Package 'zernike'

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```

addfit

Separate wavefronts

Description

Separate a set of wavefronts measured at different orientations into "polished in" and instrumental or "test stand" components.

Usage

```
addfit(..., th = 0, zcm = NULL, theta = numeric(0))
separate.wf(
  zcm,
  theta,
  maxorder = 14,
  nrow = nrow.default,
  ncol = ncol.default,
  cp = cp.default
)
```

Arguments

one or more wavefront fits as returned by psifit, vortexfit, fftfit, or wf_net. . . . th the orientation angle(s) of object under test, in degrees. Zernike coefficient matrix from addfit. zcm theta vector of angles from addfit. maxorder the maximum Zernike polynomial order of the fits. number of rows in the reconstructed wavefronts. nrow number of columns in the reconstructed wavefronts. ncol list of values describing the location and size of the wavefront as returned by ср circle.pars or pupil.pars.

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Details

The two functions addfit() and separate.wf() work together to provide a rudimentary project management capability when multiple optical tests have been run on an optical system rotated to one or more orientations. All addfit() does is extract the net Zernike coefficients in the list of values returned by psifit, vortexfit, fftfit, or wf_net. It also accepts one or more orientation angles in degrees.

The odd parameter list is to provide some flexibility in data entry. For example if several fits are available they could be entered as a group with the thetas entered as a vector the same length as the number of fits. Alternately fits could be entered one at a time. If the latter strategy is followed be sure to recycle the variable name that the return is assigned to. Also, the Zernike polynomial fits should be made to the same polynomial order. This isn't checked and will surely cause an error if different fit orders are used.

separate.wf() makes use of the properties of Zernike polynomials under rotations to disentangle the contributions from the "mirror" and the instrument or test stand to the extent possible. Least squares fits are performed for each non-axysmmetric aberration and some possibly useful summary statistics from the fits are returned in the data frames zcb and sumstats.

Value

Lists with the following elements

function addfit

zcm Zernike coefficient matrix from the zcoef.net entry in the wavefront fits, minus the first 3 elements.

function separate.wf

zcb a data frame with zernike coefficients and standard errors of estimates for intrinsic and instrumental aberrations.

sumstats a data frame with summary statistics describing the fits to each set of coefficients.

wf.mirror the estimated derotated wavefront, stored in a matrix of size nrow x ncol with class pupil. wf.inst the estimated instrumental (test stand) contribution to the wavefront.

Functions

- addfit(): Add a wavefront fit to tracking list
- separate.wf(): Separate wavefronts

aiapsi

Iterative algorithms for PSI with unknown phase shifts

Description

Three iterative algorithms for PSI with unknown phase shifts.

aiapsi 5

Usage

```
aiapsi(im.mat, phases, ptol = 0.001, maxiter=20, trace=1)
aiapsiC(im.mat, phases_init, ptol, maxiter, trace)
hkpsi(im.mat, phases, maxiter = 20, ptol = 0.001,
    trace = 1, plotprogress = TRUE)
tiltpsi(im.mat, phases, coords, ptol = 0.01, maxiter = 20, trace = 1)
tiltpsiC(im.mat, phases, coords, ptol, maxiter, trace)
```

Arguments

im.mat a *matrix* of interferogram values phases Starting guess for phase shifts

ptol Convergence criterion for phase shifts

maxiter Maximum number of iterations

trace Boolean: Print some summary data at each iteration.

plotprogress Plot some summary data for each iteration?

Also, for tiltpsi and tiltpsiC

coords Low order Zernike polynomial matrix

Details

The "variable tilt" algorithm now allows an indefinite number of low order Zernike terms to be variable between phase steps. coords can be created with zpm setting maxorder to a small value, say 4, discarding the first (dc) column and retaining as many as desired. There must be at least two columns for tilts. The third will be defocus, the next two astigmatism, the next two primary coma,

aiapsi and tiltpsi are wrappers for the calls to the C++ code in aiapsiC and tiltpsiC with sensible defaults for ptol, maxiter, and trace.

Value

A list containing the following elements:

phi The wrapped phase estimate. This is a vector as long as the number of rows in

im.mat.

mod Modulation estimate.

phases Phase shift estimates.

iter Number of iterations.

sse Sum squared error at each iteration.

Also, for tiltpsi

zcs Matrix of Zernike coefficients, with one row for each column in coords and

number of columns = number of columns of im.mat.

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Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

Zhaoyang Wang and Bongtae Han, "Advanced iterative algorithm for phase extraction of randomly phase-shifted interferograms," *Opt. Lett.* 29, 1671-1673 (2004).

Han, G-S and Kim, S-W,, "Numerical correction of reference phases in phase-shifting interferometry by iterative least squares fitting," *Applied Optics* 33, 7321-7325 (1994),

Lin, B-J et al., "An iterative tilt-immune phase-shifting algorithm," OSA conference Optical Fabrication and Testing 2010.

See Also

psifit

Description

Calculates Bath astigmatism coefficients with optional rotation of phi degrees.

Usage

```
astig.bath(D, rc, s, lambda = 632.8, phi = 0)
```

Arguments

D	Diamete
U	Diamete

rc Radius of curvature

s separation of reference and test beams

lambda Wavelength

phi angle of image horizontal relative to interferometer axis, in degrees

Details

D, rc, s, must have the same units. If those units are mm the source wavelength lambda should be in nm.

Value

The Zernike coefficients for primary astigmatism terms.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

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brcut	touw
DI CU	cpan

Branch cut algorithm for phase unwrapping

Description

Solves a modification of the assignment problem to minimize the total length of branch cuts.

Usage

```
brcutpuw(phase, pen = 0, details = FALSE)
```

Arguments

phase Matrix containing the wrapped phase map

pen Optional penalty value for connecting a residue to an edge details boolean: if TRUE return some extra details for diagnostics

Value

A matrix containing the unwrapped wavefront. If details==TRUE a named list starting with the unwrapped wavefront in puw.

Author(s)

M.L. Peck

See Also

```
qpuw idiffpuw
```

Examples

```
set.seed(1234)
PW <- wrap(matrix((0:100)*pi/10,101,101))
## need a border of NA's
PW <- cbind(rep(NA,101), PW, rep(NA,101))
PW <- rbind(rep(NA,103), PW, rep(NA,103))
PW <- PW + rnorm(103^2)
mtext(rmap(PW, plot=TRUE))
PU <- brcutpuw(PW, details=TRUE)
image(1:103, 1:103, PU$bcuts, col="blue", add=TRUE)
X11()
image(PU$puw, col=grey256, asp=1, useRaster=TRUE)</pre>
```

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circle.hough

Estimate parameters of a circle using Hough Circle Transform

Description

Uses a portion of the canny algorithm to find candidate edge points and the direction of the gradient at those points, then uses Hough Circle Transform to estimate circle parameters.

Usage

```
circle.hough(
    im,
    fw = 2,
    qt = 0.995,
    excl = 5,
    rmin = min(dim(im))/4,
    rmax = min(dim(im))/2,
    rstep = 1,
    dtheta_max = 0.5,
    dtheta_step = 0.05,
    nn = 7,
    plots = TRUE,
    details = FALSE
)
```

Arguments

im	The image to find a circle in (a modulation estimate is best)
fw	Size of Gaussian blur to smooth image
qt	Threshold to accept strong edge candidate
excl	Number of pixels to exclude around edge of frame as candidates
rmin	Minimum circle radius
rmax	Maximum circle radius
rstep	step size in constructing lookup table
dtheta_max	maximum assumed error in gradient direction
dtheta_step	increment for dtheta
plots	plot?
details	Return extra details?
number	of nearest neighbors for alternate calculation

Details

The Hough transform section first creates a lookup table of candidate radii and center points, then for each candidate edge point calculates potential centers along a fan of rays near the gradient direction. An inner join then finds matches in the lookup table and increments an accumulator vector. Highest vote at the end wins.

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Value

If details is FALSE a named list with the circle parameters

Note

This is experimental and can be very slow. A good guess for the radius is very helpful. Experimental feature: find the nn nearest neighbors of the selected trio of parameters and calculate a vote weighted mean. This is returned as rxy_alt if details is TRUE.

See Also

```
circle.pars(), pupil.pars()
```

Examples

```
example("psifit", package="zernike", ask=FALSE)
X11()
cp2 <- circle.hough(tfit$mod, rmin=round(tfit$cp$rx)-10, rmax=round(tfit$cp$rx)+10)</pre>
```

circle.pars

Pupil parameters

Description

Automatically determine the center and radius of a circular interferogram image.

Usage

```
circle.pars(im, fw=2, qt=0.995, excl=5,
    plots=TRUE, details=FALSE)
```

Arguments

im A matrix containing an image of a circular disk

fw Amount to smooth image

qt Threshold to accept an edge point, expressed as a quantile excl number of pixels around border of frame to exclude

plots Plot edge candidates and fit?

obstructed Logical: is there a central obstruction?

Details

This routine partially implements the Canny algorithm for edge detection. After optionally smoothing the input image the gradient is calculated using a Sobel filter, and edge pixels are identified by locating local maxima in the magnitude of the gradient.

The edge pixels with qt percentile largest gradients are passed to nlsrob in package robustbase lqs in package MASS to determine robustly the best fit circle.

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Value

A list with the following components:

xc X coordinate of the center of the pupil
yc Y coordinate of the center of the pupil

Universal redive of the pupil

rx Horizontal radius of the pupil ry Vertical radius of the pupil = rx obstruct Obstruction (always = 0)

Note

This routine is only effective on modulation estimates, and will almost certainly fail on interferogram images. Since data quality varies widely considerable experimentation may be needed on any given image. Increasing the smoothing parameter fw helps to suppress artifacts. Depending on how strong the actual edge is compared to artifacts qt may need to be either increased or decreased from the default value.

if details==TRUE several more pieces of data are returned. This is mostly for debugging purposes and may be eliminated in the future.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

Many routines require the pupil parameters in the form returned by circle.pars. For example psifit, fftfit, pupil, etc.

col3d OpenGL plot

Description

Returns a vector of colors similar to image() display.

Usage

```
col3d(surf, surf.col=topo.colors(256), zlim = NULL, eqa=FALSE)
```

Arguments

surf	A matrix of surface values
surf.col	Color palette for surface
zlim	Range of values to display
eqa	Equal area per color?

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Value

A vector of color values the same length as surf.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

The **rgl** package is described at http://rgl.neoscientists.org/about.shtml, and available from CRAN.

