Generative Adversarial Networks

personal favourites from 2016-2017

Botond Cseke / VW Datalab

Disclaimer: all figures are copied from the corresponding papers.

Contents

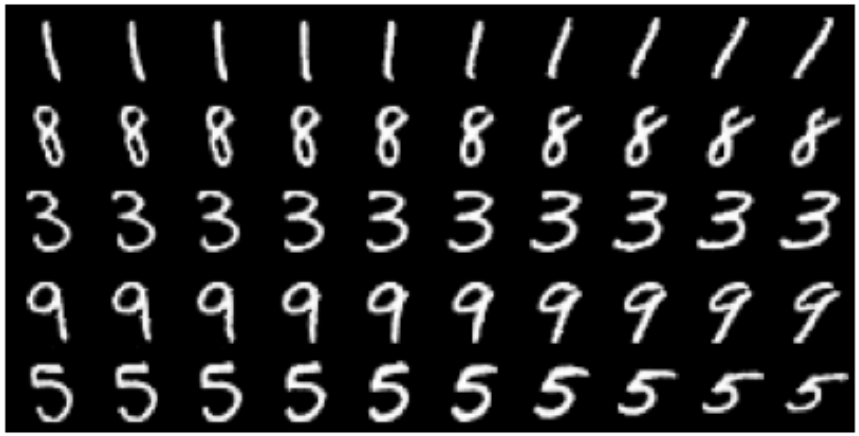
GAN models

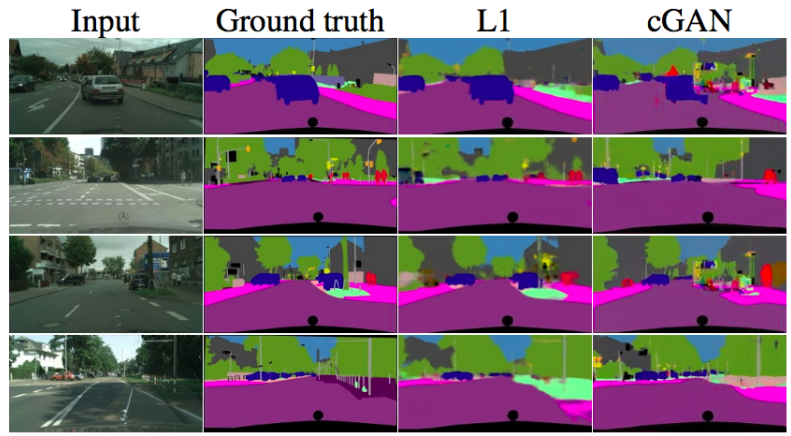
- **GAN: Generative Adversarial Nets**
- InfoGAN: learning semantic features
- CycleGAN: unpaired image-to-image translation
- f-GAN: learning neural samplers by div. min.

GAN models for Bayesian inference

- VAE: Neural variational Bayes
- ALI: Adversarially Learned Inference
- AVB: Adversarial Variational Bayes

Supervised and unsupervised learning





Unsupervised learning

- model

$$p_\theta(x) = \int p_\theta(x \mid z)\, p_0(z) dz$$

- approximation

$$p_\theta(x) \approx \hat{p}(x) = \frac{1}{S} \sum_s \delta(x - x_s)$$

Supervised learning

- model

$$p_\theta(x|u) = \int p_\theta(x \mid u, z)\, p_0(z) dz$$

- approximation

$$p_\theta(x|u) \approx \hat{p}(x|u) = \frac{1}{S} \sum_s \delta((x,u) - (x_s, u_s))$$

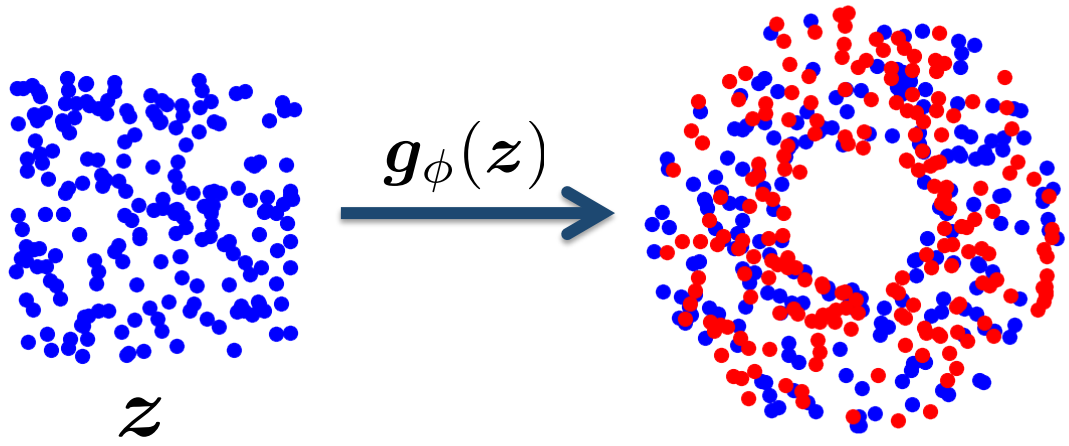


Figure: Mapping a standard distribution (blue) to approx. a data distribution (red)

Learn a random number generator G_θ to match $\hat{p}(x)$

$$x' = G_\theta(z), \quad z \sim p_0(z) \quad \text{where} \quad p_\theta(x') = \int dz\, p_0(z)\, \delta(x' - G_\theta(z))$$

Constraint on training: have access to p and p_θ only through samples

$$x \sim \hat{p}(x) \quad \text{and} \quad x' \sim p_\theta(x')$$

Limitations in learning / training

Ways to define the objectives

1. minimising any loss that at the minimum matches two distributions that is

$$\min_\theta L(p, p_\theta) \;\rightarrow\; p_\theta = p$$

2. sample based divergence minimisation approach $\min_\theta D[p_\theta(x) \,\|\, \hat{p}(x)]$

To fit θ in $p_\theta(x)$ we need the following components

1. loss functions that *use only expectations w.r.t. p_θ / MC samples*

$$E_{\hat{p}}[l(x)] \approx \frac{1}{S}\sum_s l(x_s) \quad \textit{and} \quad E_{p_\theta}[l(x')] = E_{p_0}[l(G_\theta(z))] \approx \frac{1}{S}\sum_s l(G_\theta(z_s)))$$

2. *an* autodiff framework to propagate differentials through x'

$$\frac{\partial}{\partial\theta} E_{q_\theta}[l(x')] \approx \frac{1}{S}\sum_s \frac{\partial}{\partial x} l(G_\theta(z_s)))\frac{\partial}{\partial\theta} G_\theta(z_s)$$

Generative Adversarial Nets

Ian J. Goodfellow,* Jean Pouget-Abadie,† Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair,‡ Aaron Courville, Yoshua Bengio§
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Inference as classification

1. construct two classes with real and generated data distributions

$$\mathcal{C}_1 = \{(x, 1) : \; x \sim \hat{p}(x)\} \qquad \text{and} \qquad \mathcal{C}_0 = \{(x', 0) : \; x' = G_\theta(z), \; z \sim p_0(z)\}$$

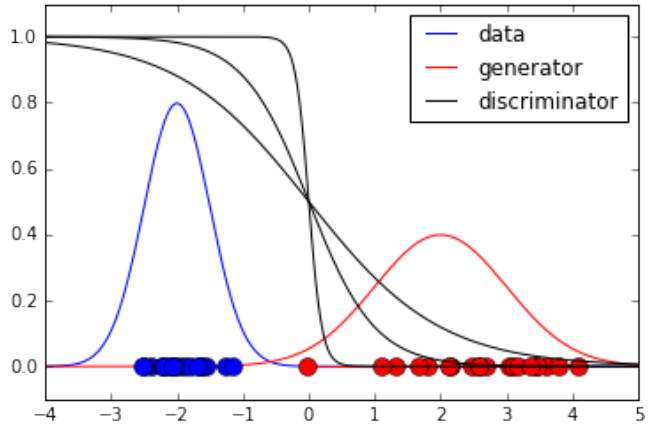
2. define the classification (quasi) empirical loss

$$L(\theta, \phi) = E_{(x,y)\sim\mathcal{C}_1}[l(D_\phi(x), y)] + E_{(x',y')\sim\mathcal{C}_0}[l(D_\phi(x'), y')]$$

Training

1. train a classifier $D_\phi(x)$ minimise the loss
2. train generator $G_\theta(z)$ to maximise loss

GAN / Adversarial classification



Inference as classification

1. construct two classes with real and generated data distributions

$$\mathcal{C}_1 = \{(x, 1) : \; x \sim p(x)\} \qquad \text{and} \qquad \mathcal{C}_0 = \{(x', 0) : \; x' = G_\theta(z), \; z \sim p(x)\}$$

