

An optimization problem Based on Integer Programming Theory

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Abstract

With the implementation of the rural revitalization strategy, promoting high-quality rural development has become a strategic requirement. A key challenge in optimizing rural development is achieving high-quality agricultural cultivation. Given fixed arable land areas and seed quality, determining optimal crop planting strategies is a critical research focus. This study takes a village in North China's mountainous region as an example, incorporating local land types, suitable crops, terrain areas, and infrastructure (traditional greenhouses and smart greenhouses). First, land is classified based on given data. Second, yields, planting costs, and sales prices of the same crop across different terrains are analyzed, with median prices used for profit comparisons. Finally, using integer programming principles and intelligent software, an optimal planting strategy for 2024 is proposed through yield models, planting area models, and maximum revenue models.

Keywords: High-quality development, Mathematical modeling, Integer programming, Optimal strategy

1. Introduction

Optimizing limited arable land resources and developing organic farming are crucial for sustainable rural economies. Selecting suitable crops and refining planting strategies can enhance efficiency, simplify field management, and reduce risks (Bian, 2025). A case study village in North China's mountainous region has 1,201 acres of open farmland (34 plots: dry flatlands, terraced fields, slopes, and irrigated land) and 20 greenhouses (16 traditional, 4 smart). Dry flatlands, terraces, and slopes support one grain crop annually; irrigated land allows one rice crop or two vegetable crops. Greenhouses enable additional planting cycles. Crop rotation and legume requirements (each plot must grow legumes at least once every three years) are critical constraints. (Lu and Gao, 2025)

Key Challenges:

- (1) Avoid overproduction: Excess yields either go to waste or are sold at 50% discount.
- (2) Address uncertainties: Variable yields, fluctuating costs, and market demands.

Management Requirements

- **Crop rotation:** No consecutive planting of the same crop.
- **Legume mandate:** Each plot must grow legumes once every three years.
- **Sales limits:** Excess yields face penalties (waste or 50% price reduction).

Model Assumptions

Assumption 1: The expected sales volume, planting costs, yield per mu, and sales prices of all crops are assumed to remain stable relative to 2023 levels. Crops planted in each season are sold in that same season.

Assumption 2: If the total production of a crop in a season exceeds its expected sales volume:

- (1) The surplus portion remains unsold and is wasted;
- (2) The surplus portion is sold at a 50% discount based on the 2023 sales price.

Issues requiring research in regional planting planning

Question 1: Based on Assumption 1 and Assumption 2, the optimal planting plan for crops in this village from 2024 to 2030 is given.

Data Extraction

- **Land details:** Plot sizes, types, and counts.
- **Crop parameters:** Yield, cost, price per land type . As the following Tables 1-5.

Table 1: Crop planting in single season

Crop name	planting season	Yield per mu/catty (slopes)	Yield per mu/catty (Terraced field)	Yield per mu/catty (dry flatland)	Planting cost/(yuan/mu)	Sales unit price/(yuan/catty)
Soybeans	single season	360	380	400	400	3.25
Black beans		450	475	500	400	7.5
Red beans		360	380	400	350	8.25
Mung bean		315	330	350	350	7.00
climbing bean		375	395	415	350	6.75
wheat		720	760	800	450	3.50
Corn		900	950	1000	500	3.00
Millet		360	380	400	360	6.75
Sorghum		570	600	630	400	6.0
Millet		475	500	525	360	7.5
Buckwheat		100	105	110	350	40
pumpkin		2700	2850	3000	1000	1.50
Sweet potato		2000	2100	2200	2000	3.25
naked oats		380	400	420	400	5.50
Barley		475	500	525	350	3.50

2. Model Construction

Key Variables:

- $S_{ijt k}$: Area of crop j planted in plot i , year t , season k .
- $Q_{j t k}$: Total yield of crop j in year t , season k .

Table 2: Crop planting in Ordinary Greenhouse Season 1

Crop name	planting season	Yield per mu/catty	Planting cost/(yuan/mu)	planting price	Planting profit
Cowpea	Ordinary Greenhouse Season1	3600	2400	8	26400
Concanavalin		2400	1200	6.75	15000
Kidney beans		3600	2400	6.5	21000
potato		2400	2400	3.75	6600
tomato		3000	2400	6.25	16350
eggplant		8000	2400	5.5	41600
spinach		3300	2700	5.65	15945
Green pepper		3000	2000	5.25	13750
Cauliflower		4000	3000	5.5	19000
cabbage		4500	3500	6.5	25750
Rape		5000	2000	5	23000
small vegetables		4000	2000	5.75	21000
cucumber		15000	3500	7	101500
lettuce		5000	2000	5.25	24250
Chili		2000	1200	7.25	13300
Water spinach		12000	5000	4.5	49000
Yellow cabbage		6000	2500	4.5	24500
Celery		6600	1100	4	25300

