manual(values = portfolio_colors, breaks = series_levels,
36                     limits = series_levels, drop = FALSE) +
37   scale_x_date(expand = expansion(mult = c(0, 0.12))) +
38   scale_y_continuous(labels = dollar_format(prefix = "$")) +
39   labs(title = "Rendimiento acumulado, todas las carteras.",
40        y = "Rendimiento acumulado", x = NULL, color = "Series") +
41   theme_minimal() +
42   theme(legend.position = c(0, 1), legend.justification = c(0, 1),
43         legend.background = element_rect(fill = alpha("white", 0.6), color = NA))
```

Rendimiento acumulado, todas las carteras.



25 Conclusión.

- La diversificación reduce riesgo y permite construir portafolios con mejor relación riesgo–rendimiento que los activos individuales.
- La frontera media–varianza es una herramienta poderosa para visualizar las decisiones de inversión y comprender el intercambio entre riesgo y retorno.