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LET'S INVENT THE WHEEL

Marvin Minsky and Seymour Papert

1. The Foot of the Wheel stands Still
2. Bumps transforms and envelopes
3. Levers
4. The Bicycle Wheel
5. Achilles and the Tractor
6. "The BIG Wheel"
7. Is this Math, Physics, Engineering or What?

There is no force, however great —
→ will stretch a thread — however fine, —
into a horizontal line **HORIZONTAL TO**
that will be absolutely straight. **WHAT?**

DAY ONE: THE FOOT OF A WHEEL STANDS STILL

which Authors

AUTHORS: This little book is about a great adventure we just had.

CRITIC: I love adventure stories. What happened to you? Was it dangerous? Were you scared?

AUTHORS: (Blushing) Oh, well no, it wasn't that kind of adventure. Just an intellectual adventure. We've been inventing --

CRITIC: (Disappointed) Well, cheer up. The world would never get along unless people invented new things. What did you invent?

AUTHORS: Well -- it wasn't really anything new. In fact it was invented before. We've been inventing the wheel.

CRITIC: The Wheel! Did you say THE WHEEL? Ha-Ha! Listen to that. These guys have been re-inventing the wheel. (Collapses in fits of laughter)

AUTHORS: Wouldn't you like us to tell you about it?

CRITIC: You've got to be kidding. There can't be anything to tell. Everyone knows all about the wheel.

AUTHORS: (Plucking up courage) Well, Critic, could you explain how the wheel works?

CRITIC: Of course I could! Its round and it rolls.

AUTHORS: O.K. Lets dig into it and you'll see what we mean. Let's imagine this old stone age guy pulling a big rock.

ILLUSTRATION

STONEY pulling a rock, puffing and panting

CRITIC: He's having trouble.

AUTHORS: Well, if he only would invent the wheel he could put the rock on a cart and pull it along with the greatest of ease--

ILLUSTRATION

STONEY pulling a cart, whistling.

CRITIC: Sure, everybody knows that. So what?

AUTHORS: Everyone knows that it works. But only a few people know why it works. Our adventure was in exploring all sides of why the wheel works and what people think about it. Most people think they understand it, but each one really only knows a piece of the story -- and hardly anyone knows the best parts.

CRITIC: Tell me something I don't know about the wheel!

AUTHORS: Did it ever strike you that the wheel would help our stone age guy just by standing still?

CRITIC: Of course not. A wheel does no good when its standing still. I already told you, it helps him by rolling. By rolling, man, that's moving, not standing still

AUTHORS: OK, the cart is moving forward. Sure, the wheel is turning. Of course, its rolling. But the important thing is, where the wheel is touching the ground, it is standing still.

CRITIC: Get out of here!

AUTHORS: Cool it, Critic. We're going to do an EXPERIMENT.

Here's a wheel with three spots.

A red triangle

A green square

A blue circle

ILLUSTRATION

first picture of wheel

We are going to roll the wheel a little. Watch the spots!

ILLUSTRATION

second picture of wheel

CRITIC: That's amazing. The triangle moved a lot, the square a little, and the circle hardly at all! What's the trick?

AUTHORS: No trick, just mathematics.

CRITIC: Mathemagic, I'd call it. There has to be a trick. Look: the wheel is going forward, but if one part of it stands still, the whole wheel would be stuck. Something is wrong!

AUTHORS: Let's look at it another way, by comparing it with another similar situation.

CRITIC: What's similar to a wheel but not a wheel?

AUTHORS: Walking.

CRITIC: Are you going to tell me you also re-invented walking?

AUTHORS: No; but something about it.

AUTHORS: Do your feet move forward when you walk?

CRITIC: I'm not answering any more of your tricky questions! -- Oh well, O.K., Yes, of course they are.

AUTHORS: Did you say "THEY ARE" or "IT IS"?

CRITIC: They, of course. Two feet!

AUTHORS: Most of the time one foot is in the air and one is on the ground.

CRITIC: Yeah. So what.

AUTHORS: The one on the ground isn't moving! Then after a while you pick it up and put it down further ahead.

CRITIC: So a part can be standing still --. O.K., I see what you mean. I still feel there's something crazy about it, but I'll let you say that the foot of the wheel is standing still even though the wheel is moving. But so what, anyway?

AUTHORS: That's a nice way to put it: the "foot of the wheel". Wish we had thought of that.

ILLUSTRATION

DRAWING of feet walking

AUTHORS: And notice another thing -- when the foot of the wheel starts

moving, it starts going STRAIGHT UP!

CRITIC: That's really amazing. It doesn't drag along or scrape even a little. I simply don't understand how it goes UP when the wheel goes ALONG, even though I see it before my eyes.

AUTHORS: Let's get our stone age man back on stage:

ILLUSTRATION

STONEY pulling his rock

Why is Stoney having such a hard time pulling his rock?

CRITIC: Because its heavy -- of course.

AUTHORS: That's true, of course. But perhaps we'd understand Stoney's problem better if we asked HOW being heavy makes a thing harder to move. After all, if Stoney would invent the wheel and build a cart, the rock would still be just as heavy. But he wouldn't have to pull so hard. What would change?

ILLUSTRATION

[Stoney pulling his cart (again)]

(long silence)

CRITIC: Of course! Friction. The rock drags on the ground because of friction. In fact, Stoney doesn't really need a cart. If Stoney POLISHED the rock -- AND the ground -- the rock would slide more easily.

AUTHORS: Especially if he greased them as well. So, great critic, you have broken the rules of your trade and have made an idea instead of tearing ten ideas into pieces. That is a perfect solution to Stoney's problem:

ILLUSTRATION

[two pictures]

Rough Surface=====Polished, Lubricated surface

Stoney sweats and puffs
as he works against
FRICTIONAL DRAG

The rock slides easily
because there is
NO FRICTIONAL DRAG

CRITIC: (blushing) Stop teasing me. I didn't really think that dumb idea would be a solution for Stoney.

AUTHORS: We aren't teasing you. It really is a good solution. Of course Stoney can't use the idea just like that. But it's got the right idea in it. Just let's dig it out.

CRITIC: How? And we seem to have forgotten about the wheel.

AUTHORS: WHO has forgotten about the wheel? WE haven't.

CRITIC: (looking around) That leaves me -- you must be trying to tell me something.

CRITIC: (daydreaming) I know only two ways to get rid of frictional drag. One is to polish and lubricate. The other is to not move at all. Stoney could stop his puffing by standing still -- but then he wouldn't get anywhere. You don't move by standing still. -- WHAT AM I SAYING? -- That's what these crazy authors have been trying to tell me. You CAN get somewhere by standing still -- sort of -- if you are round and rolling along -- So if Stoney had a ROUND rock -- or a big round log -- he could roll it and have no frictional drag.

ILLUSTRATION

Picture. Stoney pushing log along at 5 miles per hour.
Head of log is moving ten miles per hour.
Center rolling at five mph.
Foot of log is standing still.

CRITIC: The log is rolling along. But where it touches the ground it is standing still. So there is no frictional drag!

CRITIC: Now, what about the big rock? Aha! He could put the rock on the log! So this is what he does:

ILLUSTRATION

picture of rock on a track of logs

CRITIC: So the rock goes forward with no drag, no sweat.

CRITIC: The foot of the logs are not moving, so there is no drag there. The tops of the logs move just as fast as the bottom of the rock, so there is no drag there!

AUTHORS: You got it. But there is still a bug

CRITIC: A bug? I should have known it. You guys will change the subject as soon as I say anything sensible. I thought we were talking about wheels, not pugs.

AUTHORS: Cool it, cool it. Not that kind of bug. A bug in a good idea is a little something that makes the idea not quite work.

CRITIC: Are you saying my idea is a bad idea?

AUTHORS: No, a very good idea with a bug.

CRITIC: Well, you are wrong. Its a great idea and it will work. In fact I just remembered that that's how the old Egyptians got the rocks to the pyramids. But I re-invented it all by myself.

AUTHORS: We believe you. But there is still a bug. The rock will run ahead of its rollers!

CRITIC: Let's see. (He tries a little experiment with a box and six pencils.)

ILLUSTRATION

sequence showing box rolling ahead and falling off

CRITIC: Well, you are right. Hmmm.

