

Machine Learning

Introduction, Data, Modeling

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octobre 2020







Supervised

Unsupervised

Reinforcement



Course structure

Three full days:

- 2, 7, 16 octobre

Course structure

- Email jeff@p27.eu
- Github: https://github.com/mlweek/MLWeek_v3

Curse of Dimensionality

Machine learning is not magic

Machine learning is mathematics

Mostly, it's these maths:

- Probability
- Statistics
- Linear algebra
- Optimisation theory
- Differential calculus

Unless you want to, we'll skip the maths.

Probability

Probability

events

- independent
- dependent

Statistics

What is Statistics

- ① Identify a question or problem.
- ② Collect relevant data on the topic.
- ③ Analyze the data.
- ④ Form a conclusion.

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Sadly, sometimes people forget 1.

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- ② Collect relevant data on the topic.
- ③ Analyze the data.
- ④ Form a conclusion.

Statistics is about making 2–4 efficient, rigorous, and meaningful.

OpenIntro Statistics, 2nd edition, D. Diez, C. Barr, M. Çetinkaya-Rundel, 2013.

What is data science?

(Exercise: Is this the same question as the last slide?)

- ① Define the question of interest
- ② Get the data
- ③ Clean the data
- ④ Explore the data
- ⑤ Fit statistical models
- ⑥ Communicate the results
- ⑦ Make your analysis reproducible

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What the public thinks.

What is data science?

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- ⑥ Communicate the results
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Where we spend most of our time.

What is data science?

(Exercise: Is this the same question as the last slide?)

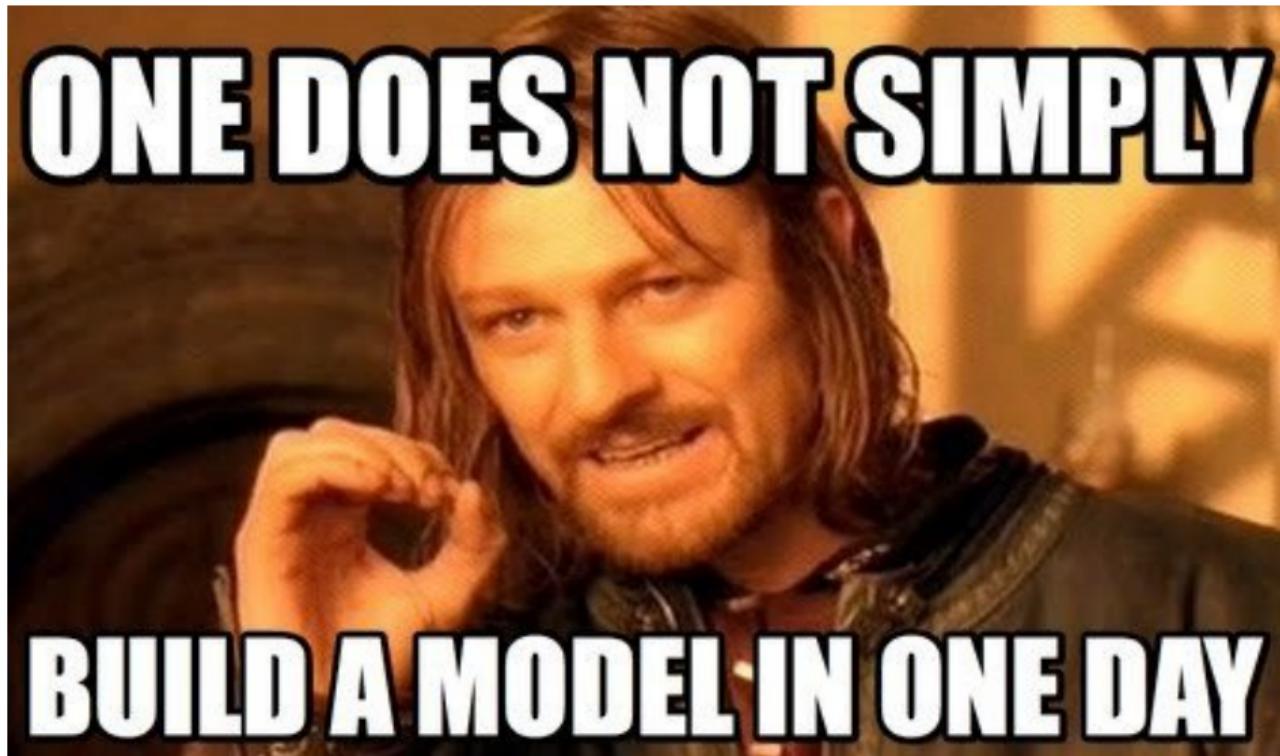
- ① Define the question of interest
- ② Get the data
- ③ Clean the data
- ④ Explore the data
- ⑤ Fit statistical models
- ⑥ Communicate the results
- ⑦ Make your analysis reproducible

The easiest part to forget.

What is data science?

*[http://simplystatistics.org/2015/03/17/
data-science-done-well-looks-easy-and-that-is-a-big-
problem-for-data-scientists/](http://simplystatistics.org/2015/03/17/data-science-done-well-looks-easy-and-that-is-a-big-problem-for-data-scientists/)*

What is data science?



Anecdote

Some properties of anecdote:

- is data
- haphazardly collected
- is generally not representative
- sometimes result of selective retention
- does not accumulate to be representative
- might be true (by chance)
- is ok to use as hypothesis, but be clear that hypothesis is anecdote

Study Types

- Observational
- Experimental

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- Experimental

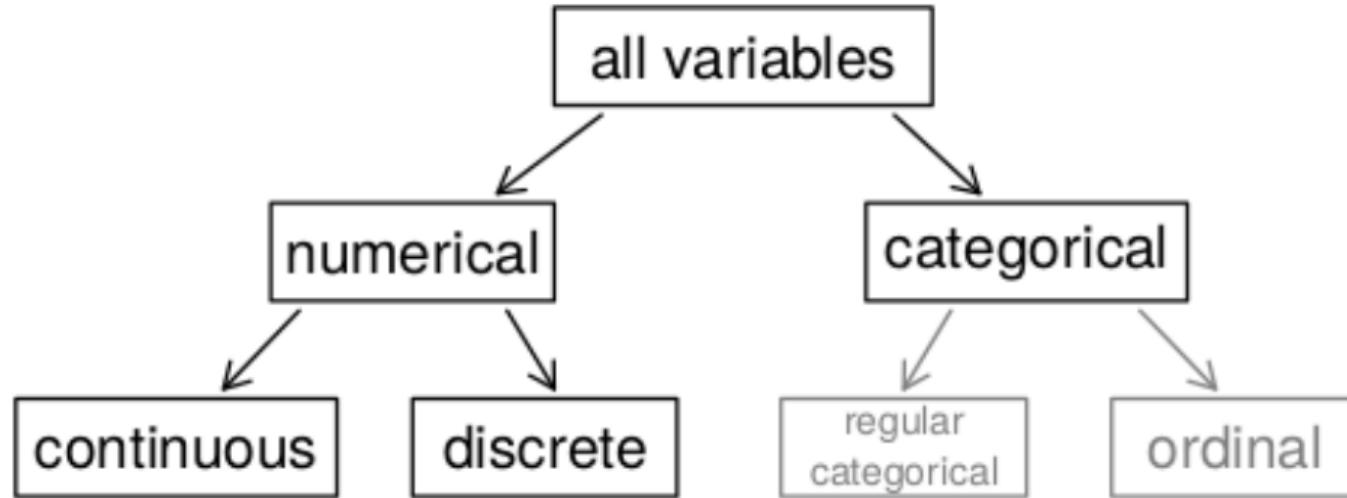
What can go wrong?

