Practical Session 4

1. Aim of the Session

In this session, you will learn how to find the partial derivative of functions and the integral functions using **SymPy**. You will also learn how to find the global minimum and maximum using **SymPy**. In addition, you will learn more data manipulation with **Pandas**.

2. Preparation

- Click the **window icon** on the bottom left corner:
- Search **Anaconda Navigator** by typing **Anaconda** from the search box under All Programs.
- Click on **Anaconda 3.6 Navigator**
- Click **Launch** under Jupyter from Anaconda Navigator;
- Once the jupyter notebook is open, what you can see are files and/or folders under C:\Docs
- Click **New** from Jupyter, then select **Python 3** from the pop-up list. You should see a new page with a blank cell, where you can type in Python code.
- Click File and select Rename from the pop-up list, you may give the file a name you prefer, for example, practical4.

3. Solving Calculus Problems (continuous)

- Calculating partial derivatives
  For partial differentiation, we assume only one variable varies while the others are fixed. Consider the function $f(m,n) = m^2n + 10mn$, the partial differentiation of $f(m,n)$ with respect to m is:

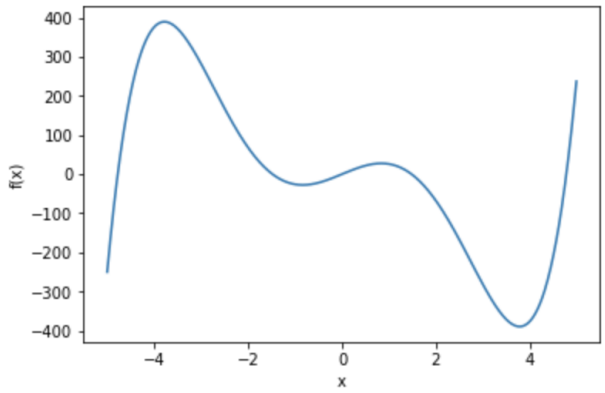
```
m = Symbol('m')
n = Symbol('n')
f = m**2*n +10*m*n
df = Derivative(f,m)
df.doit()
```

```
2*m*n + 10*n
```

  Task1:

  Calculate the partial differentiation of $f(m,n)$ with respect to n. First write down your answer on a paper. Then compare the result obtained from Python with your answer.

- Higher-order derivatives and finding the maxima and minima
  Consider the function $f(x) = x^5 - 25x^3 + 50x$, and the following graph shows the function when x is in the range of [-5, 5]. As one can see, the function has local minimum, local maximum, global minimum and global maximum values. Recall that the first-order derivative of a function is zero at the local or global maximum or minimum, that is $f'(x) = 0$. Solutions of $f'(x) = 0$ are called critical points of the function.



Task2:

Calculate the first-order derivative of $f(x) = x^5 - 25x^3 + 50x$ using **SymPy** and save the result in the variable say, **dy**

Now you can solve $f'(x) = 0$ and find the critical points by doing the following:

```
from sympy import solve

critical_points = solve(dy)
critical_points
```

```
[-sqrt(-sqrt(185)/2 + 15/2),
 sqrt(-sqrt(185)/2 + 15/2),
 -sqrt(sqrt(185)/2 + 15/2),
 sqrt(sqrt(185)/2 + 15/2)]
```

To find out the global maximum or minimum, you need to calculate the second derivative of the function. To do so, you can enter 2 as the third argument of **Derivative()**:

```
dy2 = Derivative(y, x,2).doit()
dy2
```

```
10*x*(2*x**2 - 15)
```

You can find the value of the second derivative by substituting the value of each critical point in place of x. If the resulting value is less than zero, the point is a local maximum; if the value is greater than zero, it is a local minimum. If the resulting value is zero, then we cannot deduce anything about whether the critical point x is a local minimum, maximum, or not.

```
dy2.subs({x:critical_points[0]}).evalf()
```

```
113.738286404346
```

```
dy2.subs({x:critical_points[1]}).evalf()
```

```
-113.738286404346
```

```
dy2.subs({x:critical_points[2]}).evalf()
```

```
-514.357465393284
```

```
dy2.subs({x:critical_points[3]}).evalf()
```

```
514.357465393284
```

It can be seen that critical points 1 and 4 correspond to the local minimum; critical points 2 and 3 corresponding to the local maximum.

