

Competitive Influence Maximization Across Social Networks

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Editors: Nianyin Zeng, Ram Bilas Pachori and Dongshu Wang

Abstract

The proliferation of Web 2.0 technologies has significantly reshaped information propagation dynamics across social media platforms. While existing studies extensively analyze influence maximization within single-platform environments, competitive propagation dynamics across multiple interconnected social networks remain underexplored. Addressing this research gap, we define the Competitive Influence Maximization Across Social Networks (CIMASN) problem and introduce a novel Competitive Independent Cascade Model (CICM) that incorporates competitive influences propagating simultaneously across multiple platforms. A greedy algorithm is proposed for effective seed node selection under this competitive scenario, validated through extensive experiments on both real-world and synthetic datasets. Results demonstrate that our model and algorithm significantly outperform traditional approaches, highlighting the necessity and effectiveness of modeling competitive propagation dynamics across multiple social networks.

Keywords: influence maximization, influence propagation model, greedy algorithm, multiple social networks

1. Introduction

With the maturation and widespread adoption of Web 2.0 technologies, social media platforms such as Twitter, WeChat, and Weibo have significantly transformed traditional ways of obtaining and disseminating information. These platforms feature massive user bases, complex data structures, and exponential propagation capabilities, enabling viral marketing through cascading diffusion to become a prevalent and highly effective advertising strategy. Consequently, social networks have become major arenas for commercial promotion and governmental propaganda. Influence diffusion models form the micro-foundation for understanding propagation mechanisms, with effective and secure information dissemination representing the ultimate goal and central focus of research.

Recent years have witnessed considerable exploration into global information propagation mechanisms from multiple sources, predictive modeling of information dissemination, and the selection of influential seed node sets aiming to maximize diffusion scope and speed. These studies have significantly advanced our understanding of influence propagation in social networks. However, interactions between competing influences frequently arise within these networks, leading to increasing interest in analyzing competitive influence propagation mechanisms and related optimization

models across social networks. Despite these advances, studies focusing on competitive influence propagation across multiple social networks remain limited.

In real-world scenarios, users typically engage with multiple social networks simultaneously—such as Douban for book, movie, and music sharing; Zhihu for question and answer; Weibo for microblogging; and TikTok for short videos—to fulfill diverse informational and social needs. Therefore, investigating influence propagation across multiple platforms is of great significance. Nonetheless, the current literature lacks comprehensive studies addressing competitive influence propagation across different social networks.

Our primary contributions include: (1) Defining the Competitive Influence Maximization Across Social Networks (CIMASN) problem. (2) Developing a seed set mining algorithm for CIMASN based under the Competitive Independent Cascade Model (CICM). (3) Validating the proposed algorithm’s effectiveness through experiments on both real and synthetic networks, demonstrating superior performance compared with classic seed set selection algorithms.

2. Related Work

The problem of influence maximization has been extensively studied in social network research, continuously enriched by pioneering and innovative approaches. [Kempe et al. \(2003\)](#) first formulated the influence maximization problem in their seminal work, which has since been widely explored across disciplines such as computer science, statistical physics, and information science. [Leskovec et al. \(2007\)](#) addressed inefficiencies inherent in greedy algorithms through submodularity and proposed the Cost-Effective Lazy Forward (CELF) algorithm. Research by [Min et al. \(2020\)](#) considered user attributes within online social networks, specifically incorporating users’ preferred engagement topics. Additionally, [Li et al. \(2020\)](#) posited that influences generated over long network distances might be negligible, proposing an influence propagation algorithm based on the hop count between nodes. Other studies, such as [Zhang et al. \(2022\)](#), utilized detailed network topology information to enhance heuristic algorithms. [D’Angelo et al. \(2020\)](#) performed a comparative analysis of adaptive versus non-adaptive strategies, demonstrating that adaptive strategies yield significantly better results, although non-adaptive strategies are simpler in terms of design and implementation. [Feng et al. \(2020\)](#) employed adaptive methodologies while accounting for uncertainty in seed selection and diffusion, addressing limitations in propagation numbers observed in real-world networks. [Erkol et al. \(2020\)](#) extended influence maximization into sequential networks, leveraging network topology along with varying information granularity levels. [Li et al. \(2022\)](#) argued that information dissemination is influenced by user emotional intensity and cluster credibility, thus proposing an emotion-based attributed influence maximization framework. While these studies thoroughly investigate seed set mining for influence maximization based on classical propagation models, they fall short when addressing influence maximization across multiple social networks.