2. define the classification (quasi) empirical loss

$$\begin{aligned} L(\theta, \phi) &= E_{(x,y)\sim\mathcal{C}_1}[l(D_\phi(x), y)] + E_{(x',y')\sim\mathcal{C}_0}[l(D_\phi(x'), y')] \\ &= E_{(x,y)\sim\mathcal{C}_1}[l(D_\phi(x), y)] + E_{(z',y')\sim\mathcal{C}_0}[l(D_\phi(G_\theta(z)), y')] \end{aligned}$$

Training

1. minimise the loss

$$\min_\phi L(\theta, \phi)$$

2. maximise loss

$$\max_\theta \min_\phi L(\theta, \phi)$$

show video

GAN / Possible loss functions

1. Bernoulli classification likelihood

$$L(p, p_\theta) = E_{(x,y)\sim\mathcal{C}_1}[-y \log D_\phi(x) - \cancel{(1-y)\log(1 - D_\phi(x))}]$$
$$\cancel{E_{(x',y')\sim\mathcal{C}_0}[-y' \log D_\phi(x')} - (1-y')\log\big(1 - D_\phi(x')\big)]$$

$$= E_{x\sim p(x)}[-\log D_\phi(x)] + E_{x'\sim p_\theta(x)}[-\log(1 - D_\phi(x')]$$

where we have $D^* = p/(p + p_\theta)$ and thus $L(p, p_\theta) = D_{JS}[p_\theta \,||p] + \text{const.}$

For $D_\phi(x) = \sigma(r_\phi(x))$ we get $r^*_\phi = \log p - \log p_\theta$.

2. classification as regression based likelihoods (LS-GAN)

$$L(p, p_\theta) = E_{(x,y)\sim\mathcal{C}_1}[\big(D_\phi(x) - y\big)^2] + E_{(x',y')\sim\mathcal{C}_0}[(D_\phi(x') - y'])^2]$$
$$= E_{x\sim\hat{p}(x)}[\big(D_\phi(x) - 1\big)^2] + E_{x'\sim p_\theta(x')}[D_\phi(x')^2]$$

Similar iff. solution $D^* = p/(p + p_\theta)$.

GAN / Training algorithm

Optimisation

$$\min_{\theta} \max_{\phi} E_{x \sim \hat{p}(x)}[\log D_{\phi}(x)] + E_{z \sim p_0(z)}[\log(1 - D_{\phi}(G_{\theta}(z))]$$

Stochastic gradient ascent-descent

1. sample $x_s \sim \hat{p}(x)$ and $z_s \sim \mathcal{N}(0, I_d)$ or $z_s \sim \mathcal{U}_d(0, 1)$
2. MC approximation of the objective

$$L = \frac{1}{N} \sum_i \log D_{\phi}(x_i) + \log(1 - D_{\phi}(G_{\theta}(z_i))$$

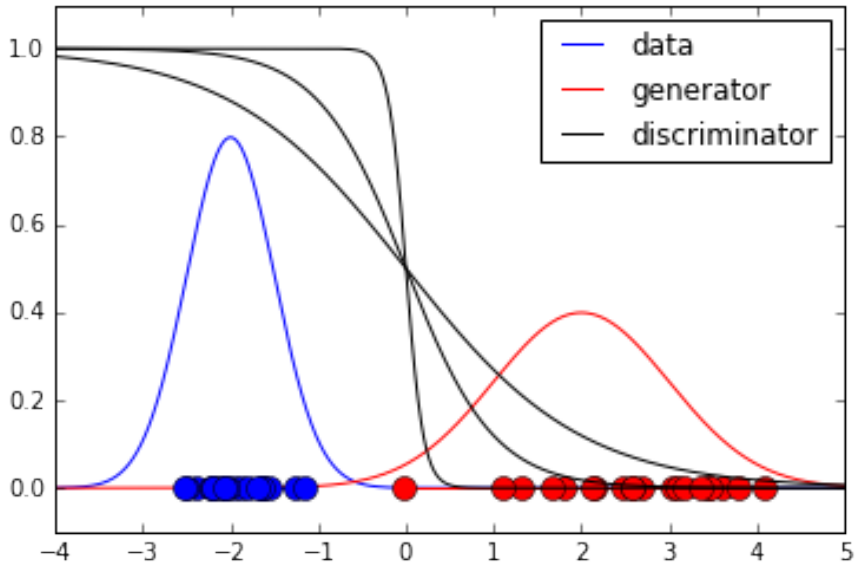
3. gradient ascent on ϕ

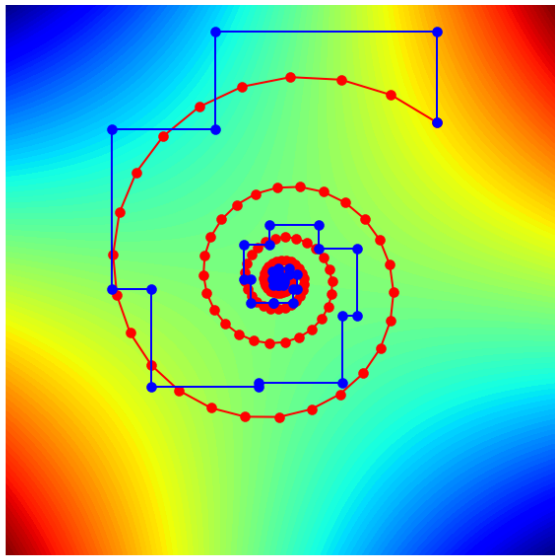
$$\phi^{(t+1)} = \phi^{(t)} + \eta_{\phi} \frac{1}{N} \sum_i \partial_{\phi} \log D_{\phi}(x_i) + \partial_{\phi} \log(1 - D_{\phi}(G_{\theta}(z_i))$$

4. gradient descent on θ

$$\theta^{(t+1)} = \theta^{(t)} - \eta_{\theta} \frac{1}{N} \sum_i \partial_{\theta} \log(1 - D_{\phi}(G_{\theta}(z_i))$$

GAN / Training Algorithm





1. sample $x_i \sim \hat{p}(x)$ and $z_i \sim \mathcal{N}(0, I_d)$ or $z_i \sim \mathcal{U}_d(0, 1)$
2. MC approximation of the objective

$$L(\theta, \phi) \approx \frac{1}{N} \sum_i \log D_\phi(x_i) + \log(1 - D_\phi(G_\theta(z_i))$$

3. gradient ascent on ϕ (K-steps)

$$\phi^{(t+1)} = \phi^{(t)} + \eta_\phi \frac{1}{N} \sum_i \frac{\partial_\phi D_\phi(x_i)}{D_\phi(x_i)} + \frac{\partial_\phi D_\phi(G_\theta(z_i))}{1 - D_\phi(G_\theta(z_i))}$$

4. gradient descent on θ

$$\theta^{(t+1)} = \theta^{(t)} + \eta_\theta \frac{1}{N} \sum_i \frac{\partial_x D_\phi(G_\theta(z_i))}{1 - D_\phi(G_\theta(z_i))} \cdot \partial_\theta G_\theta(z_i) \quad \text{flip to fix}$$

GAN / Details

1. convincing results on digits and faces dataset
2. show interpolation results as a proof that the network is not memorizing

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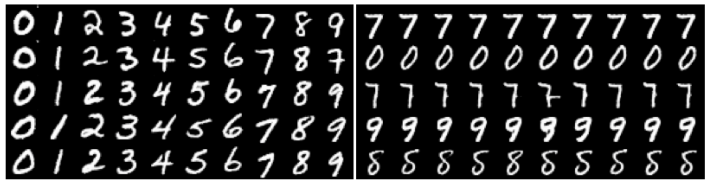
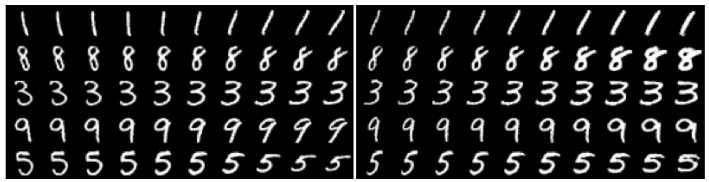
InfoGAN

InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets

Xi Chen[†‡], **Yan Duan**[†‡], **Rein Houthooft**[†‡], **John Schulman**[†‡], **Ilya Sutskever**[‡], **Pieter Abbeel**[†‡]
† UC Berkeley, Department of Electrical Engineering and Computer Sciences
‡ OpenAI



(a) Varying c_1 on InfoGAN (Digit type) (b) Varying c_1 on regular GAN (No clear meaning)



(c) Varying c_2 from -2 to 2 on InfoGAN (Rotation) (d) Varying c_3 from -2 to 2 on InfoGAN (Width)

A version of GAN that

1. maximises the mutual information between a small subset of the latent variables and the observation
2. disentangles writing styles from digit shapes on the MNIST dataset, pose from lighting of 3D rendered images

