A Proof of The Changepoint Detection Threshold Conjecture in Preferential Attachment Models

Hang Du HANGDU@MIT.EDU
Department of Mathematics, Massachusetts Institute of Technology

Shuyang Gong GONGSHUYANG@STU.PKU.EDU.CN
School of Mathematical Sciences, Peking University

Jiaming Xu JIAMING.XU868@DUKE.EDU
The Fuqua School of Business, Duke University

Editors: Nika Haghtalab and Ankur Moitra

Abstract

We investigate the problem of detecting and estimating a changepoint in the attachment function of a network evolving according to a preferential attachment model on n vertices, using only a single final snapshot of the network. Bet et al. (2023) show that a simple test based on thresholding the number of vertices with minimum degrees can detect the changepoint when the change occurs at time $n - \Omega(\sqrt{n})$. They further make the striking conjecture that detection becomes impossible for any test if the change occurs at time $n - o(\sqrt{n})$. Kaddouri et al. (2024) make a step forward by proving the detection is impossible if the change occurs at time $n - o(n^{1/3})$. In this paper, we resolve the conjecture affirmatively, proving that detection is indeed impossible if the change occurs at time $n - o(\sqrt{n})$. Furthermore, we establish that estimating the changepoint with an error smaller than $o(\sqrt{n})$ is also impossible, thereby confirming that the estimator proposed in Bhamidi et al. (2018) is order-optimal.[1]

Keywords: Preferential attachment models, Changepoint detection

Acknowledgments

The authors are grateful for inspiring discussions on the preferential attachment models with Simiao Jiao, Weijia Li, and Xiaochun Niu. J. Xu also thanks the organizers of the conference "Statistical and Probabilistic Analysis of Random Networks and Processes", where he first learned about the changepoint detection threshold conjecture.

H. Du is partially supported by an NSF-Simons research collaboration grant (award number 2031883). S. Gong is partially supported by National Key R&D program of China (No. 2023YFA1010103) and NSFC Key Program (Project No. 12231002). Part of the work is carried out while S. Gong is in residence at the Mathematical Sciences Research Institute in Berkeley, California, during the Spring 2025 semester. J. Xu is supported in part by an NSF CAREER award CCF-2144593.

1. Extended abstract. Full version appears as [arXiv:2502.00514, v3]

References

Louigi Addario-Berry, Luc Devroye, Gbor Lugosi, and Vasiliki Velona. Broadcasting on random recursive trees. *Annals of Applied Probability*, 2021.

Sayan Banerjee and Shankar Bhamidi. Root finding algorithms and persistence of jordan centrality in growing random trees. *The Annals of Applied Probability*, 32(3):2180–2210, 2022.

Sayan Banerjee and Xiangying Huang. Degree centrality and root finding in growing random networks. *Electronic Journal of Probability*, 28:1–39, 2023.

Sayan Banerjee, Shankar Bhamidi, and Iain Carmichael. Fluctuation bounds for continuous time branching processes and evolution of growing trees with a change point. *The Annals of Applied Probability*, 33(4):2919–2980, 2023.

Albert-László Barabási and Réka Albert. Emergence of scaling in random networks. *Science*, 286 (5439):509–512, 1999.

Anna Ben-Hamou and Vasiliki Velona. Inference in balanced community modulated recursive trees. *Bernoulli*, 31(1):457–483, 2025.

Gianmarco Bet, Kay Bogerd, Rui M Castro, and Remco van der Hofstad. Detecting a late change-point in the preferential attachment model. *arXiv preprint arXiv:2310.02603*, 2023.

Shankar Bhamidi, Jimmy Jin, and Andrew Nobel. Change point detection in network models: Preferential attachment and long range dependence. *The Annals of Applied Probability*, 28(1): 35–78, 2018.

Monika Bhattacharjee, Moulinath Banerjee, and George Michailidis. Change point estimation in a dynamic stochastic block model. *Journal of Machine Learning Research*, 21(107):1–59, 2020. URL http://jmlr.org/papers/v21/18-814.html.

Stéphane Boucheron, Gábor Lugosi, and Pascal Massart. *Concentration Inequalities - A Nonasymptotic Theory of Independence*. Oxford University Press, 2013. ISBN 978-0-19-953525-5. doi: 10.1093/ACPROF:OSO/9780199535255.001.0001. URL https://doi.org/10.1093/acprof:oso/9780199535255.001.0001.

Anna M Brandenberger, Luc Devroye, and Marcel K Goh. Root estimation in galton–watson trees. *Random Structures & Algorithms*, 61(3):520–542, 2022.

Simon Briend, Francisco Calvillo, and Gábor Lugosi. Archaeology of random recursive dags and cooper-frieze random networks. *Combinatorics, Probability and Computing*, 32(6):859–873, 2023.

Simon Briend, Christophe Giraud, Gábor Lugosi, and Déborah Sulem. Estimating the history of a random recursive tree. *arXiv preprint arXiv:2403.09755*, 2024.

Sbastien Bubeck, Elchanan Mossel, and Mikls Rcz. On the influence of the seed graph in the preferential attachment model. *IEEE Transactions on Network Science and Engineering*, 2(1): 30–39, 2015.

Sbastien Bubeck, Luc Devroye, and Gbor Lugosi. Finding adam in random growing trees. *Random Structures & Algorithms*, 50(2):158–172, 2017a.

Sbastien Bubeck, Ronen Eldan, Elchanan Mossel, and Mikls Rcz. From trees to seeds: on the inference of the seed from large trees in the uniform attachment model. *Bernoulli*, 23(4A):2887–2916, 2017b.

Daniel Cirkovic, Tiandong Wang, and Xianyang Zhang. Likelihood-based inference for random networks with changepoints. *arXiv preprint arXiv:2206.01076*, 2022.

