Non-Euclidean High-Order Smooth Convex Optimization

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Abstract

We develop algorithms for the optimization of convex objectives that have Hölder continuous q-th derivatives by using a q-th order oracle, for any $q \geq 1$. Our algorithms work for general norms under mild conditions, including the ℓ_p-settings for $1 \leq p \leq \infty$. We can also optimize structured functions that allow for inexactly implementing a non-Euclidean ball optimization oracle. We do this by developing a non-Euclidean inexact accelerated proximal point method that makes use of an *inexact uniformly convex regularizer*. We show a lower bound for general norms that demonstrates our algorithms are nearly optimal in high-dimensions in the black-box oracle model for ℓ_p-settings and all $q \geq 1$, even in randomized and parallel settings. This new lower bound, when applied to the first-order smooth case, resolves an open question in parallel convex optimization.

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