InfoGAN / Loss function

Add mutual entropy regularisation to the GAN objective

1. main idea: $z' = (z, c) \sim p(z)p(c), x' = G(z') = G_\theta(z, c)$ such that
   1. z comes from the usual base distribution
   2. "code distribution": c encodes semantic features by ME

$$I(c, x') = \int p(x'|c)p(x') \log \frac{p(x'|c)}{p(x')p(c)}$$

2. bounds for the intractable parts–use a KL bound

$$\begin{aligned} I(c, G_\theta(z, c)) &= H(c) + E_{x \sim G_\theta(z,c)}[E_{c' \sim p(c|x')}[\log p(c'|x')]] \\ &= H(c) + E_{x' \sim G_\theta(z,c)}[D[p(c|x') \| Q_\eta(c|x')]] \\ &\quad + E_{x' \sim G_\theta(z,c)}[E_{c' \sim p(c|x')}[\log Q_\eta(c'|x')]] \\ &\geq H(c) + E_{x' \sim G_\theta(z,c)}[E_{c' \sim p(c|x')}[\log Q_\eta(c'|x')]] \\ &\geq H(c) + E_{c \sim p(c), x' \sim G_\theta(z,c)}[\log Q_\eta(c'|x')]] \qquad (\equiv L_{MI}(\theta, \eta)) \end{aligned}$$

InfoGAN / optimisation

Augmented GAN objective

$$L_{iGAN}(\theta, \phi) = E_{x \sim \hat{p}(x)}[\log D_\phi(x)] + E_{(z,c) \sim p(z,c)}[\log(1 - D_\phi(G_\theta(z, c)))] - \lambda I(c, G_\theta(z, c))$$

Joint objective and optimisation

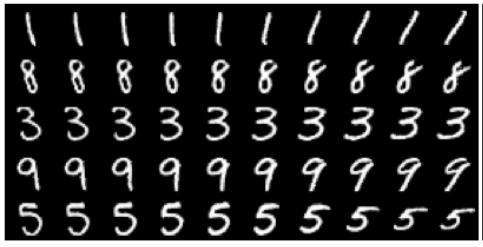
1. objective
$$L(\theta, \eta, \phi) = L_{GAN}(\phi, \theta) - \lambda \, L_{MI}(\theta, \eta)$$
2. optimisation
$$\min_{\theta, \eta} \max_{\phi} \left\{ L_{GAN}(\phi, \theta) - \lambda \, L_{MI}(\theta, \eta) \right\}$$
3. training tricks
    1. D_ϕ and Q_η share an image feature layer that only D_ϕ trains!
    2. $\lambda \approx 1$ larger learning rate for G_θ than for D_ϕ
    3. trying to keep the objective and regulariser on the same scale help is most cases

Results

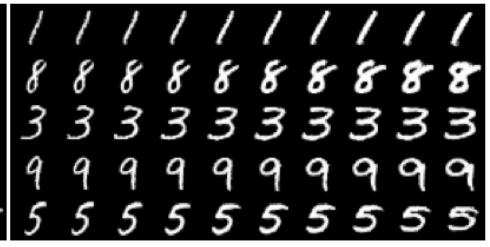
MNIST handwritten digits dataset

(a) Varying c_1 on InfoGAN (Digit type)

(b) Varying c_1 on regular GAN (No clear meaning)



(c) Varying c_2 from -2 to 2 on InfoGAN (Rotation)



(d) Varying c_3 from -2 to 2 on InfoGAN (Width)

Faces dataset

(a) Azimuth (pose)

(b) Elevation

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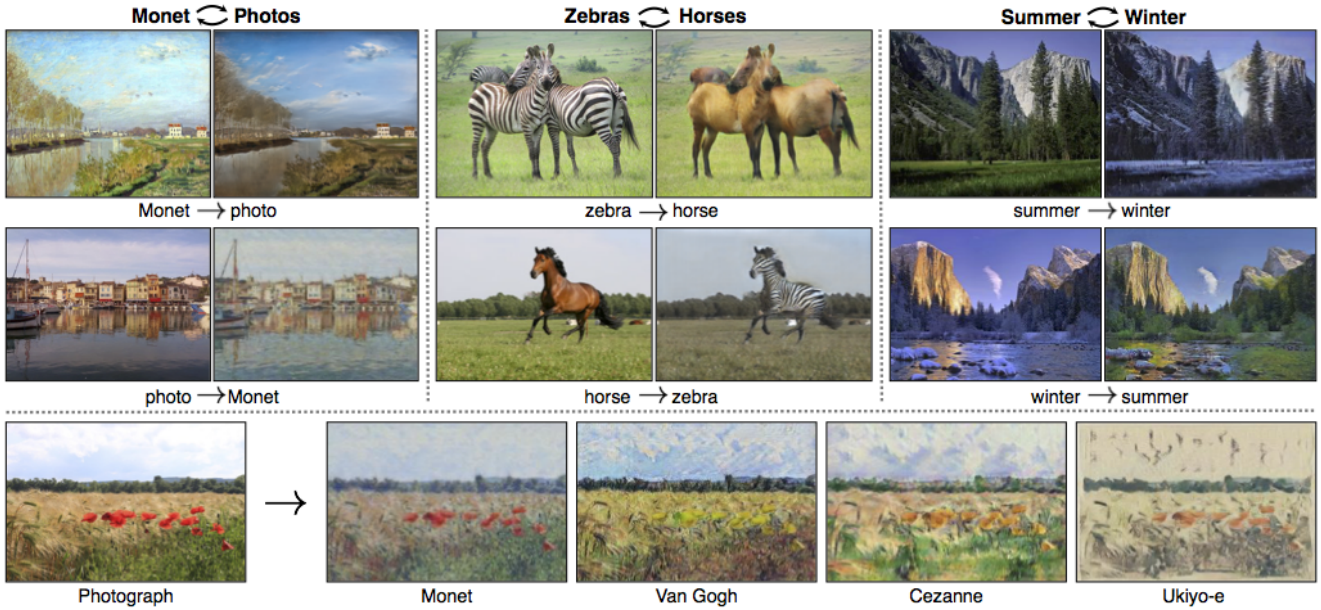
CycleGAN

Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks

Jun-Yan Zhu* Taesung Park* Phillip Isola Alexei A. Efros
Berkeley AI Research (BAIR) laboratory, UC Berkeley

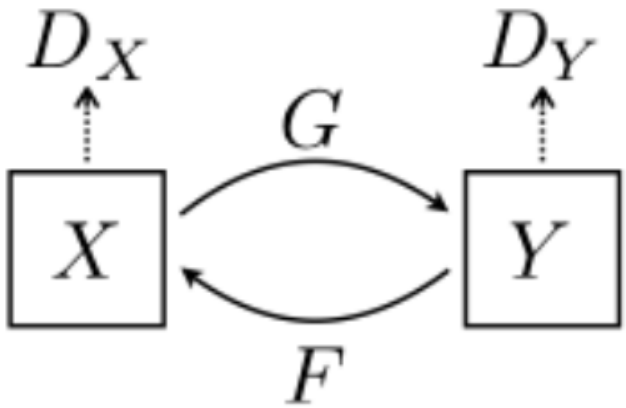


Problem statement

1. find a mapping $G : X \to Y$, such that the distribution of images $G(X)$ is indistinguishable from the distribution Y using an adversarial loss.
2. highly under-constrained mapping: we couple it with an inverse mapping $F : Y \to X$ enforce $F(G(X)) = X$ (and vice-versa)

CycleGAN / Loss functions I

Equivalent domains → symmetric GAN losses on empirical distributions / data domains



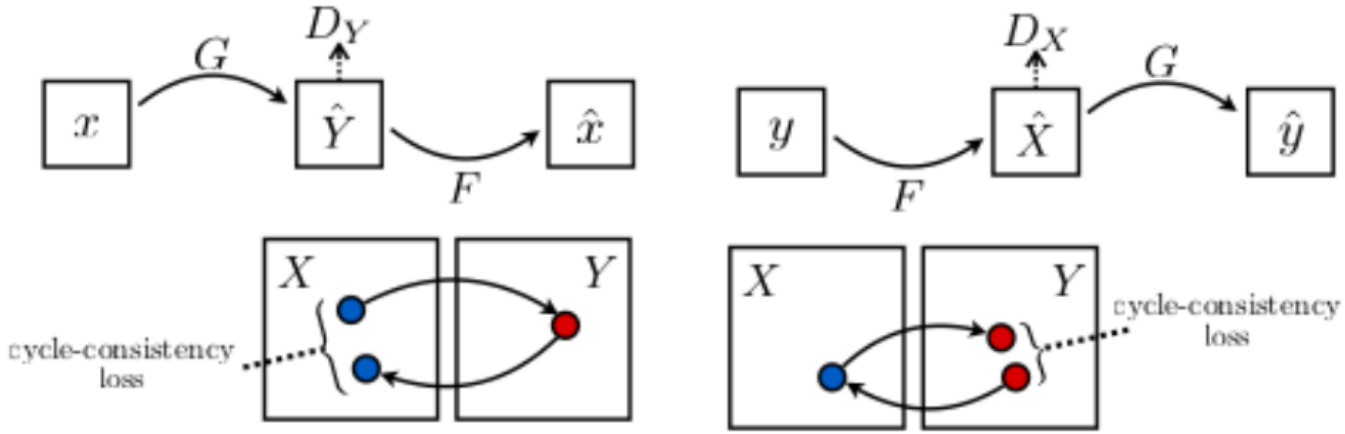
Generator space is a data domain space → a GAN in each domain

$$L_Y(G, D_Y, X, Y) = E_{y \sim \hat{p}_Y(y)}[\log D_Y(y)] + E_{x \sim \hat{p}_X(x)}[\log(1 - D_Y(G(x)))]$$
$$L_X(F, D_X, X, Y) = E_{x \sim \hat{p}_X(x)}[\log D_X(x)] + E_{y \sim \hat{p}_Y(y)}[\log(1 - D_X(F(y)))]$$

