Zad 7.

środa, 17 maja 2023 21:25

Zadanie 7. Stosując transformatę Laplace'a znajdź rozwiązania następujących zagadnień:

a) $y' - y = te^t$, $y(0) = 0$;

b) $y'' + y = \sin t$, $y(0) = 1$, $y'(0) = 2$;

c) $y'' + y = t\sin t$, $y(0) = 1$, $y'(0) = 2$;

d) $y'' - 5y' + 4y = e^{2t}$, $y(0) = 1$, $y'(0) = -1$.

$$\left.\begin{array}{l}\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0) \\ \mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)\end{array}\right\} \text{ważne}$$

a) $y' - y = te^t$, $y(0) = 0$

$\mathcal{L}\{e^t\} = \frac{1}{s-1}$

$\left(\frac{1}{s-1}\right)' = -\frac{1}{(s-1)^2}$

$$\mathcal{L}\{y'\} - \mathcal{L}\{y\} = \mathcal{L}\{te^t\}$$

$$s\mathcal{L}\{y\} - y(0) - \mathcal{L}\{y\} = \frac{1}{(s-1)^2}$$

$$\mathcal{L}\{y\}(s-1) = \frac{1}{(s-1)^2}$$

$$\mathcal{L}\{y\} = \frac{1}{(s-1)^3} = \left(\frac{1}{(s-1)}\right)^3 = \left(\mathcal{L}\{e^t\}\right)^3 \ldots ?$$

b) $y'' + y = \sin t$ $y(0) = 1$ $y'(0) = 2$

$$\mathcal{L}\{y''\} + \mathcal{L}\{y\} = \mathcal{L}\{\sin t\}$$

$$s^2\mathcal{L}\{y\} - sy(0) - y'(0) + \mathcal{L}\{y\} = \frac{1}{s^2+1}$$

$$\mathcal{L}\{y\}(s^2+1) - s - 2 = \frac{1}{s^2+1}$$

$$\mathcal{L}\{y\} = \frac{1}{(s^2+1)^2} + \frac{s+2}{s^2+1} = \frac{1}{(s^2+1)^2} + \frac{s}{s^2+1} + \frac{2}{s^2+1}$$

$$\mathcal{L}\{y\} = \frac{1}{(s^2+1)^2} + \mathcal{L}\{\cos t\} + \mathcal{L}\{2\sin t\}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+1)^2}\right\} = \mathcal{L}^{-1}\left\{\mathcal{L}\{\sin t\} \cdot \mathcal{L}\{\sin t\}\right\} \overset{8.6}{=}$$

$$\int_0^t \sin(t-v)\sin v \, dv = \frac{1}{2}\int_0^t \cos(t-2v) - \cos t \, dv$$

$$\ldots\left(\frac{1}{(s^2+1)^2}\right) \ldots$$

$$= \int_0^t \sin(t-v)\sin v \, dv = \frac{1}{2}\int_0^t \cos(t-2v) - \cos t \, dv$$

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$$= \ldots = \frac{1}{2}(\sin t - t\cos t)$$

$$y(t) = \cos t + 2\sin t + \frac{1}{2}(\sin t - t\cos t)$$