7.2 Experiment (B): Parity violation at β decay

7.2.1 Tasks

1. Energy calibration with a ^{22}Na source.
2. Confirmation of parity violation in β decay.
3. Estimation of the degree of polarization of the bremsstrahlung produced by the β particles.
4. Estimation of the longitudinal polarization of the electrons emitted in the β decay.

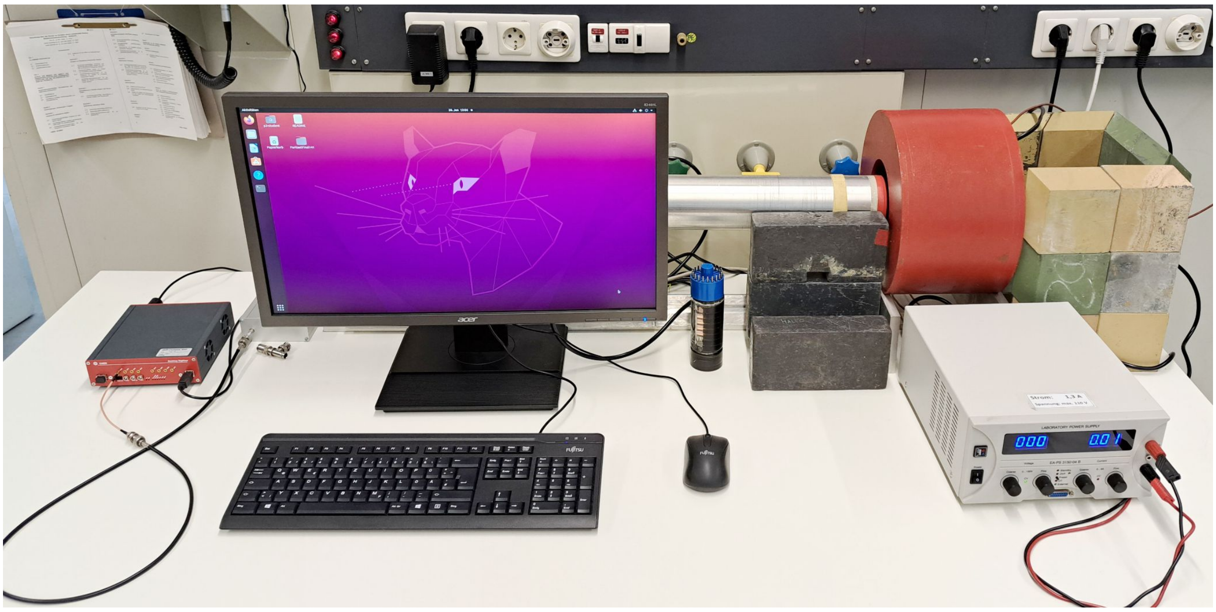

Figure 7.5: Setup of the parity violation experiment.

7.2.2 Introduction

In this experiment, an important conservation law is tested with the simplest of means. Before the experiment, however, some theoretical effort is necessary. Namely, from the measured circular polarization of the γ quanta, the longitudinal one of the electrons has to be inferred unambiguously. For this purpose it is necessary to calculate the corresponding Feynman diagrams. It turns out that the uniqueness is given only for large energies. For this reason only bremsstrahlung-quanta with energies larger than 1 MeV are counted. In principle even simpler is the Wu experiment, because the recourse to theory is not necessary. The parity violation follows directly from the measured asymmetry of the electrons. However, the experiment itself requires a little more effort because of the low temperatures and the high magnetic fields.

7.2.2.1 The parity

The parity operation is the inversion of coordinates. For a state with the wave function $\Psi(\vec{r})$ in the spatial representation is valid

$$P \cdot \Psi(\vec{r}) = \Psi(-\vec{r}) \tag{7.15}$$

The parity operation is equivalent to the spatial reflection in a plane, since it can be represented as the product of this reflection S and a subsequent rotation by 180°, $D(180°)$.

$$P = S \cdot D(180°) \tag{7.16}$$

The equivalence is a consequence of the invariance of the quantum mechanical expectation values at rotations (conservation of angular momentum).

