

Local Hurst index timing strategy

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Abstract

This paper proposes a technical analysis strategy based on the Hurst index to predict stock price trends in uncertain markets. As a robust timing tool requiring minimal assumptions, the Hurst index effectively captures market memory effects. We apply this method to the CSI 300, mathematically analyze its properties, and empirically validate its profitability.

Keywords: stock trading timing strategy, Hurst index, Quantitative Trading

1. Introduction

1.1. Research significance and status at home and abroad

The stock market's complexity and information overload often lead to costly trading errors. To address this, researchers have developed quantitative strategies and market theories to improve decision-making and stabilize returns.

Traditional securities analysis mainly comprises fundamental and technical analysis. Fundamental analysis assesses factors influencing stock prices but has limited short-term predictive power, while technical analysis forecasts trends using mathematical models of historical price patterns. In recent years, machine learning models have been widely adopted in stock price forecasting and quantitative investment due to their superior fitting capabilities (Deng, 2022). Unlike traditional security analysis, time series methods have advanced stock price forecasting from subjective analysis to model-based prediction. Ye and Cao (2001) studied market efficiency by introducing hurst index. Mandelbrot and Wallis (1969) pioneered R/S analysis for financial time series . Peters (1994) validated biased random walks in U.S. markets. Lu (2014) enhanced CSI 300 timing strategies using

Hurst index. Wang (2006) identified persistent cycles ($H \approx 0.6$) in Chinese markets. Zhang and Fu (2010) demonstrated fractal characteristics in China's stocks.

In this paper, the combination of the V-statistic and the local Hurst index is used to generate trading signals, which is able to capture the aggregation of market volatility and long memory in an integrated manner, both to capture short-term volatility anomalies and to verify the credibility of the long-term trend and to reduce false signals.

1.2. Study feasibility analysis

This paper proposes a Hurst index-based stock timing strategy under EMH and FMH frameworks. Whereas EMH assumes perfect market efficiency with symmetric information, FMH introduces fractal characteristics such as information asymmetry and price memory. The Hurst index (H) quantifies these memory effects: $H \neq 0.5$ signals predictable trends, while $H = 0.5$ indicates random walk behavior. The study follows a structure of theoretical foundation, empirical validation, and conclusion derivation.

2. Theories, Models and Methods

2.1. Two market hypothesis

2.1.1. EFFICIENT MARKET HYPOTHESIS (CHEN, 2005)

Eugene Fama's Efficient Market Hypothesis (EMH) posits that asset prices instantly reflect all information. It requires three conditions: (1) immediate price adjustments, (2) free information flow, and (3) equal data access. Core assumptions include random price movements (Brownian motion) and rational investors. While recognizing real-world irrationalities, EMH argues perfect information doesn't ensure predictability, as new data constantly updates forecasts. Fama classified EMH into three forms (weak, semi-strong, strong), each affecting market timing strategy feasibility.

Weak type effective

Under the weak type hypothesis, market price reflects all past price info. Hence, no arbitrage exists, and analyzing past prices is futile. Timing strategies based on past prices will fail, leaving no technical arbitrage space. Fundamental analysis may still yield excess profits.

Semi-strong type effective

Under the semi-strong efficient hypothesis, prices fully reflect public info on a company's prospects, making intelligence-based predictions futile. In short, semi-strong markets render fundamental analysis ineffective, and technical analysis useless, but profits can still be made from undisclosed info.

Strong type effective

The strong-form EMH claims stock prices fully reflect all information, creating perfectly efficient markets with random walks and total transparency, making prices unpredictable. However, behavioral economics challenges this view on investor irrationality, and speculative bubbles show price-value gaps, highlighting EMH's limitations.

The Hurst index strategy assumes market inefficiency, leveraging price memory effects that contradict EMH's random walk theory. Empirical analysis identifies predictable patterns in historical data, validating its fractal market theory basis over efficient markets.

2.1.2. FRACTAL MARKET HYPOTHESIS (ALIJANI ET AL., 2021; NING AND LIU, 2021; LAMPHIERE ET AL., 2021; HE ET AL., 2021)

The Fractal Market Hypothesis (FMH) emerged to address EMH's limitations in modeling real markets, particularly in explaining seasonal/weekend effects. Mandelbrot introduced fractal geometry to analyze complex patterns like coastlines, later adapted by Peters to financial markets. FMH emphasizes: (1) liquidity dynamics, (2) investor time horizons, and (3) tail risk modeling through fractal distributions.