See Also

```
plot.pupil
```

convolve2d

2D convolution

Description

General 2D convolution using FFTs

Usage

```
convolve2d(im, kern)
```

Arguments

im A matrix representing an image

kern the convolution kernel

Value

The filtered matrix im.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
gblur. Called by circle.pars.
```

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crop

Crop an array

Description

Crop a matrix or 3D array. Main application is to trim excess pixels from an image array, wavefront, etc.

Usage

```
crop(img, cp, npad = 20, nxy=NULL)
```

Arguments

img Array to be cropped.

cp A list describing the pupil boundary.

npad Amount of padding to leave around the edge.

Number pixels per side in cropped array.

Details

cp is the list provided by circle.pars.

Value

im The cropped array cp Revised value of cp

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

fftfit

Fourier transform interferogram analysis

Description

High level routines for FFT analysis of interferograms.

Usage

```
fftfit(imagedata, cp = NULL,
    sl = c(1, 1), filter = NULL, taper = 2,
    options = psfit_options())
```

fftfit 13

Arguments

imagedata A matrix containing the interferogram

cp A list describing the pupil boundary, as returned by pupil.pars

sl Position of sidelobe in the form c(x,y)

filter Size of background filter around DC

taper Size of taper applied to edge of half plane cut

options a list of parameters passed to other functions. See psfit_options

.

Details

If is.null(filter) (the default), pick.sidelobe will be called to select a Fourier domain sidelobe and background filter size.

If is.null(cp) circle.pars is applied to the modulation to estimate the pupil parameters.

See wf_net for details of the process of creating net and smoothed wavefronts from raw unwrapped wavefront maps.

Value

A list with the following components:

phase Wrapped phase map

mod The estimated modulation

cp A list describing the pupil boundary

cp.orig The precropped value of cp

wf.net Net unsmoothed wavefront; a matrix of class "pupil"

wf.smooth Net smoothed wavefront

wf.residual Difference between net wavefront and polynomial fit

fit Return value from fitzernikes
zcoef.net Net Zernike coefficients from fit

Note

These functions are based largely on the work of Roddier and Roddier (1987).

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

References

Roddier, C. and Roddier, F. 1987, **Interferogram analysis using Fourier transform techniques**, *Applied Optics*, vol. 26, pp. 1668-1673.

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See Also

```
vortexfit, wf_net, pupil.pars, pick.sidelobe.
```

Examples

```
# just run the example for 'vortexfit()'
example(vortexfit, package="zernike", ask=FALSE)
```

FFTUtilities

FFT Utilities

Description

Miscellaneous utilities for working with 2D images in the Fourier domain.

Usage

```
wftophase(X, lambda=1)
padmatrix(X, npad, fill = mean(X, na.rm=TRUE))
submatrix(X, size = 255)
fftshift(X)
.up2(nr, nc=nr)
```

Arguments

X	A matrix
lambda	Value of the wavelength, in the same units as X
npad	Size of padded matrix
fill	Values to be assigned to padded matrix elements
size	Size of returned matrix
nr	A number
nc	A number

Details

wftophase computes the complex phase from wavefront values.

padmatrix pads a matrix to size npad x npad, placing the original matrix in the lower left hand corner of the padded matrix.

submatrix extracts a size x size matrix from the center of a larger matrix.

fftshift shuffles the quadrants of a matrix around to put the DC element (1,1) in the center of the transformed matrix, with spatial frequencies increasing to the right and up.

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Value

A matrix transformation of the input matrix X.

.up2 returns the next higher power of 2 than max(nr, nc).

Note

These low level routines are used by several higher level functions that operate in the Fourier domain.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
startest, fftfit.
```

fitzernikes Least Squares fit to Zernike polynomials	
--	--

Description

Performs a least squares fit of a specified set of Zernike polynomials to a vector of wavefront measurements.

Usage

```
fitzernikes(wf, rho, theta, eps=0, phi = 0, maxorder = 14, isoseq = FALSE)
```

Arguments wf

rho	A vector of radial coordinates.
theta	A vector of angular coordinates, in radians.
eps	Obstruction fraction of annular aperture.
phi	Orientation of the image, in degrees
maxorder	Maximum Zernike polynomial order
isoseq	Boolean: use ISO/ANSI sequencing

A vector of wavefront values

Details

wf, rho, and theta must be the same length.

As of version 3.7.0 Zernike polynomials in ISO/ANSI sequence can be used through a call to zpm_cart.

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Value

The coefficients of the least squares fit using qr.solve

Note

With the standard ordering of Zernikes Z0 is the piston term, Z1 and Z2 are x and y tilts, Z3 is defocus, etc.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
zpm, zpm_cart, psifit, fftfit, vortexfit, wf_net.
```

foucogram

Simulate a Foucaultgram

Description

Simulates the appearance of a wavefront under the Foucault test.

Usage

```
foucogram(wf, edgex = 0, phradius = 0, slit = FALSE,
  pad = 4, gamma = 1, map = FALSE, lev = 0.5)
```

Arguments

wf An object of class pupil containing wavefront values edgex lateral position of knife edge

phradius radius of light source

slit Logical: Is source a slit or pinhole?

pad pad factor for FFT

gamma Gamma value for graphics display

map Logical: Overlay contours from wavefront map?

lev Increment for contour levels, if used

Details

The default value of 0 for phradius simulates a monochromatic point source. Try values in the range 10-30 to suppress diffraction effects.

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Value

A matrix of intensity levels in the simulated image.

Note

The key approximations here are treating the light source as monochromatic and spatially coherent, which is usually not the case for an extended source. Also, Fraunhofer diffraction theory is used.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

See http://home.netcom.com/~mpeck1/astro/foucault/ext_foucault.pdf for an outline of the mathematical treatment of an extended source.

See Also

pupil

gblur Gaussian blur

Description

Blur an image by fw pixels

Usage

```
gblur(X, fw=0, details=FALSE)
```

Arguments

X A matrix representing an image

fw Width of the Gaussian convolution kernel, in pixels

details Return convolution kernel?

Details

fw is the standard deviation of the Gaussian.

Value

The filtered matrix X.

gol_welsch

Note

the details option is mostly for debugging purposes and may go away.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

convolve2d

gol_welsch

Golub-Welsch method to find quadrature points and weights for Gauss-Legendre quadrature

Description

Calculates the nodes and weights for Legendre polynomials shifted to the interval (eps^2, 1).

Usage

```
gol_welsch(eps, qwts)
```

Arguments

eps obstruction fraction $0 \le eps < 1$

qwts an input R vector with length equal to the number of quatrature points.

Details

If N is the maximum polynomial order to be evaluated qwts should be at least of length N/2 + 1. Quadratures will then be nominally exact.

Value

a vector of quadrature nodes the same length as qwts in the open interval (eps^2, 1). qwts will be overwritten with the quadrature weights.

See Also

```
Called by zapm() and zapm_iso().
```

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gpcapsi	Generalized Principal components algorithm for phase shifting inter- ferometry

Description

A generalized principal components algorithm for phase shifting interferometry developed by the author. This is the "low level" implementation.

Usage

```
gpcapsi(im.mat, ptol = 0.001, maxiter = 20, trace = 1)
gpcapsiC(im.mat, ptol, maxiter, trace)
```

Arguments

im.mat Matrix containing the unmasked pixels from a set of interferograms.

ptol Convergence tolerance for phase shifts

maxiter Maximum number of iterations

trace Print progress of nonlinear solver every trace iterations. Use trace=0 for silent

operation.

Details

gpcapsi is a wrapper to the C++ call in gpcapsiC.

Value

A list with the following items:

phi Estimated wrapped phase.
mod Estimated modulation.
phases Estimated phase shifts.

snr An estimate of the S/N of the interferograms.

eigen Eigenvalues of the crossproduct matrix

Note

This is the low level interface to the algorithm. The matrix im.mat should contain the unmasked pixel values from the input interferogram array. No checks are made for valid data. This should normally be called through the high level function psifit.

Author(s)

M. L. Peck

20 gradzpm_cart

See Also

pcapsi psifit

Description

Calculate Zernike polynomial values and Cartesian gradients in ISO/ANSI sequence for a set of Cartesian coordinates.

Usage

```
gradzpm_cart(x, y, maxorder = 12L, unit_variance = FALSE, return_zpm = TRUE)
```

Arguments

x a vector of x coordinates for points on a unit disk.

y a vector of y coordinates.

maxorder the maximum radial polynomial order (defaults to 12).

unit_variance logical: return with orthonormal scaling? (default false)

return_zpm logical: return Zernike polynomial matrix? (default true)

Details

Uses the recurrence relations in the above publication to calculate Zernike polynomial values and their directional derivatives in Cartesian coordinates. These are known to be both efficient and numerically stable.

Columns are in ISO/ANSI sequence: for each radial order n >= 0 the azimuthal orders m are sequenced m = -n, -(n-2), ..., (n-2), n, with sine components for negative m and cosine for positive m. Note this is the opposite ordering from the extended Fringe set and the ordering of aberrations is quite different. For example the two components of trefoil are in the 7th and 10th column while coma is in columns 8 and 9 (or 7 and 8 with 0-indexing). Note also that except for tilt and coma-like aberrations (m=1) non-axisymmetric aberrations will be separated.

All three matrices will have the same dimensions on return. Columns 0 and 1 of dzdx will be all 0, while columns 0 and 2 of dzdy are 0.

Value

a named list with the matrices zm (optional but returned by default), dzdx, dzdy.

References

Anderson, T.B. (2018) Optics Express 26, #5, 18878 https://doi.org/10.1364/0E.26.018878 (open access)

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See Also

zpm() uses the same recurrence relations for polar coordinates and extended Fringe set ordering, which is the more common indexing scheme for optical design/testing software.

zpm_cart() calculates and returns the Zernike polynomial values only.

Examples

```
rho <- seq(0.2, 1., length=5)
theta <- seq(0, 1.6*pi, length=5)
rt <- expand.grid(theta, rho)
x <- c(0, rt[,2]*cos(rt[,1]))
y <- c(0, rt[,2]*sin(rt[,1]))
gzpm <- gradzpm_cart(x, y)</pre>
```

gray256

8 bit Grayscale

Description

A vector of gray scale levels

Usage

```
gray256
grey256
```

Value

```
Defined as gray256 <- grey(seq(0,1,length=256))
```

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>.
```

hypot

Hypotenuse

Description

The Euclidean length of a vector

Usage

```
hypot(x)
```

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Arguments

x a vector

Value

the length of the vector

Author(s)

M.L. Peck

Examples

```
hypot(c(1,2))
```

idiffpuw

Phase unwrapping by Integrating DIFFerences

Description

Simple path following algorithm for two dimensional phase unwrapping.

Usage

```
idiffpuw(phase, mask = phase, ucall = TRUE, dx = NULL, dy = NULL)
```

Arguments

phase A matrix of wrapped phase values

mask Matrix the same size as phase indicating masked pixels

ucall Boolean: User call?dx Matrix of x differencesdy Matrix of y differences

Details

mask indicates pixels that shouldn't be unwrapped. In the simplest (default) case these are just pixels where phase is undefined.

Value

if(ucall), a matrix of class "pupil" with unwrapped wavefront values, otherwise a list with items:

puw Unwrapped phase

uw Matrix indicating pixels that have been unwrapped.

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Note

brcutpuw calls rmap first to check for the presence of residues. If there are none idiffpuw is guaranteed to work and is called to do the phase unwrapping.

If there *are* residues broutpuw creates a mask then calls idiffpuw to unwrap unmasked portions of the phase map.

This function is user callable as well; use a call of the form idiffpuw(phase).

