Fall 2021 – Project 1: Martingale Report

Michael Lukacsko
mlukacsko3@gatech.edu

QUESTION AND ANSWERS

1.1 Question 1

In Experiment 1, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots:

Answer: To get an initial idea of how this question should be answered, I looked to the output seen in Figure1 attached below. Based on the charts data, it would appear the gambler has a 100% probability of winning $80 in each episode of 1000 sequential bets. The driving factor in this example is that the gambler has an unlimited bankroll. Because of the gamblers bottomless pockets, he/she would only need to win a total of 80 bets to have a total profit of $80 according to the martingale betting strategy. Given that the probability of winning one spin of the roulette wheel is 18/38, we can use binomial distribution to see the probability of winning 80 times within 1000 sequential bets as:

$P(X) = \left(\frac{n!}{X!(n-X)!}\right) \cdot p^x \cdot (1-p)^{n-X}$

where 'n' is the number of trials, 'p' is the probability of success, and 'X' is the number of successes. Substituting in our values, we get n = 1000, P = 18/38 = .474, and X = 80. Solving for $P(80) = \left(\frac{1000!}{80!(1000-80)!}\right) \cdot 0.474^{80} \cdot (1-0.474)^{1000-80}$

we see that our probability will be incredibly close to 1.00, or 100%. Looking at the graph, it looks like all 10 episodes reach the $80 winning amount around spin 180. Using this as our 'n' number, we solve for:

$$P(80) = \left(\frac{180!}{80!\,(180-80)!}\right) \cdot 0.474^{80} \cdot (1-0.474)^{180-80}$$

This shows that the gambler will net $80, or win 80 times, 97.24% after only 180 sequential bets.

1.2 Question 2

In Experiment 1, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Answer: To determine the expected value of winnings after 1000 sequential bets, we can use the expected value/weighted average formula. It can be expressed as $E(V) = \sum xi * P(xi)$ where E(V) is our expected value, 'Xi' is the expected outcome, and 'P(xi)' is the theoretical probability. We can substitute in a 1 for P(xi) because we know, based on the previous question, that our expected outcome is 1. We also know the expected outcome is $80, so this will represent 'Xi'. Plugging in our numbers and solving for E(V) gives us $E(V) = 80 * 1$. Hence, the expected value of winnings, after 1000 sequential bets, is $80. Referencing the attached Figure2 graph, we see that the mean winnings after the same 1000 sequential bets is also $80, thus proving that our expected value calculation above is correct.

1.3 Question 3

In Experiment 1, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

Answer: Looking to Figure 2 below, it is evident that the mean+stdev and mean-stdev vary drastically until the mean total of winnings reaches $80 at around 180 spins. The variations in mean-stdev and mean+stdev can be accounted for because the probability of winning and/or loosing is unpredictable until the gambler is close to winning the $80. However, our standard deviation reaches 0 and the 3 lines (mean, mean_stdev, and mean-stdev) converge once the gambler reaches $80 dollars in winning. As was proved back in question 1, there is a 100% probability that our simple gambling simulator will win, hence the standard deviation lines will always converge as the number of sequential bets increases.

1.4 Question 4

In Experiment 2, based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets. Thoroughly explain your reasoning for the answer using the experiment output. Your explanation should NOT be based on estimates from visually inspecting your plots.

Answer: The betting strategy used in questions 1-3 clearly worked very well. Given an unlimited bankroll, our gambler is almost guaranteed to reach the goal of winning $80 given 1000 sequential spins. When the gambler is limited to a $256 bankroll, one of two things can happen. The gambler will either win the $80 or loose the $256. Using a more mathematical method to determine the estimated probability of winning $80 within 1000 sequential bets, we let 'P' be the probability of winning and 1-'P' be the probability of not winning $80, or loosing the $256 bankroll. We can show that using $(P * 80) + \big((1 - P) * -256\big) = -40$ where -40 is the mean winnings based on Figure 4. Solving for 'P', we get .643. Hence, our odd of winning would be about 64% and not winning at about 36%.

1.5 Question 5

In Experiment 2, what is the estimated expected value of winnings after 1000 sequential bets? Thoroughly explain your reasoning for the answer.

