Zad 8.

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Zadanie 8. Stosując transformatę Laplace'a znajdź rozwiązania następujących zagadnień:

a) $\begin{cases} x' = 12x + 5y, & x(0) = 0, \\ y' = -6x + y, & y(0) = 1; \end{cases}$

b) $\begin{cases} x' = x - y - e^{-t}, & x(0) = 0, \\ y' = 2x + 3y + e^{-t}, & y(0) = 0. \end{cases}$

a) $A = \begin{pmatrix} 12 & 5 \\ -6 & 1 \end{pmatrix}$ bez Laplace'a bo chcę sobie przećwiczyć

$\chi_A(\lambda) = \begin{vmatrix} 12-\lambda & 5 \\ -6 & 1-\lambda \end{vmatrix} = (12-\lambda)(1-\lambda) + 30 = 12 - 13\lambda + \lambda^2 + 30 =$

$= \lambda^2 - 13\lambda + 42$

$\Delta = 169 - 168 = 1$

$\lambda_1 = \frac{13-1}{2} = 6 \quad \lambda_2 = 7$

$\begin{pmatrix} 6 & 5 \\ -6 & -5 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0 \Rightarrow \begin{cases} 6u_1 + 5u_2 = 0 \\ -6u_1 - 5u_2 = 0 \end{cases}$

$u_1 = -\frac{5}{6}u_2 \quad u = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$

$\begin{pmatrix} 5 & 5 \\ -6 & -6 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad v_1 = v_2$

$v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

rozw. ogólne $= c_1 e^{6t}\begin{pmatrix} -5 \\ 6 \end{pmatrix} + c_2 e^{7t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$

$t = 0 \quad \begin{cases} -5c_1 + c_2 = 0 \\ 6c_1 + c_2 = 1 \end{cases}$

$-11c_1 = -1 \quad \begin{cases} c_1 = \frac{1}{11} \\ c_2 = \frac{5}{11} \end{cases}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}e^{6t}\begin{pmatrix} -5 \\ 6 \end{pmatrix} + \frac{5}{11}e^{7t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

b) $\begin{cases} x' = x - y - e^{-t} \end{cases}$ $x(0) = 0$

b) $\begin{cases} x' = x - y - e^{-t} \\ y' = 2x + 3y + e^{-t} \end{cases}$ $\quad x(0) = 0$, $y(0) = 0$

$$\mathcal{L}\{x'\} = s\mathcal{L}\{x\} - x(0)$$

$$\begin{cases} s\mathcal{L}\{x\} - x(0) = \mathcal{L}\{x\} - \mathcal{L}\{y\} - \mathcal{L}\{e^{-t}\} \\ s\mathcal{L}\{y\} = 2\mathcal{L}\{x\} + 3\mathcal{L}\{y\} + \mathcal{L}\{e^{-t}\} \end{cases}$$

$$\begin{cases} \mathcal{L}\{x\} \cdot (s-1) = \mathcal{L}\{y\} - \mathcal{L}\{e^{-t}\} \\ \mathcal{L}\{y\} \cdot (s-3) = 2\mathcal{L}\{x\} + \mathcal{L}\{e^{-t}\} \end{cases}$$

$$\mathcal{L}\{e^{-t}\} = \frac{1}{s+1}$$

$$\mathcal{L}\{x\} = \frac{\mathcal{L}\{y\}}{s-1} - \frac{1}{(s-1)(s+1)}$$

$$\mathcal{L}\{y\} = \frac{2\mathcal{L}\{x\}}{s-3} + \frac{1}{(s+1)(s-3)} =$$

$$= \frac{2\mathcal{L}\{x\} \cdot (s+1) + 1}{(s+1)(s-3)}$$

$$\mathcal{L}\{x\} = \frac{2\mathcal{L}\{x\}(s+1) + 1}{(s+1)(s-3)(s-1)} - \frac{1}{(s-1)(s+1)}$$

$$\mathcal{L}\{x\} = \frac{2\mathcal{L}\{x\}}{(s-3)(s-1)} + \frac{1}{(s+1)(s-3)(s-1)} - \frac{1}{(s-1)(s+1)}$$

$$(s-3)(s-1)\mathcal{L}\{x\} - 2\mathcal{L}\{x\} = \frac{1}{s+1} - \frac{s-3}{s+1}$$

$$\mathcal{L}\{x\}\left[(s-3)(s-1) - 2\right] = \frac{s-2}{s+1}$$

$$\parallel$$

$s^2 - 4s + 1$

$$\mathcal{L}\{x\} = \frac{s-2}{(s+1)(s^2-4s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2-4s+1}$$

$$As^2 - 4As + A + Bs^2 + Bs + Cs + C = s - 2$$

$$s^2(A+B) + s(-4A+B+C) + A + C = s - 2$$

$$\begin{cases} A+B=0 \\ -4A+B+C=1 \\ A+C=-2 \end{cases}$$

$$B = -A = \frac{1}{2}$$

$$-4A - A - 2 - A = 1 \Rightarrow A = -\frac{1}{2}$$

$$C = -2 - A = -\frac{3}{2}$$

$$x = -\frac{1}{2}e^{-t} + \mathcal{L}^{-1}\{\text{gówno aughh arabfunny.mp3}\}$$