Zad 5.

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Zadanie 5. Oblicz transformaty Laplace'a funkcji :

a) t^n,

b) $t^n e^{at}$,

c) $t \sin at$,

d) $t^2 \cos at$,

e) $t^k e^{at} \cos bt$,

f) $t^k e^{at} \sin bt$,

g) $\frac{\sin t}{t}$,

h) $\frac{\cos at - 1}{t}$,

i) $\frac{e^{at} - e^{bt}}{t}$.

g) $y = \frac{\sin t}{t}$ $\quad \mathcal{L}\{\frac{\sin t}{t}\}(s) = \int_s^\infty \mathcal{L}\{\sin t\}(u)\, du =$

$$= \int_s^\infty \frac{1}{u^2+1}\, du = \operatorname{arctg}(u) \Big|_{u=s}^{\infty} = \frac{\pi}{2} - \operatorname{arctg}(s)$$

a) $y = t^n$ $\quad \mathcal{L}\{1\}(s) = \int_0^\infty e^{-st} \cdot 1\, dt = \frac{1}{s}$

$$\mathcal{L}\{t^n \cdot f\}(s) = \left[\frac{d^n}{ds^n} \mathcal{L}\{f\}(s)\right](-1)^n$$

$$\mathcal{L}\{t^n\}(s) = \left[\frac{d^n}{ds^n} \mathcal{L}\{1\}(s)\right](-1)^n = \frac{d^n}{ds^n} \frac{1}{s} \cdot (-1)^n =$$

$$= \frac{n!}{s^{n+1}}$$

b) podobnie

$$\mathcal{L}\{e^{at}\}(s) = \frac{1}{s-a}$$

$$\mathcal{L}\{t^n e^{at}\}(s) = \frac{n!}{(s-a)^{n+1}}$$

c) $\mathcal{L}\{t \sin at\}(s) = ?$

$$\mathcal{L}\{\sin at\}(s) = \frac{a}{s^2+a^2}$$

$$\frac{d}{ds} \frac{a}{s^2+a^2} = -\frac{2as}{(s^2+a^2)^2}$$

$$\mathcal{L}\{t \sin at\}(s) = \frac{2as}{(s^2+a^2)^2}$$

definicja $\mathcal{L}$

$$\mathcal{L}\{f\}(s) = \int_0^\infty e^{-st} f(t)\, dt$$

$$\mathcal{L}\{f\}(s) = \int_0^\infty e^{-st} F(t)\, dt$$

$$|F(t)| \leq ce^{\omega t}$$

wzrost podw.

$$\mathcal{L}\{\sin \omega t\} = \frac{\omega}{s^2+\omega^2}$$

$$\mathcal{L}\{\cos \omega t\} = \frac{s}{s^2+\omega^2}$$

przydatne

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{1\} = \frac{1}{s}$$

również

$$\mathcal{L}\{t^n f\}(s) = \left[\frac{d^n}{ds^n} \mathcal{L}\{f\}(s)\right] (-1)^n$$

$$\mathcal{L}\{\frac{f}{t}\}(s) = \int_s^\infty \mathcal{L}\{f\}(u)\, du$$