- Balance equation for mRNA & protein are typically given by:

$$\dot{m}_i = r_{x,i} u_i - (\mu + \theta_{m,i}) m_i + \lambda_i \quad \ldots \text{mRNA}$$
$$\dot{p}_i = r_{L,i} w_i - (\mu + \theta_{p,i}) p_i \quad \ldots \text{protein}$$

where $r_{x,i}$ = kinetic transcription rate, u_i = transcription control variable, μ = dilution term, $\theta_{m,i} / \theta_{p,i}$ = degradation constant, m_i / p_i = mRNA & protein concentration, & λ_i = background transcription

- To account for dilution term, assuming abstract volume $\mathcal{B}$, material balance of intracellular species is given by

$$\frac{d}{dt} \int_{\mathcal{B}} x_i \, d\mathcal{B} = \int_{\mathcal{B}} (\ldots) \, d\mathcal{B}$$

- Assuming well mixed, we get $\frac{d}{dt}(x_i \mathcal{B}) = (\ldots) \mathcal{B}$

$$\downarrow$$

$$\dot{x}_i = (\ldots) - \underbrace{x_i \mathcal{B}^{-1} \dot{\mathcal{B}}}_{\text{dilution term}}$$

- Assuming $\mathcal{B} = V_L$, $\mathcal{B}^{-1}\dot{\mathcal{B}} = V_L^{-1} \dot{V}_L$, since cell-free system

- However, volume is constant at 15 μL, so $\dot{V}_L = 0$, thus $\mathcal{B}^{-1}\dot{\mathcal{B}} = 0$, which means there is no dilution term in this system. Thus,

$$\dot{m}_i = r_{x,i} u_i - \theta_{m,i} m_i - \lambda_i$$

and
$$\dot{p}_i = r_{L,i} w_i - \theta_{p,i} p_i$$