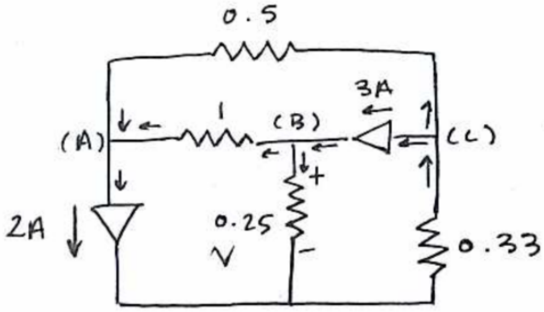
Problem Set 03: Solutions

BME253L - Fall 2025

2025-09-21

> Warning
>
> These solutions have been adapted from previous years, but I haven't had the chance to 100% guarentee their accuracy. If anything seems off, please don't hesitate to ask questions on Ed!

Problem 1



$$(A)\quad 2A = \frac{V_C - V_A}{0.5} + \frac{V_B - V_A}{1} = -3V_A + V_B + 2V_C = 2A$$

$$(B)\quad \frac{V_B - V_A}{1} + \frac{V_B}{0.25} = 3A = 5V_B - V_A$$

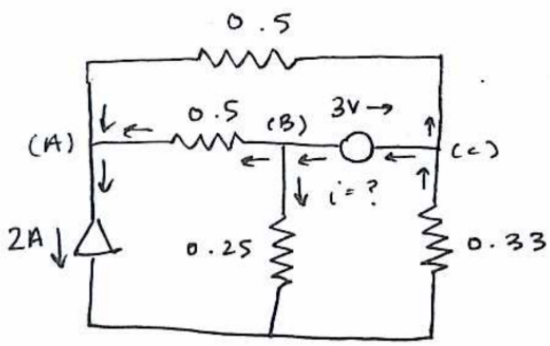
$$(C)\quad 3A + 2\left(\frac{V_C - V_A}{0.5}\right) = 3\left(\frac{-V_C}{0.33}\right)$$

$$\Rightarrow 3A = 2V_A - 5V_C$$

$$V = V_B$$

$$V_B = \frac{\begin{vmatrix} -3 & 2 & 2 \\ -1 & 3 & 0 \\ 2 & 3 & -5 \end{vmatrix}}{\begin{vmatrix} -3 & 1 & 2 \\ -1 & 5 & 0 \\ 2 & 0 & -5 \end{vmatrix}} = \frac{17}{50} = \boxed{0.34\,V}$$

Problem 2



$V_c = V_B + 3$

(A) $2A = \frac{V_c - V_A}{0.5} + \frac{V_B - V_A}{0.5} \Rightarrow 2V_c + 2V_B - 4V_A = 2A$

(B) $\frac{V_B - V_A}{0.5} + i + \frac{V_B}{0.25} = 0 \Rightarrow 6V_B - 2V_A + i = 0$

(C) $\frac{V_c - V_A}{0.5} - i = \frac{-V_c}{0.33} \Longrightarrow 2V_A - 5V_c + i = 0$

3 unknowns:

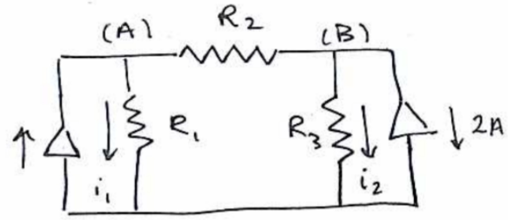
① $-4V_A + 4V_B + 0i = -4$

② $-2V_a + 6V_B + i = 0$

③ $2V_A - 5V_B + i = 15$

$$i = \frac{\begin{vmatrix} -4 & 4 & -4 \\ -2 & 6 & 0 \\ 2 & -5 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 4 & 0 \\ -2 & 6 & 1 \\ 2 & -5 & 1 \end{vmatrix}} = \frac{-232}{-28} = \boxed{8.28\ A}$$

Problem 3



$R_1 = 3\,\Omega$

$R_2 = 1\,\Omega$

$R_3 = 6\,\Omega$

(A) $1A = \frac{V_A}{3} + \frac{V_A - V_B}{1} \Rightarrow 3 = 4V_A - 3V_B \Rightarrow V_B = \frac{4}{3}V_A - 1$

(B) $\frac{V_A - V_B}{1} = \frac{V_B}{6} + 2 \Rightarrow 6V_A - 7V_B = 12$

Solve for V_A: $6V_A - 7\left(\frac{4}{3}V_A - 1\right) = 12 \Rightarrow \left(\frac{18}{3} - \frac{28}{3}\right)V_A = 5$

