

Learning from unknown information sources*

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Abstract

When an agent receives information from a source whose accuracy might be either high or low, standard theory dictates that she update as if the source has medium accuracy. In a lab experiment, subjects deviate from this benchmark by reacting less to uncertain sources, especially when the sources release good news. This pattern is validated using observational data on stock price reactions to analyst earnings forecasts, where analysts with no forecast records are classified as uncertain sources. A theory of belief updating where agents are insensitive and averse to uncertainty in information accuracy can explain these results.

Keywords— Belief updating, ambiguity, compound risk, earnings forecasts

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1 Introduction

People often need to incorporate new information for decision-making when they are uncertain about the accuracy of its source. For example, in response to a financial report issued by a new analyst, investors need to decide how to adjust their portfolios, but they do not know the analyst’s expertise. Politicians often have to rely on media and polling agencies to learn about their constituents’ needs, but they may not know the exact biases of these intermediaries. Moreover, in the current era where online health information becomes increasingly abundant and important for public health, a survey among older web users shows that while 90% of them look for health information online, only 52% believe that they can distinguish high-quality sources from low-quality ones (Tennant et al., 2015). These examples suggest that understanding how people react to news with uncertain accuracy is important for understanding real outcomes.

Standard economic theory assumes that when the accuracy of a piece of information is uncertain, agents are able to correctly deduce its *expected accuracy* and update their beliefs solely based on that expectation. As an example, consider a bet with two possible outcomes. The agent receives a report on the outcome but she does not know its accuracy: the probability that the report is correct conditional on the true outcome might be either 90% or 50%. The two possible levels of accuracy are equally likely, and the true one is independent from the realization of the outcome. According to standard theory, the agent is able to calculate that the report is correct 70% of the time in expectation, and her belief about the outcome of the bet should react to this report as if she knew with certainty that it is 70% accurate.

1.1 Main results

Using both experimental data from the lab and observational data on stock price reactions to analyst earnings forecasts, this paper provides the first evidence on how uncertainty in information accuracy affects belief updating. In part of a lab experiment, I present subjects with bets and inform them about the winning odds. I elicit subjects’ certainty equivalents (CEs) for each bet after they receive a report on its outcome. I refer to a report as *uncertain information* if its accuracy might be either high (ψ_h) or low (ψ_l), and I define its corresponding *simple information* as a report whose accuracy is known to be the midpoint, $(\psi_h + \psi_l)/2$. When subjects receive uncertain information, sometimes they know that the two possible accuracy levels are equally likely (*compound information*), and at other times they do not know their relative likelihood (*ambiguous information*). In my experiment, the effects of compound uncertainty and ambiguity prove to be qualitatively similar, so I will refer to them jointly using the generic term “uncertainty.”

The main experimental result concerns the marginal effects of uncertainty in information accuracy on posterior beliefs. Compared to the case of simple information, subjects’ beliefs move

less in the direction of the realized report when its accuracy is uncertain. In other words, uncertain information leads to *under-reaction*. Moreover, the under-reaction is more pronounced for good news than for bad news. This suggests that uncertainty in information accuracy on average leads to *pessimism* in posterior beliefs.

The same patterns also emerge when I examine stock market reactions to analyst earnings forecasts. The forecast accuracy of financial analysts is more unpredictable if they do not have a proven forecast record for the stock. I find that in response to good news (upward forecast revisions) issued by these analysts, the immediate stock price reactions are on average followed by larger positive price drifts. This phenomenon implies that investors' immediate reactions to good news are less sufficient when the news is issued by analysts without records. By contrast, the sufficiency of reactions to bad news does not depend on whether the issuing analyst has a forecast record or not. These findings are consistent with my experimental results that people's reactions to information from uncertain sources are less sufficient and more pessimistic, and they show that these phenomena are even present in a high-stake, real-world environment.

1.2 Theories

Most theories of reactions to uncertain information start with uncertainty attitudes, which describe willingness to bet on events whose probabilities are uncertain. This starting point is natural because the correctness of news from a source with uncertain accuracy is an event with uncertain probability. Previous research has shown two empirical regularities about typical uncertainty attitudes: *uncertainty-induced insensitivity* and *uncertainty aversion*. To illustrate, consider an event whose probability might be either high (p_h) or low (p_l) and compare the willingness to bet on this event to the alternative scenario where the event's probability is known to be the midpoint, $(p_h + p_l)/2$. Uncertainty-induced insensitivity refers to the pattern whereby the willingness to bet changes less as p_h and p_l change when the probability is uncertain. It captures the psychological intuition that people internalize the probabilities less as they become more complex. On the other hand, uncertainty aversion refers to an additional force that lowers the willingness to bet on this uncertain event. It reflects the tendency to be over-pessimistic in the face of uncertainty. As a basis of our theoretical framework, I use the Choquet expected utility (CEU) model ([Schmeidler, 1989](#)) which can capture insensitivity and uncertainty aversion.

When agents react to information from a source with uncertain accuracy, their uncertainty attitudes toward this source's accuracy could manifest themselves in a number of different ways. Consider an agent who evaluates a bet after hearing an uncertain source's report on its outcome. Suppose that the agent is averse to uncertainty in information accuracy. Then, one possibility is that the uncertainty aversion leads her to be pessimistic about the outcome of the bet conditional on the

report. (The belief-updating rule that leads to this possibility is known as *Full Bayesian updating*,¹ which is different from Bayesian updating in the classical sense.) Alternatively, the agent may be pessimistic about the ex-ante value of information (*Dynamically consistent updating*).² A third possibility is that after hearing the report, the agent becomes certain about one of the accuracy levels because the other ones are less likely to be true given the report (*Maximum likelihood updating*).³

These possibilities have different testable implications in the empirical settings of this paper, and Full Bayesian updating coupled with uncertainty-induced insensitivity and uncertainty aversion stands out as being most in line with the evidence described above. Intuitively, uncertainty-induced insensitivity causes agents to partially ignore information from unknown sources, regardless of whether it is good news or bad news. This tendency leads to under-reaction. For the part of information that is not ignored, under Full Bayesian updating, uncertainty-averse agents over-estimate the source accuracy when the news is bad but under-estimate it when the news is good. This asymmetry leads to pessimism about the value of the bet after hearing the news.

1.3 Attitudes toward uncertain information accuracy and uncertain economic fundamentals

Prior research on uncertainty attitudes has mostly studied how people evaluate prospects when they do not know the *prior over payoff-relevant events* (henceforth *prior* or *economic fundamental*).⁴ For example, investors may need to evaluate a complex financial asset when it is difficult to know the distribution of its returns. Uncertainty attitudes toward economic fundamentals are conceptually different from uncertainty attitudes toward information accuracy because the uncertain probability distributions are over different dimensions of the state space.⁵ A natural question to ask is how these two kinds of uncertainty attitudes relate to each other. A strong relationship would warrant extrapolating from what we know about uncertain economic fundamentals to the under-studied domain of uncertain information accuracy. Otherwise, domain-specific research would be necessary to understand learning from unknown information sources.

To measure our lab subjects' uncertainty attitudes toward economic fundamentals, I elicit their CEs of *uncertain bets* whose winning odds might be either high or low and compare them to CEs of *simple bets* whose odds are known. The comparison confirms that typical uncertainty attitudes toward economic fundamentals exhibit insensitivity and uncertainty aversion, which is qualitatively

¹Jaffray (1992); Pires (2002); Eichberger et al. (2007).

²Hanany and Klibanoff (2007).

³Dempster (1967); Shafer (1976); Gilboa and Schmeidler (1993).

⁴The question was raised in Keynes (1921), Knight (1921), and Ellsberg (1961), and has since received immense theoretical attention. For theoretical surveys, see Machina and Siniscalchi (2014) and Gilboa and Marinacci (2016). Trautmann and van de Kuilen (2015) provide a survey of empirical evidence.

⁵These dimensions are referred to as *issues* in the literature.

similar to uncertainty attitudes toward information accuracy in the aggregate.⁶

However, the aforementioned similarity completely breaks down when we focus on individual subjects. At the individual level, I construct tests for the correlations between attitudes toward different kinds of uncertainty. These tests are valid under a variety of preference models and updating rules. The results show that there is almost zero correlation between attitudes toward uncertainty in information accuracy and uncertainty in priors. This stark finding suggests that knowing a person’s preference between simple and complex assets does not help predict how she reacts differently to information from known and unknown sources. It also resonates with existing evidence showing that uncertainty attitudes vary with issues.⁷

1.4 Related literature

Theoretical studies have proposed various criteria for belief updating under uncertainty (e.g. [Dempster, 1967](#); [Shafer, 1976](#); [Jaffray, 1992](#); [Hanany and Klibanoff, 2007](#)). In my empirical settings, these theories differ in their predictions on the marginal effects of uncertainty on posterior beliefs. This allows me to test between them.

Three recent experimental studies investigate certain aspects of ambiguous information. In a contemporaneous project, [Epstein and Halevy \(2021\)](#) study belief updating with ambiguous information when the prior is compound. Using a between-subject design, they find that more subjects violate the martingale property of belief updating⁸ under ambiguous information than under a piece of simple information. More recently, [Shishkin and Ortoleva \(2021\)](#) focus on ambiguous neutral information (i.e. information whose accuracy is a midpoint-preserving spread of 50%) and study both belief updating and the demand for information. [Kellner et al. \(2020\)](#) study communication with ambiguous language and find evidence consistent with hedging against ambiguity. In contrast to these three studies, my experiment allows for the separate identification of under-reaction and pessimism caused by uncertain information accuracy. In addition, I consider both compound and ambiguous information.⁹

Two previous experimental projects study phenomena related to uncertain information accuracy. [Fryer Jr et al. \(2019\)](#) find that subjects tend to update their beliefs about political issues in the direc-

⁶Empirical studies providing evidence of uncertainty-induced insensitivity and uncertainty aversion include [Abdellaoui et al. \(2011, 2015\)](#); [Dimmock et al. \(2015\)](#); [Baillon et al. \(2018\)](#); [Anantanasuwong et al. \(2019\)](#). Theoretical models that capture these two patterns include [Ellsberg \(2015\)](#); [Chateauneuf et al. \(2007\)](#); [Gul and Pesendorfer \(2014\)](#).

⁷E.g. [Heath and Tversky \(1991\)](#); [Fox and Tversky \(1995\)](#).

⁸Loosely speaking, the martingale property of belief updating states that there exists a probability distribution over messages such that for every event, the expectation of posteriors equals the prior.

⁹Complementary to the research on uncertain information accuracy, experiments studying the effect of uncertain priors on belief updating include [Corgnet et al. \(2012\)](#), [Ert and Trautmann \(2014\)](#), [Moreno and Rosokha \(2016\)](#), [Baillon et al. \(2017\)](#), and [Ngangoué \(2020\)](#).

tions of their priors after reading ambiguous research summaries. In a social learning experiment, [De Filippis et al. \(2018\)](#) present subjects with two pieces of information: a private signal about the true state and the belief of a predecessor (who only has a private signal). When the private signal is absent or confirms the predecessor’s belief, subjects account for the predecessor’s belief in a Bayesian manner. By contrast, when the private signal contradicts the predecessor’s belief, subjects under-weight the latter. The authors interpret their result using a model where subjects treat their predecessors’ beliefs as ambiguous information.¹⁰ My experiment differs from these two studies as I examine the effects on belief updating when information accuracy changes from being *objectively* simple to *objectively* uncertain. In addition, the context of my experiment rules out ego- or ideology-driven motivated reasoning as the driving force of the results.

More broadly, my paper is related to the fast-growing literature on belief-updating biases, such as under-reaction (e.g. [Edwards, 1968](#); [Möbius et al., 2014](#)) and asymmetric updating (e.g. [Eil and Rao, 2011](#); [Möbius et al., 2014](#); [Coutts, 2019](#); [Barron, 2020](#)). [Benjamin \(2019\)](#) surveys this literature and concludes that evidence on the directions of belief-updating biases is mixed. Although most experimental studies on these topics focus on people’s reactions to objectively simple information, people may still perceive the information as uncertain to varying degrees due to inattention or bounded rationality. If this is true, then my paper suggests that perceived uncertainty in information accuracy may be a moderator for these belief-updating biases. Indeed, in a subsequent experiment, [Enke and Graeber \(2020\)](#) find evidence supporting this conjecture.¹¹

In real-world settings, two studies find patterns that can be explained by certain models of learning from ambiguous information. [Epstein and Schneider \(2008\)](#) calibrate the US stock price movement in the month after 9/11 to a model of asset pricing with ambiguous news and find that the fit is superior to a Bayesian model. [Kala \(2017\)](#) studies how rainfall signals affect Indian farmers’ agricultural decisions and find support for the robust learning model of [Hansen and Sargent \(2001\)](#). These papers do not study how the degree of uncertainty in information accuracy affects the sufficiency of reactions to news, which is what I focus on in the analysis of stock price reactions to analyst earnings forecasts.

There is a vast body of literature on stock market reactions to analyst reports in accounting and finance.¹² [Gleason and Lee \(2003\)](#) find that stock price under-reaction is less pronounced for analysts who are recognized by the *Institutional Investor* magazine. [Zhang \(2006\)](#) shows that the market under-reacts more to forecast revisions on firms whose fundamentals are more difficult to

¹⁰Other social learning experiments that study how subjects learn from others’ actions include [Nöth and Weber \(2003\)](#); [Çelen and Kariv \(2004\)](#); [Goeree et al. \(2007\)](#). However, these experiments typically observe a subject’s action only once, and the action space is usually binary.

¹¹[Enke and Graeber \(2020\)](#) find in a part of their experiment that subjects’ posterior beliefs are more compressed toward the prior (50%, 50%) if the information accuracy is compound as opposed to simple. Under their belief elicitation mechanism, this phenomenon is consistent with both under-reaction and pessimism.

¹²For surveys, see [Kothari et al. \(2016\)](#); [Bradshaw et al. \(2017\)](#).

learn. Complementary to these studies, my paper focuses on the uncertainty of analysts' accuracy, and I find that it only exacerbates under-reaction for good news. [Mikhail et al. \(1997\)](#) and [Chen et al. \(2005\)](#) study how analysts' experience and forecast records affect the market's immediate reactions to their forecasts, although they do not study the sufficiency of these reactions.

1.5 Paper structure

The rest of the paper is organized as follows. Section 2 describes the design of all parts of the lab experiment. Section 3 presents theories of belief updating with uncertain information accuracy and Section 4 presents the corresponding experimental results. In Section 5, I show experimental results related to uncertain priors over the payoff-relevant events and compare them to results on uncertain information accuracy. Section 6 presents supporting evidence using observational data on stock market reactions to analyst earnings forecasts. Finally, Section 7 concludes.

2 Experimental design

I ran a lab experiment at the Econ Lab at University of California, Santa Barbara on May 9 and 14-16, 2018. A total of 165 subjects were recruited using ORSEE ([Greiner, 2015](#)) to participate in eleven sessions which lasted on average 90 minutes.

2.1 Environment

Each session of the experiment comprises 29 rounds. Each round is framed as a race between a red horse and a blue horse, with no ties being possible. In each round, there are two payoff-relevant events, *Red* and *Blue*, corresponding to the color of the winning horse. In some rounds, subjects may also receive a piece of additional information about the outcome of the race framed as an analyst report. The report they receive is either "Red horse won" or "Blue horse won." I will refer to the former as a *good report* and the latter a *bad report* for *Red*. For *Blue*, the designation of good and bad reports is reversed. The uncertainty across rounds is independent.

The 29 rounds are grouped into five parts, which are summarized in Table 2.1. In the three parts with "simple prior", subjects in each round know with certainty the prior probability distribution over the payoff-relevant events, i.e. the winning odds of the two horses. For example, subjects may be told that the red horse has a 70% chance of winning and the blue horse has a 30% chance. What differs across these three parts is whether subjects receive an analyst report after they get to know the prior, and – if they do – whether the accuracy of information source is uncertain. Subjects do not receive any report in Part 1, whereas in Parts 2 and 3, they do. In Part 2, the reports are *simple information*, which means that subjects know their accuracy levels – denoted by ψ – with certainty.

	Order	Prior (<i>Red</i> , <i>Blue</i>)	Info accuracy
Part 1: Simple prior, No Info	1	(50%, 50%)	-
	2	(60%, 40%)	-
	3	(70%, 30%)	-
Part 2: Simple prior, Simple info	1	(50%, 50%)	70%
	2	(60%, 40%)	60%
	3	(70%, 30%)	70%
	4	(70%, 30%)	50%
Part 3: Simple prior, Uncertain info one compound block one ambiguous block	1	(50%, 50%)	90% or 50%
	2	(60%, 40%)	90% or 30%
	3	(70%, 30%)	90% or 50%
	4	(70%, 30%)	90% or 10%
Part 4: Uncertain prior, No info one compound block one ambiguous block	1	(90%, 10%) or (30%, 70%)	-
	2	(90%, 10%) or (10%, 90%)	-
	3	(90%, 10%) or (50%, 50%)	-
Part 5: Uncertain prior, Simple info one compound block one ambiguous block	1	(90%, 10%) or (50%, 50%)	70%
	2	(90%, 10%) or (10%, 90%)	70%
	3	(90%, 10%) or (50%, 50%)	60%
	4	(90%, 10%) or (30%, 70%)	50%

Table 2.1: Experimental parts and rounds

For instance, subjects may be told in a round that the analyst report is 70% accurate. This means that conditional on the true outcome of the horse race, the analyst report is correct 70% of the time and incorrect 30% of the time. In Part 3, subjects know that the information is at one of two possible accuracy levels, ψ_h or ψ_l ($\psi_h > \psi_l$), but they do not know which one. For example, they may be told that the analyst report is either 90% accurate or 50% accurate. In half of the rounds (grouped together in one block), subjects know that the two possible accuracy levels are equally likely to be the true one. I refer to this situation as subjects receiving *compound information*. In the other rounds (also grouped in a block), the distribution over the two possible accuracy levels is unknown, and I refer to the information as *ambiguous information*. The realization of the true accuracy level is independent from the horse race outcome.

In Parts 4 and 5, subjects are told in each round that the payoff-relevant events are distributed according to one of two possible priors. For example, the prior probability of *Red* might be either 50% or 90%. In half of the rounds (grouped together in one block), subjects know that the two possible priors are equally likely to be true (“compound prior”), whereas in the others, they do not know the distribution over them (“ambiguous prior”). Subjects do not receive any analyst report in Part 4, whereas in Part 5, they receive reports that are simple information.

There are three simple priors of *Red* in the experiment: 50%, 60%, and 70%. There are also

three accuracy levels of simple information: 50%, 60%, and 70%. The uncertain priors and uncertain information accuracy are midpoint-preserving spreads of their simple counterparts.

The order between rounds within each part (or each block in Parts 3, 4 and 5) is fixed. The order between the five parts varies across sessions. Within Parts 3, 4 and 5, the order between the compound and ambiguous blocks also varies across sessions. Table B.3 summarizes the orders in the eleven sessions. In Appendix B.2, I show that order effects are not a driving force of the main empirical results.

