Jamming in the Kitchen - A Cool Gateway to Physics Research

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I. INTRODUCTION

Imagine running into the kitchen looking for a quick, tasty snack, when a bottle of jelly beans catches your eye. You grab the bottle and flip it over to pour some out. Much to your surprise and frustration, the jelly beans get stuck at the opening. It takes some poking or a shake of the bottle to get them flowing out into your hands. This and similar experiences in trying to get a 'granular material' through a narrow orifice highlights a fascinating, unsolved problem in physics. Believe it or not, the development of a simple and correct formula that describes the rate at which these materials flow out of an orifice has been a subject of study for over fifty years. Why is this such a big deal? The problem is not about our sweet tooth for jelly beans, rather it is about effectively moving various granular materials (such as sand, rocks, grains, etc.) through orifices in a variety of industrial processes. This work presents a simple experiment to measure the mass flow rate of granular particles under different conditions. This experiment is designed for implementation at the high school level, providing a hands-on introduction to granular flow, jamming, and to the rich dynamics of these common yet complex systems.

Consider a discharge of grains. The flow rate, which we will denote as W, is the amount of mass, ΔM, that flows over a short period of time, Δt, such that $W \equiv \frac{\Delta M}{\Delta t}$. This rate often depends on the orifice size. Assume the grains are all spherical with diameter d_p and are placed in a container with an orifice of diameter D_o. One should expect that the flow will be slower if each grain, or a small collection of them, is comparable in size to the orifice. This is in fact true, and is quantified by an effective diameter $(D_0 - kd_p)$, where k is a constant. Early work by W.A. Beverloo and colleagues [1] proposed that the flow rate depends on this effective diameter as

$$W = C\rho_b\sqrt{g}\left(D_0 - kd_p\right)^{\frac{5}{2}}. \tag{1}$$

This equation has come to be known as Beverloo's Law. Here, ρ_b is the average number of grains per unit volume and g is the acceleration due to gravity. C is a parameter that helps fix how rapidly the grains flow out in the steady state. The value of C must be determined experimentally. Beverloo found that $0.55 < C < 0.65$. The constant k, which we introduced in the effective diameter, takes into account the shapes of the grains. The value of k has been a subject of disagreement. Nedderman and Laohakul [2] found $1 < k < 2$, though there are exceptions like sand which has a value of 2.9 [3].

Beverloo's Law suggests that mass flow follows a 5/2 power law dependence on the effective diameter. Subseqent models have been built off of Beverloo's Law. In particular, Mankoc et al. [3] have recently moved to including an exponential term and empirically suggest eliminating the k constant. Defining the dimensionless quantity $R \equiv D_o/d_p$, they propose

$$W = C^{'}\left(1 - \frac{1}{2}e^{-b(R-1)}\right)(R-1)^{\frac{5}{2}}. \tag{2}$$

Here, C' absorbs C, ρ_b, and g from (1) as well as the particle mass. The effective diameter simplifies to the form $(R-1)$. This model introduces a new parameter b, which is related to the density of particles near the orifice. The value of b must be determined empirically.

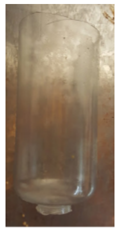
These models are meant to best describe a steady flow situation where $R >> 1$. Jamming occurs when $R \to 1$. The orifice becomes blocked by the arrangement of particles, so all flow ceases. We will treat both regimes, but focus on the march toward jamming. In section II we describe the setup and operation of this experiment. In section III, we present the results of our run of the experiment, what trends to expect, and a comparison to the predictions of Beverloo's Law and the Mankoc equation. We discuss some motivations behind these equations and implications of this experiment in section IV.

II. EQUIPMENT AND METHOD

We are interested in how the flow rate depends on the particle diameter and the orifice diameter. The particle diameter can be varied by using different types of particles. We tested yellow mustard seeds ($d_p \approx 2cm$), whole black peppercorns ($d_p \approx 5cm$), and sesame seeds ($d_p \approx 2.75cm$). Each of these can be obtained at a local grocery store. If the container used to carry out the experiment with has a fixed outlet, one must devise a way to be able to vary the orifice diameter. We constructed "orifice slides": small sheets of leftover paper or plastic with circular holes of the desired D_o to be tested cut out of them. These can be affixed over the container outlet with tape, then removed and swapped at will. It is possible that the flow rate might be affected by the geometry of the container, though this is not accounted for in the flow rate models. One can test containers with different shapes and sizes. For example, we crafted four containers from common household items: a 20oz pop bottle, a large and small saline bottle, and a shampoo bottle; these are shown in Fig. 1. We used the pop bottle.

