

Models for Sequential Data

Machine Learning 2020
mlvu.github.io

the plan

part 1: Sequences

Markov models

Embedding models

Word2Vec

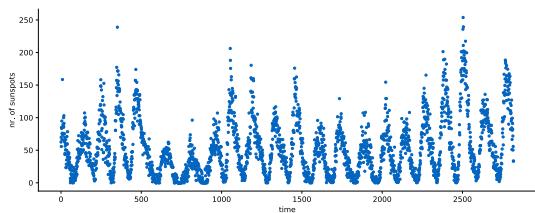
part 2:

Recurrent Neural Nets

LSTMs

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numeric 1-dimensional



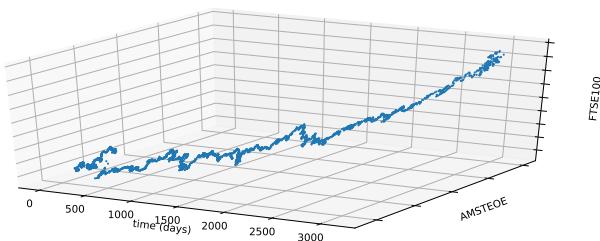
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Before we start looking at different models to learn from sequential data, it pays to think a little bit about the different types of sequential datasets we might encounter.

As with the traditional setting (a table of independently sampled instances) we can divide our features into numeric and discrete.

A single 1D sequence might look like this. We could think of a stock price over time, traffic to a webserver, or atmospheric pressure over Amsterdam. In this case, the data shows the number of sunspots observed over time.

numeric n-dimensional



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Sequential numeric data can also be multidimensional. In this case, we see the closing index of the AEX and the FTSE100 over time. This data is a sequence of 2D vectors.

symbolic (categorical) 1-dimensional

the, cat, sat, on, the, mat

t, h, e, _, c, a, t, _, s, a, t, _, o, n, _, t, h, e, _, m, a, t

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symbolic n-dimensional

the/ART cat/NOUN sat/VERB on/PREP
the/ART mat/NOUN



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single sequence vs set-of-sequences

Dear recipient, Avanjar Technologies announces the beginning of a new unprecedented global employment campaign	spam
revisor yell wheres busherry twenties	
Due to company's exploding growth Avanjar is expanding business to the European region.	
During last employment campaign over 1500 people worldwide took part in Avanjar's business	
and now we are looking for more. We are offering you	
more opportunities to earn extra money working with Avanjar Technologies.	
BIRMINGHAM, AL 35203 *** LASER PRINTER TONER CARTRIDGES*** ***FAX AND COPIER TONER***	
ATLANTA GA 30303 *** LASER PRINTER TONER CARTRIDGES*** ***FAX AND COPIER TONER***	
CHECK OUT OUR NEW CARTRIDGE PRICES .	
On 08/09/02 15:26 (PDT) Craig Pender wrote:	
> Well, I'm not sure what that means b/c - you have to transmit the signature, plus now the entire ...	ID.
> Plus the rights to all intellectual property contained in any	
> email message you submit (I guess technically there are no bits)	
> > is the right idea)	
If you own a travel related website, why not submit your site to our directory. Just select the appropriate category and	
subcategory and enter your title	
description.	
Click here to start: http://www.holiday-travel-directory.com	
.....	
I know, I assume you're going to report this to the mm folks? Yes, I will, sometime, after I look at the mm sources	
and see what they have managed to break, ...why.	
But we really want ewnbt to operate with all the versions of enh that	
exist, don't we? The patch to have ewnb do the right thing, whether	
this bug exists, or not, is trivial, so I'd suggest including it.	
Hello I am emailing you as we found your holiday property at [Source] and would love you to list it on our website for	
free:	
http://www.holidayrentals.org	
This web site now has over 1000 holiday rentals listed in less than 2 months,	
Hello again self-catering.co.uk was recently launched and if you have not yet added your property for free then please	
do as we have had many properties added in the past	
few days.	

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If the elements of our data are discrete (analogous to a categorical feature), it becomes a sequence of symbols.

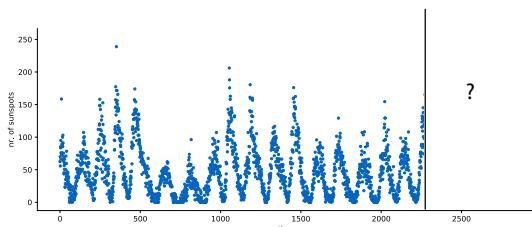
Language is a prime example. In fact, we can model language as a sequence in two different ways: as a sequence of words, or as a sequence of characters.

We can also encounter sequences with multiple categorical features per timestamp, like music, or tagged language. These can often be difficult to represent.

Then there is the question of what we're trying to predict. Do we have one sequence per instance, and are trying to, for instance, classify the sequences? That's what we see here: each email is a sequence (of words or characters).

The instances themselves do not have a very sequential ordering (emails do have a timestamp, but this ordering is usually ignored).

single sequence



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An entirely different setting is one where the dataset as a whole is a sequence, and the instances are ordered.

In that case, we often want to predict the future values of the sequence based on what we've seen in the past. To keep things simple, let's stick with 1D numeric data, like our sunspots example.

single sequence: feature extraction

1 3 5 3 4 2 3 4 5 5 4 3 2 1 3 2 2

t-3	t-2	t-1	t
5	2	3	3
3	5	2	3
3	3	5	2
5	3	3	5
4	5	3	3
2	4	5	3
3	2	4	5
3	3	2	4
1	1	3	2
2	3	3	3

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Think about the REAL-WORLD use case.

One simple way to achieve this, is to translate it to a classic regression problem by representing each point by the m values before it. This gives us a table with a target label (the value at time t) and m features (the m preceding values). If we successfully train a regression model on this task, we can then use it to predict future values of the sequence.

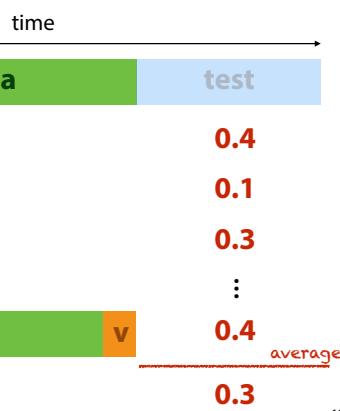
Many other features are possible: mean over the whole history. Mean over the last 10 points, variance one the last 10 points, etc.

But, there's a problem. If we shuffle this data, and pick a test and training set, the classifier is trained on data that happens in the future to data it will be tested on!

Whenever you have questions about how to approach something like this, it's best to think about the real world setting where you might apply your trained model.

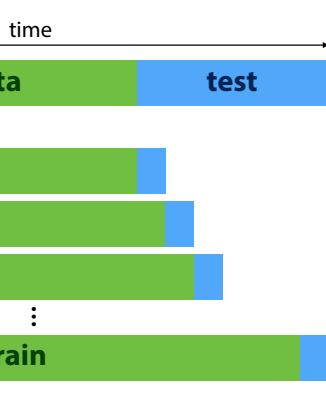
In this case, we are going to apply the prediction model on the three most recent values, to predict the next value. The test data should simulate that situation. That means that the training data should not contain instances that occur later than the test data.

walk-forward validation



To check your algorithm, to do cross validation.

testing



Note, only if the target labels have a meaningful ordering in time. In spam classification, for instance, this is not necessary (emails do have a timestamp, but seeing an email from the future does not usually give you an unrealistic advantage).

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summary: sequential data

Sequences: consisting of numbers, vectors or symbols

Dataset: consisting of a **sequence per instance**, or a **sequence of instances**.

For a sequence of instances, careful with your test and validation

Both can be converted to fit classic machine learning through feature extraction.

Rest of the lecture: **1D symbolic sequences**.

Extension to nD and/or numeric is often trivial

But, often it's better to take a model that can consume sequences natively.

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modeling sequences: probability

$$\begin{aligned} p(\text{"congratulations you have won a prize"}) \\ = p(W_1=\text{congratulations}, W_2=\text{you}, W_3=\text{have}, \\ W_4=\text{won}, W_5=\text{a}, W_6=\text{prize}) \end{aligned}$$

$$p(W_1, W_2, W_3, W_4, W_5, W_6)$$

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We'll start by looking at some ways to model sequences in probabilistic terms.

When modelling probability, we usually break the sequence up into its **tokens** (in this case the words of the sentence) and model each as a random variable. Note that these random variables are decidedly *not* independent.

This leaves us with a joint distribution over 6 variables, which we would somehow like to model and fit to a dataset.

slide 22, Probabilistic Methods 1

$$p(x, y) = p(x | y)p(y)$$

Remember this rule from the first Probabilistic Methods lecture. If we have a joint distribution over more than two variables on the left, we can apply this rule multiple times.

chain rule of probability

$$\begin{aligned} p(W_4, W_3, W_2, W_1) \\ = p(W_4, W_3, W_2 | W_1)p(W_1) \\ = p(W_4, W_3 | W_2, W_1)p(W_2 | W_1)p(W_1) \\ = p(W_4 | W_3, W_2, W_1)p(W_3 | W_2, W_1)p(W_2 | W_1)p(W_1) \end{aligned}$$

$$p(\text{prize} | \text{a, won, have, you, congratulations})$$

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This gives us the **chain rule of probability** (not to be confused with the chain rule of calculus, which is entirely different), which is often used in modelling sequences.

The chain rule allows us to break a joint distribution on many variables into a product of conditional distributions. In sequences, we often apply it so that each word becomes conditioned on the words before it.

