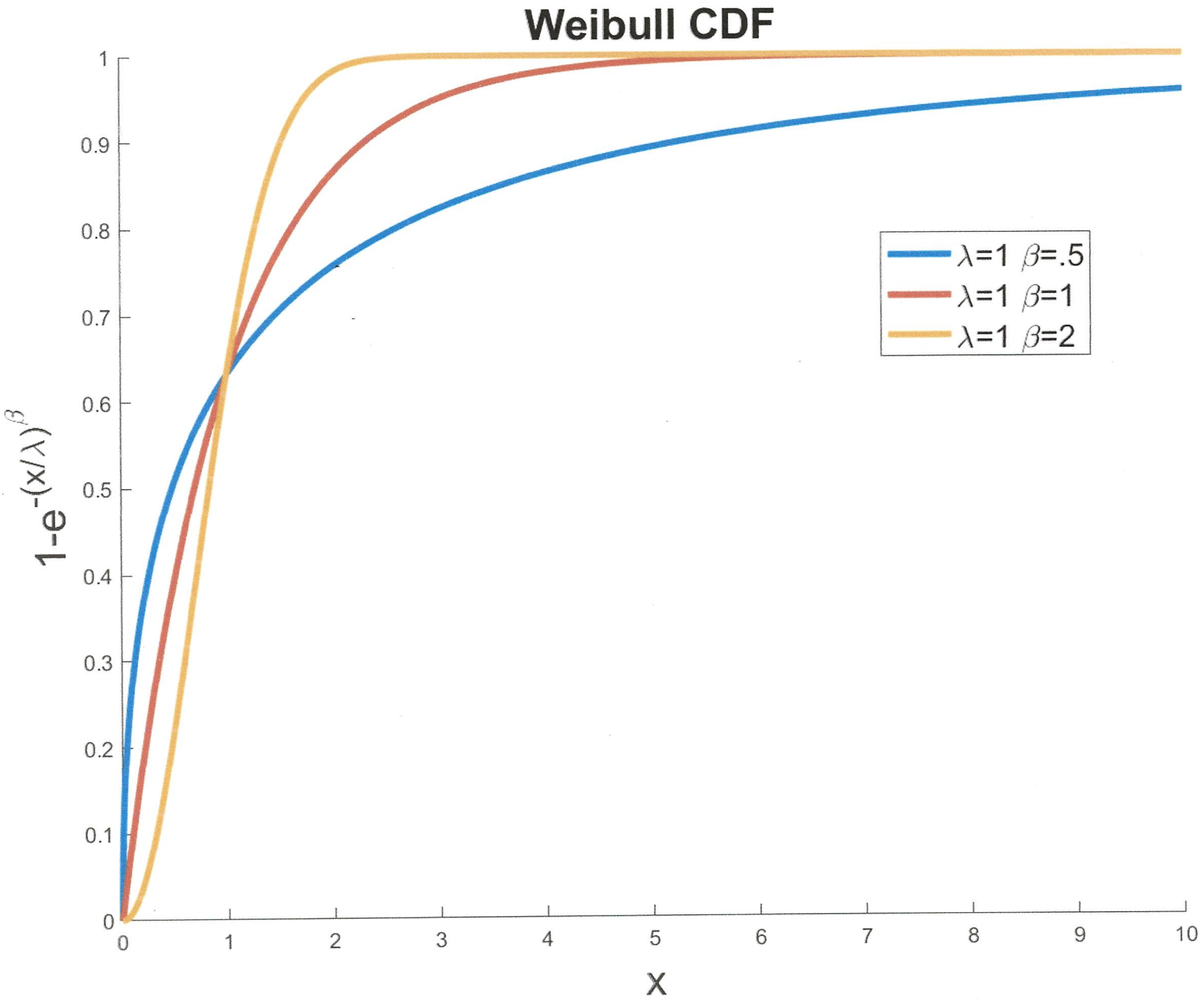
1

a



Weibull CDF
$1-e^{-(x/\lambda)^\beta}$
x
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7
8
9
10
λ=1 β=.5
λ=1 β=1
λ=1 β=2

(1).