Answer: Using the expected value/weighted average formula from question 1, we can calculate the expected value of winning after 1000 sequential bets as $E(V) = \sum xi * P(xi) +\ xj * P(xj)$. Here, we include the weighted average of losing the $256 bankroll as well. Plugging in our expected probability figures from the previous question, we get $E(V) = 80 * .64 +\ .36 * -256$. Solving for our expected value, E(V), our gamblers expected value of winning after 1000 sequential bets would be $-40.96.

1.6 Question 6

In Experiment 2, do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases? Thoroughly explain why it does or does not.

Answer: Unlike the mean, mean_stdev, and mean-stdev in question 3, the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) do not converge at any point as the number of sequential bets increases toward our 1000 bets when the gambler is limited to $256. In fact, the upper standard deviation line and lower standard deviation line appear to reach a maximum/minimum value before stabilizing. This is due again to the fact that the gambler does has a limited bankroll of $256, Referencing Figure 4 below, it can be concluded that the standard deviation is relatively small at the beginning but reaches a maximum value and stabilizes at an estimated amount of about $150 around 180 spins.

1.7 Question 7

What are some of the benefits of using expected values when conducting experiments instead of simply using the result of one specific random episode?

Answer: As with all betting scenarios, there is no certainty for the gambler. One advantage of using expected values when conducting experiments instead of simply using the result of one specific random episode is that the probability of each possible outcome is taken into consideration and uses the information gathered to calculate the expected value. This practice takes uncertainty into account which, as we can see from the graphs, is an ever present elements of betting. Another advantage is that it is simple to understand, and the numbers are simple to calculate. This makes it easy for the person interpreting the expected value to see the whole picture/entire distribution using a single figure.

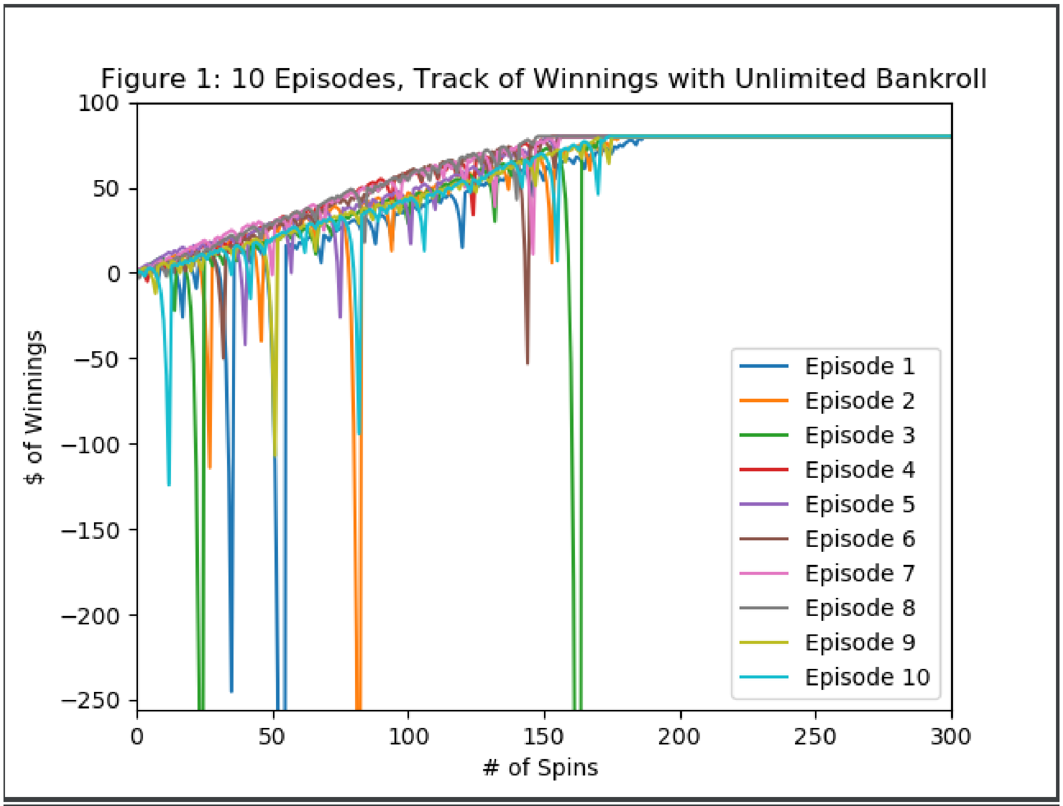

Figure 1: 10 Episodes, Track of Winnings with Unlimited Bankroll
$ of Winnings
# of Spins
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300
Episode 1
Episode 2
Episode 3
Episode 4
Episode 5
Episode 6
Episode 7
Episode 8
Episode 9
Episode 10