This tells us that if we build a model that can estimate the probability $p(x|y, z)$ of a word x based on the words y, z that precede it, we can then *chain* this estimator to give us the joint probability of the whole sentence x, y, z .

$$\log p(\text{sentence}) = \sum_{\text{word}} \log p(\text{word} | \text{all words before word})$$

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In other words, we can rewrite the probability of a sentence as the product of the probability of each word, conditioned on its history. If we use the log probability, this becomes a sum.

Note that applying the chain rule in a different order would allow us to condition any word on any other word, but conditioning on the history fits well with the sequential nature of the data, and will allow us to make some useful simplifications later.

language model

$p(W | \text{the, man, fell, out, of, the})$

window

aquarium

the man fell out of the ...

pool

cycling

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A perfect language model would encompass everything we know about language: the grammar, the idiom and the physical reality it describes. For instance, it would give **window** a very high probability, since that is a very reasonable way to complete the sentence. **Aquarium** is less likely, but still physically possible and grammatically correct. A very clever language model might know that falling out of a **pool** is not physically possible (except under unusual circumstances), so that should get a lower probability, and finally **cycling** is ungrammatical, so that should get very low probability (perhaps even zero).

The problem is most sentences of this length will never have been seen before in their entirety. A simple way to get a basic model is **to limit how far back we look in the sentence**.

Markov assumption

$$p(\text{prize} | \text{a, won, have, you, congratulations}) = p(\text{prize} | \text{a, won})$$

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This is called a **Markov assumption**. It's a bit like the naive Bayes assumption: we know it's incorrect, but it still yields a very usable model. The number of words we retain in the conditional is called the order of the Markov model: this is a Markov assumption for a second-order Markov model.

Markov model

$$\begin{aligned} p(\text{prize, a, won, have, you, congratulations}) \\ = & p(\text{prize} | \text{a, won}) \cdot p(\text{a} | \text{won, have}) \cdot p(\text{won} | \text{have, you}) \\ & p(\text{have} | \text{you, congratulations}) \cdot p(\text{you} | \text{congratulations}) \cdot p(\text{congratulations}) \end{aligned}$$

$$p(\text{prize} | \text{a, won}) \approx \frac{\#\text{"won a prize"}}{\#\text{"won a"}}$$

2nd order Markov model

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Using the Markov assumption and the chain rule together, we can model a sequence as limited-memory conditional probabilities. These probabilities can then be very simply estimated from a large dataset of text (called a **corpus**).

To estimate the probability of **prize** given "**won a**" we just count how often "won a prize" occurs as a proportion of the times "**won a**" occurs. In other words, how often "**won a**" is followed by **prize**.

These n-word snippets are called **n-grams**. "**won a prize**" is a **trigram**, and "**won a**" is a **bigram**.

This type of language model is often called a **Markov model**, because of the Markov assumption of limited memory. The size of the memory is referred to as the **order** of the Markov model. The higher the order of your model, the more you can model,

but the more data you'll need for accurate estimates if the probability.

sequential sampling from a language model

start with a small seed sequence $s = [w_1, w_2, w_3]$ of tokens.

loop:

Sample next word w according to $p(W = w | w_1, w_2, \dots)$

append w to s

One interesting thing we can do with a Markov model, is to sample from it, step by step. We start with a seed of a few words, and then work out the probability distribution over the next word, given the last n words. We sample from this distribution, append the sample to our text and repeat the process.

Note that with the Markov assumption, we only need the last n elements of the sequence to work out the probabilities.

Sequential sampling is also known as **autoregressive sampling**. In the context of Markov models, the sampling process is often called a **Markov chain**.

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sequential sampling: Markov chain

Go thy
ways, wench; serve them, joining their best friend to ransom straight,
And make him dead, deceased, she's dead, she's good, thou hast thou dost thou
dost excuse.

Is thy son to church?

CAPULET Ready to love now
Doth grace that I beseech your high majesty.

SIR WALTER BLUNT, with tears: mine ear. Prithee,
tell her you both of Rome of your will: I
pray you, daughter, he is my soul, he proclaim'd
By Richard that thou dead:
Then, as herbs, grace himself an hour.

source <http://www.schmipsum.com/> 22

Here is a bit of text sampled from a Markov model trained on the works of Shakespeare.

(We're essentially training a very simple probabilistic predictor in the fashion of slide 9, and then sampling from it sequentially)

sequence classification

$p(\text{spam} | \text{"congratulations, you have won a prize"})$

Let's now look at a sequence-per-instance example: the spam classification task we saw earlier. We'll see how to approach this with a Markov model.

We are after the probability of the class, given the contents of the message.

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Bayes classifier

$$p(\text{spam} | W_1, \dots, W_n) \propto p(W_1, \dots, W_n | \text{spam}) p(\text{spam})$$

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We'll train a generative/Bayes classifier and use Bayes rule to flip around the probabilities.

$p(\text{spam})$ can be estimated as the proportion of spam emails in our data set. For the probability of the message given the class, we'll use our language model.

conditional on class

$$\begin{aligned} p(\text{prize}, \text{a}, \text{won}, \text{have}, \text{you}, \text{congratulations} | \text{spam}) \\ = p(\text{prize} | \text{a}, \text{won}, \text{spam}) p(\text{a} | \text{won}, \text{have}, \text{spam}) \\ p(\text{won} | \text{have}, \text{you}, \text{spam}) p(\text{have} | \text{you}, \text{congratulations}, \text{spam}) \\ p(\text{you} | \text{congratulations}, \text{spam}) p(\text{congratulations}, \text{spam}) \end{aligned}$$

$$p(\text{prize} | \text{a}, \text{won}, \text{spam}) \approx \frac{\#\text{spam} \text{ "won a prize"} }{\#\text{spam} \text{ "won a"}}$$

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We use the chain rule and the Markov assumption to define the probability that a message occurs (given that it's **spam**).

We then estimate the different conditional probabilities by computing the relative frequencies of bigrams and trigrams, as before, but we compute them only over the **spam** part of our data.

algorithm: 2nd-order markov model

training:

for each class **c**:

for **n** in 1, 2, 3:

count **n-grams** in all text belonging to class **c**

classification:

given text w_1, \dots, w_m

$$p(c | w_1, \dots, w_m) \propto p(w_1, \dots, w_m | c) p(c)$$

$$p(w_1, \dots, w_m | c) =$$

$$p(w_1 | c) p(w_2 | w_1, c) \prod_{i \in [3, \dots, m]} p(w_i | w_{i-1}, w_{i-2}, c)$$

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Markov modeling: final comments

• 0-order Markov model: Naive Bayes

For spam classification higher orders don't improve performance. For other tasks they do.

• Short documents are vastly more likely than long ones

Doesn't matter for classification. In other settings, conditioning on length may be necessary.

• Laplace smoothing

Same as before: adapt the estimator by adding pseudo-observations

$$p(\text{prize} | \text{a}, \text{won}) \approx \frac{1 + \#\text{ "won a prize"} }{|V| + \#\text{ "won a"}}$$

V is the vocabulary (the set of all words known to the language model).

As before, we can give the pseudo observations a smaller weight than 1, to have less impact on the estimate.

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the man fell **out of the** ... **pool**
gazebo
cycling

“congratulations you have won a prize”

$p(\text{spam} | \text{"congrats, a reward has been awarded to you"})$

These kinds of probability models, and many other models for symbolic sequential data, is that they model symbols as entirely separate objects. They don't exploit the fact that some words have similar meanings or syntactic functions.

For instance, the Markov assumption helps us with the man falling out of the pool, because “out of the *pool*” is a common phrase, even if nobody falls out of it. However, for rarer cases like “out of the *gazebo*” vs “out of the *cycling*” it may be that both trigrams never appear in the data. In that case, we might still say that the first is more likely because it at least ends in a *noun*, which is expected.

Similarly, if our training data give us little information about the message shown here, we would want a model that can conclude that the words in the sentences are very similar to words that do occur in the spam training corpus.

To deal with both cases, we need to somehow model word *similarity*. There are many ways to do this. One popular choice is to use an **embedding model**.

embedding models

Model object \mathbf{x} by embedding vector $e_{\mathbf{x}}$.

If e_x and e_y are “similar,” so are x and y

Learn the parameters of $\{e_x\}$ from data.

Cf: latent vectors from autoencoders, PCA, etc.

Embedding models take a set of discrete objects, and assign vectors to them. These vectors are then learned from data, based on some loss over the embeddings.

(This is a bit like mapping images to latent vectors in an autoencoder, except we learn the latent representation directly, instead of learning a function from some representation into the late representation).

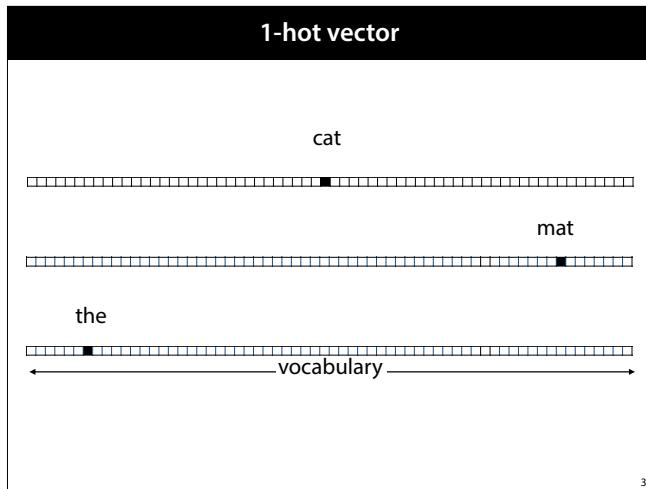
distributional hypothesis

Words that occur in the same context often have similar meanings.

The **distributional hypothesis** provides us with a sound way to train these embeddings.