Results in

$$\min_{G,F} \max_{D_X, D_Y} L_Y(G, D_Y, X, Y) + L_X(F, D_X, X, Y)$$

CycleGAN / loss functions II



Cycle constraints

1. motivation
   - large complexity $\rightarrow$ network can map the same set of input images to any random permutation
   - adversarial losses alone cannot guarantee that the learned function can map an individual input x_i to a desired output y_i
2. formulation

$$L_{cycle}(G, F) = E_{x \sim \hat{p}_X(x)}[\| F(G(x)) - x \|_1] + E_{y \sim \hat{p}_Y(y)}[\| G(F(y)) - y \|_1]$$

CycleGAN / Loss function III

Joint loss and optimisation

- overall loss

$$L(G, F, D_X, D_Y) = L_Y(G, D_Y, X, Y) + L_X(F, D_X, X, Y) + \lambda\, L_{cycle}(G, F)$$

- total loss

$$\min_{G,F} \max_{D_X, D_Y} L(G, F, D_X, D_Y)$$

Training tricks

1. use 0/1 regression loss instead of the Bernoulli GAN

$$L_Y(G, D_Y, X, Y) = E_{y \sim \hat{p}_Y(y)}[(D_Y(y) - 1)^2] + E_{x \sim \hat{p}_X(x)}[D_Y(G(x))^2]$$
$$L_X(F, D_X, X, Y) = E_{x \sim \hat{p}_X(x)}[(D_X(x) - 1)^2] + E_{y \sim \hat{p}_Y(y)}[D_X(F(y))^2]$$

2. Practical tricks
   - update discriminators D_X and D_Y using a history of 50 generated images
   - fix $\lambda = 10$ for all
   - ADAM with decaying learning rate

CycleGAN / Experiments I

Cityscapes

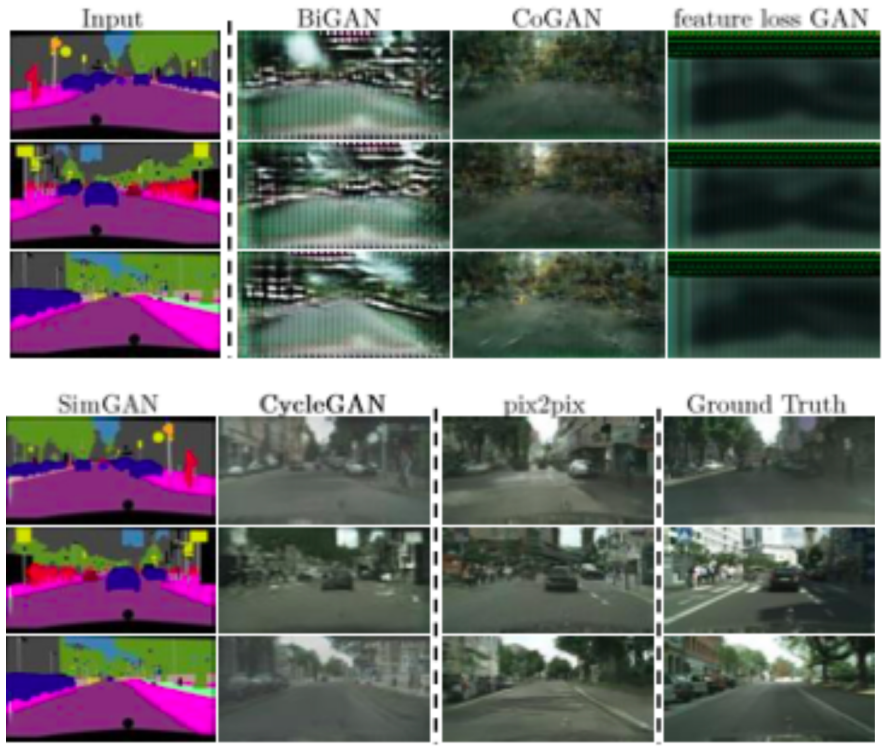
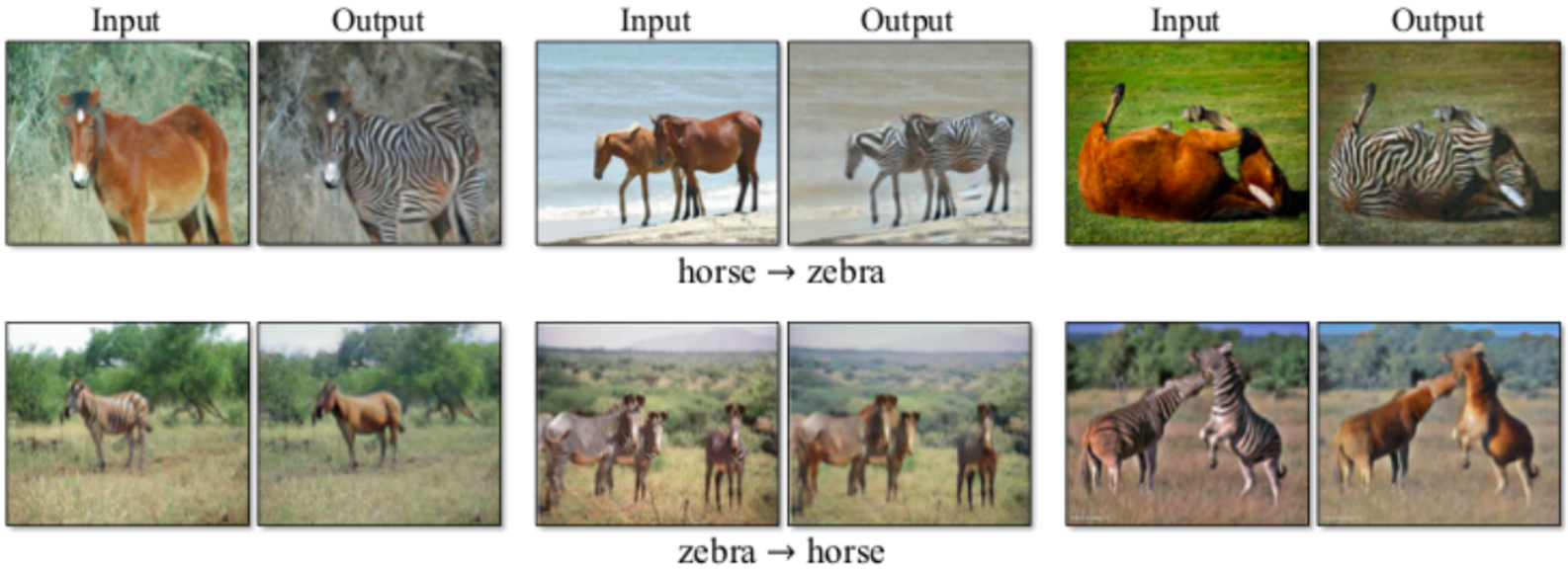