$V_A = -\frac{3}{2}\,V$

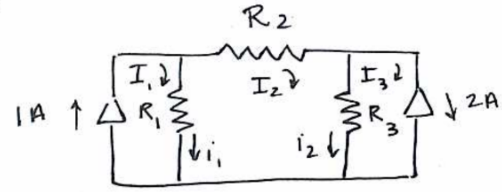
Solve for V_B: $V_B = \frac{4}{3}\left(-\frac{3}{2}\right) - 1$

$V_B = -3V$

$i_1 = \frac{V_A}{R_1} = \frac{-\frac{3}{2}V}{3\,\Omega} = \boxed{-0.5A}$

$i_2 = \frac{V_B}{R_3} = \frac{-3V}{6\,\Omega} = \boxed{-0.5A}$

Problem 4



$R_1 = 3\,\Omega$

$R_2 = 1\,\Omega$

$R_3 = 6\,\Omega$

$i_1 = I_1 - I_2$

$i_2 = I_2 - I_3$

$I_1 = 1A$

$I_3 = 2A$

$$0 = -I_1R_1 + (R_1+R_2+R_3)I_2 - R_3I_3$$

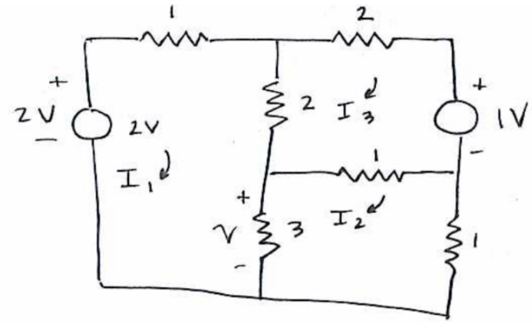
$$\hookrightarrow I_2 = \frac{(1A)(3\Omega) + (6\Omega)(2A)}{3+1+6} = \frac{15}{10}$$

$I_2 = 1.5A$

$$i_1 = 1A - 1.5A = \boxed{-0.5A}$$

$$i_2 = 1.5A - 2A = \boxed{-0.5A}$$

Problem 5



$V = (I_1 - I_3) \cdot 3\Omega$

① $2V = 6i_1 - 3i_2 - 2i_3$

② $0 = -3i_1 + 5i_2 - i_3$

③ $-1V = -2i_1 - i_3 + 5i_3$

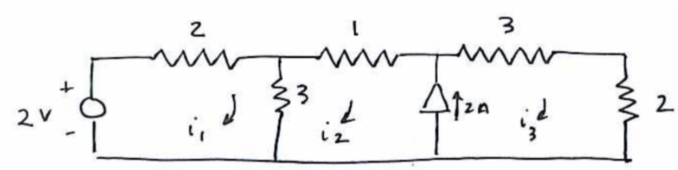
$$i_1 = \frac{\begin{vmatrix} 2 & -3 & -2 \\ 0 & 5 & -1 \\ -1 & -1 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -3 & -2 \\ -3 & 5 & -1 \\ -2 & -1 & 5 \end{vmatrix}} = \frac{35}{67} A$$

$$i_2 = \frac{\begin{vmatrix} 6 & 2 & -2 \\ -3 & 0 & -1 \\ -2 & -1 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -3 & -2 \\ -3 & 5 & -1 \\ -2 & -1 & 5 \end{vmatrix}} = \frac{22}{67} A$$

$$V = \left(\frac{35}{67} - \frac{22}{67}\right) \cdot 3\Omega$$

$$V = \frac{39}{67} V = \boxed{0.58 V}$$

Problem 6



① $5i_1 - 3i_2 - 0i_3 = 2$

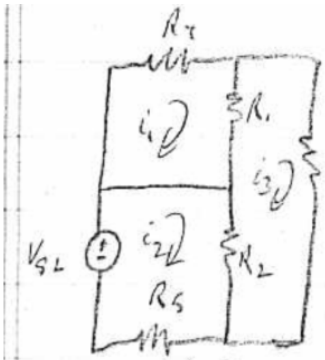
② $0i_1 - i_2 + i_3 = 2A$ (current source)

③ $-3i_1 + 4i_2 + 5i_3 = 0$

$$-3\left[\frac{2V + 3i_2}{5}\right] + 4I_2 + 5[2A + i_2] = 0$$

$$\boxed{\begin{aligned} I_2 &= -1.22A \\ I_3 &= 0.78A \\ I_1 &= -0.332A \end{aligned}}$$

