Asian option pricing

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INTRODUCTION

- In finance, an option is a contract which gives the buyer (the owner) the right, but not the obligation, to buy or sell an underlying asset or instrument at a specified strike price on or before a specified date. The seller has the corresponding obligation to fulfill the transaction that is to sell or buy if the buyer (owner) "exercises" the option. The buyer pays a premium to the seller for this right.

- An Asian option is an option whose payoff is determined by the average underlying price over some preset time period.

- As option's optimal price depend on future values of the underlying asset, we must simulate those in order to fix our option's price.

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PRICING MODEL

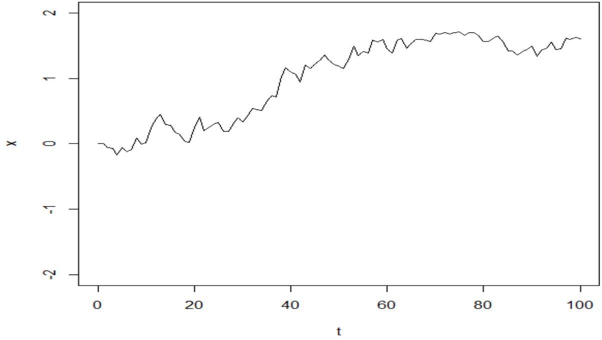
- An asian option price is given by :

$$C = \mathbb{E}\left[f(S)\right] = \mathbb{E}\left[e^{-rT}\left(\frac{1}{k}\sum_{i=1}^{k} S(t_i) - K\right)\right] \tag{1}$$

  - Where S is the vector containing the underlying asset's values over time
    r stands for the interest rate
    T is the contract's term
    K is the option's strike
    S is a random process
  - Here we can see the link between the option price and the underlying asset's average value over a given period of time (here between t_1 and t_k)
- We will now simulate this value for different processes S and different Monte Carlo methods in order to reduce our estimator's variance.

Brownian motion simulation

- A brownian motion (also called wiener process) is a random process W_t which is characterized by four facts :
  - $W_0 = 0$
  - W_t is almost surely continuous
  - W_t has independent increments
  - $W_t - W_s \backsim \mathcal{N}(0, t-s)$ for $0 \leq s \leq t$



Estimation using standard Monte Carlo

- The Black & Scholes model for option pricing gives :

$$S(t) = exp(\mu - \sigma^2/2)t + \sigma W_t \tag{2}$$

- Our first set of paremeters are :
  - $T = 1$
  - $r = 0.005$
  - $\sigma = 0.3$
  - $K = 5$
  - $k = 20$
  - $t_i = i/20$

  We intend to make them vary in the end in order to figure out their importance on the pricing model's outcome.

Estimation using standard Monte Carlo

- Our first results using standard Monte Carlo :

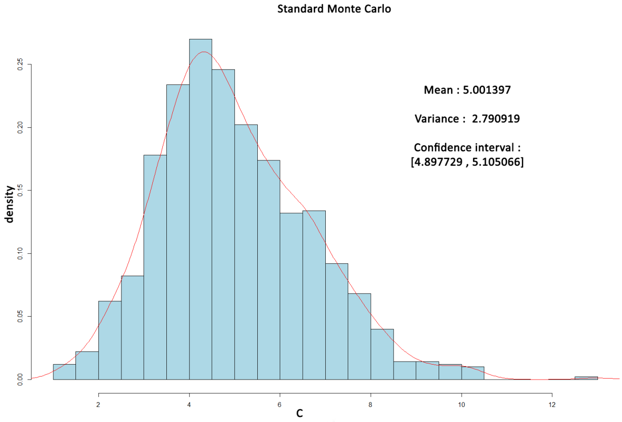


Figure: C estimator using standard Monte Carlo
$S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

ESTIMATION USING ANTITHETIC VARIATES

- The increments of our simulated brownian motion follow $\mathcal{N}(0, \sigma)$, which is symetric. Therefore $-W$ follows the same law of probability as W; for that reason, it is going to be our antithetic variate.

ESTIMATION USING ANTITHETIC VARIATES

- Our results using antithetic variates :

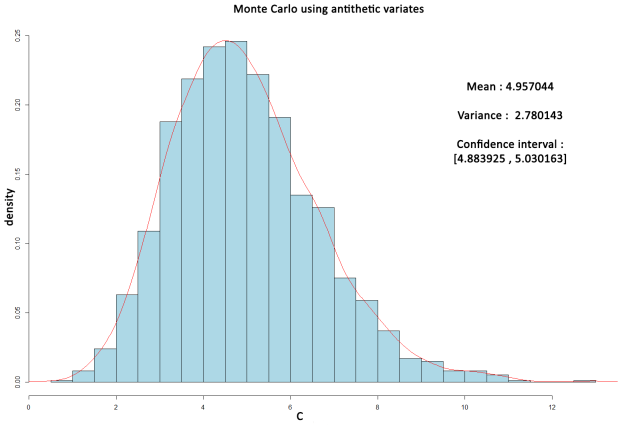


Figure: C estimator using antithetic variates
$S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

Estimation using Antithetic variates

- Here we notice a slightly reduced variance and similar mean.

