Bayes correlated equilibria, no-regret dynamics in Bayesian games, and the price of anarchy

Kaito Fujii FUJIIK@NII.AC.JP
National Institute of Informatics

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This paper investigates equilibrium computation and the price of anarchy for Bayesian games, which are the fundamental models of games with incomplete information proposed by Harsanyi (1967, 1968a,b). In normal-form games with complete information, it is known that efficiently computable no-regret dynamics converge to correlated equilibria (Foster and Vohra, 1997; Hart and Mas-Colell, 2000; Blum and Mansour, 2007), and the price of anarchy for correlated equilibria can be bounded for a broad class of games called smooth games (Roughgarden, 2015). However, as surveyed by Forges (1993), there are various non-equivalent extensions of correlated equilibria, which are collectively called *Bayes correlated equilibria*. For example, *communication equilibria* are naturally realized by introducing a mediator who can bidirectionally communicate with players (Myerson, 1982; Forges, 1986). Other extensions include *strategic-form correlated equilibria* (SFCEs) and *agent-normal-form correlated equilibria* (ANFCEs), which are defined as correlated equilibria of complete-information interpretations of Bayesian games called the strategic form and agent normal form, respectively.

In this paper, we present *the intersection of communication equilibria and ANFCEs* as an equilibrium concept that is efficiently computable and has PoA bounds for various games. Our contributions are summarized as follows.

- This paper proposes a variant of swap regret, which we call *untruthful swap regret*. If each player minimizes it in repeated play of Bayesian games, the empirical distribution of these dynamics is guaranteed to converge to this intersection class.
- We propose an efficient algorithm for minimizing the untruthful swap regret with regret upper bound $O(\sqrt{T \max\{\log|\Theta_i|, |A_i|\log|A_i|\}})$, where T is the number of rounds, $|\Theta_i|$ is the number of types, and $|A_i|$ is the number of actions.
- We show that this upper bound is tight in terms of the number of types by providing a problem instance for which no algorithm can achieve untruthful swap regret better than $\Omega(\sqrt{T \log|\Theta_i|})$.
- For smooth games and mechanisms, we extend existing lower bounds on the price of anarchy based on the smoothness arguments from Bayes–Nash equilibria to equilibria obtained by the proposed dynamics. [1]

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