Zad 13.

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Zadanie 13. Rozwiąż równania

a) $2ty\,\mathrm{d}t + (t^2 - y^2)\,\mathrm{d}y = 0$,

b) $e^{-y}\,\mathrm{d}t - (2y + te^{-y})\,\mathrm{d}y = 0$,

c) $\left(t - y\cos\frac{y}{t}\right)\,\mathrm{d}t + t\cos\frac{y}{t}\,\mathrm{d}y = 0$.

a) $\underbrace{2ty}_{M} + \underbrace{(t^2 - y^2)}_{N} y' = 0$

$$\frac{\partial\varphi}{\partial t} = 2ty \qquad \frac{\partial\varphi}{\partial y} = t^2 - y^2$$

$$\int 2ty\,dt = yt^2 + c_1(y) = \varphi(t,y)$$

$$\int t^2 - y^2\,dy = t^2 y - \frac{y^3}{3} + c_2(t) = \varphi(t,y)$$

$$\frac{\partial\varphi}{\partial y} = t^2 - y^2 = \frac{\partial\varphi}{\partial y}\left(yt^2 + c_1(y)\right) =$$

$$= t^2 + c_1'(y)$$

$$c_1'(y) = y^2$$

$$c_1(y) = \frac{y^3}{3} + C$$

$$\varphi(t,y) = yt^2 + \frac{y^3}{3} + C$$

czyli $\left(yt^2 + \frac{1}{3}y^3 + c\right)' = 0$

$$yt^2 + \frac{1}{3}y^3 = \tilde{C}$$

b) $e^{-y}\,\mathrm{d}t - (2y + te^{-y})\,\mathrm{d}y = 0$,

$$\frac{\partial\varphi}{\partial t} = e^{-y}$$

$$\int e^{-y}\,dt = e^{-y}t + c(y) = \varphi(t,y)$$

$$\frac{\partial\varphi}{\partial y} \ldots \frac{\partial}{\partial y}\left(e^{-y}t + c(y)\right) =$$

$$\int e^{-y}\,dt = e^{-y}t + c(y) = \varphi(t,y)$$

$$\frac{\partial \varphi}{\partial y} = -(2y + te^{-y}) = \frac{\partial}{\partial y}\left(e^{-y}t + c(y)\right) =$$

$$= -e^{-y}t + c'(y)$$

$$c'(y) = -2y$$

$$c(y) = -\int 2y\,dy = -y^2$$

$$\varphi(t,y) = -e^{-y}\cdot t - y^2$$

$$-e^{-y}t - y^2 = c$$

$$-y\cdot\frac{1}{t^2}$$

c) $\left(t - y\cos\frac{y}{t}\right)\,\mathrm{d}t + t\cos\frac{y}{t}\,\mathrm{d}y = 0.$

$$\frac{\partial \varphi}{\partial t} = t - y\cos\frac{y}{t}$$

$$\int t - y\cos\frac{y}{t}\,dt = \frac{t^2}{2} - \int y\cos\frac{y}{t}\,dt + c(y) =$$

$$= \frac{t^2}{2} - y\underbrace{\int\cos\frac{y}{t}\,dt}_{= I} + c(y) = (*)$$

$$I = \int\cos\left(y\cdot\frac{1}{t}\right)dt = \left|\begin{matrix} u = \frac{y}{t} \\ du = -y\cdot\frac{1}{t^2}\,dt\end{matrix}\right| =$$

$$= -\int\cos u\cdot\frac{t^2}{y} = -\frac{1}{y}\int\cos u\cdot t^2 =$$

$$= -\frac{1}{y}\left[\sin u\, t^2 - 2\int\sin u\, t\right] =$$

$$= -\frac{1}{y}\Big[\sin u\cdot t^2 - 2\big[-\cos u\, t + \underbrace{\int\cos u}_{\sin u}\big]\Big] =$$

$$= -\frac{1}{y}\left[t^2\sin u + 2t\cos u - 2\sin u\right] =$$

$$= -\frac{\sin u\,(t^2-2)}{y} - \frac{2t\cos u}{y}$$

$$(*) = \frac{t^2}{2} + \sin u\,(t^2-2) + 2t\cos u + c(y) =$$

$$= \frac{t^2}{2} + \sin\left(\frac{y}{t}\right)\cdot(t^2-2) + 2t\cos\left(\frac{y}{t}\right) + c(y)$$

$(*) = \frac{t^2}{2} + \ldots$

$$= \frac{t^2}{2} + \sin\left(\frac{y}{t}\right) \cdot (t^2-2) + 2t\cos\left(\frac{y}{t}\right) + c(y)$$

$$\frac{\partial \varphi}{\partial y} = t\cos\left(\frac{y}{t}\right) = \frac{\partial}{\partial y}(*) =$$

$$= \cos\left(\frac{y}{t}\right) \cdot \frac{1}{t}(t^2-2) - 2t\sin\left(\frac{y}{t}\right) \cdot \frac{1}{t} + c'(y) =$$

$$= t\cos\left(\frac{y}{t}\right) - \frac{2}{t}\cos\left(\frac{y}{t}\right) - 2\sin\left(\frac{y}{t}\right) + c'(y)$$

$$c'(y) = \frac{2}{t}\cos\left(\frac{y}{t}\right) + 2\sin\left(\frac{y}{t}\right) \quad / \cdot \int dy$$

$$c(y) = -\frac{2}{t}\sin\frac{y}{t} \cdot t + 2\cos\left(\frac{y}{t}\right) \cdot t + c$$

$$c(y) = 2t\cos\left(\frac{y}{t}\right) \quad (?)$$