CRITIC: That's nothing. I can debug my ideas faster than you can criticise them. All Stoney needs is an assistant who will pick up the logs left behind and bring them to the front. So the rock will keep going.

ILLUSTRATION

pictures

STONEY's ASSISTANT: (Sweating) I've had enough of this. Stoney doesn't sweat pulling the rock any more. But I sweat carrying the logs. I guess I'll automate that. Why don't I tie the logs to the rock -- then it can drag the logs with it.

CRITIC: Wow! You're right. That's the wheel. We've just invented the wheel. I mean we've invented the cart. The wheel rolls like the log. And the cart drags the wheels along.

AUTHORS: (Pompously) So let's sum it all up by seeing where the frictional drag is:

Rough rock, standing still

Rough rock, moving

No frictional drag

Lots of frictional drag where
the rock touches ground

Very smooth, lubricated rock Rolling logs,
on very smooth ground, no drag. No frictional drag.

CRITIC: So -- if the axle turns with the wheels, then two wheels and an axle is just like a log. Wait!! That's no improvement! If the axle is not turning then the wheel rubs against the axle where they touch. So there is frictional drag. All we did is move the drag from the ground up to the hub. Hmm. That's no good. So my idea is worth nothing. I'll have to go back to being a critic.

ILLUSTRATION

Picture:

Wheel and Axle, turning and rolling forward.
Foot of wheel standing still, no drag.
Hub turning, but Axe Not Turning. Friction here.

STONEY: What's wrong with you. You are too much of a critic of your own idea. You really did help us by moving the friction up there.

CRITIC: How?

AUTHORS: Do you remember your first idea -- that Stoney should polish the rock and the road?

CRITIC: (Encouraged) Wow. You are right -- I mean I am right -- after all. We've moved the friction to a new place, where we can polish and lubricate the surfaces.

CRITIC: So that's what wheels do: they move the friction from a bad place to a good place. Friction on the ground is bad because you can't polish the ground and because its too much trouble to polish every rock you want to move. But we can polish a wheel bearing, one time for all!

STONEY: And you know, if you forget to polish the old axle -- or don't do it too well -- the wheel will do it for you! Every time the wheel turns is polishes the surfaces a little more. So it gets better and better.

CRITIC: So that's what "running it in" means. Of course, after a while it will start getting worse when too much is polished away and it's wearing out---

AUTHORS: STOP. You are falling into being a critic again and finding all sorts of bad things. Let's first find out some more good things about wheels.

STONEY: I know another good thing about wheels. Remember how I was dragging the rock? Then I put it on the cart with wheels? Well, the

first time the rock was touching the ground everywhere. So it scraped and dragged a lot. With the cart, the wheels only touch four places, so they scrape less.

ILLUSTRATION

FRICITION EVERYWHERE - - - FRICITION ONLY AT FEET OF WHEELS

CRITIC: (Excited) No! No! No!! I have a counter-example to that theory.

STONEY: Counter-example? Theory? What's all that about?

CRITIC: Let's put the rock on four spikes:

ILLUSTRATION

(rock on four spikes)

CRITIC: Do you really think that you could pull this more easily than the plain old rock?

STONEY: I didn't say I could! Of course not!

CRITIC: You did so say you could.

STONEY: I did not.

CRITIC: You did so. You said: the wheels are good because they touch at four places instead of touching everywhere.

STONEY: That's what I said. I was talking about a wheel; I said nothing about spikes.

CRITIC: But your theory was that touching at a few places was better than touching everywhere. So if your theory is good, the spikes must be good.

STONEY: Oh Man -- (turns to authors) -- What do you say about all that?

AUTHORS: Sorry, we weren't listening. We were re-inventing the ball bearing. Or rather, we suddenly understood that the critic invented the ball bearing.

CRITIC: I did? When?

AUTHORS: Well, remember your plan to put the rock on logs?

STONEY'S ASSISTANT: That was the worst plan I ever heard.. I had to run back and forth and back and forth... But then I invented the wheel by making the cart drag its own log with it. That critic didn't invent

anything.

AUTHORS: But It wouldn't be so bad if there were always more logs in front.

ILLUSTRATION

rock on infinite row of logs

STONEY: There are no logs where I want to go.

AUTHORS: What if there were logs all around the earth? You could drag your rock right around.

ILLUSTRATION

earth surrounded by logs

CRITIC: I see what you're getting at. If you put little logs between the hub and the axle -- like this

ILLUSTRATION

(picture. Ball (roller) bearing.

then -- then there's no frictional drag at all! It's magic!

STONEY'S ASSISTANT: It's putting me out of a job. Well, someone will have to polish the balls.

AUTHORS: Yes, a bad thing about inventions is that they cause changes in the kinds of jobs people do. A good thing is that they tend to remove mainly the jobs that no one likes very much because they are so hard or boring.

STONEY: I'll say.

CRITIC: You'll never automate critics.

Other topics

This is a comment and editing section -- not part of the paper.

Mill-Wheels

A pair of wheels rotating against each other produces a huge crushing force. This will grind brittle particles to fine powder. Note that it is not a "grinding" action. The two surfaces approach each other with no sliding action; there is no wear on the wheels when they are not crushing, and little wear when they are. Of course, the huge forces must appear at the axle-bearings, and those bearings must be very strong and rigid because they bear the full crushing forces.

Tires and Shock-absorbers.

The pneumatic tire as a static smoothing device. If you model it as a large sequence of independently-sprung (constant-pressure/force) "feet" then you get an "inverse mechanical advantage". A bump under one of the feet raises the tire by

$$h*a/A$$

where h is the pebble's height, a is its horizontal cross-section area, and A is the tire's area touching the ground. Note that $h*a$ is in fact the pebble's volume! Incidentally, $A = W/p$ where W is the car's weight and p is the tire's pressure. You therefore make your ride smoother by letting air out of the tires until the area on the road is large. Or, you get the same effect by loading down your car with huge weights.

Shock-absorbers are completely different!

By having a mass-and-spring system, one can smooth out bumps by letting the wheel actually make a large motion over a bump, but using the mass/spring filter to do dynamic smoothing over time.

The Vibrating String

In order that a string produce a good musical note, it must vibrate with a good, reliable, constant pitch. It would not sound good if it made different pitches when plucked or hit with different strengths. Fortunately, a stretched elastic string vibrates with very nearly the same frequency when plucked lightly or strongly. This is because of a "mathematical phenomenon" called harmony about which we can write another whole book. I'll just say that it's a fascinating phenomenon.

Waves along a rope.

A difference, and... now a problem... I also sometimes have a string, acceleration equals gravity! What? Velocity is zero?

difference between tension vectors is difference between tangents is second (space) derivative. Wave equation in words?

Bubbles and surface tension.

Another example: bubble tension. a) local surface curvature idea. b) global hemisphere calculation. Force is tension times equator length/cross section area. This is inverse radius! Same as curvature or second derivative.

Some consequences about bubbles. Force very large inside small bubble. Can't blow small bubbles. They dissolve their gases in solution, too. Superheated liquids can thus exist. Invent bubble pressure pump device. Boundary conditions. Tension. Reflection from bound end... In bubble, tension is intrinsic.

DAY TWO: BUMPS

CRITIC: I have a great new wheel problem. All our talk yesterday was about wheels on hard flat roads. What happens if there is a real bump. Why doesn't the wheel get wedged?

ILLUSTRATION

AUTHORS: It does get wedged. What's your problem?

CRITIC: (Laughing) Is that a bug or a counter example?

AUTHORS: What?

CRITIC: I agree that very bad obstacles will really wedge the wheel. But that is an interesting problem about how the wheel deals with less bad obstacles. Like, here is an obstacle that is absolutely disastrous for Stoney's old pre-wheel method.

ILLUSTRATION

Step in ground Stoney about to be stuck. What can Stoney do?

STONEY: I know what I'll do. I'll use a LEVER to lift the rock over onto the ledge. LEVER and then BLOWN UP PICTURE OF LEVER. BUMPS

CRITIC: We wouldn't need a lever if we had a big enough wheel. You could just roll it over the bump.

AUTHORS: Well why would that help? You have to lift the weight over the bump in any case, don't you? How does the wheel help?

CRITIC: It's somehow keeps it from getting stuck. Wheels make things move more smoothly.

AUTHORS: Why do you think a wheel gives a smoother ride?