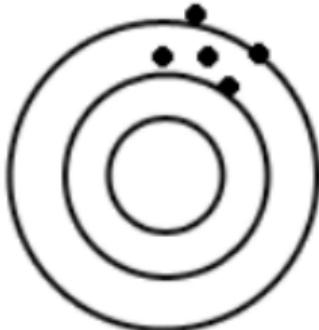
- Forgetting that association \neq causation
- Not random
- Confounding variables

Variables and statistics

- Input: features (“explanatory variables”)
- Output: response variable
- Training set (learn parameters)
- Test Set (check learned parameters)
- Validation set (check learned hyperparameters)
- Cross validation (and jack knife and ...)
- Bias and variance (picture coming up)

Variable types





High bias, low variance



Low bias, high variance



High bias, high variance



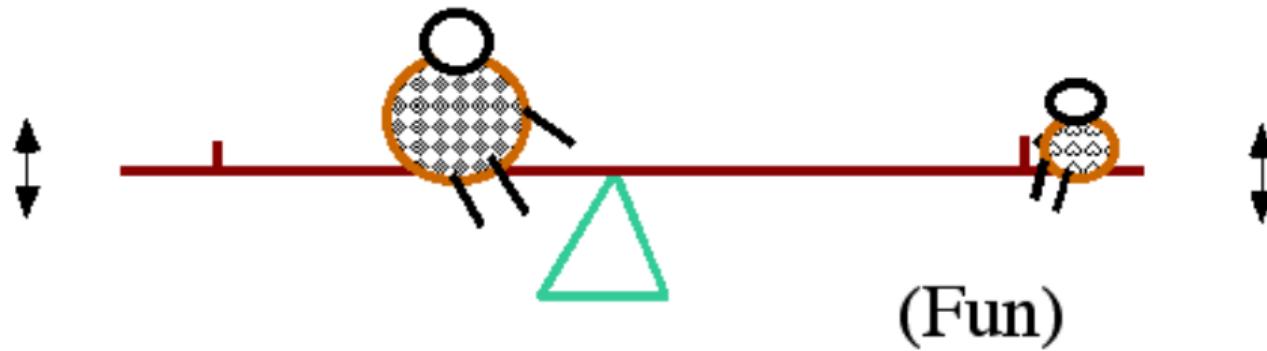
Low bias, low variance

Mean

- Weighted and unweighted
- Centroid to physicists

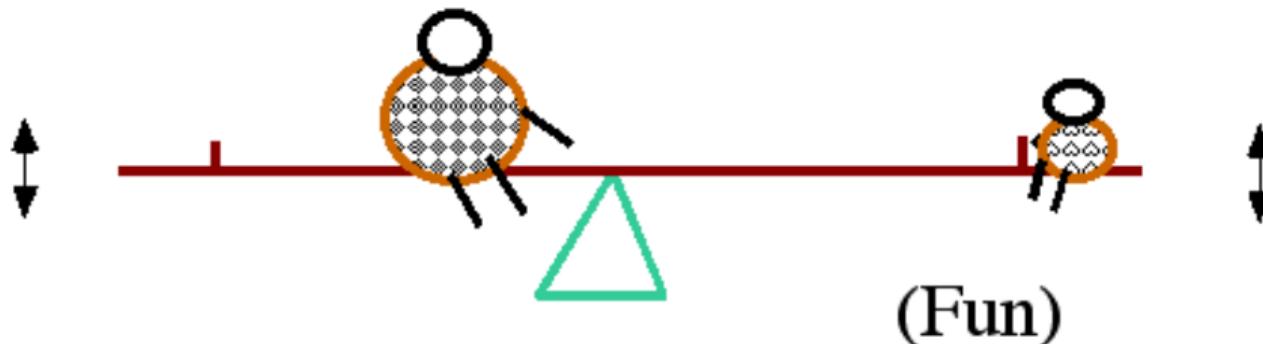
Mean

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Mean

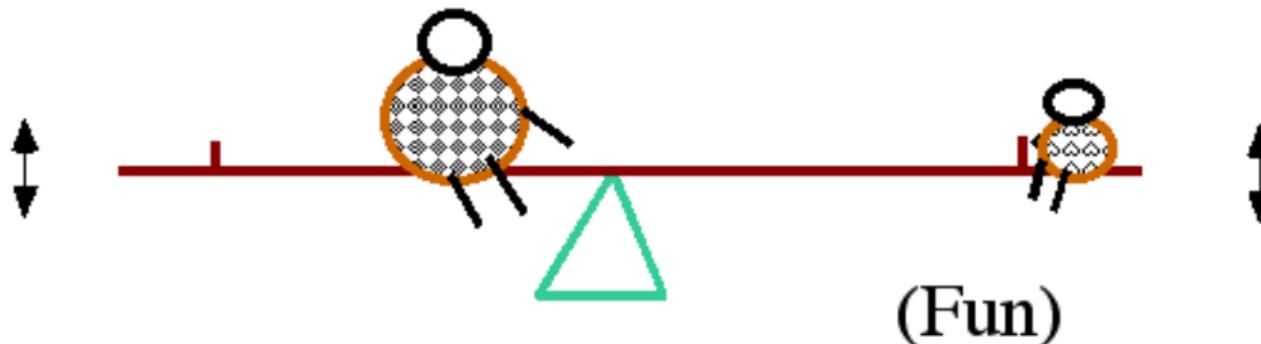
- Weighted and unweighted
- Centroid to physicists



$$\mu = E(X) = \sum w_i x_i = \mathbf{w} \cdot \mathbf{x}$$

Mean

- Weighted and unweighted
- Centroid to physicists



$$\mu = E(X) = \sum \Pr(X = x_i)x_i$$

Mean

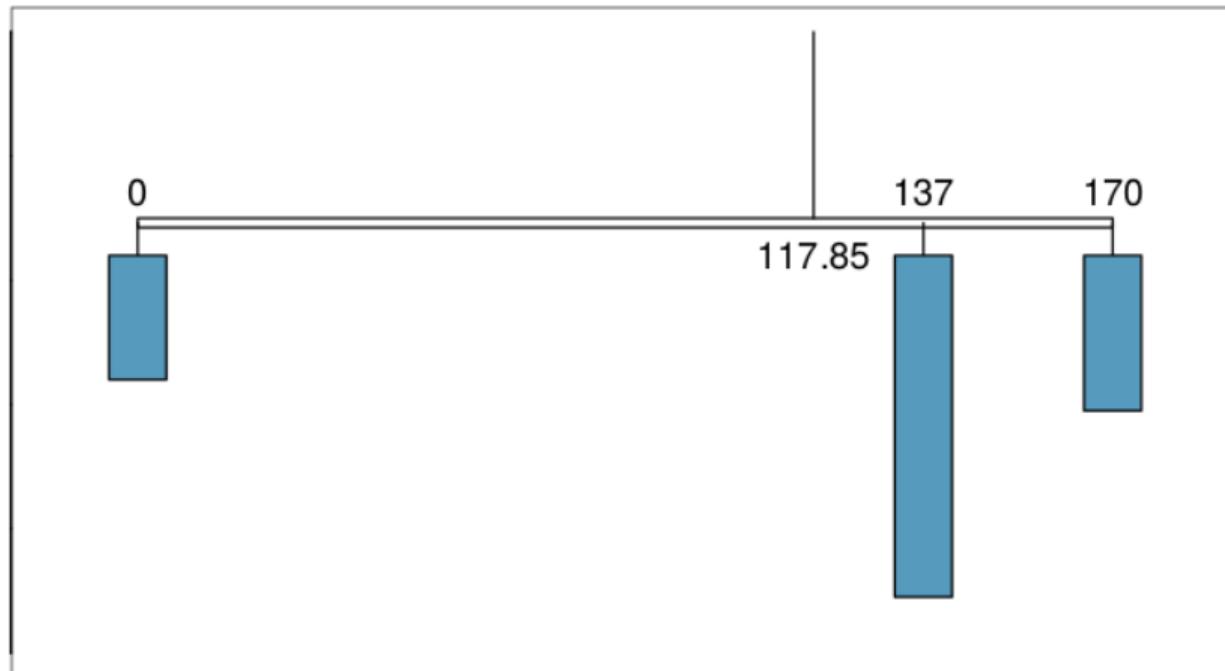
- Weighted and unweighted
- Centroid to physicists