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>. Thanks to Steve Koehler for programming ideas to considerably speed up the algorithm.

References

Ghiglia, D.C., and Pritt, M.D., 1998, **Two-Dimensional Phase Unwrapping**, New York: Wiley & Sons, Inc., ISBN 0-471-24935-1.

See Also

rmap, brcutpuw

10	dxv	I I\A/

Compiled code via Rcpp for Itoh's method of phase unwrapping

Description

Called by brcutpuw() for fast phase unwrapping

Usage

```
id_dxy_uw(nr, nc, phase, mask, dx, dy, uw)
```

Arguments

nr	number of rows in phase matrix
nc	number of columns in phase matrix
phase	phase matrix converted to vector

mask matrix of mask values converted to vector dx wrapped phase differences in x direction dy wrapped phase differences in y direction

Details

This is called by brcutpuw() through idiffpuw() but is also user callable. Wrapped phase values and differences are divided by 2*pi before input making the input values in the range [-1/2, 1/2). In brcutpuw() the mask indicates areas outside the interferogram area and lines of branch cuts

id_uw

Value

a vector with the unwrapped phase

Author(s)

```
M.L. Peck (mlpeck54 -at- gmail.com)
```

See Also

```
brcutpuw(), idiffpuw()
```

id_uw

Compiled code via Rcpp for Itoh's method of phase unwrapping

Description

```
Called by idiffpuw() for fast phase unwrapping
```

Usage

```
id_uw(nr, nc, phase)
```

Arguments

nr number of rows in phase matrix
nc number of columns in phase matrix
phase phase matrix converted to vector

Details

This is called by idiffpuw() but is also user callable. Wrapped phase values are divided by 2*pi before input making the input values in the range [-1/2, 1/2). In brcutpuw() the mask indicates areas outside the interferogram area and lines of branch cuts

Value

a vector with the unwrapped phase

Author(s)

```
M.L. Peck (mlpeck54 -at- gmail.com)
```

See Also

```
brcutpuw(), idiffpuw()
```

load.images 25

|--|--|

Description

Loads image files in jpeg, tiff, png, or raw format. load.pgm provides legacy support for reading files in pgm format.

Usage

```
load.images(files, channels=c(1,0,0), scale=1, FLIP=FALSE) load.pgm(files, imdiff=NULL)
```

Arguments

files A vector of character strings with file names

channels channel weights

scale scale factor for image resize FLIP flip image left for right?

Details

set FLIP=TRUE to reverse mirror imaged interferograms.

Any file extension other than jpg, jpeg, tif, tiff, or png is assumed to be in RAW format. Supported raw formats are determined by libraw and may not be up to date

Value

An array containing the contents of the image files.

Note

load.pgm is the original load.images included for legacy support of greyscale portable anymap files.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

26 Ispsi

lspsi

Phase Shifting Interferometry

Description

Least squares fitting of phase shifted interferograms.

Usage

```
lspsi(images, phases, wt = rep(1, length(phases)))
lspsiC(images, phases, wt)
```

Arguments

images An array containing the interferogram images

phases A vector of phase shifts wt A vector of weights

Details

images is a 3 dimensional array with dimensions nrow x ncol x length(phases), where nrow and ncol are the number of rows and columns in the individual interferogram images.

1spsi reshapes the image array into a matrix and calls 1spsiC which in turn calls the compiled C++ routine.

Value

A list containing the following components:

phi Estimated wrapped wavefront phase.

mod Estimated modulation

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
psifit
```

makezlist.iso 27

makezlist.iso

Construct list of ZP indexes in ISO/ANSI sequence with sine terms first

Description

Construct list of ZP indexes in ISO/ANSI sequence with sine terms first

Usage

```
makezlist.iso(maxorder = 12)
```

Arguments

maxorder

maximum radial and azimuthal order

Value

a named list with n=radial indexes, m=azimuthal, and t indicating trig function to apply

Examples

```
zlist.iso <- makezlist.iso(maxorder=6)
zlist <- makezlist(0, 6)</pre>
```

norm_zpm

Normalize matrix of Zernike polynomial values.

Description

Convert a matrix of Zernike polynomial values from unit scaled to unit variance aka orthonormal form.

Usage

```
norm_zpm(uzpm, maxorder = 12L)
```

Arguments

uzpm matrix of Zernike polynomial values

maxorder the maximum radial order.

Details

This is intended only for ISO/ANSI ordered matrices. The only check performed is that the number of columns in the matrix matches the expected number given by the argument maxorder. This is called by gradzpm_cart() and zpm_cart() if unit_variance is set to true in the respective function calls.

28 pcapsi

Value

matrix in orthonormal form.

pcapsi Vargas et al.'s Principal Components method for PSI

Description

Compute the phase using the Principal components algorithm.

Usage

```
pcapsi(im.mat, bgsub = TRUE, group_diag = "v")
```

Arguments

im.mat A matrix of interferogram values

bgsub Boolean - subtract the pixelwise mean as background estimate?

pcalg controls treatment of singular values of the data matrix

Details

Images are input into an array by load.images. This must be reshaped into a matrix for this function. Also, a mask should be applied if available prior to the call.

Value

A list containing the following elements:

phi The wrapped phase estimate. This is a vector as long as the number of rows in

im.mat.

mod Modulation estimate.
phases Phase shift estimates.

wt implied least squares weights.

snr An estimate of the signal to noise ratio in the input data.

eigen Singular values of the crossproduct matrix.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

pick.sidelobe 29

References

J. Vargas, J. Antonio Quiroga, and T. Belenguer, "Phase-shifting interferometry based on principal component analysis," *Opt. Lett.* **36**, 1326-1328 (2011) http://www.opticsinfobase.org/ol/abstract.cfm?URI=ol-36-8-1326

J. Vargas, J. Antonio Quiroga, and T. Belenguer, "Analysis of the principal component algorithm in phase-shifting interferometry," *Opt. Lett.* **36**, 2215-2217 (2011) http://www.opticsinfobase.org/ol/abstract.cfm?URI=ol-36-12-2215

See Also

```
psifit,
```

pick.sidelobe

Select an interferogram sidelobe in the Fourier domain

Description

Interactively locate the center of a first order sidelobe in the FFT of an interferogram, and mark the width of the background filter.

Usage

```
pick.sidelobe(imagedata, logm=FALSE, gamma=3)
```

Arguments

imagedata A matrix containing an interferogram image logm Logical: pass fn="logMod" to plot.cmat?

gamma yalue for display

Details

Uses the basic graphics utility locator.

Value

A list with the following components:

The coordinates c(x,y) of the selected sidelobe

filter Estimated size of background filter

Note

The high level FFT interferogram analysis routine fftfit requires the approximate location of the intended first order interferogram sidelobe to be specified.

30 plot.cmat

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
fftfit,
```

plot.cmat

Plot a complex matrix

Description

Plot a real valued function of a complex matrix

Usage

```
plot.cmat(X, fn = "Mod", col = grey256,
cp=NULL, zoom=1, gamma=1, ...)
```

Arguments

Χ	A complex valued matrix
fn	A function returning a real value
col	Color palette for graph
ср	pupil parameters as returned by pupil.pars
zoom	zoom factor for display
gamma	gamma value for display
	Other parameters to pass to image.default

Details

In addition to the functions described in complex fn can be assigned the values "logMod", which will call an internally defined function returning the value log(1+Mod(X)), "Mod2" to plot the power spectrum, and "logMod2" to plot the logarithm of the power spectrum.

If the parameter cp is passed axes will display spatial frequencies in cycles per pupil radius.

Value

none

Note

This is used primarily for displaying FFT's of interferograms. In the case of an interferogram in which the background has not been removed use fn="logMod" to make the first order sidelobes visible.

plot.pupil 31

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
pick.sidelobe, fftfit.
```

plot.pupil Pupils and wavefronts

Description

Plot and summary methods for objects of class "pupil".

Usage

```
plot.pupil(wf, cp=NULL, col = topo.colors(256), addContours = TRUE, cscale = FALSE,
        eqa=FALSE, zlim=NULL, ...)
summary.pupil(wf)
```

Arguments

wf	An object of class "pupil"
ср	Pupil parameters; a list as returned by pupil.pars
col	Color palette for plot
addContours	Logical: add contour lines?
cscale	Add a color scale legend?
eqa	Perform an "equal area" plot?
zlim	z limits to pass to image
	Additional parameters to pass to image.default

Details

These give simple plot and summary methods for objects of class pupil.

If eqa is TRUE, each color in the palette will be used for an equal number of pixels (as opposed to representing an equal interval). Note: the color scale (when cscale == TRUE) may be inaccurate if a very small number of colors are used.

Value

none

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

32 plotn

See Also

```
pupil, pupilrms, pupilpv, strehlratio, pupil.pars.
```

plotn

Wavefront comparison plots

Description

Plot an arbitrary number of wavefronts and all differences.

Usage

```
plotn(..., labels = NULL, addContours=FALSE, wftype = "net", col = rygcb(400), qt = c(0.01, 0.99))
```

Arguments

... List of wavefront estimates as returned by wf_net.

labels Labels to identify the wavefronts.

addContours Boolean to add contours to top row plots

wftype If the inputs are from wf_net, one of "net", "smooth", "residual".

col Color palette for top row of plot

qt Quantiles of differences to plot in comparisons.

Details

... can be any number of objects containing wavefront estimates as returned for example by wf_net.

Wavefronts are displayed on the top row, and differences of all pairs on subsequent rows. Grayscale is used to render the difference plots, and the color palette given in col is used for the wavefronts.

Value

none

Author(s)

```
M.L.\ Peck < mpeck <
```

See Also

```
plot.pupil wf_net
```

plotxs 33

Plot cross-sections (profiles) through a wavefront map.

Description

Plots an arbitrary number of cross-sections through a wavefront map, with one highlighted.

Usage

```
plotxs(wf, cp, theta0 = 0, ylim = NULL, N = 4, n = 101, col0 = "black", col = "gray", lty = 2)
```

Arguments

wf	A matrix of wavefront values.
ср	List of pupil parameters as returned by pupil.pars.
theta0	Angle of highlighted profile, in degrees.
ylim	range of heights to plot.
N	Number of cross sections.
n	Number of points for each cross section.
col0	Highlight color.

col Cross section color.

ty Line type for plots.

Details

The cross sections are equally spaced in angle from 0 to pi*(N-1)/N. Any angle can be specified for the highlighted profile at theta0.

Value

none

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

plot.pupil is the main wavefront plotting routine.

psfit_options

psfit_options

Options for PSI and FFT based fitting routines

Description

Get and optionally set parameters controlling various aspects of PSI algorithms, Zernike polynomial fitting, and data display

Usage