Table 3: Crop planting in Irrigated land Season 1

Crop name	planting season	Yield per mu/catty	Planting cost/(yuan/mu)	planting price	Planting profit
Cowpea	Irrigated land Season 1	3000	2000	8	22000
Concanavalin		2000	1000	6.75	12500
Kidney beans		3000	2000	6.5	17500
potato		2000	2000	3.75	5500
tomato		2400	2000	6.25	13000
eggplant		6400	2000	5.5	33200
spinach		2700	2300	5.65	12955
Green pepper		2400	1600	4.75	9800
Cauliflower		3300	2400	5.5	15750
cabbage		3700	2900	6.5	21150
Rape		4100	1600	5	18900
small vegetables		3200	1600	5.75	16800
cucumber		12000	2900	7	81100
lettuce		4100	1600	5.25	19925
chili		1600	1000	7.25	10600
Water spinach		10000	4100	4.5	40900
Yellow cabbage		5000	2000	4.5	20500
celery		5500	900	4	21100

- D_{jtk} : Expected sales volume of crop j planted in year t and season k .
- P_j : Normal sales price of crop j .
- P'_j : Non-normal sales price of crop j , If the surplus is unsold and wasted, $P'_j = 0$, else, $P'_j = 0.5P_j$.
- C_j : Planting cost of crop j .
- A_i : Total area of plot i .
- Y_j : Yield per mu of crop j .

Table 4: Crop planting in Smart greenhouse Season 2

Crop name	planting season	Yield per mu/catty	Planting cost/(yuan/mu)	planting price	Planting profit
Cowpea	Smart greenhouse Season 2	3200	2640	9.6	28080
Concanavalin		2200	1320	8.1	16500
Kidney beans		3200	2640	7.8	22320
Potato		2200	2640	4.5	7260
Tomato		2700	2640	7.5	17610
Eggplant		7200	2640	6.6	44880
Spinach		3000	3000	6.9	17700
Green pepper		2700	2200	6.8	16160
Cauliflower		3600	3300	6.6	20460
Cabbage		4100	3850	7.8	28130
Rape		4500	2200	6	24800
small vegetables		3600	2200	6.9	22640
Cucumber		13500	3850	8.4	109550
Lettuce		4500	2200	6.3	26150
Chili		1800	1300	8.7	14360
Water spinach		11000	5500	5.4	53900
Yellow cabbage		5400	2750	5.4	26410
Celery		6000	1200	4.8	27600

Table 5: Crop planting in Irrigated land Season 2 and Ordinary Greenhouse Season 2

Crop name	planting season	Yield per mu/catty	Planting cost/(yuan/mu)	planting price(yuan)	Planting profit(yuan)
Chinese cabbage	Irrigated land Season 2	5000	2000	2.5	10500
white radish		4000	500	2.5	9500
Carrot		3000	500	3.25	9250
Elm mushroom	Ordinary Greenhouse Season 2	5000	3000	57.5	284500
shiitake mushroom		4000	2000	19	74000
Bailing mushroom		10000	10000	16	150000
Morchella		1000	10000	100	90000

- R_t : Maximum revenue obtained from crop planting in year t .

Objective Function:

According to the profit maximization rule (Shi et al.), the maximum profit function can be obtained: Maximize revenue

$$R_t = \max \left\{ \sum_{ijtk} [\min(Q_{j,t,k}, D_{j,t,k}) * P_j + \max(Q_{j,t,k} - D_{j,t,k}, 0) * P'_j] - \sum_{ijtk} C_j Q_{j,t,k} \right\} \quad (1)$$

Constraints:

- **Land area limits:**

$$\sum_j S_{i,j,t,k} < A_i, \forall i, t \in N^+, k = 1, 2. \quad (2)$$

- **Crop-land compatibility:** Flat dry land/terraced land/hillside land: only food crops can be grown (beans belong to food).
- **Watered land:** plant vegetables (non-Chinese cabbage, white radish, carrot) in the first season, and plant one of Chinese cabbage, white radish, and carrot in the second season.