$$\mu = E(X) = \int xf(x) dx$$

<http://telescopes.stardate.org/images/research/teeter-totter/TT4.gif>

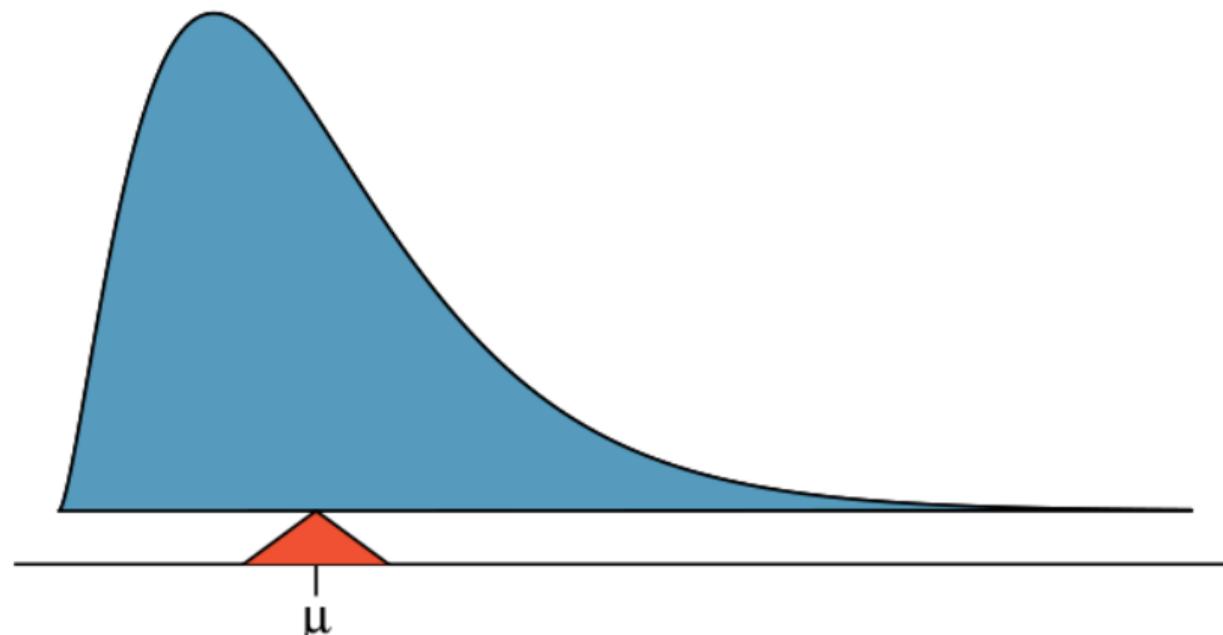
Mean

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Mean

- Weighted and unweighted
- Centroid to physicists



Population statistics

Mean is just a sum.

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

This is a special case ($w_i = 1$) of the weighted mean:

$$\mu = \frac{1}{\sum w_i} \cdot \sum_{i=1}^N w_i x_i$$

Population statistics

Deviation is distance from mean.

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\text{Deviation of } x_i = \mu - x_i$$

Population statistics

Variance is the mean square of deviations

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

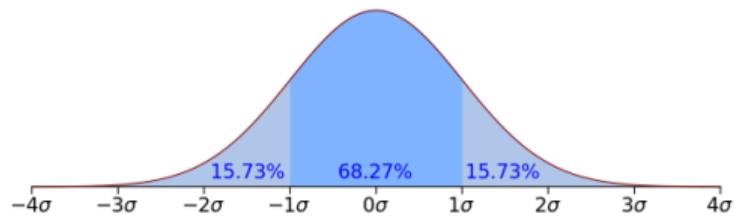
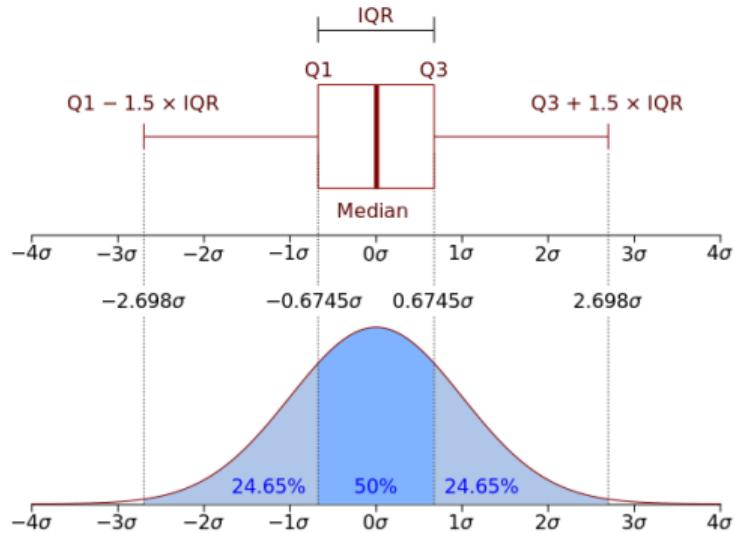
$$\text{Var}(X) = \sigma^2 = \frac{1}{N} \sum_{i=1}^N (\mu - x_i)^2$$

Population statistics

Standard deviation is square root of variance

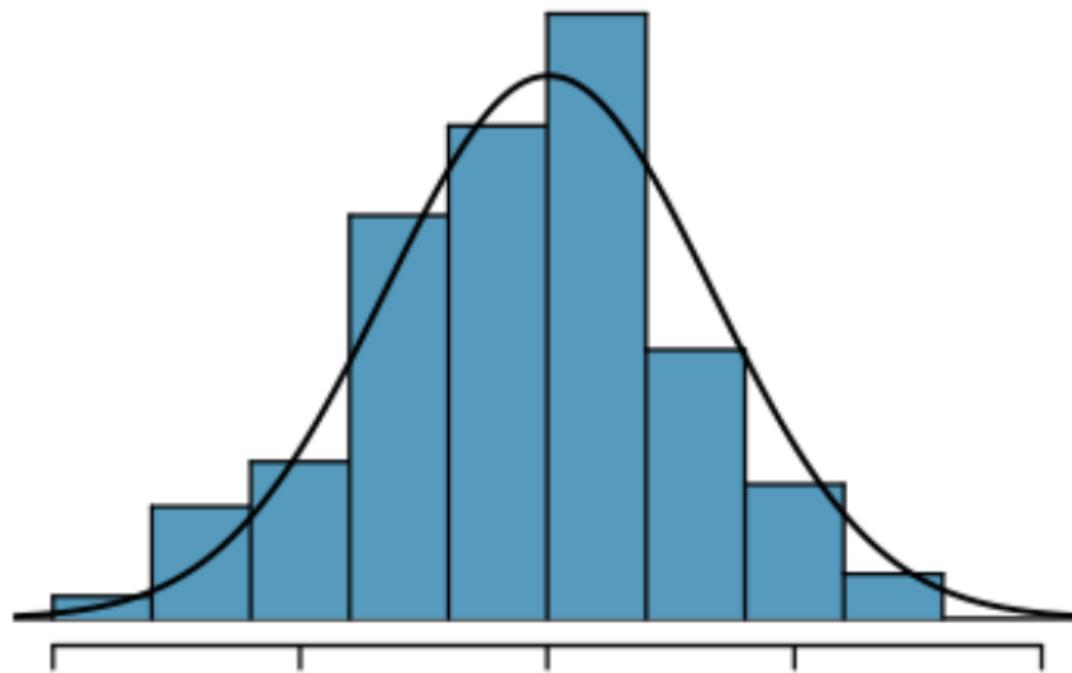
$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

$$\sigma = \sqrt{\text{Var}(X)} = \sqrt{\sigma^2} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\mu - x_i)^2}$$



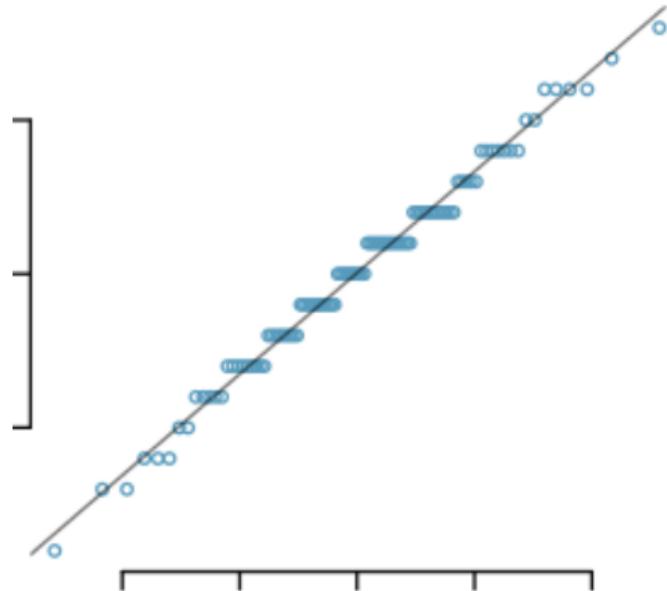
Evaluating Normal Approximations

Easy technique 1: visually compare to normal plot.