Alice Contat, Nicolas Curien, Perrine Lacroix, Etienne Lasalle, and Vincent Rivoirard. Eve, adam and the preferential attachment tree. *Probability Theory and Related Fields*, 190(1):321–336, 2024.

Harry Crane and Min Xu. Inference on the history of a randomly growing tree. *Journal of the Royal Statistical Society Series B: Statistical Methodology*, 83(4):639–668, 2021.

Nicolas Curien, Thomas Duquesne, Igor Kortchemski, and Ioan Manolescu. Scaling limits and influence of the seed graph in preferential attachment trees. *Journal de lcole PolytechniqueMathmatiques*, 2:1–34, 2015.

Maria Deijfen, Henri Van Den Esker, Remco Van Der Hofstad, and Gerard Hooghiemstra. A preferential attachment model with random initial degrees. *Arkiv för matematik*, 47:41–72, 2009.

Luc Devroye and Tommy Reddad. On the discovery of the seed in uniform attachment trees. *Internet Mathematics*, pages 75–93, 2019.

Farida Enikeeva and Olga Klopp. Change-point detection in dynamic networks with missing links. *Operations Research*, 2025.

Fengnan Gao and Aad van der Vaart. On the asymptotic normality of estimating the affine preferential attachment network models with random initial degrees. *Stochastic Processes and their Applications*, 127(11):3754–3775, 2017.

Fengnan Gao and Aad van der Vaart. Statistical inference in parametric preferential attachment trees. *arXiv preprint arXiv:2111.00832*, 2021.

Fengnan Gao, Aad van der Vaart, Rui Castro, and Remco van der Hofstad. Consistent estimation in general sublinear preferential attachment trees. *Electronic Journal of Statistics*, 11(2):3979 – 3999, 2017. doi: 10.1214/17-EJS1356. URL https://doi.org/10.1214/17-EJS1356.

John Haigh. The recovery of the root of a tree. *Journal of Applied Probability*, 7(1):79–88, 1970.

Varun Jog and Po-Ling Loh. Analysis of centrality in sublinear preferential attachment trees via the crump-mode-jagers branching process. *IEEE Transactions on Network Science and Engineering*, 4(1):1–12, 2016.

Varun Jog and Po-Ling Loh. Persistence of centrality in random growing trees. *Random Structures and Algorithms*, 2017.

Ibrahim Kaddouri, Zacharie Naulet, and Élisabeth Gassiat. On the impossibility of detecting a late change-point in the preferential attachment random graph model. *arXiv preprint arXiv:2407.18685*, 2024.

Justin Khim and Po-Ling Loh. Confidence sets for the source of a diffusion in regular trees. *IEEE Transactions on Network Science and Engineering*, 4(1):27–40, 2016.

Nitish Korula and Silvio Lattanzi. An efficient reconciliation algorithm for social networks. *Proc. VLDB Endow.*, 7(5):377388, January 2014. ISSN 2150-8097. doi: 10.14778/2732269.2732274. URL https://doi.org/10.14778/2732269.2732274.

Gbor Lugosi and Alan S. Pereira. Finding the seed of uniform attachment trees. *Electronic Journal of Probability*, 24:1–15, 2019.

Abram N Magner, Jithin K Sreedharan, Ananth Y Grama, and Wojciech Szpankowski. Times: Temporal information maximally extracted from structures. In *Proceedings of the 2018 World Wide Web Conference*, pages 389–398, 2018.

Yury Polyanskiy and Yihong Wu. *Information theory: From coding to learning*. Cambridge university press, 2025.

Miklós Z Rácz and Anirudh Sridhar. Correlated randomly growing graphs. *The Annals of Applied Probability*, 32(2):1058–1111, 2022.

Devavrat Shah and Tauhid Zaman. Rumors in a network: Who's the culprit? *IEEE Transactions on information theory*, 57(8):5163–5181, 2011.

Devavrat Shah and Tauhid Zaman. Finding rumor sources on random trees. *Operations research*, 64(3):736–755, 2016.

Remco Van Der Hofstad. *Random Graphs and Complex Networks (Volume One)*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, 2016.

Remco Van Der Hofstad. *Random Graphs and Complex Networks (Volume Two)*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, 2024.

Venugopal V Veeravalli and Taposh Banerjee. Quickest change detection. In *Academic press library in signal processing*, volume 3, pages 209–255. Elsevier, 2014.

Daren Wang, Yi Yu, and Alessandro Rinaldo. Optimal change point detection and localization in sparse dynamic networks. *The Annals of Statistics*, 49(1):203–232, 2021.

Heng Wang, Minh Tang, Youngser Park, and Carey E Priebe. Locality statistics for anomaly detection in time series of graphs. *IEEE Transactions on Signal Processing*, 62(3):703–717, 2013.

Yihong Wu and Jiaming Xu. Statistical problems with planted structures: Information-theoretical and computational limits. *arXiv preprint arXiv:1806.00118*, 2018.

Liyan Xie, Shaofeng Zou, Yao Xie, and Venugopal V Veeravalli. Sequential (quickest) change detection: Classical results and new directions. *IEEE Journal on Selected Areas in Information Theory*, 2(2):494–514, 2021.

Jean-Gabriel Young, Guillaume St-Onge, Edward Laurence, Charles Murphy, Laurent Hébert-Dufresne, and Patrick Desrosiers. Phase transition in the recoverability of network history. *Physical Review X*, 9(4):041056, 2019.

Zifeng Zhao, Li Chen, and Lizhen Lin. Change-point detection in dynamic networks via graphon estimation. *arXiv preprint arXiv:1908.01823*, 2019.