

# A Quantum Game Model and Simulation Study on Collaborative Operation of Container Sea-Rail Intermodal Transport

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**Editors:** Nianyin Zeng, Ram Bilas Pachori and Dongshu Wang

## Abstract

Under the new development pattern of “double circulation” and the continuous promotion of the Belt and Road Initiative, the demand for container sea-rail intermodal transport is increasing day by day, and the efficiency and stability of its collaborative operation become the key. This paper focuses on this, and constructs a quantum game model for the coordinated operation of container sea rail intermodal transport. The model brings the relevant stakeholders such as shipping enterprises and railway transport enterprises into the game framework, and analyzes the benefits under different strategy combinations. Through the design of simulation experiments, the model is verified. The results show that this model can accurately describe the decision-making behavior and synergy effect of various stakeholders, provide a new theory and method for optimizing the collaborative operation of sea rail intermodal transport, and help to improve the overall operation efficiency and service level.

**Keywords:** Container sea-rail intermodal transport; Quantum game model; Collaborative operation; Simulation

## 1. Introduction

Under the dual context of global supply chain restructuring and the deepening advancement of the Belt and Road Initiative(BRI), efficient and green development in the transportation sector is receiving significant attention. Sea-rail intermodal transportation, as an environmentally friendly and energy-efficient transport mode, has become a crucial choice for enhancing transportation capacity and optimizing transport structures due to its advantages of large volume and relatively low costs, playing an important driving role in the economic development of regions along sea-rail intermodal corridors.

In recent years, scholars worldwide have conducted extensive and in-depth research on sea-rail intermodal transportation. In terms of organizational coordination, research has mainly focused on the collaborative management of intermodal systems and the analysis of factors constraining their development. [Chen and Zhang \(2021\)](#) used the method of system analysis to describe its subsystems, and analyzed the port container sea-rail intermodal system composed of external structure and internal structure. [Jin and Yang \(2022\)](#) constructed a container sea-rail intermodal coordination evaluation system from five dimensions: information connectivity, unified technical standards, infrastructure connection, transport organization innovation, and policy support guidance. (2019)

[Aldona et al. \(2019\)](#) emphasized that information exchange is the core element in improving sea-rail intermodal connectivity, which helps fully leverage the advantages of maritime and railway transportation to achieve efficient cargo movement. [Duan et al. \(2019\)](#) adjusted profit distribution schemes based on differences in resource inputs among sea-rail intermodal alliance members, finding that alliance profits are related to the number of enterprises but not to the nature of participation. [Liu et al. \(2018\)](#) pointed out that the lack of aligned interests between railway operators and local governments hindered the development of rail-water inter-modal transport in China. [Chen et al. \(2018\)](#) used Bertrand game theory to demonstrate that cooperation between ports generates higher benefits compared to competition. [Zhang and Shi \(2018\)](#) considering the numerous participants and complex interest games in container sea-rail inter-modal transportation, proposed establishing a multi-party joint participation model to coordinate various interests. [Ding \(2018\)](#) based on synergy theory, explored the importance and process of coordination among various elements and subsystems of the collection and distribution systems. [Mo \(2017\)](#) suggested improving container transportation structures to promote comprehensive development across transport modes. [Wang et al. \(2020\)](#) summarized the factors that affect the configuration of sea rail intermodal loading and unloading equipment for container terminals. [Sun and Luo \(2020\)](#) analyzed the problems in the development of port container sea rail intermodal transportation from the aspects of railway transportation, port operation, shipping companies, and the construction of sea rail intermodal transportation information systems.