For an arbitrary state $|a\rangle$ with defined parity, the eigenvalue equation is

$$P\,|a\rangle = \pi_a\,|a\rangle \tag{7.17}$$

and, since the inversion twice leads back to the initial state

$$P^2\,|a\rangle = |a\rangle \tag{7.18}$$

Therefore applies

$$P^2 = 1, \qquad P = P^{-1}, \qquad \pi_a = \pm 1 \tag{7.19}$$

The eigenvalues of the parity operator are +1 and -1. In the first case we speak of even parity or symmetric state, in the second of odd parity or asymmetric state.

The expected value of an operator with defined parity transforms under the parity operation as follows.

$$POP^{-1} = \pi_0\,O \tag{7.20}$$

examples are the operators of location $\vec{r}$, momentum $\vec{p}$, spin $\vec{\sigma}$ and angular momentum $\vec{\ell}$.

$$\begin{aligned} P\vec{r}P^{-1} &= -\vec{r} \\ P\vec{p}P^{-1} &= -\vec{p} \\ P\vec{\sigma}P^{-1} &= +\vec{\sigma} \\ P\vec{\ell}P^{-1} &= +\vec{\ell} \end{aligned} \tag{7.21}$$

Here you can see that there are operators which change their sign under the parity transformation.

An operator is invariant under a transformation if its expectation value does not change. If an operator is invariant, the physical quantity for which it stands obeys a conservation law. Examples include the law of conservation of momentum, which follows from invariance with respect to translations, and the law of conservation of angular momentum, which follows from invariance with respect to space-*rotations*. Similarly, parity is said to be preserved if the expectation value of an operator is invariant with respect to space-*mirroring*.

If one wants to check whether a theory is parity-preserving, one must measure the expectation values of such operators which are sensitive to the mirroring: these are the so-called pseudoscalars. They are quantities which are scalar, i.e., rotationally invariant, but change sign upon mirroring. According to Eq. 7.21 such quantities can be formed as the scalar product of a polar and an axial vector. Exactly such expectation values, the products of one momentum and one angular momentum, have been measured when parity conservation was tested in the weak interaction. Pseudoscalars must necessarily be zero in a parity-preserving theory. Conversely, the observation of a non-zero expectation value is sufficient for the violation of parity.

In Wu's experiment, the β^- emission from polarized ^{60}Co nuclei was measured. The ^{60}Co was aligned by cooling to very low temperatures in a strong magnetic field. This makes the energy difference of states with different magnetic quantum numbers large compared to the energy of thermal motion. Therefore, only the lower states are occupied, which have large magnetic quantum numbers and describe nuclei aligned in the direction of the magnetic field. In the ideal case ($T = 0$ or $B = \infty$), all nuclei are in the lowest state, which corresponds to complete polarization. The result of Wu's measurement was: the β decay of polarized nuclei is not isotropic, the β^- particles are emitted preferentially *anti-parallel* to the nuclear spin (which is parallel to the magnetic field) of the ^{60}Co.

This result is not compatible with a parity-preserving theory, in which the world is mirror-invariant. In this case, exactly as many particles should have been emitted parallel to the field. From theory the angular distribution of the electron should be,

$$W\left(\vec{p}_\mathrm{e}, \vec{J}\right) = 1 + \frac{v}{c} \cdot P \cdot A \cdot \frac{\vec{p}_\mathrm{e} \cdot \vec{J}}{|\vec{p}_\mathrm{e}| \cdot \left|\vec{J}\right|} \tag{7.22}$$

where P is the degree of polarization of the ^{60}Co, A is the asymmetry parameter containing the matrix elements and the coupling constants of the weak interaction, $\vec{p}_\mathrm{e}$ is the momentum of the electron and $\vec{J}$ is the spin of the nucleus. As can be seen, the pseudoscalar $\left(\vec{p}_\mathrm{e} \cdot \vec{J}\right)$ is responsible for the occurrence of an asymmetry. Note also that in a parity preserving theory the angular distribution must be *necessarily* isotropic. However, this condition is not *sufficient*, because the asymmetry parameter A can be zero by chance.

