Zad 2.

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Zadanie 2. Rozwiąż równania liniowe:

a) $y' + y\cos t = 0$,
b) $y' + t^2 y = 1$,
c) $y' + t^2 y = t^2$,
d) $y' + \frac{2t}{1+t^2} y = \frac{1}{1+t^2}$,
e) $y' + y = te^t$.

a) $y' + y\cos t = 0 \quad / \cdot e^{\int \cos t \, dt} = e^{\sin t}$

$$y' e^{\sin t} + e^{\sin t} y \cos t = 0$$

$$(y e^{\sin t})' = 0$$

$$y e^{\sin t} = c$$

$$y = \frac{c}{e^{\sin t}}$$

b) $y' + t^2 y = 1 \quad / \cdot e^{\int t^2 dt} = e^{\frac{t^3}{3}}$

$$\left(y e^{\frac{t^3}{3}}\right)' = e^{\frac{t^3}{3}} \quad / \cdot \int$$

$$y = e^{-\frac{t^3}{3}} \cdot \int e^{\frac{t^3}{3}} dt$$

c) c) $y' + t^2 y = t^2$,

$$y' = t^2(1-y)$$

$$\int \frac{1}{1-y} dy = \int t^2 dt = \frac{t^3}{3} + c$$

$$-\int \frac{1}{y-1} dy = -\ln|y-1| = \frac{t^3}{3} + c \quad / \cdot \exp$$

$$|y-1| = e^{-\frac{t^3}{3}} \cdot c$$

$$y-1 = \pm e^{-\frac{t^3}{3}} \cdot c$$

$$y = 1 \pm e^{-\frac{t^3}{3}} \cdot c$$

$$y = 1 \pm e^{-\frac{t^3}{3}} \cdot c$$

d) d) $y' + \frac{2t}{1+t^2} y = \frac{1}{1+t^2}$,

$$a(t) = -\frac{2t}{1+t^2} \qquad f(t) = \frac{1}{1+t^2}$$

$$y' + \frac{2t}{1+t^2} y = \frac{1}{1+t^2} \quad / \cdot e^{\int \frac{2t}{1+t^2}} = e^{\ln|1+t^2|} = |1+t^2| > 0$$

$$\left(y|1+t^2|\right)' = \frac{|1+t^2|}{1+t^2} = 1 \quad / \int$$

$$y(1+t^2) = t + c$$

$$y = \frac{t+c}{1+t^2}$$

e) e) $y' + y = te^t$.

$$a(t) = -1 \qquad f(t) = te^t$$

$$y' + y = te^t \quad / \cdot e^{\int 1 dt} = e^t$$

$$(ye^t)' = te^{2t} \quad / \int$$

$$ye^t = \int te^{2t} dt = -\int \frac{1}{2} e^{2t} dt + \frac{1}{2} e^{2t} t =$$

$$= -\frac{1}{2} \cdot \frac{1}{2} e^{2t} + \frac{1}{2} e^{2t} t + c =$$

$$= e^{2t}\left(\frac{1}{2} t - \frac{1}{4}\right) + c$$

$$y = e^t\left(\frac{1}{2} t - \frac{1}{4}\right) + \frac{c}{e^t}$$