Robust random graph matching in Gaussian models via vector approximate message passing

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Abstract

In this paper, we focus on the matching recovery problem between a pair of correlated Gaussian Wigner matrices with a latent vertex correspondence. Although Polynomial-time algorithms for graph matching have been studied in the line of work (Barak et al. (2019); Ding et al. (2021); Fan et al. (2023a,b); Ganassali and Massoulié (2020); Ganassali et al. (2024a); Mao et al. (2021, 2023a); Ganassali et al. (2024b); Mao et al. (2023b); Ding and Li (2025+, 2023)), many of the efficient algorithms used to achieve matching recovery are believed to be fragile in the sense that adversarially modifying a small fraction of edges could fool the algorithm into outputting a result which deviates strongly from the true underlying matching. Thus, we are particularly interested in a robust version of this problem such that our observation is a perturbed input $(A + E, B + F)$ where (A, B) is a pair of correlated Gaussian Wigner matrices and E, F are adversarially chosen matrices supported on an unknown $\epsilon n * \epsilon n$ principle minor of A, B, respectively. We propose a vector approximate message passing (vector AMP) algorithm that succeeds in polynomial time as long as the correlation ρ between (A, B) is a non-vanishing constant and $\epsilon = o\left(\frac{1}{(\log n)^{20}}\right)$.

The main methodological inputs for our result are the iterative random graph matching algorithm proposed in Ding and Li (2025+, 2023) and the spectral cleaning procedure proposed in Ivkov and Schramm (2025). To the best of our knowledge, our algorithm is the first efficient random graph matching type algorithm that is robust under any adversarial perturbations of $n^{1-o(1)}$ size.[1]

Keywords: random graph matching; robust algorithm; approximate message passing.

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