1. (a) True

$h(t)$ is called impulse respose.
$h(t)$ replesents the output of system at time t due to a unit-impulse input occurring at $t=0$ when the system is intally relaxed

when $h(t)$ is output of $\delta(t)$,
$h(t-\tau)$ is output of $\delta(t-\tau)$
this mean $h(t)$ is time-invarient

when $x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$
using superposition property of linear system

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

so convolution integral assume the system $h(t)$ to be linear, time-invariant and initially at rest state condition

(b) False

$$h_p(t) = \begin{cases} \sum_{i=0}^{M-N} C_i \delta^{(i)}(t) & (M \geq N) \\ 0 & (M < N) \end{cases}$$

(c) True, Period; 6

$\sin\frac{2}{3}\pi t$ has period $T_1 = 3$

$\frac{1}{2}\cos 3\pi t$ has period $T_2 = \frac{2}{3}$

$3\alpha = \frac{2}{3}\beta$ if integer α, β exist

then $T = 3\alpha = \frac{2}{3}\beta$

if $\alpha = 2$, $\beta = 9$ $\quad 3\alpha = \frac{2}{3}\beta = 6$

So period $T = 6$

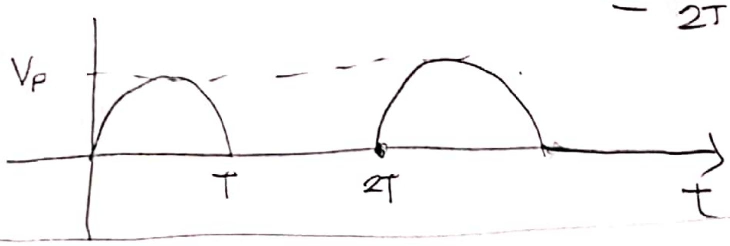
(d) False

$$V_{DC_{half}} = \frac{1}{2T}\int_0^T V_p \sin\frac{2\pi}{2T}t\, dt$$

$$= \frac{V_p}{2T}\frac{T}{\pi}\left[-\cos\frac{\pi}{T}t\right]_0^T$$

$$= \frac{V_p}{\pi}$$

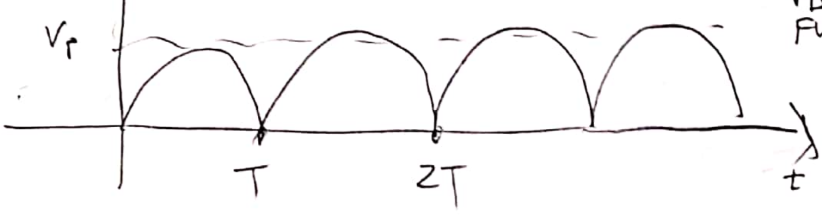
half wave



Full wave

$$V_{DC_{Full}} = \frac{1}{T}\int_0^T V_p \sin\frac{2\pi}{2T}t\, dt = \frac{2V_p}{\pi}$$

$$V_{DC_{Full}} = 2\cdot V_{DC_{half}}$$



so the strength of DC component of a full-wave rectifier is 2 as that of a half-wave rectifier.

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2. a) $y(t) = x(t)\,\exp(x(t))$

i) $y_1(t) = x_1(t)\,e^{x_1(t)}$

$y_2(t) = x_2(t)\,e^{x_2(t)}$

$$y_1(t) + y_2(t) = x_1(t)\,e^{x_1(t)} + x_2\,e^{x_2(t)}$$

$$\neq (x_1(t) + x_2(t))\,e^{x_1(t)+x_2(t)}$$

$\therefore$ non linear.