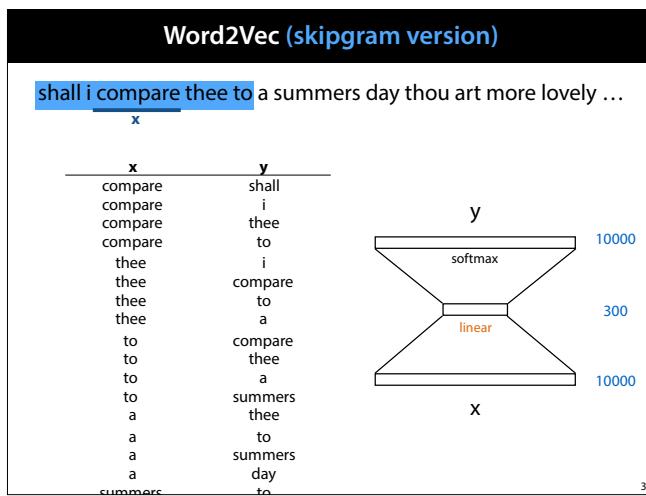
To paraphrase: if we have some word X, and we create a probability distribution P modelling how likely a word Y is to occur in the context of X, we can use that distribution P as a proxy for the *meaning* of X.

The algorithm we will discuss, Word2Vec, provides learns a simple mapping from the embedding to a probability distribution.



To start with, we'll represent words as atomic objects: in a single, very large one-hot vector.

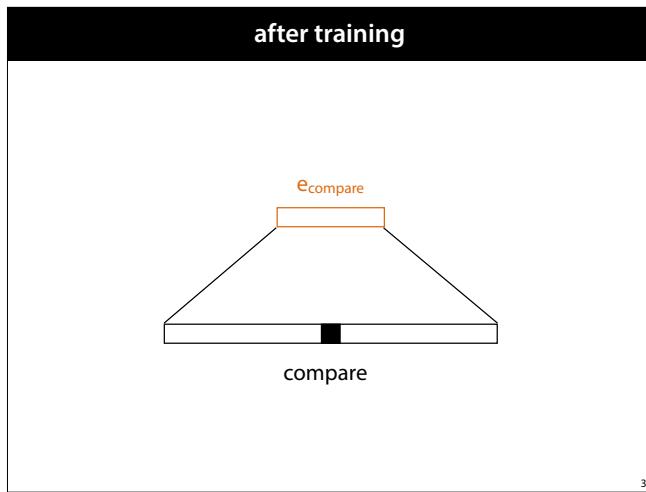
These are very big, but we can usually implement things in a clever way so that we won't explicitly need to store these in memory.



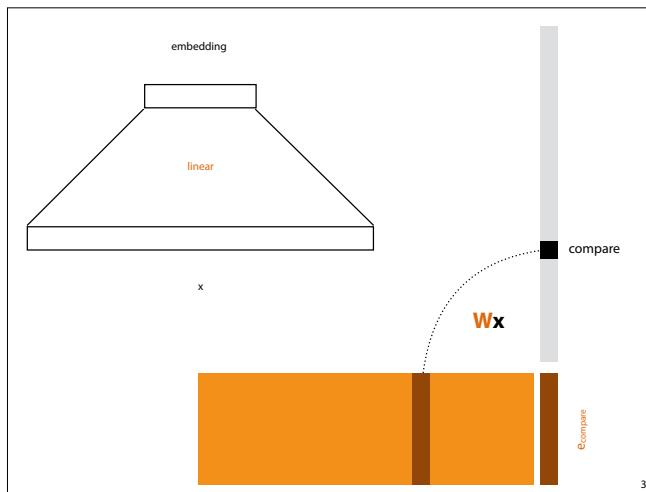
We slide a context window over the sequence. The task is to predict the distribution $p(y|x)$: that predict which words are likely to occur in the context window given the middle word.

We create a dataset of word pairs from the entire text and feed this to a very simple two-layer network. This is a bit like an autoencoder, except we're not reconstructing the output, but predicting the *context*.

The softmax activation over 10k outputs is very expensive to compute, and you need some clever tricks to make this feasible (called *hierarchical softmax* or negative sampling). We won't go into them here.



After training, we discard the second layer, and use only the embeddings produced by the first layer.

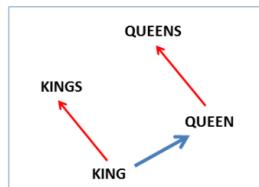
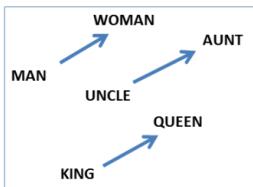


Because the input layer is just a matrix multiplication, and the input is just a one-hot vector, what we end up doing when we compute the embedding for word i , is just extracting the i -th column from \mathbf{W} .

In other words, we're not really training a function that computes an embedding for each word, we are actually learning the embeddings directly: every element of every embedding vector is a separate parameter.

$$v(\text{king}) + v(\text{woman}) - v(\text{man}) \approx v(\text{queen})$$

"feminine" vector



(Mikolov et al., NAACL HLT, 2013)

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word2vec summary

Word2Vec creates **embedding vectors for words**.

In the embedding space distances and directions reflect semantic meaning.

Word2Vec embeddings are a great starting point for deep learning projects on natural language.

Standard W2V embeddings can be downloaded from Google.

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break



by Josh Millard
via Jim Davis



by Josh Millard
via Jim Davis

Garfield generated by a Markov model

source: <https://blog.codinghorror.com/markov-and-you/>

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sequences in deep learning

Sequence models: operate on inputs of *different lengths* (using the same weights).

input: raw sequence data

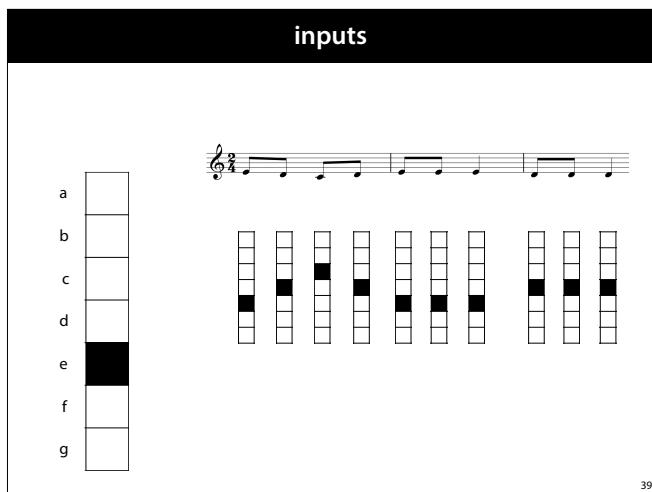
deep learning is *end-to-end* learning

output: classification, regression, token prediction

layers: sequence-to-sequence

Famously, Word2Vec produces not just an informative embedding, where similar words are close together, but for many concepts, there seems to be a kind of algebraic structure, similar to the smile vector example from the autoencoder.

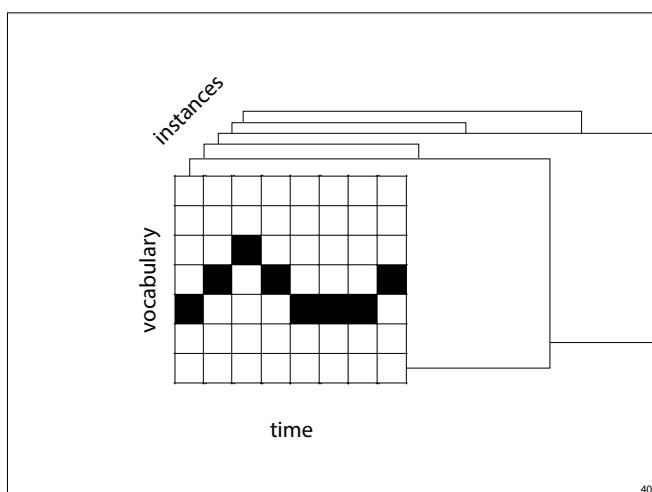
38



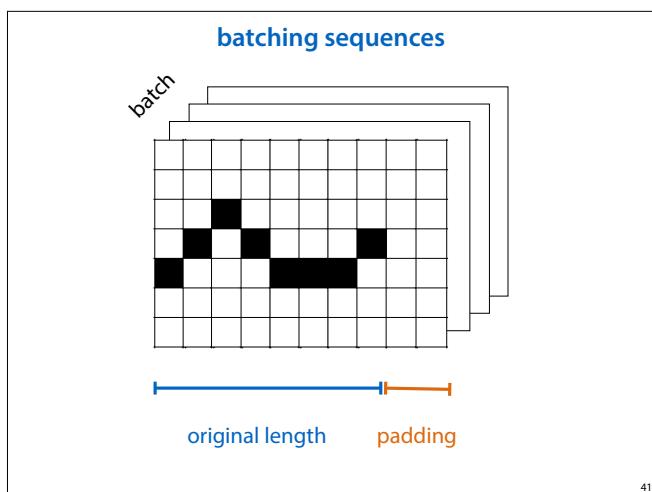
As we've seen, when we want to do deep learning, our input should be represented as a tensor. Preferably in a way that retains all information (i.e. we want to learn from the raw data, or something as close to it as possible).