Quantum game theory was proposed by [Eisert et al. \(1999\)](#) quantified non-zero sum games in the same year, discovering the superiority of quantum theory in game applications. With the continuous deepening of research in the quantum field, the integration of quantum theory and game theory has been further developed. For example, [Chang \(2023\)](#) obtained the optimal price and profit under decentralized and centralized decision-making by establishing classical and quantum game models. [Te'eni et al. \(2023\)](#) hypothetically ask whether quantum games inspired by population dynamics can benefit from unique features of quantum mechanics such as entanglement and nonlocality. [Li et al. \(2021\)](#) extended the classical strategy space to the quantum strategy space, considering maximum entanglement and separable quantum environment, and based on this, solved the waste problem in the food supply chain. [Liu et al. \(2024\)](#) constructed a quantum supervised game model to conduct an evolutionary analysis of the strategic choices of stock market investors and regulatory authorities, providing a guarantee for effective supervision of market manipulation behavior. [Frckiewicz et al. \(2022\)](#) tested pure Nash equilibrium in a quantum game that extended the two-dimensional classical two matrix game. At present, research on quantum games mainly focuses on theoretical exploration of game theory and the application of quantum game theory in the fields of finance and food supply chain ([Ikeda and Aoki, 2022](#)). As key stakeholders, railway transport enterprises and shipping enterprises need to cooperate to promote system development, but their conflicting interests objectively exist. Their behaviors within the system influence and constrain each other while being affected by other factors, creating a game-theoretic situation. Meanwhile, the bullwhip effect caused by incomplete information in cooperation has not been sufficiently studied. Against this backdrop, this paper adopts quantum game analysis to explore coordination mechanisms that encourage full cooperative engagement and attempt to solve the “Prisoner’s Dilemma,” aiming to provide new theoretical perspectives and practical references for the collaborative operation of container sea-rail intermodal transportation.

## 2. Model Construction

### 2.1. Basic Assumptions

Assumption 1: This study focuses on a sea-rail intermodal transportation collaborative system led by a shipping enterprise (S) in cooperation with a railway transport enterprise (R).

Assumption 2: All participants in the sea-rail intermodal collaborative system adhere to the homo economicus assumption, aiming to maximize individual profit while simultaneously considering the system's overall profit maximization.

Assumption 3: Participants within the system exhibit bounded rationality due to information asymmetry and risk uncertainty, further constrained by external natural and market environments.

Assumption 4: This study considers only pure-strategy Nash equilibrium in the game model construction, though mixed-strategy solutions follow analogous reasoning.

Assumption 5: The strategic behaviors of shipping and railway transport enterprises are mutually independent; their respective strategy sets influence only their own initial states, with analysis confined to interaction dynamics between the two core carriers.

Assumption 6: The model exclusively examines collaborative revenue and shared operational costs between the carriers, holding other variables constant. Sea-rail intermodal profits and total costs remain fixed and are allocated via predetermined coefficients.

### 2.2. Parameter Settings

The parameters and related concepts used in the modeling analysis are defined in Table 1.

Table 1: Parameter Variables and Their Definitions

Parameters	Definitions
$e_i$	shipping enterprises and railway transport enterprises invest effective resources and elements (i.e., level of effort), $0 \leq e_i \leq 1$
$\mu_i$	Cost Coefficients for Shipping and Railway Transport Enterprises, $\mu_i \geq 0$
$A$	Output Coefficient of Cooperative Transportation, $A \geq 0$
$\alpha$	Elasticity coefficient, $0 \leq \alpha \leq 1$
$\beta$	Profit Distribution Coefficient, $0 \leq \beta \leq 1$
$\varepsilon$	Random Disturbance Term, $\varepsilon \leq 1$
$C_i$	Input costs of shipping enterprises and railway transport enterprises in collaborative transportation
$R_i$	Expected benefits for shipping enterprises and railway transport enterprises operators in collaborative transportation
$\delta$	Degree of interdependence between railway transport enterprises and railway transport enterprises
$\varphi_i$	Degree of quantum strategy adoption by shipping enterprises and railway transport enterprises
$\theta_i$	Level of effort exerted by shipping enterprises and railway transport enterprises in collaborative transportation

### 2.3. Game Model

In a quantum environment, the game process between shipping enterprises and railway transport enterprises consists of the following five steps:

(1) The two extreme states of “full effort” and “no effort” correspond to the polarized states  $|0\rangle$  and  $|1\rangle$ , respectively. It is assumed that both parties initially adopt a “full effort” strategy, meaning their initial quantum state is  $|00\rangle$  (the first digit represents the shipping enterprise's quantum state, and the second digit represents the railway transport enterprise's quantum state).

(2) The default entanglement matrix for the game participants is:

$$\hat{J} = \exp(i\frac{\delta}{2}\Phi_x \otimes \Phi_x) = \cos\frac{\delta}{2} \cdot I + i\sin\frac{\delta}{2} \cdot \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad (1)$$

Here,  $\Phi_x = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$  is a modified Pauli-x matrix, and  $I$  is a 4\*4 identity matrix.