ii) $y(t) = x(t)\,e^{x(t)}$

$y(t)$ is only affected by current input $x(t)$

$\hookrightarrow$ memoryless

iii) $y_1(t) = x_1(t)\,e^{x_1(t)}$

$x_2(t) = x_1(t - t_0)$

$y_2(t) = x_1(t-t_0)\,e^{x_1(t-t_0)}$

$= y_1(t_0 - t) = x_1(t-t_0)\,e^{x_1(t-t_0)}$

$y_2(t) = y_1(t_0 - t)$

$\hookrightarrow$ time-invariant

iv) $y(t) = x(t)\,e^{x(t)}$

$\hookrightarrow \therefore$ $y(t)$ is only affected by current input $x(t)$

$\hookrightarrow$ causal

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(b) $y(t) = x(t-2)\,x(t+1)$

i) $y_1(t) = x_1(t-2)\,x_1(t+1)$

$y_2(t) = x_2(t-2)\,x_2(t+1)$

$y_1(t) + y_2(t) = x_1(t-2)x_1(t+1) + x_2(t-2)x_2(t+1)$

$\neq \left(x_1(t-2)+x_2(t-2)\right)\cdot\left(x_1(t+1)+x_2(t+1)\right)$

$= x_1(t-2)x_1(t+1) + x_2(t-2)x_2(t+1)$
$+ x_1(t-2)x_2(t+1) + x_2(t-2)x_1(t+1)$

↳ non linear

ii) $y(t) = x(t-2)\,x(t+1)$

$y(t)$ is affected by past input $x(t-2)$

↳ Memory system

iii) $y_1(t) = x_1(t-2)\,x_1(t+1)$

$x_2(t) = x_1(t-t_0)$

$y_2(t) = x_1(t-t_0-2)\,x_1(t-t_0+1)$

$y_1(t-t_0) = x_1(t-t_0-2)\,x_1(t-t_0+1)$

$y_2(t) = y_1(t-t_0)$ → time-invarient

iv) $y(t) = x(t-2)\,x(t+1)$

$y(t)$ is affected by future input $x(t+1)$

↳ non causal

(c) $\frac{dy}{dt} + 2y(t) = x^2(t)$

$$e^{2t}\frac{dy}{dt} + 2e^{2t}y(t) = e^{2t}x^2(t)$$

$$\frac{d}{dt}\left(e^{2t}y(t)\right) = e^{2t}x^2(t)$$

$\boxed{y(t_0) = \frac{c}{e^{2t_0}}}$

$$e^{2t}y(t) = \int_{t_0}^{t} e^{2\tau}x^2(\tau)\,d\tau + c \quad (\underline{t \geq t_0})$$

$$y(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau}x^2(\tau)\,d\tau + ce^{-2t}$$

i) $y_1(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau}x_1^2(\tau)\,d\tau + ce^{-2t}$

$y_2(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau}x_2^2(\tau)\,d\tau + ce^{-2t}$

$y_1(t)+y_2(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau}\left(x_1^2(\tau) + x_2^2(\tau)\right)d\tau + 2ce^{-2t}$

$\neq e^{-2t}\int_{t_0}^{t} e^{2\tau}\left(x_1^2(\tau) + 2x_1(\tau)x_2(\tau) + x_2^2(\tau)\right)d\tau + ce^{-2t}$

↳ non linear

ii) $y(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau}x^2(\tau)\,d\tau + ce^{-2t}$

$y(t)$ is affected past input from $x(t_0)$ to $x(t)$

So memory system

iii) $y_1(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau} x_1^2(\tau)\,d\tau + Ce^{-2t}$

$x_2(t) = x_1(t - t_\alpha)$

$y_2(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau} x_1^2(\tau - t_\alpha)\,d\tau + Ce^{-2t}$

$y_1(t - t_\alpha) = e^{-2(t-t_\alpha)}\int_{t_0}^{t-t_\alpha} e^{2\tau} x_1^2(\tau)\,d\tau + Ce^{-2(t-t_\alpha)}$

generally, $y_2(t) \neq y_1(t - t_\alpha)$ $\rightarrow$ $\boxed{\text{time-varient}}$

but if $C=0$ and when $t < t_0$ $x_1(t) = 0$

then time-invarient

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iv) $y(t) = e^{-2t}\int_{t_0}^{t} e^{2\tau} x_1^2(\tau)\,d\tau + Ce^{-2t}$ $(t \geq t_0)$

