Low-dimensional adaptation of diffusion models: Convergence in total variation

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Editors: Nika Haghtalab and Ankur Moitra

Abstract

This paper presents new theoretical insights into how diffusion generative models adapt to low-dimensional structure in data distributions. We study two widely used samplers — the denoising diffusion probabilistic model (DDPM) and the denoising diffusion implicit model (DDIM) — and analyze their convergence behavior under the assumption of accurate score estimates. Our main result shows that both DDPM and DDIM require at most $O(k/\varepsilon)$ iterations (up to logarithmic factors) to generate samples that are ε-close to the target distribution in total variation distance, where k captures an intrinsic low-dimensional structure of the distribution. Importantly, our theory holds without assuming smoothness or log-concavity. These results provide the first rigorous guarantees for the low-dimensional adaptation capability of DDIM-type samplers, and significantly improve upon prior TV-based convergence bounds for DDPM. Our analysis also highlights the role of discretization coefficients in exploiting low-dimensional structure, and establishes lower bounds that justify the optimality of commonly used parameter choices originally proposed by Ho et al. (2020); Song et al. (2020). [1]

Keywords: Low-dimensional structure, DDPM, DDIM, Score-based generative model, Stochastic localization.

Acknowledgments

Y. Chen is supported in part by the Alfred P. Sloan Research Fellowship, the AFOSR grant FA9550-22-1-0198, the ONR grants N00014-22-1-2354 and N00014-25-1-2344, the NSF grants 2221009 and 2218773, and the Amazon Research Award. We thank Yuting Wei and Yuchen Wu for their helpful discussions.

1. Extended abstract. Full version appears as [arXiv:2501.12982, v2]

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