CRITIC: Obvious! A small wheel goes into all the little holes and bumps. . . a big wheel just goes over them. Look:

ILLUSTRATION

Caption: small wheel travelling along Caption: Big wheel.

AUTHORS: What happens when you go over a pebble with big wheels and small wheels?

CRITIC: Each wheel has to climb up as high as the pebble; so big wheels and small wheels make the same bump.

ILLUSTRATION

The wheel must climb up this much. Any wheel. Big or small! pebble top level, road level.

AUTHORS: We don't agree. AUTHORS: I'm beginning to suspect that you have never thought very much about bumps.

CRITIC: What is there to think about them? The fewer bumps the better. Big bumps hurt and break things. But you can't just stay in bed all the time to avoid them. They're part of life. There. I've thought about bumps for you. AUTHORS: Well, please explain to me exactly how a bicycle wheel would go over a brick. Can you sketch the path of the axle as the wheel rolls over this bump?

ILLUSTRATION

CRITIC: It's simple. It just goes over it like this:

ILLUSTRATION

AUTHORS: That doesn't look very "smooth"!

CRITIC: I guess not. It would be just as bumpy as the road, and I don't really believe that. Anyway another thing wrong with it is that it jumps up instantly. Nothing can move a real distance in no time. I guess it must go up and down more like this:

ILLUSTRATION

AUTHORS: That's a lot better, but I think you're still just guessing. Why don't you figure out exactly what happens. Try drawing pictures of the wheel at different times in going over the brick.

CRITIC:

ILLUSTRATION

CRITIC: Aha. The path of the axle seems to be sort of curved in places. but the bump isn't curved. Amazing. That must be how the wheel makes things go smoother.

AUTHORS: Much better. But you still haven't shown exactly what happens!

CRITIC: O.K., I'll try. But it's getting boring. CRITIC (After a long time). Boy, this is complicated, but I think I've really got it.

First, of course, the axle goes straight until the heel hits the bump

ILLUSTRATION

AUTHORS: Right.

CRITIC: Then the wheel turns around on the point of the bump!

ILLUSTRATION

The axle actually moves along part of a circle!

AUTHORS: Excellent! It isn't easy to see that. That's what I really wanted you to discover.

CRITIC: Right. It took me a long time to realize that the wheel has a new foot. A point on the bump-corner that is now the wheel's foot. And that point AUTHORS: It is strangely like walking. You might say that the wheel is not moving. Anyway, as long as the wheel is still climbing up the step, that foot stays in the same place. So the axle has to move in a circle. It was hard to think of because I am used to a wheel turning on its axle. I never had to think about a wheel turning around one of its edge points.

AUTHORS: Wheels usually turn around edge points. They hardly ever turn on their axles.

CRITIC: WHAT DID YOU SAY?

AUTHORS: Nothing, really. Just a private joke. Well, you are coming along fine. What does our wheel do next?

CRITIC: Once it gets on top of the bump, it just rolls so the axle goes straight along for a while.

ILLUSTRATION

Then it goes down. Just as before, its foot is fixed on the bump-corner and the axle moves down in a part of a circle.

ILLUSTRATION

and then, finally, it rolls straight away.

AUTHORS: Now can you explain why it's easier to roll over the bump with a wheel.

CRITIC: Yes, I think so. It's like going up a ramp or a hill, instead

of crashing into a step or a cliff. And also, of course, there's still no frictional drag because the foot of the wheel never moves.

AUTHORS: Great. I just have one more question. Why is it easier to go over the bump with a large wheel than with a small wheel?

CRITIC: Well, if the wheel is too small, it simply can't put its foot on the bump's corner. It's just too high a "step".

ILLUSTRATION

ILLUSTRATION It's about ten times as high as the bump before it will roll over it without much trouble. SMOOTHING AND ENVELOPES. To see how different sizes of wheels affect smoothness, we plot how each goes over a rough road.

ILLUSTRATION Radius 2 and Radius 3 Do continuous case with use circle and show critical wheel phenomenon. The Hinge is a wheel

The rough road as seen by the wheel-car: a "transformation."

ILLUSTRATION showing the Road with small wheel, pretty bumpy. The little bumps almost gone. Smooth ride. With a rubber tire, even smoother, but the hard wheel itself does . . . ILLUSTRATIONS CAPTION: small wheel climbing a pebble. Notice the steepness of the climb angle. CAPTION: A big wheel climbs to the same height to cross the pebble, but its center goes up a longer but gentle slope!! This makes the ride gentler. It also shows how to see the mechanical advantage from another angle.

CRITIC: What a pun!

AUTHORS: What?

DAY THREE: LEVERS

STONEY: I want to show you something. you can think of the wheel as a lever for going over bumps.

CRITIC: That's silly. A wheel is round, a lever is straight. A wheel "rolls," a lever stays put.

STONEY: Well, look at this; you can think of the rim of the wheel as a lever.

ILLUSTRATIONS That really shows why a big wheel makes it easier than a little wheel. AUTHORS: Yes, that is a nice way to look at it, although it is hard to see exactly where the forces are working.

CRITIC: Well, I still have a problem. Just how good is a wheel as a lever. I see why a big wheel is better than a small one, but how could I tell just how much weight Stoney could lift using a wheel?

AUTHORS: How good is a LEVER as a lever?

CRITIC: I WAS ASKING A SENSIBLE QUESTION. WHY ARE YOU ASKING A SILLY ONE?

AUTHORS: It's not really silly. You must be sure you really understand the lever before you can expect to understand something like a lever. Here: how hard would you have to push to lift this 1,000 lb weight?

ILLUSTRATION

CRITIC: Just ten pounds. Because my side of the lever is 100 times longer, I can lift 100 times as much as without the lever.

STONEY: Real men don't need cheap tricks like that.

AUTHORS: Brute force doesn't solve problems. It takes brains.

STONEY: Then why don't you just talk the rock into moving.

AUTHORS: It's easier to bet you that you couldn't lift it.

STONEY: Sure i can. Wanna see?

AUTHORS: Later, perhaps. By the way, what are the disadvantages of the lever.

STONEY: Even I know that. To lift it 1 foot, you'd have to move the end of the lever 100 feet. (In fact, besides your lever would be bigger than a telephone pole. You wouldn't even be able to lift the end of

your lever. Hah. Hah.

AUTHORS: Of course we're talking about a mathematical lever, and mathematicians don't have to worry about all those little practical details.

CRITIC: Anyway I'm getting bored. Now that we remember how a lever works, how about answering my question about how hard we must push a wheel.

AUTHORS: Here's a huge wheel, 100 inches in radius, and a one-inch high bump. My truck weighs 21,000 lbs. How hard do I push?

ILLUSTRATION

CRITIC: I don't know.

AUTHORS: Well, how far does the truck move forward as it rises one inch over the bump?

CRITIC: About ten inches, just from looking at your diagram. Great. At last I understand it completely. It really is just like a lever. I have to move ten times as far to lift it one inch but I only need to push one-tenth as hard as I'd have to lift without the wheel. I could lift 1,000 lbs with that wheel, and only push with 100 lbs of force.

AUTHORS: That's the idea, alright.

VIGNETTE We did not want to discourage Critic by telling him that there is still a bug in his argument. Actually the wheel is only about half that good. The reason is seen in the diagram of the axle-

path: ILLUSTRATION The Critic imagined a straight ramp from P to Q, in which the slope is 1 to 10. But, as you can see, the far end of the ramp is horizontal, while its beginning is actually TWICE as steep. (Angle A and angle B are equal, because of the parallel lines, and angle A and angle C are equal because they are both formed by the same chord of a circle.) So angle B = angle C, and the ramp angle B plus C is just twice angle B, which is what the critic thought. The force required is therefore at least twice as much as he thought: it would take about 200 pounds. The CRITIC was right "on the average," though, because it needs more force at first, less later. How hard must you push on the Axle to make a wheel go over a Bump if the load is 1,000 pounds?

ILLUSTRATION: Radius of Wheel, Height of Bump, etc.

CRITIC: How did you figure those out?

AUTHORS: We'll talk about it later.

ILLUSTRATION: The Claw Hammer as a Wheel/Lever This is a good example of all we have seen. If you imagine the claw hammer as part of a wheel

then

1: The leverage is high. You couldn't pull a big nail straight out by yourself but it's easy with the claw hammer. Good leverage. In fact, Infinite.