```
psfit_options(...)
```

Arguments

crop

colors topo.colors(256) refine **TRUE** "qual" puw_alg fringescale 1 NULL wt **TRUE** bgsub maxiter 20 1e-04 ptol 1 trace 2 nzcs zc0 6:7 satarget c(0, 0)astig.bath c(0, 0)maxorder 14 uselm **FALSE FALSE** isoseq **FALSE** usecirc sgs nthreads parallel::detectCores()/2 plots TRUE

FALSE

psfit_options 35

Details

Calling psfit_options with an empty argument list returns the default values of the options used in psifit and wf_net as itemized above. The list can be modified directly or by passing argument value pairs to the function call.

Parameters you might want to change include:

satarget sets the target SA for "numerical nulling." This is a vector of length 2 setting the target values of primary and 5th order SA.

ptol sets convergence tolerances for iterative PSI algorithms. These have different definitions and different values may be suitable for different algorithms. A value around 0.01 is appropriate for tiltpsi.

The number of variable Zernike terms in the algorithm tiltpsi is controlled by nzcs. Set it to 3 to include defocus, 5 to include primary astigmatism, 7 to include coma.

maxorder sets the maximum Zernike polynomial order for wavefront fitting. It must be even and at least 6. The default generally produces a good wavefront representation but you may want to experiment with higher order fits.

Parameters isoseq and usecirc control which sets of Zernike (annular) polynomials are called for wavefront fitting. If Zernikes in ANSI/ISO sequence are desired set isoseq = TRUE. The default FALSE uses extended Fringe ordering. Set usecirc = TRUE if you want to use Zernike circle polynomials even for obstructed wavefronts.

puw_alg specifies the phase unwrapping algorithm. Current options are "qual", "brcut", "lpbrcut", and "lp".

A new and somewhat experimental feature from version 3.6.0 of the package is threaded computation of Zernike polynomial matrices. The number of threads used is set with the option nthreads. The default is to use half the number of cores detected because on CPUs that support multi- or hyperthreading the number of cores reported by detectCores() is double the number of physical cores. Setting nthreads = 1 will turn off threading. Different C++ routines are used for the matrix fill in the unthreaded (zpmC) and threaded (zpmCP) cases. Speed improvements, if any, may vary.

If you don't like the default color palette there are many other choices. If you like rainbows rygcb defined in this package produces a relatively perceptually uniform version that's well suited for display on an RGB monitor.

Value

A named list with the current values of the arguments.

Author(s)

M.L. Peck <mlpeck54@gmail.com>

36 psifit

Description

High level function for Least squares analysis of phase shifted interferograms.

Usage

```
psifit(images, phases, cp = NULL, satarget = NULL, psialg = "ls", options = psfit_options())
```

Arguments

images An array containing the interferogram images

phases A vector of phase shifts

cp A list describing the pupil boundary, as returned by pupil.pars

satarget Target 4th and 6th order SA coefficients in non-null tests of aspheres

psialg String identifying the PSI algorithm to use

options a list of options

Details

images is a 3 dimensional array with dimensions nrow x ncol x length(phases), where nrow and ncol are the number of rows and columns in the individual interferogram images.

The current values recognized for psialg are

```
Is least squares with known phase shifts

aia the "advanced iterative algorithm" aiapsi

pc1 pca with pcalg="pc1"

pc2 pca with pcalg="pc2"

pc3 pca with pcalg="pc3"

gpc my generalized PC algorithm in gpcapsi

pc1thenaia first pcapsi then aiapsi

pc1thentilt first pcapsi then tiltpsi

gpcthentilt first gpcapsi then tiltpsi

tilt tiltpsi
```

psifit 37

Value

A list with the following components

phi wrapped phase estimate mod modulation estimate

phases phase shifts

cp the interferogram boundary

wf.net net, unfiltered wavefront (see wf_net)

wf.smooth Zernike fit wavefront

wf.residual the difference

fit Coefficients of Zernike fit to wavefront

zcoef.net Net Zernike coefficients

extras any extra data returned by low level functions

Author(s)

```
M.L. Peck <mlpeck54@gmail.com>
```

See Also

```
lspsi, aiapsi, tiltpsi, gpcapsi, pcapsi
```

```
## reuse the files from the demo for an example of two stage fitting
  ## using gpca then tiltpsi
require(zernike)
fpath <- file.path(find.package(package="zernike"), "psidata")</pre>
files <- scan(file.path(fpath, "files.txt"), what="character")</pre>
for (i in 1:length(files)) files[i] <- file.path(fpath, files[i])</pre>
# load the images into an array
images <- load.images(files)</pre>
# parameters for this run
source(file.path(fpath, "parameters.txt"))
# phase shifts
phases <- wrap((0:(dim(images)[3]-1))/frames.per.cycle*2*pi)</pre>
phases <- switch(ps.dir, ccw = -phases, cw = phases, phases)</pre>
# target SA coefficients for numerical null.
sa.t <- sconic(diam,roc,lambda=wavelength)</pre>
zopt <- psfit_options()</pre>
```

38 pupil

```
zopt$satarget <- sa.t
zopt$ptol <- 0.01
tfit <- psifit(images, phases, psialg="gpcthentilt", options=zopt)</pre>
```

pupil

Pupils and wavefronts

Description

Create a pupil object and optionally fill it with a wavefront. For our purposes a "pupil" is defined to be a matrix representation of a circular or annular aperture. Simple plot and summary methods are also provided.

Usage

Arguments

zcoef	A vector of Zernike coefficients
maxorder	Maximum Zernike polynomial order
zlist	List of indexes the same length as zcoef
isoseq	ZPs in ISO/ANSI sequence
phi	Amount to rotate image, in degrees
piston	Constant to add to wavefront values
nrow	Number of rows in output matrix
ncol	Number of columns in output matrix
ср	A list with items $xc - x$ coordinate of central pixel, $yc - y$ coordinate of central pixel, $rx - x$ radius in pixels, $ry - y$ radius in pixels, obstruct - central obstruction fraction.

pupil 39

Details

plot.pupil and summary.pupil provide simple plot and summary methods for objects of class "pupil".

pupil. arb will accept an arbitrary list of Zernike indexes.

pupil requires a complete set of Zernikes as returned by zpm or zpm_cart.

Default values nrow.default, ncol.default and cp.default are now (as of version 3.7.0) stored in the package environment. These can be used or overridden by setting values in the user's global environment.

Version 3.7.0 of this package makes some fairly significant changes in pupil from previous releases. First, zlist is no longer used to indicate the set of Zernikes used. Instead the maximum radial order is passed as maxorder, defaulting to 14. The complete set of Zernike polynomial values including piston is calculated with either zpm or zpm_cart if isoseq is TRUE. The vector zcoef, if non-null, must be either the same length as the number of columns in the zernike matrix with a null value for piston, or one shorter with a non-null value for piston. The vector zcoef.net returned by wf_net has the first, piston, element removed so a non-null piston value must be passed as an argument.

Value

A matrix of size nrow x ncol. The matrix is assigned to the class "pupil". NAs are used to fill the matrix outside the defined area of the pupil.

Note

The parameter cp is used to define the dimensions of the pupil. See pupil.pars for details.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
zpm, zpm_cart, wf_net, pupilrms, pupilpv, strehlratio, pupil.pars, circle.pars.
```

```
\label{eq:wf-pupil} $$ wf <- pupil(zcoef=rnorm(length(makezlist(0, 14)$n), 0, 0.01)) $$ plot(wf, addContours=FALSE) $$ summary(wf)
```

40 pupil.pars

|--|--|

Description

Interactively determine the center, radius, and obstruction fraction of a circular or annular interferogram image.

Usage

```
pupil.pars(im = NULL, obstructed = FALSE)
```

Arguments

im A matrix containing an interferogram image obstructed Logical: is there a central obstruction?

Details

In pupil.pars, if the image has already been plotted im can be NULL, which is the default.

Value

A list with the following components:

xc X coordinate of the center of the pupil
yc Y coordinate of the center of the pupil
rx Horizontal radius of the pupil
ry Vertical radius of the pupil
obstruct Obstruction

Note

pupil.pars uses the basic graphics library routine locator to interactively mark the edge of the pupil, and optionally the edge of the obstruction. After right clicking to terminate locator() a least squares fit is performed to the marked points to determine the center and radius of the pupil.

Note that all routines that make use of Zernikes implicitly assume a circular pupil, or an annular one with small obstruction. We allow rx != ry for imaging sensors with non-square aspect ratios.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

Many routines require the pupil parameters in the form returned by pupil.pars. For example psifit, fftfit, pupil, etc.

pupil.rhotheta 41

Description

Calculate matrixes of polar coordinates for pupil's.

Usage

```
pupil.rhotheta(nrow, ncol, cp)
```

Arguments

nrow	Number of rows in interferogram images
ncol	Number of columns in interferogram images
ср	A list describing the pupil boundary, as returned by pupil.pars

Value

A list with the following components:

rho A matrix of radial coordinates
theta A matrix of angular coordinates

Note

My Zernike polynomial routines work in polar coordinates, which this function provides. Also, NA's are used to fill the matrix outside the pupil boundary, making the returned values convenient for selecting pixels inside interferograms.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
Zernike, pupil.
```

42 pupilrms

pupilrms

Wavefront statistics

Description

Compute basic statistics of wavefronts stored in "pupil" objects.

Usage

```
pupilrms(pupil)
pupilpv(pupil)
strehlratio(rms)
```

Arguments

pupil A matrix of class "pupil" rms An rms wavefront error

Value

Estimates of the RMS and P-V wavefront errors. strehratio calculates Mahajan's approximation to the Strehl ratio.

Note

pupilrms simply calculates the standard deviation of finite values in the matrix pupil. This is a crude, but usually accurate enough estimate of the true RMS wavefront error.