The procedure is simple. Take the container with the desired orifice and mount it so that it is oriented vertically above a collection bowl. Then, initiate flow by releasing the particles through the container. We measured the flow rate by placing a collection bowl on a standard spring kitchen scale, and tracking the scale reading over time. We did this by placing the scale next to a stopwatch and recording them. Most error here is systematic. With a spring scale, the reading will bounce as the grains fall onto it. Further, the camera's frame rate limits the precision of the stop watch reading.

(a) Bottom of a pop bottle



(b) Saline bottle

(c) Shampoo bottle

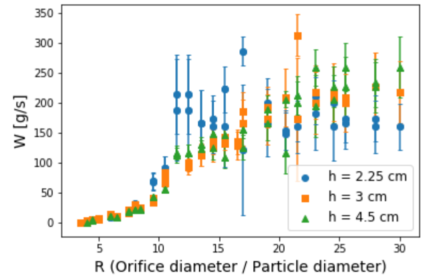
FIG. 1. Possible containers for housing particle flow.

III. RESULTS & ANALYSIS

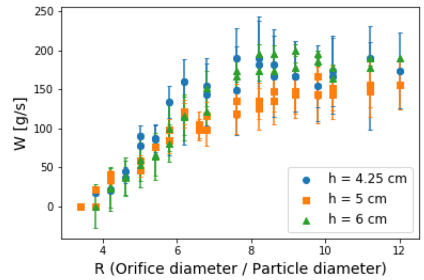
The models we introduced describe how flow rate depends on the particle and orifice size. They do not explicitly account for other factors which might affect the rate, such as having a larger pile of particles or adding some sort of lubricant to the container. We will first put these two factors to the test. Then, we will compare our data to Beverloo's Law and the model by Mankoc et al.

A. Column Height

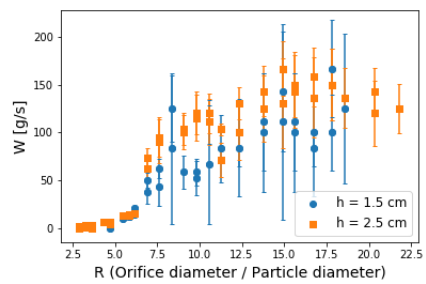
The flow can be initiated in several ways. One can pour them through the container so they flow out as they fall. One can also block the outlet, pile the particles inside the container, then unblock the outlet. The latter method tests how column height, h, affects flow rate. The column height is the total height of the level pile, measured from the bottom of the pile. For our gravity driven flow, we expect h will not significantly affect the flow rate. We tested this for each particle type using the container in Fig. 1a. The mustard seeds were tested at $h = 2.25, 3,$ and $4.5\ cm$, the peppercorns at $h = 4.25, 5,$ and $6\ cm$, and the sesame seeds at $h = 1.5$ and $2.5 cm$. Our results are shown in Fig. 2. Although the flow rates vary by up to $\pm 50\ g/s$ at fixed R, we observed no significant difference caused the change in column height.



(a) Mustard Seeds



(b) Whole Peppercorns



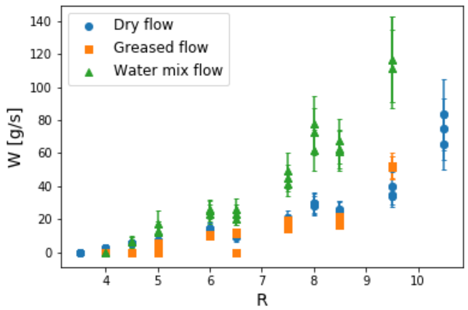
(c) Sesame Seeds

FIG. 2. Measuring mass flow rate as a function of R for different seeds with different h being tested. For each data set, there is insignificant difference between the trends for each h tested, for that particular seed, so we agree that the column height does not appear to significantly impact the flow rate W.

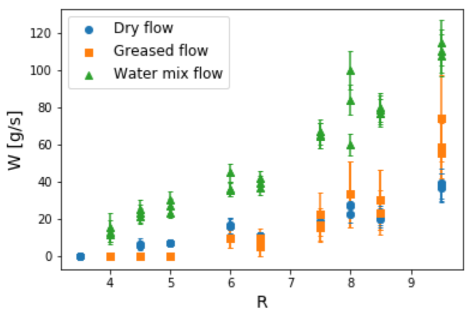
B. Flow Catalysts

Lubricants and liquids can alter friction of the container walls and between individual particles. This might affect the flow rate by changing C or ρ_b, and may be related to the viscosity, or relative abundance or densities of the particle and catalyst. We tested the effects of adding grease (WD-40) and water to a flow of mustard seeds. The results are shown in Fig. 3a for the pop bottle and Fig. 3b for the large saline bottle.

