Project3: Support Vector Machine

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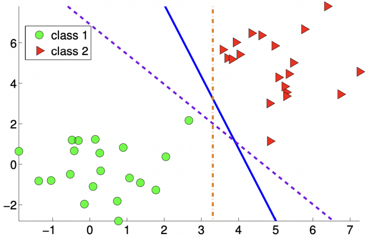
Presentation outline

Motivation

Support Vector Machine (SVM) is a discriminant algorithm used to classify data points in different classes. In our case, we will use it in case of sentiment analysis. In other words, we will use SVM to classify whether a sentiment related to a movie is positive or negative based on reviews that we have in our dataset.

Problem



Details

- Find the right support vector
- Compute the margin using the supports
- classify each data point

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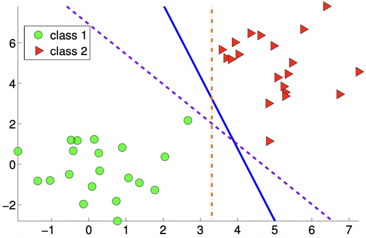

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Objective

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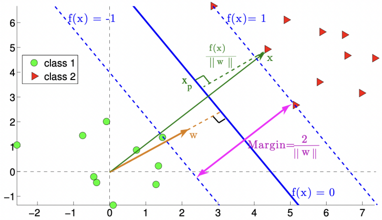
Implementing the optimized form of Support Vector Machine (SVM).

Methodology

Let $\mathcal{D} = \{(x_i, y_i) \in \mathcal{X} \times \{-1, 1\}\}$ the set of labeled points



Details

- $f(x) = w^T.x + b$
- The margin is equal to $M = \frac{2}{\|w\|}$

Methodology

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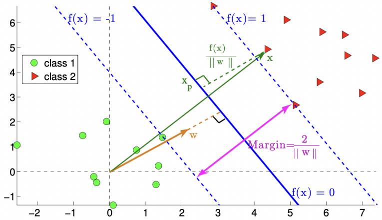

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Methodology

We are going to maximize the margin $M = \frac{2}{||w||}$ to be large as possible.

Primal problem

$$\begin{cases} min_{w,b}\frac{1}{2}||w||^2 \\ s.t\ y_i(w^Tx_i + b) \geq 1\ \forall i \end{cases} \quad (Primal\ \ problem) \tag{2.1}$$

Let derive the dual problem from (2.1)
The Lagrangian associated to the primal problem (2.1) is given by:

Lagrangian

$$L(w, b, \alpha) = \frac{1}{2}||w||^2 - \sum_{i}^{n} \alpha_i \left[y_i(w^Tx_i + b) - 1 \right] \tag{2.2}$$

Methodology

Partial derivative

$$\frac{\partial L}{\partial w} = w - \sum_{i}^{n} \alpha_i y_i x_i \tag{2.3}$$

$$\frac{\partial L}{\partial b} = -\sum_{i}^{n} \alpha_i y_i \tag{2.4}$$

Solve $\partial L = 0$

$$\partial L = 0 \implies \begin{cases} w = \sum_i^n \alpha_i y_i x_i \ (a) \\ \sum_i^n \alpha_i y_i = 0 \ (b) \end{cases}$$

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Methodology

Dual problem

(b) - (a) in 2.2 We have

$$L = -\frac{1}{2}\sum_{i}^{n}\sum_{j}^{n}\alpha_j y_j x_j^T y_i x_i \alpha_i + \sum_{i}^{n}\alpha_i \qquad (2.5)$$

Dual problem

Let $Q = (Q_{ij})$ where $Q_{ij} = y_j y_i x_j^T x_i$

$$\begin{cases} L = -\frac{1}{2}\alpha^T Q\alpha + 1^T\alpha \\ s.t\ y^T\alpha = 0 \ and \ \alpha \geq 0 \end{cases} \qquad (2.6)$$

Methodology

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Methodology

Dual problem

So to find w and b we need first to find the value of α, it should be the solution of the optimization problem below.

$$\begin{cases} min_{\alpha}\frac{1}{2}\alpha^{T}Q\alpha - 1^{T}\alpha \\ s.t\ y^{T}\alpha = 0\ and\ \alpha \geq 0\ (Dual\ \ problem) \end{cases} \tag{2.7}$$

Methodology

Lets calculate b.

Compute b

$$y_s(wx_s + b) = 1 \quad (2.8)$$

By substituting (a) in (2.8) :

$$y_s(\sum_{m\in S} \alpha_m y_m x_m . x_s + b) = 1 \quad (2.9)$$

Methodology

compute b

$$y_s^2\Big(\sum_{m\in S} \alpha_m y_m x_m . x_s + b\Big) = y_s \tag{2.10}$$

with $y_s^2 = 1$

$$b = \frac{1}{N_s} \sum_{s\in S}\Big(y_s - \sum_{m\in S} \alpha_m y_m x_m . x_s\Big) \tag{2.11}$$

Results and discussion

After testing our model on the test data, we obtained an accuracy of 0.83This seems to be a good result considering the amount of data we had at our disposal.
The link of the github repository :
https://github.com/mohammedelabbas/SVM.gits

Conclusion

Finally, we can say that the SVM algorithm is efficient for this kind of task. We also saw how we could transform our initial primal problem to a dual problem, which allowed us to approach the problem in a better way and thus use the cvxopt package to help us solve the quadratic problem that was posed to us with the dual form of the SVM.

References

Jurasfky, Daniel and Martin, James H, "An introduction to natural language processing, computational linguistics, and speech recognition", 2000, Pearson Education, Inc

THANKS FOR YOUR

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ATTENTION !!!