2: The nail comes straight up! Of course only at first. If you keep pulling it will curve over. From this you can see that it would be useless to glue someone's car wheels to the ground as a practical joke! The leverage of the wheel would easily break any glue. No use nailing the rim down either.

STONEY: Say, you've been using your wheel-lever very stupidly! Did you realize that if you push the top of the wheel you can move twice the load as you can by pushing the art? Look here:

ILLUSTRATION Caption: Stoney's Way and Usual Way

AUTHORS: Amazing. Of course. That way, the handle of the "wheel-lever" is twice as long. So you get twice the lifting force.

CRITIC: You have to push twice as far. But I suppose it's often worth it.

STONEY: Certainly, for people of limited strength, there's no other way.

AUTHORS: So that means we only need blank lbs to raise a 1,000 pound load over a plank in bump with a 100 inch radius wheel. (They all celebrate)

AUTHORS: There's still more.

CRITIC: You're kidding.

AUTHORS: Who sees another lever in the last picture?

CRITIC: You mean like the puzzle-pictures where you have to find ten animals?

AUTHORS: Exactly. There are levers and wheels in everything. It's a way of seeing tohrld. Here it is:

ILLUSTRATION

CRITIC: Amazing, indeed. The cart-body itself is a lever. The wheel going over the bump only has to lift 300 lbs, to raise the 1,000 pound load.

STONEY: You don't need that lever idea, actually. You could just think of 300 lbs on each eel.



CRITIC: Of course you don't save any "work" in the long run. BOTH wheels BOTH wheels have to go over the bump.

AUTHORS: But it's twice as easy each time.

STONEY: So it's good for weak people.

AUTHORS: So with the cart, you only need to push blank lbs to get 1,000 lbs over a one.inch bump.

CRITIC: You're still using Stoney's trick of pushing on the top of the wheel, aren't you?

AUTHORS: Yes. We can use all the different tricks at once, if they are "independent."

CRITIC: Hey! Why not use a lot of wheels, like 10 of them, and only push a tenth as hard?

AUTHORS: O.K., try it with 3, now ILLUSTRATION

CRITIC: That's strange. It got worse! For the middle wheel I have to use full force. Why is that?

STONEY: Because you did something stupid. When the middle wheel hits the bump, the whole weight of the load lies directly on it.

AUTHORS: Yes. We lost the cart-lever. Too bad.

CRITIC: I feel sad. I thought I had a real good idea, but I have to throw it away because it's wrong.

AUTHORS: You don't have to throw it away. It really was a good idea, but it has a few bugs. If a roof leaks you don't have to build a new house -- and you can build with ideas and fix them just as you can build with and repair things. What was the bug in your idea?

CRITIC: When you lift the middle wheel, the whole cart comes off the ground. Hmmm. Maybe the thing is too rigid.

AUTHORS: Yes, indeed. How about this one:

ILLUSTRATION

CRITIC: I think that works. You have put back the cart lever, to put 1/3 of the eight in front and 2/3 in back.

STONEY: Yes, and then you split the 2/3 into two wheels, using the cart-lever.

AUTHORS: Right. So each wheel has $1/3$ the weight now and you only have to push $1/3$ as hard to get each wheel over the bump.

CRITIC: And you could keep doing at: here is another version where you only need a quarter of the force ILLUSTRATION

CRITIC: And here is a way to get one-sixteenth! ILLUSTRATION

STONEY: yuk. Those people will go to any extreme to avoid honest work.

AUTHORS: In fact, they use something very much like that for the little wheels that support the treads in "caterpillar" tractors and tank treads.

VIGNETTE: Springs and Suspensions The last few cart models became complicated because we had to find a way to spread out the load's weight over several wheels. The problem was partly the rigidity of the cart-body, just as the critic complained. Another, simpler way to get around this is to use springs! ILLUSTRATION

You need to get "long" springs, so that their force doesn't change much as go over a bump. This is why air-inflated tires are so good also. For small pebbles we get a large mechanical advantage.

There are other, quite different reasons why springs help make smooth rides!

(Dynamics. How does tire support axle?)

CRITIC: Well, if that's such a good idea why don't they use it more.

AUTHORS: I suppose there's some other bug we haven't thought of.

STONEY: There sure is! Imagine trying to steer it around a turn.

CRITIC: Well, how do tanks and tractors go around turns?

STONEY: It's easy for them. They simply scrape and slide and sometimes tear out part of the road and turn it.

AUTHORS: Yes. They aren't very popular with the other citizens. One last remark. A cart usually has 4 wheels, not 2.

CRITIC: So each wheel actually can have just $1/4$ the weight. You'd need only blank lbs force to push it over our bump, pushing at the top of the wheel.

STONEY: Provided the wheels have proper springs to distribute the weight evenly when there is a bump. If the cart is too rigid, then only two or three wheels will rest on the ground while you go over the bump, and you won't get your mechanical advantage of 4 times the force.

DAY FOUR: BICYCLE WHEELS

AUTHORS: Hi Critic, we've invented a game just for people like you.

CRITIC: What kind of game?

AUTHORS: A critical game. We'll show you a picture, you criticize it. In case you haven't guessed it's a picture of a wheel. A bicycle wheel. Now tell us what's wrong with it.

ILLUSTRATION / CRITIC: Well, it's not very round.

AUTHORS: Great, we'll chalk up one point for you. What else is wrong? Criticize as much as you like. / ILLUSTRATION

CRITIC: Well . . . let's see. Bicycle wheels aren't so flat. The spokes come out, sort of . . . and criss-cross. Something like this. ILLUSTRATION

AUTHORS: O.K. you get 5 points for that. What else is wrong?

CRITIC: There aren't enough spokes. ~~But~~

AUTHORS: Another point. There is another thing wrong worth ten whole points. Now, to help you find it we'll make another drawing. Perfectly round, the proper number of spokes, even a tire. Only to keep it simple we're leaving out the ~~third~~^{~dun-dun~} part and just drawing it flat.

ILLUSTRATION / CRITIC: I don't know what you are looking for. There's no value for pumping the tire.

AUTHORS: No. . . it's nothing like that. It is ~~a very essential aspect~~ ^{has to do with} of how the wheel actually works. Especially the back wheel. ~~So if you~~ ^{Let's think about} ~~drawn~~ ^{it's simplified} ILLUSTRATION

CRITIC: The rider pushes on the pedal. The pedal turns that cog wheel. The cog wheel pulls on the chain. The chain turns the hub. The hub makes the whole wheel go round with it.

AUTHORS: ~~Let's think about~~ ^{Now let's think about} how the hub makes the wheel go round.

CRITIC: It pulls on the spokes and the spokes pull on the rim. ~~but there's only one spoke~~ ^{Let's look at} our picture ~~with just one spoke~~ ^{drawn} ILLUSTRATION

CRITIC: I see the point! The hub is pushing the spoke sideways instead of pulling along it! The spoke should be tied on something like this: ^{to the hub}

ILLUSTRATION instead of like this ILLUSTRATION Quickly get me a real bicycle wheel that I can see if I'm right.

so

AUTHORS: You are right ILLUSTRATION -- Photo of bicycle wheel.

ILLUSTRATION -- Photo of cart wheel with wooden spokes connected radially.

I did notice that they were - sort of, crisscross -
but I didn't think about why.

CRITIC: ~~fact~~ really is amazing. I've seen a thousand bikes and I never noticed how the spokes attached. Are you sure they are always tied on that way. . . what could we call it?

AUTHORS: Let's call this way of tying the spokes "tangential"

ILLUSTRATION and this way "radial" ILLUSTRATION But before we go on let's clean out some bugs. Can you see any?

~~lot of bugs~~

CRITIC: Well now that you mention it I see a terrible bug. What if the hub tries to turn the wheel the other way? If the spoke is tied tangentially like this ILLUSTRATION and turns this way, the spoke will just bend.



AUTHORS: So what do you think the wheel makes do about that?

CRITIC: Of course. . . they have some spokes in each direction. so whatever way it wants to turn half the spokes are working. an the other half are really doing nothing. ILLUSTRATION Caption: Two spokes. So that's another reason why the spokes criss-cross. So that bug is debugged out of our worries. But I see another one. . . which you've been hiding from me by the way you made the last few drawings. You draw how the spoke is connected the hub tangentially. But what about how it is connected to the rim?