Evaluating Normal Approximations

Easy technique 2: normal probability plot.



Also known as a quantile-quantile plot.

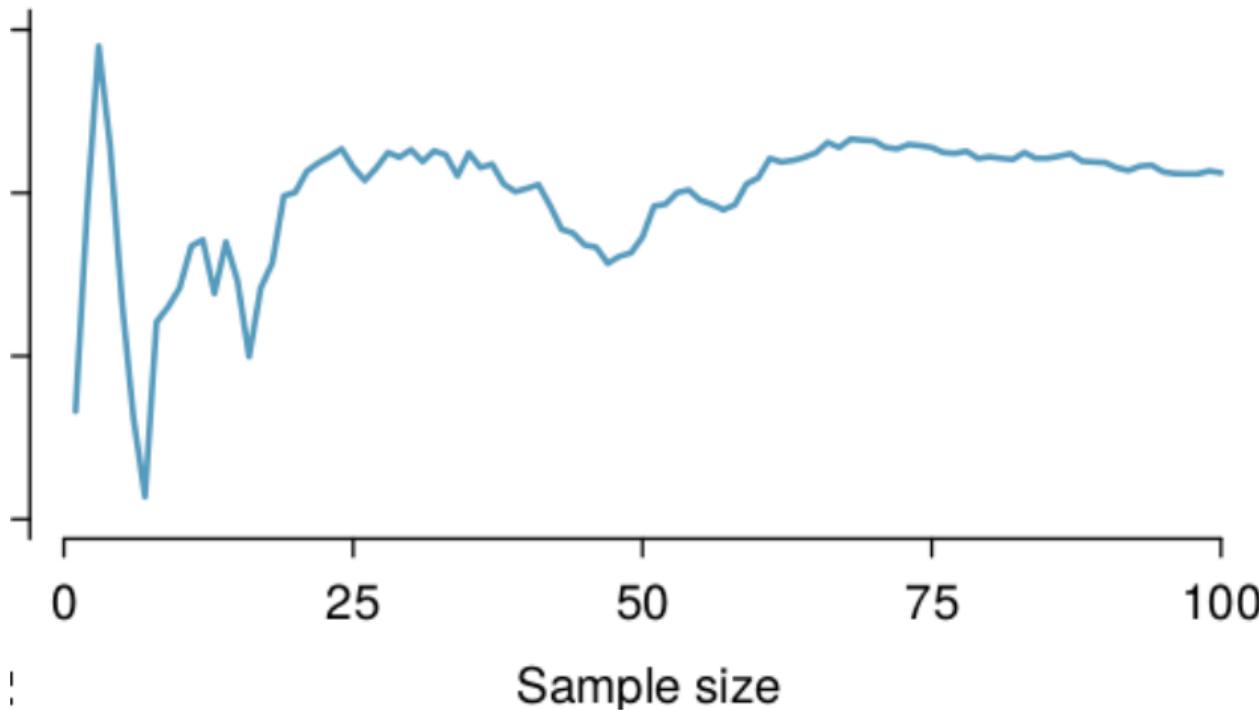
$$\bar{x} \neq \mu$$

Inference Concepts

Running mean. Sequence of partial sums (divided by number in sum).

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Sampling variation. Change of \bar{x} from one sample to the next.

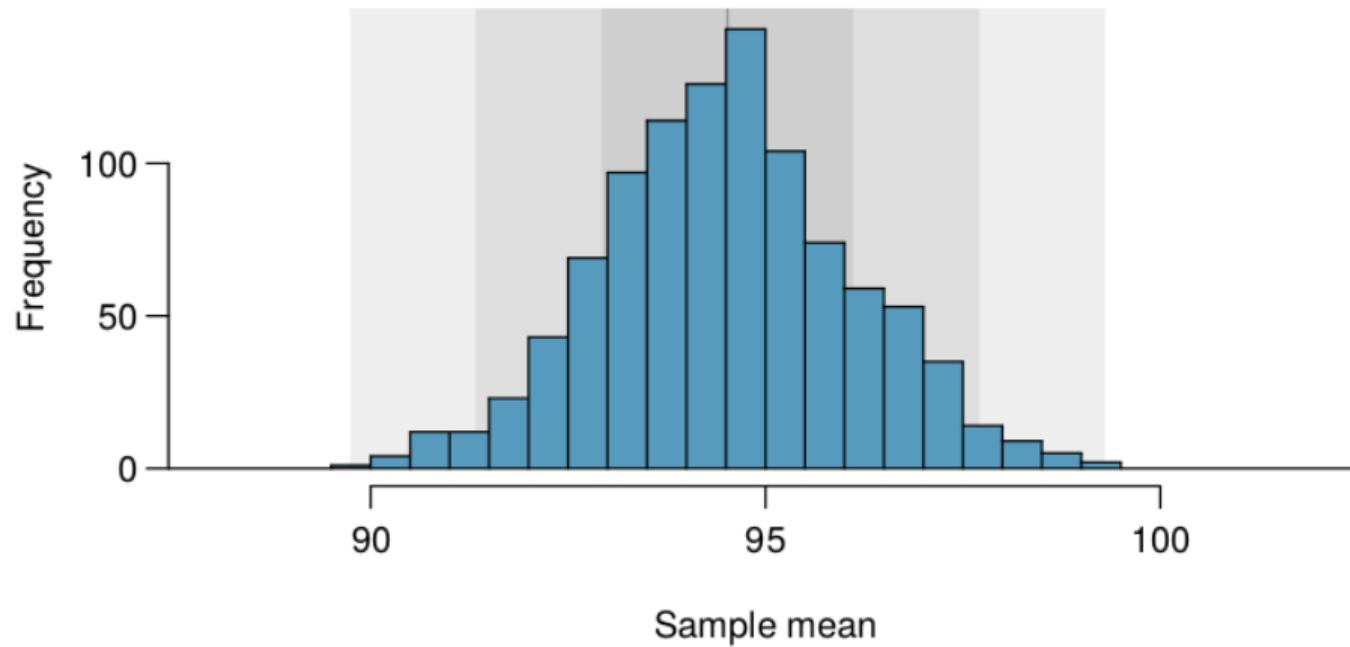
Inference Concepts

Running mean. Sequence of partial sums (divided by number in sum).

Sampling variation. Change of \bar{x} from one sample to the next.

Sampling distribution. The distribution of possible point samples of a fixed size from a given population.

Sampling distribution



Confidence intervals

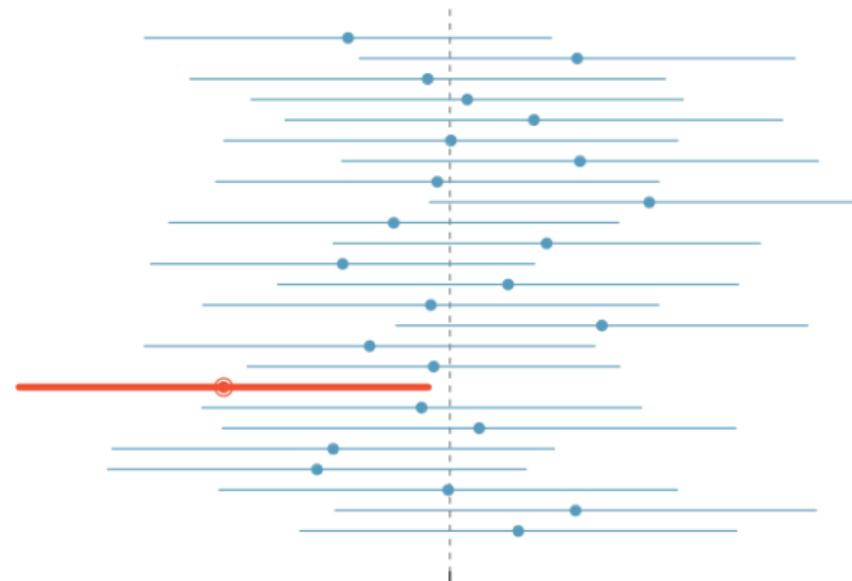
Sample n points, choose an interval around the sample mean.