[He et al. \(2012\)](#) extended the classical Linear Threshold (LT) model, introducing a competitive linear threshold model capable of capturing competitive propagation dynamics between positive and negative influences within social networks. They formulated a novel problem termed Influence Blocking Maximization (IBM), proved that the IBM objective function satisfies submodularity, and proposed efficient approximation algorithms to solve the IBM problem. [Guo et al. \(2024\)](#) proposed an influence maximization algorithm that integrates group trust and local topological features to enhance propagation efficiency and effectiveness in social networks. [Yang et al. \(2024\)](#) developed BIM-DRL, a deep reinforcement learning framework that jointly models entity correlation

and seed selection to achieve balanced influence propagation in multi-entity social networks. Recently, Yunfei et al. (2018) tackled simultaneous propagation scenarios involving multiple mutually reinforcing influences, proposing the Associated Influence Maximization problem and presenting a parallel seed set mining algorithm. Sha and Zhu (2025) developed a reaction–diffusion model utilizing the Laplacian matrix to analyze rumor propagation patterns across various network structures, providing insights into optimal control strategies for information dissemination. Although these studies significantly contributed to multi-influence propagation models, they did not account for cross-network propagation dynamics.

3. Problem Definition

Given a set of m social networks $G = \{G^1, \dots, G^m\}$, each social network $G^x = (V^x, E^x)$ consists of a set of nodes V^x representing users and a set of edges E^x representing friendship relations among users in network G^x . In these m social networks, users can exist concurrently across multiple networks. For example, node v_i^y in social network G^y may represent the same individual as node v_j^x in social network G^x .

Within these networks, two competing influences, denoted as P and I , propagate among users. If a user accepts influence P , they become P -active permanently. Conversely, if a user accepts influence I , they become I -active permanently. If a user has not accepted any influence, they remain in an inactive state. Hence, each user exclusively exists in one of three states: inactive, P -active, or I -active.

Let P represent our influence and I represent the competitor’s influence across the m social networks. We aim to identify an initial set of seed users S^P with size $|S^P| = k$ to initiate influence P propagation. Under a specific propagation model, the goal is to maximize the total number of P -active users across all m social networks. This problem is defined as Competitive Influence Maximization Across Social Networks (CIMASN). It is important to note that if the same user appears in multiple networks, they are counted only once when aggregating the active states. Studying CIMASN provides valuable insights into the mechanisms and optimization strategies for cross-platform influence propagation.

4. Competitive Independent Cascade Model

Building upon the classical Independent Cascade (IC) model, we propose the Competitive Independent Cascade Model (CICM) to better reflect real-world competitive influence propagation scenarios across social networks. The propagation mechanism operates in discrete time steps $t = 0, 1, 2, \dots$. Each node v_i^x in any network G^x occupies one of three states: inactive, P -active, or I -active. The propagation probability from node v_i^x to its neighboring node v_j^x within network G^x is calculated based on node closeness as follows:

$$p(v_i^x, v_j^x) = \frac{\eta(v_i^x, v_j^x)}{\sum_{v_l^x \in N(v_j^x)} \eta(v_l^x, v_j^x)} \quad (1)$$

where $N(v_j^x)$ denotes the set of neighbors for user v_j^x , and

$$\eta(v_i^x, v_j^x) = |N(v_i^x) \cap N(v_j^x)| + 1 \quad (2)$$

represents the number of common neighbors between v_i^x and v_j^x , with the additional term $+1$ accounting for the direct connection between v_i^x and v_j^x . Consequently, $\eta_{(v_i^x, v_j^x)}$ is always a positive integer.

Given the initial seed sets S^P for influence P and S^I for influence I , all nodes in S^P start in a P -active state, and all nodes in S^I start in an I -active state at $t = 0$. Let S_t^P denote the set of P -active users at time step t , and S_t^I denote the set of I -active users at time step t . Clearly, $S_0^P = S^P$ and $S_0^I = S^I$. For each subsequent time step $t \geq 1$, sets S_t^P and S_t^I are first initialized as S_{t-1}^P and S_{t-1}^I , respectively. Then, at time step $t - 1$, each newly P -activated node v_i^x attempts to activate its inactive neighbors v_j^x with probability $p_{(v_i^x, v_j^x)}$. If node v_j^x is successfully activated by influence P , it is added to S_t^P . Importantly, if the newly activated node v_j^x corresponds to the same user appearing in other networks, the corresponding nodes in those networks are also set to P -active. Influence I propagates similarly. This iterative process continues until no new nodes become activated, signaling the end of the propagation.