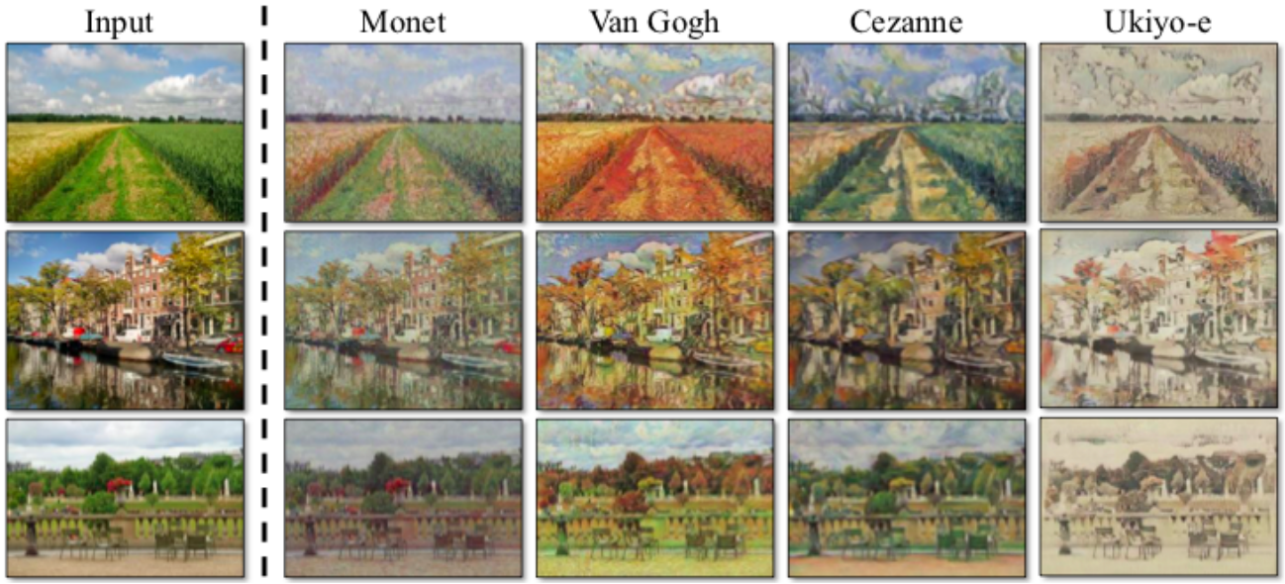
Figure: Different variants of the method for mapping labels $\rightarrow$ photos trained on cityscapes.

Evaluation metrics

- Amazon Mechanical Turk
- comparing overlaps to ground truth using standard semantic segmentation metrics
- inception score [explain idea only]

CycleGAN / Experiments II





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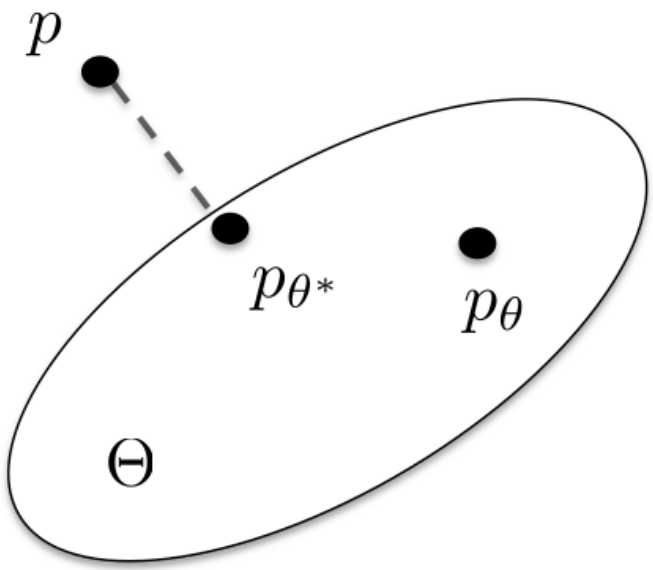
f-GAN / Approximating probability densities



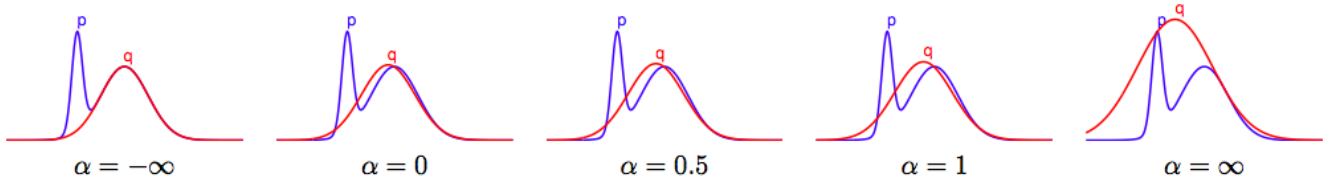
Parametric approximation of pdfs

- target distribution $p(x)$ given only through a finite set of samples

$$\hat{p}(x) = \frac{1}{n} \sum_i \delta(x - x_i), \quad x_i \sim p(x)$$

- p_θ limited to $\theta \in \Theta$
- what should the loss be?

f-GAN / Divergence measures



(Minka, 2015)

Comparing distributions by using divergence measures

- α divergence

$$\mathbb{D}_\alpha[p \parallel q] = \frac{1}{(1-\alpha)\alpha}\left(1 - \int dx\; p(x)^\alpha q(x)^{1-\alpha}\right)$$

- important special cases

$$\mathbb{D}_0[p \parallel q] = \int dx\; q(x) \log \frac{q(x)}{p(x)} \quad \text{and} \quad \mathbb{D}_1[p \parallel q] = \int dx\; p(x) \log \frac{p(x)}{q(x)}$$

- Jensen-Shannon (max of the GAN objective w.r.t. D_ϕ)

$$\mathbb{D}_{JS}[p \parallel q] = \mathbb{D}_0\left[p \parallel \frac{p+q}{2}\right] + \mathbb{D}_0\left[q \parallel \frac{p+q}{2}\right]$$

- others: total variation, etc.

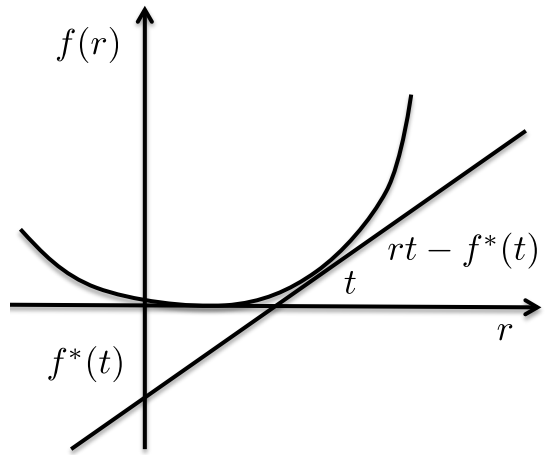
Sample based divergence minimisation (f-GAN)

Divergence measure (f convex, $f(1) = 0$)

$$\mathrm{D}_f\left[q_\phi||p\right] = \int d\boldsymbol{x}\, q_\phi(\boldsymbol{x}) f\Big(\frac{p(\boldsymbol{x})}{q_\phi(\boldsymbol{x})}\Big)$$

Lower bound [Nguyen et al.; 2010]

$$\begin{aligned}\mathrm{D}_f\left[q_\phi||p\right] &\geq \int d\boldsymbol{x}\, q_\phi(\boldsymbol{x})\Big[\frac{p(\boldsymbol{x})}{q_\phi(\boldsymbol{x})} t(\boldsymbol{x}) - f^*\big(t(\boldsymbol{x})\big)\Big] \\ &= \mathrm{E}_{\boldsymbol{x}\sim p}\left[t(\boldsymbol{x})\right] - \mathrm{E}_{\boldsymbol{x}'\sim q_\phi}\left[f^*\big(t(\boldsymbol{x}')\big)\right]\end{aligned}$$



$$f^*(t) = \sup_r\{rt - f(r)\}$$
$$f(r) = \sup_t\{rt - f^*(t)\}$$

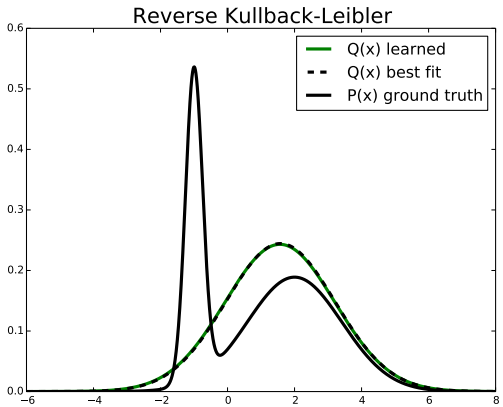
f-GAN objective

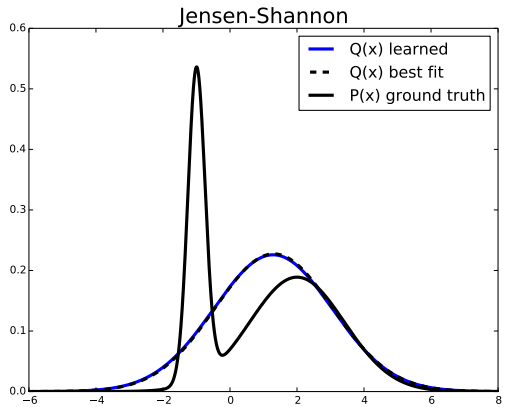
$$\min_\phi \max_\omega \; \mathrm{E}_{\boldsymbol{x}\sim p}\left[t_\omega(\boldsymbol{x})\right] - \mathrm{E}_{\boldsymbol{x}'\sim q_\phi}\left[f^*\big(t_\omega(\boldsymbol{x}')\big)\right]$$