A 95% confidence interval means if we sample repeatedly, about 95% of the samples will contain the population mean.

Confidence intervals

Sample n points, choose an interval around the sample mean.

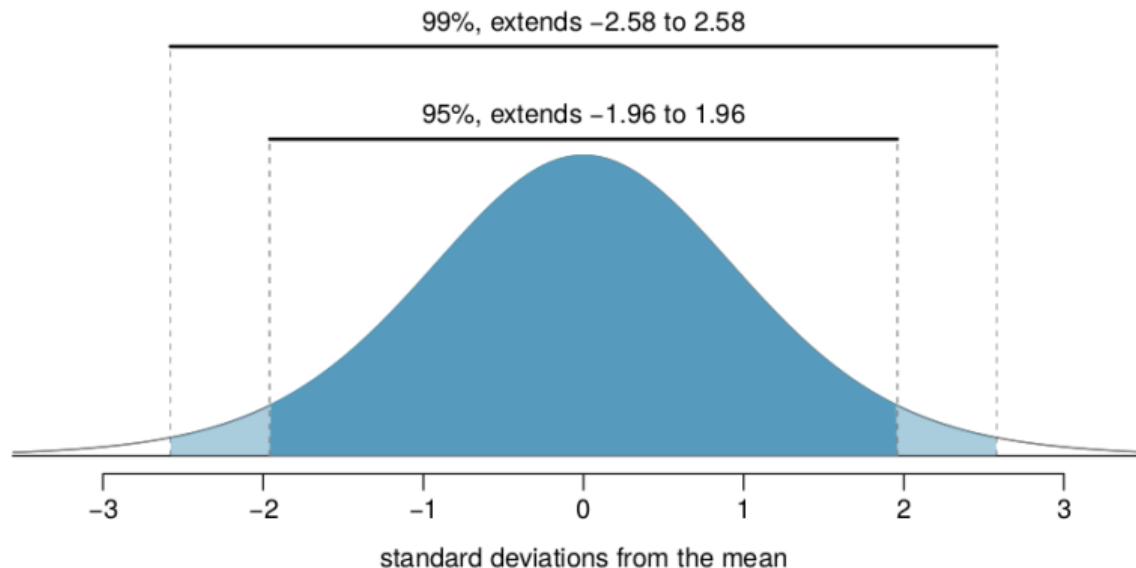
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Confidence intervals

Sample n points, choose an interval around the sample mean.

A 95% confidence interval means if we sample repeatedly, about 95% of the samples will contain the population mean.



Linear Algebra

Linear algebra: basics

$$v = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} \in \mathbb{R}^n$$

This is all mostly for convenience (not getting lost). Remember weighted averages?

$$\mu = w^T \cdot x$$

Linear algebra: basics

$$A = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{bmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix}$$
$$= \left\{ \begin{array}{ccc} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{array} \right\} \in \mathbb{R}^{n \times n}$$

Linear algebra: basics

$$u + v = \begin{pmatrix} u_1 + v_1 \\ u_2 + v_2 \\ \vdots \\ u_n + v_n \end{pmatrix}$$

Linear algebra: basics

$$\alpha \mathbf{v} = \begin{pmatrix} \alpha v_1 \\ \alpha v_2 \\ \vdots \\ \alpha v_n \end{pmatrix} \quad (\alpha \in \mathbb{R})$$

Linear algebra: basics

$$\| v \| = \sqrt{v_1^2 + \cdots + v_n^2}$$

Linear algebra: basics

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= u_1 \cdot v_1 + \cdots + u_n \cdot v_n \\ &= \| \mathbf{u} \| \| \mathbf{v} \| \cos \theta \end{aligned}$$

Linear algebra: basics

$$C = A + B \iff c_{ij} = a_{ij} + b_{ij}$$

$$C = AB \iff c_{ij} = \sum_k a_{ik} b_{kj}$$

$$A = B^T \iff a_{ij} = b_{ji}$$

$$AA^{-1} = A^{-1}A = \text{diag}(1)$$

Linear algebra: transformations

$$Ax = y \quad f = T_A : \mathbb{R}^n \rightarrow \mathbb{R}^n$$

$$x = A^{-1}Ax = A^{-1}y \quad f^{-1} = T_{A^{-1}} : \mathbb{R}^n \rightarrow \mathbb{R}^n$$

Linear algebra: transformations

B is a basis for V iff any of these conditions are met:

- B is a minimal generating set of V
- B is a maximal set of linearly independent vectors
- Every vector $v \in V$ can be expressed in a unique way as a sum of $b_i \in B$

(The conditions are equivalent.)

Linear algebra: transformations

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Bases are not unique.

Linear algebra: transformations

Eigenvectors, eigenvalues:

$$Av = \lambda v$$

Linear algebra: transformations

Eigenvectors, eigenvalues:

$$Av = \lambda v$$

$$Av = \lambda 1 v \iff (A - \lambda 1)v = 0$$

Linear algebra: transformations

Eigenvectors, eigenvalues:

$$Av = \lambda v$$

Some matrices are diagonalisable. Then

$$A = Q\Lambda Q^{-1} \quad \text{with } \Lambda = \begin{bmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & 0 \\ 0 & 0 & \lambda_n \end{bmatrix}$$

$$\text{and } Q = \begin{bmatrix} | & & | \\ v_1 & \cdots & v_n \\ | & & | \end{bmatrix}$$

Linear algebra: transformations

video time



Questions?

