Fall 2022 *Instructors: Jacob Abernethy, Bhuvesh Kumar, Guanghui Wang*

ECE 8803: Online Decision Making in Machine Learning

Homework 2

Released: Nov 3 *Due: Nov 11, 11:59 pm ET*

General rubric (on mandatory problems) For a problem worth X points, give yourself:

- All X points if you were able to solve all of the sub-parts of the problem correctly, with fully written answers.
- 75% of X points if you got all but one sub-part exactly right, or (for the overall problem) you got the main idea correct, but made a small mistake in the final steps.
- 50% of X points if you got half of the sub-parts right, or (for the overall problem) you got the high level main idea, but were not able to execute it in the correct manner.
- 25% of X points if you got 1/4-th of the sub-parts right, or (for the overall problem) if you had the right first step, but were not able to proceed beyond that.
- 0% of X points if you did not attempt the problem or get any of the main ideas.

Problem 1 (Multiplicative weights = Follow-the-Regularized-Leader). 15 points

(a) There is a unique answer to this question. You do not need to prove that the maximum was 0.5, and the minimum was at $0, 1$: the correct answer suffices.

(b) As mentioned in the solutions, there are multiple valid answers for the function $f(\cdot)$. Any function of the form specified in the solution gets full credit, along with a formal justification.

(c) Same as part (b) for the choice of $f(\cdot)$. You need to argue that the objective function is convex for full credit.

(d) Same as part (a): you can use the visual observation from the figure in part (a) to make this argument. You do not need to show formally that the entropy function is maximized at $p = 0.5$.

(e) For full credit, you need to argue that i) the objective function is convex (showing that the Hessian is positive semi-definite is the easiest way to do this), ii) solve the unconstrained minimum, iii) project it into the set of all probability vectors. You could also solve steps ii) and iii) by taking the Lagrangian if you so chose.

Problem 2 (New algorithms through regularization). 20 points

(a) Like in problem 1 part (a), you do not need to formally argue the identity of the maximum and minimum. The plot with the correct values reported suffices.

(b) The correct plot and correct answer suffices for full credit on this question. There is only one answer.

(c) As specified in the question, you are free to use the starter code provided in the notebook or directly borrow from the code provided in the .ipynb file. Any code that provides the correct implementation gets full credit.

(d) Again, any code that provides the correct implementation and gets the curves provided in the solution gets full credit. Because you are evaluating expected algorithm performance, the solution curves are unique.

(e) The correct answer + justification that shows that you understood the exploitation-randomization principle will get full credit for this question.

(f) For full credit on this question, you need to: i) argue convexity of the objective function, ii) solve for Pet using the quadratic formula, and iii) pick the right solution out of the quadratic formula that gives Pet between 0 and 1.

Problem 3 (Lower bound for online linear optimization (OLO)). 15 points

(a) There is a unique answer to this question following the hint.

(b) Either the geometric or algebraic solution that was provided gets full credit.

(c) The answer to this question is unique and combines parts (b) and (a).

(d) This was a challenge question! If you got it correct, pat yourself on the back. If you couldn't figure it out, don't feel bad. The provided answer is the simplest solution. If you have an alternative solution that you believe could work, please come see me at OH or send me and GTA Guanghui Wang a Piazza private post with your solution. We are happy to look through it and assist.

Problem 4 (Online gradient descent with a time-varying, data-dependent learning rate) 30 points

(a) The correct answer suffices for full credit: justification is not needed.

(b) The provided solution is the simplest approach; alternative algebra is fine as long as each of the individual steps is correct and is rigorously justified.

(c) The provided solution is the simplest approach; alternative algebra is fine as long as each of the individual steps is correct and is rigorously justified. Your solution should use the first-order definition of convexity in one of the steps.

(d) The provided solution is unique. While the hint suggested working out the case T = 3 first to get intuition, this is optional, not required.

(e) The provided solution is unique. Constants that are higher than in the solution provided are fine and suffice for full credit.

(f) The correct proof for the regret bound together with the correct answer to the posed question (about whether the bound is better or worse) suffices for full credit.