Zad 2.

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M5.2. $\boxed{\text{1 punkt}}$ Wykazać, że zachodzi wzór rekurencyjny

$$f[x_0, x_1, \ldots, x_k] = \frac{f[x_1, x_2, \ldots, x_k] - f[x_0, x_1, \ldots, x_{k-1}]}{x_k - x_0} \qquad (k = 1, 2, \ldots),$$

przy czym $f[x_j] = f(x_j)$.

$$f[x_0, x_1, \ldots, x_k] := \sum_{i=0}^{k} \frac{f(x_i)}{\prod_{j=0, j\neq i}^{k}(x_i - x_j)}.$$

$$\frac{f[x_1, x_2, \ldots, x_k] - f[x_0, x_1, \ldots, x_{k-1}]}{x_k - x_0} =$$

$$= \left(\sum_{i=1}^{k} \frac{f(x_i)}{\prod_{\substack{j=1 \\ j\neq i}}^{k} (x_i - x_j)} - \sum_{i=0}^{k-1} \frac{f(x_i)}{\prod_{\substack{j=0 \\ j\neq i}}^{k-1} (x_i - x_j)} \right) \cdot \frac{1}{(x_k - x_0)} =$$

przyjrzyjmy się jak składniki tych sum zmieniają się dla kolejnych i

1) dla $i=0$ mamy

$$\frac{f(x_0)}{\prod_{j=1}^{k-1}(x_0 - x_j) \cdot (x_k - x_0)} = \frac{f(x_0)}{\prod_{j=1}^{k-1}(x_0 - x_j) \cdot (x_0 - x_k)} = \frac{f(x_0)}{\prod_{\substack{j=0 \\ j\neq 0}}^{k}(x_0 - x_j)}$$

2) dla $i=k$ mamy

$$\frac{f(x_k)}{\prod_{\substack{j=1 \\ j\neq k}}^{k}(x_k - x_j)} \cdot \frac{1}{(x_k - x_0)} = \frac{f(x_k)}{\prod_{\substack{j=0 \\ j\neq k}}^{k}(x_k - x_j)}$$

$j \neq k$ $\qquad\qquad$ $j \neq n$

3) dla $0 < i < k$ mamy

$$\left[\frac{f(x_i)}{\prod\limits_{\substack{j=1 \\ j \neq i}}^{k} (x_i - x_j)} - \frac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{k-1} (x_i - x_j)} \right] \cdot \frac{1}{(x_k - x_0)} =$$

$$= \frac{f(x_i)(x_i - x_0) - f(x_i - x_k)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{k} (x_i - x_j)} \cdot \frac{1}{(x_k - x_0)} =$$

$$= \frac{f(x_i) \cdot x_i - f(x_i) \cdot x_i + f(x_i)(x_k - x_0)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{k} (x_i - x_j)} \cdot \frac{1}{(x_k - x_0)} =$$

$$= \frac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{k} (x_i - x_j)}$$

razem mamy zatem

$$\frac{f(x_0)}{\prod\limits_{\substack{j=0 \\ j \neq 0}}^{k} (x_0 - x_j)} + \sum_{i=1}^{k-1} \frac{f(x_i)}{\prod\limits_{\substack{j=0 \\ j \neq i}}^{k} (x_i - x_j)} + \frac{f(x_k)}{\prod\limits_{\substack{j=0 \\ j \neq k}}^{k} (x_k - x_j)} =$$

$$= \sum_{i=0}^{k} \frac{f(x_i)}{\prod\limits_{j=0}^{k} (x_i - x_j)} \overset{(\text{def.})}{=} f[x_0 \ldots x_k] \qquad \square$$

$$\sum_{i=0} \prod_{\substack{j=0 \\ j\neq i}} (x_i - x_j) \qquad \square$$