In this experiment, another, equally simple pseudoscalar is measured, the scalar product of the spin and momentum of the β-particle.

$$H = \frac{\vec{\sigma} \cdot \vec{p}}{|\vec{\sigma}| \cdot |\vec{p}|} \tag{7.23}$$

This is the projection of the electron spin onto the direction of flight and is called longitudinal polarization or helicity. If a finite value is measured for the helicity of a β-particle, the parity is violated.

7.2.2.2 The polarization of particles and photons

For a particle with spin $\vec{S}$, the polarization P with respect to a distinct direction (z-axis) is defined as the ratio of the expected value of the spin operator along this direction to the magnitude of the spin

$$P = \frac{\langle \vec{S}_z \rangle}{S} \tag{7.24}$$

In it $\vec{S}_z$ is the z-component of the spin operator $\vec{S}$. The spin space of a particle with spin S has $2S + 1$ dimensions. As basis vectors in this space can be chosen the $2S + 1$ orthonormal states $|S, S_z\rangle$, which are at the same time eigenstates of $\vec{S}^2$ and $\vec{S}_z$, for which thus holds

$$\begin{aligned} \vec{S}^2 \left|S, S_z\right\rangle &= S\left(S+1\right)\left|S, S_z\right\rangle \\ \vec{S}_z \left|S, S_z\right\rangle &= S_z \left|S, S_z\right\rangle \\ -S \leq\ & S_z \leq +S \end{aligned} \tag{7.25}$$

For a particle with $S = 1/2$, the representation of the spin operator $\vec{S}$ by the Pauli matrices $\vec{\sigma}$ is common. It is

$$\vec{S} = \frac{1}{2}\,\vec{\sigma} = \frac{1}{2}\,(\sigma_x, \sigma_y, \sigma_z) \tag{7.26}$$

with

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{7.27}$$

In this representation, the base states $|S, S_z\rangle$ are.

$$|1/2, +1/2\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ und } |1/2, -1/2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{7.28}$$

These states describe the alignment parallel and anti-parallel to the z-axis, respectively. The general state is a superposition of the two basic states

$$\Psi = \begin{pmatrix} a_+ \\ a_- \end{pmatrix} = a_+ \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + a_- \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \text{mit} \qquad a_+^2 + a_-^2 = 1 \tag{7.29}$$

a_+^2, a_-^2 are the probabilities of the states with parallel or anti-parallel orientation. The polarization of this state is

$$P = \frac{\langle S_z \rangle}{S} = \langle \sigma_z \rangle = \langle \Psi | \sigma_z | \Psi \rangle = a_+^2 - a_-^2 \qquad \text{mit} \qquad -1 \leq P \leq +1 \tag{7.30}$$

The maximum values belong to parallel ($a_- = 0, P = +1$) and anti-parallel ($a_+ = 0, P = -1$) configuration to the z-axis, respectively. These are the cases of complete polarization *with respect to the z-axis.* For any *other* state, a direction can be found in which the particle is completely polarized.

This is valid for *one* particle. For a entirety of *many* particles, e.g. a particle beam, the polarization is still defined by Eq. 7.24, but now the averaging goes also over the total number of all particles. In this case there is not any more a pure state, so the expectation value is no longer given by Eq. 7.30, but by

$$P = \langle \bar{\sigma_z} \rangle \sum_{S_z=-1/2}^{S_z=+1/2} p_{S_z} \langle S, S_z | \sigma_z | S, S_z \rangle \tag{7.31}$$

Here, p_{S_z} is the probability of the occurrence of a pure state with a given S_z and it is

$$p_{+1/2} + p_{-1/2} \equiv p_+ + p_- = 1 \tag{7.32}$$

Inserting Eq. 7.27 and Eq. 7.28, we obtain for the polarization

$$p = p_+ - p_- \tag{7.33}$$

The probabilities p_+ and p_- can be determined by measuring the relative number of particles N_+ and $N-$ in the pure states, respectively. Finally, one obtains for the polarization

$$P = \frac{N_+}{N_+ + N_-} - \frac{N_-}{N_+ + N_-} = \frac{N_+ - N_-}{N_+ + N_-} \tag{7.34}$$

Absolute polarization exists when all particles are oriented parallel ($N_- = 0$, $P = +1$) or anti-parallel ($N_+ = 0$, $P = -1$) to the z-axis. The polarization vanishes if there are equal numbers pointing in each direction ($N_+ = N_-$). If it is not totally polarized ($|P| < 1$), no other direction can be found with respect to which the polarization is absolute.