2.2 Decisions and payment

Each subject receives a \$5 show-up fee, and – if they finish the experiment – a \$10 completion fee. The amounts of bonus they receive depend on their decisions in the experiment. At the end of each round, I elicit subjects' certainty equivalents (CEs) of a bet on *Red* and a bet on *Blue*.¹³ A bet on an event pays out \$20 if it is realized and \$0 otherwise. To ensure the incentive compatibility of the CE elicitation, I use a variant of the Becker-DeGroot-Marschak (BDM) mechanism (Becker et al., 1964). Moreover, only one randomly selected bet counts for the bonus. Specifically, a price between \$0 and \$20 is randomly selected. If a subject's CE for the bet that counts for the bonus is higher than the price, then her bonus will equal the payout of that bet; otherwise, her bonus will equal the price. In the first two sessions, the original version of the BDM mechanism was implemented and subjects were asked to write down their minimum selling prices for each bet on paper.¹⁴ In the other nine sessions, the BDM mechanism was implemented through a multiple price list programmed using oTree (Chen et al., 2016), where a subject makes a series of binary choices between receiving the bet and receiving a certain amount of money incrementing from \$1 to \$19 in steps of \$1. The CE is inferred to be the minimum certain amount that the subject chooses over the bet.¹⁵ After subjects report their CEs in a round, they do not receive any feedback until the very end of the experiment.

2.3 Implementation of randomization

To encourage subjects to consider each bet and each price in isolation (Baillon et al., 2015) and establish the credibility of the random incentive mechanism, the randomization is conducted publicly before the first round of each session. Specifically, each subject draws two envelopes from two bags, one from each. One envelope contains the bet that will count for bonus and the other contains the price of the bet (row in the multiple price list).¹⁶

¹³I elicit CEs instead of probability equivalents so that the tasks resemble real-life financial decisions instead of pure mathematical questions. In addition, CEs are arguably easier for subjects to understand.

¹⁴A total of 38 observations from three subjects in these two sessions are missing due to illegibility.

¹⁵Multiple switching between the left and right sides of the list is not allowed.

¹⁶If subjects use the random incentive system between rounds to hedge against uncertainty, then

In each round, each binary event is determined by a random draw from a deck of ten cards numbered from 1 to 10, with one card for each number. To determine which event realizes, a small number on the drawn card corresponds to *Red* being the realized event and a large number corresponds to *Blue*. The threshold number is determined by the true prior over the events. For example, suppose that the true prior of *Red* is 70%. Then the red horse wins if a number between 1 and 7 is drawn, and the blue horse wins if the number is between 8 and 10. In rounds with additional information, the analyst report is correct if the number drawn from a second deck of cards is small, and incorrect if the number is large. The threshold number corresponds to the true accuracy level of the report. Another deck of cards is used in rounds with two possible priors. If the two priors are equally likely, then which prior is the true prior depends on whether the draw from this deck is between 1 and 5. If the distribution over the two priors is unknown, then the threshold number that determines the true prior is not disclosed to the subjects.¹⁷ When the information accuracy is uncertain, the true accuracy is determined in a similar fashion.¹⁸ After drawing the cards, the experimenter announces the realized report to the subjects, and then the subjects report their CEs for the red bet and blue bet.

2.4 Logistics

Subjects watch instructional videos at the outset of the experiment and before each part. After each video, screenshots and scripts are distributed to subjects on paper for their reference. Before proceeding to the first round of each part, subjects answer several comprehension questions to demonstrate that they understand the instructions. Both the videos and the comprehension questions take extra care to ensure that subjects understand the statistical meaning of priors and information accuracy, although there is no mention of any updating rules. The experiment ends with an unincentivized survey. The instructional videos, their scripts and sample screenshots of the rounds can be found on [my website](#).

3 Theory of belief updating with uncertain information accuracy

In this section, I analyze theories about belief updating with uncertain information accuracy.

¹⁷To mitigate the concern that the experimenter manipulates the threshold number ex post, subjects are told that the threshold number is printed on a paper and they are welcome to inspect it after the experiment.

¹⁸Instructions are framed such that the uncertainty about true prior or the uncertainty about the accuracy level of the information is always resolved first. In the first two sessions, to determine the true prior or the true accuracy level of information, a card is drawn from a deck of eight cards instead of ten. The uncertainty is resolved by whether the number drawn is even or odd.

3.1 A model of uncertainty attitudes

Theories of belief updating with uncertain information accuracy typically start with a model of uncertainty attitudes, which describes how agents evaluate prospects when the probability distribution over events is uncertain.¹⁹ In this paper, I will use a Choquet expected utility (CEU) model (Schmeidler, 1989) to capture attitudes toward compound uncertainty and ambiguity.

In the state space S , there is an event E and its complement E^c . The (objective) probability of E is either p_h or p_l , with $p_h \geq p_l$ and $p_h + p_l \geq 1$. The probability of E^c is either $1 - p_h$ or $1 - p_l$. An act assigns a simple lottery to each event. I identify the simple lotteries assigned to E and E^c by their (von Neumann-Morgenstern) utility indices, denoted by u_1 and u_2 , respectively.

Under CEU, each event is assigned a value ν in $[0, 1]$ called *capacity*. Capacities are a generalization of probability measures. Like probabilities, capacities satisfy the following two conditions:

- $\nu(\emptyset) = 0$ and $\nu(S) = 1$;
- If $E_1 \subseteq E_2$, then $\nu(E_1) \leq \nu(E_2)$.

Unlike probabilities, capacities of sets are allowed to be non-additive.

Given a vNM utility function and a capacity function, the utility of an act for a CEU agent is

$$\begin{cases} \nu(E)u_1 + (1 - \nu(E))u_2, & \text{if } u_1 \geq u_2 \\ (1 - \nu(E^c))u_1 + \nu(E^c)u_2, & \text{if } u_1 < u_2 \end{cases}. \quad (1)$$

If $\nu(E) = \frac{p_h + p_l}{2}$ and $\nu(E^c) = 1 - \nu(E)$, then the CEU preference coincides with a standard expected utility (EU) preference that treats p_h and p_l symmetrically.

The capacities $\nu(E)$ and $\nu(E^c)$ can be rewritten as follows.

$$\nu(E) = W\left(\frac{p_h + p_l}{2}; \varepsilon, \alpha\right) := (1 - \varepsilon)\left(\frac{p_h + p_l}{2} - \alpha\right) + \varepsilon \cdot 0.5, \quad (2)$$

$$\nu(E^c) = W\left(1 - \frac{p_h + p_l}{2}; \varepsilon, \alpha\right), \quad (3)$$

where $\varepsilon = 1 - \frac{\nu(E) - \nu(E^c)}{p_h + p_l - 1}$ and $\alpha = \frac{1 - \nu(E) - \nu(E^c)}{2(1 - \varepsilon)}$.²⁰ Figure 3.1 illustrates geometrically how ε and α are derived from $\nu(E)$ and $\nu(E^c)$. To interpret, $1 - \varepsilon$ captures how sensitive the capacities are to the objective probabilities, so ε is a measure of uncertainty-induced insensitivity. If $\varepsilon = 0$, then the capacities are fully sensitive; as ε approaches 1, the capacities become less sensitive. On the other hand, α measures how far the average capacity of the two events fall short from their average objective

¹⁹Gilboa and Marinacci (2016); Machina and Siniscalchi (2014) provide surveys on models of uncertainty attitudes.

²⁰If $p_h + p_l = 1$, I set ε to be 0. When $p_h + p_l \neq 1$, I assume that $\nu(E) > \nu(E^c)$ so that $\varepsilon < 1$ and α is well-defined.

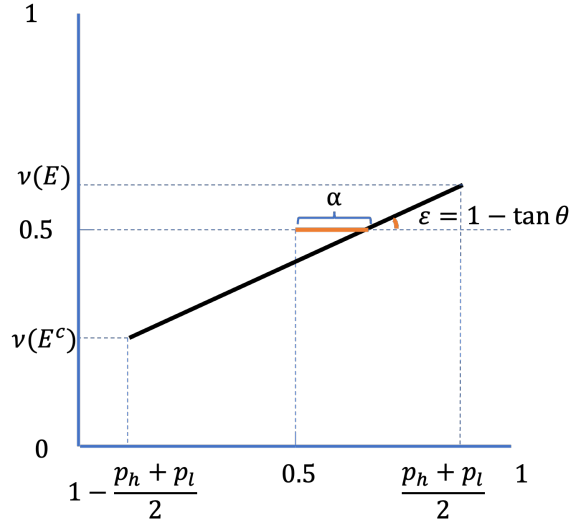


Figure 3.1: Illustration of uncertainty-induced insensitivity and uncertainty aversion in a CEU model

probability (which is $1/2$), normalized by the sensitivity of the capacities to objective probabilities. Therefore, α is a measure of uncertainty aversion, with $\alpha = 0$ representing uncertainty neutrality. Equations (2) and (3) indicate that ε and α fully capture a CEU agent's uncertainty attitude toward E and E^c . Intuitively, when evaluating an event, a CEU agent first assign ε weight to 0.5 , which is the average objective probability of the two events E and E^c . The rest of the weight is assigned to the objective probability $\frac{p_h + p_l}{2}$ shaded by the degree of uncertainty aversion α . Importantly, ε and α are specific to the events E and E^c . In my experimental setting, for example, uncertainty attitudes may depend on whether the events are outcomes of the horse race or the correctness of the report. The values of ε and α can also differ between ambiguous and compound events and can vary with the events' objective probabilities.

3.2 Updating rules

In problems of belief updating with uncertain information accuracy, uncertainty attitudes may manifest themselves in a number of different ways depending on the updating rule. Consider the following setup in which an agent chooses between a bet and a certain amount of utils. There are two payoff-relevant events, G and B . The bet pays out 1 util if G occurs and 0 util otherwise. Let p be the probability of G . Before an agent makes the choice, she receives an additional piece of binary information $m \in \{g, b\}$.

This setup emulates the main parts of the experiment. One util corresponds to \$20. For the red bet, event G is *Red*, event B is *Blue*, report g is “Red horse won” and report b is “Blue horse won.”

For the blue bet, the mapping is reversed.

I define the *evaluation* of a bet as the amount of utils u such that the agent is indifferent between the bet and u . If the accuracy of the information is $Pr(g|G) = Pr(b|B) = \psi$ with certainty, then after observing the realized report, a Bayesian EU agent will evaluate the bet by the Bayesian posterior belief on G : $u(g) = Pr^{Bayes}(G|p, g, \psi) := \frac{p\psi}{p\psi + (1-p)(1-\psi)}$, $u(b) = Pr^{Bayes}(G|p, b, \psi) := \frac{p(1-\psi)}{p(1-\psi) + (1-p)\psi}$.

In an *uncertain information* problem, the prior probability of G is still simple, but the accuracy of additional information is either ψ_h or ψ_l . The two levels of accuracy satisfy $0 < \psi_l < \psi_h < 1$ and $\psi_h + \psi_l \geq 1$. Which accuracy level is true is uncorrelated with the payoff-relevant events. If the two accuracy levels are equally likely (as in compound information), then a Bayesian EU agent will evaluate the bet by the Bayesian posterior belief on G :

$$u(m) = Pr^{Bayes}\left(G|p, m, \frac{\psi_h + \psi_l}{2}\right), \quad m \in \{g, b\}. \quad (4)$$

The Bayesian EU agent will have the same conditional evaluations when the information is ambiguous if she treats the two accuracy levels symmetrically based on the principle of insufficient reason.²¹

For an agent who is not a Bayesian EU maximizer, the conditional evaluations of bets in an uncertain information problem depend on her uncertainty attitudes toward information accuracy and her belief-updating rule. I assume a CEU agent and use ε and α to capture her uncertainty attitudes about the events “the information is correct/incorrect.”²² I will analyze three major non-Bayesian belief-updating rules. These three updating rules are most widely used in applied work, have clear psychological intuitions, and are the basic elements of many other rules.²³ For each updating rule, I will examine how choices conditional on uncertain information deviate from those conditional on simple information. I will also investigate how uncertainty attitudes for information accuracy (i.e. ε and α) affect these choices. Proofs of results in this subsection can be found in Appendix C.2.

3.2.1 Full Bayesian updating

In an uncertain information problem, Full Bayesian updating dictates that the evaluation of a bet conditional on a good report is given by

$$u(g) = Pr^{Bayes}(G|p, g, W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)) \quad (5)$$

²¹Alternatively, one can apply Bayes’ rule by first calculating one Bayesian posterior for each accuracy level and then taking their average weighted by the updated likelihood of each accuracy level. This procedure is equivalent to applying Bayes’ rule to the midpoint accuracy.

²²The capacity of every event in the state space that is relevant for belief updating can be derived under minimal assumptions. See Appendix C.1 for details.

²³I study other updating rules (e.g. [Klibanoff et al., 2009](#); [Gul and Pesendorfer, 2018](#); [Cheng and Hsiaw, 2018](#)) in the 2020 version of this paper ([Liang, 2020](#)).

and conditional on a bad report it is

$$u(b) = Pr^{Bayes}(G|p, b, W(\frac{\psi_h + \psi_l}{2}; \varepsilon, -\alpha)). \quad (6)$$

These formulas can be derived from [Eichberger et al. \(2007\)](#) and they admit a simple interpretation. The agent behaves as if she updates by applying Bayes' rule to the prior and her subjective accuracy, which is a distortion of the midpoint accuracy $\frac{\psi_h + \psi_l}{2}$. The subjective accuracy puts ε -weight on 50%, which leads to under-reaction to news. The rest of the weight is assigned to the midpoint accuracy minus or plus α , depending on which accuracy level leads to a more pessimistic Bayesian posterior *given the realized report*. Intuitively, an agent who is averse to uncertainty in information accuracy ($\alpha > 0$) worries that good reports' accuracy is low but bad reports have high accuracy. An extreme form of pessimism can occur if $\frac{\psi_h + \psi_l}{2} - \alpha < 50\%$. In this case, even the evaluation conditional on a good report is (weakly) lower than the prior p .

The following proposition summarizes the predictions of Full Bayesian updating.

Proposition 1 *Suppose that a CEU agent uses Full Bayesian updating. In an uncertain information problem:*

1. *if $\varepsilon = 0$ and $\alpha = 0$, then her conditional evaluations coincide with the Bayesian evaluations conditional on simple information with accuracy level $\frac{\psi_h + \psi_l}{2}$;*
2. *as α increases, there is more pessimism, i.e., the conditional evaluations decrease;*
3. *as ε increases, there is more under-reaction, i.e., the conditional evaluations become closer to p .*

3.2.2 Dynamically consistent updating

In an uncertain information problem, under Dynamically consistent updating ([Hanany and Klibanoff, 2007](#)), the evaluation of the bet conditional on report $m \in \{g, b\}$ is given by

$$u(m) = Pr^{Bayes}(G|p, m, \max\{W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha), 50\%\}). \quad (7)$$

Unlike Full Bayesian updating, the as-if subjective information accuracy under Dynamically consistent updating is the same regardless of the realized report. Specifically, the weight on 50% is always ε and the rest of the weight is always assigned to $\frac{\psi_h + \psi_l}{2} - \alpha$ so long as the subjective information accuracy is not lower than 50%.

The interpretation of this formula is that an agent who uses Dynamically consistent updating evaluates her contingent plan of choices *before the realization of information*. If the agent is averse

to uncertainty in information accuracy ($\alpha > 0$), then she will prefer to under-react to information so that her ex-ante payoff is less dependent on the realization of that uncertainty.

Proposition 2 *Suppose a CEU agent uses Dynamically consistent updating. In an uncertain information problem:*

1. *if $\varepsilon = 0$ and $\alpha = 0$, then the conditional evaluations coincide with the Bayesian evaluations conditional on information with accuracy level $\frac{\psi_h + \psi_l}{2}$;*
2. *as either ε or α increases, there is more under-reaction, i.e., the conditional evaluations become closer to p .*

3.2.3 Maximum likelihood updating

In an uncertain information problem, Maximum likelihood updating (Gilboa and Schmeidler, 1993) selects only the accuracy level(s) that is mostly likely given the realized report, then the agent conducts Full Bayesian updating using the selected accuracy level(s). Since reports that confirm the prior are more likely to be accurate than not, agents update too much to them. By a similar logic, they update too little to reports that go against the prior. Formally, if $p \neq 50\%$, then the evaluation of the bet conditional on a good report is given by

$$u(g) = \begin{cases} Pr^{Bayes}(p, g, \psi_h), & \text{if } p > 50\% \\ Pr^{Bayes}(p, g, \psi_l), & \text{if } p < 50\% \end{cases} \quad (8)$$

and conditional on a bad report it is

$$u(b) = \begin{cases} Pr^{Bayes}(p, b, \psi_l), & \text{if } p > 50\% \\ Pr^{Bayes}(p, b, \psi_h), & \text{if } p < 50\% \end{cases}. \quad (9)$$

If $p = 50\%$, then the predictions of Maximum likelihood updating coincide with those of Full Bayesian updating.

The following proposition summarizes the properties of Maximum likelihood updating.

Proposition 3 *Suppose a CEU agent uses Maximum likelihood updating. In an uncertain information problem:*

1. *If $p \neq 50\%$, the conditional evaluations of the bet exhibit confirmation bias relative to those conditional on simple information with accuracy $\frac{\psi_h + \psi_l}{2}$. That is, evaluations update more if information confirms the prior, less if it contradicts the prior. The measures of uncertainty attitudes, ε and α , do not affect the conditional evaluations.*

Theory	Aversion ($\alpha > 0$)	Insensitivity ($\varepsilon > 0$)
Full Bayesian updating	Pessimism	Under-reaction
Dynamically consistent updating	Under-reaction	Under-reaction
Maximum likelihood updating	$p \neq 50\%$: Confirmation bias (α and ε are irrelevant) $p = 50\%$: coincide with FBU	

Table 3.1: Summary of theoretical predictions in uncertain information problems

2. If $p = 50\%$, conditional evaluations under Maximum likelihood updating coincide with those under Full Bayesian updating.

3.2.4 Summary of theoretical implications

Consider a CEU agent who is insensitive and averse to uncertainty in information accuracy: $\varepsilon > 0$ and $\alpha > 0$. Taking Bayesian learning with the corresponding simple information as the benchmark, Table 3.1 summarizes the predictions of the updating rules that I have discussed so far. If $\varepsilon = 0$ and $\alpha = 0$, then all theories except Maximum likelihood updating coincide with the benchmark.

4 Experimental results on belief updating with uncertain information accuracy

The previous section presented three theories of belief updating with uncertain information, which generate a total of three different patterns: under-reaction, pessimism, and confirmation bias. The left panel of Figure 4.1 illustrates what each of these patterns implies about the comparisons between belief updating with simple and uncertain information. *Neutral news* is defined as reports whose (midpoint) accuracy is 50%, and *good (bad) news* as non-neutral news that says a bet will win (lose). Under-reaction and pessimism have the same directional predictions for good news but opposite ones for bad news. For neutral news, under-reaction predicts that uncertain information accuracy does not have an effect on posteriors, whereas pessimism predicts that posteriors conditional on uncertain information will be lower. The directional prediction of confirmation bias depends on the prior: the posteriors conditional on uncertain information are higher given high priors and lower given low priors.

