

# Problem Set 03: Solutions

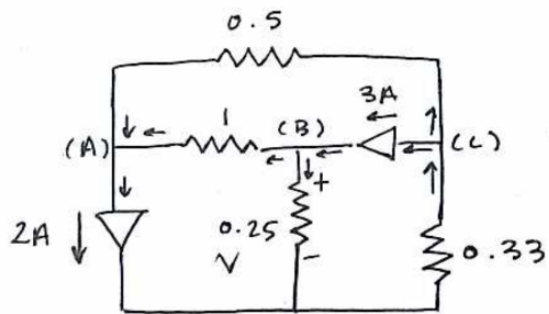
BME253L - Fall 2025

2025-09-21

## Warning

These solutions have been adapted from previous years, but I haven't had the chance to 100% guarantee their accuracy. If anything seems off, please don't hesitate to ask questions on Ed!

# Problem 1



$$(A) \quad 2A = \frac{V_C - V_A}{0.5} + \frac{V_B - V_A}{1} = -3V_A + V_B + 2V_C = 2A$$

$$(B) \quad \frac{V_B - V_A}{1} + \frac{V_B}{0.25} = 3A = 5V_B - V_A$$

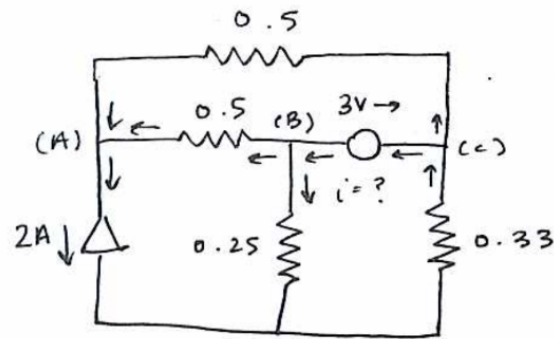
$$(C) \quad 3A + 2\left(\frac{V_C - V_A}{0.5}\right) = 3\left(\frac{-V_C}{0.33}\right)$$

$$\Rightarrow 3A = 2V_A - 5V_C$$

$$V = V_B$$

$$V_B = \frac{\begin{vmatrix} -3 & 2 & 2 \\ -1 & 3 & 0 \\ 2 & 3 & -5 \end{vmatrix}}{\begin{vmatrix} -3 & 1 & 2 \\ -1 & 5 & 0 \\ 2 & 0 & -5 \end{vmatrix}} = \frac{17}{50} = \boxed{0.34V}$$

## Problem 2



$$V_C = V_B + 3$$

$$(A) \quad 2A = \frac{V_C - V_A}{0.5} + \frac{V_B - V_A}{0.5} \Rightarrow 2V_C + 2V_B - 4V_A = 2A$$

$$(B) \quad \frac{V_B - V_A}{0.5} + i + \frac{V_B}{0.25} = 0 \Rightarrow 6V_B - 2V_A + i = 0$$

$$(C) \quad \frac{V_C - V_A}{0.5} - i = -\frac{V_C}{0.33} \Rightarrow 2V_A - 5V_C + i = 0$$

3 unknowns:

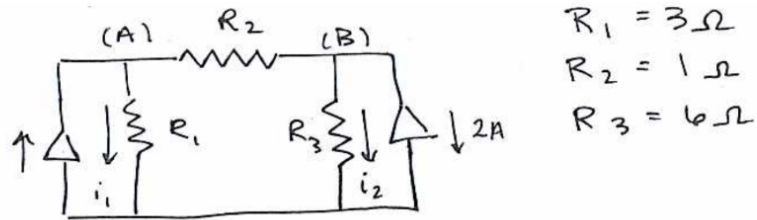
$$① \quad -4V_A + 4V_B + 0i = -4$$

$$② \quad -2V_A + 6V_B + i = 0$$

$$③ \quad 2V_A - 5V_B + i = 15$$

$$i = \frac{\begin{vmatrix} -4 & 4 & -4 \\ -2 & 6 & 0 \\ 2 & -5 & 15 \end{vmatrix}}{\begin{vmatrix} -4 & 4 & 0 \\ -2 & 6 & 1 \\ 2 & -5 & 1 \end{vmatrix}} = \frac{-232}{-28} = \boxed{8.28A}$$