```
summary.pupil calls these functions.
```

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

Schroeder, D.J. 2000, Astronomical Optics, 2nd Edition, Academic Press, chapter 10.

See Also

```
summary.pupil.
```

```
zcoef <- rnorm(length(makezlist()$n), 0, 0.01)
wf <- pupil(zcoef=zcoef)
plot(wf)
summary(wf)
sqrt(crossprod(zcoef)) # A more accurate estimate of RMS</pre>
```

PVr 43

PVr Zygo's "robust" PV

Description

A peak to valley error estimate that reduces the effect of noise and artifacts

Usage

```
PVr(wf.zfit, wf.residual)
```

Arguments

wf.zfit matrix containing the smoothed Zernike fit wavefront

wf.residual matrix of the difference between the raw wavefront and the Zernike fit. These

values are returned by wf_net()

Details

no check is performed on the wavefronts, so it's the user's responsibility to make sure these come from the same source

Value

the estimated PVr

References

Evans, C. (2009) Optical Engineering 48(4), 43605. https://doi.org/10.1117/1.3119307

qpuw

Quality guided algorithm for phase unwrapping

Description

Quality guided algorithm for two dimensional phase unwrapping.

Usage

```
qpuw(phase, qual)
```

Arguments

phase A matrix of wrapped phase values

qual A matrix of quality values the same size as phase.

44 q_uw

Value

puw A matrix of class "pupil" with the unwrapped wavefront.

Note

This is a straightforward implementation of the quality guided algorithm of G&P.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

Ghiglia, D.C., and Pritt, M.D., 1998, **Two-Dimensional Phase Unwrapping**, New York: Wiley & Sons, Inc., ISBN 0-471-24935-1.

See Also

```
idiffpuw, brcutpuw
```

q_uw

Compiled code via Rcpp for quality guided phase unwrapping

Description

Called by qpuw() for fast quality guided phase unwrapping

Usage

```
q_uw(nr, nc, phase, qual)
```

Arguments

nr number of rows in phase matrix
nc number of columns in phase matrix
phase phase matrix converted to vector
qual quality matrix converted to vector

Details

This is called by qpuw() but is also user callable. Wrapped phase values are divided by 2*pi before input making the input values in the range [-1/2, 1/2).

Value

a vector with the unwrapped phase

rescale 45

Author(s)

M.L. Peck (mlpeck54 -at- gmail.com) with valuable programming advice from Steve Koehler

See Also

```
qpuw(), idiffpuw()
```

rescale

Rescale an image.

Description

Rescale a matrix containing a bitmapped image using bilinear interpolation.

Usage

```
rescale(img, scale)
```

Arguments

img A matrix with image data.

scale Scale factor.

Details

A value <1 will shrink the image.

Value

A matrix containing the rescaled image data.

Note

NA's are OK.

Author(s)

 $M.L. \ Peck < mpeck 1@ix.netcom.com >$

See Also

Called by load. images if necessary.

rmap

rmap	Utilities for phase unwrapping	
	comment for France mon of France	

Description

Utility functions for use in 2D phase unwrapping.

Usage

```
rmap(phase, dx = NULL, dy = NULL, plot = FALSE, ...) wrap(phase)
```

Arguments

phase	Matrix of wrapped phase values
dx	Matrix of x differences
dy	Matrix of y differences
plot	Boolean: plot residue positions?
	additional arguments for image

Details

dx and dy must have the same dimension as phase.

Value

```
In rmap if plot == TRUE

nr the number of residues identified in the map

otherwise

phase wrapped phase returned by wrap

residues Matrix the same size as phase with residues marked as + or - 1.
```

Note

These are primarily intended for internal use but can be used interactively. Calling rmap(phase, plot=TRUE) will plot the positions of residues and return nothing. If (plot==FALSE) in the call to rmap a matrix the same size as phase is returned with residues identified with values of +1 or -1.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>. Steve Koehler is responsible for the efficient implementation of the wrap function.

rygcb 47

References

Ghiglia, D.C., and Pritt, M.D., 1998, **Two-Dimensional Phase Unwrapping**, New York: Wiley & Sons, Inc., ISBN 0-471-24935-1.

See Also

Called by brcutpuw.

rygcb

A better rainbow.

Description

Produces a rainbow color palette with colors ranging from "red" to "blue" or "magenta". Perceptual uniformity should be superior to R's rainbow.

Usage

```
rygcb(n)
rygcbm(n)
```

Arguments

n

Number of color levels

Details

The palette is created using colorRampPalette.

Value

A vector of colors.

Note

The call to colorRampPalette sets space="Lab" and interpolate="spline" with the intent of creating a more perceptually uniform rainbow.

Author(s)

M.L. Peck

See Also

grey256

48 rzernike_ann

Examples

```
plotsp <- function(spectrum) {
    sl <- length(spectrum)
    rgbv <- col2rgb(spectrum)
    plot((0:(sl-1))+0.5, rgbv[1,], type="l", col="red", xlim=c(0,sl),ylim=c(0,300),xlab="Index",ylab="Channel value"
    points((0:(sl-1))+0.5, rgbv[2,], type="l", col="green")
    points((0:(sl-1))+0.5, rgbv[3,], type="l", col="blue")
    grid()
    rect(0:(sl-1), 260, 1:sl, 300, col=spectrum, density=NA)
    }
    plotsp(rygcb(400))
    X11()
    plotsp(rygcbm(500))</pre>
```

rzernike_ann

Radial Zernike Annular polynomials

Description

Create a matrix of Zernike Annular polynomial values in extended Fringe sequence for a set of polar coordinates.

Usage

```
rzernike_ann(rho, eps, n, m, xq, qwts)
```

Arguments

rno	a vector of radial coordinates.
eps	the obstruction fraction $0 \le eps < 1$.
n	the maximum radial order required
m	azimuthal order
xq	nodes for Gauss-Legendre quadrature
qwts	weights for Gauss-Legendre quadrature

Details

To the author's knowledge no recurrence relations for radial Zernike annular polynomials have been published, even though several are well known for the closely related Zernike circle polynomials. However the m=0 polynomials representing axially symmetric aberrations are just shifted Legendre polynomials with an easily derived recurrence relation. This routine makes use of that fact to generate recurrence relations for arbitrary polynomial indexes using chebyshev's algorithm with modified moments. The modified moments are calculated using Gauss-Legendre quadrature. If enough quadrature nodes were chosen the quadrature is nominally exact, as are the resulting annular Zernike values.

rzernike_ann_direct 49

Value

A length(rho) x (n-m)/2+1 column matrix of radial Zernike Annular polynomial values evaluated at the input radial coordinates. The radial indexes are in increasing order from m, m+2, ..., n.

See Also

This function is called by zapm() and zapm_iso().

rzernike_ann_direct

Radial Zernike annular polynomials from formulas

Description

The first 15 radial Zernike annular polynomials copied from Table 1 of Mahajan (1994)

Usage

```
rzernike_ann_direct(rho, eps, j)
```

Arguments

rho a vector of radial coordinates.

eps the obstruction fraction $0 \le eps < 1$.

j index of the value to fetch

Details

This function is included for testing and reference only.

Value

A vector of radial Zernike annular polynomial values at the coordinates rho for index j

References

Mahajan, V. N. (1994). Zernike Annular Polynomials and Optical Aberrations of Systems with Annular Pupils. *Supplement to Applied Optics* **33**, 8125-8127.

See Also

This function is called by <code>zapm_direct()</code> and <code>zapm_iso_direct()</code>. Complete sequences to arbitrary order can be calculated with <code>rzernike_ann()</code> and <code>rzernike_ann_128()</code>.

50 sconic

Description

twice the radial height difference between a sphere and conic surface

Usage

```
sconic(D, rc, b = -1, eps = 0., lambda = 632.8, nmax = 6)
```

Arguments

D	Diameter (mm)
rc	Radius of curvature (mm)
b	conic constant (default -1)
eps	obstruction fraction (default 0)
lambda	source or test wavelength (nm)
nmax	maximum polynomial order

Details

For the typical use case D and rc should be in millimeters and lambda in nanometers.

If eps > 0 Zernike annular coefficients will be computed and returned.

Value

Zernike polynomial coefficients. The default returns the 4th and 6th order coefficients in a length two vector.

Note

This estimates twice the radial distance between a sphere and conic surface with same paraxial radius of curvature, and returns Zernike coefficients of polynomial expansion. Intended for "numerical nulling" when testing an asphere at center of curvature, and should be more accurate than the vertical height difference calculated by zconic for that purpose.

Author(s)

M.L. Peck

See Also

zconic

startest 51

Examples

```
2.*zconic(1000,5000)
sconic(1000,5000)
```

startest

Star test simulator

Description

Simulates a star test.

Usage

```
startest(wf=NULL, zcoef=NULL, maxorder=14L, phi=0,
lambda = 1, defocus=5, cp=NULL,
obstruct=NULL, npad = 4,
gamma=2, psfmag=2, displaymtf=TRUE, displaywf=FALSE)
```

Arguments

wf A matrix of class pupil containing wavefront values

zcoef Vector of Zernike coefficients

maxorder maximum Zernike polynomial order

phi Angle to rotate wavefront

lambda Wavelength, in same units as coefficients

defocus Amount of defocus in waves

cp pupil parameters
obstruct Obstruction fraction
npad Pad factor for FFT

gamma Gamma value for graphics display

psfmag Magnification factor for in focus PSF display

displaymtf Logical: Display MTF?

displaywf Logical: Display calculated wavefront?

Details

If wf is NULL the wavefront is calculated from the the Zernike coefficients (which should be non-NULL).

Value

A list with the following components:

psf The in focus point spread function.

otf The complex optical transfer function, a complex matrix of size pupilsize.

mtf The modulation transfer function, a real matrix of size pupilsize.

52 summary.wf_zfit

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

```
Born, M. and Wolf, E. 1999, Principles of Optics, 7th Edition, Cambridge University Press. Suiter, H. R., 1994, Star Testing Astronomical Telescopes, Willman-Bell, Inc.
```

See Also

```
Zernike, pupil.
```

```
summary.wf_zfit
```

Methods for class "wf_zfit"

Description

Summary, print, plot, report, and invert methods for the returned list of values from psifit(), fftfit(), or vortexfit()

Usage

```
## S3 method for class 'wf_zfit'
summary(wffit, digits = 3, printnow = TRUE)
```

Arguments

```
wffit the return values from one of the fringe analysis routines or wf_net()
digits number of digits to display in print and summary methods
printnow send output to console?
... values passed to plot.pupil()
```

Details

The report method combines the plot, summary, and print methods into an html document and displays it in a browser. The invert method negates the values of wavefronts and Zernike coefficients and returns the adjusted input

Value

summary and print methods return data frame with wavefront summaries and Zernike coefficients

synth.interferogram 53

Description

Compute and display a synthetic interferogram.

Usage

Arguments

wf	A matrix of wavefront values
zcoef	A vector of Zernike coefficients
maxorder	Maximum Zernike polynomial order
nr	Number of rows in the output matrix
nc	Number of columns in the output matrix
ср	A list describing the pupil boundaries, as created by pupil.pars
phi	Amount to rotate the wavefront, in degrees
addzc	A 4-vector with piston, tilt, and defocus terms to be added
fringescale	Fringe scale. Should be 1 for single pass, 0.5 for double, etc.
plots	Logical: Plot the interferogram?

Details

Either wf or zcoef must be non-null, but not both. If zcoef is specified maxorder must be as well. Additional piston, tilt, and defocus terms can be added to the calculated wavefront using addzc.

Value

A matrix of intensity levels in the calculated interferogram, assigned class "pupil".

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

pupil.

54 turbwf

Examples

```
# create a list of zernikes
zcoef <- rnorm(length(zlist.fr$n), mean=0, sd=0.01)
iwf <- synth.interferogram(zcoef=zcoef, maxorder=12)
X11()
# show again with some tilt
iwf <- synth.interferogram(zcoef=zcoef, maxorder=12, addzc=c(0,5,5,0))</pre>
```

turbwf

Kolmogorov Turbulence

Description

Simulates the optical effects of atmospheric turbulence using Noll's (1976) calculation of the covariance matrix of Zernike polynomials under Kolmogorov turbulence.

Usage