(3) Here, we only discuss the equilibrium of the EWL quantum game under a two-parameter scenario, and thus assume the strategy space takes the form of:

$$\hat{U}(\theta_i, \phi_i) = \begin{pmatrix} e^{i\phi_i} \cos\frac{\theta_i}{2} & \sin\frac{\theta_i}{2} \\ -\sin\frac{\theta_i}{2} & e^{-i\phi_i} \cos\frac{\theta_i}{2} \end{pmatrix} \quad (2)$$

Here,  $\theta_S$  is taken for  $\theta_i$ , and  $\theta_R$  and  $\theta_i \in [0\pi]$  represent the degree of cooperative effort between the two parties; while  $\varphi_i$  adopts  $\varphi_S, \varphi_R$ , and  $\varphi_i \in [0\pi/2]$  is taken to represent the quantum strategy intensity of the collaborating parties.

(4) The untangling matrix can be derived from the entanglement matrix.

$$\hat{J}^+ = \cos\frac{\delta}{2} \cdot I - i\sin\frac{\delta}{2} \cdot \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \cos\frac{\delta}{2} & 0 & 0 & -i\sin\frac{\delta}{2} \\ 0 & \cos\frac{\delta}{2} & i\sin\frac{\delta}{2} & 0 \\ 0 & i\sin\frac{\delta}{2} & \cos\frac{\delta}{2} & 0 \\ -i\sin\frac{\delta}{2} & 0 & 0 & \cos\frac{\delta}{2} \end{pmatrix} \quad (3)$$

The resulting state is given by:

$$|\psi_f\rangle = \hat{J}^+ \cdot [U_S(\theta_S\varphi_S) \otimes U_R(\theta_R\varphi_R)] \cdot \hat{J} |00\rangle \quad (4)$$

After quantum state collapse, the final state becomes a basis vector in the tensor product space (i.e., the individual quantum states), with the corresponding probability being:

$$\begin{cases} P_{00} = [\cos^2(\varphi_S + \varphi_R) + \sin^2(\varphi_S + \varphi_R) \cdot \cos^2\delta] \cos^2\frac{\theta_S}{2} \cos^2\frac{\theta_R}{2} \\ P_{01} = [\cos^2\varphi_S + \sin^2\varphi_S \cdot \cos^2\delta] \cos^2\frac{\theta_S}{2} \sin^2\frac{\theta_R}{2} + [\sin^2\varphi_R \cdot \sin^2\delta] \sin^2\frac{\theta_S}{2} \cos^2\frac{\theta_R}{2} \\ P_{10} = [\cos^2\varphi_R + \sin^2\varphi_R \cdot \cos^2\delta] \sin^2\frac{\theta_S}{2} \cos^2\frac{\theta_R}{2} + [\sin^2\varphi_S \cdot \sin^2\delta] \cos^2\frac{\theta_S}{2} \sin^2\frac{\theta_R}{2} \\ P_{11} = [\sin^2(\varphi_S + \varphi_R) \cdot \sin^2\delta] \cos^2\frac{\theta_S}{2} \cos^2\frac{\theta_R}{2} + \sin^2\frac{\theta_S}{2} \sin^2\frac{\theta_R}{2} \end{cases} \quad (5)$$

(5) The calculation yields  $P_{00} + P_{01} + P_{10} + P_{11} = 1$ , from which the expected payoff functions for both parties can be derived.

The shipping enterprise's expected profit is:

$$\begin{aligned} ER_S &= A_S P_{00} - C_S P_{01} + 0 \cdot P_{10} + 0 \cdot P_{11} \\ &= A_S [1 - \sin^2(\varphi_S + \varphi_R) \sin^2\delta] \cos^2\frac{\theta_S}{2} \cos^2\frac{\theta_R}{2} \\ &\quad - C_S [\cos^2\varphi_S + \sin^2\varphi_S \cdot \cos^2\delta] \cos^2\frac{\theta_S}{2} \sin^2\frac{\theta_R}{2} \\ &\quad - C_S \sin^2\varphi_R \sin^2\delta \sin^2\frac{\theta_S}{2} \cos^2\frac{\theta_R}{2} \end{aligned} \quad (6)$$

The railway transport enterprise's expected profit is:

$$\begin{aligned}
 ER_R &= A_R P_{00} + 0 \cdot P_{01} + C_R P_{10} + 0 \cdot P_{11} \\
 &= A_R [1 - \sin^2(\varphi_S + \varphi_R) \sin^2 \delta] \cos^2 \frac{\theta_S}{2} \cos^2 \frac{\theta_R}{2} \\
 &\quad - C_R [\cos^2 \varphi_R + \sin^2 \varphi_R \cdot \cos^2 \delta] \cos^2 \frac{\theta_R}{2} \sin^2 \frac{\theta_S}{2} \\
 &\quad - C_R \sin^2 \varphi_S \sin^2 \delta \sin^2 \frac{\theta_R}{2} \cos^2 \frac{\theta_S}{2}
 \end{aligned} \tag{7}$$

It can be seen that the only factor that affects the expected return of both parties is the entanglement degree  $\delta$ , which can be specified in advance to affect the choice of strategies of both parties, so as to maximize the expected return of all partners.