### Problem 3



$$(A) \quad 1A = \frac{V_A}{3} + \frac{V_A - V_B}{1} \Rightarrow 3 = 4V_A - 3V_B \Rightarrow V_B = \frac{4}{3}V_A - 1$$

$$(B) \quad \frac{V_A - V_B}{1} = \frac{V_B}{6} + 2 \Rightarrow 6V_A - 7V_B = 12$$

$$\text{Solve for } V_A: \quad 6V_A - 7\left(\frac{4}{3}V_A - 1\right) = 12 \Rightarrow \left(\frac{18}{3} - \frac{28}{3}\right)V_A = 5$$

$$V_A = -\frac{3}{2}V$$

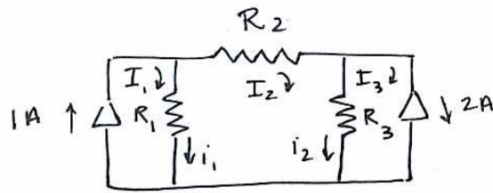
$$\text{Solve for } V_B: \quad V_B = \frac{4}{3}\left(-\frac{3}{2}\right) - 1$$

$$V_B = -3V$$

$$i_1 = \frac{V_A}{R_1} = \frac{-\frac{3}{2}V}{3\Omega} = \boxed{-0.5A}$$

$$i_2 = \frac{V_B}{R_3} = \frac{-3V}{6\Omega} = \boxed{-0.5A}$$

#### Problem 4



$$R_1 = 3\Omega$$

$$R_2 = 1\Omega$$

$$R_3 = 6\Omega$$

$$i_1 = I_1 - I_2$$

$$i_2 = I_2 - I_3$$

$$I_1 = 1A$$

$$I_3 = 2A$$

$$0 = -I_1 R_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3$$

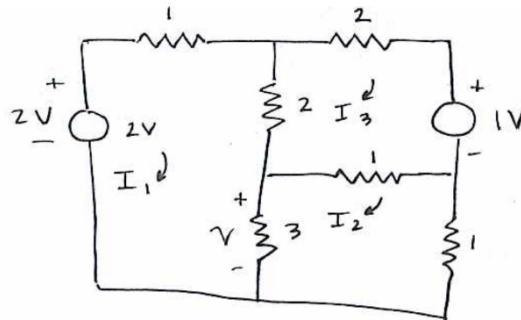
$$\Rightarrow I_2 = \frac{(1A)(3\Omega) + (6\Omega)(2A)}{3 + 1 + 6} = \frac{15}{10}$$

$$I_2 = 1.5A$$

$$i_1 = 1A - 1.5A = \boxed{-0.5A}$$

$$i_2 = 1.5A - 2A = \boxed{-0.5A}$$

### Problem 5



$$V = (I_1 - I_3) \cdot 3\Omega$$

$$\textcircled{1} 2V = 6i_1 - 3i_2 - 2i_3$$

$$\textcircled{2} 0 = -3i_1 + 5i_2 - i_3$$

$$\textcircled{3} -1V = -2i_1 - i_3 + 5i_2$$

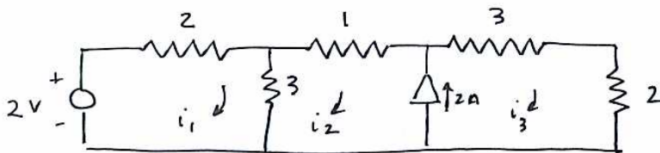
$$V = \left( \frac{35}{67} - \frac{22}{67} \right) \cdot 3\Omega$$

$$V = \frac{39}{67} V = \boxed{0.58 V}$$

$$i_1 = \frac{\begin{vmatrix} 2 & -3 & -2 \\ 0 & 5 & -1 \\ -1 & -1 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -3 & -2 \\ -3 & 5 & -1 \\ -2 & -1 & 5 \end{vmatrix}} = \frac{35}{67} A$$

$$i_2 = \frac{\begin{vmatrix} 6 & 2 & -2 \\ -3 & 0 & -1 \\ -2 & -1 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -3 & -2 \\ -3 & 5 & -1 \\ -2 & -1 & 5 \end{vmatrix}} = \frac{22}{67} A$$

