Zadanie 1. Rozwiąż równania o rozdzielonych zmiennych:

a) $\sqrt{y^2+1} = tyy'$, b) $ty' + y = y^2$, c) $\sqrt{2y-1} = y'$.

a) $\sqrt{y^2+1} = tyy'$

$$\frac{1}{t} = \frac{y}{\sqrt{y^2+1}} y'$$

$$\int \frac{y}{\sqrt{y^2+1}} \, dy = \ln t + c$$

$$\int \frac{y}{\sqrt{y^2+1}} \, dy = \left| \begin{array}{l} u = y^2+1 \\ du = 2y \, dy \end{array} \right| =$$

$$= \int \frac{y}{\sqrt{u}} \frac{du}{2y} = \frac{1}{2} \cdot \int \frac{1}{\sqrt{u}} \, du =$$

$$= \frac{1}{2}\left(2u^{\frac{1}{2}}\right) = u^{\frac{1}{2}} = (y^2+1)^{\frac{1}{2}} = \ln|t| + c$$

$$y^2 + 1 = (\ln|t| + c)^2$$

$$y = \pm\sqrt{(\ln t + c)^2 - 1}$$

b) $ty' + y = y^2$

$$ty' = y(y-1)$$

$$\int \frac{1}{y(y-1)} \, dy = \ln t + c$$

$$-\frac{1}{y} + \frac{1}{y-1} = \frac{-y+1+y}{y(y-1)}$$

$$\int \frac{1}{y(y-1)} \, dy = \int -\frac{1}{y} + \frac{1}{y-1} \, dy =$$

$$= -\int \frac{1}{y} + \int \frac{1}{y-1} = -\ln y + \ln(y-1) = \ln t + c$$

$$\ln \frac{y-1}{y} = \ln t + c \quad /\exp$$

$$\frac{y-1}{y} = ct$$

$$1 - \frac{1}{y} = ct$$

$$\frac{1}{y} = 1 - ct \implies y = \frac{1}{1-ct}$$

c) $\sqrt{2y-1} = y'$

$$1 = \frac{y'}{\sqrt{2y-1}}$$

$$\int \frac{1}{\sqrt{2y-1}}\, dy = t + c$$

$$\int \frac{1}{\sqrt{2y-1}}\, dy = \left| \begin{array}{c} u = 2y-1 \\ du = 2\, dy \end{array} \right| =$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{u}}\, du = \sqrt{u} = t + c$$

$$u = (t+c)^2$$
$$2y - 1 = (t+c)^2$$
$$y = \frac{1}{2}\left[(t+c)^2 + 1 \right]$$