TRY IT. ILLUSTRATION Caption: This is tangential. What about this? Is it tangential or radial? Can you invent a wheel with real tangential connections at both ends? TRY its job.

PROBLEM: Imagine what might happen if you had a wheel with radially connected spokes. ILLUSTRATION VIGNETTE on possible effects with radial spokes. VIGNETTE on other situations like the spokes; stretched wires, etc. HOW DO THE WHEELS HOLD THE BIKE UP?

AUTHORS: How do the spokes hold up the Bicycle and Rider?

AUTHORS: How do spokes help?

CRITIC: !!! It's obvious. Anyone can see that. I admit that it was surprising that wheels are levers. But surely there's nothing interesting about spokes.

AUTHORS: Let's see if you really can explain it. Suppose the bike is standing still. All the weight of the bike and rider rests on the hub.

You have to show me how the spokes hold the hub up.

CRITIC: Alright, I'll play your game. What do the spokes do: the bottom spokes are obviously the most important for holding the hub up, but the other ones near them help a little. The spokes at the side can't help at all -- I see -- they can only pull sideways and that can't help hold it up. I guess it IS interesting! The spokes at the top aren't used much either, but -- oh, I see, it's like Stoney's logs -- when the wheel rolls they will come down and help later.

AUTHORS: That's good, but it has some serious bugs! Let's see how your theory actually works out. Can we simplify it by looking at a wheel with just four spokes? ILLUSTRATION

CRITIC: It doesn't look strong enough.

AUTHORS: It takes 200 pounds to break a spoke wire. Anyway, what would happen if we took out the top spoke? CRITIC: Wow! It would collapse instantly! ILLUSTRATION

AUTHORS: But if you took out the bottom spoke?

CRITIC: Hmm. It would... it might... did you say a spoke could hold 200 pounds? Why it would just hang there! ILLUSTRATION I begin to see. You CAN'T use a spoke to push up very hard, because when you PUSH on a wire it just bends. But you can PULL a wire very hard before it breaks.

AUTHORS: Do you mean to tell me that a bike HANGS FROM THE TOP OF ITS WHEEL instead of being held up from the bottom?

CRITIC: Exactly. Isn't that amazing!!

AUTHORS: How about the other parts of your theory, now that we have fixed this bug

CRITIC: The Stoney-and-log part is still basically O.K. Except now it's the BOTTOM spokes that aren't needed, until the rolling brings them to the top! CRITIC: Now I see what you mean about a theory with a bug not being "wrong". The general idea is still good, even though I had to change "bottom" to "top" and "push" to "pull".

I wish teachers knew about "bugs". Then they wouldn't have to say an idea is wrong just because it doesn't work, or gets some incorrect answers, or even has some things the exact opposite of what they should be. The "basic idea" could still be valuable.

AUTHORS: What about the side-spokes?

CRITIC: Are you suggesting that there is still another bug left in my theory? Let's see. We'd better imagine the four-spoke wheel with the

bottom AND the side spokes taken out.

ILLUSTRATION Note: Shouldn't we be able to use the paper disc model? Caption: tear, crumple. Point: The TEAR means a lot of force. The CRUMPLE means only a little force!

CRITIC: This is crazy. Do you realize that we are now talking about a "wheel" with just one spoke! Anyway, it doesn't look good. If i know my bicycle rimsthe whole thing will just squash! ILLUSTRATION I'll take sure you can't hang all your weight on a helpless bicycle wheel-rim.

AUTHORS: And so, this shows that . . . ?

CRITIC: It shows that if you make ridiculous experiments you get ridiculous results. Wait -- Aha! I see. The side-spokes DO have a purpose. They keep the wheel from collapsing. They keep it frombulging out sideways!

AUTHORS: How?

CRITIC: (Thinking for longer-than-usual before opening his mouth.) Say, I think I really understand this. It's not really important for them to attach to the hub. The important thing is that they attach TO EACH OTHER to connect the opposite sides of the wheel together, so it can't bulge out! They might as well be made of one wire going straight across. ILLUSTRATION

AUTHORS: So, as far as the side-spokes are concerned, the hub is just a handy place for attaching them together.

CRITIC: Eactly. Well, this also keeps the bicycle from swinging back-and-forth while it hangs from the top. Say! This really IS amazing. The spokes and the hub are doing all sorts of things at the same time. And while the wheel is rolling, each spoke keeps changing what it does!

CRITIC: I don't know what this will do to my mind. I look at a "simple wheel," now I see a very complicated machine. I used to think my typewriter was complicated, but now a wheel looks even worse. How will I ever be able to face a bicycle again? I'm even afraid to think about a spring, or a pencil, or anything! How can a scientist STAND understanding things so well.

AUTHORS: Well it's really not so bad. The secret is (whispers) you only have to really understand it once. You don't have to understand it all EVERY TIME!

(They both think about this for a long time.)

CRITIC: I have a question. When you asked me how the spokes hold up the bicycle, I said that the spokes at the sides can't help because they only pull sideways so they can't lift. But if spoke-wire is as strong

as you say, couldn't they help a little. As soon as you put some weight on the hub, the top spokes surely stretch a little, and then the side spokes will start to help.

AUTHORS: You are right, in a way. They will indeed start to help a little. But, we think, incredibly little. Let's try to find out whether this is actually a significant effect. How much do you suppose a bicycle hub goes down when you get on the bike?

CRITIC: Boy, not very much. You certainly can't see it. It must be less than an eighth of an inch or we'd probably notice it move.

AUTHORS: O.K., a bike wheel is about 24 inches across, and let's suppose the hub sinks $1/8$ inch. That's about one part in a hundred. Now here's my plan: let's look at another version -- suppose we tie a wire tightly from one house to another, a hundred feet away.

ILLUSTRATION See if you can move the knot in the center, first across, and then down.

CRITIC: I'll try it, but I know the answer already. I'll be able to pull it down a couple of feet without any trouble, but I couldn't move it even a single inch across.

AUTHORS: Right. Why is there such a difference?

CRITIC: Isn't it obvious? It's much easier to bend a wire than to stretch a wire. To move it across, you have to stretch the wire and that's hard. To pull it up you only have to bend it and that's easy.

CRITIC: Wait. Doesn't the rope have to stretch some even in the "up" motion? ILLUSTRATION

CRITIC: Help. Now I'm not sure. It certainly has to stretch -- because it has to go more distance. I wish I knew how much it has to stretch. You're mathematicians -- please solve it for me.

AUTHORS: Well, just this once. It certainly has to stretch some. Let's see how much (calculates for a minute) -- (mumbles . . . "up one foot, in 50 feet . . . similar triangles . . . so it lengthens $1/50$ of 1 foot . . . about $1/4$ inch in each half. . .) ILLUSTRATION Well it only stretches $1/2$ inch altogether. That's less than half of a thousandth of the wire's length! One hundred feet is 1,200 inches. It's one part in 2,400.

CRITIC: You're kidding. We pulled it a whole foot. That's one-hundredth of the length.

AUTHORS: But it simply didn't have to stretch that much. Here. Let's draw the picture accurately! ILLUSTRATION

CRITIC: Amazing. The height is $1/50$ of the length, but the stretch is only about $1/50$ of the HEIGHT! So it's $1/2,500$ of the length.

CRITIC: But something about this still bothers me. I see that we only have to stretch the wire one-half inch. But even in 100 feet, it ought to be hard to stretch this wire to be $1/2$ inch longer. It looks pretty strong.

AUTHORS: This is bicycle-spoke wire! It's about $1/12$ th of an inch thick (.080) and it's very strong indeed. If we used the same steel to make a wire an inch thick, it would hold 300,000 pounds without breaking. This little wire can hold (mumbles again) about $(3 \times 10 \text{ to the } 5) \times (4 \times 10 \text{ to the } 2) \pi = 12 \text{ dot } \pi \text{ dot } 10 = 400 \text{ lbs.}$

CRITIC: Well, you haven't answered my question. How hard should it be to pull it $1/2$ " longer. AUTHORS: (Looks up "piano wire" in his Handbook of Chemistry & Physics, in the section on "Elastic Constants of Solids".) Well, I don't know exactly which alloy of steel this is, but I do know that spoke wire is one of the strongest, and according to this, the strongest steel wires are (mumbling again. . .) elastic constant is thirty million, per square inch. Its area . . . oh, we just figured that out -- about one two-hundredth of a square inch and we're stretching it one twenty-five hundredth of its length. . . (he scribbles: 30 times 30 to the sixth over 200 times $2,500 = 60$) about sixty pounds. Anyway, that shows it won't break. CRITIC: Great. That proves that either your theory is wrong or your Handbook, or both! Didn't you notice that I pulled the wire down with one finger? Sixty pounds my foot.