### 3. Simulation Analysis

#### 3.1. Without considering entanglement situations

Assume that the output coefficient  $R_S = 200$  and cost coefficient  $C_S = 60$  of a shipping enterprises are given. The enterprise's profit is plotted in Figure 1 using MATLAB 2022B.

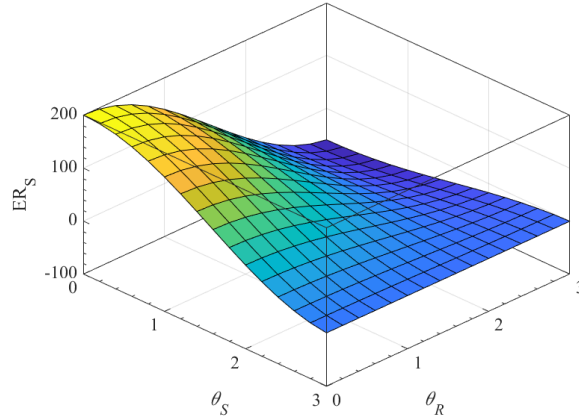


Figure 1: Three dimensional image of shipping enterprise revenue and effort level of both parties without considering entanglement.

As can be seen from Figure 1, when  $\theta_R$  is close to 0, the revenue  $ER_S$  of shipping enterprises will decrease with the increase of  $\theta_S$  more obviously; When  $\theta_R$  is close to  $\pi$ , the relationship between the revenue  $ER_S$  of shipping enterprise and its own effort is not easy to judge, and it is impossible to observe the relationship between the two intuitively. Therefore, the income surface is cut along five different values between the effort degree  $\theta_R = 0$  and  $\theta_R = \pi$  to obtain a two-dimensional section of the shipping enterprise's income  $ER_S$  and its own effort degree  $\theta_S$ , as shown in Figure 2. Among them, figure 3 further details the impact on shipping enterprises when  $\theta_R=0$  to  $\theta_R = \pi$ .

It can be seen from Figure 2 and figure 3 that when  $\theta_R$  increases and approaches  $\frac{\pi}{2}$ ,  $ER_S$  decreases with the increase of  $\theta_S$ , that is, the revenue  $ER_S$  of shipping enterprises increases with the increase of their own efforts, and the investment in the process of sea-rail intermodal collaborative transportation is directly proportional to the revenue; In the process that  $\theta_R$  increases from  $\frac{3\pi}{4}$  to  $\pi$ ,  $ER_S$  increases with the increase of  $\theta_S$ , that is, the revenue of shipping decreases with the increase of their own efforts; With the increase of  $\theta_R$  value,  $ER_S$  will decrease with the decrease of  $\theta_S$ , and will

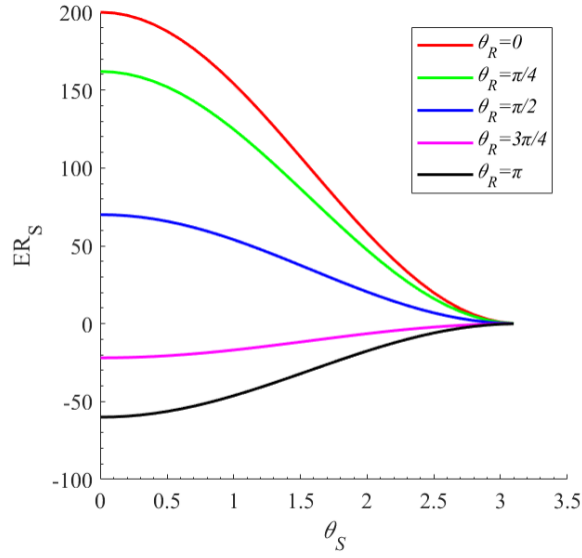


Figure 2: The Impact of Effort Level from  $\theta_R = 0$  to  $\theta_R = \pi$  on the Revenue of Shipping Enterprises in the non entangled state.