(b) $F((k+1)\lambda) - F(k\lambda)$

$1-e^{-((k+1)\lambda/\lambda)^{\beta}} - \left(1-e^{-(k\lambda/\lambda)^{\beta}}\right)$

$1-e^{-(k+1)^{\beta}} - 1 + e^{-k^{\beta}}$

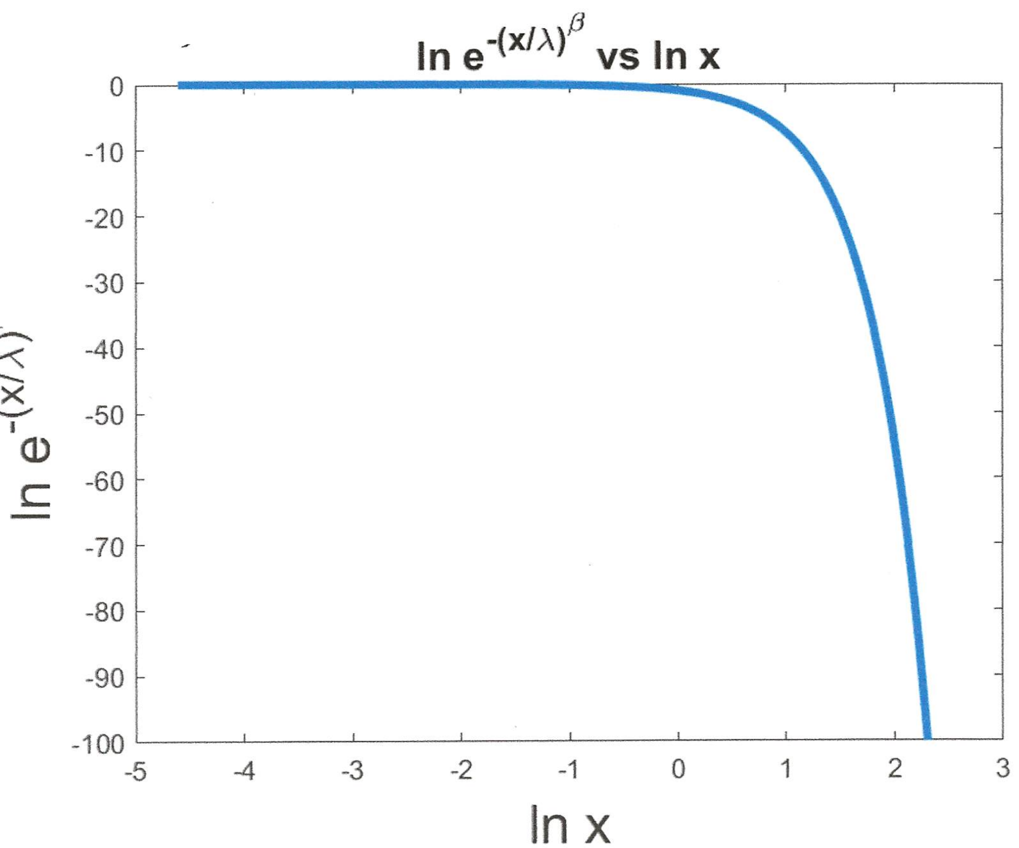
$$\boxed{e^{-k^{\beta}} - e^{-(k+1)^{\beta}} = P[k\lambda < X < (k+1)\lambda]}$$

$$\boxed{P[X>k\lambda]} = 1 - P[X \leq k\lambda]$$

$= 1 - \left(1-e^{-(k\lambda/\lambda)^{\beta}}\right)$

$$= \boxed{e^{-k^{\beta}}}$$

(c)



$\ln e^{-(x/\lambda)^{\beta}}$ vs $\ln x$
$\ln e^{-(x/\lambda)^{\beta}}$
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-5
-4
-3
-2
-1
0
1
2
3
ln x

(2)