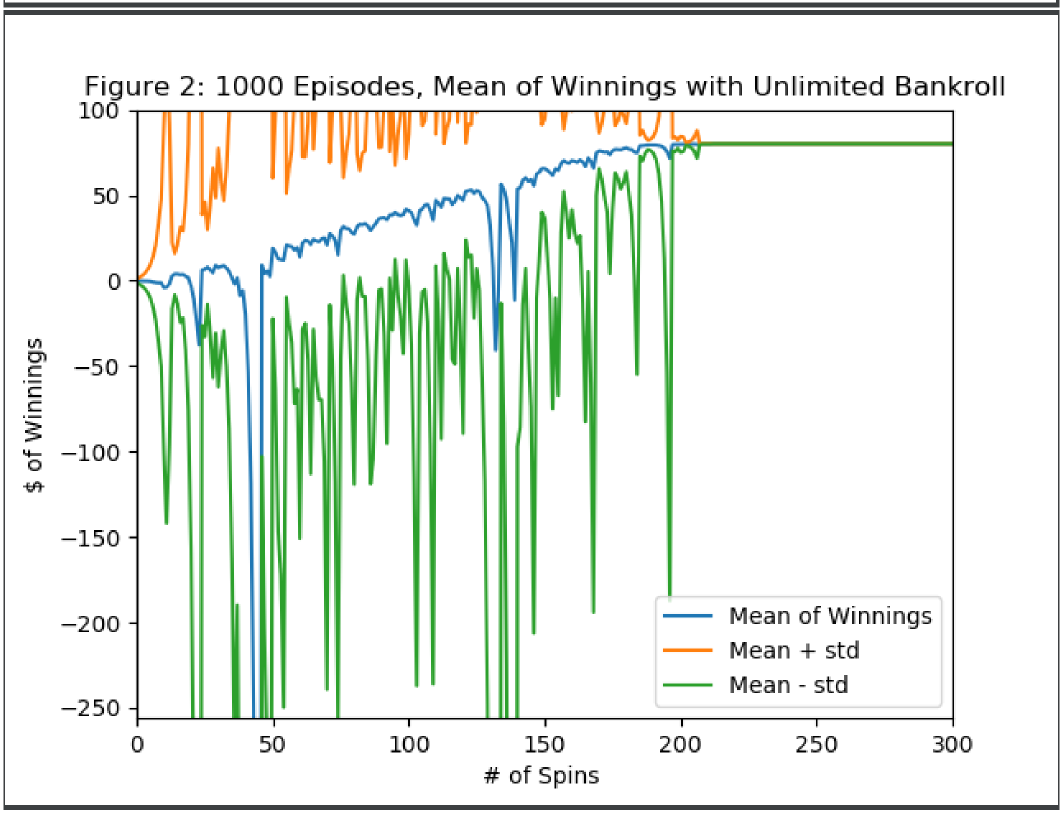

Figure 2: 1000 Episodes, Mean of Winnings with Unlimited Bankroll
$ of Winnings
# of Spins
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300
Mean of Winnings
Mean + std
Mean - std