These considerations are valid in the non-relativistic regime, where the direction of the momentum and the direction of the spin are not coupled. Therefore, the z-direction can be chosen arbitrarily with respect to the momentum direction, so that a purely transverse polarization is possible for the single electron as well as for the electrons at large. In relativistic Dirac theory, on the other hand, the electron always has a spin component in the direction of momentum, i.e. longitudinal polarization, whose magnitude depends on the velocity. It goes down to zero with the velocity and becomes absolute when the velocity approaches the speed of light.

The situation is somewhat different for the γ quantum, which has spin 1. In general, according to Eq. 7.25, three states are necessary for such a particle to describe the polarization. In the chosen representation, these are the projections along the z-axis with $S_z = -1, 0, +1$. For the γ quantum, which has the rest mass zero and therefore moves with the speed of light, this is not valid. In this case, there is no transverse spin. There are only two alignments, either in the direction of the momentum or in the opposite direction. These alignments correspond to the circular polarization in classical optics. A *single* γ quantum is always fully circularly polarized and if the spin is in the direction of momentum, the polarization is right-circular.

$$P_\mathrm{C} = \frac{N_+ - N_-}{N_+ + N_-} \tag{7.35}$$

N_+ and N_- are the numbers of right and left circularly polarized quanta in the beam, respectively. $P_\mathrm{C} = 1$ means that all quanta are right circularly polarized.

Linear polarization of a *single* γ-quantum does not exist. However, for the *whole number of quanta*, it is possible if there are fixed phase relations between the circular polarizations of the quanta. This is also in analogy to classical optics.

7.2.3 Principle of the measurement

7.2.3.1 The polarization of Bremsstrahlung

In this experiment, the longitudinal polarization of electrons emitted during β-decays is determined by measuring the polarization of the bremsstrahlung produced when the electrons decelerate in matter. In this process, the polarization of the electrons is partially transferred to the photons, and the polarization state of the bremsstrahlung is determined by the electrons. There are three cases to distinguish.

a) The electrons are unpolarized: then the bremsstrahlung is linearly polarized. The polarization is largest at the low energy end of the bremsstrahlung spectrum. As the energy increases, the linear polarization decreases and disappears at the high end.

b) The electrons are transversely polarized: In this case, the polarization is elliptical, since a circular component is added to the linear one. This is also energy dependent and largest at low energies. It goes towards zero when the photon energy goes towards the endpoint energy of the β-spectrum.

c) The electrons are longitudinally polarized: The bremsstrahlung is circularly polarized. The polarization is always larger than that in b) and shows the opposite behaviour. It increases sharply with increasing energy of the photons, and takes the highest values at the endpoint energy. This is shown in Fig. 7.6 for the bremsstrahlung of electrons emitted by ^{90}Y. At the endpoint energy, the bremsstrahlung is practically completely polarized.

The circular polarization has the same sign as the helicity. Thus, for β^--particles which have negative helicity, the bremsstrahlung is left circularly polarized. This suggests that during deceleration, helicity is transferred. The magnitude of the transfer depends only slightly on the energy of the electron, but strongly on the energy of the γ quantum, as can be seen from Fig. 7.7. The transfer is plotted as a function of the relative photon energy for two very different electron