### Problem 6



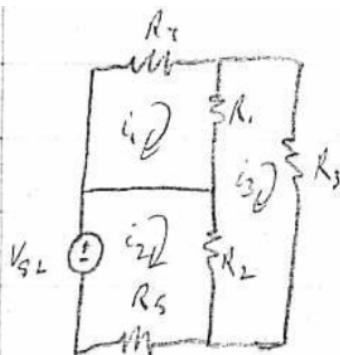
$$\textcircled{1} 5i_1 - 3i_2 - 0i_3 = 2$$

$$\textcircled{2} 0i_1 - i_2 + i_3 = 2A \text{ (current source)} \quad \left\{ \begin{array}{l} -3 \left[ \frac{2V + 3i_2}{5} \right] + 4i_2 + 5[2A + i_2] = 0 \end{array} \right.$$

$$\textcircled{3} -3i_1 + 4i_2 + 5i_3 = 0$$

$$\boxed{\begin{array}{l} I_2 = -1.22 A \\ I_3 = 0.78 A \\ I_1 = -0.332 A \end{array}}$$

# Problem 7



## HW #4 Solutions

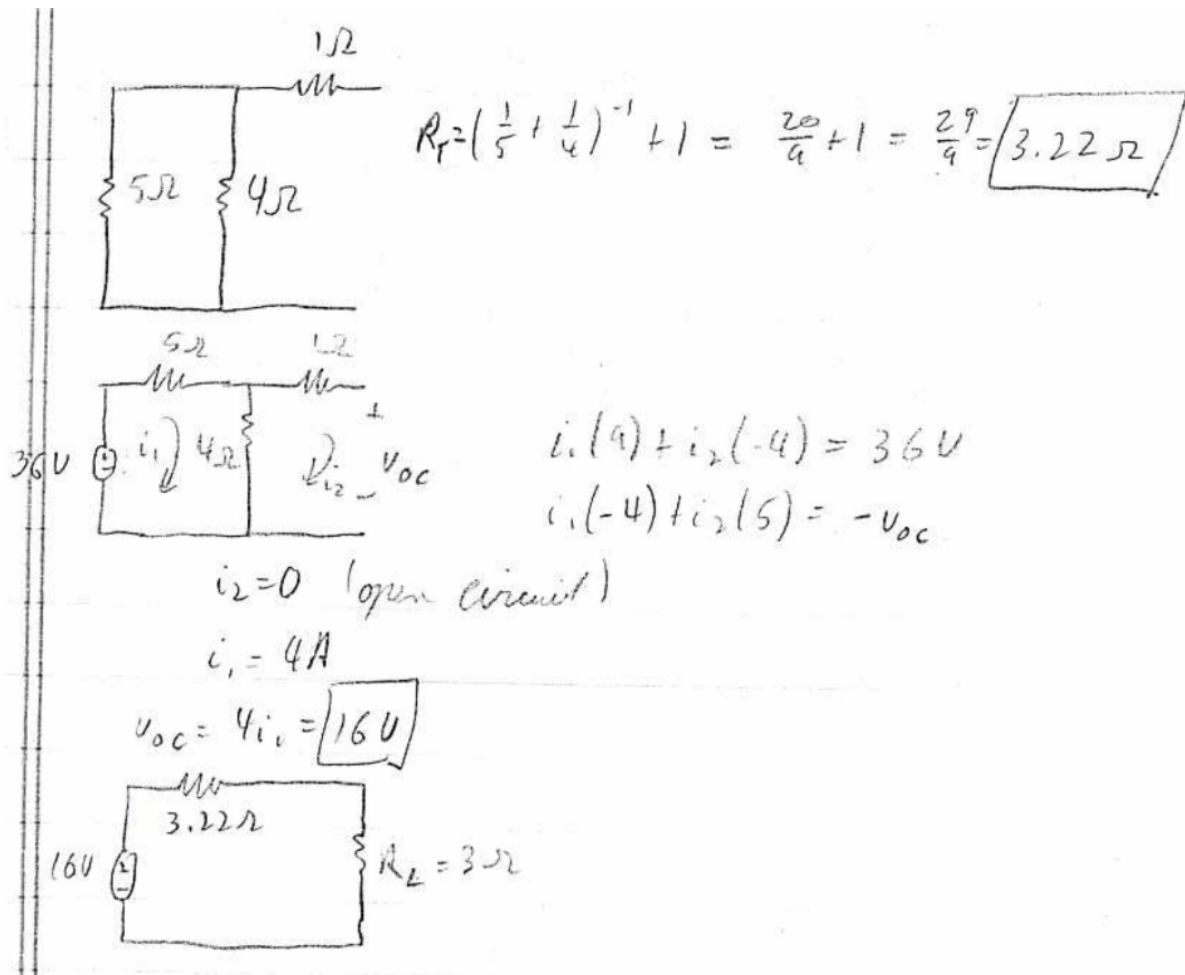
$$i_1 (R_4 + R_1) + i_3 (-R_1) = 0$$