The global maximum and minimum of $f(x)$ in the interval [-5, 5] is attained either at a critical point or at one of the endpoints of the domain ($x = -5$ and $x = 5$). To find

the global minimum, you must compute the value of $f(x)$ at the critical point 1, critical point 4, $x = -5$ and $x = 5$. Among these points, the place where $f(x)$ has the largest value must be the global minimum.

```
x_min=-5
x_max=5

y.subs({x:critical_points[0]}).evalf()
```

```
-27.6014252624441
```

```
y.subs({x:critical_points[3]}).evalf()
```

```
-389.535828035730
```

```
y.subs({x:x_min}).evalf()
```

```
-250.000000000000
```

```
y.subs({x:x_max}).evalf()
```

```
250.000000000000
```

As can be seen, the global minimum of $f(x)$ is about -389.54 at critical point 4.

Now please spend a couple of minutes in thinking carefully about how the minimum and maximum values of a function have been found. Can you remember the procedure? If not, you need to go back and have a look at this section again. Otherwise, you may continue.

- Finding the integrals of functions

  Consider to find the integral $\int sxdx$. You can find the integral by creating an object of the **Integral** class first as follows:

```
from sympy import Integral, Symbol

s = Symbol('s')
x = Symbol('x')
Integral(s*x,x)
```

```
Integral(s*x, x)
```

Similar to **Limit** and **Derivative** classes, you can now evaluate the integral using the **doit()** method:

```
Integral(s*x,x).doit()
```

```
s*x**2/2
```

To find the definite integral, for example, $\int_5^0 sxdx$, you can simply specify the variable with the lower and upper limits:

```
from sympy import Integral, Symbol

s = Symbol('s')
x = Symbol('x')
Integral(s*x,(x,0,5)).doit()
```

Task3:

Calculate the integrate $\int_6^2 x\,dx$

1) Do it by yourself and write your answer on a paper
2) Finish it by using **SymPy** and compare with your result.

4. Gradient Ascent Algorithm

Download demo_gradient_ascent.ipynb from Canvas. Open it from your **jupyter** notebook.

This is a generic program for gradient ascent in a simple version.

Task4:

1) Enter function $x^5 - 30 \times x^3 + 50 \times x$, and the variable to differentiate with respect to is x.
2) Observe how the maximum value changes when you enter different initial values. For example, you may enter the first initial value -2, and then 0.5.

5. Datasets

During learning on this module, you need to apply what you have learnt on various datasets in order to understand and memorise processes. This section shows you how and where you can obtain some datasets for you to use.

- Scikit-learn toy datasets

  Scikit-learn (http://scikit-learn.org/stable) is the core of data science operations on Python. You will learn more about it later in this module. The Scikit-learn toy datasets are included in the Scikit-learn package. You can load these datasets directly into Python by the import command as follows:

```
from sklearn import datasets

iris = datasets.load_iris()
```

  where iris is one of the toy datasets. To see which datasets have been included and the details about each dataset, you can view here: https://scikit-learn.org/stable/datasets/index.html .

  All Scikit-learn datasts provide the following methods:

  - .DESCR: a description of the data. For example: iris.DESCR
  - .data: This shows all the data. For example: iris.data
  - .feature_names: This shows the names of the features. For example: iris.feature_names
  - .target: This contains the target values expressed as values or numbered classes. For example: iris.target
  - .target_names: This shows the names of the classes in the target. For example: iris.target_names
  - .shape: this is a method that you can apply to both .data and .target. For example: iris.data.shape

  Task5:

  Load the Boston dataset from the Scikit-learn toy datasets. Understand the dataset by using the methods in the above list.