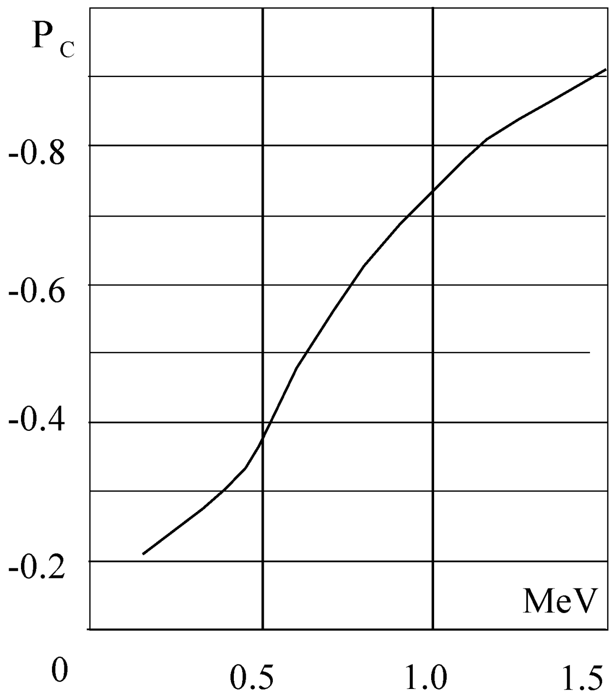

Figure 7.6: Circular polarisation of bremsstrahlung of ^{90}Y.

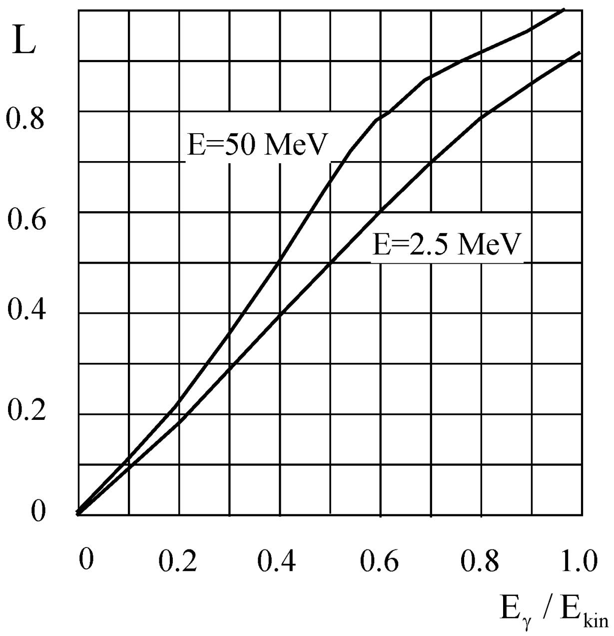

Figure 7.7: Helicity transfer as a function of the energy of the photon.

energies. The two curves are not very different, but both increase sharply with photon energy. At the upper end of the bremsstrahlung spectrum, the transfer is complete.

The reason for this is the conservation of angular momentum. At energies in the range of a few 100 keV, the spin of the electron is completely opposite to the momentum before the collision. At the endpoint energy, the brems-quantum is preferentially emitted forward in the momentum direction, with the electron coming to rest and flipping the spin. The angular momentum changes by a whole unit in the process. The photon carries away this angular momentum.

An aggregate of β-particles emitted into a certain solid angle cannot be transversely polarized. Although every single electron has a transversal component (except in the extreme relativistic case), the directions are statistically distributed in space, so that the average value of all electrons vanishes. This is not true for the longitudinal component, for which there is always a finite value in a limited solid angle.

7.2.3.2 The measurements of the circular polarization of the γ-quanta

The longitudinal polarization of γ quanta is experimentally determined by Compton scattering from polarized electrons. This is possible because the effective cross section contains a term that depends on the alignment of the spins of the scattering particles. The polarization dependent Compton cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{r_0^2}{2} \cdot \frac{k^2}{k_0^2} \cdot (\Phi_0 + f \cdot P_\mathrm{C} \cdot \Phi_\mathrm{C}) \tag{7.36}$$

r_0 is the classical electron radius, k_0 the momentum of the incident and k that of the scattered photon, P_C the circular polarization and f the degree of polarization of the electrons. In the experiment, magnetized iron is used as the scatterer. Thus, $f = 2/26$ is the proportion of aligned electrons to the total number of electrons.