Content of the fractal market hypothesis

The Fractal Market Hypothesis outlines four principles: (1) Diverse investors—short-term traders exploit volatility; long-term investors stabilize markets. (2) Investors analyze data differently: short-term traders use technical analysis; long-term investors use fundamentals. (3) Liquidity from varied horizons ensures market stability. (4) Market predictability weakens with economic cycle ties—weak ties highlight short-term factors; strong ties raise cyclical risks.

2.1.3. CSI 300 INDEX VALIDITY TEST

proving the CSI 300 index is not efficient is necessary before introducing Hurst index-based timing strategies, providing theoretical support for further study. In an efficient market, the return of stocks roughly follows a normal distribution:

$$X \sim N(\mu, \sigma^2), Y = \frac{(X - \mu)}{\sigma} \sim N(0, 1)$$

The paper evaluates validity by analyzing CSI 300 returns distribution (2002-2020) and assessing normality through distribution patterns. The Jarque-Bera (JB) test checks if skewness/kurtosis match normal distributions.

The kurtosis coefficient (K) measures distribution curve aggregation, with higher values indicating steeper probability density curves. The JB statistic uses fourth-order moments. The kurtosis formula is:

$$K = \frac{\widehat{\mu_4}}{\widehat{\sigma^4}} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^{\frac{4}{2}}}$$

Under normality, the JB statistic approximates 0. In non-normal cases, JB increases regardless of skewness/kurtosis. This paper uses the JB statistic (skewness + kurtosis) to test CSI 300 returns normality (2002-2020).

$$JB = \frac{S^2}{\frac{6}{n}} + \frac{(K - 3)^2}{\frac{24}{n}}$$

In the above equation, n is the total number of samples. The calculation results of the JB statistic using the above definition are shown in Table 1.

Table 1: JB statistic

| Time | Amount of data | Mean | Standard deviation | Kurtosis | Skewness | JB |
|-----------|----------------|-----------|--------------------|----------|----------|--------|
| 2010-2020 | 4653 | -0.000389 | 0.007946 | 8.7933 | -1.5061 | 151225 |

The histogram of the probability density distribution is shown in Figure 1.

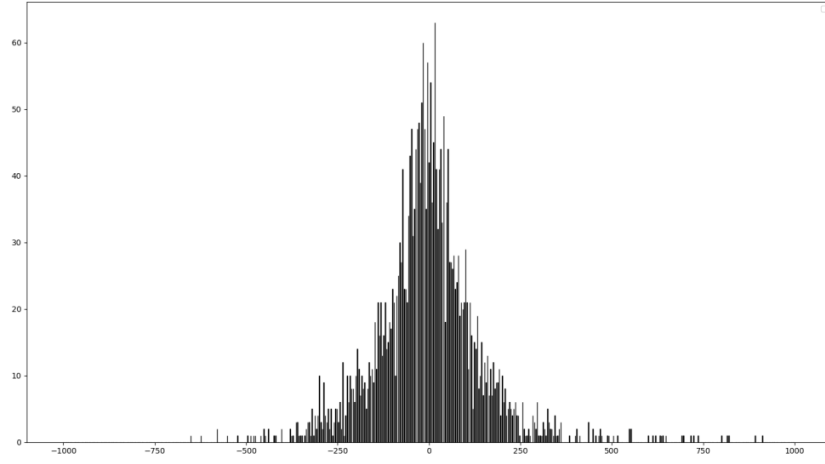


Figure 1: Histogram of yield probability density distribution

The JB statistic $\neq 0$, showing the CSI 300 index's 2010-2020 daily returns deviate from normality (left-skewed, peaked). This indicates the Chinese stock market violates EMH, allowing Hurst index-based strategies to predict price trends.

2.1.4. FRACTAL STUDY OF CSI 300 INDEX

Fractal theory suggests market prices incompletely reflect information, creating predictable patterns ($H \neq 0.5$) rather than random walks. This underpins Hurst index-based strategies, validated by our CSI 300 analysis (2010-2020).

2.1.5. SELF-SIMILARITY OF CSI 300 INDEX

This paper aims to verify the scale-invariance and autocorrelation of the CSI 300 index by observing log returns on daily, monthly, and weekly scales. The result is shown Figure 2, Figure 3 and Figure 4.

The table 2 shows CSI 300 index log return trends (daily, weekly, monthly) from 2010-2020. It shows high similarity across scales, suggesting self-similarity.