- **Ordinary greenhouse:** first season vegetables, second season edible fungi.
- **Smart greenhouse:** Vegetables can be grown in both seasons (non-restricted varieties).
- **Rotation and legume requirements:** The same plot cannot be planted in the same season for two consecutive years, namely,

$$S_{i,j,t,k} * S_{i,j,t+1,k} = 0. \quad (3)$$

- **Legume Crop Constraints:** Plant legume crops at least once every three years for each plot. For plots i . Plant legume crops for at least one of the three years in t , t plus 1, t plus 2.
- **Planting concentration constraints:** The number of plots planted per crop per season does not exceed the number of arable plots contained in the selected plot type (to avoid too much dispersion). The planting area of each crop in a single plot is not less than the minimum area of all plots (Zhou, 2025).
- **Sales and overproduction penalties:** Non-normal sales price of crop j , If the surplus is unsold and wasted, $P'_j = 0$, if the excess is sold at a reduction of 50% of the 2023 sales price,

$$P'_j = 0.5P_j \quad (4)$$

3. Conclusions

According to the experimental data given in Table 1-5, the following experimental results can be obtained.

Result1: Yields of the same crops were related to the type of cultivated land, the details as the Figure 1.

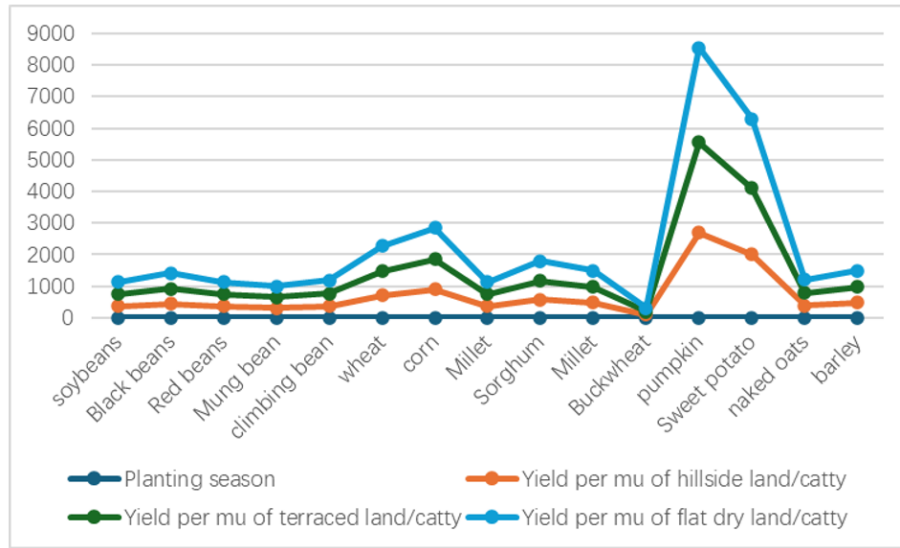


Figure 1: The profit change trend

The experimental results show that the profit change trend of the same crop is the same in the same land; according to the profit maximization (Han and Zeng, 2025), the priority of planting crops that need to be planted can be determined by the height of the cusp in the figure.

Result2: Crops are planted within the same plot, and the planting priority has not changed. As the Figure 2.

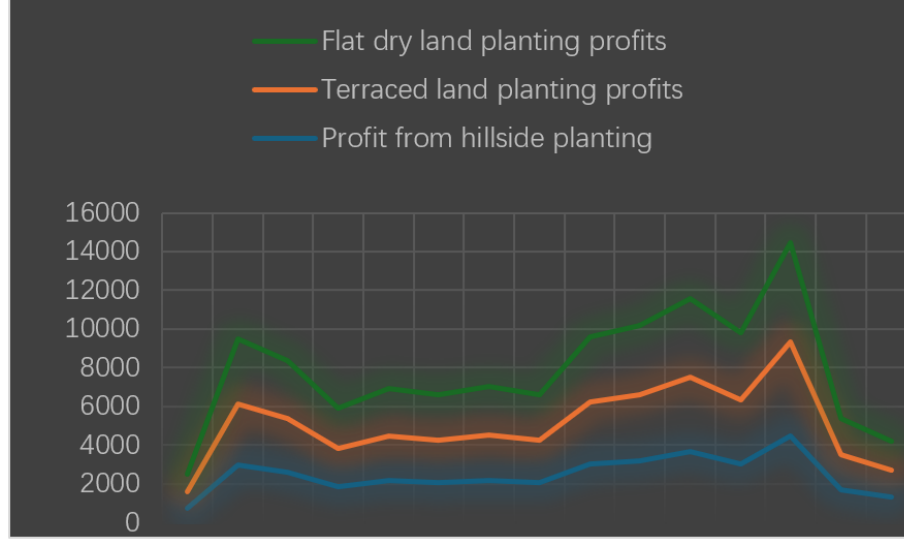


Figure 2: The planting priority

The experimental results help us determine how to plant. The planting priority is determined according to the height of the cusp in the figure.