The right panel of Figure 4.1 test these patterns by showing the CEs of bets with simple priors (henceforth *simple bets*) conditional on good news, bad news, and neutral news. Additional statistical tests – including within- and between-subject *t*-tests – can be found in Table B.4. Perhaps the most salient empirical pattern is that the mean CEs conditional on uncertain good news are lower than the mean CEs conditional on their simple counterpart for every combination of prior and (midpoint)

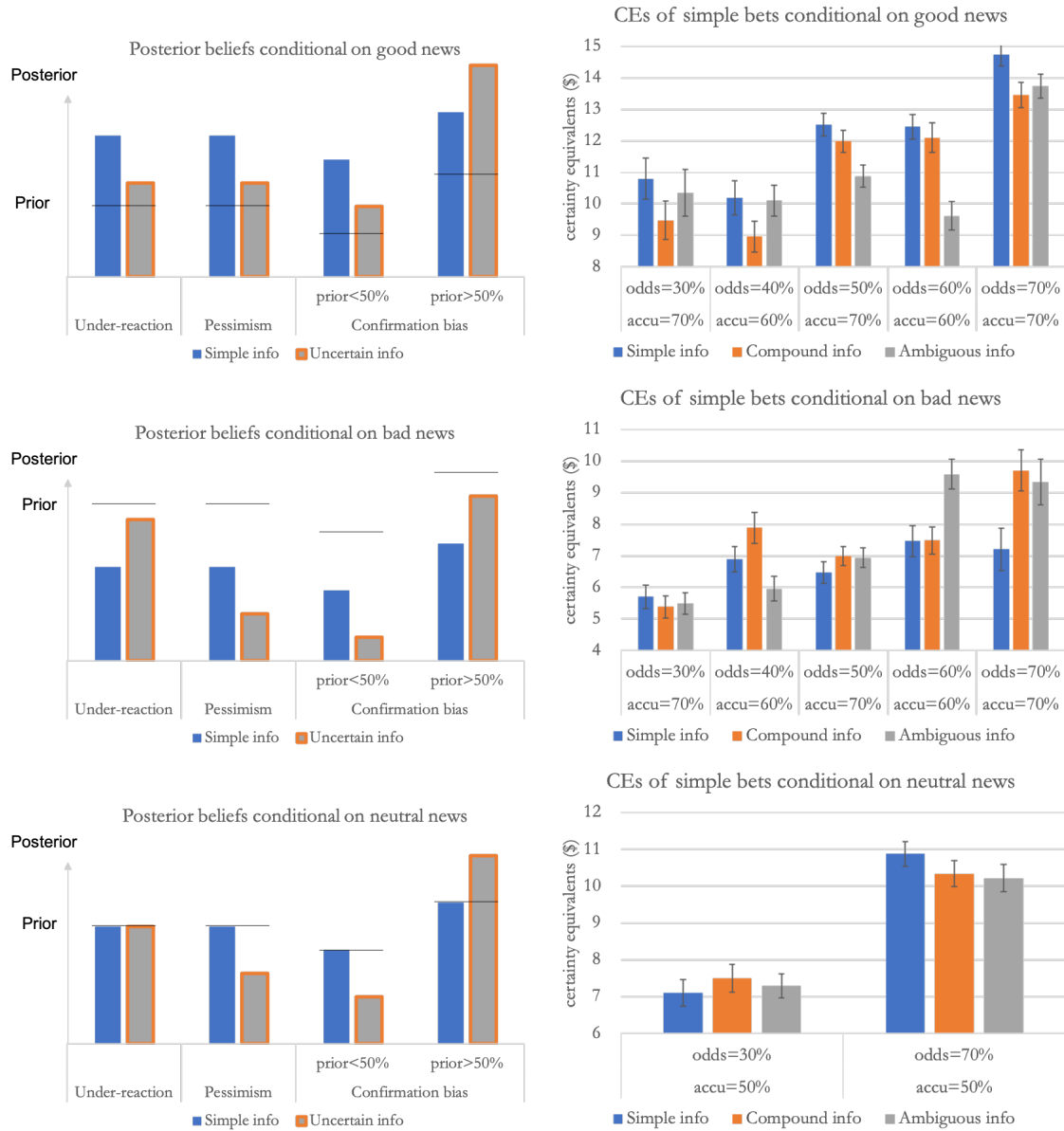


Figure 4.1: Simple priors with simple and uncertain information

Notes: The left panel of this figure illustrates what under-reaction, pessimism, and confirmation bias each predicts about the comparisons between belief updating with simple and uncertain information. Neutral news refers to any report whose (midpoint) accuracy is 50%. Good (Bad) news is a good (bad) report that is non-neutral news. The right panel compares the mean CEs of simple bets conditional on simple, compound and ambiguous information in the experiment. Each group of bars corresponds to a combination of prior and information. For example, “odds=30%, accu=70%” in the upper right graph represents tasks where the prior is 30% and the information is good news with 70% (midpoint) accuracy. Error bars represent 95% confidence intervals.

information accuracy. This is consistent with the predictions of both under-reaction and pessimism, but not consistent with confirmation bias.

For bad news, the mean CEs conditional on compound and ambiguous information are higher compared to simple information in three out of five comparisons, and they are slightly lower or mixed in the other two. Since under-reaction and pessimism generate opposite directional predictions for bad news, one interpretation of the results is that both under-reaction and pessimism are at force, although the results are less uniform than those of good news. In addition, if we focus only on CEs conditional on ambiguous information, the results are consistent with confirmation bias, which is the prediction of Maximum likelihood updating.

The mean CEs of a 70% odds bet conditional on compound and ambiguous neutral news are significantly lower than that conditional on simple neutral news. For a 30% odds bet, the mean CEs conditional on compound and ambiguous neutral news are statistically indistinguishable from that conditional on simple neutral news. Again, the results are consistent with the combination of under-reaction and pessimism, but not with confirmation bias. Taken together, the empirical patterns resemble the prediction of Full Bayesian updating the most.²⁴

To further demonstrate the under-reaction and pessimism caused by uncertain information accuracy, for each round with an analyst with uncertain accuracy (henceforth *uncertain information round*), I define *absolute pessimists/optimists* and *absolute under-/over-reactors*, two pairs of mutually-exclusive categories, and then show that the former in each pair prevails.

First, I introduce some notations. Let $CE(p, m, \psi_h \text{ or } \psi_l)$ denote the conditional CE of a bet in an uncertain information round, where p is the prior of the bet, $m \in \{g, b\}$ indicates whether the report says that the bet will win or lose, and the third argument represents the possible accuracy levels of the information. Analogously, let $CE(p, m, \psi)$ be the conditional CE of a bet in a round with simple prior and simple information. Then, in an uncertain information round with non-neutral news, define the *uncertainty premium* of a bet as

$$Pm(p, m, \psi_h \text{ or } \psi_l) := CE(p, m, \frac{\psi_h + \psi_l}{2}) - CE(p, m, \psi_h \text{ or } \psi_l). \quad (10)$$

Note that $Pm(p, m, \psi_h \text{ or } \psi_l)$ may be missing for some subjects because calculating it requires that $CE(p, m, \frac{\psi_h + \psi_l}{2})$ is available in the data. In the rounds with neutral news, I do not distinguish between the contents of the report. The uncertainty premium of a bet in these rounds is defined as

$$Pm(p, -, 90\% \text{ or } 10\%) := CE(p, m', 50\%) - CE(p, m, 90\% \text{ or } 10\%), \quad (11)$$

²⁴While compound uncertainty and ambiguity often have the same directional effects on CEs, their magnitudes are different in many cases. In Appendix E, I compare the magnitudes of their effects at the subject level.

		Bet that the report says will win	
		+	-
Bet that the report	+	Absolute pessimist	Absolute over-reactor
says will lose	-	Absolute under-reactor	Absolute optimist

Table 4.1: Classification of subjects in an uncertain information round

Notes: This table summarizes the classification of subjects in an uncertain information round. To be classified into any of the four categories, the uncertainty premium of at least one bet in the round needs to be non-zero. For rounds with neutral information, I do not classify subjects as absolute over-/under-reactors.

where m and m' are the realized reports in the respective rounds.

Now I can define the categories, which are summarized in Table 4.1. A subject is an absolute pessimist in an uncertain information round if the uncertainty premiums of the two bets in this round are both weakly positive and at least one of them is strictly positive. Analogously, a subject is an absolute optimist if the two uncertainty premiums are both weakly negative and at least one is strictly negative.

In an uncertain information round where the midpoint information accuracy is not 50%, if a subject's uncertainty premium for the bet that the report says will win is weakly positive, her uncertainty premium for the other bet is weakly negative, and at least one of the two is not zero, then I call this subject an absolute under-reactor. By contrast, if the bet that the report says will win has a weakly negative uncertainty premium, the other one has a weakly positive premium, and at least one is not zero, then this subject is called an absolute over-reactor. In rounds with neutral news, I do not classify subjects into these two categories.

Table 4.2 shows the percentages of each category in each uncertain information round. In every round, there are more absolute under-reactors than absolute over-reactors. In all but one rounds, there are more absolute pessimists than absolute optimists. In aggregate, the percentage of absolute under-reactors is significantly higher for both compound and ambiguous information rounds. The aggregate percentage of absolute pessimists is also higher than that of absolute optimists, and the difference is significant for ambiguous information rounds.²⁵

In summary, my experimental results show that uncertainty in information accuracy leads to under-reaction and pessimism. These two patterns are most consistent with the prediction of

²⁵In Appendix B.1, I consider two alternative categories: absolute confirmation bias and absolute contradiction bias. These two categories overlap with absolute over-/under-reactors, as an absolute over-reactor in a round with a confirmatory report is classified into the category of absolute confirmation bias. In all but one round, there are fewer absolute confirmation-biased subjects than absolute contradiction-biased subjects. This result together with the comparisons between mean CEs of bets suggests that uncertainty in information accuracy does not lead to prevalent confirmation bias, with the possible exception of ambiguous bad news.

Prior (<i>Red, Blue</i>)	Midpoint Information accuracy	Type of information	Absolute pessimists	Absolute optimists	<i>p</i> -value %(Abs. pess.) =%(Abs. opt.)	Absolute under-reactors	Absolute over-reactors	<i>p</i> -value %(Abs. under.) =%(Abs. over.)	N
(50%, 50%)	70%	Ambiguous	31.5%	19.4%	0.029	44.2%	18.2%	0	165
(60%, 40%)	60%	Ambiguous	31.0%	18.3%	0.128	43.7%	18.3%	0.007	71
(70%, 30%)	70%	Ambiguous	25.5%	23.4%	0.768	40.4%	19.1%	0.008	94
(70%, 30%)	50%	Ambiguous	25.5%	19.7%	0.31	-	-	-	137
Aggregate		Ambiguous	28.5%	20.1%	0.01	43.0%	18.5%	0	
(50%, 50%)	70%	Compound	21.2%	25.5%	0.425	43.0%	22.4%	0.001	165
(60%, 40%)	60%	Compound	27.4%	23.6%	0.586	36.8%	24.5%	0.107	106
(70%, 30%)	70%	Compound	27.6%	16.3%	0.057	39.8%	26.0%	0.059	123
(70%, 30%)	50%	Compound	27.7%	19.7%	0.172	-	-	-	137
Aggregate		Compound	25.6%	21.5%	0.164	40.4%	24.1%	0	

Table 4.2: Classification of subjects in each uncertain information round

Notes: This table shows the percentages of subjects that are classified into the four categories in each uncertain information round. Only subjects who face comparable belief-updating problems in the uncertain information round and its corresponding simple information round are counted. In the rows under “Aggregate”, I calculate the percentage of instances that subjects are classified into each category, aggregated across the four or three rounds that are relevant for that category. The *p*-values are computed using Pearson’s chi-square goodness-of-fit tests.

uncertainty-induced insensitivity and uncertainty aversion together with Full Bayesian updating.

5 Relationship with uncertainty attitudes toward economic fundamentals

Previous research on uncertainty attitudes typically studies Ellsberg urns, compound lotteries, or complex financial assets. A common feature among these objects is that the probability distribution that is uncertain is over the payoff-relevant events. This is in contrast to unknown information sources where the uncertain probability distribution is over the correctness of the information. Understanding the relationship between uncertainty attitudes toward distributions over payoff-relevant events (henceforth *priors* or *economic fundamentals* for short) and uncertainty attitudes toward information accuracy informs the theoretical question of whether uncertainty attitudes are universal or issue-specific. Practically, it also tells us whether it is appropriate to make predictions about reactions to unknown information sources using our knowledge about evaluations of assets with uncertain economic fundamentals.

To study subjects' uncertainty attitudes toward priors, I compare the CEs of uncertain bets in Part 4 of the experiment to the CEs of simple bets in Part 1. Consistent with prior studies, subjects exhibit uncertainty aversion and uncertainty-induced insensitivity when they evaluate bets with uncertain odds. I also compare evaluations of uncertain and simple bets conditional on simple information (Part 5 and Part 2). Even with information, subjects still exhibit uncertainty aversion, but I do not find evidence of uncertainty-induced insensitivity. Details of the results and the theoretical justifications of the comparisons are relegated to Appendix A.

These results show that uncertainty attitudes toward information accuracy and priors are qualitatively similar in the aggregate. However, in order to determine whether they manifest the same behavioral trait, we need to investigate their correlations at the individual level. If such correlations are strong and significant, then we can rather safely use knowledge about an agent's attitude toward one kind of uncertainty to make predictions about her attitudes toward the others; otherwise, extrapolation is not warranted and we would need to study them separately.²⁶

Correlation analysis is challenging because different combinations of updating rules and uncertainty attitudes can generate similar behavior. Without knowing the updating rule to which a subject adheres, it is sometimes difficult to pin down her uncertainty attitudes. For example, suppose that a CEU subject exhibits under-reaction to news but no pessimism in an uncertain information problem. Then, this behavior is consistent with $\varepsilon > 0$, $\alpha = 0$ and Full Bayesian updating, but it is also

²⁶See Appendix E for a similar individual-level analysis of the relationship between compound and ambiguity attitudes.

consistent with $\varepsilon \geq 0$, $\alpha > 0$ and Dynamically consistent updating. To circumvent this identification issue, I focus attention on tests of correlation that are valid under CEU preferences and all three updating rules considered so far. I informally describe these tests below and present the results. Details about their theoretical derivation and implementation can be found in Appendix D.

One such test is based on the following property of the CEU preferences. Suppose that an agent's uncertainty attitudes toward priors and information accuracy are described by the same insensitivity and uncertainty aversion measures. Then, if her CE of a simple bet with 70% odds is higher than that of its corresponding uncertain bet (odds = 90% or 50%), she must also evaluate a 50% odds simple bet higher after hearing a 70%-accurate simple good news than after hearing the corresponding uncertain good news (accuracy = 90% or 50%). The converse is also true. To see why this one-to-one mapping between the two CE comparisons holds, note that with the same ε and α for both information accuracy and priors, under any of the three updating rules, both comparisons boil down to comparing 70% and $W(70\%; \varepsilon, \alpha)$. As a result, the prevalence of this one-to-one mapping in the data can serve as a measure of the similarity between the two kinds of uncertainty attitudes at the individual level.

I compute the correlation between the directions of the aforementioned two CE comparisons using the experimental data. The coefficient is 0.08 (p -value = 0.28) for compound uncertainty and 0 (p -value = 0.99) for ambiguity. These results suggest that uncertainty attitudes toward priors and information accuracy are not similar on the individual level.

There are three potential objections to this interpretation of the results. First, the CEs in the first comparison are unconditional while those in the second one are conditional. Hence, it could be the act of updating that alters the manifestation of uncertainty attitudes. To control for this confound, in another test I replace the unconditional CEs in the first comparison with CEs of the same bets conditional on simple neutral news. The results are unaffected: the correlation coefficient is 0 (p -value = 0.97) for compound uncertainty and 0.03 (p -value = 0.71) for ambiguity. Second, one may worry that the noise in the data might dilute any correlations and render them undetectable. To address this concern, in a third test I compute the correlation between the unconditional CE comparison that appears in the first test and the CE comparison conditional on simple neutral news that appears in the second test. Both CE comparisons are driven by subjects' uncertainty attitudes toward priors and hence should be positively correlated. Indeed, the correlation coefficient is 0.15 (p -value = 0.05) for compound bets and 0.26 (p -value = 0) for ambiguous bets, both being significantly positive. This result shows that the lack of correlation in the first two tests is not an artifact of measurement errors. A third concern is that the lack of correlation might be driven by inattentive or "confused" subjects. In Appendix D.1, I repeat the tests within a subsample of subjects who adhere well to some basic rationality properties, and the results remain qualitatively unchanged.

Taken together, the results suggest that subjects have distinct uncertainty attitudes toward priors

and information accuracy.

6 Suggestive evidence from the stock market

In this section, I complement the experimental results with evidence from the US stock markets. Consistent with the lab findings, I show that stock prices react less sufficiently to analyst earnings forecasts with more uncertain accuracy. In addition, the decrease in reaction sufficiency only occurs for good news, but not for bad news. These empirical patterns suggest that the experimental findings on learning from unknown information sources are externally valid and economically important.

Brokerage houses hire financial analysts to conduct research on publicly-traded companies and issue forecasts on their earnings. A large body of literature in accounting and finance has studied the information content of analysts' forecasts, the market reactions to them, and the factors that affect the sufficiency of these reactions (Kothari et al., 2016). Following this literature, I use data from three sources: quarterly earnings forecasts and earnings announcements from the Institutional Broker Estimate System (I/B/E/S) detail history file, stock returns from the Center for Research in Security Prices (CRSP), and firm characteristics from Compustat. I require stocks to be common shares (share codes 10 or 11) on the AMEX, NYSE, or NASDAQ (exchange codes 1, 2, or 3), and I exclude stocks with prices lower than \$1 or market capitalization smaller than \$5 million. I restrict attention to earnings forecasts for quarters between January 1, 1994 and June 30, 2019,²⁷ but in order to construct analyst characteristics such as experience, I use data dated back to January 1, 1984.

The setting of analyst earnings forecasts and the stock market provides an opportunity to study the effect of uncertain information accuracy on market reaction. In Appendix F.1, I prove that when the accuracy of an earnings forecast is uncertain, the reaction of an investor's earnings expectation will be insufficient and biased downward if she is a CEU agent with typical uncertainty attitudes ($\varepsilon > 0$ and $\alpha > 0$) and uses Full Bayesian updating. To the extent that stock price movement reflects the change in investors' earnings expectations, the stock price reactions to forecasts with uncertain accuracy will exhibit the same patterns.

To empirically test this prediction, a key challenge is that I do not observe the exact degree of uncertainty that investors perceive about the accuracy of each analyst forecast. In light of this issue, I use whether the issuing analyst has a proven forecast record for the stock as a proxy for the perceived uncertainty in his report's accuracy. Specifically, at a point in time, an analyst has a proven forecast record for a stock if she has issued a quarterly earnings forecast on this stock before *and* the actual earnings of that quarter have been announced. This proxy is valid because prior research has shown that forecast accuracy is stock-specific and persistent (Park and Stice, 2000), past forecast accuracy

²⁷I do not include observations that date further back in time because the announcement dates recorded in I/B/E/S often differed from the actual dates by a couple of days prior to the early-1990s.