Features and Modeling

Vector spaces

Vector spaces

Features are dimensions

Feature extraction

Feature engineering

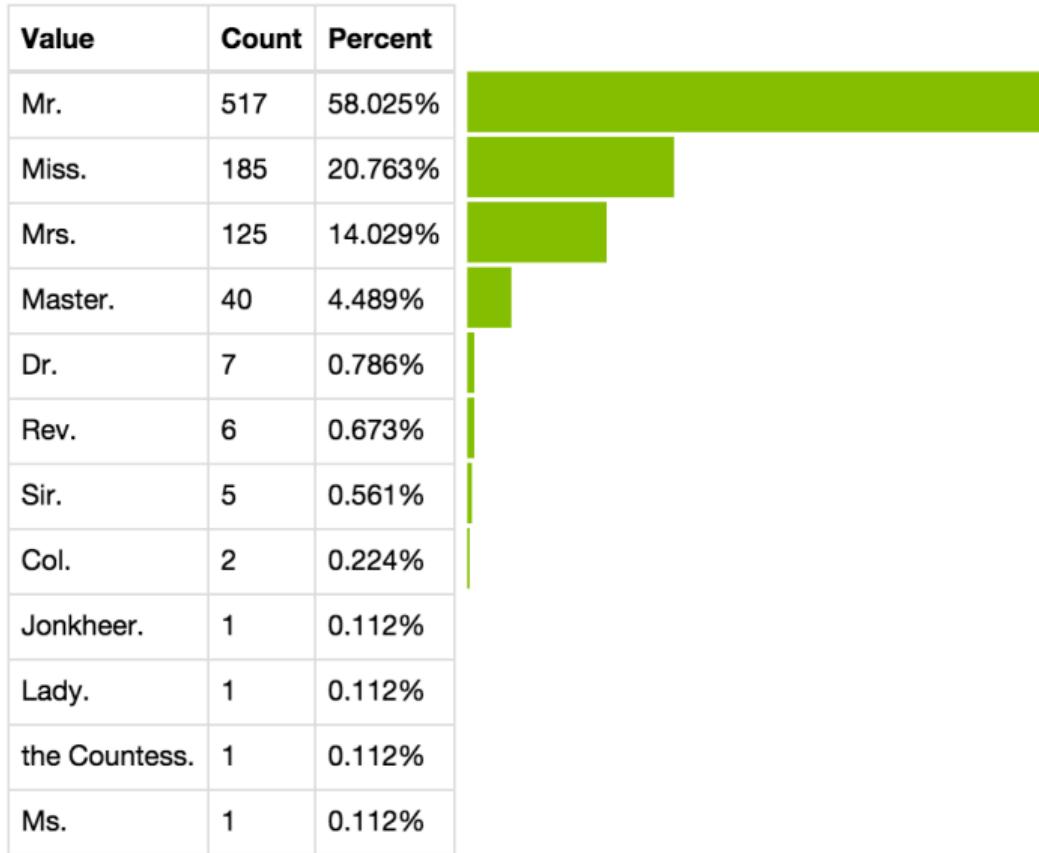
Feature extraction

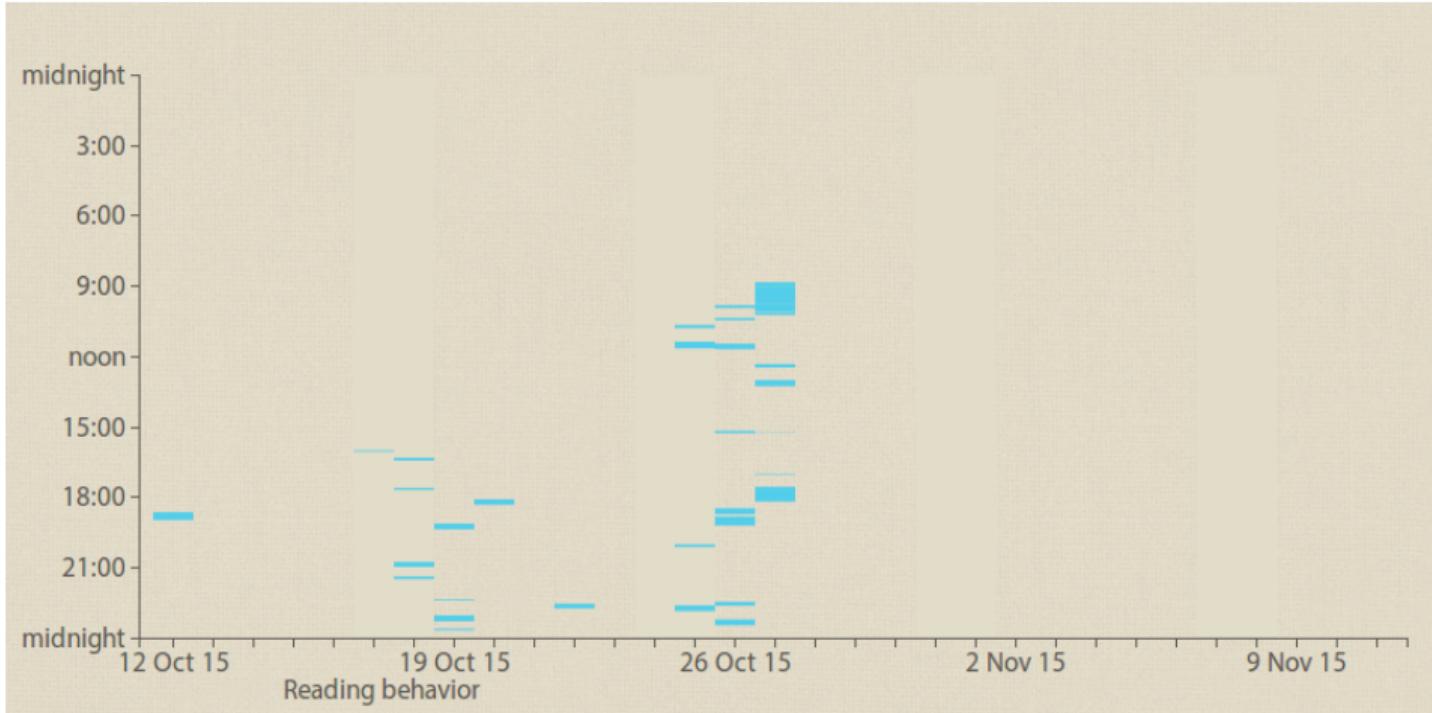
Feature engineering

Synthetic features

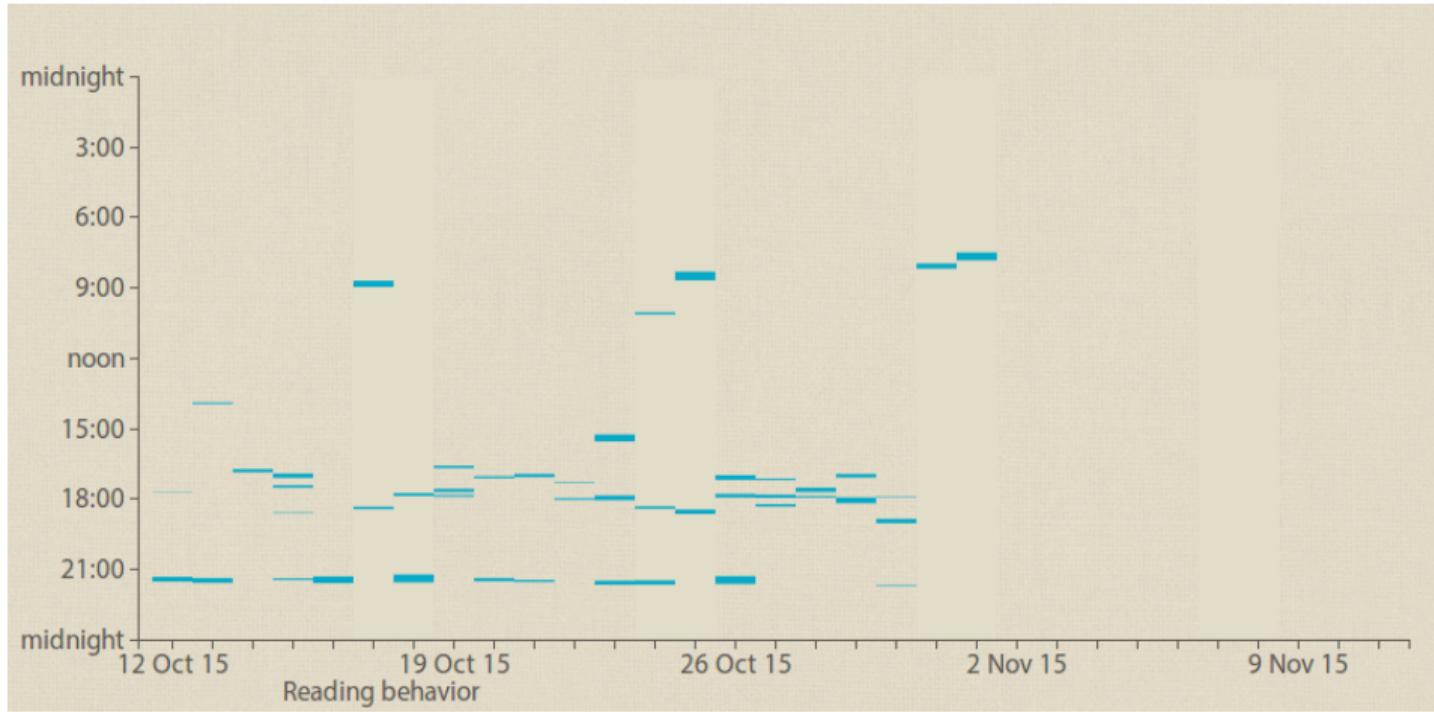
Feature Engineering

- ① Brainstorm
- ② Pick some
- ③ Make them
- ④ Evaluate
- ⑤ Repeat





Jellybooks



Jellybooks

One of K = one-hot encoding

Text features

Bag of words

- Corpus (documents)
- Vocabulary (set of unique words)
- Words

Text features

Bag of words

- Order doesn't matter
- Stop words
- Stemming (*racinisation, désuffixation*)
- Lemmatisation (*transformer en lemme*)