For both containers, adding the grease did not seem to change the flow rate compared to when nothing was added. We did observe that the greased flow jammed at higher R than the dry flow. The mixture of seeds and water did have a higher flow rate at all R.



(a) Pop bottle



(b) Saline bottle

FIG. 3. We compare the flow rate of mustard seeds after adding grease and after mixing the seeds into water. This was tested with two containers.

The seeds were poured into the container from a cup. For both the greased and water mixture sets, some seeds got stuck to the walls of both the cup and container. The water mixture was first stirred to prevent settling or clumping, but this did not prevent seeds from sticking. It would be interesting to explore testing of different viscosities to see if the flow rate has a simple relationship with viscosity. Similarly, we have not explored the behavior of greased or wet grains for the case of $R << 1$. This may be an interesting regime to explore the effects of jamming in future work.

C. Data Modelling

We ran several trials of mustard seed flow through the pop bottle. In Fig. 4, we compare this data to Beverloo's Law on a logarithmic scale. The data exhibits a linear trend that plateaus for around $\ln(R) \geq 2.5$. The linearity corresponds to the steady flow regime heading towards jamming, while the turn off occurs when the flow begins to saturate.

We restrict our attention to the linear regime. Beverloo's Law best describes flow here, but has trouble near jamming. On this logarithmic scale, the 5/2 power law takes the form of a line with slope $a_B = 5/2 = 2.5$. For the march toward jamming in $ln(R) \leq 2.5$, our data best fits a line of slope $a = 2.88 \pm 0.26$. This slope is 1.46σ higher than a_B. In the strict steady flow regime where Beverloo holds best, $2 \leq ln(R) \leq 2.8$, the data is fit with slope $a = 2.51 \pm 0.24$ which is consistent with the 5/2 power law model.

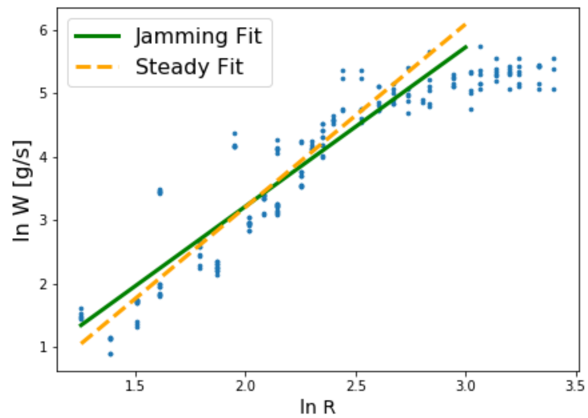


FIG. 4. Plotting our flow rate data on a log scale. The solid line is a fitted slope of $a = 2.88 \pm 0.26$ and the dashed line is a fitted slope of 2.51 ± 0.24. The latter, fit in the steady flow regime, is consistent with Beverloo's Law.

Next, we compare our data to the model of Mankoc et. al [3]. Upon taking the natural logarithm and simplifying, (2) reduces to

$$\ln(W) = \ln(2) + b\,(R - 1) \tag{3}$$

Here, b is now the only parameter, and this is similar to the equation of a line. In principle, b ought to be independent of R. In Fig. 5 we fit our data to (3) to get $b = 0.3695 \pm 0.0110$. This is larger than the value reported in Mankoc et. al [3] of $b = 0.051$. Since b is related to the grain density near the orifice, the discrepancy is likely due to our different experimental setups. The value may depend strongly on apparatus properties like curvature and geometry within the container, the force driving the flow, or the presence of a catalyst. It would be an interesting task for students to test several of these factors for their effect on b.

IV. DISCUSSION

After having analyzed these flow rate equations, it is natural to wonder where they came from. When tackling an unsolved problem, physicists are guided by prior experience. Sometimes, this amounts to making a guess and adjusting it based on the results of an experiment. Problems involving rates, like a jelly bean flow, often depend on power laws or exponential behavior. We guess one dependence, fit the data to it, then adjust accordingly