$P(1in) = P(X \leq 1) = \frac{1}{10}$

$P(3in) = P(1 < X \leq 3) = \frac{2}{10}$

$P(5in) = P(3 < X \leq 5) = \frac{2}{10}$

$P(\text{greater } 5) = P(X > 5) = \frac{5}{10}$

$E[\text{points}] = \left(\frac{1}{10}\right)10 + \left(\frac{2}{10}\right)5 + \left(\frac{2}{10}\right)3 + \left(\frac{5}{10}\right)0$

$= \frac{13}{5} = 2.6$ points

(3) CDF $= \begin{cases} 1-e^{-\lambda x} & x>0 \\ 0 & \le 0 \end{cases}$

$$P(X>2) = 1-(1-e^{-\frac{1}{2}(2)})$$
$$= e^{-1}$$

$$\boxed{P(X>2\,hr) \quad .3679 = e^{-1}}$$

(5) exponential distribution is memoryless... so conditional doesn't matter

$$P(X>s+t \mid X>t) = \frac{P(X>s+t \text{ and } X>t)}{P(X>t)}$$

$$= \frac{P(X>s+t)}{P(X>t)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda t}} = e^{-\lambda s-\lambda t+\lambda t}$$

$$= e^{-\lambda s}$$

$t=9 \quad s=1$

$$\boxed{\text{thus} \quad P(X\geq 10 \mid X\geq 9) = e^{-\frac{1}{2}} = .6065}$$

(4.) $\overset{(Y)}{X} = -\ln(4-4\overset{(X)}{U})$

$U \sim [0,1]$

$$f_X(x)\left|\frac{dx}{dy}\right|\Bigg|_{U=g^{-1}(x)}$$

$$e^{-x} = 4-4U \qquad f_X(x) = 1 \text{ (uniform dist)}$$

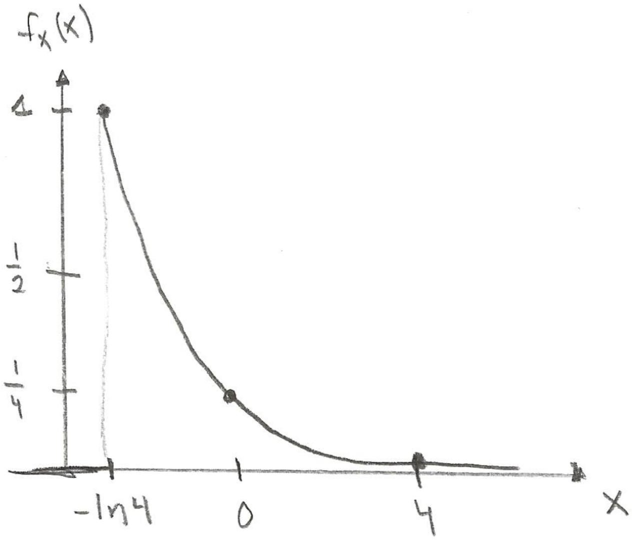
$$4U = -e^{-x} + 4$$

$$U = 1 - \frac{e^{-x}}{4}$$

$$\frac{du}{dx} = \frac{e^{-x}}{4}$$

(a) $\boxed{\text{pdf of } X = \frac{e^{-x}}{4} \quad \text{from } -\ln 4 \le x \le \infty \quad (-\ln 0 = \infty) \quad 0 \text{ otherwise}}$

(b)



(5)

$\mathbb{E}$[split containing p]

$$\int_0^p (1-u)\,du + \int_p^1 u\,du$$

$$\left[u - \frac{u^2}{2}\right]_0^p + \left[\frac{u^2}{2}\right]_p^1$$

$$p - \frac{p^2}{2} + \frac{1}{2} - \frac{p^2}{2}$$

$$p - p^2 + \frac{1}{2}$$

$$\boxed{\mathbb{E}\left[\begin{matrix}\text{length of}\\ \text{piece w/}\\ p\end{matrix}\right] = p(1-p) + \frac{1}{2}}$$