Estimation using Control variates

- We wish to minimize C's variance. For this purpose we chose $\overline{exp(W_T)}$ to be our control variate.
- We now have as an estimator for C :

$$C'_i = C_i - \rho\overline{exp(W_T)} \tag{3}$$

Where C_i is the i^{th} simulation for C and $\overline{exp(W_T)}$ the empirical mean of the i^{th} simulation for $exp(W_T)$ with duration T.

- An optimal value for b is:

$$b* = \frac{Cov(C, \overline{exp(W_T)})}{Var(\overline{exp(W_T)})} \tag{4}$$

ESTIMATION USING CONTROL VARIATES

- Our results using control variates :

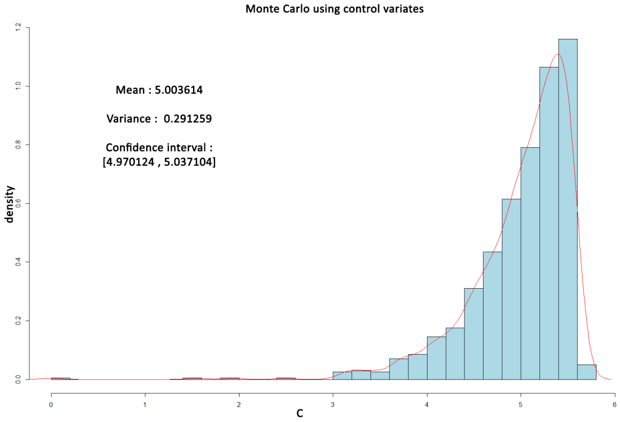


Figure: C estimator using control variates
$S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

Estimation using Control variates

- When the antithetic variates method did not reduce our estimator's variance so much, the control variates method dramatically influenced its precision! With this second technique, the standard deviation is ten times smaller!

ESTIMATION USING QUASI-MONTE CARLO

- Instead of using standard Monte Carlo and simulating W_t with pseudorandom sequence we will now simulate W_t with low discrepancy sequences such as Sobol sequence or Halton sequence.

ESTIMATION USING QUASI-MONTE CARLO

- Our results using quasi monte carlo :

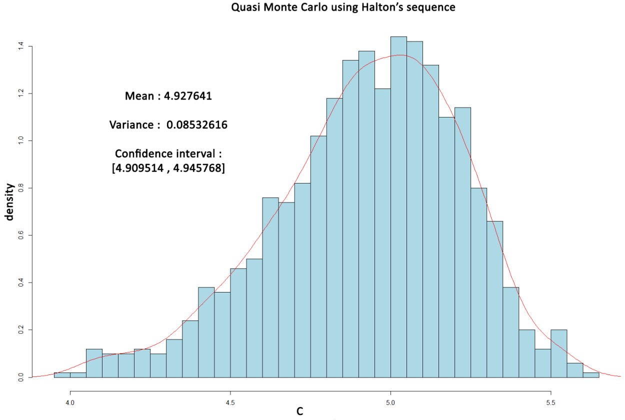


Figure: C estimator using control variates
$S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

ESTIMATION USING QUASI-MONTE CARLO

- It seems our best results are obtained using Quasi Monte Carlo method to generate our brownian motions.

The Euler Scheme

- It is often hard to simulate precisely pricing models. An approximation of pricing models can be calculated with the Euler scheme.
- We would like to measure the impact of discretization of the Black & Scholes model on the output of the different methods we have used so far.

THE EULER SCHEME

- Let X be a stochastic process, the Euler Scheme gives us :

$$\overline{X_0} = X_0$$

$$\overline{X_{t_i}} = X_{t_{i-1}} \left(1 + \mu \frac{T}{k} + \sigma (W_{t_i} - W_{t_{i-1}}) \right)$$

- μ: mean; T: contract's term, k: number of division of the time T; W: Brownian motion.
- We apply the four different methods to the new data, discretized by the Euler Scheme.

Estimation using standard Monte Carlo

- Our results using standard Monte Carlo on discretized Black & Scholes:

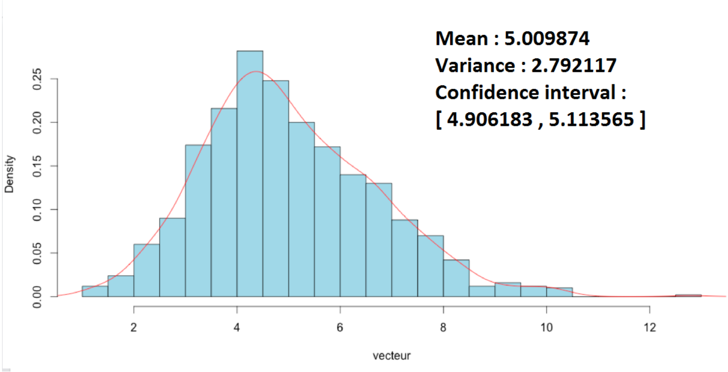


Figure: C estimator of discrete Black & Scholes using standard Monte Carlo $S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

Estimation using standard Monte Carlo

- We notice that the distance between the true value and the calculated mean (5.010) is larger in the model below than in the regular model (5.001); the variance (2.792) in the discretized model is quite similar to the variance (2.791) in the regular one. The bias is still positive.