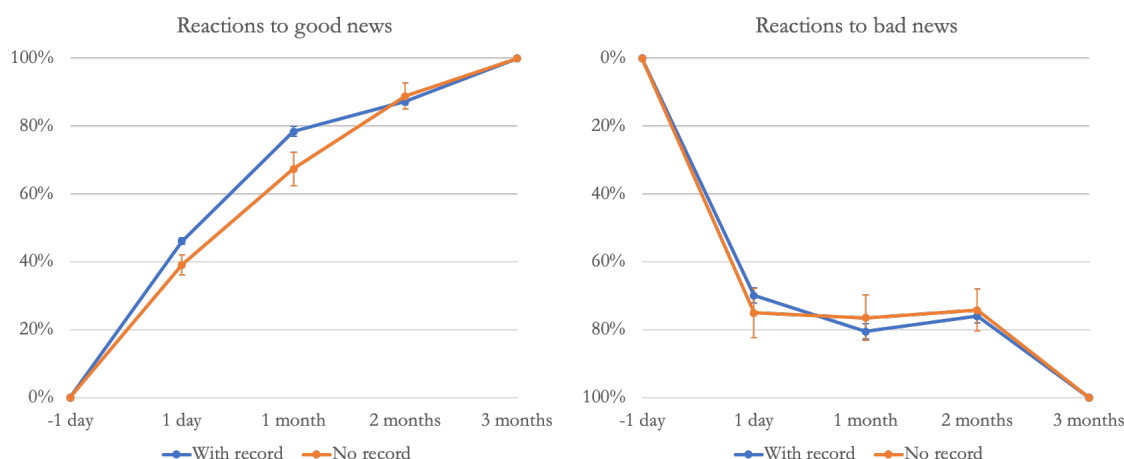


Figure 6.1: Reactions to forecast revisions

Notes: This figure shows the average size-adjusted returns from one trading day before the forecast announcement to one trading day, one month, and two months after the forecast announcement, normalized by the average three-month returns. The left and right panels plot reactions to upward and downward forecast revisions, respectively. Error bars represent standard errors calculated using the delta method.

predicts future accuracy better than many other analyst attributes (Brown, 2001; Hilary and Hsu, 2013), and investors learn about an analyst's forecast accuracy from her forecast record (Chen et al., 2005). I will henceforth refer to forecasts issued by analysts without (stock-specific) forecast records as "no-record forecasts" and the rest as "with-record forecasts."

In order to identify the stock price reaction to a specific analyst earning forecast, I need to mitigate the confounds of other news events that happen around the same time as the forecast announcement. To do this, I only include observations where there is neither earnings announcement from the company nor earnings forecast announcements by any other analyst on the same company on the forecast announcement day. Moreover, I restrict attention to forecast revisions, which can be naturally classified into good news and bad news. Following Gleason and Lee (2003), I define good news as an upward revision, which is a forecast that is higher than the issuing analyst's previous forecast on the same quarterly earnings. Analogously, bad news is defined as a downward forecast revision. This leaves us with a final sample of 1,025,823 forecasts issued by 12,815 analysts on 10,712 stocks.

Descriptive results show clear patterns that support the hypotheses. Figure 6.1 plots the average size-adjusted returns²⁸ from one trading day before the forecast announcement to one trading day, one month, and two months after the forecast announcement, normalized by the average three-month returns. Assuming that reactions to forecasts are complete after three months, the proportion of

²⁸Size-adjusted returns are the stock's buy-hold returns minus the equal-weighted average returns of stocks in the same size decile in the same period.

reactions that happen in a shorter period is a measure of reaction sufficiency in that period. The left panel shows that for good news, the one-day, one-month and two-month reactions are less sufficient for no-record forecasts. By contrast, for bad news, there is almost no difference in sufficiency between reactions to no-record and with-record forecasts (as is shown in the right panel). These results suggest that investors react insufficiently and pessimistically to no-record forecasts.²⁹

In addition to the uncertainty in accuracy, no-record and with-record forecasts also differ in other dimensions, which need to be controlled for in a regression analysis in order to identify the effect of uncertainty. Table F.2 provides the definitions for the control variables in the regression, which include characteristics of the forecasts, the issuing analysts, the stocks covered, and the information environment. Table F.3 lists their summary statistics.³⁰ No-record forecasts on average have larger realized forecast errors. The companies they cover tend to be smaller, have higher and more volatile past returns and lower book-to-market ratios, and are followed by fewer analysts. The analysts without past records follow fewer stocks and industries.

The main specification of the regression analysis is as follows.

$$\begin{aligned}
 Ret[2, 64]_i = & \eta_0 + \eta_1 Ret[-1, 1]_i + \eta_2 NoRecord_i + \eta_3 GoodNews_i \\
 & + \eta_4 NoRecord_i \cdot GoodNews_i + \eta_5 Ret[-1, 1]_i \cdot GoodNews_i + \eta_6 Ret[-1, 1]_i \cdot NoRecord_i \\
 & + \eta_7 Ret[-1, 1]_i \cdot NoRecord_i \cdot GoodNews_i + Controls_i + Controls_i \cdot Ret[-1, 1]_i + TimeFE_i + \varepsilon_i.
 \end{aligned}
 \tag{12}$$

The dependent variable $Ret[2, 64]_i$ is the size-adjusted stock returns in the [2,64]-trading day period after forecast i is announced (64 trading days are roughly three months), and $Ret[-1, 1]_i$ is the immediate price reaction to forecast i in the [-1,1]-trading day window. The correlation between the immediate price reactions and the price drifts that ensue is a measure of the sufficiency of the market's reactions to analysts' forecasts. This is because if immediate price reactions are on average followed by drifts in the same (opposite) direction, then the immediate reactions must be insufficient (excessive). $NoRecord_i$ and $GoodNews_i$ are indicator variables for no-record forecasts and good news as previously defined. By including the interactions between $Ret[-1, 1]$, $NoRecord$ and $GoodNews$, this specification can measure how much the sufficiency of stock price reactions to forecasts varies with the issuing analyst's record and the direction of forecast revision. In addition, I include controls on the characteristics of the forecast, the issuing analyst, the stock covered, and the information environment, as well as their interactions with $Ret[-1, 1]$. Year-quarter dummies are also included to control for unobserved time fixed effects on returns. In view of the descriptive results that immediate stock price reactions to no-record forecasts are less sufficient especially for

²⁹The summary statistics for unnormalized returns in windows with different lengths are in Table F.1.

³⁰Table F.4 provides summary statistics for all earnings forecasts issued between January 1, 1994 and June 30, 2019, including those that do not meet our data selection criteria.

Dependent Var: Ret[2,64]	(1)	(2)	(3)	(4)
Ret[-1, 1]	0.0215 (0.0336)	0.0173 (0.0333)	0.343*** (0.100)	0.335** (0.100)
NoRecord	-0.000671 (0.00287)	-0.00225 (0.00277)	0.000814 (0.00213)	0.000584 (0.00205)
NoRecord \times Ret[-1, 1]	-0.0435 (0.0626)	-0.0430 (0.0622)	-0.0281 (0.0474)	-0.0311 (0.0465)
GoodNews	0.0113*** (0.00243)	0.0111*** (0.00211)	0.0107*** (0.00189)	0.0107*** (0.00177)
GoodNews \times Ret[-1, 1]	0.0605† (0.0351)	0.0569 (0.0349)	0.0480 (0.0294)	0.0452 (0.0293)
NoRecord \times GoodNews	0.00421 (0.00269)	0.00440† (0.00262)	0.00102 (0.00253)	0.00123 (0.00247)
NoRecord \times GoodNews \times Ret[-1, 1]	0.150* (0.0624)	0.150* (0.0626)	0.122† (0.0626)	0.124* (0.0620)
Controls	N	N	Y	Y
Controls \times Ret[-1,1]	N	N	Y	Y
Year-Quarter FE	N	Y	N	Y
Observations	1001418	1001417	894004	894004
R^2	0.001	0.010	0.004	0.014

Table 6.1: Sufficiency of stock market reactions to forecast revisions

Notes: This table reports the results of Regression (12). The dependent variable $Ret[2, 64]$ is the size-adjusted stock returns in the [2,64]-trading day period after a forecast is announced, and $Ret[-1, 1]$ is the immediate price reaction to a forecast. The variable *NoRecord* indicates that a forecast is issued by an analyst with no stock-specific forecast record. The variable *GoodNews* indicates an upward forecast revision. Control variables are characteristics of the forecast, the issuing analyst, the stock covered, and the information environment, summarized in Table F.2. Three-dimensional (stock, analyst, year-quarter) cluster-robust standard errors in parentheses. † $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

good news, we expect the coefficient on the triple interaction, η_7 , to be positive.

Table 6.1 shows the results from the regression analysis. Across the four specifications that differ on the set of controls and fixed effects, the coefficients on $NoRecord \times Ret[-1, 1]$ and $NoRecord$ are small and insignificant, suggesting that whether the issuing analyst of a forecast has a past record does not affect the sufficiency of immediate stock price reactions for bad news. By contrast, the coefficient on $Ret[-1, 1] \times NoRecord \times GoodNews$ is consistently positive and significant. To interpret the magnitudes of the coefficients, the ratio between the price drift in the [2,64]-trading day window and the immediate reaction is larger for no-record good news than for with-record good news by around 10 percentage points. Taken together, the results imply that investors' reactions to earnings forecasts with more uncertain accuracy are more insufficient and pessimistic.

In Appendix F.3, I examine the robustness of the regression results. In Table F.5, I show that the signs of the coefficients are robust to changing the price drift window of the left-hand side variable in Specification (12). The effect sizes tend to increase as the drift window becomes longer, suggesting that the insufficiency of the immediate reactions are gradually corrected. Table F.6 shows the regression results for different cuts of the data. The results are robust when I only consider “high-innovation” forecast revisions, “isolated” forecasts, and forecasts announced after January 1, 2004.³¹ The main effect appears not to be solely driven by forecasts on small-cap stocks, as the magnitude (although not the statistical significance) of the coefficient on the triple interaction term remains when I exclude all stocks with market capitalization smaller than \$2 billion. However, this coefficient vanishes if I only include large-cap stocks (market capitalization > \$10 billion), which may be due to the high concentration of sophisticated investors in those stocks and their low transaction costs. I also consider a specification that includes the interactions between year-quarter dummies and $Ret[-1, 1]$, and the results remain robust. Table F.7 reports the results of regressions that replace $Ret[-1, 1]$ and its interactions terms in Specification (12) with *Revision* and its interactions terms. The variable *Revision* is the difference between an analyst’s revised forecast on earnings per share and the previous forecast, normalized by the stock price two trading days prior to the announcement of the revision. The results from this specification are similar: the price drift per unit of *Revision* is larger for no-record good news than for with-record good news, although the difference is small and insignificant for bad news.

In sum, stock price reactions to earnings forecasts are less sufficient if they are issued by analysts with no forecast record. This phenomenon only happens for good news but not for bad news. These results corroborate the experimental finding that uncertainty in information accuracy leads to under-reaction to news and pessimism.

7 Conclusion

This paper studies the effects of uncertainty in information accuracy on belief updating using a controlled lab experiment and observational data from the stock market. In the experiment, a mean/midpoint preserving spread in the information accuracy leads to subjects reacting less to the

³¹Following Gleason and Lee (2003), a forecast revision is high-innovation if it falls outside the range between the issuing analyst’s previous forecast and the previous consensus (the consensus is the average of all forecasts available at the time). High-innovation forecast revisions are likely to contain new information as they are not simply herding toward the consensus. “Isolated” forecasts are observations where there is neither earnings announcement from the company nor forecast announcements by any other analysts on the same company in the three-day window centered on the forecast announcement day. This filter further eliminates concerns that other news events might be driving $Ret[-1, 1]$. The focus on the period after 2004 is because a host of regulations on the financial analyst industry came into effect in 2002/03 (Bradshaw et al., 2017), and the quality of forecast announcement time data in I/B/E/S improved after 2004 (Hirshleifer et al., 2019).

information. Moreover, the under-reaction is more pronounced for good news than for bad news. The same two patterns also emerge in the stock market. I show that stock prices under-react more to earnings forecasts issued by analysts with no proven forecast record, and the under-reaction only occurs for good news but not for bad news. I examine the predictions of a wide variety of theories and find that a theory that combines Full Bayesian updating with uncertainty aversion and uncertainty-induced insensitivity best captures the empirical results.

In the experiment, I compare the effects of uncertain information accuracy to those of uncertain priors. Uncertainty in priors leads to pessimism, and in problems without belief updating it also induces insensitivity. Although the aggregate effects of uncertain information accuracy and uncertain priors are mostly similar, subjects' attitudes toward these two kinds of uncertainty are uncorrelated. The lack of correlation lends support to the view that uncertainty attitudes depend on the relevant issues. Practically, it also suggests that knowing a person's attitude toward assets with unknown fundamentals does not help to predict her reactions to information from unknown sources.

This paper raises several questions for future research. First, given that the empirical settings in this paper are purely monetary, it remains to be explored how uncertain information accuracy interacts with non-financial concerns such as ideology and ego utility. Second, as belief updating is closely linked to demand for information ([Ambuehl and Li, 2018](#)), what are the determinants of demand for uncertain information? Third, as this paper shows that attitudes toward uncertain priors and uncertain information are uncorrelated, what are the moderating factors of these two distinct attitudes? Finally, given that signals from an uncertain information source are not only relevant for the payoff-relevant events but also inform the accuracy of the source itself, it would be interesting to study how people learn about a source's accuracy from its own signals.

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A Details on uncertainty attitudes toward priors

In this section, I present the theories and experimental evidence on the evaluations of bets with uncertain priors in detail.

A.1 Evaluating uncertain bets with no updating

In this section, I compare subjects' evaluations of uncertain bets and simple bets, both without any analyst report. The comparison shows that subjects' uncertainty attitudes toward priors exhibit uncertainty aversion and uncertainty-induced insensitivity, just like their attitudes toward information accuracy.

To use a similar theoretical framework as in Section 3, consider an agent choosing between a bet and a certain amount of utils. There are two payoff-relevant events, G and B . The bet pays out 1 util if G occurs and 0 util otherwise. If the probability of G is known to be p , then a standard EU agent will evaluate the bet by $u = p \cdot 1 + (1 - p) \cdot 0 = p$. If the event G has a compound probability, i.e. its probability is either p_h or p_l , each with equal chance, then a standard EU agent will evaluate the bet by $u = \frac{p_h + p_l}{2}$. The same evaluation also applies to the case of ambiguous probability if a standard EU agent treats p_h and p_l symmetrically under the principle of insufficient reason.

By contrast, if an agent's uncertainty attitudes toward priors are captured by a CEU preference, then she evaluates the uncertain bet by

$$u = W\left(\frac{p_h + p_l}{2}; \varepsilon, \alpha\right)$$

and applies her risk preference to translate utils to CEs.

Figure A.1 shows the CEs of simple, compound and ambiguous bets in Parts 1 and 4 of my experiment where subjects do not receive additional information.³² The mean CEs of simple bets are lower than their expected values except when the odds of winning is 30%. This is consistent with Prospect Theory (Kahneman and Tversky, 1979).

My main focus in this section, however, is on the comparison between CEs of uncertain bets and simple bets. The mean CEs of uncertain bets are lower than their simple counterparts for medium

³²Since the red and blue bets in a (50%, 50%) horse race are both bets with a 50% chance of winning, I take the average of the CEs of the two bets to be the CE of a 50% odds bet. In the simple round whose prior is (50%, 50%) and in its two corresponding uncertain rounds, 82% of the subjects report the same CE for the red and blue bets, which is in line with results in previous studies. See Table C. VI of Chew et al. (2017) for a meta-study. Moreover, the deviations from color neutrality are not significantly different from zero.

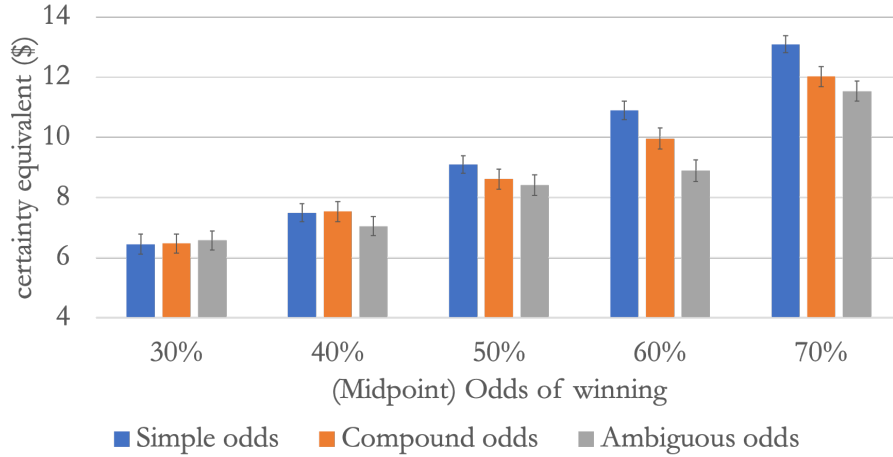


Figure A.1: CEs of bets without additional information

Notes: The figure shows the mean CEs of bets without belief updating. Each group of bars represents the three tasks that share the same (midpoint) odds of winning. Error bars represent \pm one standard error.

and high odds. Nevertheless, the gaps vanish for bets with low odds (30% for ambiguous bets, 30% and 40% for compound bets), indicating that the evaluations of uncertain bets are less sensitive to the winning odds than those of simple bets. These patterns confirm that the subjects' uncertainty attitudes toward priors exhibit uncertainty aversion and uncertainty-induced insensitivity,³³ just like their uncertainty attitudes toward information accuracy.

A.2 Evaluating uncertain bets conditional on simple information

While the experimental tasks used to illustrate uncertainty attitudes toward information accuracy feature belief updating, those analyzed in Section A.1 do not. To control for belief updating, I also study uncertainty attitudes toward priors by comparing evaluations of uncertain bets and simple bets after subjects update in response to simple information.

A.2.1 Theoretical setup

In an *uncertain prior* problem, the prior probability of G is either p_h or p_l with $p_l < p_h$, but the accuracy of additional information is known to be $\psi \geq 0.5$. As in the previous section, I will apply several belief-updating rules to the CEU model and compare their predictions in uncertain prior problems. Proofs of results in this subsection can be found in Appendix C.2.

³³For similar empirical patterns, see Abdellaoui et al. (2011, 2015); Dimmock et al. (2015); Baillon et al. (2018); Anantanasuwong et al. (2019).

A.2.2 Full Bayesian updating

Under Full Bayesian updating, a CEU agent updates by applying Bayes' rule to the prior capacity of G and the realized signal. Hence, given the uncertainty attitudes toward priors, the Full Bayesian evaluation of a bet conditional on report $m \in \{g, b\}$ is

$$u(m) = Pr^{Bayes}(G|W(\frac{p_h + p_l}{2}; \varepsilon, \alpha), m, \psi).$$

The insensitivity parameter ε is responsible for the degree of under-weighting of priors in belief updating, and the aversion parameter α corresponds to pessimism.

The following proposition summarizes the predictions of Full Bayesian updating in uncertain prior problems.