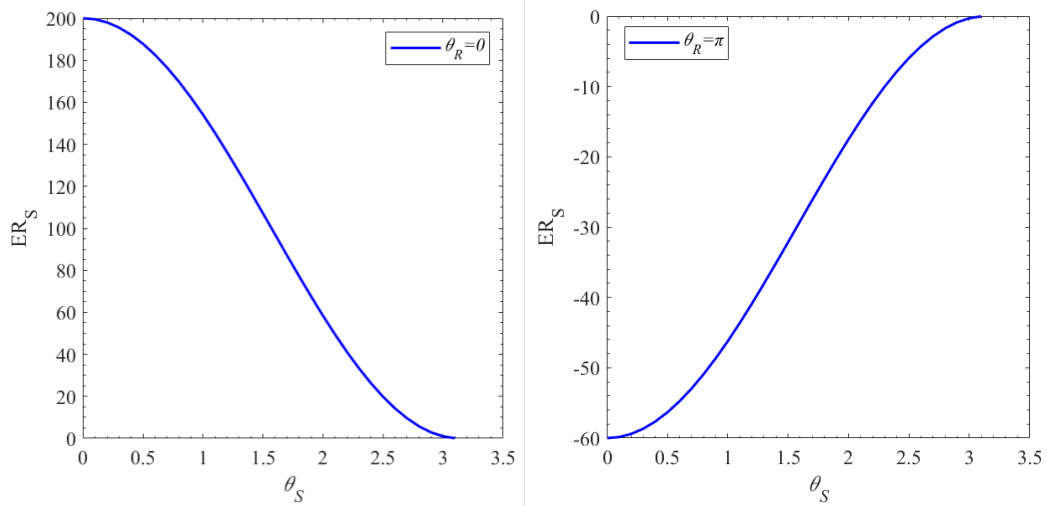


Figure 3: Effect of effort of railway transport enterprises on the revenue of shipping enterprises in the non entangled state when  $\theta_R = 0$ ,  $\theta_R = \pi$ .

eventually decrease with the decrease of  $\theta_S$ , indicating that the revenue  $ER_S$  of shipping enterprises will also increase with the increase of effort  $e_R$  of railway transport enterprises. Therefore, as long as the efforts of railway transport enterprises reach a certain threshold, shipping enterprises will tend to adopt the full effort strategy.

### 3.2. Considering entanglement situations

Under the numerical assumption  $R_S = 200$ ,  $C_S = 60$  that disregards entanglement degree and in conjunction with the railway transport enterprise's adoption of a generic quantum strategy, the shipping enterprise's revenue function is plotted as shown in Figure 4.

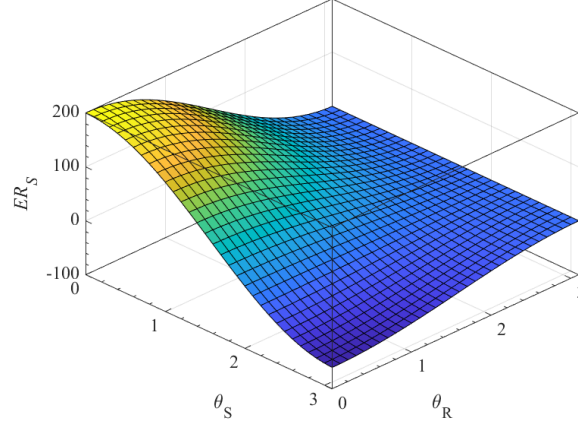


Figure 4: Three dimensional image of the revenue of shipping enterprises and effort level of both parties when considering entanglement.

As can be seen from Figure 4, when  $\theta_R$  is close to 0, the revenue  $ER_S$  of the shipping enterprise gradually decreases with the increase of  $\theta_S$ , and the decreasing trend is more obvious, while when  $\theta_R$  is close to  $\pi$ , the changing trend of  $ER_S$  with  $\theta_S$  cannot be seen intuitively from this image. In order to more clearly show the change trend of the income curve, some two-dimensional views are drawn, as shown in Figure 5 and Figure 6.