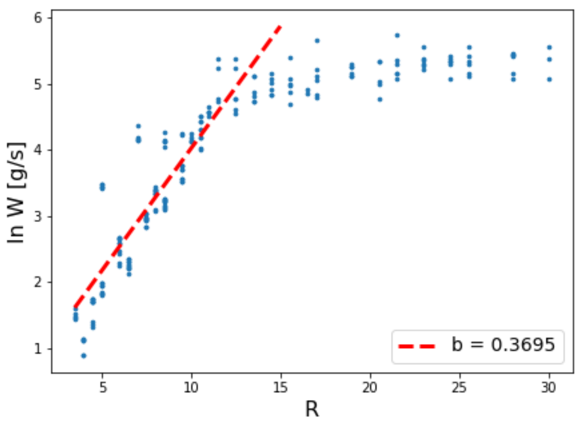
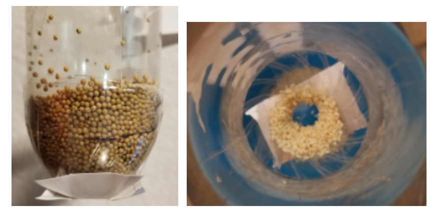


FIG. 5. Here we plot our flow rate data on a semi-log scale to fit it according to (3). The best fit of our data gives $b = 0.3695$. In comparison, Mankoc et al. found $b = 0.051$ [3], which would not lie along our data at all.

until we get the most ideal fit. In the case of granular matter flow, Beverloo et. al [1] tried a power law fitting which worked out well. This fit broke down very close to jamming, however. Mankoc et al. included a saturation function in the form of an exponential, which greatly improved the fit close to jamming. Solutions of other problems suggested trying these fits and it should be no surprise that they ended up fitting the data for this problem quite well.

Step back into the kitchen and pick the bottle of jelly beans back up. Hopefully, there will now be less frustration after the beans get stuck at the opening. Since W sharply increases with increasing R, obeying a mix of power law and exponential behavior, finding an alternate bottle with a larger opening should be the obvious course of action. We have shown that column height does not affect the flow rate; it ought to make no difference whether your bottle is full or half empty. This is a manifestation of the Janssen effect [4], in which the container walls support a portion of the weight of the granular layers via friction. This is a behavior unique to granular solids. Similarly, if you are the type of person who enjoys mixing their jelly beans in water, you can expect that they will flow out of the bottle slightly faster that way.

A closer look inside the bottle reveals that there are different ways in which the jelly beans can get stuck. The primary type is the materialization of the effective diameter, seen in Fig. 6a. As $R \to 1$, the particle or bulk of particles becomes as large as the outlet, so they build up above the orifice but cannot flow through. This jamming was independent of column height. We also observed particles building up around the orifice, shown in Fig. 6b, forming a wall that prevents further flow. This is due to static friction along the container walls. Particles sliding along the walls are slowed to a stop before flowing through the orifice.



(a) (b)

FIG. 6. We encountered two different types of jamming. 6a shows a side view of jamming when the effective diameter is small. 6b shows jamming due to particles building up on the container around the orifice.

V. CONCLUSION

Beverloo's Law is most effective in the steady flow regime, having a $a_B = 2.5$ power law slope. In this regime, our data is best fit with $a = 2.51 \pm 0.24$, consistent with Beverloo. Jamming occurs at small R, equivalent to a small effective diameter. Beverloo's Law has trouble extending to this regime. We have shown that the best fit slope is $a = 2.88 \pm 0.26$ in this jamming regime. Mankoc et al. [3] derived a corrected version of Beverloo's Law, which we simplified in (3). This equation fits our data well through both the steady and jamming regimes. Our fit gives a value $b = 0.3695 \pm 0.0110$, larger than the $b = 0.051$ reported in the original paper.

The primary motivation of this work was to create a simple experiment to introduce high school level physics students to granular flows. This problem is not commonly presented in current curriculums, despite having implications that can be noticed in a student's everyday life. We have designed a simple method that can be easily implemented. Students can use this experiment to learn how jamming arises, explore the behavior of greased, wet, or other conditions of grains near jamming, and test how container geometry affects the flow rate through its orifice. We have shown that this experiment is low-cost, as materials can be crafted from household and recycled materials. Laboratory-grade equipment would certainly field a more precise examination of the flow rate behavior. However, we have shown that fascinating problems can nevertheless be explored in the kitchen with everyday objects.

[1] W. Beverloo, H. Leniger, and J. van de Velde, Chemical Engineering Science **15**, 260 (1961).
[2] R. Nedderman and C. Laohakul, Powder Technology **25**,

91 (1980).
[3] C. Mankoc, A. Janda, R. Arevalo, J. M. Pastor, I. Zuriguel, A. Garcimartin, and D. Maza, Granular Matter **9** (2007), https://doi.org/10.1007/s10035-007-0062-2.
[4] F. Ebrahimi, T. Azizpour, and H. Maleki, Phys. Rev. E **82**, 031302 (2010).