Zad 11.

środa, 18 stycznia 2023 10:01

M11.11. 3 punkty. Włącz komputer i uzasadnij odpowiedź. Obliczyć z dokładnością do $\varepsilon = 10^{-4}$ wartość całki

$$I(f) = \int_0^\infty \cos^2(x) e^{-x} = 3/5.$$

In numerical analysis **Gauss–Laguerre quadrature** (named after Carl Friedrich Gauss and Edmond Laguerre) is an extension of the Gaussian quadrature method for approximating the value of integrals of the following kind:

$$\int_0^{+\infty} e^{-x} f(x)\,dx.$$

In this case

$$\int_0^{+\infty} e^{-x} f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

where x_i is the i-th root of Laguerre polynomial $L_n(x)$ and the weight w_i is given by[1]

$$w_i = \frac{x_i}{(n+1)^2 [L_{n+1}(x_i)]^2}.$$

The **closed form** is

$$L_n(x) = \sum_{k=0}^{n} \binom{n}{k} \frac{(-1)^k}{k!} x^k.$$

zad11.py > ...

```python
from sympy import *

def gauss_laguerre(n, func):
    x = Symbol("x")
    poly = Poly(laguerre(n, x))
    roots = poly.all_roots()
    x_i = [r.evalf(20) for r in roots]
    w_i = [(r/(((n+1) * laguerre(n+1, r))**2)).evalf(20) for r in roots]
    res = 0
    for (w, x) in zip(w_i, x_i):
        res = res + w * func(x)
    return res

def f(x):
    return cos(x)**2

print(gauss_laguerre(18, f))
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

```
mluczynski@mluczynski:~/Desktop/kochane studia$ python3 zad11.py
0.60000565882260595957
mluczynski@mluczynski:~/Desktop/kochane studia$
```