Zad 8

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M3.8. $\boxed{\text{2 punkty}}$ Udowodnij, że metoda iteracyjna:

$$x_{n+1} = \frac{x_n\,(x_n^2 + 3R)}{3x_n^2 + R}$$

jest zbieżna sześciennie do $\sqrt{R}$.

Rzeczy do spełnienia

1. $\varphi(\sqrt{R}) = \sqrt{R}$

$$\varphi(\sqrt{R}) = \frac{\sqrt{R}\,(R+3R)}{3R+R} = \sqrt{R} \qquad \text{git}$$

2. $\varphi'(\sqrt{R}) = 0 \longleftarrow$ z tego mamy też zbieżności lokalną bo jak $\varphi'(\sqrt{R}) = 0$ to w otoczeniu $\sqrt{R}$ $|\varphi'(x)| < 1$

$\varphi'(x) =$

Derivative

$$\frac{\partial}{\partial x}\left(\frac{x^3 + 3Rx}{3x^2 + R}\right) = \frac{3(R - x^2)^2}{(R + 3x^2)^2}$$

$$\varphi'(\sqrt{R}) = \frac{3(R-R)^2}{(R+3R)^2} = 0 \qquad \text{git}$$

3. $\varphi''(\sqrt{R}) = 0$

$\varphi''(x) =$

Derivative

$$\frac{\partial}{\partial x}\left(\frac{3(R - x^2)^2}{(R + 3x^2)^2}\right) = \frac{48Rx(x^2 - R)}{(R + 3x^2)^3}$$

$$\varphi''(\sqrt{R}) = \frac{48R\sqrt{R}\,(R-R)}{(R+3R)^3} = 0 \qquad \text{git}$$

4. $\varphi'''(\sqrt{R}) \neq 0$

$\varphi'''(x) =$

Derivative

$$\frac{\partial}{\partial x}\left(\frac{48Rx(-R + x^2)}{(R + 3x^2)^3}\right) = -\frac{48R(R^2 - 18Rx^2 + 9x^4)}{(R + 3x^2)^4}$$

$$\varphi'''(\sqrt{R}) = -\frac{48R(R^2 - 18R^2 + 9R^2)}{(R+3R)^4} = -\frac{48R(-8R^2)}{4^4R^4} =$$

$$\varphi''(VK) = -\frac{}{1\ \cancel{2}\ 2\cancel{4}\overset{3}{\cancel{6}}\,(R+3R)^4} = -\frac{}{4^4 R^4} =$$

$$= \frac{\cancel{8}\cdot\cancel{48}\,R^3}{\underset{1}{\cancel{4}}\cdot\underset{\cancel{2}}{\cancel{4}}\cdot\underset{1}{4}\cdot\underset{2}{\cancel{4}}\,R^4} = \frac{1}{2R} \neq 0$$