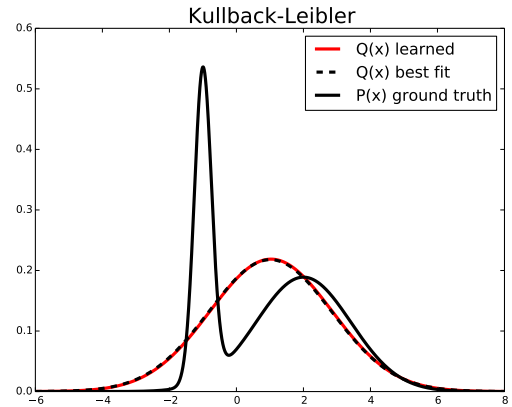
Training: stochastic gradient ascent-descent using mini-batches

f-divergences and conjugate duals

Name	$D_f(P\|Q)$	Generator $f(u)$	$t^*(x)$
Kullback-Leibler	$\int p(x) \log \frac{p(x)}{q(x)} \, \mathrm{d}x$	$r \log r$	$1 + \log \frac{p(x)}{q(x)}$
Reverse KL	$\int q(x) \log \frac{q(x)}{p(x)} \, \mathrm{d}x$	$-\log r$	$-\frac{q(x)}{p(x)}$
Jensen-Shannon	$\frac{1}{2} \int p(x) \log \frac{2p(x)}{p(x)+q(x)} + q(x) \log \frac{2q(x)}{p(x)+q(x)} \, \mathrm{d}x$	$-(r+1) \log \frac{1+r}{2} + r \log r$	$\log \frac{2p(x)}{p(x)+q(x)}$

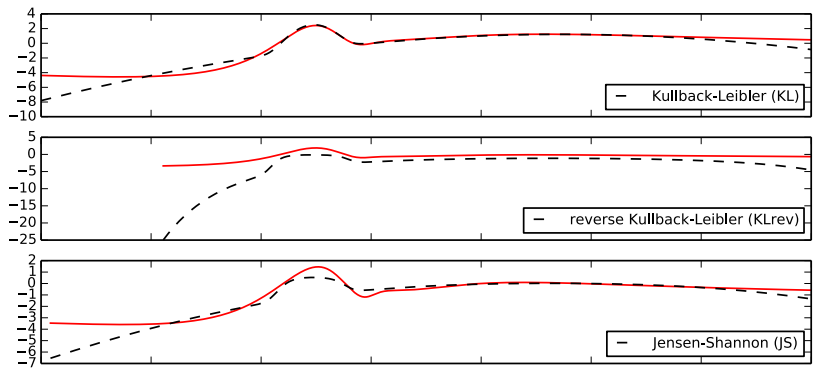






Optimal variational function

$$t^*(\boldsymbol{x}) = f'\left(\frac{p(\boldsymbol{x})}{q_\phi(\boldsymbol{x})}\right)$$



f-GAN practical considerations

Name	Output activation h_f	dom_{f^*}	Conjugate $f^*(t)$	$f'(1)$
Kullback-Leibler (KL)	v	$\mathbb{R}$	$\exp(t-1)$	1
Reverse KL	$-\exp(-v)$	$\mathbb{R}_-$	$-1-\log(-t)$	-1
Jensen-Shannon	$\log(2)-\log(1+\exp(-v))$	$t<\log(2)$	$-\log(2-\exp(t))$	0

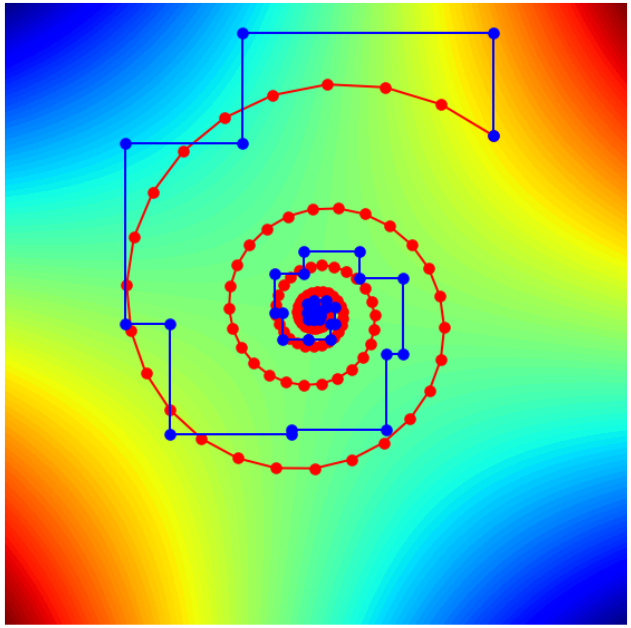
Use $t_\omega(\boldsymbol{x}') = h_f(w_\omega(\boldsymbol{x}'))$ to map into f^* input domain

$$L(\theta,\omega) = \mathrm{E}_{\boldsymbol{x}\sim p}\left[h_f(v_\omega(\boldsymbol{x}))\right] - \mathrm{E}_{\boldsymbol{z}\sim q_0}\left[f^*\left(h_f(v_\omega(g_\theta(\boldsymbol{z}))\right)\right]$$

Synchronous gradient ascent-descent

$$\theta_{t+1} = \theta_t - \boldsymbol{\Gamma}_{\theta,t}\,\partial_\theta \tilde{L}(\theta_t,\omega_t)$$

$$\omega_{t+1} = \omega_t + \boldsymbol{\Gamma}_{\omega,t}\,\partial_\omega \tilde{L}(\theta_t,\omega_t)$$

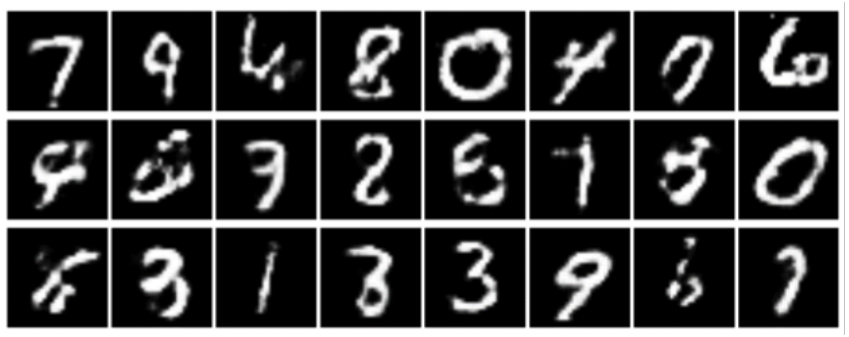


MNIST

- Model from the original GAN work [Goodfellow et al.; 2014]: simple 2/3 layer NN
- Evaluation using KDE log-likelihoods
  - build KDE on model samples, evaluate on real data
  - known shortcomings, but popular in other works (old approach)

Training divergence	KDE $\langle LL \rangle$ (nats)	$\pm$ SEM
Kullback-Leibler	416	5.62
Reverse Kullback-Leibler	319	8.36
Pearson χ^2	429	5.53
Neyman χ^2	300	8.33
Squared Hellinger	-708	18.1
Jeffrey	-2101	29.9
Jensen-Shannon	367	8.19
GAN	305	8.97
Variational Autoencoder	445	5.36
KDE MNIST train (60k)	502	5.99

Kullback-Leibler



Kullback-Leibler (reverse)

Hellinger

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- **VAE: Neural variational Bayes**
- ALI: Adversarially Learned Inference
- AVB: Adversarial Variational Bayes

VAE / Unsupervised Bayesian learning through amortisation

Auto-Encoding Variational Bayes

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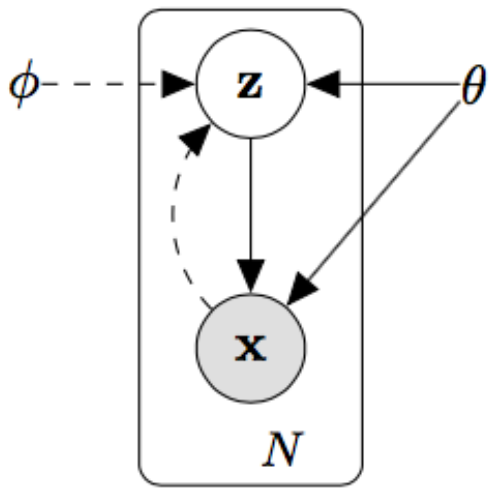
- Bayesian unsupervised learning

$$p_\theta(x) = \int dz\, p_\theta(x|z)\, p_0(z)$$

- variational Bayes approximations for likelihood optimisation and posterior inference
- unifies VB with auto-encoders to simplify inference $\rightarrow$ scaling it to large data sets



VAE / bounding the likelihood function

Variational bound

$$\begin{aligned}
\log p_\theta(x_i) &= \log \int dz_i \; p(x_i|z_i)\, p_0(z_i) \\
&= \log \int dz_i \; p(x_i|z_i) \; \frac{p_0(z_i)}{q_i(z_i)} q_i(z_i) \\
&\geq \int dz_i \; q_i(z_i) \; \log \frac{p(x_i|z_i) p_0(z_i)}{q_i(z_i)} \\
&= \mathbb{E}_{q_i}[\log p_\theta(x_i|z_i)] - \mathbb{D}[q_i(z_i) \parallel p_0(z_i)] \qquad \equiv L(\theta, q_i : x_i)
\end{aligned}$$