- UCI Machine Learning Repository

  You can also download datasets from the UCI webpage (https://archive.ics.uci.edu/ml/index.php)

6. Data Manipulation with **Pandas**

The **pandas** package (http://pandas.pydata.org) deals with everything that **NumPy** cannot do. It allows you to handle complex tables of data of different types and time series [2]. You can import **pandas** and check its version as follows:

```
import pandas as pd
pd.__version__
```

- Fast and easy data loading

  The most common format for machine learning data files is CSV files. There are a number of ways to load a CSV file in Python (https://machinelearningmastery.com/load-machine-learning-data-python/). Let's start with opening a CSV using pandas. For example:

```
import pandas
url = "http://aima.cs.berkeley.edu/data/iris.csv"
names = ['sepal_length','sepal_width', 'petal_length','petal_width', 'target']
iris_data = pandas.read_csv(url, sep =',', decimal='.', header=None, names=names)
```

  As can be seen, you can specify the name of the file, the character used as a separator, the character used for the decimal placeholder (decimal), the header and the variables names using a list.

  To get a rough idea of this data file's content, you can print the first or the last rows using the following commands:

```
iris_data.head()
```

	sepal_length	sepal_width	petal_length	petal_width	target
0	5.1	3.5	1.4	0.2	setosa
1	4.9	3.0	1.4	0.2	setosa
2	4.7	3.2	1.3	0.2	setosa
3	4.6	3.1	1.5	0.2	setosa
4	5.0	3.6	1.4	0.2	setosa

```
iris_data.tail()
```

	sepal_length	sepal_width	petal_length	petal_width	target
145	6.7	3.0	5.2	2.3	virginica
146	6.3	2.5	5.0	1.9	virginica
147	6.5	3.0	5.2	2.0	virginica
148	6.2	3.4	5.4	2.3	virginica
149	5.9	3.0	5.1	1.8	virginica

Task6:

Type the following commands line by line and observe what you get from each line:

```
iris_data['sepal_length']
iris_data['target']
iris_data.head
print(iris_data.shape)
```

As you can see, the last column 'target' is in the string format. If you want to convert it to numerical labels, you may try the following:

```
labels =pd.factorize(iris_data.target)[0]
```

```
labels
```

```
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
       2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2], dtype=int64)
```

- Convert a Scikit-learn dataset to a **Pandas** dataset

To use more functions from Pandas to deal with data, sometimes you may want to convert a Scikit-learn dataset to a Pandas dataset. For example;

```
import numpy as np
import pandas as pd
from sklearn.datasets import load_iris

iris = load_iris()

newdata = pd.DataFrame(data= np.c_[iris['data'], iris['target']],
                       columns= iris['feature_names'] + ['target'])
```

where np.c_ is the concatenate function from the numpy module. np.c_ concatenates iris[‘data’] and iris[‘target’] arrays. For Pandas column argument concatenates the iris['feature_names'] list and the string list [‘target’].

And you can type the following to check the data type:

```
newdata.dtypes
```

```
sepal length (cm)    float64
sepal width (cm)     float64
petal length (cm)    float64
petal width (cm)     float64
target               float64
dtype: object
```

Further, if you want to change the data type, for example, you may want to change ‘target’ from float64 to the integer type (Int64), you can type the following:

```
newdata['target'] = newdata['target'].astype('Int64')
```

Now if you type **newdata.dtypes**, you should get something similar to the following:

```
newdata.dtypes
```

```
sepal length (cm)    float64
sepal width (cm)     float64
petal length (cm)    float64
petal width (cm)     float64
target                 Int64
dtype: object
```

You will learn more Data Manipulation with **Pandas** in next session.

References

1. Amit Saha: Doing math with Python: use programming to explore algebra, statistics, calculus and more!
2. Alberto Boschetti and Luca Massaron: Python data science essentials: become an efficient data science practitioner by thoroughly understanding the key concepts of Python. 2015 Packt Publishing