Result3: Taking into account the maximization of profits and state policy (Hu et al., 2023), with the help of Deepseek, the order in which crops are planted can be determined, Next is the planting strategy for some crops in 2024:

Plot Name	Year	Season Season	Crop Number	Crop Name	Planting Area/Mu	Expected Sales Volume/Pounds	Actual Output/Pounds	Excess Handling Method	Revenue /Yuan
A1	2024	Single Season	6	Wheat	80	64,000	64,000	No Excess	179,200
D1	2024	First Season	21	Tomato	15	36,000	36,000	No Excess	108,000
D1	2024	Second Season	35	Chinese Cabbage	15	75,000	75,000	No Excess	37,500
E1	2024	First Season	18	Sword Bean	0.6	1,440	1,440	No Excess	7,920
E1	2024	Second Season	38	Elm Mushroom	0.6	3,000	3,000	No Excess	150,000

Result 4: Based on the optimization model, using the integer programming theory, (Zeng, 2025) Based on the model:

$$R_t = \max \left\{ \sum_{ijtk} [\min(Q_{j,t,k}, D_{j,t,k}) * P_j + \max(Q_{j,t,k} - D_{j,t,k}, 0) * P'_j] - \sum_{ijtk} C_j Q_{j,t,k} \right\} \quad (5)$$

Combining the data given in Tables 1-5, with the help of MATLAB and Deepseek, 2024 Total revenue: RMB 1,234,567 (Gao et al., 2001).

4. Model Evaluation

The integer programming framework effectively integrates multi-year, multi-plot, and multi-crop variables (Wang, 2024). However, annual adjustments are recommended to account for market volatility and climate impacts. There are still many topics that can be discussed in the article. For example, considering factors such as price changes, market demand, and policy subsidies for crops, there will still be new research results. Due to the limited space, we can continue to study in this direction in the future and continuously optimize the model (Zhang and Wu, 2025).

References

- Mengke Bian. Research on mathematical programming algorithm for production operation management optimization. *Journal of Heihe University*, 16(05):185–188, 2025. ISSN 1674-9499.
- Tian Gao, Mengguang Wang, Lixin Tang, and Jianhai Song. Transfer flow optimization model: Special 0-1 linear integer programming problem. *Control and Decision*, (S1):705–708, 2001. ISSN 1001-0920. doi: 10.13195/j.cd.2001.s1.65.gaot.016.
- Shuai Han and Jiwei Zeng. Research on remote tower staffing optimization based on integer programming. *Journal of Civil Aviation*, 9(02):57–60, 2025. ISSN 2096-4994.
- Fei Hu, Xisheng Zhan, and Jian Tu. Comprehensive decision-making of transmission network planning scheme based on optimization theory. *Electrical Switch*, 61(01):17–21+25, 2023. ISSN 1004-289X.
- Zhengguang Lu and Peng Gao. Design and practice of primary logistics optimization model for refined oil products based on mixed integer programming. *International Petroleum Economy*, 33(02):100–106, 2025. ISSN 1004-7298.
- Ruoyu Shi, Liping Chen, Jingjie Xie, and Wenzheng Huang. Optimization of subway train energy consumption based on mixed integer programming. *Railway communication signal engineering technology*, 22.
- Yulong Wang. Extension and application of ecological agriculture technology in crop planting. *Grain, Oil and Feed Technology*, (07):183–185, 2024. ISSN 2096-8515.
- Xianke Zeng. Research on the location model and algorithm of urban logistics distribution center. *Logistics Technology*, 48(05):28–30, 2025. ISSN 1002-3100. doi: 10.13714/j.cnki.1002-3100.2025.05.007.
- Jingwen Zhang and Wenjuan Wu. Mathematical model of agricultural water distribution planning for improving crop yield. *Journal of Southwest University (Natural Science Edition)*, 47(05): 234–241, 2025. ISSN 1673-9868. doi: 10.13718/j.cnki.xdzk.2025.05.020.
- Le Zhou. Power plant scheduling optimization technology and application based on integer programming. *China's new technology and new product*, (06):78–80, 2025. doi: 10.13612/j.cnki.cntp.2025.06.043.