Table 2: Market trends on daily, monthly and weekly yield

| time scale | Hurst index (H) | fractal dimension ($D=2-H$) | market state |
|---------------|-----------------|-------------------------------|---------------------------------|
| daily yield | 0.58 | 1.42 | Trend enhancement (persistence) |
| weekly yield | 0.62 | 1.38 | Trend enhancement |
| monthly yield | 0.60 | 1.40 | Trend enhancement |

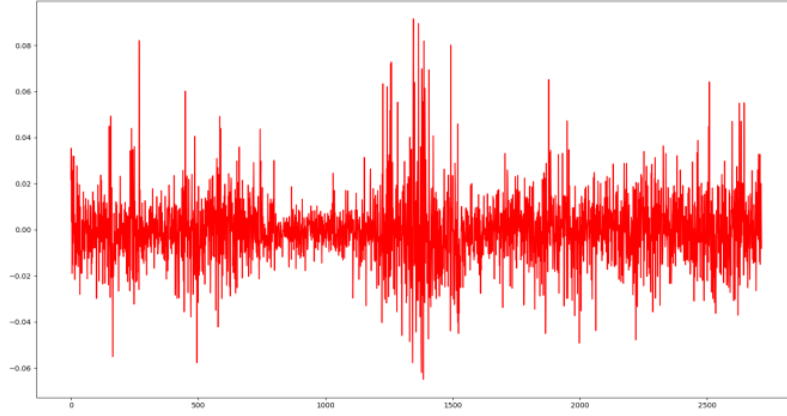


Figure 2: Daily yield curve

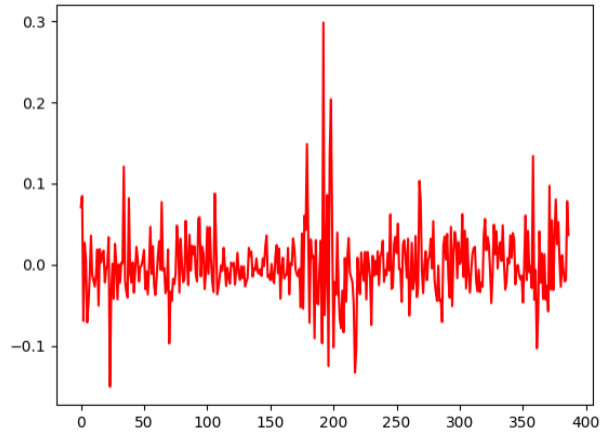


Figure 3: Weekly Return Curve

2.1.6. HURST INDEX ESTIMATION METHOD AND TIMING STRATEGY BASED ON R/S ANALYSIS METHOD

The Hurst index can be estimated via multiple methods, with R/S analysis favored for its nonparametric nature (avoiding normal distribution assumptions) (He et al., 2021; Rong et al., 2021). This robustness suits financial time series analysis, defined as $(R/S)_n = C \cdot n^H$ (H = Hurst exponent, C = constant).

$$\left(\frac{R}{S}\right)_n = C \cdot n^H$$

Calculating the Hurst index, We estimate the Hurst exponent through the following steps (Figure 5):

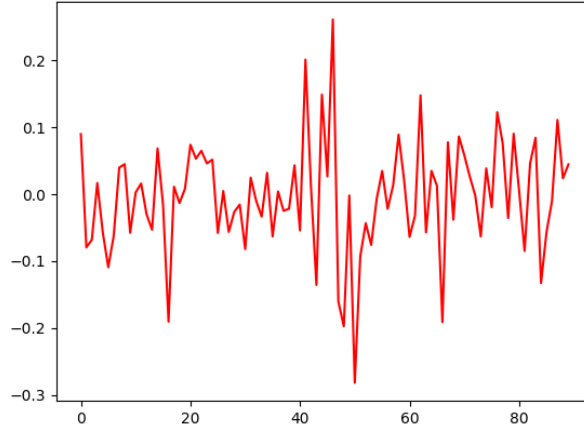


Figure 4: Monthly Return Curve

1. Input Preparation:

Compute log returns: $S_t = \log \left(\frac{P_{t+1}}{P_t} \right), t = 1, 2, \dots, M - 1$

2. Subinterval Processing:

- Partition series into A segments of length n
- Calculate cumulative deviations: $X_{ka} = \sum_{i=1}^k (N_{ia} - e_a) \quad k = 1, 2, \dots, n$

3. Rescaled Range Calculation:

For each subinterval:

- Compute range $R_a = \max(X_{ka}) - \min(X_{ka}), k = 1, 2, \dots, n$
- Calculate standard deviation $S_a = \sqrt{\frac{1}{n} \sum_{k=1}^n (X_{ka} - e_a)^2}$

4. Hurst Estimation:

Fit via least squares: $\lg \left(\frac{R}{S} \right)_n = \lg(a) + H \lg(n)$

$\lg(a)$ and H is Value to be estimated.

