Zad 8.

poniedziałek, 5 grudnia 2022 22:08

8. Za pomocą metody anihilatorów oblicz $s_n = \sum_{i=1}^{n} i2^i$ rozwiązując zależność $s_n = s_{n-1} + n2^n$.

$$s_n - s_{n-1} = n2^n$$

anihilatorem $\langle n2^n \rangle$ jest $(E-2)^2$

$$s_n - s_{n-1} = 0$$
$$s_{n+1} - s_n = 0$$
$$E\langle s_n \rangle - \langle s_n \rangle = \langle 0 \rangle$$
$$(E-1)\langle s_n \rangle = \langle 0 \rangle$$

mamy $(E-1)(E-2)^2 \langle s_n \rangle = (E-2)^2 \langle n2^n \rangle = \langle 0 \rangle$

zatem $s_n = \alpha 1^n + \beta 2^n + \gamma n 2^n$

$$s_1 = \sum_{i=1}^{1} i2^i = 1 \cdot 2 = 2 = \alpha + 2\beta + 2\gamma$$
$$s_2 = 2 + 2 \cdot 4 = 10 = \alpha + 4\beta + 8\gamma$$
$$s_3 = 2 + 8 + 24 = 34 = \alpha + 8\beta + 24\gamma$$

$$\begin{cases} 2 = \alpha + 2\beta + 2\gamma \\ 10 = \alpha + 4\beta + 8\gamma \\ 34 = \alpha + 8\beta + 24\gamma \end{cases}$$

$$\begin{cases} 8 = 2\beta + 6\gamma \\ 24 = 4\beta + 16\gamma \end{cases}$$

$$8 = 4\gamma \Rightarrow \begin{cases} \gamma = 2 \\ \beta = -2 \\ \alpha = 2 \end{cases}$$

zatem

$$s_n = 2 - 2^{n+1} + n2^{n+1} =$$
$$= 2 + (n-1)2^{n+1}$$