5. Solution Frameworks

Under the Competitive Independent Cascade Model (CICM), the problem of identifying the initial seed set to maximize influence propagation (CIMASN) is at least NP-hard. Clearly, if we consider a special case with only a single social network ($m = 1$) and an empty initial seed set for the competing influence I (i.e., $S^I = \emptyset$), then the CIMASN problem reduces to Kempe's classical Influence Maximization (IM) problem under the IC model. Since the IM problem is known to be NP-hard, it follows that CIMASN is also NP-hard. To address this computational complexity, we employ a greedy strategy to solve the CIMASN problem.

For clarity in formulating our algorithm, we define the expected influence spread of influence P , given the seed sets S^P and S^I , as $\sigma^P(S^P | S^I)$. Hence, the CIMASN problem can be formally stated as:

$$\arg \max_{|S^P|=k} \sigma^P(S^P | S^I) \quad (3)$$

The proposed greedy algorithm operates as follows: Initially, set $S^P = \emptyset$. Then iteratively evaluate each node v in the inactive state across all m networks, calculating the marginal gain of influence propagation if v were added as a seed node:

$$\sigma^P(S^P \cup \{v\} | S^I) - \sigma^P(S^P | S^I) \quad (4)$$

Since accurately computing the influence spread is NP-hard, we employ Monte Carlo (MC) simulations to estimate these marginal gains. The node v with the highest marginal gain is selected and added to the seed set S^P . The process repeats iteratively until k seed nodes are selected. This procedure is summarized in Algorithm 1.

For a social network G^l with $|V^l|$ nodes and $|E^l|$ edges, each candidate node requires MC rounds of simulation to estimate its influence. Therefore, the overall computational complexity for selecting k seed nodes is given by: $O(k \cdot MC \cdot \sum_{l=1}^m |V^l| \cdot |E^l|)$

Table 1: Statistical Information of Real Networks.

Dataset	User(Nodes)	Relationship(Edges)	Average Degree
Facebook	4,039	88,234	21.8
Twitter	75,879	508,837	6.7

Algorithm 1 Greedy Algorithm for Seed Set Selection

Input: Social networks $G = \{G^1, \dots, G^m\}$, competitor's seed set S^I , budget k , simulation rounds MC

Output: Optimal seed set S^P

$S^P \leftarrow \emptyset$

Compute influence propagation probabilities between all node pairs in the m networks based on Equation 1

for $i \leftarrow 1$ **to** k **do**

for each inactive node v in all networks **do**

Temporarily set v to P -active

Simulate CICM propagation across networks $\{G^1, \dots, G^m\}$

Count the number of newly activated P -active nodes across all networks

Ensure the same user across different networks is only counted once

Repeat for MC iterations to get the average marginal gain defined in Equation 4

end

Identify node v with the highest marginal gain:

$v = \arg \max_u [\sigma^P(S^P \cup \{u\} \mid S^I) - \sigma^P(S^P \mid S^I)]$

Update seed set: $S^P \leftarrow S^P \cup \{v\}$

end

return S^P

6. Experimental Study

To validate the effectiveness of our proposed model and algorithm, we conducted experiments on both real-world and synthetic social networks. The real-world datasets were obtained from Facebook and Epinions using publicly available APIs, with details summarized in Table 1. Synthetic networks were generated using the NetworkX toolkit, as described in Table 2.

Since user alignment across different social networks is beyond the scope of this paper, we adopted a simplified approach: randomly selecting 1% of nodes from the Facebook dataset and pairing them with an equal number of randomly selected nodes from the Epinions dataset, treating matched pairs as the same user across different platforms. In synthetic datasets, 0.5% of nodes from synthetic network 1 were paired with nodes in synthetic network 2, another 0.5% paired with synthetic network 3, and an additional 0.5% simultaneously paired across all three networks. This setup constructs more complex cross-network scenarios. The resulting datasets are referred to as Real Networks (RN) and Synthetic Networks (SN), respectively.

To validate the effectiveness of our proposed CICM model, we compared it with the classical IC model, which neither considers cross-network propagation nor competitive influences. The seed

Table 2: Statistical Information of Synthetic Networks.