Here is an example: to encode a simple monophonic musical sequence, we just one-hot encode the notes, and encode note sequence as a matrix: one dimension for time, one dimension for the notes. We can do the same thing for characters.

source: <https://violin-sheetmusic.org>

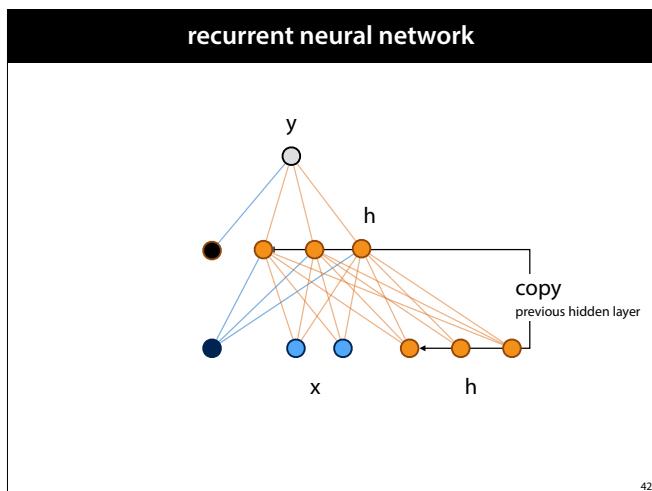


If we have multiple sequences of different lengths, this leads to a data set of matrices of different sizes.



In principle, this is not a problem, we want to build models that can deal with sequences of any length (and can generalise over sequences of variable lengths).

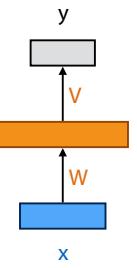
However, within a batch it's usually required that all sequences have the same length. Often, we put sequences of similar length together (for instance by sorting the data by length) and then pad the shorter sequences with zero vectors, so that all sequences are the same length.



One of the most popular neural networks for sequences is the **recurrent neural network**. This is a generic name for any neural network with cycles in it.

The figure shows a popular configuration. It's a basic fully connected network, except that its input x is extended by three nodes to which the hidden layer is copied.

visual shorthand



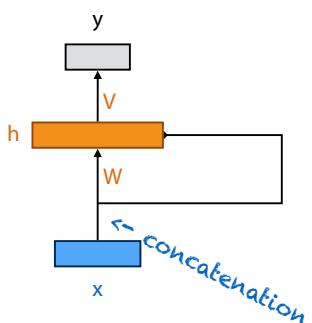
Adapted from
<http://colah.github.io/posts/2015-08-Understanding-LSTMs/>

To keep things clear we will adopt this visual shorthand: a rectangle represents a vector of nodes, and an arrow feeding into such a rectangle annotated with a weight matrix represents a fully connected transformation.

We will assume bias nodes are included without drawing them.

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visual shorthand

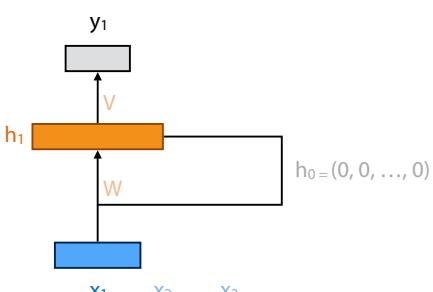


A line with no weight matrix represents a copy of the input vector. When two lines flow into each other, we concatenate their vectors.

Here, the added line copies h , concatenates it to x , and applies weight matrix W .

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RNNs on sequences

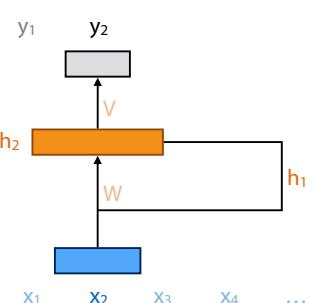


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We can now apply this neural network to a sequence. We feed it the first input, x_1 , result in a first value for the hidden layer, h_1 , and retrieve the first output y_1 .

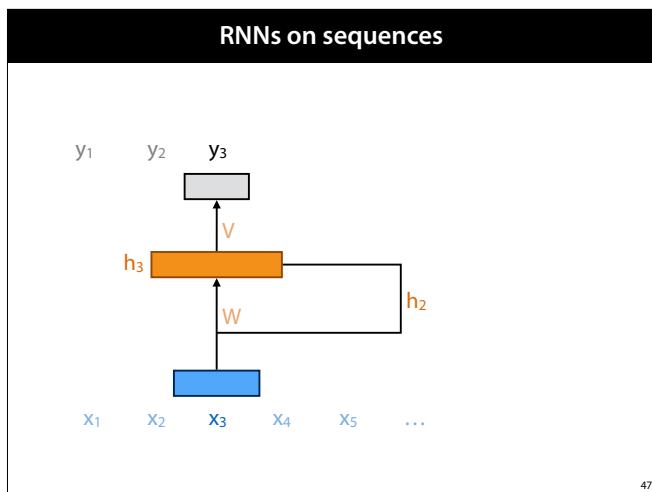
The hidden nodes are initialise to zero, so at first the network behaves just like a fully connected network.

RNNs on sequences

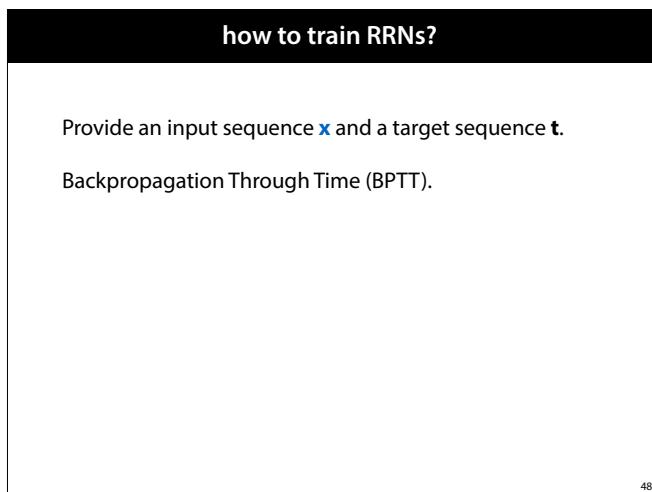


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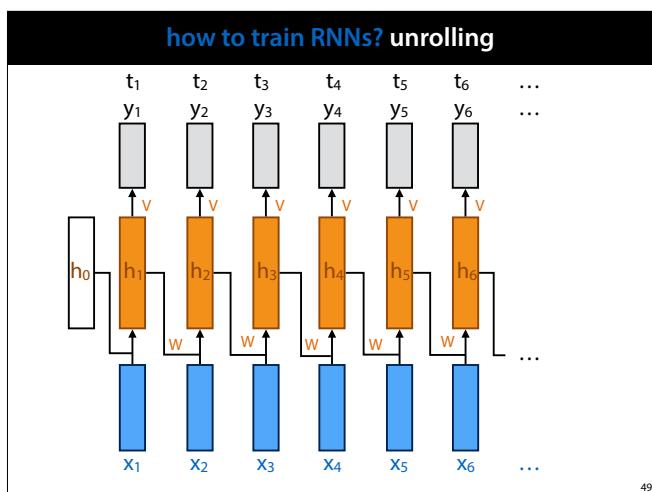
We then feed the second input in the sequence, x_1 . We now receive the *previous* hidden layer, h_1 , concatenate it to the input, and multiply it by W , to produce the second hidden layer h_2 . This is multiplied by V to produce the second output.



And so on.



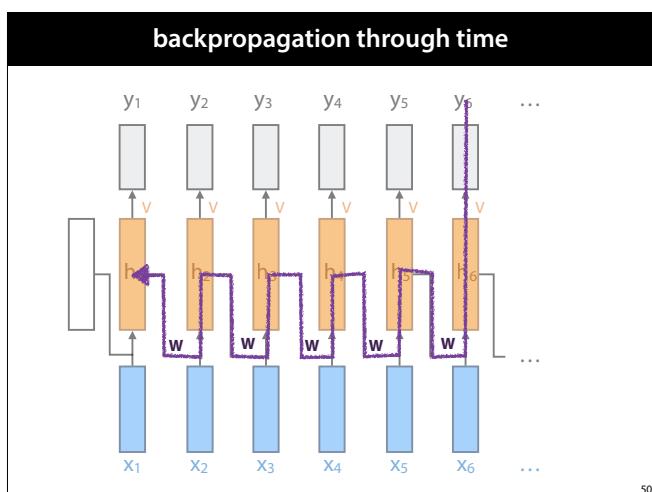
In principle, we



Instead of visualising a single small network, applied at every time step, we can **unroll** the network. Every step in the sequence is applied in parallel to a copy of the network, and the recurrent connection flows from the previous copy to the next.

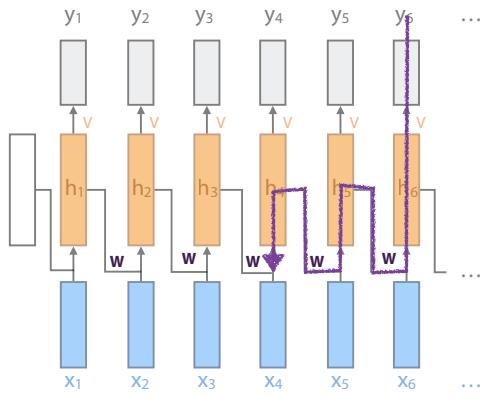
Now the whole network is just one big, complicated **feedforward net**, that is, a network *without* cycles. Note that we have a lot of shared weights, but we know how to deal with those.

The hidden layer is initialised to the zero vector.



Now the whole network is just one big, complicated feedforward net. Note that we have a lot of shared weights, but we know how to deal with those.

truncated backpropagation through time



In *truncated* backpropagation through time, we limit how far back the backpropagation goes, to save memory. The output is still entirely dependent on the whole sequence, but the weights are only trained based on the last few steps. Note that the weights are still affected everywhere, because they are shared between timesteps.

RNNs: thing to note

sequence-to-sequence model.

no Markov assumption (potentially infinite memory).

fixed set of weights, variable input length.

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what can we use RNNs for?