AUTHORS: My theory's O.K. You forgot one thing. When you stretched the wire one-half inch, with one finger, how much did your finger move?

CRITIC: One foot, of course. (Thinks) But. . . you mean? Good Grief! You don't mean. . . there's another lever here? The wire is stretched one-half inch. I can't see it. Where is it?

STONEY: That means he only pulled on it with two and a half pounds. But I want to know where the lever is, too.

AUTHORS: It's the wheel-bump principle again! Look at it sideways: ILLUSTRATION If you imagine a hundred-foot wheel rolling over a quarter-inch bump, you get a "mechanical advantage" of fifty!

CRITIC: Incredible. I'm having some trouble seeing how to use your diagram, but I think I get the general idea. So to pull along the wire with 60 pounds, I only have to pull DOWN with a little more than 1 pound.

AUTHORS: Exactly. Except it's more like two pounds because we have to push two "lever-wheels". Or is it one-half pound? (Starts mumbling again. . .)

CRITIC: Oh, never mind, for heaven's sake. The important thing is that it's a very small force. Who cares about a factor of two, or one-half. A pound or two must be right, because that's what it felt like when my finger plied it.

AUTHORS: Anyway, I'm actually a bit surprised to find the side-spokes even contributing a pound of support.

STONEY: We don't really know that, because we don't really know how much the hub sags.

But if each spoke near the top holds less than 60 pounds -- and I'm sure of that -- we know that the side spokes can't contribute more than a few percent of the support.

AUTHORS: Now let's go back to spokes. Now can you think of another reason why he wheels are made like B and not like A? ILLUSTRATION of A and B wheels.

CRITIC: I certainly do, now. If you stepped very hard on the pedal, you'd get many thousands of pounds of stretching force on the spokes.

AUTHORS: That's right. In fact, even with the super-strong steel the bicycle people use for spokes, they'd just stretch until it turned into an "along" wheel! And when you stopped pedalling --

CRITIC: Then they'd be stretched so badly the whole wheel would be loose. All the weight would hang on the top of the rim -- the side-spokes would let the wheel bulge --

AUTHORS: And the whole thing would probably crumple as soon as you hit your first bad bump.

DAY FIVE: A BIG WHEEL

AUTHORS: In the picture you see a big wall, a door in the wall and a man called Mr. A. looking through the door. To be quite sure you see about where Mr. A. is and when the wall is and where the door is I have drawn it all from three points of view. CRITIC: Sure, What are you fussing about . . . there's a building with a small door and a guy behind the wall looking through the door. What's it got to do with a wheel?

AUTHORS: Good argument. So let's bring in a wheel. In front of the wall a wheel is rolling along. Mr. A can't see it because its close to the wall and behind it.

CRITIC: So I see. After a while the wheel will get to the door and Mr. A will see it.

AUTHORS: You sure are bright today! So we'll give you a real problem. Draw what the wheel will look like when Mr. A first sees it framed in the door.

ILLUSTRATION Caption: This is a door without a wheel. Draw in the wheel as it first appears.

CRITIC: Easy. Like this: ILLUSTRATION Or rather first like this ILLUSTRATION then ILLUSTRATION then ILLUSTRATION, etc.

AUTHORS: genius! So let's do another one.

CRITIC: Easier or harder?

AUTHORS: You tell us afterwards. In some ways much easier, in some ways much harder.

CRITIC: O.K. I'm listening.

AUTHORS: Some problem except the wheel is 100 miles high.

CRITIC: How high?

AUTHORS: A hundred miles.

CRITIC: Well, he'll see the wheel over the top of the wall.

AUTHORS: Just a bug! Make the wall 200 miles high and a thousand long.

CRITIC: How about this: ILLUSTRATION Caption: edge of wheel

AUTHORS: Well then -- show us when the wheel goes behind the wall.

CRITIC: does ILLUSTRATION Caption: like this going high up.

AUTHORS: But when does it go at the bottom? Here. CRITIC: It can't go under the ground so that's wrong. (yelling) I DON'T MIND BEING WRONG. STOP STARING AT ME. O.K. . . . I'll have another shot. How's this* ILLUSTRATION

AUTHORS: We won't say if it's right or wrong. We'll just ask you one question; if you take a six foot long piece of a hundred mile wheel, what will it look like?

CRITIC: I guess it'll be pretty straight. Gee. . . so I have another bug. It's got to be straight. So it's like this: ILLUSTRATION

AUTHORS: Progress. . . but . . .

CRITIC: HOLD IT. Don't tell me. Don't even ask me. I know if I try to dot in the part of the wheel behind the wall it hits the ground too soon: ILLUSTRATION So it's flatter. Like this: ILLUSTRATION

CRITIC: (Excited) But that still hits the ground too soon. My God! If it's a hundred mile wheel it must hit the ground blankd miles away from the door. So it's got to be very flat! Now!! Like this: ILLUSTRATION

You can't even see it's not completely flat!! So the wheel comes in like this: ILLUSTRATIONS Like a curtain coming down!!!

AUTHORS: You get our prize. But we still have a question.

CRITIC: Oh no. . . not another bug in my so beautiful theory. . . well, what is it?

AUTHORS: What if the wehel came from the other direction? How would it look?

CRITIC: So there is a bug. My theory makes the wheel look just the same if it's rolling from left to right or right to left. That can't be! There must be something wrong. But what is it?

AUTHORS: We don't know.

CRITIC: (Mumbling) Left, right, up, down, curtain right left. . . BUT NO, there's nothing wrong. Here's an analogy. A man steps on a bug: ILLUSTRATION Does the bug know what way the man is walking? Does it care? All it sees is the foot coming down -- pretty well straight down. ILLUSTRATION Same thing if a tiny bug gets run over by a big truck. He just sees the black rubber coming straight down like the sky is falling on his head. He doesn't know if the truck is coming or going.

DAY 6: ACHILLES AND THE TRACTOR

CRITIC: We've just reinvented the wheel.

AUTHORS: What?

CRITIC: We've proved the wheel is impossible!

AUTHORS: But. . . they exist.

CRITIC: Illusion -- pure illusion . . .

AUTHORS: (Pulling themselves together) Well, let's hear your "proof".

CRITIC: It's very mathematical.

AUTHORS: I don't doubt it!

CRITIC: Do you admit that if a wheel existed it would touch the ground at just one point?

AUTHORS: They do exist and they touch the ground at just one point . . . that is to say if the ground is really flat.

CRITIC: Please control yourself. Can't you follow a HYPOTHETICAL REASONING.

AUTHORS: O.K. The wheel WOULD touch the ground at just one point if it EXISTED.

CRITIC: Let's call that point X. Now what can you say about X?

AUTHORS: It's the third letter from the end of the alphabet. . .

CRITIC: No silly. . . the point named X not the letter X.

AUTHORS: Well, what should we say? You told us lots of things . . . like it stands still . . . sorry, like it WOULD stand still IF a wheel COULD exist.

CRITIC: Also it supports the WHOLE weight of the wheel.

AUTHORS: That's pretty good of it.

CRITIC: But that's the ABSURDUM of my proof. One point COULDN'T support all that weight. It would have an infinite force on it and would break.

AUTHORS: Yeah. . . that's puzzling. . . but it does it for only one INSTANT of time. So I guess some other point becomes X before X gets a

chance to break. So you see. . . I'VE RE-RE-DISCOVDRED the wheel.

CRITIC: But what if the wheel sinot rollingAUTHORS: Well. . . in that case. . . well we need another way out . . .

CRITIC: The trouble with you is that you are always looking for PRACTICAL ways out. You hav enever understood the spirit of PH ILOSOPHY, of PURE REASON. Now take Zeno, for example, he woulreally appreciate my proof instead of looking for a way out of it. . .