$$\Phi_0 = 1 + \cos^2\theta + (k_0 - k) \cdot (1 - \cos\theta) \tag{7.37}$$

contains the dependence of the effective cross section on the scattering angle θ without taking the spins into account and leads to the Klein-Nishima formula.

$$\Phi_\mathrm{C} = -(1 - \cos\theta) \cdot [(k_0 + k) \cdot \cos\theta \cdot \cos\psi + k \cdot \sin\theta \cdot \sin\psi \cdot \cos\phi] \tag{7.38}$$

is the polarization dependent part. Where ψ is the angle between $\vec{k}_0$ and the electron spin $\vec{S}$ and ϕ between the $\left(\vec{k}_0 \cdot \vec{S}\right)$ plane and the $\left(\vec{k}_0 \cdot \vec{k}\right)$ plane.

It can be seen that Φ_C changes the sign when the electron spin flips, because then ψ changes into $\psi + \pi$. In the experiment, the magnetization direction of the stray magnet is reversed (pole reversal). Let N_+ be the number of scattered photons when the electron spin is approximately parallel to the incident quantum ($0 \leq \psi < \pi/2$), N_- the corresponding number for anti-parallel alignment ($\pi \leq \psi < 3\pi/2$), then the relative count rate difference when the spin is reversed is

$$E = \frac{N_- - N_+}{N_- + N_+} = f \cdot P_\mathrm{C} \cdot \frac{\Phi_\mathrm{C}^-}{\Phi_0} \tag{7.39}$$

here Φ_C^- is Φ_C for ($\pi \leq \psi < 3\pi/2$). An asymmetry E occurs only when both electron polarization and circular polarization of photons are different from zero. The factor Φ_C^-/Φ_0 depends on the photon energy and on the geometry of the arrangement and can therefore be made large by clever arrangement.

In Fig. 7.8 Φ_C^-/Φ_0 is plotted as a function of the scattering angle θ for different γ energies (in units of electron rest mass). The curves are valid for $\psi = 0$. It is seen that the magnitude of Φ_C^-/Ψ_0 increases with photon energy. That is why it is convenient to use only the high-energy

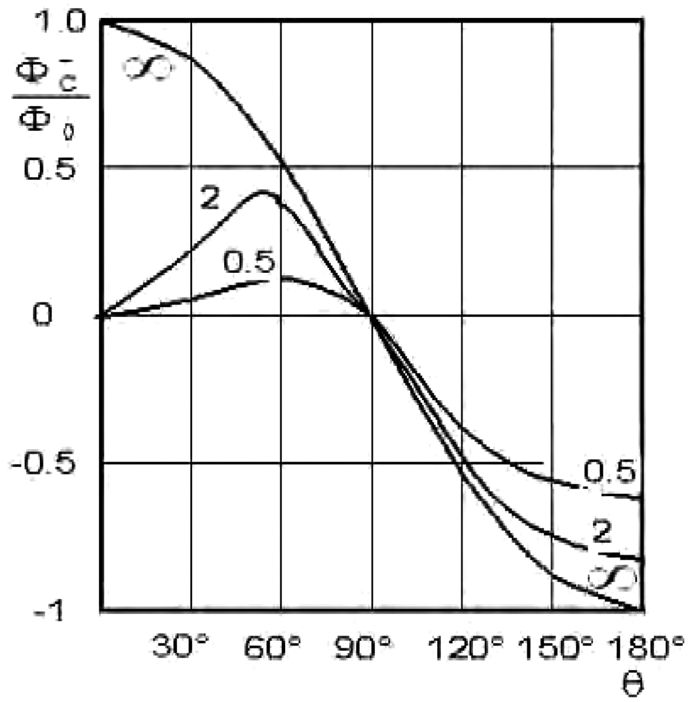

Figure 7.8: Φ_C^-/Φ_0 as function of the scattering angle for different photon energies.