$$i_2 (R_2 + R_5) + i_3 (-R_2) = V_{s2}$$

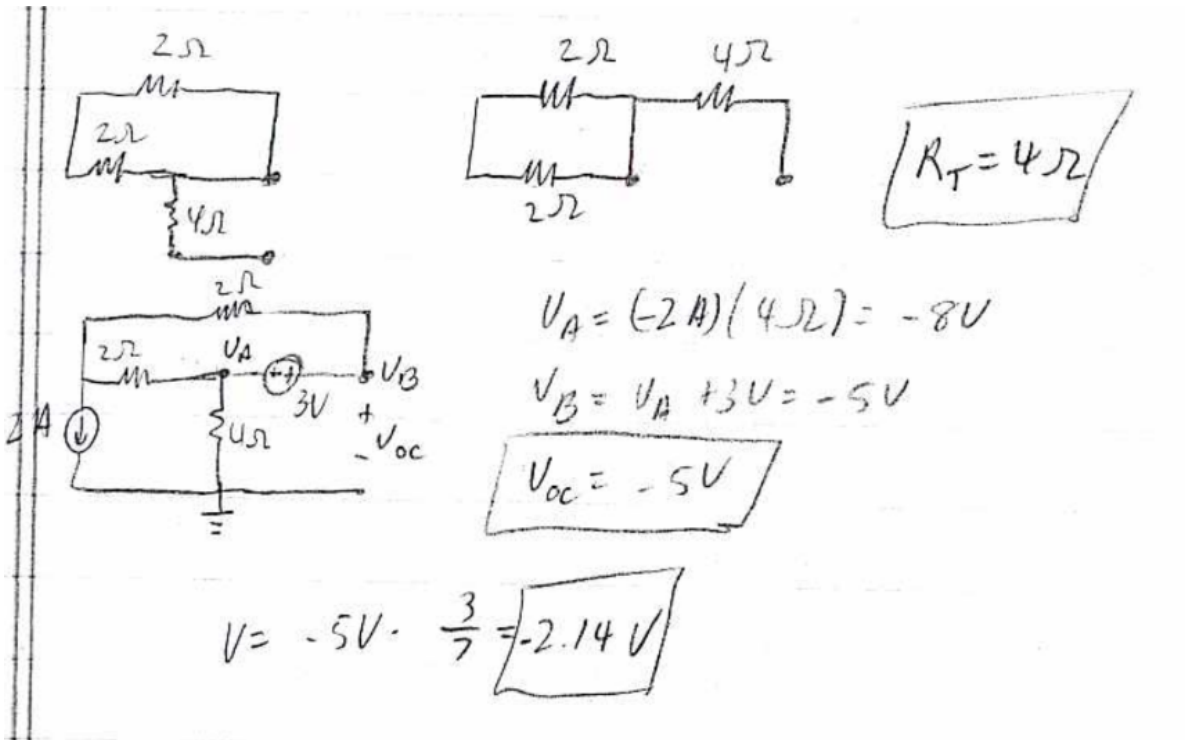
$$i_1 (-R_1) + i_2 (-R_2) + i_3 (R_1 + R_2 + R_3) = 0$$

$$i_3 = \frac{\begin{vmatrix} R_1 + R_4 & 0 & 0 \\ 0 & R_2 + R_5 & V_{s2} \\ -R_1 & -R_2 & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + R_4 & 0 & -R_1 \\ 0 & R_2 + R_5 & -R_2 \\ -R_1 & -R_2 & R_1 + R_2 + R_3 \end{vmatrix}} = \frac{\begin{vmatrix} 8 & 0 & 0 \\ 0 & 6 & 450 \\ -7 & -5 & 0 \end{vmatrix}}{\begin{vmatrix} 8 & 0 & -7 \\ 0 & 6 & -5 \\ -7 & -5 & 22 \end{vmatrix}} = \boxed{32 \text{ A}}$$