AUTHORS: Who is Zeno?

CRITIC: He's a Greek ho proved that you can never get to your destination because before you did so you'd have to go half-way and before you did that you'd have to go half-way there and so on. . .

AUTHORS: What doyou mean . . . "and soWhat about it?

CRITIC: Don't you see you'd have to do an infinite number of things.

AUTHORS: Wow. Tah's great. We never realized we could do an infinitenumber of things.

CRITIC: No, it's not! You can't. That's just Zeno's argument. NO ONE can do an infinite number of things. But you MUST do so in order to move. So you can't move.

AUTHORS: Maybe YOU can't move. We can.

CRITIC: You really don't understand. It's not a practical argument. It's a PARADOX. Just like my argument about wheels. Of course, I KNOW they do exist. I was just being pPH ILOSOPHICAL when I proved they don't.

AUTHORS: While you were spouting all that stuff about philosophers we just realized that you really have somethng. Wheels DON'T really exist!

CRITIC: If they don't exist, what do they do?

AUTHORS: Roll, man, roll. that's what they do. But seriously the really perfect wheels can't exist.

CRITIC: Why not?

AUTHORS: Well you told us why not! Because there would be an infinite force on the foot of the wheel. But instead of breaking the wheel goes a little flat. So it really looks like this:

ILLUSTRATION

CRITIC: What if it's made of something really rigid?

AUTHORS: Why don't you philosophers take yourselves more seriously? Your own argument proves that its got to go a little flat. Just a tiny little.

CRITIC: How litt?

AUTHORS: You'll have to peep in the vignetrte box for that. Meantime I realized that if you wanted to put a very big load on the wheel maybe you would deliberately make it very flat. Maybe like this: ILLUSTRATION Is that possible? How could you do it?

CRITIC: You can't do it. It wouldn't be a wheel. More like a caterpillar tractor . . . like a tank. . . That's not a wheel.

AUTHORS: You know sometimes you aren't as dumb as you seem to be! You've just made usunderstand tractors. All wheels are a bit of a tractor and really all tractors are like wheels. It's the same phenomenon. . .

CRITIC: What about that business of the foot of the wheel standing still. No part of a tank stands still!

AUTHORS: Do you think that the middle of the bottom part of the caterpillar track is moving?

ILLUSTRATION

Is this part moving on this? What about this?

CRITIC: Well.

IS THIS MATHEMATICS? OR PHYSICS? OR WHAT?

CRITIC: I'd like to ask you a question.

AUTHORS: Sure. Go ahead.

CRITIC: What is it we've been doing in all these conversations?

ZUTHORS: What do you mean by "what are we doing?" . . . talking. . . trying to understand about wheels . . . thinking. . .

CRITIC: But I mean, what do we CALL what we are talking about . . . is this physics or math or engineering or philosophy. . . I can't quite place it.

AUTHORS: Sure! not. But don't blame US if people chop up knowledge into such chunks. Blame the schools or the book-publishers or the professors' trade unions. We're just trying to understand something.

CRITIC: Do you mean that we don't need to divide up knowledge into subjects? But then everyone would have to know everything!

AUTHORS: That would be great, if one could know everything! Unfortunately it seems to be impossible so we do need people with special knowledge. For example, when we were wondering about how strong spokes and fly-wheels could be we asked an expert on strengths of materials. But we still think that everyone should know how to get to understand something like the wheel. So it doesn't belong to anyone's speciality. But why did you ask?

CRITIC: I nearly forgot why I asked. The reason was very specific. I got the feeling that you were hiding something from me. For example you guys know how to calculate exactly how big a pebble can be squeezed under a wheel. . . well, I'd like to be able to know the mathematics as well as the talking part of all this. So I'd like you to lend me the formula or equation or whatever you use.

AUTHORS: So you think mathematics is formulas and equations. We think mathematics is a way of understanding phenomena. If you understand the ideas properly you can figure out the formulas for yourself. . .

CRITIC: STOP. I'm almost sorry I asked. You are talking just like my old math teachers at school. Whenever you ask them how to do something, instead of just telling you they try to make you "understand." So please. . . won't you just tell me the formula? If you like I'll do a trade: you tell me the formula and afterwards I'll let you make me understand it.

AUTHORS: You win. Here's the formula: HEIGHT = (DISTANCE) to the two divided by DIAMETER

CRITIC: That's a funny formula. I expected to see x's and y's instead of words.

AUTHORS: Well, if you like you could call it $h = d^2/D$. Sometimes we think it like that ourselves, but we prefer mnemonic names to short one letter symbols.

CRITIC: But you didn't tell me how to use this formula? Heights of what?

AUTHORS: We thought you didn't want to understand it!

CRITIC: Come on! You know what I mean.

AUTHORS: Sure, critic. Let's draw a picture of a . . .

CRITIC: What! What else?

AUTHORS: A wheel, a road and a stone ILLUSTRATION The stone is wedged under the wheel. If you know how close to the foot it can get (that's distance) you can figure out how high the stone is (that's HEIGHT). . . if you also know the size of the wheel (that's DIAMETER = 100) example the diameter is 100 cm, the stone jams when it's 10 cms fro the foot of the wheel (50 DISTANCE = 10). How high is the stone? HEIGHT = (DISTANCE)² divided by DIAMETER = 10 to the 2 divided by 100 = 100 divided by 100 = 1

CRITIC: You must be teasing me. I could have measured the stone without the wheel!

AUTHORS: Well, you can use the formula in reverse. Let's suppose that you measured the stone and found it was 1/2 cm high. How far from the stone is the foot of the wheel when the wheel first touches the stone? Easy: HEIGHT = 1/2 DISTANCE IS UNKNOWN.

DIAMETER is still 100. The formula is still: HEIGHT = (DISTANCE)² divided by DIAMETER. So $1/2 = (\text{DISTANCE})^2 / 100$. So (DISTANCE)² must be $1/2 \times 100$ which is 50. So DISTANCE must be quotient 50, which is about 7.

CRITIC: While I was watching you from across the table i realized that if I draw your picture upside down I can use your formula for something else, much more interesting than wheels and pebbles.

AUTHORS (Sceptically): Oh yes. . . what?

CRITIC: ILLUSTRATION Now the circle is the EARTH instead of being a wheel and the blsaack blob is a mountain and the road is a line of sight and the question is how far away can I see a three mile high mountain?

AUTHORS (Impressed): GEE. . . the problems are isomorphic.

CRITIC (Ignoring the funny word): . . . it's just exactly the same problem so the same formula should work: HEIGHT is 3, I don't know distance but I know $3 = (\text{DISTANCE})^2$ divided by 100 so $\text{DISTANCE}^2 = 300$ so $\text{DISTANCE} = \text{quotient } 300$ so that's about 17. Seventeen miles . . . something must be wrong. I could see it from much farther away than that. . . unless there was a tree in the way or a fog.

AUTHORS: You forgot to. . .

CRITIC: You keep quiet. I'll figure it out, . . . there's the bug, I forgot to change DIAMETER. It's not 100, it ought to be the diameter of the earth. . . that's about 8,000 miles. So let's try again: $3 = (\text{DISTANCE})^2$ divided by 8,000 so $(\text{DISTANCE})^2$ to the 2 = 24,000. So $\text{DISTANCE} = \text{quotient } 24,000$. HELP! How do I calculate that?

AUTHORS: Lots of ways. Use a table. . . or just an approximation like: quotient 24,000 is quotient 24 X quotient 1,000 and quotient 24 is about 5 and quotient 1,000 is about 50. So that's 5X30 or 150. You can see your mountain from 150 miles away. (Next morning)

CRITIC: Hey you guys. I've found out how to use your formula for lots of stuff besides wheels and horizons. It's really the most useful formula I ever knew.

AUTHORS: (Patronizingly) We'd really love to see your new results.

CRITIC: Did you ever have a problem like this: you are going from city A to city B and you want to go via city C. How much further is it? Do you know you can use your formula to calculate that?

ILLUSTRATION Caption: The Straight Line Distance from A to B is 1,000 miles. C is 20 miles off the route. How far is the trip A to C to B?