Proposition 4 *Suppose that a CEU agent uses Full Bayesian updating. In an uncertain prior problem:*

1. *if $\varepsilon = 0$ and $\alpha = 0$, then her conditional evaluations coincide with the Bayesian conditional evaluations given a simple prior $\frac{p_h + p_l}{2}$;*
2. *as α increases, the conditional evaluations decrease;*
3. *as ε increases, the evaluation conditional on a good report becomes closer to ψ and that conditional on a bad one becomes closer to $1 - \psi$.*

A.2.3 Dynamically consistent updating

In an uncertain prior problem, the conditional evaluations under Dynamically consistent updating are the same as those under Full Bayesian updating. Under Dynamically consistent updating, an agent who is averse to uncertainty ($\alpha > 0$) prefers to make choices so that her ex-ante payoff is less dependent on the realization of that uncertainty. When the uncertainty is in priors, mitigating ex-ante payoff exposure to uncertainty requires refraining from taking the bet. This coincides with Full Bayesian updating under which an uncertainty-averse agent tries to mitigate ex-post payoff exposure to uncertainty. The following proposition summarizes the results.

Proposition 5 *In an uncertain prior problem, Dynamically consistent updating has the same predictions as Full Bayesian updating for a CEU agent.*

A.2.4 Maximum likelihood updating

In uncertain prior problems, the prior(s) that is most likely to generate the realized report is selected and updated under Maximum likelihood updating. Since good news is more likely to be generated

Theory	Aversion ($\alpha > 0$)	Insensitivity ($\varepsilon > 0$)
Full Bayesian updating & Dynamically consistent updating	Pessimism	Under-weighting of priors
Maximum likelihood updating	$\psi \neq 50\%$: Over-reaction to news (α and ε are irrelevant) $\psi = 50\%$: coincide with FBU	

Table A.1: Summary of theoretical predictions in uncertain prior problems

from high priors and bad news from low priors, agents will over-react to news. Formally, if $\psi \neq 50\%$, then the evaluation of the bet conditional on a good report is given by

$$u(g) = Pr^{Bayes}(p_h, g, \psi)$$

and that conditional on a bad one is

$$u(b) = Pr^{Bayes}(p_l, b, \psi).$$

If $\psi = 50\%$, then the conditional evaluations coincide with Full Bayesian updating.

The following proposition summarizes the properties of Maximum likelihood updating.

Proposition 6 *Suppose a CEU agent uses Maximum likelihood updating. In an uncertain prior problem:*

1. *if $\psi \neq 50\%$, the conditional evaluations of the bet exhibit over-reaction relative to those given the simple prior $\frac{p_h + p_l}{2}$. The measures of uncertainty attitudes, ε and α , do not affect the conditional evaluations.*
2. *if $\psi = 50\%$, conditional evaluations under Maximum likelihood updating coincide with those under Full Bayesian updating.*

A.2.5 Summary of theoretical implications

Consider a CEU agent whose attitudes toward uncertain priors fall in the typical range: $\varepsilon > 0$ and $\alpha > 0$. Taking Bayesian learning with the corresponding simple prior as the benchmark, Table A.1 summarizes the predictions of the three updating rules I have discussed so far. The left panel of Figure A.2 illustrates what the three main predictions, under-weighting of priors, pessimism, and over-reaction to news, each implies about the comparisons between belief updating with simple and uncertain priors.

If $\varepsilon = 0$ and $\alpha = 0$, then all theories except Maximum likelihood updating coincide with the benchmark.

A.2.6 Experimental results

The right panel of Figure A.2 show the CEs of simple, compound and ambiguous bets conditional on simple information. Additional statistical tests – including within- and between-subject t -tests – are in Table A.4. Among the twelve combinations of prior and information accuracy, the mean conditional CE given the compound prior is lower than its simple counterpart in eight comparisons, and the mean conditional CE given the ambiguous prior is lower in seven comparisons. This suggests – albeit not strongly – that uncertain priors lead to pessimism in the conditional CEs. There is no clear pattern of either under-weighting of priors or over-reaction to news.

Similar as in the comparison between simple information and uncertain information, I define *absolute pessimists/optimists* and *absolute prior under-/over-weighters* for each uncertain prior round, and then compare their relative prevalence.

In an uncertain prior round, if the prior of a bet might be either p_h or p_l , the realized report is $m \in \{g, b\}$, and the information accuracy ψ is not 50%, then define the uncertainty premium of this bet in this round as

$$Pm(p_h \text{ or } p_l, m, \psi) := CE(\frac{p_h + p_l}{2}, m, \psi) - CE(p_h \text{ or } p_l, m, \psi).$$

If $\psi = 50\%$, then I define the uncertainty premium as

$$Pm(p_h \text{ or } p_l, -, 50\%) := CE(\frac{p_h + p_l}{2}, m', 50\%) - CE(p_h \text{ or } p_l, m, 50\%),$$

where m and m' are the realized reports in the respective rounds.

The classification of subjects is summarized in Table A.2. Same as in an uncertain information round, a subject is classified as an absolute pessimist in an uncertain prior round if the uncertainty premiums for the two bets in this round are both weakly positive and at least one of them is strictly positive. An absolute optimist, on the other hand, is a subject whose uncertainty premiums for the two bets in this round are both weakly negative but not both zero.

In uncertain prior rounds where the midpoint prior is not (50%, 50%), a subject is classified as an absolute prior under-weighter if the uncertainty premium of the red bet is weakly positive, that of the blue bet is weakly negative, and one of the two is not zero.³⁴ Analogously, a subject is a prior over-weighter if the uncertainty premium of the red bet is weakly negative, that of the blue bet is weakly positive, and one of the two is not zero. I do not classify prior under-/over-weighters for rounds where the midpoint prior is (50%, 50%).

Table A.3 shows the percentages of each of the four categories in all eight rounds with uncertain priors and simple information. In all rounds but one, there are more absolute pessimists than

³⁴Recall that the red bet always has a (midpoint) prior weakly higher than 50%.

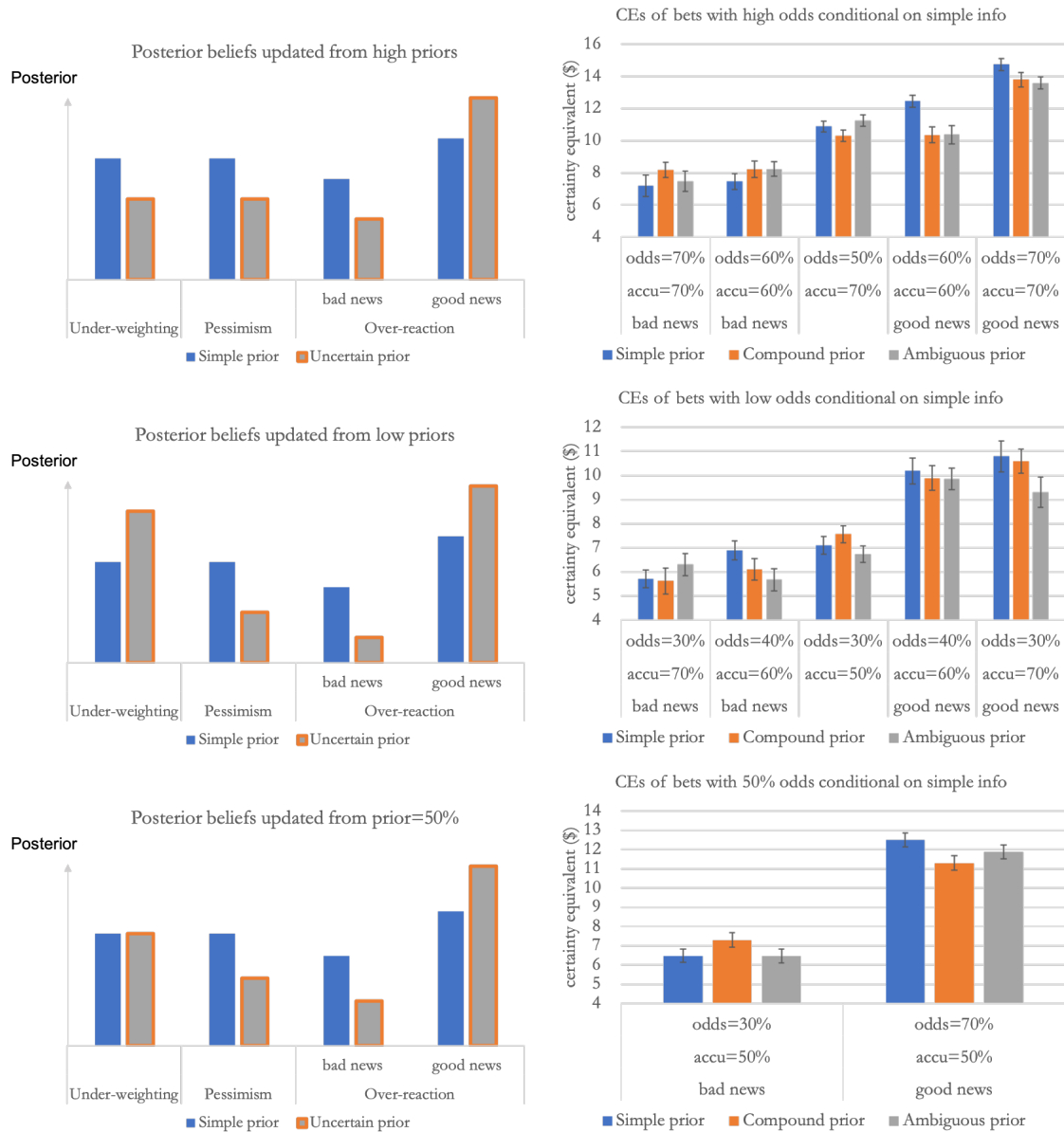


Figure A.2: Simple and uncertain priors with simple information

Notes: The left panel of this figure illustrates what under-weighting of priors, pessimism, and over-reaction to news each predicts about the comparisons between belief updating with simple and uncertain priors (high, low, and medium priors refer to priors that are higher, lower, and equal to 50%). The right panel compares the mean CEs of simple, compound, and ambiguous bets conditional on simple information in the experiment. Each group of bars correspond to a combination of prior and information. For example, “odds=70%, accu=70%, bad news” in the upper right graph represents tasks where the (midpoint) prior is 70% and the information is bad news with 70% accuracy. Error bars represent +/- one standard error.

		Red bet	
Uncertainty premium		+	-
Blue bet	+	Absolute pessimist	Absolute prior over-weighter
	-	Absolute prior under-weighter	Absolute optimist

Table A.2: Classification of subjects in an uncertain prior round

Notes: This table summarizes the classification of subjects in an uncertain prior round. To be classified into any of the four categories, the uncertainty premium of at least one bet in the round needs to be non-zero. For rounds whose midpoint prior is (50%, 50%), I do not classify subjects as absolute prior under-/over-weighters.

optimists, and the percentage of the former aggregated across rounds is also significantly higher than the latter for both compound and ambiguous prior rounds. This further confirms that uncertain priors lead to pessimism. By contrast, there is not strong evidence of either the under- or over-weighting of priors. In three out of six rounds, there are more absolute prior under- than over-weighters, whereas the opposite is true in the other three rounds.

Taken together, my experimental results suggest that in problems with belief updating, uncertainty in priors leads to pessimism. This pattern is consistent with the combination of uncertainty aversion and either Full Bayesian updating or Dynamically consistent updating. Under-weighting of priors – which is the prediction of uncertainty-induced insensitivity together with these two updating rules – is not borne out in the data.

Midpoint prior (<i>Red, Blue</i>)	Information accuracy	Type of prior	Absolute pessimists	Absolute optimists	<i>p</i> -value %(Abs. pess.) =%(Abs. opt.)	Absolute prior under-weighters	Absolute prior over-weighters	<i>p</i> -value %(Abs. negl.) =%(Abs. over.)	N
(50%, 50%)	70%	Amb	31.1%	21.3%	0.084	-	-	-	164
(60%, 40%)	60%	Amb	32.5%	24.7%	0.366	27.3%	33.8%	0.466	77
(70%, 30%)	70%	Amb	29.0%	16.7%	0.032	41.3%	31.9%	0.196	138
(70%, 30%)	50%	Amb	27.0%	18.4%	0.104	29.4%	35.6%	0.331	163
Aggregate		Amb	29.5%	18.8%	0.001	33.3%	33.9%	0.9	
(50%, 50%)	70%	Comp	32.3%	22.0%	0.072	-	-	-	164
(60%, 40%)	60%	Comp	39.3%	15.4%	0	35.9%	26.5%	0.198	117
(70%, 30%)	70%	Comp	21.3%	25.5%	0.67	31.9%	40.4%	0.493	47
(70%, 30%)	50%	Comp	25.2%	22.1%	0.569	33.1%	30.7%	0.695	163
Aggregate		Comp	30.5%	20.8%	0.002	33.9%	30.6%	0.449	

Table A.3: Classification of subjects in each uncertain prior round

Notes: This table shows the percentages of subjects that are classified into the four categories for each uncertain prior round. Only subjects who face comparable belief-updating problems in the uncertain prior round and its corresponding simple prior round are counted. In the rows under "Aggregate", I calculate the percentage of instances subjects are classified into each category, aggregated across the four or three rounds that are relevant for that category. The *p*-values are computed using Pearson's chi-square goodness-of-fit tests.

Prior and info	Type of prior	within-subject		between-subject		
		$\overline{CE}(simp) - \overline{CE}(unc)$	N	$\overline{CE}(simp) - \overline{CE}(unc)$	N(simp)	N(unc)
odds=30%, accu=70% bad news	compound ambiguous	-0.47 (0.52) -0.21 (0.58)	32 95	0.01 (0.99) -2.39 (0.06)	111 111	39 11
odds=40%, accu=60% bad news	compound ambiguous	-0.11 (0.84) 0.9 (0.26)	57 30	1.8 (0.03) 1.28 (0.08)	73 73	32 44
odds=30%, accu=50% neutral news	compound ambiguous	-0.45 (0.18) 0.42 (0.16)	163 163			
odds=40%, accu=60% good news	compound ambiguous	0.63 (0.29) 0.34 (0.61)	60 47	-0.75 (0.57) 0.33 (0.71)	91 91	16 42
odds=30%, accu=70% good news	compound ambiguous	0.13 (0.93) 1.33 (0.06)	15 43	0.09 (0.92) 1.11 (0.43)	54 54	79 16
odds=70%, accu=70% bad news	compound ambiguous	-1.73 (0.02) 0.12 (0.85)	15 43	-0.78 (0.34) -1.61 (0.25)	54 54	79 16
odds=60%, accu=60% bad news	compound ambiguous	-0.1 (0.86) 0.02 (0.97)	60 47	-0.29 (0.82) -0.4 (0.63)	91 91	16 42
odds=70%, accu=50% neutral news	compound ambiguous	0.56 (0.12) -0.34 (0.27)	163 163			
odds=60%, accu=60% good news	compound ambiguous	1.96 (0) 0.97 (0.2)	57 31	2.11 (0.02) 2.47 (0)	73 73	32 44
odds=70%, accu=70% good news	compound ambiguous	-0.03 (0.96) 0.96 (0.01)	32 95	1.74 (0.02) 2.01 (0.1)	111 111	39 11
odds=50%, accu=70% bad news	compound ambiguous	-0.88 (0.03) -0.04 (0.9)	164 164			
odds=50%, accu=70% good news	compound ambiguous	1.26 (0) 0.63 (0.1)	164 164			

Table A.4: Within- and between-subject comparison between CEs of uncertain and simple bets conditional on simple information

Notes: This table shows the differences in mean conditional CEs between uncertain prior problems and simple prior problems. Numbers in parentheses are p -values in t -tests, and N is the number of subjects included. For example, the top row of the table states that there are 32 subjects who receive bad reports both in the compound information problem and in the simple information problem where the prior is 30% and (midpoint) information accuracy 70%. Among these subjects, the difference in mean conditional CEs between these two problems is -\$0.47 and the p -value of the paired t -test is 0.52. Thirty-nine subjects receive a bad report in the compound prior problem but not in the simple problem, and there are 111 subjects who receive a bad report in the simple problem in total. The difference between the mean conditional CE of the simple bet of the latter group and that of the compound bet of the former group is 0.01, and the p -value of the unpaired t -test is 0.99.

Prior (<i>Red, Blue</i>)	Midpoint Information accuracy	Type of information	Absolute confirmation bias	Absolute contradiction bias	N
(60%, 40%)	60%	Ambiguous	21	23	71
(70%, 30%)	70%	Ambiguous	23	33	94
(70%, 30%)	50%	Ambiguous	30	56	137
(60%, 40%)	60%	Compound	36	29	106
(70%, 30%)	70%	Compound	36	45	123
(70%, 30%)	50%	Compound	28	62	137

Table B.1: Absolute confirmation bias and absolute contradiction bias

Notes: This table shows the numbers of subjects that are classified into absolute confirmation bias and absolute contradiction bias. Only subjects who face comparable belief-updating problems in the uncertain information round and its corresponding simple information round are counted.

B Additional results on the experiment

B.1 An alternative classification for behaviors in uncertain information rounds

In this section, I consider two alternative subject categories based on behaviors in uncertain information rounds: absolute confirmation bias and absolute contradiction bias. In an uncertain information round where the odds of the bets are not 50%, if a subject's uncertainty premium for the bet with higher-than-50% odds is weakly negative, her uncertainty premium for the other bet is weakly positive, and at least one of the two is not zero, then I classify this subject into the category of absolute confirmation bias. If, on the contrary, the bet with high odds has a weakly positive uncertainty premium, the other one has a weakly negative premium, and at least one is not zero, then this subject is absolute contradiction-biased. In rounds where the odds are 50-50, I do not classify subjects into these two categories.

Table B.1 shows the number of subjects in these two categories. Except in one round, there are more subjects in the category of absolute contradiction bias in every uncertain information round. This suggests that uncertainty in information accuracy does not lead to prevalent confirmation bias.

B.2 Elicitation methods, order effects, and anchor effects

In this section, I show that the experimental results are robust to different elicitation methods, order effects, and anchor effects.