Sequence to sequence:

Eg. Translating English to French

Sequence to label

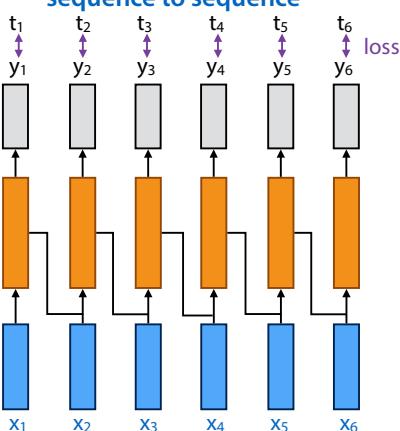
Eg. sequence classification, regression

Label to sequence

Eg. (conditional) sentence generation

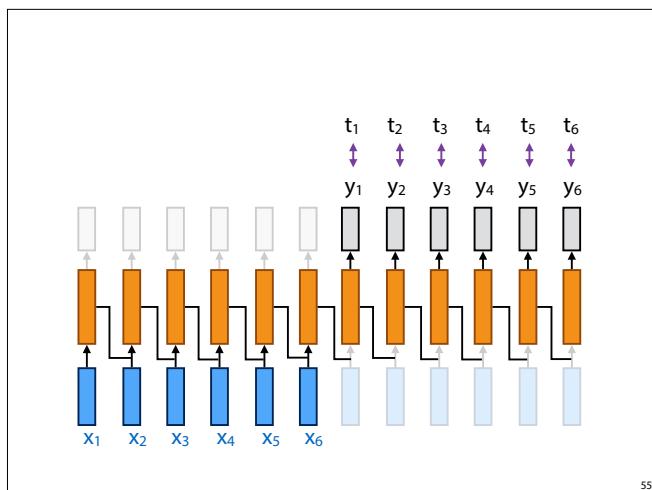
53

sequence to sequence



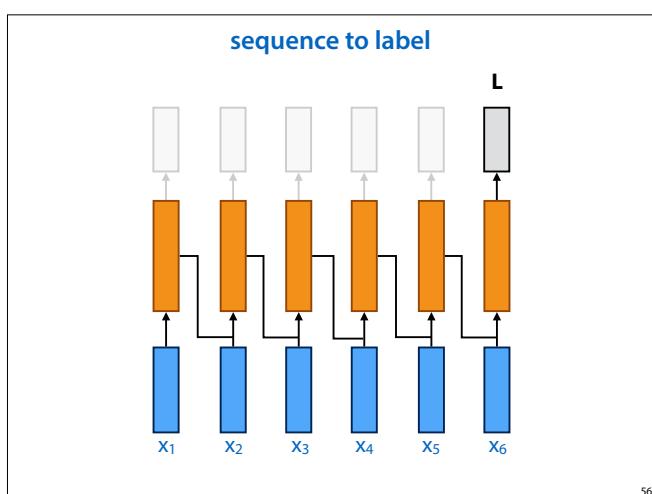
If this is the most straightforward way of doing sequence to sequence modeling. Note that the model is now forced to produce the first output before seeing the rest of the input.

54



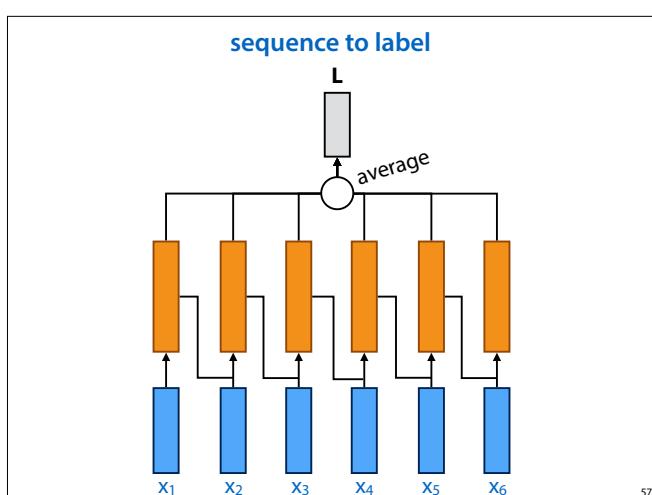
If we feed the input and output to the model like this (where the greyed out boxes represent zero vectors), the model gets to read the whole input before having to decide on an output.

Sometimes people even put a fixed number of zero vectors in between the input and the output to allow the model some "time" to process.



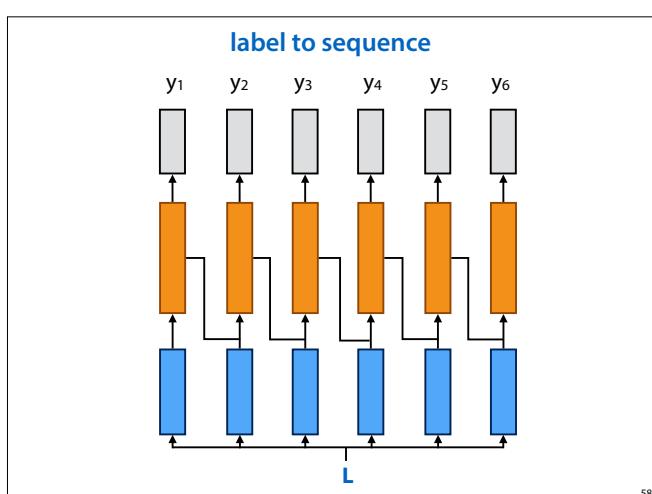
If we want to map sequences to labels, we could just ignore all outputs except the last, and take that to be the label. We can compare it to the target label and back propagate the error.

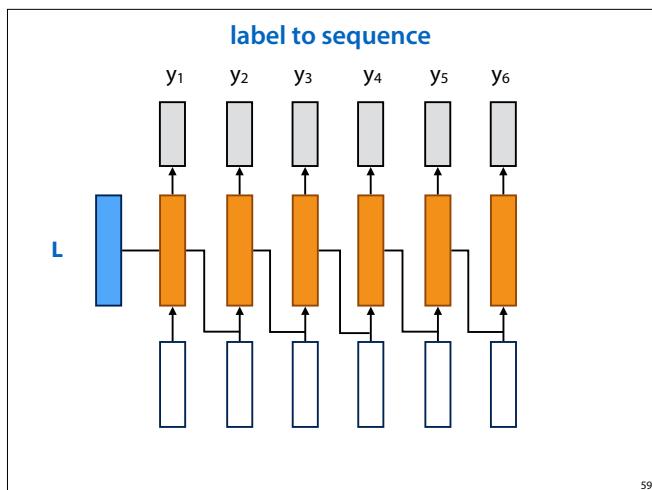
However, this puts a long distance between the first input and the label, and a very short distance between the last input and the label. This asymmetry may mean the model ignores information at the start of the sequence.



A better approach is to average the outputs over the whole sequence. This makes the distance to the label the same for every input.

Note that the model should be able to handle sequences of variable lengths: this means we can't put a fully connected layer on the top to combine the whole sequence into a single label.





We can also use the hidden layer at timestep 0 as an input for the label, and zero the sequence inputs (or use them for something else).

the problem of long-term dependence

I was born in **France**, as matter of fact in a little village near Paris, it's famous for its pain-au-chocolat, I lived there until I was 16, when I moved to Amsterdam, so I'm fluent in...

French
Dutch
Aquarium

60

Basic RNNs work pretty well, but they do not learn to remember information for very long. Technically they can, but the gradient vanished too quickly over the timesteps.

You can't have a long term memory for everything. You need to be selective, and you need to learn to select words to be stored for the long term when you first see them.

In order to remember things long term you need to forget many other things.

LSTM (1997)

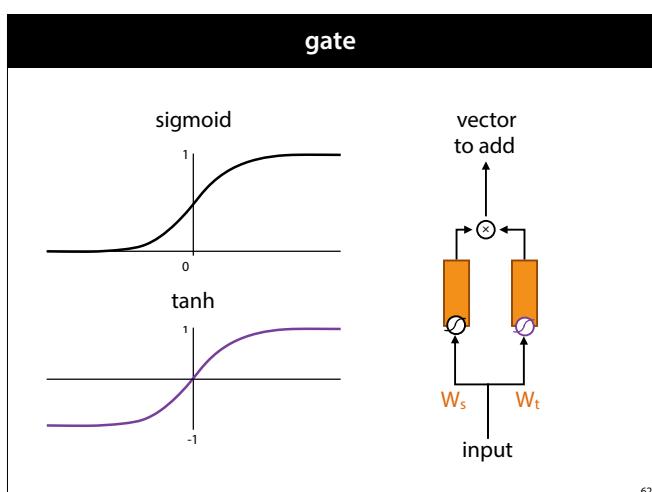
Long short-term memory

Selective forgetting and remembering, controlled by learnable "gates"

Possibly the first successful deep neural network

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An enduring solution to the problem are LSTMs. LSTMs have a complex mechanism, which we'll go through step by step, but the main component is a gating mechanism.



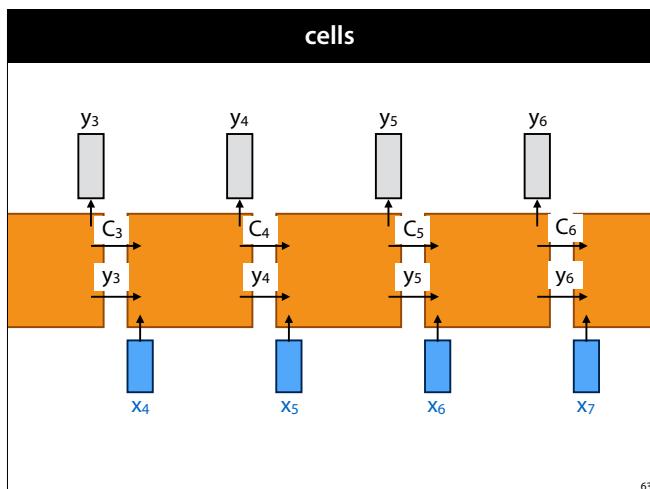
The gate combines the sigmoid and tanh activations. The sigmoid we've seen already. The tanh is just a the sigmoid rescaled so that its outputs are between -1 and 1.