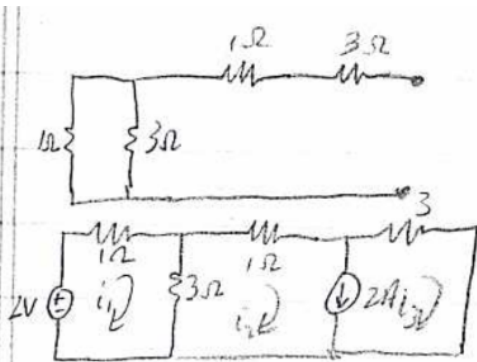
# Problem 8



# Problem 9



# Problem 10



$$R_T = \frac{3}{4} + 1 + 3 = \frac{19}{4} = \boxed{4.75 \Omega}$$

$$i_1(1+3) + i_2(-3) = 2$$

$$i_1(-3) + i_2(1+3) + i_3(3) = 0$$

$$i_2 - i_3 = 2$$

$$i_3 = i_2 - 2$$

$$i_1(-3) + i_2(4) + (i_2 - 2)(2) = 0$$

$$i_1(-3) + 7i_2 = 6$$

$$i_2 = \frac{6 + 3i_1}{7}$$

$$i_1(4) + \frac{1}{7}(6 + 3i_1)(-3) = 2$$

$$i_1(28) + (-18 - 9i_1) = 14$$

$$19i_1 = 32$$

$$i_1 = \frac{32}{19}$$

$$i_2 = 1.5789$$

$$i_3 = -0.42$$

$$\boxed{i_3 = -0.42 \text{ A}}$$

# Problem 11

Handwritten circuit diagrams and calculations for Problem 11.

The circuit consists of a  $5\Omega$  resistor in series with a parallel combination of  $1\Omega$  and  $3\Omega$  resistors. A  $2\Omega$  resistor is in parallel with the  $3\Omega$  resistor. The circuit is connected to terminals  $a$  and  $b$ .

The calculations show the use of mesh analysis to find the short-circuit current  $i_{sc}$ .

Mesh equations:

$$i_1(9) + i_2(-1) + i_3(-3) = 0$$

$$i_1(-1) + i_2(3) + i_3(-2) = 8$$

$$i_1(-3) + i_2(-2) + i_3(5) = 0$$

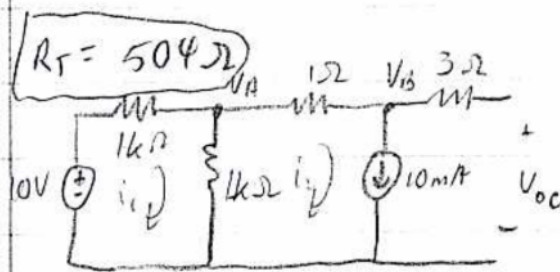
Matrix equation:

$$\begin{bmatrix} 9 & -1 & -3 & 0 \\ -1 & 3 & -2 & 8 \\ -3 & -2 & 5 & 0 \end{bmatrix} \Rightarrow \begin{matrix} i_1 = 1.6 \\ i_2 = 5.236 \\ i_3 = 3.05 \end{matrix}$$

Final result:

$$i_{sc} = i_N = 3.05 \text{ A}$$

# Problem 12



$$i_2 = 0.010 \text{ A}$$

$$i_1(2000) - i_2(1000) = 10$$

$$i_1 = 0.010 \text{ A}$$

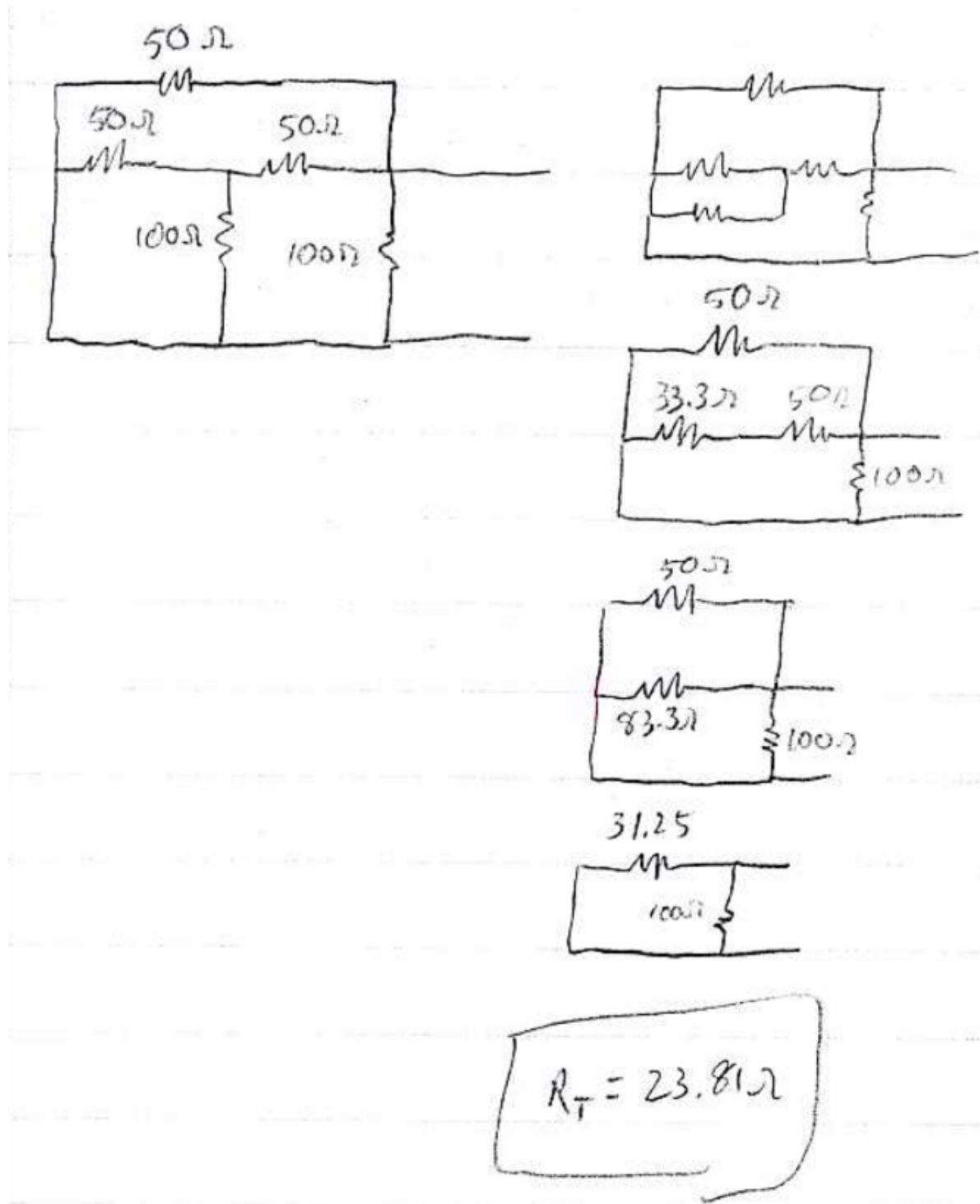
$$V_A = 10 - 0.01 \cdot 1000 = 0 \text{ V}$$

$$V_B = V_A - 0.010 \cdot 1 = -0.01 \text{ V}$$

$$V_{OC} = V_B = -0.01 \text{ V}$$



# Problem 13



Problem 14

For Maximum Power Transfer,  $R_L = R_T$

$$R_L = 8\Omega$$

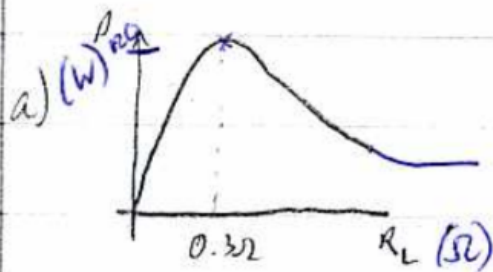
$$V_L = 6V$$

$$P_L = \frac{6^2}{8} = 4.5W$$

$$i = \frac{12}{16} = 0.75A$$

$$\% \text{ eff} = \frac{P_L}{P_s} \times 100\% = \frac{4.5W}{0.75A \cdot 12V} = 50\%$$

# Problem 15



max at  $0.3R = R_L$

$$b) P_L = \left( V_S \cdot \frac{R_L}{R_S + R_L} \right)^2 \quad R_L = V_S^2 \cdot \frac{R_L}{(R_S + R_L)^2} = V_S^2 R_L (R_S + R_L)^{-2}$$

$$\frac{dP_L}{dR_L} = V_S^2 (R_S + R_L)^{-2} + V_S^2 R_L (-2)(R_S + R_L)^{-3} = 0$$

$$V_S^2 (R_S + R_L) + V_S^2 R_L (-2) = 0$$

$$R_S + R_L - 2R_L = 0$$

$$\boxed{R_L = R_S}$$