$y(t)$ is affected by current signal

and past signal from t_0 to t

so causal system

CamScanner로 스캔하기

3. (a) $x(t) = rect\left(\frac{t}{4}\right) + \delta(t-1)$

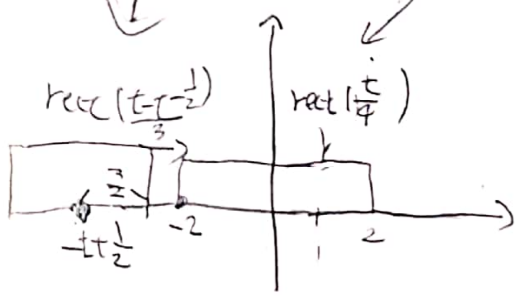
$$h(t) = rect\left(\frac{t-\frac{1}{2}}{3}\right) + \delta(t-3) + 2\delta(t)$$

$$y(t) = \left[rect\left(\frac{t}{4}\right) + \delta(t-1)\right] * \left[rect\left(\frac{t-\frac{1}{2}}{3}\right) + 2\delta(t) + \delta(t-3)\right]$$

$$= rect\left(\frac{t}{4}\right) * rect\left(\frac{t-\frac{1}{2}}{3}\right) + \delta(t-1) * rect\left(\frac{t-\frac{1}{2}}{3}\right)$$
$$+ rect\left(\frac{t}{4}\right) * 2\delta(t) + 2\delta(t-1) * \delta(t)$$
$$+ rect\left(\frac{t}{4}\right) * \delta(t-3) + \delta(t-1) * \delta(t-3)$$

$$= rect\left(\frac{t}{4}\right) * rect\left(\frac{t-\frac{1}{2}}{3}\right) + rect\left(\frac{t-\frac{3}{2}}{3}\right)$$
$$+ 2rect\left(\frac{t}{4}\right) + 2\delta(t-1)$$
$$+ rect\left(\frac{t-3}{4}\right) + \delta(t-4)$$

$$* \quad \int_{-\infty}^{\infty} rect\frac{\tau}{4} \cdot rect\left(\frac{t-\tau-\frac{1}{2}}{3}\right) d\tau$$



$-t+\frac{1}{2}+\frac{3}{2} < -2 \to 0$

$-2 < -t+\frac{1}{2}+\frac{3}{2} < 1 \to -t+4$

$1 < -t+\frac{1}{2}+\frac{3}{2} < 2 \to 3$

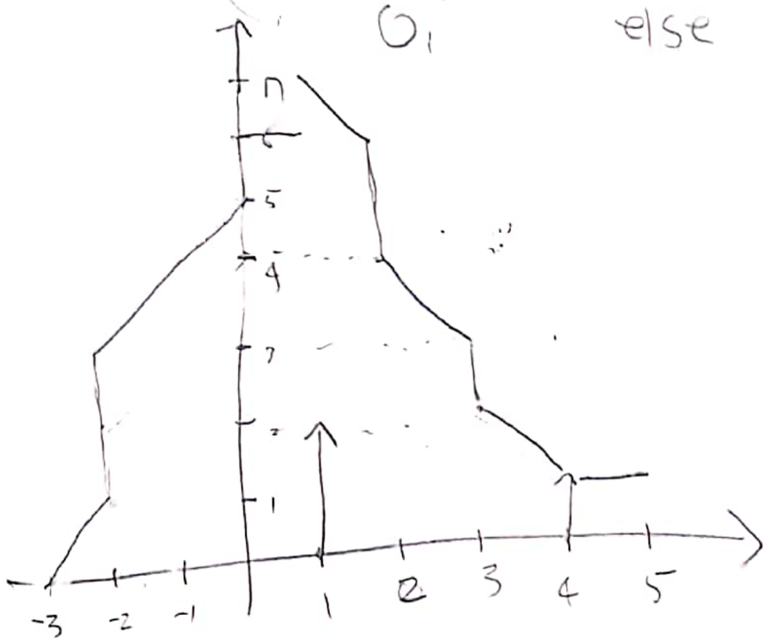
$2 < -t+\frac{1}{2}+\frac{3}{2} < 5 \to 2-(-t+\frac{1}{2}-\frac{3}{2})$