The gating mechanism takes two input vectors, and combines them using a sigmoid and a tanh activation. The gate is best understand as producing an additive value: we want to figure out how much of the input to add to some other vector (if it's import, we want to add most of it, otherwise, we want to forget it, and keep the original value).

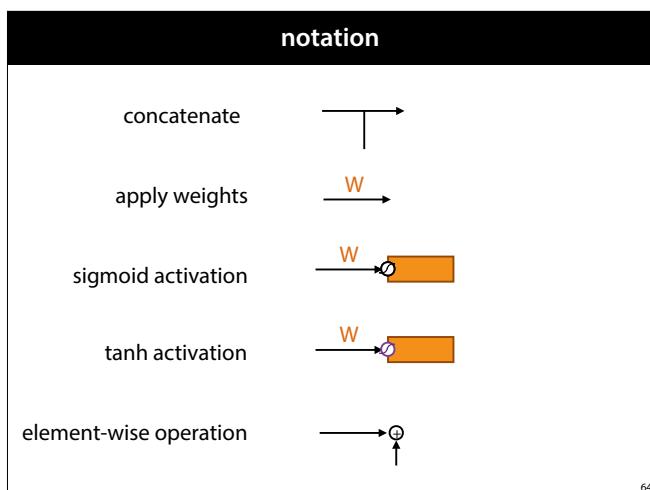
The input is first transformed by two weight metrics and then passed though a sigmoid and a tanh. The tanh should be thought of as a mapping of the input to the range [-1, 1]. This ensures that the effect of the addition vector can't be too much. The

sigmoid acts as a selection vector. For elements of the input that are important, it outputs 1, retaining all the input in the addition vector. For elements of the input that are not important, it outputs 0, so that they are zeroed out. The sigmoid and tanh vectors are element-wise multiplied.

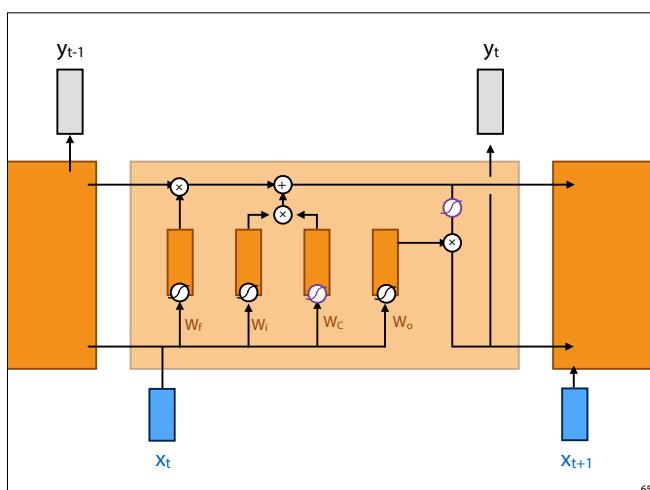
Note that if we initialise W_t and W_s to zero, the input is entirely ignored.



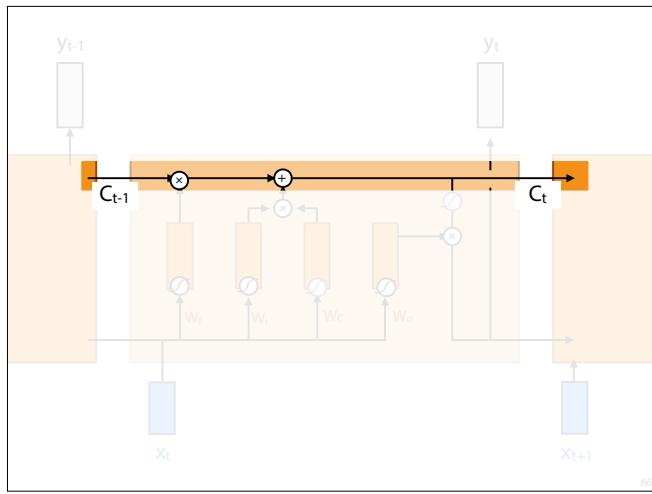
The basic operation of the LSTM is called a cell (the orange square, which we'll detail later). Between cells, there are two recurrent connections, y , the current output, and C the **cell state**.



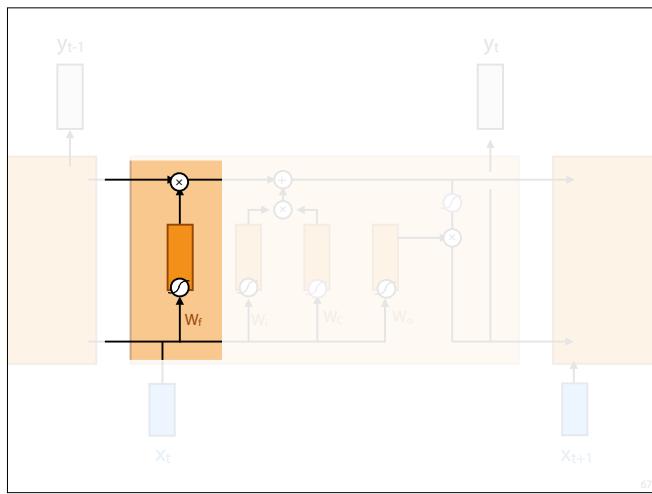
Here is our visual notation.



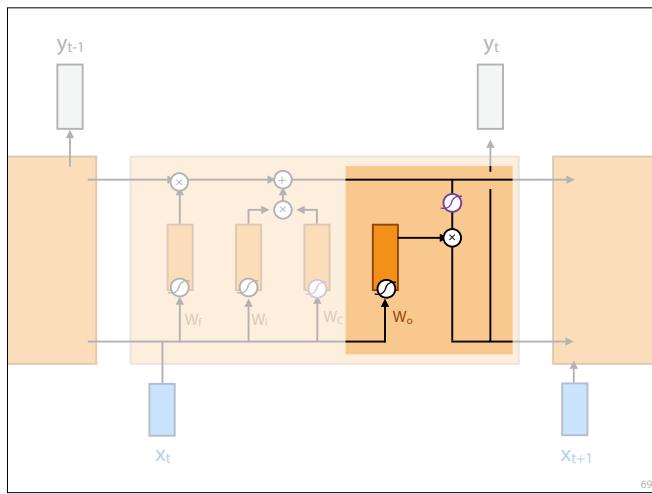
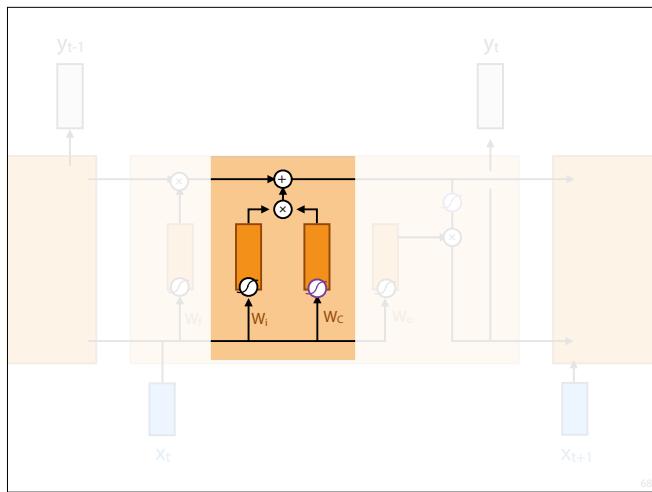
Here is what happens inside the cell. It looks complicated, but we'll go through all the elements step by step.



The first is the “conveyor belt”. It passes the previous cell state to the next cell. Along the way, the current input can be used to manipulate it.



It looks at the current **input**, concatenated with the previous **output**, and applies an element-wise scaling to the current value in the conveyor belt. Outputting all 1s will keep the current value on the belt what it is, and outputting all values near 0, will decay the values (forgetting what we've seen so far, and allowing it to be replaced by our new values in the next step).



Finally, we need to decide what to output now. We take the current value of the conveyor belt, tanh it to rescale, and element-wise multiply it by another sigmoid activated layer. This layer is sent out as the current output, and sent to the next cell along the second recurrent connection.

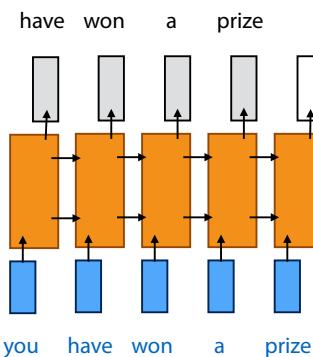
LSTM language model

$p(\text{prize} \mid \text{a, won, have, you, congratulations})$

NB: no Markov assumption!