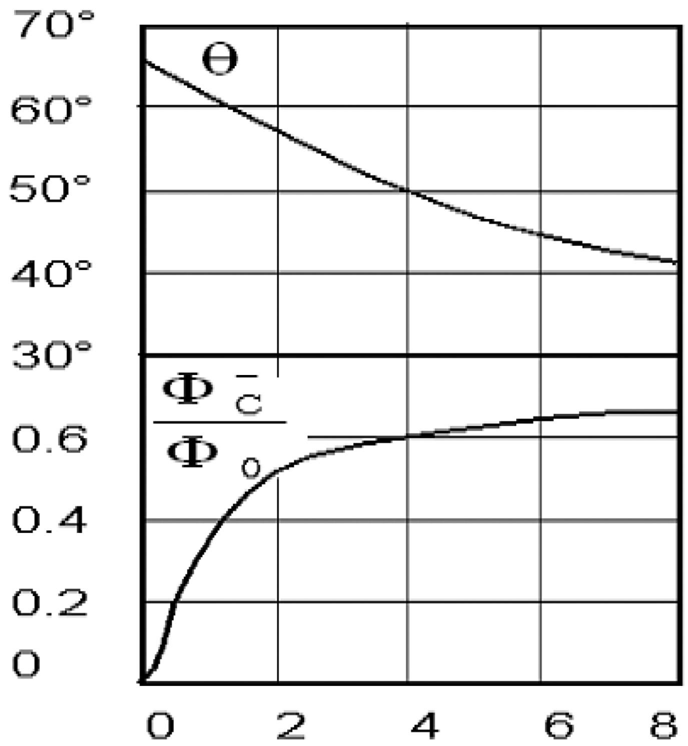

Figure 7.9: The optimal scattering angle and the corresponding ratio $\Phi_{\mathrm{C}}^-/\Phi_0$ versus the photon energy in units of the electron rest mass (511 keV).

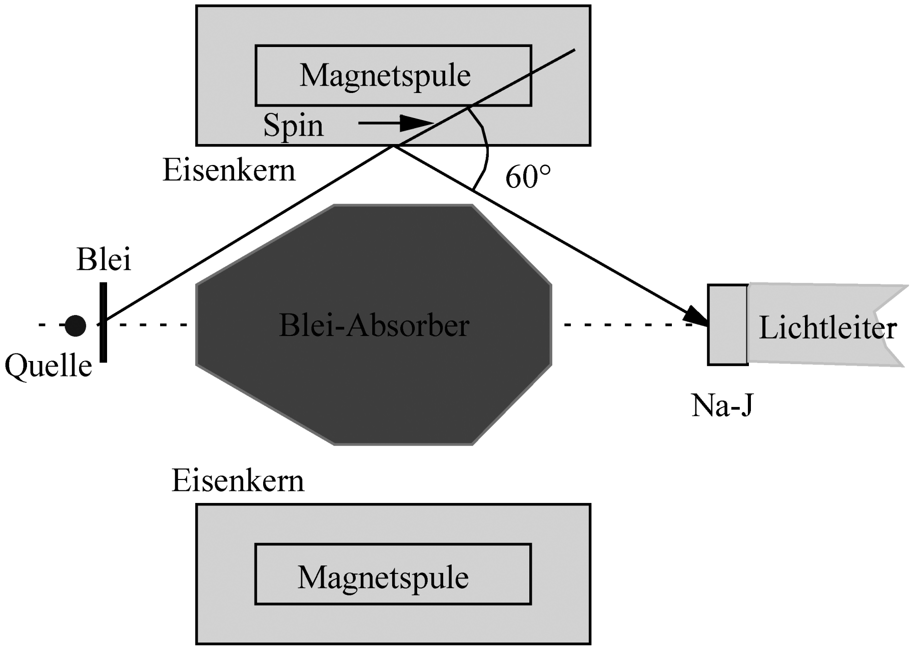

Figure 7.10: Sketch of the apparatus.

part of the spectrum. For the radioactive source ${}^{90}\mathrm{Sr} + {}^{90}\mathrm{Y}$ used in the experiment, this is the region above 1 MeV ($k_\gamma = 2$). At this energy Φ_C^-/Φ_0 is largest for backward scattering ($\theta \to 180°$) and in forward scattering in the neighbourhood of $\theta = 60°$. Backward scattering is eliminated because it has a too small effective cross section. Regarding the angle ψ it is not necessary to optimize, because its influence on the magnitude of Φ_C^-/Φ_0 is only small.