$5 < -t+\frac{1}{2}+\frac{3}{2} \to 0$

$$\Rightarrow \begin{cases} 0 & (t<-3) \\ t+3 & (-3 \le t < 0) \\ 3 & (0 \le t < 1) \\ -t+4 & (1 \le t < 4) \\ 0 & (t \ge 4) \end{cases}$$

$$x(t) = \mathrm{rect}\left(\frac{-t}{4}\right) * \mathrm{rect}\left(\frac{t-\frac{1}{2}}{3}\right) + \begin{cases} 2 & -2<t<0 \\ 3 & 0<t<1 \\ 4 & 1<t<2 \\ 2 & 2<t<3 \\ 1 & 3<t<5 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} t+3 & -3<t<-2 \\ t+5 & -2<t<0 \\ 6 & 0<t<1 \\ -t+8 & 1<t<2 \\ -t+6 & 2<t<3 \\ -t+5 & 3<t<4 \\ 1 & 4<t<5 \\ 2\delta(t-1) & t=1 \\ \delta(t-4) & t=4 \\ 0 & \text{else} \end{cases}$$



4. $x(t) = \delta(t)$

$$h''(t) + 2h'(t) + 2h(t) = \delta'(t) + 4\delta(t)$$

$$n^2 + 2n + 2 = 0$$

$$n = -1 \pm j$$

$$h_h(t) = (C_1 e^{-t}\cos t + C_2 e^{-t}\sin t)\, u(t)$$

$$(\because M < N \rightarrow h_p(t) = 0)$$

$$h'(t) = \{(C_2 - C_1) e^{-t}\cos t - [C_2 + C_1] e^{-t}\sin t\}\, u(t)$$

$$+ C_1 \delta(t)$$

$$h''(t) = (-2C_2 e^{-t}\cos t + 2C_1 e^{-t}\sin t)\, u(t)$$

$$+ (C_2 - C_1)\delta(t) + C_1 \delta'(t)$$

$$h''(t) + 2h'(t) + 2h(t) = \delta'(t) + 4\delta(t)$$

$$C_1 = 1 \qquad C_2 - C_1 + 2C_1 = 4$$

$$C_2 = 3$$

$$h(t) = (e^{-t}\cos t + 3e^{-t}\sin t)\, u(t)$$

5. $C_n = \frac{1}{2}\int_{-1}^{1} x(t)\, e^{-jn\pi t}\, dt$

$= \frac{1}{2}\int_{-1}^{0} -e^{-jn\pi t} dt + \frac{1}{2}\int_{0}^{1} e^{-jn\pi t} dt$

$= \frac{1}{2}\left(\frac{1-e^{jn\pi}}{jn\pi} + \frac{e^{-jn\pi}-1}{-jn\pi} \right)$

$= \frac{1}{jn\pi}\left(1 - \frac{1}{2}\left(e^{jn\pi} + e^{-jn\pi}\right)\right)$

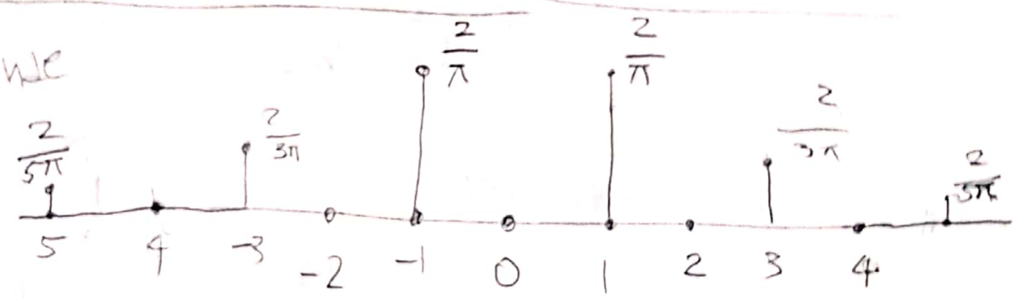
$= \begin{cases} -j\frac{2}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$

$|C_n| = \begin{cases} \frac{2}{|n\pi|} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$