Problem 7



HW #4 Solutions

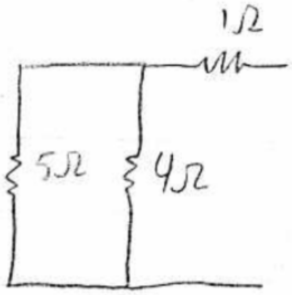
$$i_1(R_4+R_1)+i_3(-R_1)=0$$

$$i_2(R_2+R_5)+i_3(-R_2)=V_{S2}$$

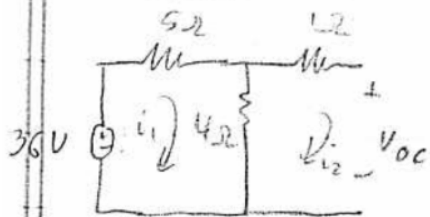
$$i_1(-R_1)+i_2(-R_2)+i_3(R_1+R_2+R_3)=0$$

$$i_3=\frac{\begin{vmatrix} R_1+R_4 & 0 & 0 \\ 0 & R_2+R_5 & V_{S2} \\ -R_1 & -R_2 & 0 \end{vmatrix}}{\begin{vmatrix} R_1+R_4 & 0 & -R_1 \\ 0 & R_2+R_5 & -R_2 \\ -R_1 & -R_2 & R_1+R_2+R_3 \end{vmatrix}}=\frac{\begin{vmatrix} 8 & 0 & 0 \\ 0 & 6 & 450 \\ -7 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 8 & 0 & -7 \\ 0 & 6 & -5 \\ -7 & -5 & 22 \end{vmatrix}}=\boxed{32A}$$

Problem 8



$$R_T = \left(\frac{1}{5} + \frac{1}{4}\right)^{-1} + 1 = \frac{20}{9} + 1 = \frac{29}{9} = \boxed{3.22\,\Omega}$$



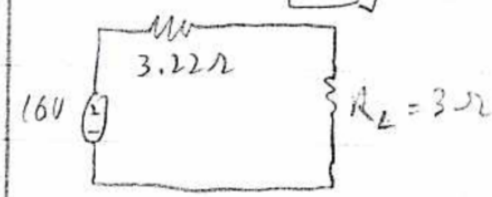
$$i_1(9) + i_2(-4) = 36V$$

$$i_1(-4) + i_2(5) = -v_{oc}$$

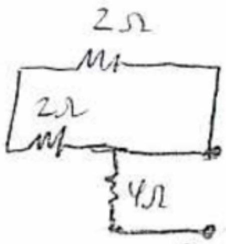
$i_2 = 0$ (open circuit)