In sessions 1 and 2, CEs are elicited as the willingness-to-accept for selling bets using a BDM

	good news					bad news					neutral news	
Prior (Midpoint) Accuracy	30%	40%	50%	60%	70%	30%	40%	50%	60%	70%	30%	70%
	70%	60%	70%	60%	70%	70%	60%	70%	60%	70%	50%	50%
	Sessions 1 and 2: CEs elicited using BDM											
Simple info		11.88	14.29	13.80	15.28	8.28	8.33	9.09	10.56		9.63	12.27
Compound info	10.69		13.52	12.91	13.56	8.88	10.13	9.70		11.25	10.84	11.26
Ambiguous info	12.16	11.44	12.44	12.50	15.63	8.00	9.19	9.34	11.31	11.44	8.94	11.50
	Sessions 1, 2, 3, and 4: Order between parts is 1-4-2-3-5											
Simple info	10.00	10.84	13.07	13.80	15.00	6.67	8.33	7.67	8.27	8.00	7.88	11.46
Compound info	9.83	10.38	13.02	12.91	13.25	6.25	10.13	7.76	6.72	10.72	8.25	11.08
Ambiguous info	12.16	10.10	11.64	11.06	14.18	5.91	6.72	7.28	10.07	11.44	7.18	11.44
	Sessions 5, 8 and 9: Order between parts is 1-4-2-5-3											
Simple info	10.70	9.23	11.76	12.93	15.33	4.73	6.07	5.82	5.47	6.70	5.91	10.36
Compound info	8.80	7.76	10.67		13.17	4.87		6.20	7.62	7.73	6.16	9.82
Ambiguous info	7.67	9.73	10.04	7.80	13.50	4.73	5.10	6.16	8.47	8.13	7.07	8.53
	Sessions 6, 7, 10 and 11: Order between parts is 1-2-3-4-5											
Simple info	12.00	10.13	12.51	11.81	14.29	5.04	6.67	5.71	8.94	7.64	7.22	10.69
Compound info		10.18	11.95	11.56	13.71	5.17	6.40	6.83	8.91		7.76	9.97
Ambiguous info	11.06	10.44	10.73	9.79	13.44	5.56	5.98	7.17	9.75	8.38	7.58	10.27
	Sessions 4, 7, 8, 9 and 11: Ambiguity before compound											
Simple info	11.62	10.09	11.61	11.19	14.10	3.96	5.85	5.58	6.98	6.69	6.16	10.41
Compound info		9.67	11.73	10.88	12.95	4.64	5.84	5.97	6.95		6.05	10.42
Ambiguous info	11.06		10.22	9.31	12.62	4.66	5.32	5.93		8.38	6.65	9.23
	Sessions 1, 2, 3, 5, 6 and 10: Ambiguity after compound											
Simple info	10.04	10.30	13.24	13.20	15.22	7.03	7.50	7.19	7.98	7.68	7.88	11.27
Compound info	9.48	8.28	12.20	12.92	14.26	6.55	9.25	7.84	8.00	9.70	8.69	10.27
Ambiguous info	9.98	10.10	11.42	10.32	14.82	6.28	7.45	7.75	9.58	9.84	7.82	11.03

Table B.2: CEs of simple bets conditional on uncertain and simple information: subsamples

mechanism. The top panel of Table B.2 shows the average CEs of simple bets conditional on three kinds of information in these two sessions. Elicited CEs are higher across the board, which is consistent with prior results on WTA elicited in BDM mechanisms (see, for example, Cason and Plott, 2014). Subjects underreact more to uncertain information, which is consistent with results in the full sample. Pessimistic reactions under uncertain information are less salient in this part of the dataset.

Recall that in the experiment there are three different orders among parts, and two orders between compound and ambiguous uncertainty within parts (Table B.3). To check for order effects, I list the mean conditional CEs of simple bets by order in Table B.2. Across all subsamples, CEs conditional on simple good news are higher than those conditional on uncertain good news. For bad news, the

Session	Order between parts	Ambiguous block first?	Number of subjects
1	1-4-2-3-5	No	16
2	1-4-2-3-5	No	16
3	1-4-2-3-5	No	13
4	1-4-2-3-5	Yes	16
5	1-4-2-5-3	No	15
6	1-2-3-4-5	No	16
7	1-2-3-4-5	Yes	16
8	1-4-2-5-3	Yes	15
9	1-4-2-5-3	Yes	15
10	1-2-3-4-5	No	11
11	1-2-3-4-5	Yes	16

Table B.3: Description of sessions

comparison between uncertain information and simple information is mostly mixed as in the full sample. The patterns for neutral news in subsamples are also similar to the full sample. These results suggest that our key results are robust to order effects.

In all three different orders among parts, Part 2 (simple prior with simple information) comes before Part 3 (simple prior with uncertain information) and Part 5 (uncertain prior with simple information). This raises the question whether subjects anchor their answers in Parts 3 and 5 to those in Part 2.

To address this concern, I first conduct a within-subject analysis by running a paired t -test between the conditional CEs in each uncertain information (prior) problem and their counterparts in the corresponding simple problem. The subjects who are included in the paired t -tests are those who receive comparable reports in the two corresponding rounds,³⁵ so their conditional CEs in the uncertain information (prior) round could potentially be anchored to their answers in the corresponding simple round. The other subjects who do not receive comparable reports are not subject to the anchor effect, and I compare the mean of their conditional CEs in the uncertain information (prior) problem to the mean conditional CE in the corresponding simple problem in an unpaired t -test, which is a between-subject analysis.

Table B.4 reports results for uncertain information problems. For subjects who receive comparable reports in the uncertain information problem and the corresponding simple problem ("within-subject"), it is apparent that uncertain information leads to under-reaction to news. There is also evidence of pessimism caused by uncertain information accuracy. First, the effect sizes are more likely to be significant for good news than for bad news. Second, in half of the comparisons with

³⁵Receiving comparable reports in two corresponding rounds means that the uncertainty premiums of the red bet and the blue bet in the uncertain information (prior) round can be calculated from data. See Section 4 and Appendix A.2.6 for the definition of uncertainty premiums.

neutral information, CEs conditional on uncertain information are significantly lower. (In the other comparisons with neutral information, the uncertain CEs are higher but the differences are not significant.) The results of the between-subject analysis are more noisy, but the overall patterns of under-reaction and pessimism remain present.

Table A.4 reports results for uncertain prior problems. Despite the noise in the results, in the majority of the comparisons in both within- and between-subject analysis, the conditional CEs of uncertain bets are lower than their simple counterparts, suggesting that uncertain priors in belief-updating problems lead to pessimism.

Taken together, the key effects of uncertain information and uncertain priors are robust to order effects and anchor effects.

Prior and info	Type of information	within-subject		between-subject		
		$\overline{CE}(simp) - \overline{CE}(unc)$	N	$\overline{CE}(simp) - \overline{CE}(unc)$	N(simp)	N(unc)
odds=30%, accu=70%	compound	1.25 (0.11)	28	0.11 (0.93)	54	16
good news	ambiguous	2.4 (0.06)	15	-0.81 (0.45)	54	32
odds=40%, accu=60%	compound	0.8 (0.23)	59	1.19 (0.27)	91	26
good news	ambiguous	1.14 (0.12)	29	0.09 (0.93)	91	31
odds=50%, accu=70%	compound	0.52 (0.14)	163			
good news	ambiguous	1.66 (0)	164			
odds=60%, accu=60%	compound	0.55 (0.25)	47	-0.24 (0.75)	73	33
good news	ambiguous	1.9 (0)	42	3.25 (0)	73	63
odds=70%, accu=70%	compound	0.86 (0.02)	95	2.08 (0.02)	111	26
good news	ambiguous	0.54 (0.1)	79	1.74 (0.03)	111	39
odds=30%, accu=70%	compound	0.16 (0.57)	95	-0.1 (0.91)	111	26
bad news	ambiguous	-0.32 (0.4)	79	0.32 (0.67)	111	39
odds=40%, accu=60%	compound	-0.28 (0.47)	47	-1.56 (0.05)	73	33
bad news	ambiguous	-0.12 (0.83)	42	1.54 (0.02)	73	63
odds=50%, accu=70%	compound	-0.59 (0.07)	163			
bad news	ambiguous	-0.47 (0.14)	165			
odds=60%, accu=60%	compound	-0.64 (0.15)	59	-1.42 (0.15)	91	26
bad news	ambiguous	-1.1 (0.16)	29	-1.67 (0.07)	91	31
odds=70%, accu=70%	compound	-1.14 (0.22)	28	-4.05 (0)	54	16
bad news	ambiguous	-0.73 (0.56)	15	-2.7 (0.01)	54	32
odds=30%, accu=50%	compound	-0.33 (0.33)	163			
neutral news	ambiguous	-0.17 (0.66)	162			
odds=70%, accu=50%	compound	0.6 (0.05)	163			
neutral news	ambiguous	0.65 (0.03)	162			

Table B.4: Within- and between-subject comparison between CEs of simple bets conditional on uncertain and simple information

Notes: This table shows the differences in mean conditional CEs between uncertain information problems and simple information problems. Numbers in parentheses are p -values in t -tests, and N is the number of subjects included. For example, the top row of the table states that there are 28 subjects who receive good reports both in the compound information problem and in the simple information problem where the prior is 30% and (midpoint) information accuracy 70%. Among these subjects, the difference in mean conditional CEs between these two problems is \$1.25 and the p -value of the paired t -test is 0.11. Sixteen subjects receive a good report in the compound information problem but not in the simple problem, and there are 54 subjects who receive a good report in the simple problem in total. The difference between the mean simple conditional CE of the latter group and the mean compound conditional CE of the former is 0.11, and the p -value of the unpaired t -test is 0.93.

C Additional results on the CEU preferences

C.1 Capacity functions in problems with belief updating

In this subsection, I derive the capacities for events in uncertain information problems and uncertain prior problems. Recall that the events “the bet will win/lose” are denoted by G and B . In addition, denote the two events “the information is correct/incorrect” by T and F . The outcome of the bet and the correctness of the information are independent by design of the experiment.

I make the following assumptions about the capacity function.

Assumption 1 *If event E 's probability is p for sure, then*

1. $\nu(E) = p$;
2. *for all event E' s.t. $E \cap E' = \emptyset$, $\nu(E \cup E') = p + \nu(E')$.*

Assumption 2 *If events E and E' are independent, then $\nu(E \cap E') = \nu(E) \cdot \nu(E')$.*

Item 1 of Assumption 1 implies that the CEU agent's evaluations of simple bets adhere to standard expected utility. Item 2 of Assumption 1 states that there is no “complementarity” between a simple event and any non-intersecting event. Assumption 2 states that capacities are multiplicative between independent events, just like probabilities. These two assumptions are sufficient for deriving the capacities for events that are relevant for updating in uncertain information problems and uncertain prior problems.

In an uncertain information problem, the probability of G is p for sure, so by item 1 of Assumption 1, $\nu(G) = p$ and $\nu(B) = 1 - p$. The probability of T is either ψ_h or ψ_l . Let the uncertainty attitudes toward information accuracy be summarized by ε , the uncertainty-induced insensitivity parameter, and α , the uncertainty aversion parameter. Then we have $\nu(T) = W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)$ and $\nu(F) = W(1 - \frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)$. Because G is independent from T , by Assumption 2, $\nu(G \cap T) = p \cdot \nu(T)$. The capacities $\nu(G \cap F)$, $\nu(B \cap T)$ and $\nu(B \cap F)$ are similarly determined. By item 2 of Assumption 1, $\nu(G \cup T) = p + \nu(B \cap T)$. The expressions for $\nu(G \cup F)$, $\nu(B \cup T)$ and $\nu(B \cup F)$ are similar. The capacities of the two events, $g = (G \cap T) \cup (B \cap F)$ and $b = (G \cap F) \cup (B \cap T)$, are not pinned down by the assumptions, but they are irrelevant for belief updating.

In an uncertain prior problem, the information accuracy is ψ for sure, so $\nu(T) = \psi$ and $\nu(F) = 1 - \psi$ by item 1 of Assumption 1. The probability of G is either p_h or p_l , so $\nu(G) = W(\frac{p_h + p_l}{2}; \varepsilon, \alpha)$ and $\nu(B) = W(\frac{1 - p_h + p_l}{2}; \varepsilon, \alpha)$, where ε and α describe the uncertainty attitude toward priors. The capacities of intersections $\nu(G \cap T)$, $\nu(G \cap F)$, $\nu(B \cap T)$ and $\nu(B \cap F)$ are pinned down by Assumption 2. The capacities of unions $\nu(G \cup T)$, $\nu(G \cup F)$, $\nu(B \cup T)$ and $\nu(B \cup F)$ are further determined by item 2 of Assumption 1.

C.2 Proofs of results on updating under CEU preferences

Proof of Propositions 1 and 4. [Eichberger et al. \(2007\)](#) defines Full Bayesian updating for capacities as follows. The capacity of event A conditional on realized report E is

$$\nu(A|E) = \frac{\nu(A \cap E)}{\nu(A \cap E) + 1 - \nu(A \cup E^c)}.$$

We can obtain the Full Bayesian conditional evaluations of bets by directly applying the definition above. For example, in an uncertain information problem, the capacity of G conditional on report g is

$$\begin{aligned} \nu(G|(G \cap T) \cup (B \cap F)) &= \frac{\nu(G \cap T)}{\nu(G \cap T) + 1 - \nu(G \cup T)} \\ &= \frac{p \cdot W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)}{p \cdot W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha) + (1 - p) \cdot (1 - W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha))} \\ &= Pr^{Bayes}(G|p, g, W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)). \end{aligned}$$

Hence, the evaluation of the bet conditional on report g is

$$u(g) = Pr^{Bayes}(G|p, g, W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)).$$

The conditional evaluation given report b and those in uncertain prior problems can be similarly derived.

The comparative statics of the conditional evaluations with respect to α and ε are straightforward.

Proof of Propositions 2 and 5. Under Dynamically consistent updating ([Hanany and Klibanoff, 2007](#)), the agent forms a contingent plan of actions before the report realizes and executes the plan resolutely after observing the report. In our example where the agent chooses between a bet and a certain amount of utils, the contingent plan denoted by $a = (a(g), a(b))$ specifies an action $a(m) \in \{Bet, Sure\}$ conditional on the good report and the bad report. Let $U(a(m), E)$ denote the utility of action $a(m)$ under payoff-relevant event E . The optimal plan maximizes utility from the ex-ante perspective. In an uncertain information problem, the ex-ante utility of an agent with a CEU preference is

$$W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha) \cdot [p \cdot U(a(g), G) + (1 - p) \cdot U(a(b), B)] + (1 - W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)) \cdot [p \cdot U(a(b), G) + (1 - p) \cdot U(a(g), B)]$$

if her plan of action is (Bet, Bet) , $(Bet, Sure)$ or $(Sure, Sure)$ and

$$W(\frac{\psi_h + \psi_l}{2}; \varepsilon, -\alpha) \cdot [p \cdot U(a(g), G) + (1 - p) \cdot U(a(b), B)] + (1 - W(\frac{\psi_h + \psi_l}{2}; \varepsilon, -\alpha)) \cdot [p \cdot U(a(b), G) + (1 - p) \cdot U(a(g), B)]$$

if her plan is $(Sure, Bet)$.

It is straightforward that the payoff of (Bet, Bet) equals p and that of $(Sure, Sure)$ equals the certain amount u . Note that both payoffs are independent from the information accuracy. The intuition is that if the agent's action is unaffected by the realization of information, then the ex-ante utility is not exposed to the uncertainty in the information. This is in contrast with the ex-post utility conditional on the realized report. The only choice that makes the ex-post conditional evaluation independent from the uncertainty in the information is choosing the certain amount of utils.

Since $W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha) \geq 1 - W(\frac{\psi_h + \psi_l}{2}; \varepsilon, -\alpha)$, it can be shown by simple algebra that $(Sure, Bet)$ always leads to lower ex-ante utility than $(Bet, Sure)$. Hence, I only need to consider (Bet, Bet) , $(Sure, Sure)$ and $(Bet, Sure)$ as the candidate optimal plans.

We know that $(Sure, Sure)$ yields a higher utility than (Bet, Bet) if and only if $u > p$. Hence, to pin down the optimal plan for each u , we only need to find the u such that $(Bet, Sure)$ is optimal. The plan $(Bet, Sure)$ yields a higher utility than $(Sure, Sure)$ if and only if

$$W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha) \cdot p \cdot (1 - u) - (1 - W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha))(1 - p) \cdot u > 0$$

$$\iff u < Pr^{Bayes}(G|p, g, W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)).$$

Similarly, $(Bet, Sure)$ yields a higher utility than (Bet, Bet) if and only if

$$u > Pr^{Bayes}(G|p, b, W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)).$$

The two inequalities can be simultaneously satisfied if and only if $\frac{\psi_h + \psi_l}{2} - \alpha > 0.5$. When this condition holds, it is easy to check that $(Bet, Sure)$ is indeed optimal in the interval between the two right-hand side expressions. If we interpret the upper and lower boundaries of the interval in which $(Bet, Sure)$ is optimal as the “conditional evaluations” given good and bad reports, respectively, then these “conditional evaluations” coincide exactly with the Bayesian conditional evaluations with the information accuracy being $W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha)$.

If $\frac{\psi_h + \psi_l}{2} - \alpha < 0.5$, then there is no u such that $(Bet, Sure)$ is optimal. Hence, the agent's optimal plan is to not respond to the information at all: she always chooses the certain amount of utils if $u > p$ and always takes the bet if $u < p$, *regardless of the realized report*.

In an uncertain prior problem, the ex-ante utility of an agent with a CEU preference is

$$W(\frac{p_h + p_l}{2}; \varepsilon, \alpha) \cdot [\psi \cdot U(a(g), G) + (1 - \psi) \cdot U(a(b), G)] + (1 - W(\frac{p_h + p_l}{2}; \varepsilon, \alpha)) \cdot [(1 - \psi) \cdot U(a(g), B) + (1 - \psi) \cdot U(a(b), B)].$$

The ex-ante expected utility of $(Sure, Sure)$ is still u but that of (Bet, Bet) is now $W(\frac{p_h + p_l}{2}; \varepsilon, \alpha)$. The ex-ante expected utility of $(Bet, Sure)$ is again always higher than that of $(Sure, Bet)$.

We know that (Bet, Bet) yields a higher ex-ante payoff than $(Sure, Sure)$ if $u > W(\frac{p_h + p_l}{2}; \varepsilon, \alpha)$. Simple algebra shows that $(Bet, Sure)$ yields a higher payoff than $(Sure, Sure)$ if and only if

$$u < Pr^{Bayes}(G|W(\frac{p_h + p_l}{2}; \varepsilon, \alpha), g, \psi)$$

and $(Bet, Sure)$ yields a higher payoff than (Bet, Bet) if and only if

$$u > Pr^{Bayes}(G|W(\frac{p_h + p_l}{2}; \varepsilon, \alpha), b, \psi).$$

The two inequalities above can always be compatible. Note that the two expressions on the right-hand side are exactly the same as the conditional evaluations in the same uncertain prior problem under Full Bayesian updating. This suggests that for uncertain prior problems, Full Bayesian updating and Dynamically consistent updating make the same predictions under CEU preferences.

The comparative statics of conditional evaluations with respect to ε and α are straightforward.

Proof of Propositions 3 and 6. In an uncertain information problem, the likelihood of report g is $p \cdot \psi + (1 - p) \cdot (1 - \psi)$. If $p > 0.5$, then the likelihood is increasing in ψ and thus ψ_h is selected. If $p < 0.5$, then ψ_l is selected upon the realization of g . Similarly, the likelihood of report b is $p \cdot (1 - \psi) + (1 - p) \cdot \psi$. If $p > 0.5$, then ψ_l is selected and if $p < 0.5$, ψ_h is selected. If $p = 0.5$, then both ψ_h and ψ_l are retained regardless of the realized report. Uncertain prior problems are similar.