70

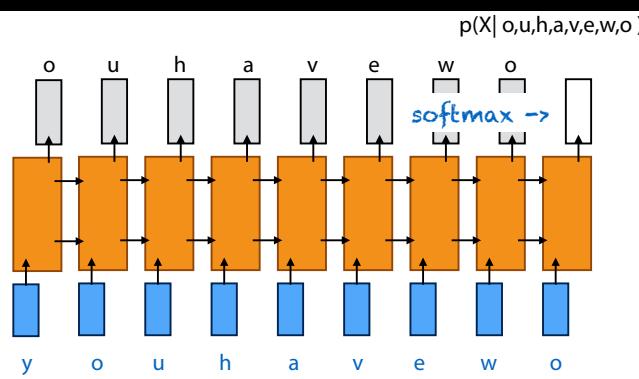
LSTM language model



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We train the language model to predict the next token in the sequence. This is easy to do: we just use the sequence shifted by 1 as the training sequence.

character level language model



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We'll do this at character level, to make it more challenging, and we ensure that the network outputs a softmax activated vector over all characters at each timestep. We can then interpret the output of the model at step t as the probability of the character at that point, conditioned on the entire history.

sequential sampling from a language model

start with a small seed sequence $s = [c_1, c_2, c_3]$ of tokens.

loop:

Sample next char c according to $p(C = c \mid c_1, c_2, \dots)$
feed the *whole* seed to the network

append c to s

also known as a *autoregressive RNN*

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Note that this time, as there is no Markov assumption, the network has to see the *whole sequence* so far to predict the next character.

some examples

source: The Unreasonable Effectiveness of Recurrent Neural Networks
Andrej Karpathy

<http://karpathy.github.io/2015/05/21/rnn-effectiveness/>

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Shakespeare

PANDARUS:

Alas, I think he shall be come
approached and the day
When little strain would be
attain'd into being never fed,
And who is but a chain and
subjects of his death,
I should not sleep.

Second Senator:

They are away this miseries,
produced upon my soul,
Breaking and strongly should be
buried, when I perish
The earth and thoughts of many

75

Remember, this is a **character level** language model.

Wikipedia

Naturalism and decision for the majority of Arab countries' capitalide was grounded by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal in the [[Protestant Immineners]], which could be said to be directly in Cantonese Communication, which followed a ceremony and set inspired prison, training. The emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known in western [[Scotland]], near Italy to the conquest of India with the conflict. Copyright was the succession of independence in the slop of Syrian influence that was a famous German movement based on a more

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Note that not only is the language natural, the wikipedia markup is also correct (link brackets are closed properly, and contain key concepts).

October 25|21]] to note, the kingdom of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known in western [[Scotland]], near Italy to the conquest of India with the conflict. Copyright was the succession of independence in the slop of Syrian influence that was a famous German movement based on a more popular servicious, non-doctrinal and sexual power post. Many governments recognize the military housing of the [[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]], that is sympathetic to be to the [[Punjab Resolution]] (PJS)[<http://www.humah.yahoo.com/guardian.cfm/7754800786d17551963s89.htm>] Official economics Adjoint for the Nazism, Montgomery was swear to advance to the resources for those Socialism's rule, was starting to signing a major tripad of aid exile.]]

77

The network can even learn to generate valid (looking) URLs for external links.

```

<page>
  <title>Antichrist</title>
  <id>865</id>
  <revision>
    <id>15900676</id>
    <timestamp>2002-08-03T18:14:12Z</timestamp>
    <contributor>
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      <id>23</id>
    </contributor>
    <minor />
    <comment>Automated conversion</comment>
    <text xml:space="preserve">#REDIRECT
[[Christianity]]</text>
</revision>
</page>

```

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Sometimes wikipedia text contains bits of XML for structured information. The model can generate these flawlessly.

LaTeX

For $\bigoplus_{i=1}^n \mathcal{F}_i$, where $\mathcal{L}_{\mathcal{O}_X} = 0$, hence we can find a closed subset H in \mathcal{H} and any sets \mathcal{F}_i on $X \setminus H$ is a closed immersion of S_i , then $U \rightarrow T$ is a separated algebraic space.

Proof. Proof of (1). It also start we get

$$S = \text{Spec}(R) = U \times_X T \times_S U$$

and the similarly in the fiber product covering we have to prove the lemma generated by $\coprod Z \times_U V \rightarrow V$. Consider the maps M along the set of points Spec_M and $T \rightarrow U$ is the fiber category of S in U in Section ?? and the fact that any U affine, we Morphism $G \rightarrow S$ such that we can find a scheme S and any open subset $W \subset U$ in $\text{Sh}(G)$ such that $\text{Spec}(R) \rightarrow S$ is smooth or an

$$U = \bigcup U_i \times_S U_i$$

which has a nonzero morphism we may assume that f_i is of finite presentation over S . We claim that $\mathcal{O}_{X,i}$ is a scheme where $x, x', x'' \in S'$ such that $\mathcal{O}_{X,x'} \rightarrow \mathcal{O}_{X,x''}$ is separated. By Algebra, Lemma ?? we can define a map of complexes $\text{GL}_{\mathcal{O}}(x'/S')$.

To prove study we see that \mathcal{F}_j is a covering of X , and T_j is an object of $\mathcal{F}_{X,j}$ for $j \geq 0$ and \mathcal{F}_j exists and \mathcal{F}_j be a presheaf of \mathcal{O}_X -modules on C as a \mathcal{F} -module. In particular $\mathcal{F} = U/J$ we have to show that

$$M^{\bullet} = T^{\bullet} \otimes_{\text{Spec}(R)} \mathcal{O}_{X,i} = i_X^*(\mathcal{F})$$

is a unique morphism of algebra stacks. Note that

$$\text{Arrows} = (\mathcal{S}/\mathcal{S})_{\text{Iperf}}, (\mathcal{S}/\mathcal{S})_{\text{Iperf}}$$

and

$$V = \Gamma(S, \mathcal{O}) \dashrightarrow (U, \text{Spec}(A))$$

is an open subset of X . Thus U is affine. This is a continuous map of X is the inverse, the groupoid scheme S .

Proof. See discussion of sheaves of sets.

The result for prove any open covering follows from the less of Example ??, we replace S by $X_{\text{proper, f.flat}}$ which gives an open subspace of X and T equal to $S_{\Sigma_{\mathcal{F}}}$, see Definition ??, Namely, by Lemma ?? we see that R is geometrically regular over S .

Lemma 0.1. Assume (3) and (3) by the construction in the description. Suppose $X = \lim(X_i)$ by the formal open covering X and a single map $\text{Proj}_X(A) = \text{Spec}(B)$ over U compatible with the complex

$$\text{Spec}(A) = \Gamma(X, \mathcal{O}_X \otimes_{\mathcal{O}_X} \mathcal{O}_X)$$

When in this case we to show that $Q = \mathcal{C}_Z X$ is stable under the following result in the second conditions of (1). Q is a closed subscheme of X implies the proof. By Definition ?? (without element is when the closed subschemes are catenary. If T is surjective we may assume that T is connected with residue fields of S . Moreover there exists a closed subscheme $C \subset X$ of X such that $X \setminus C$ is proper. Then C is defined as a closed subset of the neighborhood. Suffices to check that the following theorem

(1) T is locally of finite type. Since $S = \text{Spec}(R)$ and $Y = \text{Spec}(R)$.

Proof. This is form all sheaves of sheaves on X . But given a scheme U and a surjective étale morphism $U \rightarrow X$. Let $U \cap U_i = \coprod_{i=1, \dots, n} U_i$ be the scheme X over S at the schemes $X_i \rightarrow X$ and $U = \lim X_i$.

The following lemma subjective reconstroemps this implies that $\mathcal{F}_{n+1} = \mathcal{F}_{n+2} = \dots$

Lemma 0.2. Let X be a locally Noetherian scheme over S . $E = \mathcal{O}_{X,S}$. Set $T = \mathcal{O}_{S,C}^{\bullet}$. Since $T^{\bullet} \subset T^{\bullet}$ are nonzero over $i \leq p$ is a subset of $J_{n+1} \supset J_n$ works.

Lemma 0.3. In Situation ??, Hence we may assume that p is the next functor (??). On the other hand, by Lemma ?? we see that

$$D(\mathcal{O}_X) = \mathcal{O}_X(D)$$

where K is an F -algebra where b_{n+1} is a scheme over S .