The optimal scattering angle θ as a function of photon energy (in electron rest masses) is drawn in Fig. 7.9 together with the corresponding maximum values of Φ_C^-/Φ_0. In the experiment, the mean scattering angle is 60°. The corresponding value of Φ_C^-/Φ_0 is

$$\Phi_C^-/\Phi_0 = 0,52 \pm 0,05 \qquad \text{at} \qquad \theta = 60° \tag{7.40}$$

With the asymmetry E in Eq. 7.39 the circular polarization including the sign can be determined. Since Φ_C^-/Φ_0 is positive for forward scattering, P_C has the same sign as E. The asymmetry is positive when, for most scatterings, the electron spin is opposite to the direction of the photon incidence, i.e. pointing toward the source. In this case, since the iron has a negative magnetic moment, the magnetic field is directed away from the source.

7.2.4 Setup and implementation

Fig. 7.10 shows a drawing with a cut along the symmetry axis of the apparatus. Electrons emitted from a ${}^{90}\mathrm{Sr} + {}^{90}\mathrm{Y}$ source generate bremsstrahlung quanta in a Pb layer surrounding the source. These are first scattered in the iron core of a cylindrical magnet, whose axis is alligned in the source-detector direction, before entering the NaJ detector. The crystal is connected to a photomultiplier tube (PMT) via a long light guide so that the PMT is outside the magnetic field. This avoids false asymmetries that can occur due to the influence of opposing magnetic fields on the pulse heights. An absorber made of lead is placed in the center of the magnet to prevent unscattered quanta from travelling directly from the source to the detector. The geometry is chosen so that the average scattering angle is 60°. The photomultiplier signal is digitized by a fast ADC (CAEN Digitizer DT5725, 8x 250 MSPS). The data of the individual events are transferred to a PC (Linux) and stored in a file. Afterwards the pulses are analyzed with a Python script that determines the pulse heights. Each pulse height is converted into energy units by an energy calibration. The Python code is executed in a Jupyter notebook.

In the discussion of the curves for the optimization of Φ_C^-/Φ_0 an ideal geometry has been assumed, where the finite dimensions of source, magnet and crystal have been neglected. In reality, one cannot assume fixed values for the angles, but has to take the average over a finite range. Therefore, the given value for Φ_C^-/Φ_0 is only an approximation. For the *proof* of parity violation with $|E| > 0$, however, this is of no importance, since only the *size* of the measured polarization is influenced by it.

The measurement method is counting selected events (see Sec. 5.7.1). Only bremsstrahlung quanta whose original energy before Compton scattering are greater than 1 MeV shall be counted. Therefore, an energy calibration of the measurement system has to be performed first. For this purpose, a ^{22}Na source is available. The number of scattered quanta is measured for the two orientations of the magnetic field. One measures data runs with short measurement times, always inverting the field between the runs. The counts of two neighboring runs belong together to determine E. In this way one can notice possible fluctuations. The measuring time per run should be at least 30 s. At least 30 run pairs should be measured.

CAUTION: Before reversing the polarity of the magnetic field by swapping the banana plugs, it is essential to regulate the coil current to zero.

7.2.5 Evaluation and error calculation

All measured values belonging to one magnetic field direction are added. The asymmetry E is calculated with the sums. The error of the asymmetry is determined by assuming Poisson statistics for the counts and Gaussian error propagation to determine σ_E. The squareroot of the sums is used as the error of the summed up counts. This method yields the pure statistical error.

In a second analysis, the asymmetry is calculated for each run pair individually. Then the mean value $\langle E \rangle$ and the standard deviation of the distribution are determined. This gives the mean error of a *single measurement*. The mean error of the *mean value* is obtained by dividing by the squareroot of the number of individual measurements. Compare this error with the statistical error from the previous analysis.

7.2.6 Literature

Introduction chapters 1 – 6 in this script
β-decay: [1], [11], [27], [28], [40]
Polarization: [41], [42]
Detectors: [19]
Electronics: [21], [22]