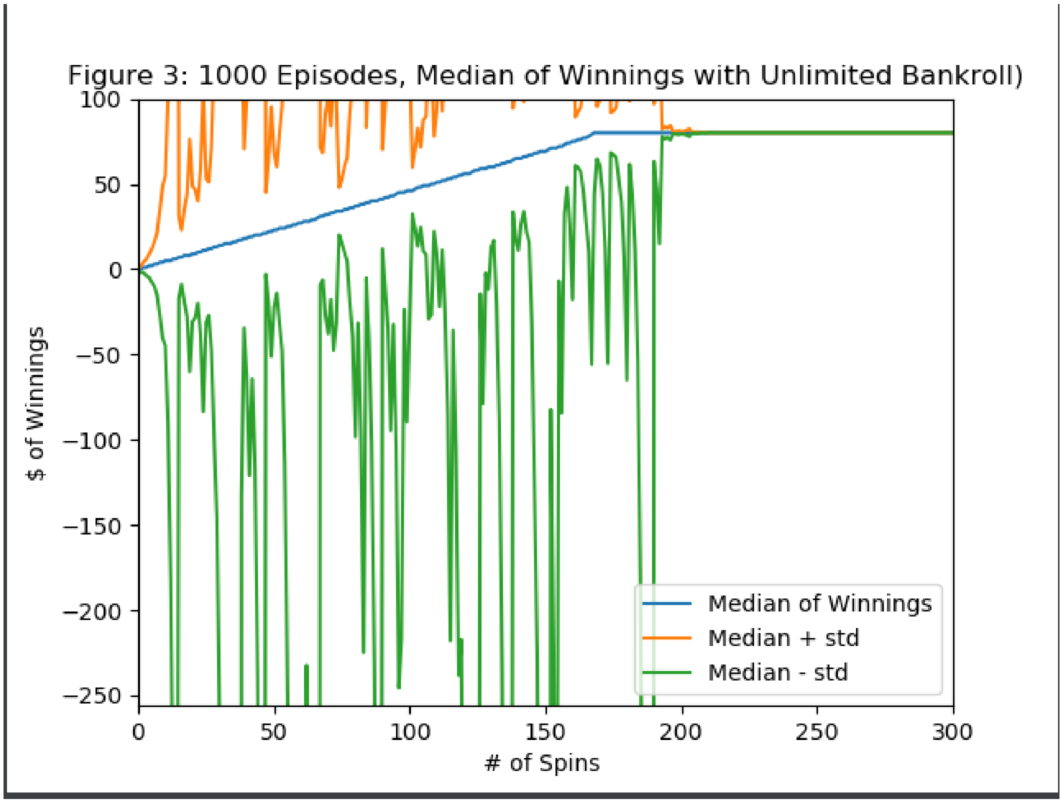
Figure 3: 1000 Episodes, Median of Winnings with Unlimited Bankroll)
$ of Winnings
# of Spins
Median of Winnings
Median + std
Median - std

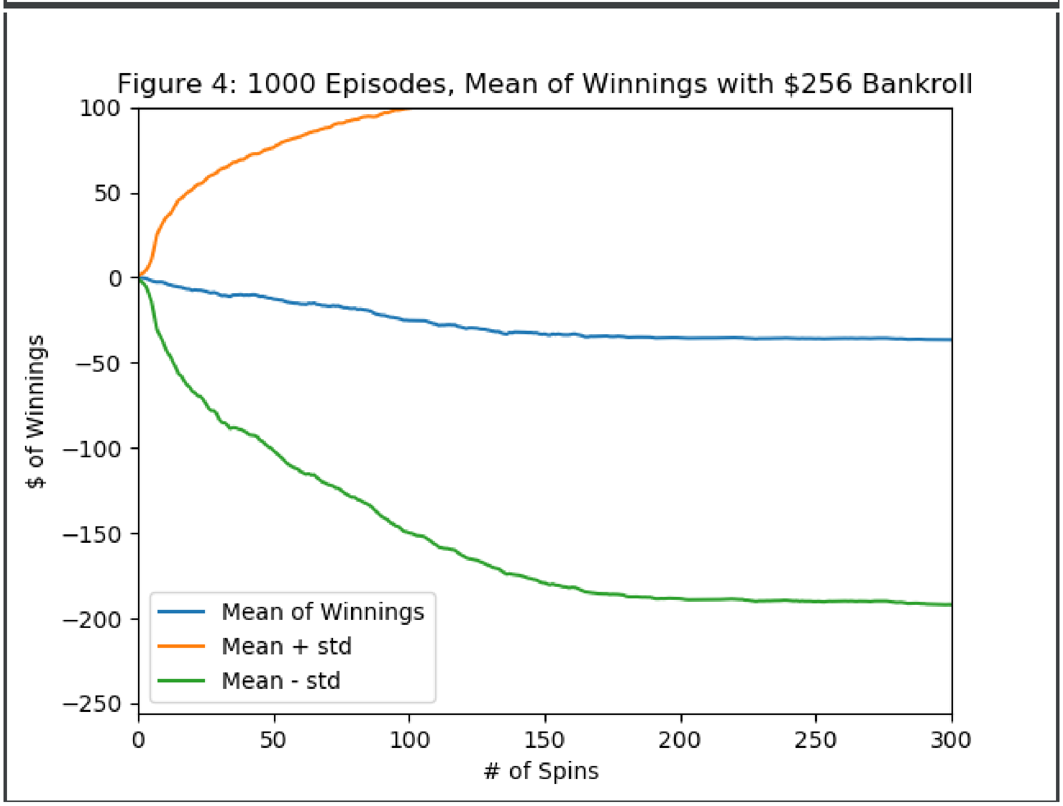
Figure 4: 1000 Episodes, Mean of Winnings with $256 Bankroll
$ of Winnings
# of Spins
Mean of Winnings
Mean + std
Mean - std