D Additional results on the correlation between different kinds of uncertainty attitudes

In this section, I derive tests of correlations between uncertainty attitudes for priors and information accuracy. The correlation tests I construct are based on the signs of uncertainty premiums. For a bet whose prior is either p_h or p_l , define the sign of its uncertainty premium in a problem without belief updating as

$$SP(p_h \text{ or } p_l) = \text{sign} \left(CE(\frac{p_h + p_l}{2}) - CE(p_h \text{ or } p_l) \right) := \begin{cases} 1, & \text{if } CE(p_h \text{ or } p_l) < CE(\frac{p_h + p_l}{2}) \\ 0, & \text{if } CE(p_h \text{ or } p_l) = CE(\frac{p_h + p_l}{2}) \\ -1, & \text{if } CE(p_h \text{ or } p_l) > CE(\frac{p_h + p_l}{2}) \end{cases}.$$

For a simple bet with uncertain information, define the sign of uncertainty premium as

$$SP(p, m, \psi_h \text{ or } \psi_l) = \text{sign}(Pm(p, m, \psi_h \text{ or } \psi_l)),$$

where $Pm(\cdot, \cdot, \cdot \text{ or } \cdot)$ is defined in Section 4. Similarly, define the sign of uncertainty premium of an uncertain bet in a problem with belief updating as

$$SP(p_h \text{ or } p_l, m, \psi) = \text{sign}(Pm(p_h \text{ or } p_l, m, \psi)),$$

where $Pm(\cdot \text{ or } \cdot, \cdot, \cdot)$ is defined in Appendix A.2.6.

The following proposition lays out the basis for the tests of correlations between different kinds of uncertainty attitudes.

Proposition 7 *Suppose that a CEU agent uses either Full Bayesian updating, Dynamically consistent updating, or Maximum likelihood updating and adapts it to the generalized Bayes' rule. Then*

1. *if the agent's attitudes toward uncertain information and uncertain priors (in problems without updating) are described by the same CEU preference, then*

$$SP(50\%, g, 90\% \text{ or } 50\%) = SP(90\% \text{ or } 50\%);$$

2. *if the agent's attitudes toward uncertain information and uncertain priors (in problems with updating) are described by the same CEU preference, then*

$$SP(50\%, g, 90\% \text{ or } 50\%) = SP(90\% \text{ or } 50\%, -, 50\%);$$

3. *if the agent's attitudes toward uncertain priors in problems with and without updating are described by the same CEU preference, then*

$$SP(90\% \text{ or } 50\%, -, 50\%) = SP(90\% \text{ or } 50\%) \text{ and } SP(10\% \text{ or } 50\%, -, 50\%) = SP(10\% \text{ or } 50\%).$$

To see why item 1 in the proposition is true, note that for a CEU agent who uses Full Bayesian updating adapted to the generalized Bayes' rule, the comparison between $CE(50\%, g, 90\% \text{ or } 50\%)$ and $CE(50\%, g, 70\%)$ boils down to the comparison between $W(70\%; \varepsilon, \alpha)$ and 70% . If the same ε and α apply to both uncertainty in information accuracy and uncertainty in priors (in problems without updating), then the same comparison between $W(70\%; \varepsilon, \alpha)$ and 70% also determines the comparison between $CE(90\% \text{ or } 50\%)$ and $CE(70\%)$. Moreover, this statement is also true if the agent uses the other two belief-updating rules. This is because the conditional CEs under Dynamically consistent updating coincide with Full Bayesian updating for good news, and those under Maximum likelihood updating are the same as Full Bayesian updating if $p = 50\%$. Similar arguments also apply to items 2 and 3. The formal proof of Proposition 7 is in Appendix D.2. In

Test	Correlation coefficient	
	Ambiguity	Compound
SP(50%, g, 90% or 50%)=SP(90% or 50%)	0 (0.99)	0.08 (0.28)
SP(50%, g, 90% or 50%)=SP(90% or 50%,-,50%)	0.03 (0.71)	0 (0.97)
SP(90% or 50%,-,50%)=SP(90% or 50%)	0.26 (0)	0.15 (0.05)
SP(10% or 50%,-,50%)=SP(10% or 50%)	0.22 (0)	0.1 (0.19)

Table D.1: Results of correlation tests

Notes: This table lists the correlation coefficients of the tests that are valid under Full Bayesian updating, Dynamically consistent updating and Maximum likelihood updating adapted to generalized Bayes' rule. Numbers in parentheses are p -values with the null hypothesis being that the correlation is zero.

the 2020 version of this paper (Liang, 2020), I show that the proposition also holds under several extensions of the smooth model and Segal's two-stage model.

I compute the correlation between the two sides of each equation in Proposition 7 to test for correlation between attitudes toward two different kinds of uncertainty. Table D.1 shows the results. While the correlations that involves attitudes toward uncertain information are all very close to zero, the correlations between attitudes toward uncertain priors with and without belief updating have larger magnitudes and, in most cases, high significance. The results remain qualitatively unchanged if I restrict the tests to a subsample of subjects who adhere well to some basic rationality properties (see Appendix D.1.) Taken together, these results imply that with or without the updating, subjects have rather consistent uncertainty attitudes toward priors. By contrast, their attitudes toward uncertainty in information accuracy are distinct from how they treat uncertain priors.

D.1 Analysis with a “more rational” subsample

Another concern is that the no-correlation results may be driven by “confused” subjects who do not even adhere to basic rationality properties. To show that this is not the case, I first consider several such rationality properties and show that subjects' adherence to them are reasonably good. Subsequently, I repeat the correlation tests within the sample of more “rational” sample and show that the results are virtually unchanged.

I consider the following four monotonicity properties in problems with simple priors and no updating: $CE(30\%) \leq CE(40\%)$, $CE(40\%) \leq CE(50\%)$, $CE(50\%) \leq CE(60\%)$, and $CE(60\%) \leq CE(70\%)$. Among all 165 subjects, 109 satisfy all of these properties, 38 satisfy three of them, 14 satisfy two, and 4 people one. I also consider two monotonicity properties in problems with simple priors and simple information: $CE(50\%, g, 70\%) \geq CE(50\%)$ and $CE(50\%, b, 70\%) \leq CE(50\%)$.³⁶

³⁶I choose these two specific properties because theoretically, any agent who has monotonic risk preference

Test	Correlation coefficient	
	Ambiguity	Compound
SP(50%, g, 90% or 50%)=SP(90% or 50%)	-0.03 (0.77)	0.12 (0.17)
SP(50%, g, 90% or 50%)=SP(90% or 50%, -, 50%)	0.07 (0.42)	0 (0.96)
SP(90% or 50%, -, 50%)=SP(90% or 50%)	0.33 (0)	0.22 (0.01)
SP(10% or 50%, -, 50%)=SP(10% or 50%)	0.21 (0.02)	0.08 (0.38)

Table D.2: Results of correlation tests (“rational” subsample)

Notes: This table lists the correlation coefficients of the tests that are valid under Full Bayesian updating, Dynamically consistent updating and Maximum likelihood updating adapted to generalized Bayes’ rule. The sample comprises 131 subjects who violate at most one of six monotonicity properties. Numbers in parentheses are p -values with the null hypothesis being that the correlation is zero.

117 subjects satisfy both inequalities and 43 subjects satisfy one of them.

To address the concern that the no-correlation results may be driven by “confused” subjects, I repeat the tests with the 131 subjects who violate at most one of the six monotonicity properties listed in the previous paragraph. Table D.2 shows that the results are qualitatively the same.

D.2 Proof of Proposition 7

Suppose an agent’s attitudes toward uncertain priors (in problems without updating) is described by a CEU preference, then her CE of an uncertain bet is given by

$$CE(p_h \text{ or } p_l) = M\left(W\left(\frac{p_h + p_l}{2}; \varepsilon, \alpha\right)\right),$$

where $M : [0, 1] \rightarrow \mathbb{R}_+$ is an increasing function that maps the (subjective) winning odds of a bet to its CE. Suppose that the same CEU preference also describes her attitudes toward uncertainty in information accuracy and that she follows the Full Bayesian updating rule adapted to the generalized Bayes’ rule. Then the agent’s CE for a simple bet is

$$CE(p, g, \psi_h \text{ or } \psi_l) = M\left(Pr^{GB}(G|p, g, W\left(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha\right))\right)$$

and uses the generalized Bayes’ rule to update beliefs should satisfy them. In addition, all subjects report $CE(50\%, g, 70\%)$ and $CE(50\%, b, 70\%)$ in the experiment. Other similar inequalities have their shortcomings. For instance, the inequality $CE(60\%, g, 60\%) \geq CE(50\%)$ also has the aforementioned theoretical appeal, but not all subjects have their $CE(60\%, g, 60\%)$ recorded in the dataset due to random signal realization.

conditional on an uncertain good report and

$$CE(p, b, \psi_h \text{ or } \psi_l) = M \left(Pr^{GB}(G|p, b, W(\frac{\psi_h + \psi_l}{2}; \varepsilon, -\alpha)) \right)$$

conditional on an uncertain bad report. Note that $M(\cdot)$ is increasing and the generalized Bayesian posterior is increasing in information accuracy conditional on a good report and decreasing conditional on a bad report. This implies that if $0 \leq y < x \leq 1$ and $x + y \geq 1$, then for any p ,

$$\begin{aligned} SP(p, g, x \text{ or } y) &= \text{sign} \left(M \left(Pr^{GB}(G|p, g, \frac{x+y}{2}) \right) - M \left(Pr^{GB}(G|p, g, W(\frac{x+y}{2}; \varepsilon, \alpha)) \right) \right) \\ &= \text{sign} \left(M \left(\frac{x+y}{2} \right) - M \left(W(\frac{x+y}{2}; \varepsilon, \alpha) \right) \right) \\ &= SP(x \text{ or } y) \end{aligned} \tag{13}$$

and

$$\begin{aligned} SP(p, g, x \text{ or } y) &= \text{sign} \left(M \left(Pr^{GB}(G|p, b, \frac{x+y}{2}) \right) - M \left(Pr^{GB}(G|p, b, W(\frac{x+y}{2}; \varepsilon, -\alpha)) \right) \right) \\ &= \text{sign} \left(M \left(Pr^{GB}(G|p, g, 1 - \frac{x+y}{2}) \right) - M \left(Pr^{GB}(G|p, g, 1 - W(\frac{x+y}{2}; \varepsilon, -\alpha)) \right) \right) \\ &= \text{sign} \left(M \left(\frac{1-y+1-x}{2} \right) - M \left(W(\frac{1-y+1-x}{2}; \varepsilon, \alpha) \right) \right) \\ &= SP(1-y \text{ or } 1-x). \end{aligned} \tag{14}$$

Suppose instead that the agent uses Dynamically consistent updating adapted to the generalized Bayes' rule. Then, the hypothesis that the same CEU preference applies to both uncertain priors (in problems without updating) and uncertain information implies that the CE of a simple bet conditional on uncertain information is

$$CE(p, m, \psi_h \text{ or } \psi_l) = M \left(Pr^{GB}(G|p, m, \max\{W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha), 0.5\}) \right),$$

which in turn implies that if $0 < p < 1$, $0 \leq y < x \leq 1$ and $x + y > 1$, then

$$\begin{aligned} SP(p, g, x \text{ or } y) &= \text{sign} \left(M \left(Pr^{GB}(G|p, g, \frac{x+y}{2}) \right) - M \left(Pr^{GB}(G|p, g, \max\{W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha), 0.5\}) \right) \right) \\ &= \text{sign} \left(M \left(Pr^{GB}(G|p, g, \frac{x+y}{2}) \right) - M \left(Pr^{GB}(G|p, g, W(\frac{x+y}{2}; \varepsilon, \alpha)) \right) \right) \\ &= \text{sign} \left(M \left(\frac{x+y}{2} \right) - M \left(W(\frac{x+y}{2}; \varepsilon, \alpha) \right) \right) \\ &= SP(x \text{ or } y) \end{aligned} \tag{15}$$

and

$$\begin{aligned}
SP(p, g, x \text{ or } y) &= \text{sign} \left(M \left(Pr^{GB}(G|p, b, \frac{x+y}{2}) \right) - M \left(Pr^{GB}(G|p, b, \max\{W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha), 0.5\}) \right) \right) \\
&= -\text{sign} \left(M \left(Pr^{GB}(G|p, g, \frac{x+y}{2}) \right) - M \left(Pr^{GB}(G|p, g, \max\{W(\frac{\psi_h + \psi_l}{2}; \varepsilon, \alpha), 0.5\}) \right) \right) \\
&= -SP(x \text{ or } y).
\end{aligned} \tag{16}$$

Finally, if the agent uses Maximum likelihood updating adapted to the generalized Bayes' rule,³⁷ then uncertainty attitudes only have a bite on the conditional CEs if the prior is 50%, in which case the prediction coincides with Full Bayesian updating. Therefore, in this scenario Equations (13) and (14) restricted to $p = 50\%$ are the implications of an agent having the same CEU preference toward uncertain priors (in problems without updating) and uncertain information.

Note that the validity of Equations (13) to (16) is independent of the agent's risk preference M and the parameters in the generalized Bayes' rule. The two sides of each equation are also constructed using non-overlapping parts of data. Hence, the correlation between the two sides of each equation constitutes a test of whether subjects' attitudes toward uncertainty in priors (without information) and uncertainty in information accuracy are correlated, *given the theories under which the equation is valid*.

There is one equation that is valid under all three theories and can be the basis of a correlation test using data from my experiment:

$$SP(50\%, g, 90\% \text{ or } 50\%) = SP(90\% \text{ or } 50\%). \tag{17}$$

Now I turn to correlations that involve attitudes toward uncertain priors in problems with updating. For a CEU agent who uses the adapted Full Bayesian updating or the adapted Dynamically consistent updating, the CE for an uncertain bet conditional on simple information is

$$CE(p_h \text{ or } p_l, m, \psi) = M \left(Pr^{GB}(G|W(\frac{p_h + p_l}{2}; \varepsilon, \alpha), m, \psi) \right).$$

Since M is increasing and the generalized Bayesian posterior is increasing in the prior, if an agent uses the same CEU model for uncertainty in priors in problems with and without belief updating, then for any $0.5 \leq \psi < 1$ and $0 \leq y < x \leq 1$,

$$SP(x \text{ or } y, m, \psi) = SP(x \text{ or } y). \tag{18}$$

³⁷The Maximum likelihood updating adapted to the generalized Bayes' rule has the same selection rule as Maximum likelihood updating under Bayes' rule. The difference is that given the selected prior(s)/information accuracy level(s), beliefs are updated using the adapted Full Bayesian updating.

If the agent uses the adapted Maximum likelihood updating, then Equation (18) is valid if $\psi = 50\%$. Hence, if I require the correlation tests in my experiment to be valid under all three theories, then they need to be based on the equations

$$SP(90\% \text{ or } 50\%, -, 50\%) = SP(90\% \text{ or } 50\%) \quad (19)$$

and

$$SP(10\% \text{ or } 50\%, -, 50\%) = SP(10\% \text{ or } 50\%) \quad (20)$$

Now suppose that an agent uses the same CEU model for uncertainty in information accuracy and uncertainty in priors (in problems with updating). If she uses the adapted Full Bayesian updating, then for any $0 < p < 1$, $0.5 \leq \psi < 1$ and x and y such that $0 \leq y < x \leq 1$ and $x + y \geq 1$,

$$SP(x \text{ or } y, m, \psi) = SP(p, g, x \text{ or } y), \quad (21)$$

and

$$SP(1 - y \text{ or } 1 - x, m, \psi) = SP(p, b, x \text{ or } y). \quad (22)$$

If she uses the adapted Dynamically consistent updating, then for any $0 < p < 1$, $0.5 \leq \psi < 1$ and x and y such that $0 \leq y < x \leq 1$ and $x + y > 1$, Equation (21) holds and

$$SP(x \text{ or } y, m, \psi) = -SP(p, b, x \text{ or } y). \quad (23)$$

If the agent uses the adapted Maximum likelihood updating, then Equations (21) and (22) hold when $p = \psi = 50\%$. Therefore, the equation that is valid under all three theories and can form a basis for a correlation test using data in my experiment is

$$SP(90\% \text{ or } 50\%, -, 50\%) = SP(50\%, g, 90\% \text{ or } 50\%). \quad (24)$$

E Individual-level relation between attitudes toward compound uncertainty and ambiguity

In this section, I examine the individual-level relation between attitudes toward compound uncertainty and ambiguity. On the one hand, compound uncertainty and ambiguity differ on whether the full probability distribution over events is explicitly specified. On the other hand, both types of uncertainty are more complex than simple risks. Hence, investigating the association between compound and ambiguity attitudes sheds light on the relative importance of “unknown unknown” and complexity in decisions under uncertainty.

If an agent treats compound and ambiguous information identically, then

$$CE^{Comp}(p, m, \psi_h \text{ or } \psi_l) = CE^{Amb}(p, m, \psi_h \text{ or } \psi_l)$$

for any prior p , report m and information accuracy ψ_h and ψ_l . Similar equations hold for uncertain priors with or without belief updating if an agent holds the same attitudes toward compound and ambiguous uncertainty in priors.

Among all cases where a subject's CE for a simple bet and its compound and ambiguous counterparts are all available, there are 39% where the CEs of the compound and ambiguous bets are identical. The analogous percentages for uncertain information and uncertain priors (in problems with updating) are 36% and 35%.³⁸ To construct benchmarks for these percentages where attitudes toward compound and ambiguous uncertainty are independent, I generate independent uniform random permutations of the compound CEs and ambiguous CEs among those that share the same corresponding simple CE.³⁹ Using the permuted data, I calculate the same three percentages as before. Among 500 simulations, the highest numbers are 22%, 23%, and 21% for uncertain priors (without updating), uncertain information, and uncertain priors (with updating), respectively. These numbers are significantly lower than the actual percentages of cases where a subject's compound CE is equal to her corresponding ambiguous CE, which implies that the match between compound and ambiguity attitudes is not merely coincidence. Furthermore, I show in Table E.1 that the result is not simply driven by cases where the corresponding simple, compound, and ambiguous CEs are all the same, as the conclusion remains even if I exclude these cases. Moreover, there are more cases where the compound CE coincides with its corresponding ambiguous CE than where either of these two matches the simple CE. Taken together, my results show that compound uncertainty and ambiguity are often treated as the same by subjects. This finding confirms and extends previous experimental evidence which focuses on uncertain economic fundamentals (Halevy, 2007; Abdellaoui et al., 2015; Chew et al., 2017; Gillen et al., 2019).

³⁸If I do not require the simple CE to be available, the percentages are 39%, 35% and 35%, respectively.

³⁹For example, there are 18 subjects who report $CE(60\%, g, 60\%) = 12$ and among these 18 subjects, there are 11 whose $CE^{Comp}(60\%, g, 90\% \text{ or } 30\%)$ is not missing. Hence, I randomly permute these 11 CEs which are conditional on compound information. Similarly, there are 13 subjects among the 18 whose $CE^{Amb}(60\%, g, 90\% \text{ or } 30\%)$ is not missing. I generate an independent random permutation of these 13 CEs conditional on ambiguous information.

	$\frac{Amb=Comp}{All}$	$\frac{Amb=Comp \neq Simp}{\neg(Amb=Comp=Simp)}$	$\frac{Simp=Amb}{All}$	$\frac{Simp=Comp}{All}$
Info accuracy	36% (22%)	22% (18%)	30%	30%
Priors (without updating)	39% (23%)	26% (16%)	29%	30%
Priors (with updating)	35% (21%)	23% (16%)	28%	26%

Table E.1: Relation between compound uncertainty and ambiguity

Notes: The first column of this table shows the percentages of cases where corresponding compound and ambiguous CEs are identical. Numbers in parentheses are the maximum of these percentages in 500 simulations where compound and ambiguous CEs are randomly permuted among those that share the same simple counterpart. The second column excludes cases where the corresponding simple, compound, and ambiguous CEs are all the same. The third column shows the proportions of cases where the ambiguous CE is equal to the corresponding simple CE, whereas the last column is analogous for the match between compound CEs and simple CEs.