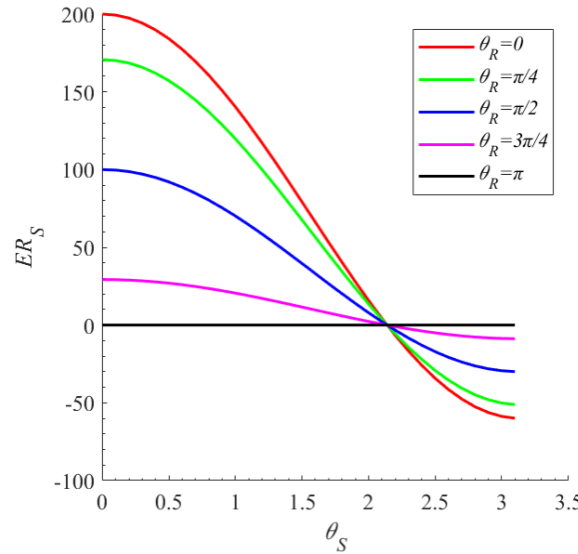


Figure 5: The impact of effort level from  $\theta_R = 0$  to  $\theta_R = \pi$  on the revenue of shipping enterprises in entangled states.

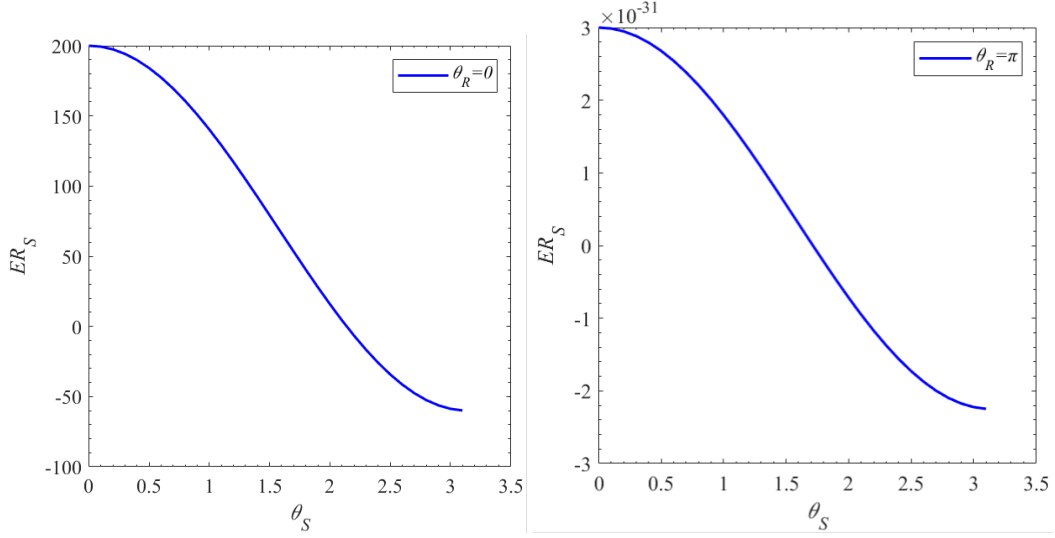


Figure 6: Effect of effort of railway transport enterprises on the revenue of shipping enterprises in the entangled state when  $\theta_R = 0$ ,  $\theta_R = \pi$ .

It can be seen from Figure 5 and Figure 6 that, unlike the case without considering entanglement, when  $\theta_R$  approaches  $\pi$  from 0, no matter how the value of  $\theta_R$  changes,  $ER_S$  will gradually decrease with the increase of  $\theta_S$ , that is, the revenue of shipping enterprises will increase with the increase of their own efforts; With the increase of  $\theta_R$ , the reduction rate tends to be gentle, that is, the decline in the effort of railway transport enterprises will reduce the synergy income of shipping enterprises, indicating that the income of shipping enterprises will increase with the increase in the effort of railway transport enterprises. It can be seen that in the maximum entangled state, for the shipping enterprises, no matter how the efforts of the railway transport enterprises change, their own income will change positively with their own efforts, but their own income will decline slightly due to the reduction of the efforts of the railway transport enterprises, but they will not bear the costs caused by the other party's "betrayal", and their efforts will still be rewarded, which has a great incentive for the shipping enterprises to choose the maximum efforts.

#### 4. Conclusion

This paper constructs a quantum game model to study the decision-making behavior and synergy effects in the collaborative operation of container sea-rail intermodal transport. By introducing shipping enterprises and railway transport enterprises as the game players, the paper analyzes the payoffs under different strategy combinations and verifies the effectiveness of the model through simulation experiments.

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