Dataset	User(Nodes)	Relationship(Edges)	Average Degree
Synthetic network 1	5,000	75,000	15
Synthetic network 2	5,000	50,000	10
Synthetic network 3	5,000	100,000	20

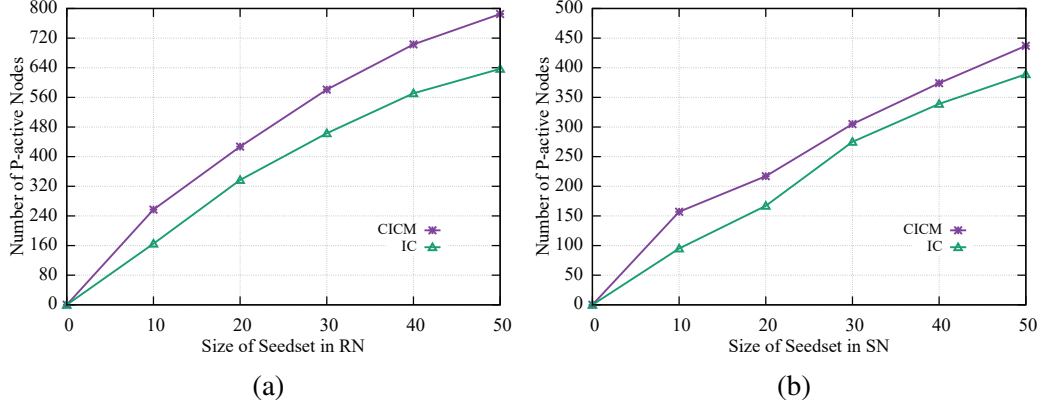


Figure 1: Influence spreads on different propagation model.

size for competitor influence I was set to 30, chosen randomly, and the HighDegree algorithm was used to select seed nodes for influence P . Experiments were performed on both real and synthetic networks, with results presented in Fig. 1. The CICM model exhibited a significantly broader influence spread compared to the classical IC model under identical initial conditions, indicating the improved ability of CICM to represent cross-platform competitive propagation.

We further evaluated the performance of the greedy seed selection algorithm (GA) by comparing it against HighDegree and Random seed selection algorithms. Again, 30 random seeds were used for competitor influence I . Seed selection for our influence P was performed using the High-

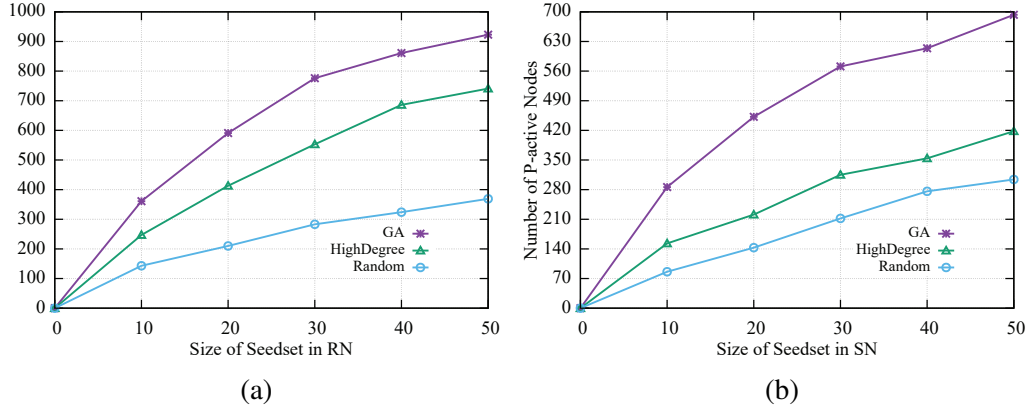


Figure 2: Comparison of influence spreads with seed sets selected by different algorithms.

Degree, Random, and our proposed GA algorithms, respectively. Experimental results shown in Fig. 2 demonstrate that the GA algorithm outperforms both HighDegree and Random algorithms, leading to a significantly larger influence spread for our influence P . These findings confirm that GA effectively identifies superior seed sets under the CICM model across multiple networks.

7. Conclusion

Considering the realistic scenario of competitive influence propagation across multiple social networks, it is critical to understand the mechanisms and optimize propagation strategies across different platforms. We formally defined the *CIMASN* problem and proposed the *CICM* to better reflect real-world competitive dynamics. Based on this model, we further developed a greedy seed set selection algorithm to maximize influence propagation across multiple networks. Experimental results conducted on real and synthetic networks verified the effectiveness of the proposed CICM model. Moreover, our GA demonstrated superior performance compared to classical seed selection algorithms. However, despite its effectiveness, the proposed greedy algorithm still suffers from high computational complexity. Future research will focus on designing more efficient algorithms to address seed selection problems under cross-network propagation scenarios.

Acknowledgments

This research is supported by Yunnan Local Undergraduate Universities Basic Research Joint Special Project (202101BA070001-257), Yunnan Province Key Laboratory of Intelligent Logistics Equipment and Systems and Yunnan Local Undergraduate Universities Basic Research Joint Special Project (202001BA070001-197).

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