```
turbwf(friedratio = 1, zlist = makezlist(2, 40), reps = 1)
```

Arguments

friedratio Ratio of pupil diameter to Fried parameter

zlist A list of Zernikes, as returned for example by makezlist

reps Number of draws to simulate

Details

The default value of zlist has 440 elements, which may be more than necessary for a reasonable representation of an "atmospheric" wavefront.

Value

A list with the following components:

zcoef.turb A reps x length(zlist\$n matrix of simulated draws of Zernike coefficients.

V Covariance matrix of the indexed Zernikes.

Note

Typos in the original source material have been corrected in the code. Note that scintillation is not modelled.

vortexfit 55

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

Noll, R.J. 1976, **Zernike polynomials and atmospheric turbulence**, *J. Opt. Soc. Am.*, Vol. 66, No. 3, p. 207.

See Also

```
Zernike, pupil.
```

Examples

```
# Simulate a single draw from a turbulent atmosphere
zcoef.turb <- turbwf(friedratio=5, zlist=makezlist(2,30), reps=1)$zcoef.turb
# Warning: this can take a while
wf <- pupil(zcoef=zcoef.turb, zlist=makezlist(2,30))
plot(wf)
summary(wf)</pre>
```

vortexfit

Vortex transform.

Description

Fringe analysis by Vortex aka Spiral Quadrature transform.

Usage

```
vortexfit(
  imagedata,
  cp = NULL,
  filter = NULL,
  fw.o = 10,
  options = psfit_options()
)
```

Arguments

imagedata matrix containing the interferogram data

cp list with circle parameters describing interferogram location. Defaults to NULL

filter size of filter to remove background

fw.o size of gaussian blur to smooth orientation estimate

options A list with general fitting and display options. See psfit_options().

56 vortexfit

Details

Implements the Vortex or spiral phase quadrature transform method of Larkin et al. (2001) https://doi.org/10.1364/JOSAA.18 including the fringe orientation estimation approach in Larkin (2005) https://doi.org/10.1364/OPEX.13.008097. Thanks to Steve Koehler for ideas on implementation details.

Value

a list with wavefront estimates, wrapped phase, modulation, etc. The return has S3 class 'wf_zfit' with plot, print, summary, and report methods.

Warning

This routine is offered as is with no license, as it may be in violation of one or more US and international patents.

See Also

This is one of two routines provided for analysis of single interferograms, along with fftfit(). This *may* be suitable for interferograms with closed fringes.

```
require(zernike)
fpath <- file.path(find.package(package="zernike"), "psidata")</pre>
fname <- "Image197.jpg"</pre>
img <- load.images(file.path(fpath, fname))</pre>
# parameters for this run
source(file.path(fpath, "parameters.txt"))
# target SA coefficients for numerical null.
sa.t <- sconic(diam,roc,lambda=wavelength)</pre>
zopt <- psfit_options()</pre>
zopt$satarget <- sa.t</pre>
# display an interferogram
if (tolower(.Platform$OS.type) == "windows") windows() else x11()
image(1:nrow(img), 1:ncol(img), img, col=grey256, asp=1,
xlab="X", ylab="Y", useRaster=TRUE)
mtext("Sample Interferogram")
if (tolower(.Platform$OS.type) == "windows") windows() else x11()
vfit <- vortexfit(img, filter=15, fw.o=10, options=zopt)</pre>
# do "classical FFT" based fit and compare results
dev.set(dev.next())
ftfit <- fftfit(img, cp=vfit$cp, sl=c(32, 0), filter=15, options=zopt)</pre>
```

wf3d.pupil 57

```
plotn(ftfit, vfit, labels=c("fft", "vortex"))
```

wf3d.pupil

OpenGL wavefront plot

Description

Interactive plot of a wavefront using the OpenGL package **rgl**. This is a 3D plotting method for objects of class "pupil".

Usage

Arguments

wf	A matrix of wavefront values
СВ	A list describing the pupil boundary

zoom.wf Zoom factor for heights surf.col Color palette for surface bg.col Background color eqa Equal area per color?

new.window If TRUE (the default), open a new rgl deice. Otherwise clears and re-uses an

existing window if available

Details

The default color palette will match the colors in the default version of plot.pupil.

Value

none

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

The **rgl** package is available from CRAN.

See Also

```
plot.pupil
```

58 wf_net

Examples

```
# create a random wavefront

wf <- pupil(zcoef=rnorm(length(makezlist()$n), mean=0, sd=0.01))
# the default method

plot(wf)

#this is more fun

wf3d(wf)</pre>
```

wf_net

Wavefront post-processing

Description

Perform post-processing of phase and modulation estimates performed by any of the fringe analysis routines

Usage

```
wf_net(phi, mod, cp, options)
```

Arguments

phi	wrapped phase map
mod	modulation estimate

cp a list describing the pupil boundary options a list of options. See psfit_options

Details

Called by psifit, vortexfit, fftfit. This now performs all of the post-processing on a calculated wrapped phase and modulus estimate from any of the fringe analysis routines in this package. This includes calculating the actual interferogram location parameters if not specified at call time, cropping if desired, unwrapping the phase, calculating a net and Zernike fit wavefront.

Value

A list with the following components:

phi the input phase, cropped if requested

mod the input modulus

cp interferogram location parameters as returned by circle.pars

cp.orig input cp

wf_net 59

wf.net Net unsmoothed wavefront; a matrix of class "pupil"
wf.smooth Net smoothed wavefront
wf.residual Difference between net wavefront and polynomial fit
fit Return value from fitzernikes
zcoef.net Net Zernike coefficients from fit

Note

As of ver. 3.7.2 the values of options\$isoseq and cp\$obstruct determine what is called to fit Zernikes. If isoseq is TRUE it's either zpm_cart or zapm_cart, with the latter called automatically if cp\$obstruct > 0. Otherwise the calls are made to zpm or zapmC.

Author(s)

M.L. Peck <mpeck1@ix.netcom.com>

```
## illustrates effect of incremental addition of an obstruction
## and standalone use of wf_net
require(zernike)
fpath <- file.path(find.package(package="zernike"), "psidata")</pre>
files <- scan(file.path(fpath, "files.txt"), what="character")</pre>
for (i in 1:length(files)) files[i] <- file.path(fpath, files[i])</pre>
# load the images into an array
images <- load.images(files)</pre>
# parameters for this run
source(file.path(fpath, "parameters.txt"))
# phase shifts
phases <- wrap((0:(dim(images)[3]-1))/frames.per.cycle*2*pi)</pre>
phases <- switch(ps.dir, ccw = -phases, cw = phases, phases)</pre>
# target SA coefficients for numerical null.
sa.t <- sconic(diam,roc,lambda=wavelength)</pre>
zopt <- psfit_options(satarget=sa.t)</pre>
psfit <- psifit(images, phases, psialg="ls", options=zopt)</pre>
# get back the raw wavefront
cp <- psfit$cp
eps <- seq(0.1, 0.5, by=0.1)
ne <- length(eps)</pre>
```

60 zapm

```
pad0 <- rep(0, ne)
  ## collect some basic data
 df.annfits \leftarrow data.frame(eps=c(0, eps), sa.null = c(sa.t[1], pad0), sa.obs = c(psfitfit[9], pad0), sa.net = c(psfitfit[9], 
                                                                                                       rms.net = c(sqrt(crossprod(psfit$zcoef.net)), pad0), defocus=c(psfit$fit[4], pad0))
  for (i in seq_along(eps)) {
            cp$obstruct <- eps[i]</pre>
            sa.t <- sconic(diam, roc, eps=eps[i], lambda=wavelength)</pre>
            zopt$satarget <- sa.t</pre>
            fiti <- wf_net(psfit$phi, psfit$mod, cp=cp, options=zopt)</pre>
        \label{eq:df.annfits} $$ df.annfits[i+1, 2:ncol(df.annfits)] <- c(sa.t[1], fiti$fit[9], fiti$zcoef.net[8], sqrt(crossprod(fiti$zcoef.net[8], sqrt(crossprod(fiti$zcoef.net[8],
  }
 rm(eps)
 attach(df.annfits)
X11()
 plot(eps, sa.null, type="b", ylim=range(df.annfits[,2:ncol(df.annfits)]), xlim=c(\emptyset, \emptyset.8), main="Trends with obstraints of the plot of th
 points(eps, sa.obs, type="b", pch=2, col=2, lty=2)
points(eps, sa.net, type="b", pch=3, col=3, lty=3)
 points(eps, rms.net, type="b", pch=4, col=4, lty=4)
 points(eps, defocus, type="b", pch=5, col=5, lty=5)
 grid()
 legend(x=0.6, y=0, legend=names(df.annfits)[-1], col=1:5, lty=1:5, pch=1:5)
  ## plot the first and last smoothed wavefronts to show difference is completely symmetrical
 plotn(psfit, fiti, wftype="smooth", qt=c(0,1))
 detach(df.annfits)
 print(df.annfits, digits=2)
```

zapm

Zernike Annular polynomials

Description

Create a matrix of Zernike Annular polynomial values in extended Fringe sequence for a set of polar coordinates.

Usage

```
zapm(rho, theta, eps, maxorder = 12L, nq = maxorder/2L + 5L)
```

Arguments

```
rho a vector of radial coordinates with eps \leq rho \leq 1.
```

theta a vector of angular coordinates, in radians.

zapm 61

```
eps the obstruction fraction 0 \le \text{eps} < 1.

maxorder the maximum radial polynomial order (defaults to 12).

ng the number of quadrature points for numerical integration
```

Details

The *radial* polynomials are calculated using recurrence relations generated numerically using chebyshev's algorithm with modified moments. See the documentation for rzernike_ann(). A formal presentation is included in the package documentation.

Value

a matrix of Zernike Annular polynomial values evaluated at the input polar coordinates and all radial orders from 0 through maxorder in Fringe sequence, with orthonormal scaling.

```
sample_az <- function(maxorder=12, eps=0.33, col=rev(zernike::rygcb(400)), addContours=TRUE, cscale=TRUE) {</pre>
  ## get coordinates for unobstructed and obstructed apertures
  cpa <- cp.default</pre>
  cpa$obstruct <- eps</pre>
  prt <- pupil.rhotheta(nrow.default,ncol.default,cp.default)</pre>
  prta <- pupil.rhotheta(nrow.default,ncol.default,cp=cpa)</pre>
  rho0 <- prt$rho[!is.na(prt$rho)]</pre>
  theta0 <- prt$theta[!is.na(prt$theta)]</pre>
  rhoa <- prta$rho[!is.na(prta$rho)]</pre>
  thetaa <- prta$theta[!is.na(prta$theta)]</pre>
  ## fill up matrixes of Zernikes and Annular Zernikes
  zm <- zpmC(rho0, theta0, maxorder=maxorder)</pre>
  zam <- zapm(rhoa, thetaa, eps=eps, maxorder=maxorder, ng=maxorder/2+5)
  ## pick a column at random and look up its index pair
  zlist <- makezlist(0, maxorder)</pre>
  i <- sample(2:ncol(zm), 1)</pre>
  n <- zlist$n[i]</pre>
  m <- zlist$m[i]</pre>
  ## fill up the wavefront representations and plot them
  wf0 <- prt$rho
  wf0[!is.na(wf0)] \leftarrow zm[,i]
  class(wf0) <- "pupil"</pre>
  wfa <- prta$rho
  wfa[!is.na(wfa)] <- zam[,i]</pre>
  class(wfa) <- "pupil"</pre>
```

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```
plot(wf0, cp=cp.default, col=col, addContours=addContours, cscale=cscale)
mtext(paste("Zernike, n =", n, " m =", m))

x11()
plot(wfa, cp=cpa, col=col, addContours=addContours, cscale=cscale)
mtext(paste("Annular Zernike, n =", n, " m =", m))

## return Zernike matrices and wavefronts invisibly
## just in case user wants to do something with them
invisible(list(zm=zm, wf0=wf0, zam=zam, wfa=wfa))
}
sample_az()
```

zapm_128

Zernike Annular polynomials, extended precision version

Description

Create a matrix of Zernike Annular polynomial values in extended Fringe sequence for a set of polar coordinates.