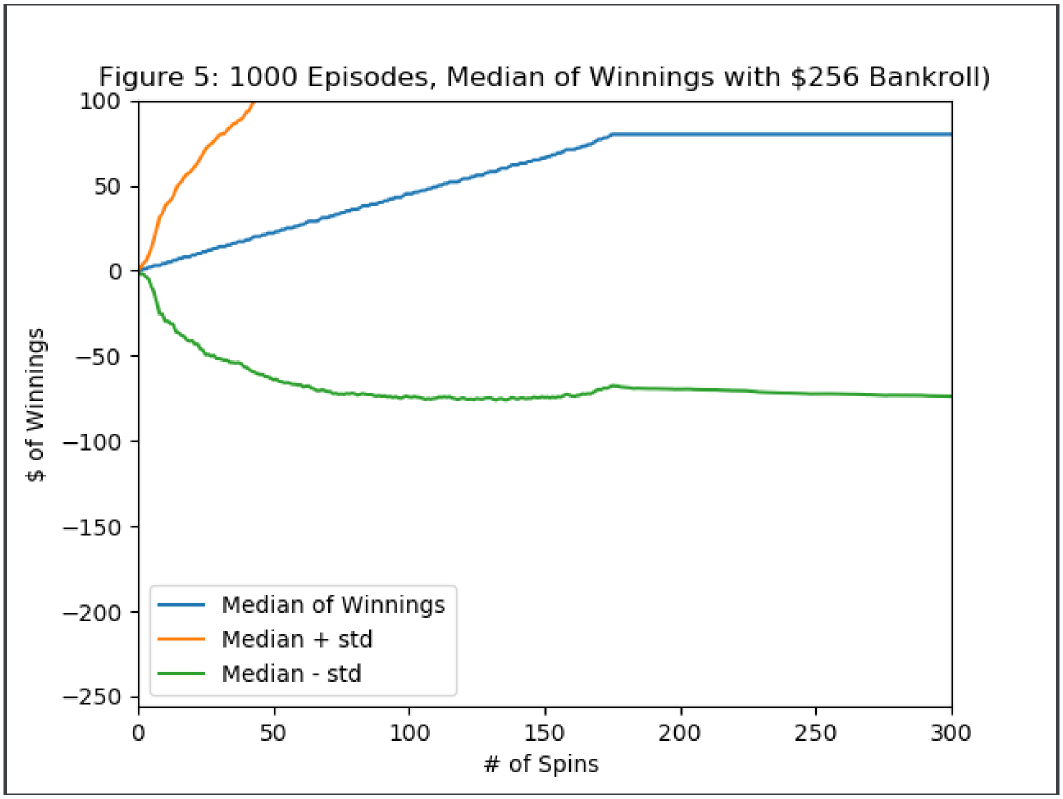

Figure 5: 1000 Episodes, Median of Winnings with $256 Bankroll)
100
50
0
−50
−100
−150
−200
−250
$ of Winnings
0
50
100
150
200
250
300
# of Spins
Median of Winnings
Median + std
Median - std