Analyzing the Hurst Index

Hurst exponent (H) measures time series memory: $H = 0.5$ =random walk (EMH), $H > 0.5$ =trend persistence, $H < 0.5$ =mean reversion. Metrics like fractal dimension ($D = 2 - H$) and correlation ($C = 2^{2H-1} - 1$) gauge market structure. H/D/C validate fractal markets: $H \neq 0.5$ ($D \neq 1.5$) enables technical trading. $H \rightarrow 1$ ($D \rightarrow 1$)=high predictability; $H \rightarrow 0.5$ ($D \rightarrow 1.5$) = efficient markets. Challenges EMH by revealing predictable price patterns.

2.1.1.7. V-STATISTIC

The V-statistic (rescaled range over window n) is a key R/S analysis tool. Peters' V-statistic tracks correlation decay to identify memory cycle endpoints. In log-log plots, three regimes emerge: flat

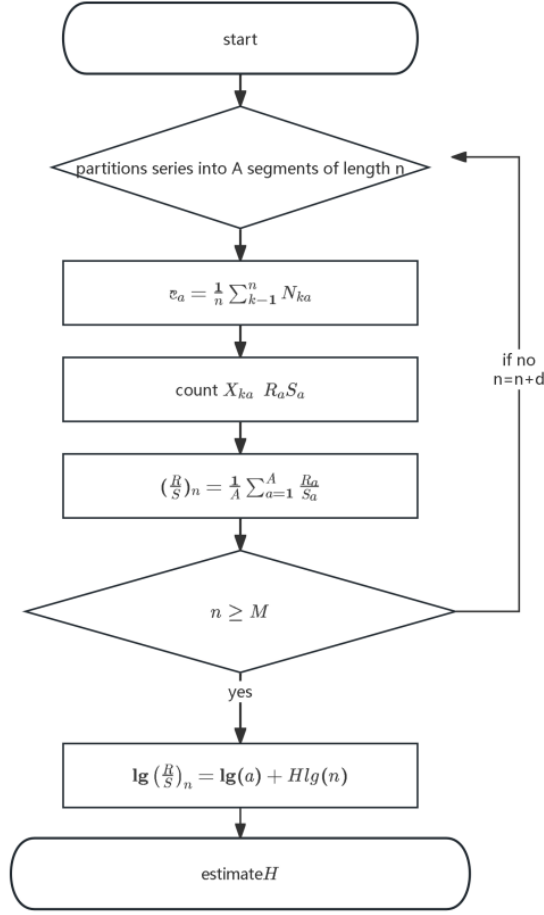


Figure 5: The process of estimating H

$(H = 0.5)$ =random walks, upward $(H > 0.5)$ =trends, downward $(H < 0.5)$ =mean reversion. These patterns detect market regime shifts, with V_n vs $\log(n)$ inflection points marking transitions to randomness.

Experiment (Zemzem and Tagina, 2023; Pourahmadi et al., 2023; Ullah et al., 2023)

This study develops a CSI 300 timing strategy using V-statistics and local Hurst index analysis on 2010-2020 daily data ($n = 4,653$). The V-statistic's inflection point (shown in Figure 6) at $\lg(n) = 2.59$ identifies a 389-day memory cycle, establishing the window size for local Hurst calculations. When $H = 0.5$, this marks the memory loss threshold where predictive power diminishes.

2.2. Local Hurst index

The local Hurst index applies R/S analysis to rolling 389-day windows of return series, generating dynamic trend estimates that capture evolving market memory effects.