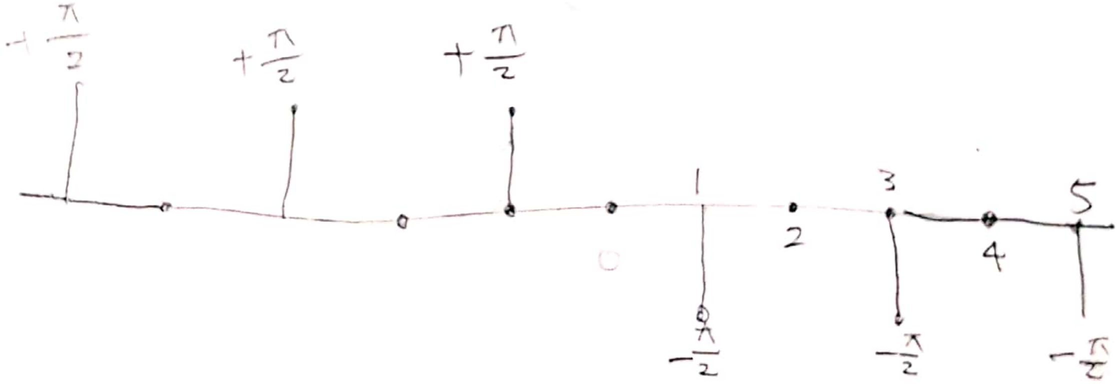
$m = 1, 2, 3$

$\measuredangle C_n = \begin{cases} -\frac{\pi}{2} & n = 2m-1 \\ 0 & n = 2m-2 \\ \frac{\pi}{2} & n = -(2m-1) \end{cases}$

magnitude



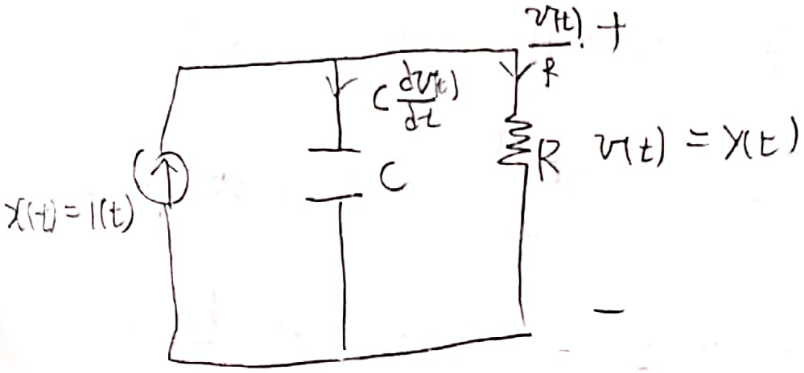
Phase



(b) $\hat{x}_N(t)$

(c) $e_N(t) = x(t) - \hat{x}(t)$

6.



$R=1$

$C=1$

a) $i(t) = C\frac{dv(t)}{dt} + \frac{v(t)}{R} \rightarrow i(t) = \frac{dv(t)}{dt} + v(t)$ (by KCL

b) let $i(t) = e^{j\omega t}$ then $v(t) = H(\omega)e^{j\omega t}$

$$e^{j\omega t} = j\omega H(\omega)e^{j\omega t} + H(\omega)e^{j\omega t}$$

$$H(\omega) = \frac{1}{1+j\omega}$$

c) Because when ω become larger, $H(\omega)$ become smaller. So this is LPF

$$|H(\omega)| = \left|\frac{1}{1+j\omega}\right| = \frac{1}{1+\omega^2}$$

d) let $i(t) = \delta(t)$ then

$$\delta(t) = h'(t) + h(t)$$

$$h_h(t) = C_1 e^{-t}u(t)$$

$$M<N \rightarrow h_p = 0 \rightarrow h(t) = h_h(t)$$

$$\delta(t) = -C_1e^{-t}u(t) + C_1e^{-t}\delta(t) + C_1e^{-t}u(t)$$

$$\therefore C_1 = 1 \quad h(t) = e^{-t}u(t)$$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty}(4\cos\tau + \cos 2\tau)e^{-(t-\tau)}u(t-\tau)d\tau$$

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$$y(t) = e^{-t}\int_{-\infty}^{t} 4\cos\tau\, e^{\tau} d\tau + e^{-t}\int_{-\infty}^{t} \cos 2\tau\, e^{\tau} d\tau$$

$$= e^{-t}\left[2e^{\tau}(\sin\tau + \cos\tau)\right]_{-\infty}^{t} + e^{-t}\left[\frac{1}{5}e^{\tau}(2\sin 2\tau + \cos 2\tau)\right]_{-\infty}^{t}$$

$$= 2\sin t + 2\cos t + \frac{2}{5}\sin 2t + \frac{1}{5}\cos 2t$$

$u(t)$?