Usage

```
zapm_128(rho, theta, eps, maxorder = 12L, nq = maxorder/2L + 5L)
```

Arguments

rho a vector of radial coordinates with eps \leftarrow rho \leftarrow 1.

theta a vector of angular coordinates, in radians.

eps the obstruction fraction $0 \le eps \le 1$.

maxorder the maximum radial polynomial order (defaults to 12).

nq the number of quadrature points for numerical integration

Details

The *radial* polynomials are calculated using recurrence relations generated numerically using chebyshev's algorithm with modified moments. See the documentation for rzernike_ann(). A formal presentation is included in the package documentation.

Value

a matrix of Zernike Annular polynomial values evaluated at the input polar coordinates and all radial orders from 0 through maxorder in Fringe sequence, with orthonormal scaling.

zapm_128 63

```
sample_az_128 <- function(maxorder=12, eps=0.33, col=rev(zernike::rygcb(400)), addContours=TRUE, cscale=TRUE) {</pre>
  ## get coordinates for unobstructed and obstructed apertures
  cpa <- cp.default</pre>
  cpa$obstruct <- eps</pre>
  prt <- pupil.rhotheta(nrow.default,ncol.default,cp.default)</pre>
  prta <- pupil.rhotheta(nrow.default,ncol.default,cp=cpa)</pre>
  rho0 <- prt$rho[!is.na(prt$rho)]</pre>
  theta0 <- prt$theta[!is.na(prt$theta)]</pre>
  rhoa <- prta$rho[!is.na(prta$rho)]</pre>
  thetaa <- prta$theta[!is.na(prta$theta)]</pre>
  ## fill up matrixes of Zernikes and Annular Zernikes
  zm <- zpmC(rho0, theta0, maxorder=maxorder)</pre>
  zam <- zapm_128(rhoa, thetaa, eps=eps, maxorder=maxorder, nq=maxorder/2+5)
  ## pick a column at random and look up its index pair
  zlist <- makezlist(0, maxorder)</pre>
  i <- sample(2:ncol(zm), 1)</pre>
  n <- zlist$n[i]</pre>
  m <- zlist$m[i]</pre>
  ## fill up the wavefront representations and plot them
  wf0 <- prt$rho
  wf0[!is.na(wf0)] <- zm[,i]
  class(wf0) <- "pupil"</pre>
  wfa <- prta$rho
  wfa[!is.na(wfa)] <- zam[,i]
  class(wfa) <- "pupil"</pre>
  plot(wf0, cp=cp.default, col=col, addContours=addContours, cscale=cscale)
  mtext(paste("Zernike, n =", n, " m =", m))
  x11()
  plot(wfa, cp=cpa, col=col, addContours=addContours, cscale=cscale)
  mtext(paste("Annular Zernike, n =", n, " m =", m))
  ## return Zernike matrices and wavefronts invisibly
  ## just in case user wants to do something with them
  invisible(list(zm=zm, wf0=wf0, zam=zam, wfa=wfa))
}
sample_az_128()
```

zapm_iso

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Zernike Annular polynomials from formulas

Description

Create a matrix of Zernike Annular polynomial values for the counterparts of primary and secondary optical aberrations.

Usage

```
zapm_direct(rho, theta, eps)
```

Arguments

rho a vector of radial coordinates with eps \leq rho \leq 1.

theta a vector of angular coordinates, in radians. eps the obstruction fraction $0 \le eps < 1$.

Details

The values are from published formulas. This function is included for testing and reference only.

Value

```
a length(rho) x 16 matrix of Zernike annular polynomial values.
```

See Also

Calls rzernike_ann_direct() for radial Zernike annular values. Use zapm() or zapm_128() for complete sequences of values to arbitrary order.

zapm_iso

Zernike Annular polynomials, ISO ordering

Description

Create a matrix of Zernike Annular polynomial values in ISO/ANSI sequence for a set of polar coordinates.

Usage

```
zapm_iso(rho, theta, eps, maxorder = 12L, nq = maxorder/2L + 5L)
```

zapm_iso 65

Arguments

rho a vector of radial coordinates with eps <= rho <= 1. theta a vector of angular coordinates, in radians. eps the obstruction fraction 0 <= eps < 1. maxorder the maximum radial and azimuthal polynomial order (defaults to 12). nq the number of quadrature points for numerical integration

Details

The *radial* polynomials are calculated using recurrence relations generated numerically using chebyshev's algorithm with modified moments. See the documentation for rzernike_ann(). A formal presentation is included in the package documentation.

Value

a matrix of Zernike Annular polynomial values evaluated at the input polar coordinates and all radial orders from 0 through maxorder in ISO/ANSI sequence, with orthonormal scaling.

```
sample_az_iso <- function(maxorder=12, eps=0.33, col=rev(zernike::rygcb(400)), addContours=TRUE, cscale=TRUE) {</pre>
  ## get coordinates for unobstructed and obstructed apertures
  cpa <- cp.default</pre>
  cpa$obstruct <- eps</pre>
  prt <- pupil.rhotheta(nrow.default,ncol.default,cp.default)</pre>
  prta <- pupil.rhotheta(nrow.default,ncol.default,cp=cpa)</pre>
  rho0 <- prt$rho[!is.na(prt$rho)]</pre>
  theta0 <- prt$theta[!is.na(prt$theta)]</pre>
  rhoa <- prta$rho[!is.na(prta$rho)]</pre>
  thetaa <- prta$theta[!is.na(prta$theta)]</pre>
  ## fill up matrixes of Zernikes and Annular Zernikes
  zm <- zpm_cart(x=rho0*cos(theta0), y=rho0*sin(theta0), maxorder=maxorder)</pre>
  zam <- zapm_iso(rhoa, thetaa, eps=eps, maxorder=maxorder, nq=maxorder/2+5)
  ## pick a column at random and look up its index pair
  zlist <- makezlist.iso(maxorder)</pre>
  i <- sample(2:ncol(zm), 1)</pre>
  n <- zlist$n[i]</pre>
  m <- zlist$m[i]</pre>
  ## fill up the wavefront representations and plot them
  wf0 <- prt$rho
  wf0[!is.na(wf0)] \leftarrow zm[,i]
  class(wf0) <- "pupil"</pre>
```

2apm_iso_128

```
wfa <- prta$rho
wfa[!is.na(wfa)] <- zam[,i]
class(wfa) <- "pupil"

plot(wf0, cp=cp.default, col=col, addContours=addContours, cscale=cscale)
mtext(paste("Zernike, n =", n, " m =", m))

x11()
plot(wfa, cp=cpa, col=col, addContours=addContours, cscale=cscale)
mtext(paste("Annular Zernike, n =", n, " m =", m))

## return Zernike matrices and wavefronts invisibly
## just in case user wants to do something with them
invisible(list(zm=zm, wf0=wf0, zam=zam, wfa=wfa))
}
sample_az_iso()</pre>
```

zapm_iso_128

Zernike Annular polynomials, ISO ordering - extended precision version

Description

Create a matrix of Zernike Annular polynomial values in ISO/ANSI sequence for a set of polar coordinates.

Usage

```
zapm_iso_128(rho, theta, eps, maxorder = 12L, nq = maxorder/2L + 5L)
```

Arguments

rho a vector of radial coordinates with eps \leq rho \leq 1.

theta a vector of angular coordinates, in radians.

eps the obstruction fraction $0 \le eps \le 1$.

maxorder the maximum radial and azimuthal polynomial order (defaults to 12).

nq the number of quadrature points for numerical integration

Details

The *radial* polynomials are calculated using recurrence relations generated numerically using chebyshev's algorithm with modified moments. See the documentation for rzernike_ann(). A formal presentation is included in the package documentation.

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Value

a matrix of Zernike Annular polynomial values evaluated at the input polar coordinates and all radial orders from 0 through maxorder in ISO/ANSI sequence, with orthonormal scaling.

Examples

}

```
sample_az_iso_128 <- function(maxorder=12, eps=0.33, col=rev(zernike::rygcb(400)), addContours=TRUE, cscale=TRU
  ## get coordinates for unobstructed and obstructed apertures
  cpa <- cp.default
  cpa$obstruct <- eps</pre>
  prt <- pupil.rhotheta(nrow.default,ncol.default,cp.default)</pre>
  prta <- pupil.rhotheta(nrow.default,ncol.default,cp=cpa)</pre>
  rho0 <- prt$rho[!is.na(prt$rho)]</pre>
  theta0 <- prt$theta[!is.na(prt$theta)]</pre>
  rhoa <- prta$rho[!is.na(prta$rho)]</pre>
  thetaa <- prta$theta[!is.na(prta$theta)]</pre>
  ## fill up matrixes of Zernikes and Annular Zernikes
  zm <- zpm_cart(x=rho0*cos(theta0), y=rho0*sin(theta0), maxorder=maxorder)</pre>
  zam <- zapm_iso_128(rhoa, thetaa, eps=eps, maxorder=maxorder, nq=maxorder/2+5)</pre>
  ## pick a column at random and look up its index pair
  zlist <- makezlist.iso(maxorder)</pre>
  i <- sample(2:ncol(zm), 1)</pre>
  n <- zlist$n[i]</pre>
  m <- zlist$mΓi]
  ## fill up the wavefront representations and plot them
  wf0 <- prt$rho
  wf0[!is.na(wf0)] \leftarrow zm[,i]
  class(wf0) <- "pupil"</pre>
  wfa <- prta$rho
  wfa[!is.na(wfa)] <- zam[,i]
  class(wfa) <- "pupil"</pre>
  plot(wf0, cp=cp.default, col=col, addContours=addContours, cscale=cscale)
  mtext(paste("Zernike, n =", n, " m =", m))
  x11()
  plot(wfa, cp=cpa, col=col, addContours=addContours, cscale=cscale)
  mtext(paste("Annular Zernike, n =", n, " m =", m))
  ## return Zernike matrices and wavefronts invisibly
  ## just in case user wants to do something with them
  invisible(list(zm=zm, wf0=wf0, zam=zam, wfa=wfa))
```

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```
sample_az_iso_128()
```

zapm_iso_direct

Zernike Annular polynomials from formulas

Description

Create a matrix of Zernike Annular polynomial values complete through radial and azimuthal orders 6 in ISO/ANSI sequence.

Usage

```
zapm_iso_direct(rho, theta, eps)
```

Arguments

rho a vector of radial coordinates with eps \leftarrow rho \leftarrow 1.

theta a vector of angular coordinates, in radians. eps the obstruction fraction $0 \le eps < 1$.

Details

The values are from published formulas. This function is included for testing and reference only.

Value

a length(rho) x 28 matrix of Zernike annular polynomial values.

See Also

Calls rzernike_ann_direct() for radial Zernike annular values. Use zapm_iso() or zapm_iso_128() for complete sequences of values to arbitrary order.

