Zad 6.

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Zadanie 6. Naszkicuj portrety fazowe następujących układów równań różniczkowych:

a) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -5 & 1 \\ 1 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

b) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} -4 & -1 \\ 1 & -6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$,

c) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 5 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$,

d) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -5 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$.

a) $\chi_A(\lambda) = \begin{vmatrix} -5-\lambda & 1 \\ 1 & -5-\lambda \end{vmatrix} = (-5-\lambda)^2 - 1 =$

$= (-6-\lambda)(-4-\lambda) = 0$

$\lambda_1 = -6 \quad \lambda_2 = -4$

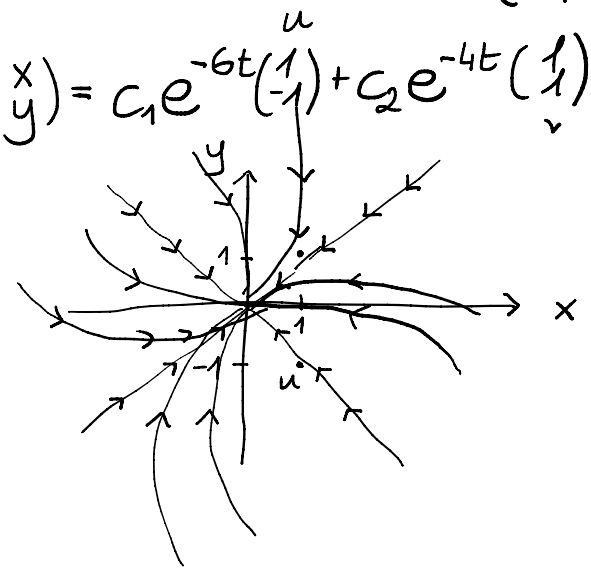
$\begin{pmatrix} -5+6 & 1 \\ 1 & -5+6 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0 \qquad u_1 + u_2 = 0$

$u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \qquad v_1 = v_2$

$v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = c_1 e^{-6t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} + c_2 e^{-4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$



b) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} -4 & -1 \\ 1 & -6 \end{pmatrix}}_{A} \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{cases} x' = -4x - y \\ y' = x - 6y \end{cases}$

$\chi_A(\lambda) = (-4-\lambda)(-6-\lambda) + 1 =$

$= 24 + \lambda^2 + 10\lambda + 1 = \lambda^2 + 10\lambda + 25 = 0$

$(\lambda+5)^2 = 0 \Rightarrow \lambda_{1/2} = -5$

$(\lambda+5)^2=0 \Rightarrow \lambda_{1/2}=-5$

$$\begin{pmatrix} -4+5 & -1 \\ 1 & -6+5 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}=0$$

$u_1-u_2=0 \qquad u=\begin{pmatrix}1\\1\end{pmatrix}$

$$\begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$$

$v_1-v_2=1 \qquad v_1=v_2+1$

$v=\begin{pmatrix}1\\0\end{pmatrix}$

$$\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}\cdot e^{5t}+\begin{pmatrix}1\\0\end{pmatrix}te^{5t}$$