CRITIC: The funny thing that has happened to me is that I can see circles all over the place. (Just like you were saying about seeing levers. . . page. . .) Like here! You probably don't even see a circle and a tangent in this situation. But I do:

ILLUSTRATION. Caption: This circle includes all the places exactly 500 miles from A. C is not on the circle. But it is on the tangent. About twenty miles from the foot F. So how far is C from C'. Look, it's just like our rock pictures. So use the formula 20 to the 2 over $1,000 = 400$ over $1,000 < 1/2$. So C is not even half a mile further than C'. Can you believe it? To go from A to B via C adds less than a mile to the whole journey.

CRITIC: But that's not the most amazing thing. I think it is much more amazing that the same ideas work for wheels and for navigation. That is

really fantastic.

AUTHORS: But the most amazing thing of all is that you worked it out. We'd call what you did true mathematics. Congratulations. Let's celebrate the winning of a new soul -- a critic who has become a mathematician. Next Day.

CRITIC: (In a rage) You bastards. You thought you could make a fool of me with that false formula.

AUTHORS: It's not false. . . it's a . . .

CRITIC: It's false. Look at this

ILLUSTRATION Caption Distance = $\pi/2$ diameter = 2. According to Formula: Height = π to the $2/8$. But I measured it and it is. . . blank. I've drawn the figure ten times. . . I'm quite sure of myself.

AUTHORS: You are quite right. . . and we should have warned you that the formula is APPROXIMATE and really very good only when D is much smaller than the diameter. Here's a table for Diameter = 100. ILLUSTRATION

CRITIC: I see . . . I nearly said that a formula is no good if it's not exactly right. But this one is pretty good for all the problems we've been doing. My whole idea of mathematics has changed. I thought it was a dull business of being exactly right about numbers now I see it's truly exciting aspect; it gives you new ways to see and think about the world.

AUTHORS: Instead of getting to be so lyrical about mathematics let's just put together a list of situations where we see the mathematical phenomenon common to the wheels, horizons and trips we've been discussing.

CRITIC: Mathematical phenomenon. . . what's that mean?

AUTHORS: That's what we like to call the kind of theory that's in all these situations. And more . . . For example think for a minute about how a guitar string can vibrate.

CRITIC: Because the player pulled it aside. . .

AUTHORS: But don't you think it is remarkable that it can move far enough sideways. After all it's a taut, stretched string. ILLUSTRATION To move sideways it has to get larger . . . it must stretch. . .

CRITIC: Wow. It's my trip principle again. The middle of the string can move sideways a whole lot without stretching the string very much at all . . . And all that story about the bicycle spikes . . . and the taut string stretched between the houses. . . and how hard it is to fix an object with one stretched string . . . wow, that is a PHENOMENAL

PHENOMENON.

AUTHORS: You haven't yet quite made your full conversion from critic . . .
BLANK COMMAND (38,6) (38,3)
you are still being a pure mathematician.

CRITIC: You mean I should want to USE tyhe phenomenon. Well, I was actually thinking about that. It seems to me you should be able to use it to make a new kind of super-lever device, something like the claw hammer with a very huge mechanical advantage. Look at this, for example. ILLUSTRATION. Caption: All circles are free pivots. You push down on A. A lot of movement of A produces a very small movement of C. Too complicated. Let's fix C to simplicity. So if you push A up and down B moves from side to side -- but a much smaller distance, so it must have a much bigger force so let's put a vice head at B.

ILLUSTRATION

AUTHORS: Good CRITIC, we are proud of you. You are a true engineer as well as an applied mathematician. Of course the thing has already been invented. . . . ILLUSTRATION

CRITIC: I've got to find something new. Let's try it the other way. . .
BLANK COMMAND (38,24) (38,21)

we'll use the phenomenon to produce a lot of movement . . . a lot of speed, rather than a lot of force. So go back to the simple picture. ILLUSTRATION. Caption: Ball here at a speed of 10 ft/sec. And this will make a movement here at a speed of 1,000 ft/sec. What a phenomenal acceleration!

STONEY: (Appearing quietly with his bow and arrow) Well I see the authors really got you. They started re-inventing the wheel and you've ended re-inventing the bow and arrow.

notes

AXLES, BEARINGS, AND LUBRICATION.

STONEY: While you guys were carrying on about making me lift 3,000 lbs -- just listening was enough to make my arm ache -- I was thinking of another way to see a wheel as a lever.

CRITIC: Lifting 3,000 lbs with a lever doesn't make your arm ache any more than lifting 100 lbs without the lever.,.

STONEY: Well that's bad enough. You couldn't do it at all!

AUTHORS: Quiet critic! Let's hear about Stoney's lever.

STONEY: Here's my picture. Can you see my lever?

ILLUSTRATION

When I pull the cart the ground makes the wheel turn. But it really just pushes on the rim, and the rim pushes on the spokes and the spokes turn the hub. Now I think the spoke is a lever.

ILLUSTRATION Caption: Hub, with friction for drag. How Stoney thinks of a spoke as a lever.

With a heavy rock and not-so-good grease, the frictional drag at the hub might be quite big. Imagine that you have to turn the hub by pushing on the spoke. Where is it better to push? At the red arrow, at the black arrow or at the blue arrow?

You get more leverage at the blue arrow. Less force will be needed.

If this is true when you push on ONE spoke it is true when the rim of the cart pushes on all the spokes. So less force is needed if the spokes are long. So a cart with big wheels needs less force to pull it than a cart with tiny wheels. ESSAY: So to reduce frictional drag, it pays to use small axles. A large wheel will spin on a very small axle even without any oil at all! In delicate instruments and machines, like watches, the bearing axles are usually very tiny -- almost points, and this makes the frictional drag very small. Most people do not know much about frictional drag, actually. For example, do you know the answer to this question* if we use the same materials, but in different sizes,

which box will be harder to drag? ILLUSTRATION Caption: wood floor
(large area rubbing on floor) (Small area rubbing on floor).

The answer is that the force is about the same for both. There is more PRESSURE on each square inch of the smaller sliding surface, so there is more frictional drag on each square inch, but it has fewer square inches -- and the two effects balance out.

We mention this because some people might think that the smaller axle is better just because it has less area -- and therefore less place for frictional drag. But in fact this is NOT the reason. The small axle and the larger axle both have the SAME FRICTIONAL DRAG force! But the better leverage of the small axle means that it takes less force at the RIM of the wheel to overcome that frictional drag. (Another way to look at it is that the smaller hub has a smaller distance to move, at the same force. Since WORK is proportional to both force and distance, and the smaller hub moves less distance, the smaller hub requires less work for each turn of the wheel.)

How does Grease help?

CRITIC: It makes the surface slippery.

ILLUSTRATION

Fluid layer -- is easy to shear. why don't surfaces break through?

Puzzles about Points Moving Backwards.

1. (Well-known) A train-wheel flange moves backward.
2. (Little Known) A wheel-foot moves back when wheel goes over a bump.
3. Spool of Thread. ILLUSTRATION.
4. Bike Pedal ILLUSTRATION. Caption: which way?

GEARS AS LEVERS

ILLUSTRATION Caption: This is the clever way to get around Stoney's "telephone pole."

GEARS AS WELL-PACKED (RE-USABLE) LEVER. 100/1! ILLUSTRATION.

BREAKING A STICK.

ILLUSTRATION. The stick breaks at the point of largest leverage -- very much as though there were a claw hammer inside it!

bow and arrow

ILLUSTRATION Very high speed just before arrow leaves string!

SUPERWHEELS. The Caterpillar-tractor and other variants. Can you invent a super-bike that can go over curbs with hardly a bump?

How Inflated Tires work as Super Wheels! Absorbing small bumps by "absorption." The contact area as a "mechanical advantage!" Because of constant pressure!!

HOW MANY SPOKES ON A BIKE WHEEL? It must seem like a lot. Why?

1. Will you agree you need at least five?

ILLUSTRATION

2. But because of the tangential principle they must go in pairs.

ILLUSTRATION. Caption: So 10 spokes.

3. But a flat wheel is too flimsy. It needs a long hub -- two sets of the above

ILLUSTRATION. Caption: So, 20 spokes.

Real Bikes begin with 8 or 9 in the first set, so end up, usually with 32 or 36.

DYNAMICS.

Perhaps a whole chapter: Penny rolling -- Why doesn't it fall.

Top Spinning -- How it bears on ground.

Gyroscope.

Yo-Yo.

Bicycles have power-steering!

How does it stay up?