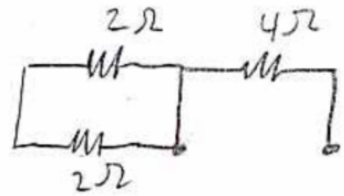
$i_1 = 4A$

$v_{oc} = 4i_1 = \boxed{16V}$

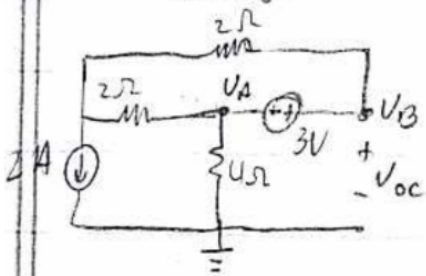


Problem 9





$\boxed{R_T = 4\,\Omega}$



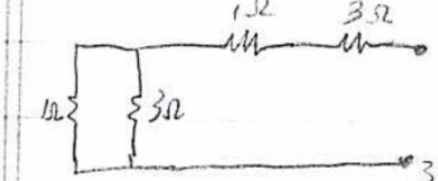
$V_A = (-2\,A)(4\,\Omega) = -8V$

$V_B = V_A + 3V = -5V$

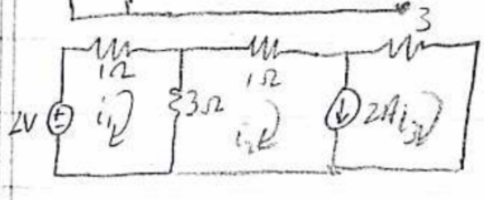
$\boxed{V_{oc} = -5V}$

$V = -5V \cdot \frac{3}{7} = \boxed{-2.14\,V}$

Problem 10



$$R_T = \frac{3}{4} + 1 + 3 = \frac{19}{4} = \boxed{4.75\,\Omega}$$



$$i_1(1+3) + i_2(-3) = 2$$

$$i_1(-3) + i_2(1+3) + i_3(3) = 0$$

$$i_2 - i_3 = 2$$

$$i_3 = i_2 - 2$$

$$i_1(-3) + i_2(4) + (i_2 - 2)(3) = 0$$

$$i_1(-3) + 7i_2 = 6$$

$$i_2 = \frac{6 + 3i_1}{7}$$

$$i_1(4) + \frac{1}{7}(6 + 3i_1)(-3) = 2$$

$$i_1(28) + (-18 - 9i_1) = 14$$

$$19i_1 = 32$$

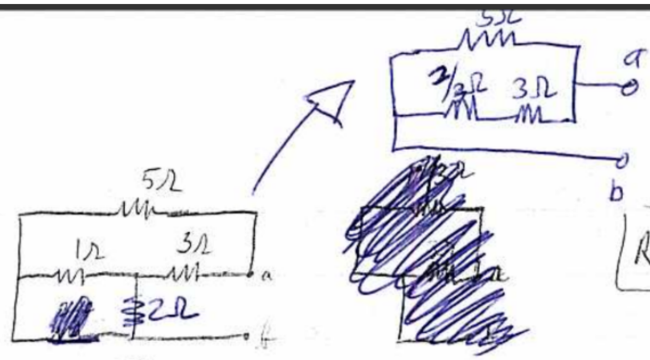
$$i_1 = \frac{32}{19}$$

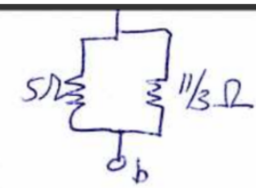
$$i_2 = 1.5789$$

$$i_3 = -0.42$$

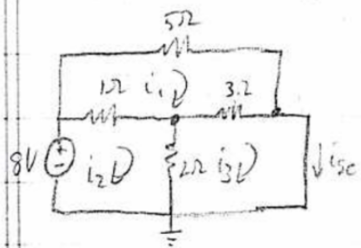
$$\boxed{i_N = -0.42\,A}$$

Problem 11





$\boxed{R_T = 2.12\,\Omega}$



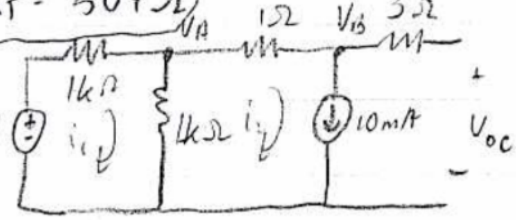
$$i_1(9) + i_2(-1) + i_3(-3) = 0$$
$$i_1(-1) + i_2(3) + i_3(-2) = 8$$
$$i_1(-3) + i_2(-2) + i_3(5) = 0$$

$$\left[\begin{array}{ccc|c} 9 & -1 & -3 & 0 \\ -1 & 3 & -2 & 8 \\ -3 & -2 & 5 & 0 \end{array}\right] \Rightarrow \begin{array}{l} i_1 = 1.6 \\ i_2 = 5.236 \\ i_3 = 3.05 \end{array}$$

