Zad 5.

środa, 28 grudnia 2022 17:43

M9.5. $\boxed{\text{1 punkt}}$ Wyznaczyć trzeci wielomian optymalny w sensie normy jednostajnej na zbiorze $\{0, 1, 2, 4, 6\}$ dla funkcji o wartościach

	x_1	x_2	x_3	x_4	x_5
x_k	0	1	2	4	6
$f(x_k)$	1	9	23	93	259

$w_3^* = ax^3 + bx^2 + cx + d$

chcemy, żeby $\{0,1,2,4,6\}$ był alternansem

$$f(x_i) - w_3^*(x_i) = (-1)^i \mathcal{E}$$

$$\begin{cases} 1 - w_3^*(0) = -\mathcal{E} \\ 9 - w_3^*(1) = \mathcal{E} \\ 23 - w_3^*(2) = -\mathcal{E} \\ 93 - w_3^*(4) = \mathcal{E} \\ 259 - w_3^*(6) = -\mathcal{E} \end{cases} \qquad \begin{cases} d - \mathcal{E} = 1 \\ a+b+c+d+\mathcal{E} = 9 \\ 8a+4b+2c+d-\mathcal{E} = 23 \\ 64a+16b+4c+d+\mathcal{E} = 93 \\ 216a+36b+6c+d-\mathcal{E} = 259 \end{cases}$$

mfw

$$\left(\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 9 \\ 8 & 4 & 2 & 1 & -1 & 23 \\ 64 & 16 & 4 & 1 & 1 & 93 \\ 216 & 36 & 6 & 1 & -1 & 259 \end{array}\right) \qquad \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 9 \\ 0 & -4 & -6 & -7 & -9 & -49 \\ 0 & -16 & -12 & -7 & 9 & -91 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & -72 & -48 & -26 & 26 & -362 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 9 \\ 0 & -4 & -6 & -7 & -9 & -49 \\ 0 & 0 & 12 & 21 & 45 & 105 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 60 & 100 & 188 & 520 \end{array}\right) \qquad \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 9 \\ 0 & -4 & -6 & -7 & -9 & -49 \\ 0 & 0 & 12 & 21 & 45 & 105 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -5 & -37 & -5 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 1 & 9 \\ 0 & -4 & -6 & -7 & -9 & -49 \\ 0 & 0 & 12 & 21 & 45 & 105 \\ 0 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -42 & 0 \end{array}\right) \qquad \begin{cases} a+b+c+d+\mathcal{E} = 9 \\ 4b+6c+7d+9\mathcal{E} = 49 \\ 4c+7d+15\mathcal{E} = 35 \\ d - \mathcal{E} = 1 \\ -42\mathcal{E} = 0 \end{cases}$$

$\mathcal{E} = 0$

$d = 1$

$w^*(x) = x^3 + 7x^2 + 1$

$e = 0$

$d = 1$

$4c + 7 = 35 \Rightarrow c = \frac{28}{4} = 7$

$4b + 42 + 7 = 49 \Rightarrow b = 0$

$a + 7 + 1 = 9 \Rightarrow a = 1$

$w_3^*(x) = x^3 + 7x^2 + 1$