F Additional results on stock market reactions to earnings forecasts

F.1 A model of asset pricing with uncertain information

In this section, I derive the effects of uncertain information accuracy on stock prices in a simple representative-agent model. The model has three dates, labeled 0, 1, and 2. The representative agent owns a share of a stock, which is a claim to a dividend d whose true amount is revealed at date 2. At date 1, a piece of information m about the dividend is realized. At date 0, the representative agent has a rational expectation about the amount of dividend, which is described by the pdf $F(d)$.

If the agent knows the information structure of m , denoted by $\psi(m|d)$, then her expectation about the dividend conditional on m adheres to Bayes' rule:

$$\mathbb{E}(d|m) = \frac{\int_d d \cdot \psi(m|d) dF(d)}{\int_d \psi(m|d) dF(d)}.$$

I now focus on the case where the agent does not know the information structure. For example, the information m may be an earnings forecast issued by an analyst who is unfamiliar to the agent. Suppose that the information structure might be either $\psi_1(m|d)$ or $\psi_2(m|d)$, and the two possibilities are equally likely. Then the Bayesian expectation about the dividend conditional on m should be

$$\mathbb{E}^{Bayes}(d|m) = \frac{\int_d d \cdot \left(\frac{\psi_1(m|d) + \psi_2(m|d)}{2} \right) dF(d)}{\int_d \left(\frac{\psi_1(m|d) + \psi_2(m|d)}{2} \right) dF(d)}.$$

In view of the experimental results in this paper, people may not follow Bayes' rule when the information structure is uncertain. Therefore, adapting the CEU preference and Full Bayesian updating to the current setting, I assume that the representative agent's conditional expectation about the dividend is given by

$$\mathbb{E}(d|m) = \frac{\int_d d \cdot \left((1 - \varepsilon) \left(\frac{\psi_1(m|d) + \psi_2(m|d)}{2} - \alpha \cdot \left(\bar{\psi}(m|d) - \underline{\psi}(m|d) \right) \right) + \varepsilon \cdot \psi_0(m) \right) dF(d)}{\int_d \left((1 - \varepsilon) \left(\frac{\psi_1(m|d) + \psi_2(m|d)}{2} - \alpha \cdot \left(\bar{\psi}(m|d) - \underline{\psi}(m|d) \right) \right) + \varepsilon \cdot \psi_0(m) \right) dF(d)}. \quad (25)$$

I assume that the pdf $\psi_0(m)$ does not depend on the true dividend d , so it represents an uninformative information structure. Between ψ_1 and ψ_2 , let $\bar{\psi}$ be the one that leads to a more optimistic expectation given message m and let $\underline{\psi}$ be that one that leads the agent to be more pessimistic:

$$\bar{\psi} = \arg \max_{\psi \in \{\psi_1, \psi_2\}} \mathbb{E}(d|m) = \frac{\int_d d \cdot ((1 - \varepsilon)\psi(m|d) + \varepsilon \cdot \psi_0(m)) dF(d)}{\int_d ((1 - \varepsilon)\psi(m|d) + \varepsilon \cdot \psi_0(m)) dF(d)}$$

and

$$\underline{\psi} = \arg \min_{\psi \in \{\psi_1, \psi_2\}} \mathbb{E}(d|m) = \frac{\int_d d \cdot ((1 - \varepsilon)\psi(m|d) + \varepsilon \cdot \psi_0(m)) dF(d)}{\int_d ((1 - \varepsilon)\psi(m|d) + \varepsilon \cdot \psi_0(m)) dF(d)}.$$

Hence, the α term in Equation (25) reflects uncertainty aversion, which makes the agent overweight the more pessimistic information structure.

Simple algebra lead to the following proposition, which is a counterpart of Proposition 1 in the stock market setting.

Proposition 8 *Assume that the representative investor has a CEU preference and uses Full Bayesian updating.*

1. *If $\varepsilon = 0$ and $\alpha = 0$, then her conditional expectations about the dividend coincide with the Bayesian expectations conditional on simple information with information structure $\frac{\psi_1 + \psi_2}{2}$;*
2. *As α increases, the conditional expectations decrease;*
3. *As ε increases, the conditional expectations become closer to the prior expectation $\int_d d dF(d)$.*

A straightforward corollary of Proposition 8 is that if $\alpha > 0$ and $\varepsilon > 0$, then the expectation conditional on good news, i.e. m such that $\mathbb{E}^{Bayes}(d|m) > \int_d d dF(d)$, is lower than the Bayesian benchmark. This is because both $\alpha > 0$ and $\varepsilon > 0$ cause the agent's expectation to deviate from the Bayesian benchmark downwards. For bad news, on the other hand, the comparison with the Bayesian benchmark is ambiguous.

To study the implications on stock prices, I assume for simplicity that the representative agent is risk neutral, does not discount the future, and only cares about the dividend at date 2. Then, the

stock price at each date is equal to the expectation on that date about the dividend. Moreover, the abnormal returns at date 2 are hence $R_2 = d - \mathbb{E}(d|m)$. In view of the corollary to Proposition 8, if m is good news, then the abnormal returns are expected to be positive. ⁴⁰

F.2 Variable definitions and summary statistics

	With record			No record		
	N	mean	sd	N	mean	sd
Good news						
Ret[-1,1]	366,050	0.00795	0.0574	31,554	0.00887	0.0717
Ret[-1,22]	365,998	0.0135	0.133	31,553	0.0153	0.164
Ret[-1,43]	365,675	0.0151	0.183	31,539	0.0202	0.231
Ret[-1,64]	364,133	0.0172	0.227	31,462	0.0227	0.293
Ret[-1,EA+1]	364,993	0.0157	0.212	31,403	0.0206	0.272
Bad news						
Ret[-1,1]	562,312	-0.00668	0.0634	46,822	-0.00957	0.0739
Ret[-1,22]	562,235	-0.00770	0.144	46,816	-0.00976	0.164
Ret[-1,43]	561,752	-0.00727	0.194	46,778	-0.00947	0.221
Ret[-1,64]	559,170	-0.00955	0.235	46,653	-0.0128	0.276
Ret[-1,EA+1]	560,105	-0.00994	0.221	46,595	-0.0112	0.272

Table F.1: Returns after forecast revisions

Notes: This table summarizes the size-adjusted returns in different time windows around the forecast announcement, separately for with-record and no-record forecasts and for good news and bad news. It includes only forecasts that meet all of the data selection criteria. "EA+1" is the 1st trading day after the announcement of the actual earnings. For the summary statistics of $Ret[-1, EA + 1]$, I exclude observations where the actual earnings announcement happens later than 190 trading days after the forecast announcement. Variable definitions are in Table F.2.

F.3 Robustness checks for results on stock market reactions to earnings forecasts

⁴⁰Epstein and Schneider (2008) introduce a recursive model where the price at date t is the expectation of the prices at date $t + 1$. Making this assumption in my setting would change the stock price at date 0 but not the other results.

Variable	Definition
Main variables	
<i>Ret</i> [t, T]	The stock's (buy-hold) returns between the t th and the T th trading day after the analyst's forecast announcement minus the equal-weighted average returns of stocks in the same size decile in the same period
<i>NoRecord</i>	Indicator variable: =0 if the analyst has issued a quarterly earnings forecast on this stock before and the actual earnings of that quarter have been announced; =1 otherwise
<i>GoodNews</i>	Indicator variable: =0 if the earnings forecast is a downward revision from the last forecast issued by the same analyst on the same stock's quarterly earnings; =1 if it is an upward revision
Controls	
<i>ForecastError</i>	Absolute forecast error is the absolute difference between a forecast and the actual earnings per share, normalized by the stock price two trading days prior to the forecast announcement. Forecast error is absolute forecast error normalized by the standard deviation of absolute forecast errors among all forecasts for the same stock-quarter
<i>StockExp/IndExp/TotExp</i>	Experience (stock-specific/industry-specific/total): number of days since the analyst's first earnings forecast on the same stock/any stock in the same industry/any stock
<i>Companies</i>	Number of stocks covered by the analyst in the same year
<i>Industries</i>	Number of industries covered by the analyst in the same year
<i>Turnover</i>	Indicator variable: =0 if the analyst has not changed brokerage house in the year; =1 otherwise
<i>Horizon</i>	Number of days between the earnings forecast and the end of the forecasted quarter
<i>DaysElapsed</i>	Number of days elapsed since the last forecast issued by any analyst on the same firm's quarterly earnings or the firm's last earnings announcement, whichever is later
<i>BrokerSize</i>	Number of analysts in the same brokerage house who cover the same stock in the same year
<i>Coverage</i>	Number of analysts covering the same firm in the same year
<i>log(MktCap)</i>	Logarithm of market capitalization at the end of last year
<i>B/M</i>	Book-to-Market ratio at the end of last year. Winsorized at the 1st and 99th percentiles
<i>PastReturns</i>	Size-adjusted returns from seven months before forecast announcement to one month before forecast announcement. Winsorized at the 1st and 99th percentiles
<i>Volatility</i>	Standard deviation of the stock's monthly returns in the 24 months before the end of the calendar year prior to the forecast announcement
<i>Volume</i>	Average monthly turnover of the stock in the past calendar year

Table F.2: Variable definitions

VARIABLES	With record			No record		
	N	mean	sd	N	mean	sd
GoodNews	943,984	0.394	0.489	81,839	0.403	0.491
ForecastError	937,015	-0.122	0.933	80,787	-0.101	0.941
StockExp	943,984	1,389	1,418	81,839	83.38	242.0
IndExp	943,984	2,379	2,081	81,839	943.2	1,476
TotExp	943,984	3,024	2,351	81,839	1,561	1,920
Companies	943,984	16.73	8.306	81,839	14.01	9.171
Industries	943,984	4.377	2.693	81,839	3.972	2.705
Turnover	943,984	0.0321	0.176	81,839	0.0377	0.191
Horizon	943,984	42.92	46.56	81,839	40.50	50.30
DaysElapsed	943,984	11.49	15.82	81,839	13.24	16.83
BrokerSize	943,984	1.072	0.275	81,839	1.281	0.509
log(MktCap)	943,801	7.826	1.845	81,815	7.151	1.790
B/M	943,784	0.518	0.397	81,815	0.467	0.377
PastReturns	929,349	0.00338	0.297	81,255	0.0455	0.361
Volume	914,191	2.239	1.850	71,545	2.155	1.901
Coverage	943,984	13.73	8.667	81,839	11.39	8.144

Table F.3: Summary statistics

Notes: This table summarizes the indicator variable *GoodNews* and the control variables, separately for with-record and no-record forecasts. It only includes observations that meet all of the data selection criteria, i.e. forecast revisions for quarters between January 1, 1994 and June 30, 2019 such that on the forecast announcement day, there is neither an earnings announcement from the company nor earnings forecast announcements by any other analyst on the same company. Variable definitions are provided in Table F.2.

VARIABLES	With record			No record		
	N	mean	sd	N	mean	sd
GoodNews	2,412,921	0.393	0.488	168,938	0.398	0.490
ForecastError	2,401,471	-0.161	0.908	167,523	-0.146	0.927
StockExp	2,412,921	1,435	1,447	168,938	75.70	231.7
IndExp	2,412,921	2,470	2,122	168,938	971.5	1,515
TotExp	2,412,921	3,134	2,414	168,938	1,592	1,969
Companies	2,412,921	16.63	7.459	168,938	13.86	8.477
Industries	2,412,921	4.290	2.564	168,938	3.843	2.574
Turnover	2,412,921	0.0262	0.160	168,938	0.0352	0.184
Horizon	2,412,921	49.18	40.83	168,938	46.17	48.98
DaysElapsed	2,412,921	5.349	12.09	168,907	7.603	15.74
BrokerSize	2,412,921	1.070	0.273	168,938	1.287	0.513
log(MktSize)	2,412,502	8.092	1.786	168,890	7.462	1.757
B/M	2,412,450	0.493	0.384	168,889	0.445	0.365
PastReturns	2,375,825	0.00198	0.288	167,802	0.0378	0.353
Volume	2,347,292	2.452	1.912	149,676	2.402	1.996
Coverage	2,412,921	15.92	9.143	168,938	13.61	8.842

Table F.4: Summary statistics (all forecast revisions between 1/1/1994 and 6/30/2019)

Notes: This table summarizes the indicator variable *GoodNews* and the control variables, separately for with-record and no-record forecasts. It includes all forecast revisions for quarters between January 1, 1994 and June 30, 2019. Variable definitions are in Table F.2.

	(1)	(2)	(3)
	Ret[2,22]	Ret[2,43]	Ret[2,EA+1]
Ret[-1, 1]	0.0179 (0.0568)	0.146† (0.0775)	0.252** (0.0808)
NoRecord	0.000717 (0.00120)	0.00170 (0.00158)	0.00116 (0.00156)
NoRecord × Ret[-1, 1]	-0.0335 (0.0227)	-0.0214 (0.0319)	-0.0520 (0.0364)
GoodNews	0.00689*** (0.000929)	0.00753*** (0.00137)	0.00939*** (0.00149)
GoodNews × Ret[-1, 1]	0.0297 (0.0190)	0.0449* (0.0222)	0.0280 (0.0275)
NoRecord × GoodNews	-0.000648 (0.00151)	0.000990 (0.00206)	0.000416 (0.00210)
NoRecord × GoodNews × Ret[-1, 1]	0.0510 (0.0326)	0.100* (0.0478)	0.122* (0.0496)
Controls	Y	Y	Y
Controls × Ret[-1,1]	Y	Y	Y
Year-Quarter FE	Y	Y	Y
Observations	895740	895168	892678
R^2	0.009	0.012	0.015

Table F.5: Sufficiency of stock market reactions to forecast revisions: different drift lengths

Notes: This table reports the results of Regression (12) with different dependent variables. $Ret[2, 22]$ and $Ret[2, 43]$ are the stock's 1-month and 2-month size-adjusted buy-hold returns starting from the 2nd trading day after the forecast announcement, respectively. "EA+1" is the 1st trading day after the announcement of the actual earnings. In the model $Ret[2, EA + 1]$, I exclude observations where the actual earnings announcement happens later than 190 trading days after the forecast announcement. Three-dimensional (stock, analyst, year-quarter) cluster-robust standard errors in parentheses. † $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Dependent Var: Ret[2,64]	(1)	(2)	(3)	(4)	(5)	(6)
Ret[-1, 1]	High innovation	Isolated	After 2004	MktCap>2b	MktCap>10b	Qtr SE&FE
	0.307* (0.139)	0.271† (0.137)	0.369** (0.131)	-0.00618 (0.158)	-0.625† (0.317)	
NoRecord	0.00213 (0.00272)	-0.00211 (0.00229)	-0.00210 (0.00243)	0.00498† (0.00267)	0.00358 (0.00281)	0.000536 (0.00205)
NoRecord × Ret[-1, 1]	-0.0667 (0.0505)	-0.0960 (0.0629)	-0.105† (0.0570)	-0.0449 (0.0783)	0.00603 (0.136)	-0.0264 (0.0458)
GoodNews	0.0120*** (0.00222)	0.0111*** (0.00182)	0.00737*** (0.00185)	0.00364* (0.00147)	0.00396* (0.00154)	0.0106*** (0.00176)
GoodNews × Ret[-1 1]	0.0440 (0.0339)	0.0715* (0.0353)	0.0163 (0.0344)	0.0646* (0.0264)	0.0633 (0.0426)	0.0522† (0.0266)
NoRecord × GoodNews	0.000813 (0.00339)	0.00146 (0.00280)	-0.000372 (0.00287)	-0.00459 (0.00360)	-0.00515 (0.00352)	0.00131 (0.00247)
NoRecord × GoodNews × Ret[-1, 1]	0.229** (0.0780)	0.153† (0.0869)	0.134† (0.0800)	0.163 (0.135)	0.0522 (0.169)	0.107† (0.0620)
Controls	Y	Y	Y	Y	Y	Y
Controls × Ret[-1,1]	Y	Y	Y	Y	Y	Y
Year-Quarter FE	Y	Y	Y	Y	Y	Y
Year-Quarter Slope Effects	N	N	N	N	N	Y
Observations	502879	571449	649583	499077	215867	894004
R ²	0.018	0.013	0.013	0.020	0.017	0.016

Table F.6: Sufficiency of stock market reactions to forecast revisions: robustness checks

Notes: This table reports the results of Regression 12 under different cuts of the data and specifications. “High innovation” restricts attention to forecasts that fall outside the range between the same analyst’s previous forecast and the previous consensus. “Isolated” focuses on observations where there is neither an earnings announcement from the company nor forecast announcements by any other analysts on the same company in the three-day window centered on the forecast announcement day. “After 2004” uses forecasts announced after Jan 1, 2004. “MktCap>2b” and “MktCap>10b” focus on stocks whose market capitalization is larger than \$2 billion and \$10 billion, respectively. “Qtr SE&FE” includes the interactions between the year-quarter dummies and $Ret[-1, 1]$, in addition to year-quarter fixed effects. Three-dimensional (stock, analyst, year-quarter) cluster-robust standard errors in parentheses. † $p < 0.10$, * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

	(1)	(2)
	Ret[-1,1]	Ret[2,64]
Revision	1.077*** (0.258)	2.281* (0.972)
NoRecord	-0.00258** (0.000812)	0.000937 (0.00297)
NoRecord × Revision	-0.113 (0.179)	-0.566 (0.480)
GoodNews	0.0116*** (0.000581)	0.00641** (0.00196)
GoodNews × Revision	0.438*** (0.118)	2.586*** (0.727)
NoRecord × GoodNews	0.00448*** (0.00110)	-0.00163 (0.00397)
NoRecord × GoodNews × Revision	-0.0416 (0.336)	2.822* (1.315)
Controls	Y	Y
Controls × Revision	Y	Y
Year-Quarter FE	Y	Y
Observations	503943	502879
R^2	0.026	0.022

Table F.7: Sufficiency of stock market reactions to forecast revisions: magnitudes of revisions

Notes: This table reports the results of the following regression.

$$\begin{aligned}
Ret[t, T]_i = & \eta_0 + \eta_1 Revision_i + \eta_2 NoRecord_i + \eta_3 GoodNews_i \\
& + \eta_4 NoRecord_i \cdot GoodNews_i + \eta_5 Revision_i \cdot GoodNews_i + \eta_6 Revision_i \cdot NoRecord_i \\
& + \eta_7 Revision_i \cdot NoRecord_i \cdot GoodNews_i + Controls_i + Controls_i \cdot Revision_i + TimeFE_i + \varepsilon_i.
\end{aligned}
\tag{26}$$

Revision is the difference between an analyst's revised forecast on earnings per share and her previous forecast, normalized by the stock price two trading days prior to the announcement of the revision. I winsorize *Revision* at the 1st and 99th percentiles. I only include "high-innovation" revision, i.e. forecasts that fall outside the range between the same analyst's previous forecast and the previous consensus. Three-dimensional (stock, analyst, year-quarter) cluster-robust standard errors in parentheses. $\dagger p < 0.10$, $*p < 0.05$, $**p < 0.01$, $***p < 0.001$