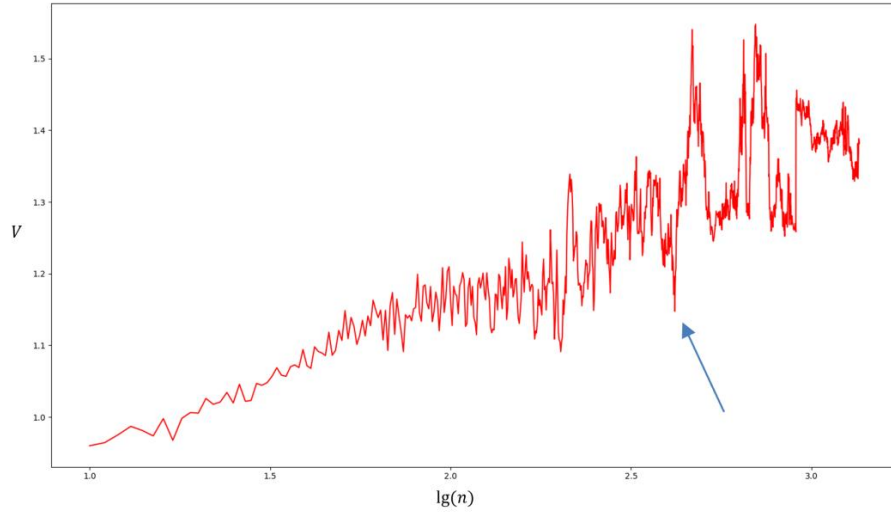


Figure 6: Graph of the relationship between V-statistic and $\lg(n)$

2.3. Buying and selling strategy

Hurst-based CSI 300 strategy uses $E(H)=0.551$ threshold. Buy signals: price up + Hurst >0.551 or market <3500 at threshold. Exit at peaks with Hurst <0.551 . 2010-2020 data validates strong CSI 300-Hurst link. Innovations: memory thresholds, dual filters, trend exits enhance signals and persistence.

2.4. Simulated Trading

This paper simulates trading from February 3, 2010, using the above-mentioned trading strategy for timing purchase and sale timing.

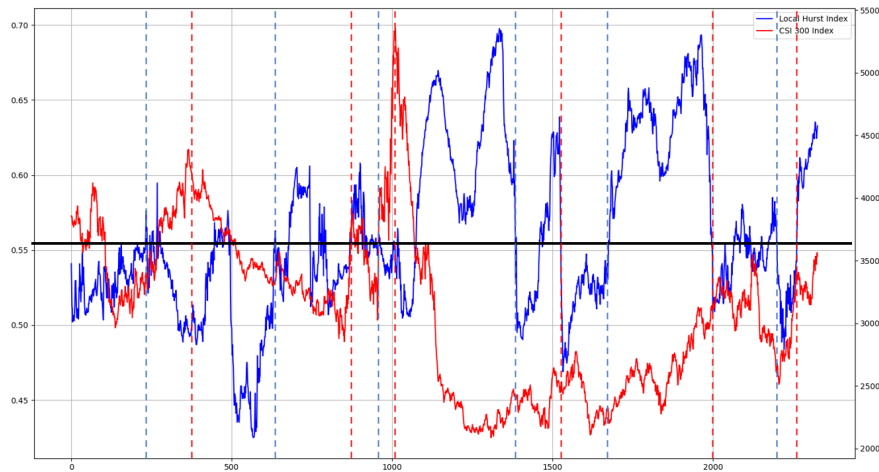


Figure 7: Buying and selling results

The trading decision results are in the figure 7. The black line is $E(H)$, blue dotted line is buy signal, and red dotted line is sell signal. From Feb 3, 2010 to Jan 1, 2020, the strategy issued 6 signals; trading records are in the table 3.

Table 3: Buy-Sell Timing Returns Table

| Purchase Date (trading day) | Index at the time of purchase | Sell Date (trading Date) | Index at the time of sale | Yield Rate |
|--------------------------------|----------------------------------|-----------------------------|------------------------------|------------|
| 241 | 3246 | 381 | 4180 | 28.21% |
| 639 | 3310 | 864 | 3614 | 9.18% |
| 963 | 3267 | 1014 | 4723 | 44.56% |
| 1391 | 2416 | 1584 | 2536 | 4.96% |
| 1674 | 2278 | 2003 | 3101 | 36.12% |
| 2210 | 2531 | 2331 | 3416 | 34.96% |

3. Summary

Hurst-based CSI 300 timing strategy yields 390% returns (2010-2020). Theoretical bases: rejects EMH via non-normal returns ($JB=151,225$), validates FMH through self-similarity. Method: combines V-statistic memory cycles (389 days) and local Hurst ($E(H)=0.551$) for signals. Empirical results confirm efficacy, but limitations (simplified stats, sample size) suggest future work via expanded features and robustness tests.

Acknowledgments

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