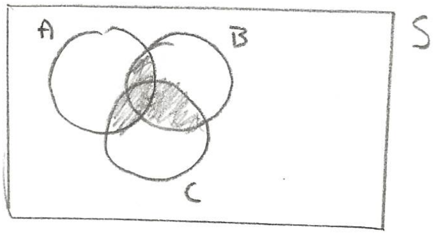
...chael Potter

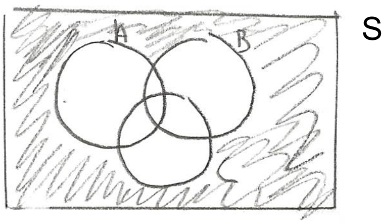
Homework 1

1

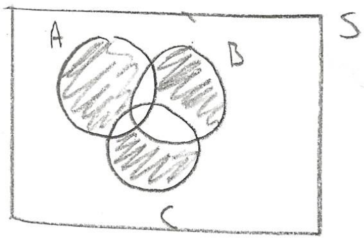
a. $(A\cap B)\cup(A\cap C)\cup(B\cap C)$



b. $(A\cup B\cup C)^c$



c. $(A\cap B^c\cap C^c)\cup(A^c\cap B\cap C^c)\cup(A^c\cap B^c\cap C)$



2

a. $\underbrace{P(A\cap B)}_{LHS} \geq \underbrace{P(A) + P(B) - 1}_{RHS}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

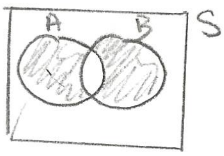
$P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$P(A) + P(B) - P(A\cup B) \geq P(A) + P(B) - 1$

$0 \leq P(A\cup B) \leq 1$ by axioms + corollaries

since $0 \leq P(A\cup B) \leq 1$, then $P(A\cap B) = \boxed{\underbrace{P(A)+P(B)-P(A\cup B)}_{LHS} \text{ must be } \geq \underbrace{P(A)+P(B)-1}_{RHS}}$

b.



$P[(A \cap B^c) \cup (A^c \cap B)]$

Axiom 3 $\quad (A \cap B^c) \cap (A^c \cap B) = \emptyset$

so $\quad P(A \cap B^c) + P(A^c \cap B)$

LHS

plug back in

$A = (A \cap B) \cup (A \cap B^c) \rightarrow P(A) = P(A \cap B) + P(A \cap B^c) \rightarrow P(A \cap B^c) = P(A) - P(A \cap B)$

$B = (A \cap B) \cup (A^c \cap B) \rightarrow P(B) = P(A \cap B) + P(A^c \cap B) \rightarrow P(A^c \cap B) = P(B) - P(A \cap B)$

Axiom 3

$\underset{\text{LHS}}{P(A \cap B^c) + P(A^c \cap B)} = P(A) - P(A \cap B) + P(B) - P(A \cap B) = P(A) + P(B) - 2P(A \cap B) = \text{RHS}$

$P(A) + P(B) - 2P(A$

(3)

$30030 = 2^1 \times 3^1 \times 5^1 \times 7^1 \times 11^1 \times 13^1$

To have an even ~~f~~factor... it must be a factor of 2

To get other even divisors, may be expressed as combinations of 2 and other prime numbers ...

① 2 $\binom{5}{0}$

② 2×3 $\binom{5}{1}$

5

$\vdots$

13

③ $2 \times \underbrace{3 \times 5}_{\text{choose next 2}}$ $\binom{5}{2}$

$\times 3 \times 7$

$\vdots$

④ $2 \times \underbrace{3 \times 5 \times 7}_{\text{choose next 3}}$ $\binom{5}{3}$

$\times 3 \times 7 \times 11$

$\vdots$

⑤ $2 \times \underbrace{3 \times 5 \times 7 \times 11}_{\text{choose next 4}}$ $\binom{5}{4}$

$\times 7 \times 11 \times 13 \times 5$

$\vdots$

⑥ $2\times$ $\underbrace{\text{all the remaining digits}}_{\text{choose next 5 digits}}$ $\binom{5}{5}$

total # of even factors $= \binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$

$= 1 + 5 + 10 + 10 + 5 \quad 1$

$\boxed{= 32 \text{ positive even integer divisors}}$

(4)

a. at least Two successive tails =

1 − (no two successive tails)

{THHT, HTHT, THTH, $\binom{4}{1}$, HHHH} = $\boxed{1 - \frac{8}{2^4} = 0.5 = P(\text{at least two successive tails})}$

two tails — one tail — no tails

total count = 2^4 (sample w/ replacement w/ order)

b. one head in first 3 tosses =
{HTTT THTT TTHT; HTTH THTH TTHH} → $\binom{3}{1} \cdot 2$

→ choose pos of head in first 3 pos
→ choose head or tail for last pos

$\boxed{P(\text{seeing exactly one head in first 3 tosses}) = \frac{3 \cdot 2}{2^4} = .3750}$

(5)

N distinguishable balls

total count N^N — N → ball i turn N; number of bins I can assign ball i

one empty box = $N \cdot (N-1) \cdot \binom{N}{2} \cdot (N-2)!$

choose 1 box out of N−1 boxes have 2 balls

one of th boxes is empty

the order of the remaining balls in the remaining boxes

which 2 balls belong in the chosen box w/ 2 b

$\boxed{P(\text{exactly one empty box}) = \frac{N \cdot (N-1) \binom{N}{2} \cdot (N-2)!}{N^N} = \frac{\binom{N}{2} N!}{N^N}}$

N undistinguishable balls

total count $\binom{N+N-1}{N-1}$ one empty box = $N \cdot (N-1)$

choose empty box

Choose which of remaining boxes has two balls

$\boxed{\frac{N \cdot (N-1)}{\binom{N+N-1}{N-1}} = P(\text{exactly one empty box})}$

(6) total count is 6^6

Three diff numbers each appear twice = $\left(\frac{6!}{2!2!2!}\right)\binom{6}{3}$

→ choose which 3 out of 6 digits appear twice

→ the specific dice that take on the digits

$\boxed{\frac{\left(\frac{6!}{2!2!2!}\right)\binom{6}{3}}{6^6} = \frac{1800}{46656} = 0.0386}$

6·5·4