Zad 3.

środa, 17 maja 2023 17:15

Zadanie 3. Znajdź rozwiązania ogólne równań

a) $y'' - 2y' + y = \frac{e^t}{t}$, b) $y'' + 4y = 2\,\mathrm{tg}\,t$, c) $y'' - 4y' + 4y = te^{2t}$,

używając metody uzmienniania parametrów.

c) $y'' - 4y' + 4y = 0$

$r^2 - 4r + 4 = 0 \quad (r-2)^2 = 0 \quad r = 2$

$y_1(t) = e^{2t} \quad y_2(t) = te^{2t}$

chcemy zgadnąć rozw. szczególne. spodziewamy się, że będzie postaci $t^n e^{2t}$

$n = 3$, wstawiamy do równania

$y(t) = t^3 e^{2t}$

$y'(t) = 3t^2 e^{2t} + 2t^3 e^{2t}$

$y''(t) = 6te^{2t} + 6t^2 e^{2t} + 6t^2 e^{2t} + 4t^3 e^{2t} =$

$= 12t^2 e^{2t} + 4t^3 e^{2t} + 6te^{2t}$

$y'' - 4y' + 4y = 12t^2 e^{2t} + 4t^3 e^{2t} + 6te^{2t} - 12t^2 e^{2t} - 8t^3 e^{2t}$

$+ 4t^3 e^{2t} = 6te^{2t}$ [6x za dużo ale gites]

$\Downarrow$

$y(t) = \frac{1}{6} t^3 e^{2t} \leftarrow$ rozw. szczególne

ogólne:

$y(t) = c_1 e^{2t} + c_2 e^{2t} \cdot t + \frac{1}{6} t^3 e^{2t}$

a) $y'' - 2y' + y = \frac{e^t}{t} = f(t)$

$y'' - 2y' + y = 0$

$r^2 - 2r + 1 = 0 \quad (r-1)^2 = 0$

$y_1(t) = e^t \quad y_2(t) = te^t$

$y(t) = c_1(t) e^t + c_2(t) \cdot te^t$

$$y(t) = c_1(t)e^t + c_2(t)\cdot te^t$$

$$\begin{pmatrix} e^t & te^t \\ e^t & e^t + te^t \end{pmatrix} \begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{e^t}{t} \end{pmatrix}$$

$$\begin{cases} e^t c_1' + te^t c_2' = 0 \\ e^t c_1' + e^t c_2' + te^t c_2' = \frac{e^t}{t} \end{cases}$$

$$\begin{cases} c_1' + tc_2' = 0 \\ c_1' + c_2' + tc_2' = \frac{1}{t} \end{cases}$$

$$c_2' = \frac{1}{t} \Rightarrow c_2 = \ln t$$

$$c_1' = -t\ln t \quad / \int$$

$$\int c_1' dt = -\int t\ln t \, dt$$

$$c_1(t) = -\int t\cdot \ln t \, dt$$

$$y(t) = \left(-\int t\cdot \ln t \, dt\right)\cdot e^t + \ln t \cdot te^t \quad \ldots ?$$

b) $y'' + 4y = 2tg(t)$

$$y'' + 4y = 0$$

$$r^2 + 4 = 0 \qquad r = \pm 2i$$

$$e^{2it} = \cos(2t) + i\sin(2t)$$

$$y_1(t) = \cos 2t \qquad y_2(t) = \sin(2t)$$

bazowe jednorodnego

$$y(t) = c_1(t)\cos 2t + c_2(t)\sin 2t$$

$$\begin{pmatrix} \cos 2t & \sin 2t \\ -2\sin 2t & 2\cos 2t \end{pmatrix} \begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} 0 \\ 2tg t \end{pmatrix}$$

$$\begin{cases} c_1' \cdot \cos 2t + c_2' \cdot \sin 2t = 0 \\ -2c_1' \sin 2t + 2c_2' \cos 2t = 2\operatorname{tg} t \end{cases}$$

$$\begin{cases} c_2' \cos 2t - c_1' \sin 2t = \operatorname{tg} t \\ c_2' \sin 2t + c_1' \cos 2t = 0 \end{cases}$$

bruh