```
sample_az_iso_direct <- function(eps=0.33, col=rev(zernike::rygcb(400)), addContours=TRUE, cscale=TRUE) {

## get coordinates for obstructed aperture
cp <- cp.default
cp$obstruct <- eps
prt <- pupil.rhotheta(nrow.default,ncol.default,cp=cp)
rho <- prt$rho[!is.na(prt$rho)]
theta <- prt$theta[!is.na(prt$theta)]

## fill up matrixes of Annular Zernikes using direct and recursive formulas

zam_dir <- zapm_iso_direct(rho, theta, eps=eps)</pre>
```

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```
zam_iso <- zapm_iso(rho, theta, eps=eps, maxorder=6)</pre>
## pick a column at random and look up its index pair
zlist <- makezlist.iso(6)</pre>
i <- sample(2:ncol(zam_iso), 1)</pre>
n <- zlist$n[i]</pre>
m <- zlist$m[i]</pre>
## fill up the wavefront representations and plot them
wf_dir <- prt$rho
wf_dir[!is.na(wf_dir)] <- zam_dir[,i]</pre>
class(wf_dir) <- "pupil"</pre>
wf_zapm <- wf_dir
wf_zapm[!is.na(wf_zapm)] <- zam_iso[,i]</pre>
class(wf_zapm) <- "pupil"</pre>
plot(wf_dir, cp=cp, col=col, addContours=addContours, cscale=cscale)
mtext(paste("Annular Zernike from formula, n =", n, " m =", m))
x11()
plot(wf_zapm, cp=cp, col=col, addContours=addContours, cscale=cscale)
mtext(paste("Annular Zernike from `zapm_iso`, n =", n, " m =", m))
cat(paste("Sample wavefronts: SD of diffs", format(sd(wf_dir-wf_zapm, na.rm=TRUE), digits=6), "; range of diffs",
     format(min(wf_zapm-wf_dir, na.rm=TRUE), digits=6),":",
     format(max(wf_zapm-wf_dir, na.rm=TRUE), digits=6),"\n\n"))
cat(paste("Zernike matrixes: SD of diffs", format(sd(zam_dir-zam_iso), digits=6), "; range of diffs",
format(min(zam_dir-zam_iso), digits=6),":",
format(max(zam_dir-zam_iso), digits=6),"\n"))
## return Zernike matrices and wavefronts invisibly
## just in case user wants to do something with them
invisible(list(zam_dir=zam_dir, zam_iso=zam_iso, wf_dir=wf_dir, wf_zapm=wf_zapm))
sample_az_iso_direct()
```

zconic

Zernike coefficients for a conic surface

Description

Calculates the radially symmetric Zernike coefficient values up to order nmax for a conic surface relative to a sphere of the same paraxial radius of curvature.

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Usage

```
zconic(D, rc, b = -1, lambda = 1e-06, nmax = 6)
```

Arguments

D Diameter

rc Radius of curvature
b Conic constant

1ambda Wavelength – defaults to 632.8 (nm)

nmax Maximum radial polynomial order. Defaults to 6.

Details

For the typical use case D and rc should be in millimeters and lambda in nanometers.

Value

A vector of length nmax/2-1 of coefficient values, in increasing radial order, n=c(4,6, ...).

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

See Also

```
Zernike, sconic
```

Examples

```
zconic(200,2000)
zconic(10, 20, b=-1.05, lambda=632.8E-9, nmax=12)
```

Zernike

Zernike Polynomials

Description

Routines for creating and manipulating Zernike polynomials.

Usage

```
Zernike(rho, theta, n, m, t)
rzernike(rho, n, m)
drzernike(rho, n, m)
```

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Arguments

rho	normalized radius, $0 \le rho \le 1$
theta	angular coordinate
n	radial polynomial order
m	azimuthal order
t	character for trig function: one of c("n", "c", "s")

Note

These functions return Zernikes scaled such that they form an orthonormal basis set for the space of functions defined on the unit circle. Note that this is not the most commonly used definition (as given e.g. in *Born and Wolf*). The definition I use is often associated with *Noll* (1976).

The function zmult can be used to convert between normalized and conventionally defined vectors of Zernike coefficients.

The basic low level functions rzernike and drzernike use numerically stable recurrence relationships for the radial Zernikes.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

References

Born, M. and Wolf, E. 1999, *Principles of Optics*, 7th Edition, Cambridge University Press, chapter 9 and appendix VII.

Noll, R.J. 1976, **Zernike polynomials and atmospheric turbulence**, *J. Opt. Soc. Am.*, Vol. 66, No. 3, p. 207.

```
http://wyant.opt-sci.arizona.edu/zernikes/zernikes.htm
http://mathworld.wolfram.com/ZernikePolynomial.html
```

See Also

```
makezlist, zlist.fr, zmult, zpm, pupil, pupilrms, pupilpv, strehlratio.
```

```
Zernike(1, 0, 4, 0, "n") # == sqrt(5)

# A slightly more complex example

rho <- seq(0, 1, length = 101)
theta <- rep(0, 101)

plot(rho, Zernike(rho, theta, 6, 0, "n"), type="1",
   ylim=c(-3.5,3.5), main="Some 6th order Zernike Polynomials")
lines(rho, Zernike(rho, theta, 5, 1, "c"), lty=2)
lines(rho, Zernike(rho, theta, 4, 2, "c"), lty=3)
lines(rho, Zernike(rho, theta, 3, 3, "c"), lty=4)</pre>
```

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zlist

Lists of Zernike polynomial indexes

Description

Ordered lists of Zernike polynomial indexes.

Usage

```
makezlist(minorder = 2, maxorder = 14)
zlist.fr
zmult(zlist = makezlist())
```

Arguments

minorder minimum value of n+m maxorder maximum value of n+m

zlist a list of the form returned by makezlist

Details

Zernike polynomials are indexed by a radial index n, an azimuthal index m, and include cosine, sine, and radial terms. These routines return lists of indexes using a popular ordering scheme for Zernike polynomials.

Value

makezlist and zlist.fr return lists with the following components:

```
n radial order
m azimuthal order
t one of c("c", "s", "n")
```

zmult returns a vector the same length as the components of zlist.

Note

```
zlist.fr is an augmented "Fringe" set of Zernike polynomials equivalent to makezlist(2,12). makezlist returns a complete list of indexes for all orders from minorder through maxorder, where "order" is the value of n+m.
```

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

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See Also

Virtually all high level functions that work with Zernike polynomials use these lists. See for example pupil, psifit, fftfit.

Examples

```
zlist <- makezlist(2,12)
zcoef <- rnorm(length(zlist))
zcoef # a vector of normalized Zernike coefficients
zcoef*zmult(zlist) # Coefficients in conventional representation
sqrt(crossprod(zcoef)) # This is the RMS error of the wavefront
# constructed from these Zernikes</pre>
```

 ${\tt zmoments}$

Zernike moments

Description

Calculate Zernike moments from a vector of coefficients

Usage

```
zmoments(zcoef, maxorder = 14)
```

Arguments

zcoef Zernike coefficients

maxorder Maximum order to return

Value

A table of the moments along with radial and azimuthal orders

References

M.L. Peck

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zpm Mat	rixes of Zernike polynomials
---------	------------------------------

Description

Create a matrix of Zernike polynomial values.

Usage

```
zpm(rho, theta, phi= 0 , maxorder = 14, nthreads=parallel::detectCores()/2)
zpmC(rho, theta, maxorder)
zpmCP(rho, theta, maxorder)
zpm.arb(rho, theta, phi = 0, zlist = makezlist())
```

Arguments

rho A vector of radial coordinates.

theta A vector of angular coordinates, in radians.

phi Orientation of the image, in degrees

zlist A list of indexes, as returned by makezlist
maxorder The maximum Zernike polynomial order
nthreads Number of threads for threaded function call

Details

rho and theta must be the same length.

Value

zpm. arb returns a matrix of size length(rho) x length(zlist\$n) with values of Zernike polynomials evaluated at the polar coordinates (rho, theta-pi*phi/180).

zpm, zpmC, and zpmCP return a matrix of size length(rho) x (maxorder/2+1) 2 of Zernike polynomial values including a piston term.

Note

These are used by various routines to make least squares fits of sets of Zernike polynomials to measured wavefront values.

zpmC is the C++ routine that does the computations for zpm. No column names are returned.

Threaded computation of the matrix is now available using zpmCP.

Author(s)

```
M.L. Peck <mpeck1@ix.netcom.com>
```

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See Also

Zernike, makezlist, zlist.fr, fitzernikes, zapm for Annular Zernikes.

zpm_cart Zernike polynomials

Description

Calculate Zernike polynomial values in ISO/ANSI sequence for a set of Cartesian coordinates.

Usage

```
zpm_cart(x, y, maxorder = 12L, unit_variance = TRUE)
```

Arguments

x a vector of x coordinates for points on a unit disk.

y a vector of y coordinates.

maxorder the maximum radial polynomial order (defaults to 12).
unit_variance logical: return with orthonormal scaling? (default true)

Details

This is the same algorithm and essentially the same code as <code>gradzpm_cart()</code> except directional derivatives aren't calculated.

Value

a matrix of Zernike polynomial values evaluated at the input Cartesian coordinates and all radial and azimuthal orders from 0 through maxorder.

```
##illustrates difference in smoothed wavefront from using zpm_cart with ISO sequence of same order
require(zernike)
fpath <- file.path(find.package(package="zernike"), "psidata")
files <- scan(file.path(fpath, "files.txt"), what="character")
for (i in 1:length(files)) files[i] <- file.path(fpath, files[i])

## load the images into an array
images <- load.images(files)

## parameters for this run
source(file.path(fpath, "parameters.txt"))</pre>
```

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```
## phase shifts

phases <- wrap((0:(dim(images)[3]-1))/frames.per.cycle*2*pi)
phases <- switch(ps.dir, ccw = -phases, cw = phases, phases)

## target SA coefficients for numerical null.

sa.t <- sconic(diam,roc,lambda=wavelength)
zopt <- psfit_options()
zopt$satarget <- sa.t
psfit <- psifit(images, phases, psialg="ls", options=zopt)

## get back the raw wavefront

wf.raw <- qpuw(psfit$phi, psfit$mod)

## This will tell wf_net to use zpm_cart instead

zopt$isoseq <- TRUE
ifit <- wf_net(wf.raw, cp = psfit$cp, options=zopt)

## plotn does a direct comparison

plotn(psfit, ifit, wftype="smooth", qt=c(0,1))</pre>
```

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