Image features

- Corners, edges (rotation invariant, but scaling can hide)
- More complex: scale space or RNN
- Point matching is easy

Image features

Problems

- Illumination
- Scale
- Rotation
- Skew (perspective)
- Data size (matrices not sparse)

python

Useful tools

- virtualenv
- pip
- ipython
- ipython notebook
- conda.pydata.org

python

Notes

- pip install -r requirements.txt
- ipython offers tab completion (vs python)
- ipython notebook opens in a browser, caches cell output but not cell state

pandas

```
import pandas as pd  
import numpy as np  
import scipy  
import matplotlib.pyplot as plt
```

pandas

Dataframe has many constructors. For example,

```
In [5]: pd.DataFrame({ 'A' : 1.,
                      'B' : pd.Timestamp('20161209'),
                      'C' : pd.Series(1,index=list(range(4)),dtype='float32'),
                      'D' : np.array([3] * 4, dtype='int32'),
                      'E' : pd.Categorical(["test","train","test","train"]),
                      'F' : 'hello' })
```

Out [5]:

	A	B	C	D	E	F
0	1	2016-12-09	1	3	test	hello
1	1	2016-12-09	1	3	train	hello
2	1	2016-12-09	1	3	test	hello
3	1	2016-12-09	1	3	train	hello

In [6]:

pandas

Viewing data

```
In [16]: dates = pd.date_range('20161209', periods=4, freq='1w')
```

```
In [17]: df = pd.DataFrame(np.random.randn(4,5), index=dates,  
columns=list('ABCDE'))
```

```
In [18]: df.head()
```

```
Out[18]:
```

	A	B	C	D	E
2016-12-11	-1.303610	-1.235823	0.621914	0.379340	-0.326934
2016-12-18	-1.218197	-1.113826	0.546314	-0.255001	-0.135573
2016-12-25	-0.124625	0.337268	-0.406295	0.587049	-0.904906
2017-01-01	-0.283182	-0.866213	0.051509	0.693037	-0.661055

```
In [19]:
```

pandas

Basic data exploration

In [19]: `df.describe()`

Out [19] :

	A	B	C	D	E
count	4.000000	4.000000	4.000000	4.000000	4.000000
mean	-0.732403	-0.719648	0.203361	0.351106	-0.507117
std	0.614672	0.721194	0.478728	0.424558	0.342755
min	-1.303610	-1.235823	-0.406295	-0.255001	-0.904906
25%	-1.239550	-1.144325	-0.062942	0.220755	-0.722018
50%	-0.750689	-0.990019	0.298912	0.483195	-0.493995
75%	-0.243543	-0.565343	0.565214	0.613546	-0.279094
max	-0.124625	0.337268	0.621914	0.693037	-0.135573

In [20] :

pandas

Select a column (series)

```
In [20]: df.loc[dates[1]]
```

```
Out[20]:
```

```
A    -1.218197
```

```
B    -1.113826
```

```
C     0.546314
```

```
D    -0.255001
```

```
E    -0.135573
```

```
Name: 2016-12-18 00:00:00, dtype: float64
```

```
In [21]:
```

pandas

Select a range

```
In [21]: df.loc[:, ['A', 'C']]
```

```
Out[21]:
```

	A	C
2016-12-11	-1.303610	0.621914
2016-12-18	-1.218197	0.546314
2016-12-25	-0.124625	-0.406295
2017-01-01	-0.283182	0.051509

```
In [22]:
```

pandas

Boolean selection criteria

```
In [23]: df[df.D > 0]
```

```
Out[23]:
```

	A	B	C	D	E
2016-12-11	-1.303610	-1.235823	0.621914	0.379340	-0.326934
2016-12-25	-0.124625	0.337268	-0.406295	0.587049	-0.904906
2017-01-01	-0.283182	-0.866213	0.051509	0.693037	-0.661055

```
In [24]:
```

pandas

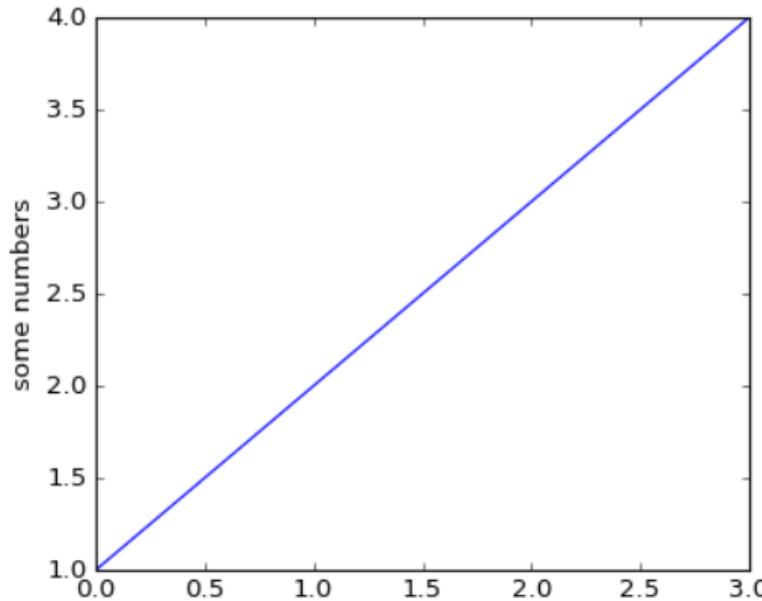
Recommended

[http://www.gregreda.com/2013/10/26/
intro-to-pandas-data-structures/](http://www.gregreda.com/2013/10/26/intro-to-pandas-data-structures/)

Plotting

Draw a line

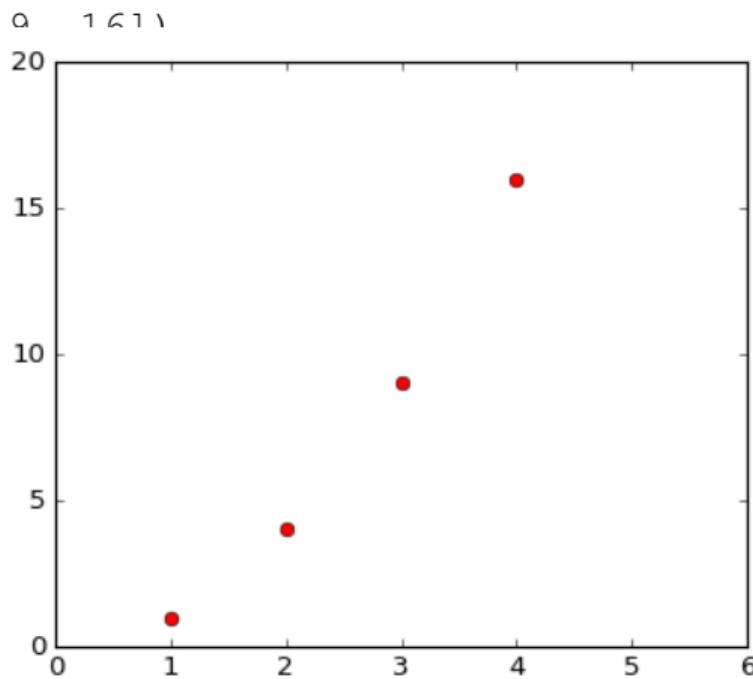
```
import matplotlib.pyplot as plt  
plt.plot([1,2,3,4])  
plt.ylabel('some numbers')  
plt.show()
```



Plotting

Draw a line

```
import matplotlib.pyplot as plt  
plt.plot([1, 2, 3, 4], [1, 4, 9, 16])  
plt.ylabel('some numbers')  
plt.show()
```