ESTIMATION USING ANTITHETIC VARIATES

- Our results using antithetic variates:

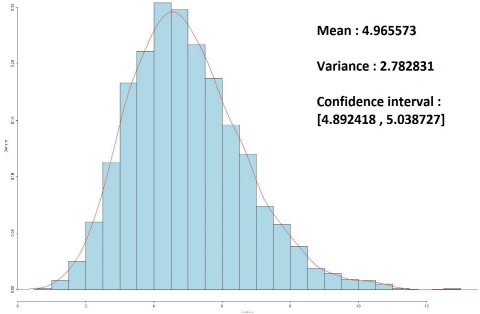


Figure: C estimator of discrete Black & Scholes using antithetic variates
$S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

ESTIMATION USING ANTITHETIC VARIATES

- The bias is negative, as in the regular model. The variances are quite alike (2.780 in the regular model; 2.783 in the discrete one) and quite close to the one found in the standard Monte Carlo method.

ESTIMATION USING CONTROL VARIATES

- Our results using control variates:

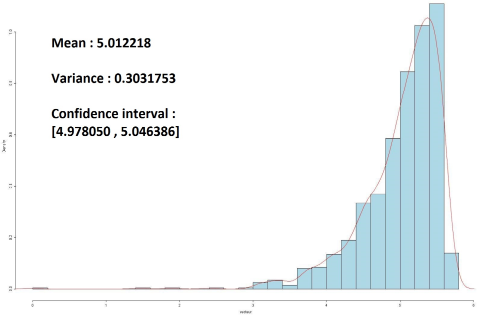


Figure: C estimator of discrete Black & Scholes using control variates
$S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

ESTIMATION USING CONTROL VARIATES

- Compared to the regular model, the distant between the mean (5.012) and the true value is greater in the discretized model than in the regular one (5.004). The variances are similarly small compared to the ones calculated via the standard Monte Carlo and the antithetic variate (0.291 for the regular model, 0.303 for the discretized one).

ESTIMATION USING QUASI-MONTE CARLO

- Our results using Quasi-Monte Carlo:

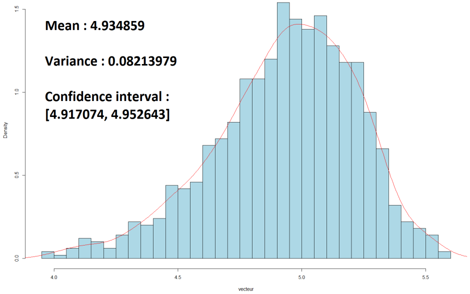


Figure: C estimator of discrete Black & Scholes using Quasi Monte Carlo $S_0 = 10$, r=0.05, k=20, T=1, $\sigma = 0.3$, K=5, number of simulation = 1000

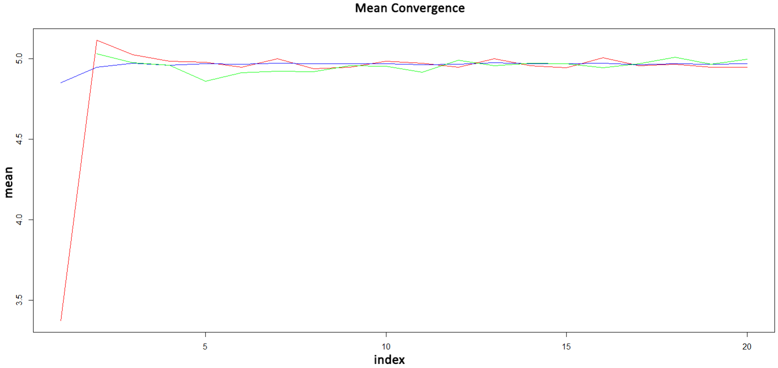
ESTIMATION USING QUASI-MONTE CARLO

- As in the regular model, the Quasi Monte Carlo provides the best (smallest) variance of all the methods. In the discretized model, it is even smaller than in the regular one (0.082 against 0.085). The bias too is smaller in the discretized model than in the regular one.

Impacts of the different methods on the mean & variance

Values of the variance in the 4 methods				
	Monte Carlo	Antithetic var.	Control var.	Quasi MC
Normal	2.791	2.780	0.291	0.085
Discretized	2.791	2.782	0.303	0.082

SPEED OF CONVERGENCE



Red curve : Standard Monte Carlo
Blue curve : Antithetic Variate Monte Carlo
Green curve : Control Variate Monte Carlo
Number of simulation : index x 500