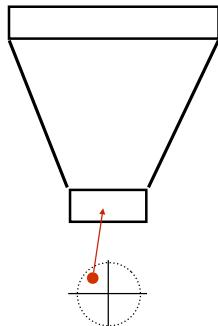
79

<https://twitter.com/deepdrumpf?lang=en>

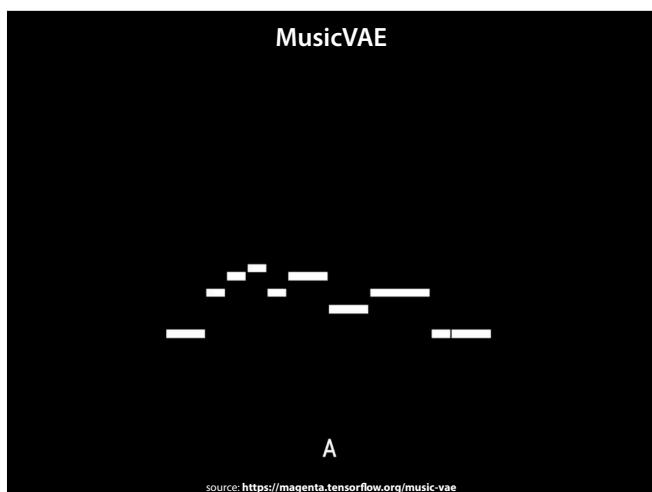
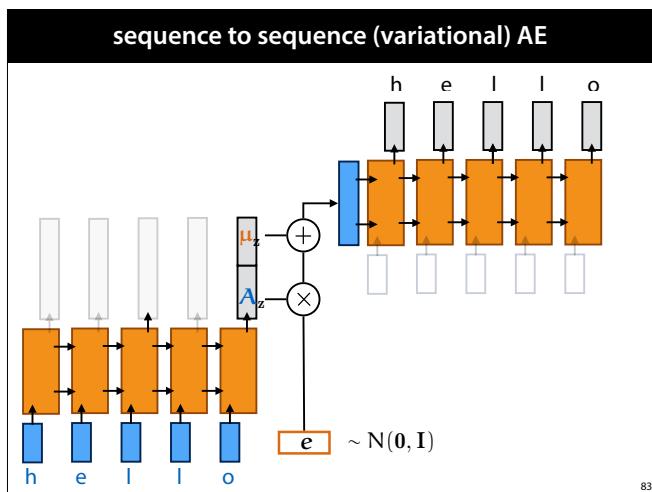
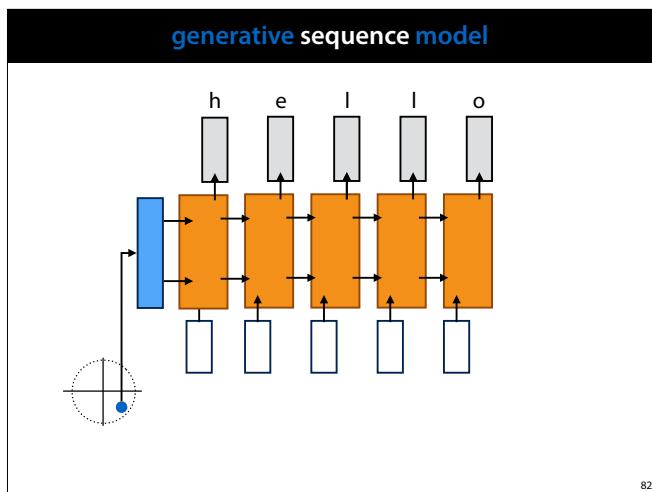


80

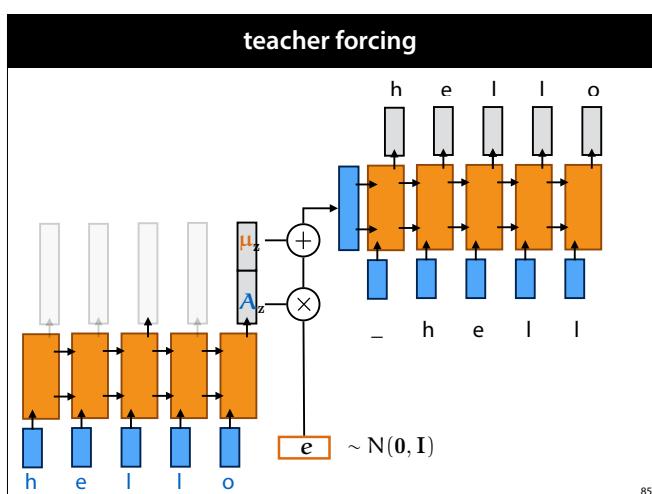
generative sequence model

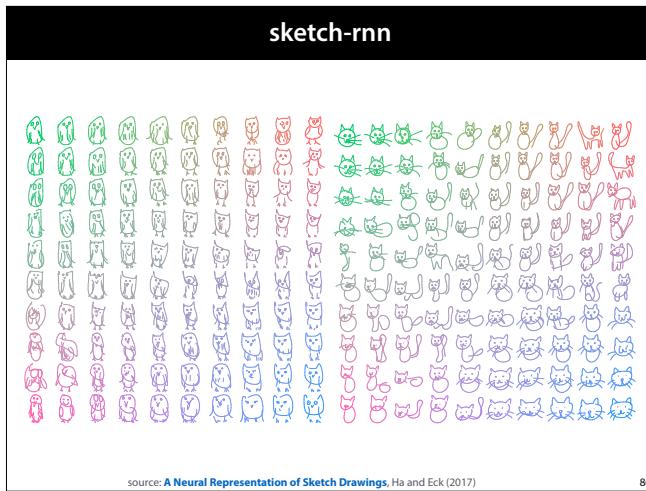


81



source: <https://magenta.tensorflow.org/music-vae>





Here we see two interpolation grids for a sequence-to-sequence VAE trained on a dataset of millions of quickly drawn sketches.

interpolation

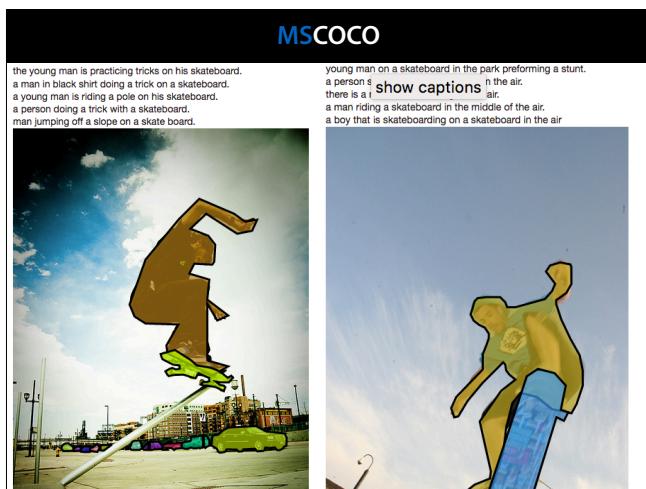
AE
i went to the store to buy some groceries . i store to buy some groceries . i were to buy any groceries . horses are to buy any groceries . horses are to buy any animal . horses the favorite any animal . horses the favorite favorite animal . horses are my favorite animal .
VAE
he was silent for a long moment . he was silent for a moment . it was quiet for a moment . it was dark and cold . there was a pause . it was my turn .

source: [Generating Sentences from a Continuous Space](#) by Samuel R. Bowman

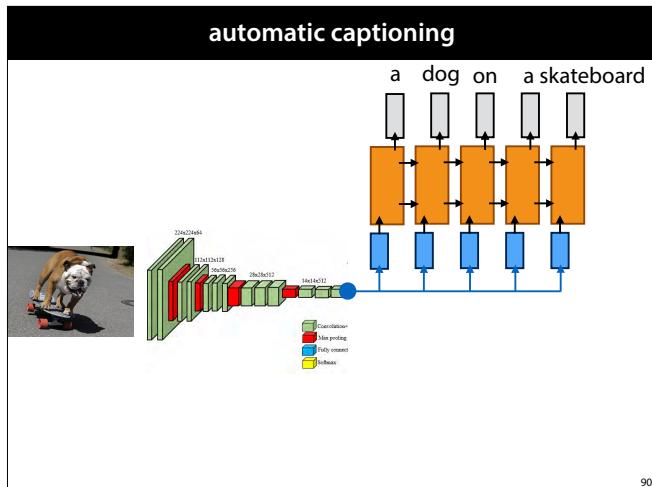
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Here is an example of *interpolation* on sentences. First using a regular autoencoder, and then using a VAE. Note that the intermediate sentences for the AE are non-grammatical, but the intermediate sentences for the VAE are all grammatical.

source: *Generating Sentences from a Continuous Space* by Samuel R. Bowman, Luke Vilnis, Oriol Vinyals, Andrew M. Dai, Rafal Jozefowicz, Samy Bengio
<https://arxiv.org/abs/1511.06349>

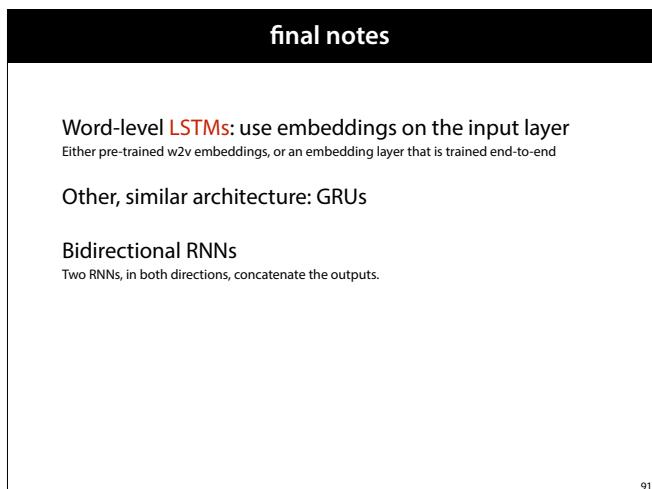


cocodataset.org

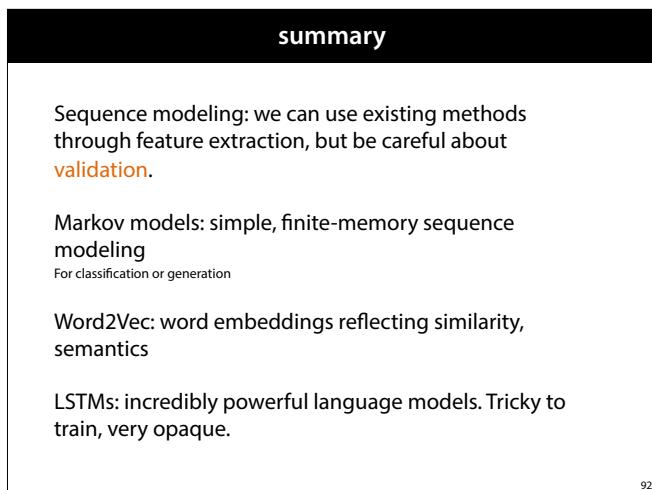


Here's a simple approach to the image captioning problem, which you . We take a pretrained image classification network (see the transfer learning slide from Deep Learning 2) and feed the output to an LSTM. We train end-to-end and use sequential sampling to generate outputs. This is a model that you **should be able to train on a high end laptop with a basic GPU.**

This will give you a pretty decently performing model. To reach the state of the art, you also need something called an "attention mechanism." But that's outside the scope of this lecture.



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