Plotting

Draw a line

```
import numpy as np
import matplotlib.pyplot as plt
```

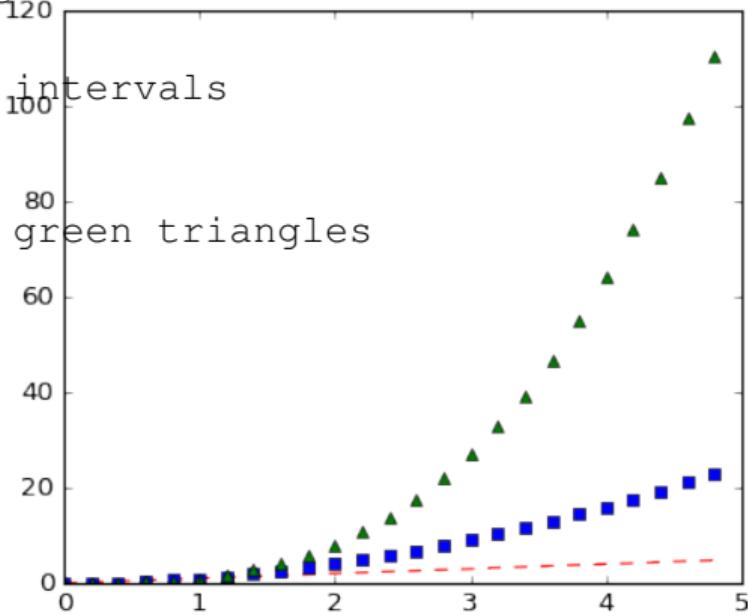
evenly sampled time at 200ms intervals

```
t = np.arange(0., 5., 0.2)
```

red dashes, blue squares and green triangles

```
plt.plot(t, t,
          'r--', t,
          t**2, 'bs',
          t, t**3, 'g^')

plt.show()
```



Plotting

Draw two curves

```
import numpy as np
import matplotlib.pyplot as plt

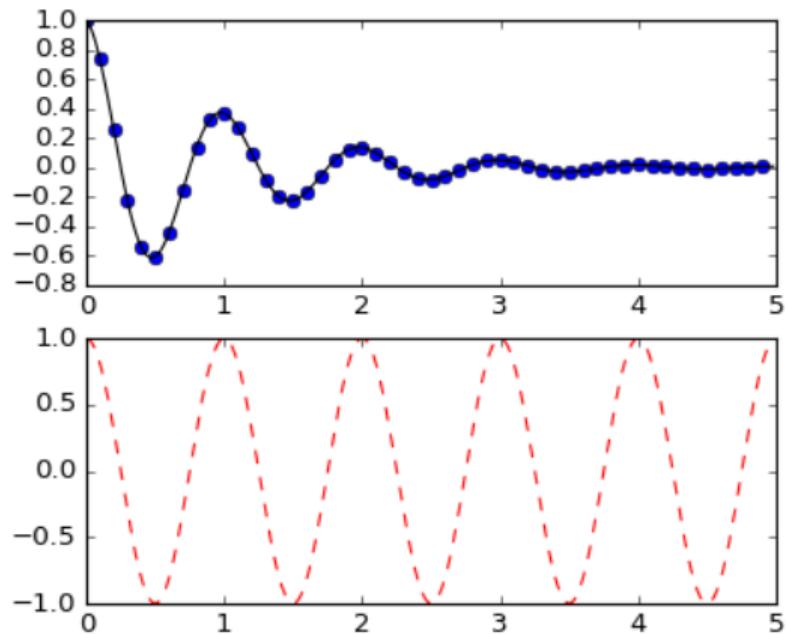
def f(t):
    return np.exp(-t) * np.cos(2*np.pi*t)

t1 = np.arange(0.0, 5.0, 0.1)
t2 = np.arange(0.0, 5.0, 0.02)

plt.figure(1)
plt.subplot(211)
plt.plot(t1, f(t1), 'bo', t2, f(t2), 'k')

plt.subplot(212)
plt.plot(t2, np.cos(2*np.pi*t2), 'r--')
plt.show()
```

Plotting



Plotting

Draw two curves

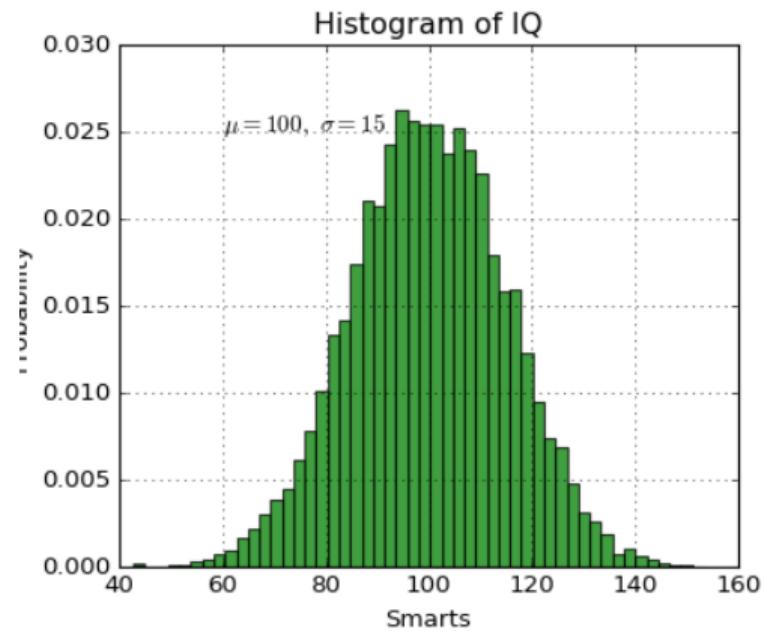
```
import numpy as np
import matplotlib.pyplot as plt

mu, sigma = 100, 15
x = mu + sigma * np.random.randn(10000)

n, bins, patches = plt.hist(x, 50, normed=1, facecolor='g', alpha=0.75)

plt.xlabel('Smarts')
plt.ylabel('Probability')
plt.title('Histogram of IQ')
plt.text(60, .025, r'$\mu=100, \ \sigma=15$')
plt.axis([40, 160, 0, 0.03])
plt.grid(True)
plt.show()
```

Plotting



Plotting

Scatter plot

http://matplotlib.org/mpl_examples/pylab_examples/scatter_demo2.py

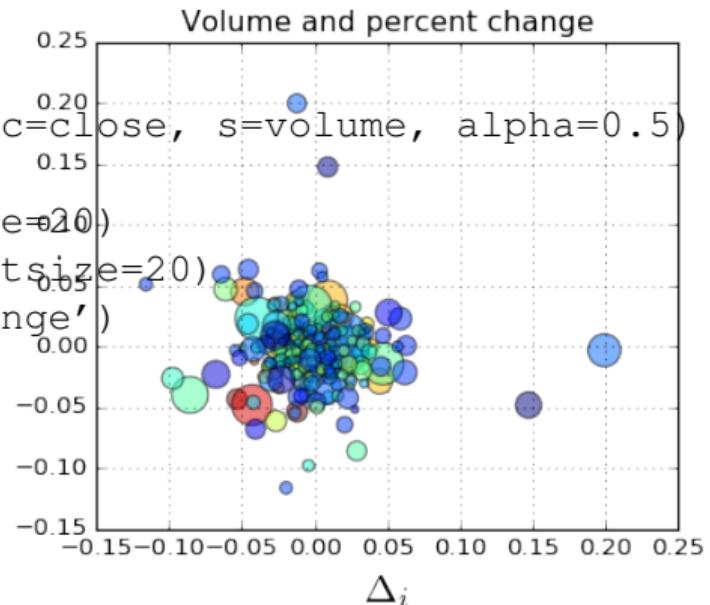
```
import numpy as np
import matplotlib.pyplot as plt

fig, ax = plt.subplots()
ax.scatter(delta1[:-1], delta1[1:], c=close, s=volume, alpha=0.5)

ax.set_xlabel(r'$\Delta_i$', fontsize=20)
ax.set_ylabel(r'$\Delta_{i+1}$', fontsize=20)
ax.set_title('Volume and percent change')

ax.grid(True)
fig.tight_layout()

plt.show()
```



Plotting

http://matplotlib.org/users/pyplot_tutorial.html

<http://matplotlib.org/users/beginner.html>



Questions?