Properties

- for each x_i we have

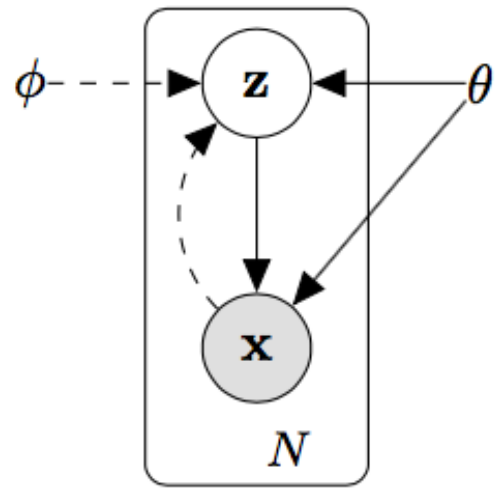
$$q_i^*(z_i) = \frac{p(x_i|z_i)\, p(z_i)}{p_\theta(x_i)} \quad \rightarrow \quad q_i^*(z_i) = p(z_i|x_i)$$

- tight bound

$$L(\theta, q_i^*; x_i) = \log p_\theta(x_i)$$

- optimisation gap

$$\log p_\theta(x_i) - L(\theta, q_i; x_i) = D[q_i(z_i) \parallel p(z_i|x_i)]$$



VAE / large scale data

Divergence bound

$$\log p_\theta(x_i) \geq L(\theta, q_i; x_i) \quad \left(= \mathbb{E}_{q_i}[\log p_\theta(x_i|z_i)] - \mathbb{D}[q_i(z_i) \parallel p_0(z_i)] \right)$$

VAE approach

- overall likelihood bound

$$\sum_i \log p_\theta(x_i) \geq \sum_i L(\theta, q_i; x_i)$$

- tight bound (many independent optimisation problems)

$$\max_\theta \sum_i \log p_\theta(x_i) \geq \max_\theta \sum_i \max_{q_i} L(\theta, q_i; x_i)$$

- amortisation and parameter sharing

$$q_i(z_i) \equiv q_\phi(z_i|x_i) \quad \text{e.g.} \quad q_\phi(z_i|x_i) = \mathcal{N}(z_i; \mu_\phi(x_i), \sigma_\phi(x_i)^2)$$

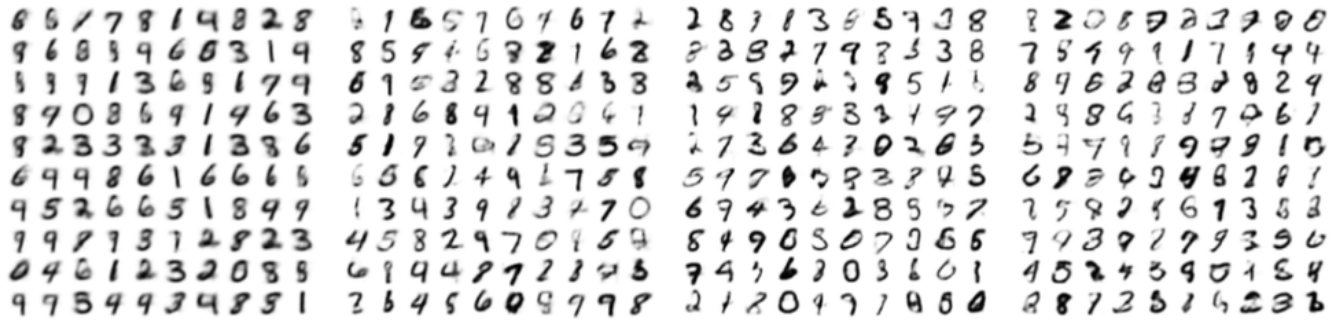
- SGD for

$$\max_\theta \max_\phi L(\theta, q_\phi(z_i; x_i); x_i)$$

- neural network parameterisation of Gaussians / reparameterisation

$$[\, \mu_\phi(x_i), \log \sigma_\phi(x_i)) \,] = \mathsf{NN}_\phi(x_i) \quad \text{and} \quad z_i^{(s)} = \mu_\phi(x_i) + \sigma_\phi(x_i)\, \epsilon_i^{(s)}$$

VAE / Experiments I



(a) 2-D latent space (b) 5-D latent space (c) 10-D latent space (d) 20-D latent space

Unsupervised learning of digits

- details of the model

$$p_0(z) = \mathcal{N}(z; 0, I_d), \quad \text{and} \quad p_\theta(x|z) = \mathcal{N}(x|\mu_\theta(z), \sigma_\phi(z)^2)$$

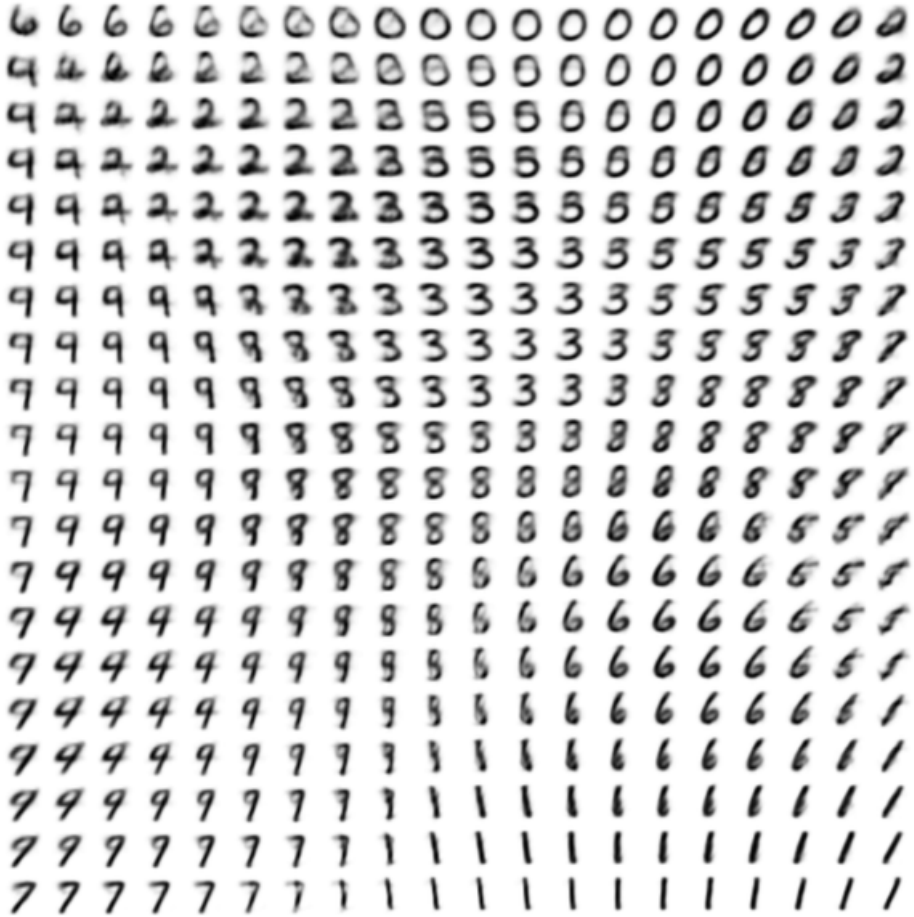
- details of the approximation

$$q_\phi(z_i|x_i) = \mathcal{N}(z_i; \mu_\phi(x_i), \sigma_\phi(x_i)^2) \quad \text{s.t.} \quad [\, \mu_\phi(x_i), \log \sigma_\phi(x_i)) \,] = \mathsf{NN}_\phi(x_i)$$

- training: SDG steps
  - sample $x_i \sim \hat{p}(x)$, sample $\epsilon_i^{(k)} \sim \mathcal{N}(0, I_d)$, $z_i^{(k)} = \mu_\phi(x_i) + \sigma_\phi(x_i)\, \epsilon_i^{(k)}$
  - approx of MC loss

$$\tilde{L}(\theta, \phi) = \frac{1}{N_x} \sum_i \Big\{ \frac{1}{N_z} \sum_j \log p_\theta(x_i|z_i) - \underbrace{\mathbb{D}[q_\phi(z_i|x_i) \,\|\, \mathcal{N}(z_i; 0, I_d)]}_{\text{analytic}} \Big\}$$

VAE / Experiments II



Structure of the latent space z

- fit a 2d latent space for z
- map 2d uniform grid $z_{i,j} = (i.j)/N, \quad i, j = 1, \ldots, N$ through normal inverse cdf
- sample / compute mean of $p_\theta(x|z_{i,j})$