$$\boxed{i_{sc} = i_N = 3.05\,A}$$

Problem 12

$\boxed{R_T = 504\,\Omega}$



$i_2 = 0.010\,A$

$i_1(2000) - i_2(1000) = 10$

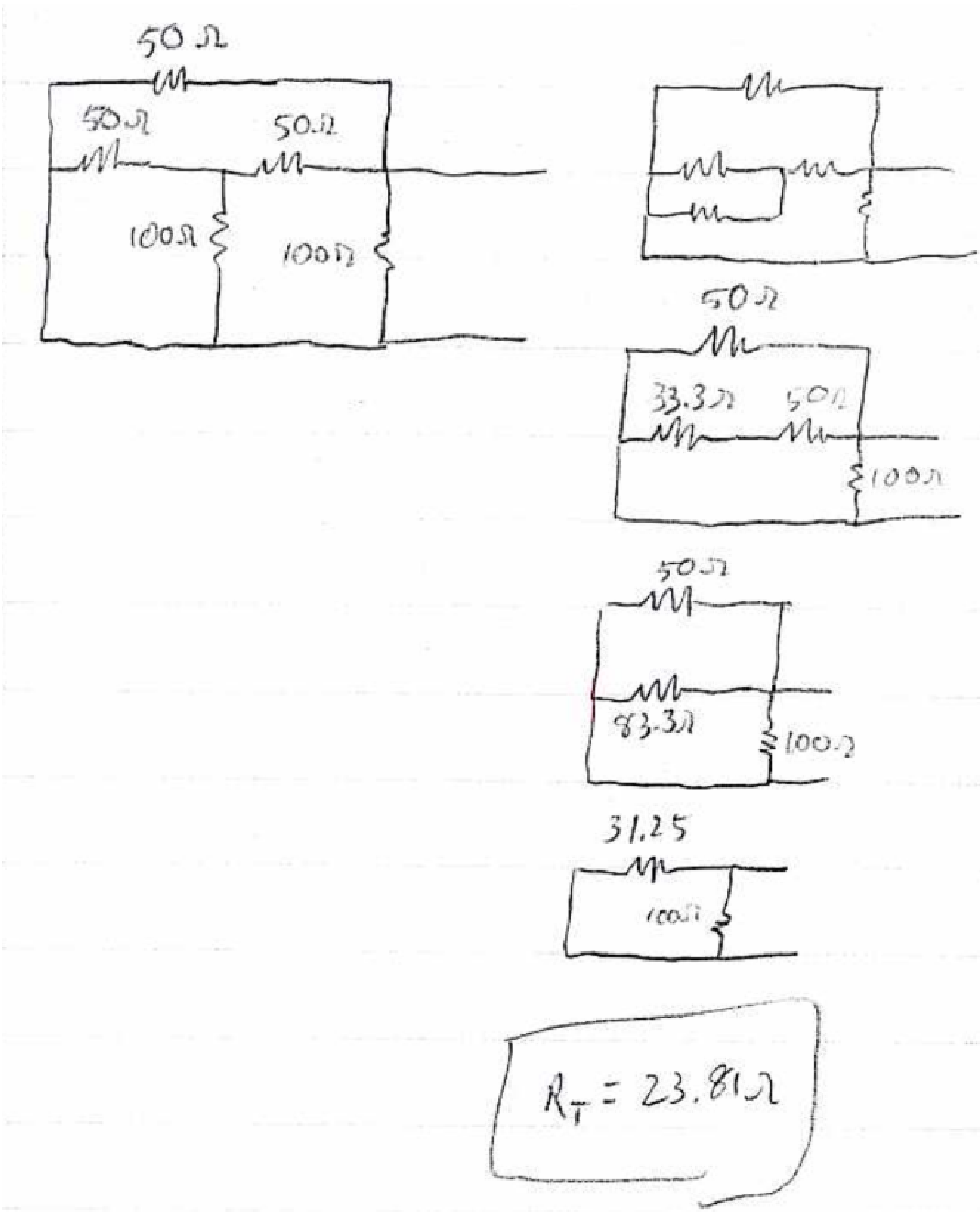
$i_1 = 0.010\,A$

$V_A = 10 - 0.01 \cdot 1000 = 0V$

$V_B = V_A - 0.010 \cdot 1 = -0.01V$

$\boxed{V_{oc} = V_B = -0.01V}$

Problem 13


50 Ω
50 Ω
50 Ω
100 Ω
100 Ω
50 Ω
33.3 Ω
50 Ω
100 Ω
50 Ω
83.3 Ω
100 Ω
31.25
100 Ω
$R_T = 23.81\,\Omega$

Problem 14

For Maximum Power Transfer, $R_L = R_T$

$\boxed{R_L = 8\Omega}$

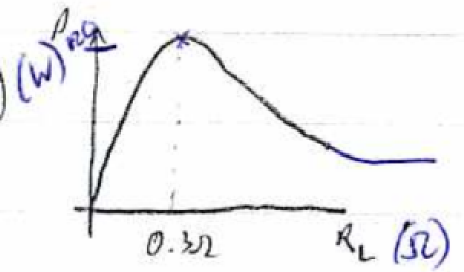
$V_L = 6V$

$P_L = \frac{6^2}{8} = \boxed{4.5W}$

$i = \frac{12}{16} = 0.75A$

$\% \text{ eff} = \frac{P_L}{P_S} \times 100\% = \frac{4.5W}{0.75A \cdot 12V} = \boxed{50\%}$

Problem 15

a) 

max at $0.3\Omega = R_L$

b) $P_L = \left(V_S \cdot \frac{R_L}{R_S + R_L}\right)^2 R_L = V_S^2 \cdot \frac{R_L}{(R_S + R_L)^2} = V_S^2 R_L (R_S + R_L)^{-2}$

$\frac{dP_L}{dR_L} = V_S^2 (R_S + R_L)^{-2} + V_S^2 R_L (-2)(R_S + R_L)^{-3} = 0$

$V_S^2 (R_S + R_L) + V_S^2 R_L (-2) = 0$

$R_S + R_L - 2R_L = 0$

$\boxed{R_L = R_S}$