

①

(1 1 1)	(2 1 1)	(3 1 1)
(1 1 2)	(2 1 2)	(3 1 2)
(1 1 3)	(2 1 3)	(3 1 3)
(1 2 1)	(2 2 1)	(3 2 1)
(1 2 2)	(2 2 2)	(3 2 2)
(1 2 3)	(2 2 3)	(3 2 3)
(1 3 1)	(2 3 1)	(3 3 1)
(1 3 2)	(2 3 2)	(3 3 2)
(1 3 3)	(2 3 3)	(3 3 3)

1	2	3
0	1	2
1	0	1
2	3	4
1	2	3
3	1	2
0	4	0
2	3	4
1	2	3

a) $P(X=1) = \frac{6}{27}$

$P(X=2) = \frac{7}{27}$

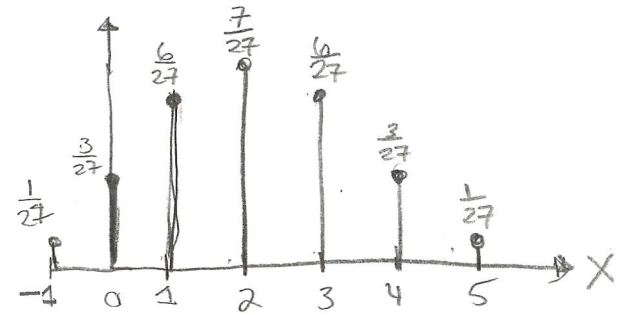
$P(X=3) = \frac{6}{27}$

$P(X=4) = \frac{3}{27}$

$P(X=5) = \frac{1}{27}$

$P(X=0) = \frac{3}{27}$

$P(X=-1) = \frac{1}{27}$



7+3

b) $E[X] = (-1)\left(\frac{1}{27}\right) + (0)\left(\frac{3}{27}\right) + (1)\left(\frac{6}{27}\right) + (2)\left(\frac{7}{27}\right) + (3)\left(\frac{6}{27}\right) + (4)\left(\frac{3}{27}\right) + (5)\left(\frac{1}{27}\right)$
 $= 2.00$

$Var(X) = \frac{1}{27}(-1-2)^2 + \frac{3}{27}(0-2)^2 + \frac{6}{27}(1-2)^2 + \frac{7}{27}(2-2)^2 + \frac{6}{27}(3-2)^2$
 $+ \frac{3}{27}(4-2)^2 + \frac{1}{27}(5-2)^2$
 $= 2.00$

c) $P(|X| \geq 0) = 1$ $P(|X| \geq 3) = \frac{10}{27}$ $P(|X| \geq K) = 1$ for $K < 0$
 $P(|X| \geq 4) = \frac{24}{27}$ $P(|X| \geq 4) = \frac{4}{27}$ $P(|X| \geq K) = 0$ for $K > 5$
 $P(|X| \geq 2) = \frac{17}{27}$ $P(|X| \geq 5) = \frac{1}{27}$

2) CDF of Geometric RV

$$\sum_{k=1}^N p(1-p)^{k-1} \quad k=1, \dots, N \quad \text{can succeed on 1}$$

$$p \sum_{k=1}^N (1-p)^{k-1}$$

$$p \left(\frac{1 - (1-p)^N}{1 - (1-p)} \right)$$

$$p \left(\frac{1 - (1-p)^N}{p} \right)$$

$$1 - (1-p)^N$$

$$1 - (1-p)^N \geq 0.99$$

$$0.01 \geq (1-p)^N$$

$$0.01 \geq \left(\frac{3}{4}\right)^N$$

$$N \geq 17$$

) so 170 seconds before expire time

(3)

$$\sum_{j=1}^{\infty} P(X \geq j) = \sum_{j=1}^{\infty} \sum_{k=j}^{\infty} P(X=k) = \sum_{k=1}^{\infty} \sum_{j=1}^k P(X=k) = \sum_{k=1}^{\infty} k P(X=k) = E[X]$$

as the double sum was over all different integer pairs (j, k)
 s.t. $1 \leq j \leq k < \infty$

(4). $P(E \text{ w/ year}) = p$

profit if A occurs

$\overset{p}{\underbrace{X-A}}$

$\overset{(1-p)}{\underbrace{X}}$

profit

$$E[\text{profit}] = \left(\frac{A}{10} \right) = p(X-A) + (1-p)X$$

$$= pX - pA + X - pX$$

$$\frac{A}{10} = X - pA$$

$$X = pA + \frac{A}{10}$$

$$X = A(p + 0.1)$$

$$(5) \quad \sum_{k=0}^3 \frac{\binom{4}{k} \binom{16}{3-k}}{\binom{20}{3}} \cdot k$$

$$= \frac{560}{1140} \cdot 0 + \frac{480}{1140} \cdot 1 + \frac{96}{1140} \cdot 2 + \frac{4}{1140} \cdot 3$$

$$E[\text{defects in sample}] = 0.600$$

The problem follows a hypergeometric distribution, so use that info...