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- **ALI: Adversarially Learned Inference**
- AVB: Adversarial Variational Bayes

Adversarially Learned Inference

ADVERSARIALLY LEARNED INFERENCE

Vincent Dumoulin[1], **Ishmael Belghazi**[1], **Ben Poole**[2]
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ADVERSARIAL FEATURE LEARNING

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A version of GAN that

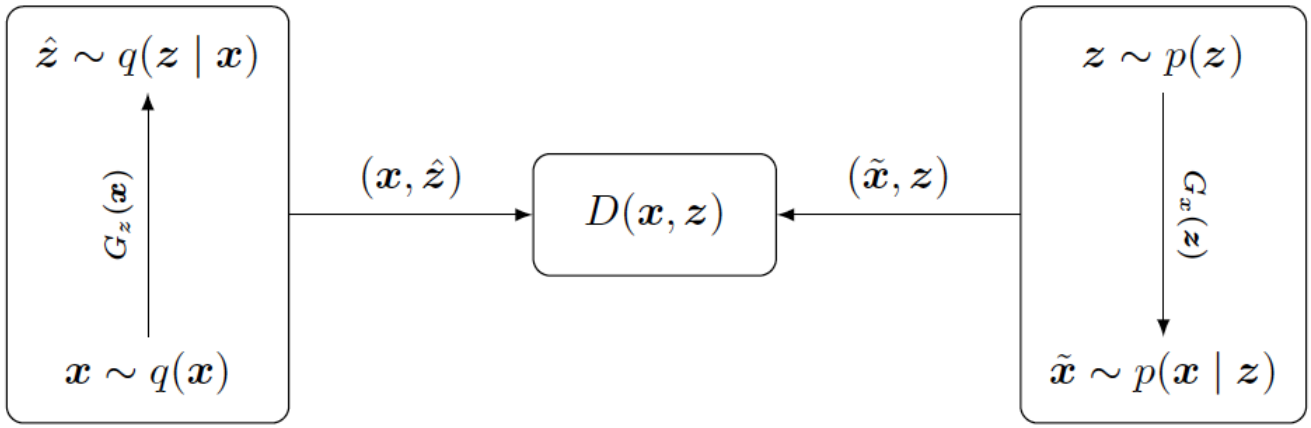
1. does Bayesian inference by directly matching densities
2. main idea: write the joint $p(x, z)$ as

$$p(x, z) = p_\theta(x|z) \times p_0(z) = q_\phi(z|x) \times \hat{p}(x)$$

2. optimise θ and ϕ such that

$$p_\theta(x|z) \times p_0(z) \approx q_\phi(z|x) \times \hat{p}(x)$$

Adversarially Learned Inference



Loss function and algorithm

1. discriminator $D_\omega(x, z)$ and loss

$$L(\{\theta, \phi\}, \omega) = \mathbb{E}_{(z', x) \sim q_\phi(z|x)\hat{p}(x)} \left[\log D_\omega(z', x) \right] + \mathbb{E}_{(x', z) \sim p_\theta(z|x)p_0(z)} \left[\log(1 - D_\omega(z, x')) \right]$$

2. Algorithm
    1. sample $z_i \sim p_0(z)$ and $x_i' \sim p_\theta(x \mid z_i)$ for $s \in \{1, \ldots, N\}$
    2. sample $x_i \sim \hat{p}(x)$ and $z_i' \sim q_\phi(z \mid x_i)$ for $s \in \{1, \ldots, N\}$
    3. do gradient ascent-descent on the MC estimate of $L(\{\theta, \phi\}, \omega)$

Adversarially Learned Inference / Results

SVHN dataset

original

learned

Interpolation in latent space of CelebA

;

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Adversarial Variational Bayes

Adversarial Variational Bayes:
Unifying Variational Autoencoders and Generative Adversarial Networks

Lars Mescheder [1] **Sebastian Nowozin** [2] **Andreas Geiger** [1 3]

Bayesian inference by density ratio matching

1. approximate the KL divergence through density ratio matching

$$\log q_\phi(z|x) - \log p_0(z) = \operatorname*{argmax}_{r_\omega} \Big\{ \mathbb{E}_{(z',x)\sim q(z'|x)\hat{p}(x)} \big[\log \sigma(r_\omega(z',x)) \big] + \mathbb{E}_{(z,x')\sim p_0(z)\hat{p}(x)} \big[\log(1-\sigma(r_\omega(z,x'))) \big] \Big\}$$

2. do max. likelihood by

$$\max_\theta \max_\phi \mathbb{E}_{(z',x)\sim q(z|x)\hat{p}(x)} \big[\log p_\theta(x|z') - r^*_\omega(z',x) \big]$$

AVB / details

L.d.r. loss

$$L(r_\omega) = \mathbb{E}_{(z,x)\sim q(z|x)\hat{p}(x)}\left[\log \sigma(r_\omega(z,x))\right] + \mathbb{E}_{(z,x)\sim p_0(z)\hat{p}(x)}\left[\log(1-\sigma(r_\omega(z,x)))\right]$$

Variation w.r.t. r_ω

$$\delta_{r_\omega} L(r_\omega) = q(z|x)\,\hat{p}(x)\,(1-\sigma(r_\omega(z,x))) - p_0(z)\,\hat{p}(x)\,\sigma(r_\omega(z,x))$$

Optimaliity condition

$$\delta_{r_\omega(z,x_i)} L(r^*_\omega) = 0 \quad \rightarrow \quad r^*_\omega(z,x_i) = \log q_\phi(z|x_i) - \log p_0(z)$$

KL divergence estimate

$$\begin{aligned}\sum_i \mathrm{D}[q_\phi(z|x) \parallel p_0(z)] &= \mathbb{E}_{(z,x)\sim q(z|x)\hat{p}(x)}[\log q_\phi(z|x) - \log p_0(z)] \\ &= \mathbb{E}_{(z,x)\sim q(z|x)\hat{p}(x)}[r^*_\omega(z,x)]\end{aligned}$$

AVB / step by step

Define posterior approximation $q_\phi(z \mid x)$ as the neural sampler

$$z = z_\phi(x, \epsilon) \quad \text{were} \quad \epsilon \sim \mathcal{N}(0, I_d)$$

Algorithmic details

1. sample $\epsilon_i \sim \mathcal{N}(0, I_d)$, $x_i \sim \hat{p}(x)$, $z_i \sim p_0(z)$
2. do gradient update on the losses
   - 2.1 update θ using

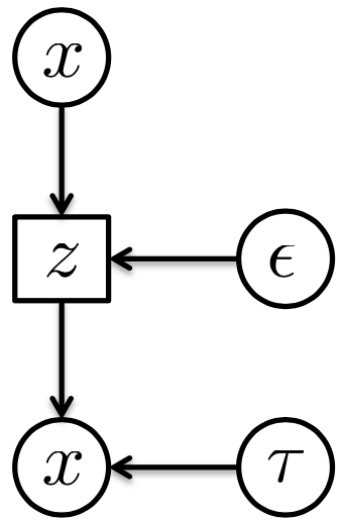
$$L(\theta) = \sum_i \log p(x_i \mid z_\phi(x_i, \epsilon_i))$$

   - 2.2 update ϕ using

$$L(\phi) = \sum_i \log p(x_i \mid z_\phi(x_i, \epsilon_i)) - r_\omega(z_\phi(x_i, \epsilon_i), x_i)$$

   - 2.3 update ω using

$$L(\omega) = \sum_i \log \sigma(r_\omega(z_\phi(x_i, \epsilon_i), x_i)) + \log\left(1 - \sigma(r_\omega(x_i, z_i))\right)$$



AVB / Experiments / MNIST

	$\log p(x) \geq$	$\log p(x) \approx$	
AVB (8-dim)	$(\approx -83.6 \pm 0.4)$	-91.2 ± 0.6	
AVB + AC (8-dim)	$\approx -96.3 \pm 0.4$	-89.6 ± 0.6	
AVB + AC (32-dim)	$\approx -79.5 \pm 0.3$	-80.2 ± 0.4	
VAE (8-dim)	-98.1 ± 0.5	-90.9 ± 0.6	
VAE (32-dim)	-87.2 ± 0.3	-81.9 ± 0.4	
VAE + NF (T=80)	-85.1	$-$	(Rezende & Mohamed, 2015)
VAE + HVI (T=16)	-88.3	-85.5	(Salimans et al., 2015)
convVAE + HVI (T=16)	-84.1	-81.9	(Salimans et al., 2015)
VAE + VGP (2hl)	-81.3	$-$	(Tran et al., 2015)
DRAW + VGP	-79.9	$-$	(Tran et al., 2015)
VAE + IAF	-80.8	-79.1	(Kingma et al., 2016)
Auxiliary VAE (L=2)	-83.0	$-$	(Maaløe et al., 2016)

AVB / Experiments / Faces

original (right) and generated (left)

interpolation results

What have we learned?

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