Problem 1. #19 from 2.1

$$\overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}}^{E} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z+x \end{pmatrix}$$

$$\overbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}}^{E^{-1}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z-x \end{pmatrix}$$

$$E \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix}$$

$$E^{-1}E \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = E^{-1} \begin{pmatrix} 3 \\ 4 \\ 8 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$$

Problem 2. #19 from 2.2

$$\begin{pmatrix} 1 & 4 & -2 \\ 1 & 7 & -6 \\ 0 & 3 & q \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ t \end{pmatrix}$$

$\text{row}_2 -= \text{row}_1$

$$\begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & -4 \\ 0 & 3 & q \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ t \end{pmatrix}$$

$\text{row}_3 -= \text{row}_2$

$$\begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & -4 \\ 0 & 0 & q+4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ t-5 \end{pmatrix}$$

Part 2-a. Singular System (q)

If $q = -4$ then the third equation is $0 = t - 5$, which exposes no information about z. Thus if $q = -4$ then the system is singular.

Part 2-b. No Solutions (t)

What value of t results in the system having no solutions?

If $q = -4$ and $t = 5$ then the third equation is $0 = 0$. Thus the value of z is irrelevant and the system has infinitely many solutions.

Part 2-c. Solution where $z = 1$

Find the solution which has $z = 1$.

$$q = -3$$
$$t = 6$$
$$\begin{pmatrix} 1 & 4 & -2 \\ 0 & 3 & -4 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 1 \end{pmatrix}$$
$$z = 1$$
$$3y - 4z = 5$$
$$y = 3$$
$$x + 4y - 2z = 1$$
$$x = -9$$
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -9 \\ 3 \\ 1 \end{pmatrix}$$

Problem 3. Graph Intersection

The graph of

$$y = a + bx + c \sin \frac{\pi x}{2}$$

passes through the points $(0, 2), (1, 6), (2, 12)$.

Each pair of points represents an equation.

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \\ 12 \end{pmatrix}$$

swap row_2 with row_3

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 12 \\ 6 \end{pmatrix}$$

That looks enough like a Gaussian Eliminated matrix. Back substitution, here we come.

$$a = 2$$
$$2 + 2b = 12$$
$$b = 5$$
$$2 + 5 + c = 6$$
$$c = -1$$
$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$$

Problem 4. #20 from 2.4

$$A = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$v = \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$$

$$Av = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 2y \\ 2z \\ 2t \\ 0 \end{pmatrix}$$

$$A^2v = \begin{pmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 4z \\ 4y \\ 0 \\ 0 \end{pmatrix}$$

$$A^3v = \begin{pmatrix} 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 8t \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$A^4v = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Problem 5. #23 from 2.4

Part 5-a.

Find a nonzero matrix A for which $A^2 = 0$.

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$
$$A^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Part 5-b.

Find a matrix that has $A^2 \neq 0$ but $A^3 = 0$.

$$A = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$A^2 = \begin{pmatrix} 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$A^3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Problem 6. #3 from 2.5

$$\begin{pmatrix} 10 & 20 \\ 20 & 50 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\text{row}_2 -= 2\text{row}_1$

$$\begin{pmatrix} 10 & 20 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$y = -0.2$

$10x + 20(-0.2) = 1$

$x = 0.5$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0.5 \\ -0.20 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 20 \\ 20 & 50 \end{pmatrix} \begin{pmatrix} t \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$\text{row}_2 -= 2\text{row}_1$

$$\begin{pmatrix} 10 & 20 \\ 0 & 10 \end{pmatrix} \begin{pmatrix} t \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$z = 0.1$

$10t + 20(0.1) = 0$

$t = -0.2$

$$\begin{pmatrix} t \\ z \end{pmatrix} = \begin{pmatrix} -0.2 \\ 0.1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} x & t \\ y & z \end{pmatrix} = \begin{pmatrix} 0.5 & -0.2 \\ -0.2 & 0.1 \end{pmatrix}$$

Problem 7. #3 from 2.6

Problem 8. Upper Triangular Squares

```
julia> {triu(ones(n,n))^2 for n=1:5}
5-element Array{Any,1}:
 1x1 Array{Float64,2}:
 1.0
 2x2 Array{Float64,2}:
 1.0  2.0
 0.0  1.0
 3x3 Array{Float64,2}:
 1.0  2.0  3.0
 0.0  1.0  2.0
 0.0  0.0  1.0
 4x4 Array{Float64,2}:
 1.0  2.0  3.0  4.0
 0.0  1.0  2.0  3.0
 0.0  0.0  1.0  2.0
 0.0  0.0  0.0  1.0
 5x5 Array{Float64,2}:
 1.0  2.0  3.0  4.0  5.0
 0.0  1.0  2.0  3.0  4.0
 0.0  0.0  1.0  2.0  3.0
 0.0  0.0  0.0  1.0  2.0
 0.0  0.0  0.0  0.0  1.0
```

It appears that the bottom triangle remains zeros but the upper triangle increases towards to the top-right where it reaches $n+1$. When trying to understand why this is, I found it helpful to think about splitting up the rows. The first row of A^2 is $A[1;:] * A$. $A^2[2;:] = A[2;:] * A$, etc.

Problem 9. Order Exploration

Cool.

```
julia> [mean([order(rperm(j)) for i=1:10000]) for j=1:15]
15-element Array{Float64,1}:
  1.0
  1.4963
  2.1783
  2.7911
  3.9452
  4.5654
  6.2151
  7.3896
  9.043
 10.8293
 13.6837
 15.7355
 19.0345
 22.0164
 24.9219
```

I found that the average order depends greatly upon the size of the matrix. This makes some intuitive sense as their is more space to for the P to wander around before finding its way back to I.

Problem 10. Speedy Computers

$$\underbrace{f(N)}_{\text{fp.computations}} = \frac{2}{3}\underbrace{N}_{\text{vars}}{}^3$$
$$N_{\text{blue}} = 12,681,215$$
$$\underbrace{S_{\text{blue}}}_{\text{speed of IBM}} = 1$$
$$S_{\text{ALAM}} = 2$$
$$\frac{f(N)}{S} = T$$
$$\frac{f(N_{\text{blue}})}{S_{\text{blue}}} = T = \frac{f(N_{\text{ALAM}})}{S_{\text{ALAM}}}$$
$$f(N_{\text{blue}}) = \frac{f(N_{\text{ALAM}})}{2}$$
$$\frac{2}{3}{N_{\text{blue}}}^3 = \frac{{N_{\text{ALAM}}}^3}{3}$$
$$\left(2{N_{\text{blue}}}^3\right)^{\frac{1}{3}} = N_{\text{ALAM}}$$
$$N_{\text{ALAM}} \approx 15,977,329 \text{ variables.}$$