Zad 4.

4. Znajdź zwartą postać ciągu a_n określonego wzorem:

$$a_0 = 1, a_1 = 0, a_n = \frac{a_{n-1} + a_{n-2}}{2}$$

$$a_n = \frac{a_{n-1} + a_{n-2}}{2}$$

$$2a_n = a_{n-1} + a_{n-2}$$

$$2a_n - a_{n-1} - a_{n-2} = 0$$

$$2a_{n+2} - a_{n+1} - a_n = 0$$

$$2\langle a_{n+2} \rangle - \langle a_{n+1} \rangle - \langle a_n \rangle = 2E^2 \langle a_n \rangle - E \langle a_n \rangle - \langle a_n \rangle =$$

$$= (2E^2 - E - 1) \langle a_n \rangle = \langle 0 \rangle$$

$$(2E^2 - E - 1) = (E - 1)(E + \tfrac{1}{2})$$

zatem ogólną postacią jest

$$a_n = \alpha 1^n + \beta (-\tfrac{1}{2})^n$$

dla $a_0 = 1, a_1 = 0$

$$\begin{cases} 1 = \alpha + \beta \\ 0 = \alpha - \tfrac{1}{2}\beta \end{cases} \Rightarrow \begin{cases} 1 = \tfrac{3}{2}\beta \\ \beta = \tfrac{2}{3} \\ \alpha = \tfrac{1}{3} \end{cases}$$

zwarta postać: $a_n = \tfrac{1}{3} + \tfrac{2}{3} \cdot (-\tfrac{1}{2})^n$