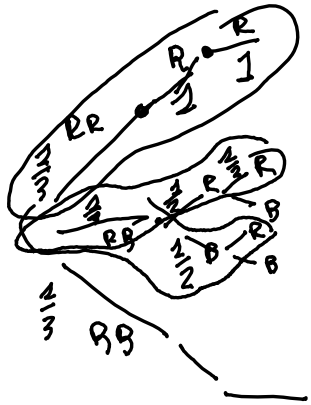
2022-06-07

1) $\boxed{RR}$ $\boxed{RB}$ $\boxed{BB}$



2) prima pallina R

$\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{2}$ ✓

formula prob totale
||
cammino su albero

3) $R_1 \Rightarrow R_2$?

$P(R_2|R_1)\, P(R_1) = \underbrace{P(R_1 \cap R_2)}_{\text{cammino su albero}} \Rightarrow$

$\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{2}\, x = \frac{5}{\cancel{12}\ 6}$

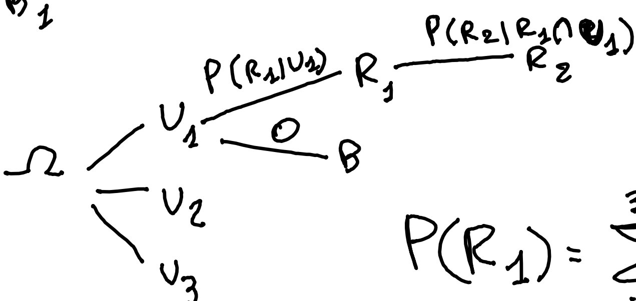
$$P(R_1|R_2) = \frac{P(R_2|R_1)P(R_1)}{P(R_2)}$$

$$P(R_2) = \frac{1}{3} + \left(\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}\right)\cancel{2} = \frac{1}{2}\ \left(\frac{2}{6}+\frac{1}{6}\right)$$

U_i = "si sceglie l'urna i"

R_i

B_1



formula prob totale

$$P(R_1) = \sum_{i=0}^{3} P(U_i \cap R_1) =$$
$$= \sum_{i=0}^{3} P(R_1|U_i)\,P(U_i)$$

$$P(R_1 \cap R_2) = \sum P(U_i \cap R_1 \cap R_2) =$$
$$= P(R_2|R_1 \cap U_i)\cdot P(R_1|U_i)\,P(U_i)$$

$$P(R_2) = \sum P(U_i \cap R_1 \cap R_2) + \sum P(U_i \cap B_1 \cap R_2)$$

2) dado non truccato (4 facce) lanciato due volte

X = prodotto di valori apparsi nei due lanci

Y = valore massimo che appare nei due lanci

$S_X = \{1, 2, 3, 4, 6, 8, 9, 12, 16\}$

$(x,x) = \frac{1}{4^2}$

X	1	2	3	4	6	8	9	12	16
P_X	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

$P(X>6) = \cancel{\frac{6}{16}}\ \frac{3}{8}$

$S_Y = \{1, \dots 4\}$

$X \backslash Y$	1	2	3	4
1	$\frac{1}{16}$	0		
2	0	$\frac{2}{16}$		
3			$\frac{2}{16}$	
4				$\frac{2}{16}$
6				
8				$\frac{2}{16}$
9				
12				$\frac{2}{16}$
16				$\frac{1}{16}$

		X	Y
1	1	~~1~~	~~1~~
1	2	~~2~~	2
1	3	~~3~~	~~3~~
1	4	~~4~~	~~4~~
2	1	~~2~~	~~2~~
2	2	4	2
2	3	6	3
2	4	8	4
3	1	~~3~~	~~3~~
3	2	6	3
3	3	9	3
3	4	~~12~~	~~4~~
4	1	~~4~~	~~4~~
4	2	8	4
4	3	~~12~~	~~4~~
4	4	~~16~~	~~4~~

3) $f_X(x) = \alpha e^{-|x|} = \begin{cases} \alpha e^{x} & x \leq 0 \\ \alpha e^{-x} & x \geq 0 \end{cases}$

affinché f_X sia effettivamente una densità =>

$$\int_{-\infty}^{0} \alpha e^{x} + \int_{0}^{+\infty} \alpha e^{-x} = 1$$

$$\alpha [e^{x}]_{-\infty}^{0} - \alpha [e^{-x}]_{0}^{+\infty} = 1$$

$$\alpha [1 - \cancel{\tfrac{1}{e^{\infty}}}] - \alpha [\cancel{\tfrac{1}{e^{\infty}}} - 1] = 1$$

$$\alpha + \alpha = 1 \Rightarrow \boxed{\alpha = \tfrac{1}{2}}$$

- integrali =>
  formule immediate base (anche delle derivate)
  - derivate diverse
  - metodo parti
- formalizzazione diagramma ad albero
- rivedersi appunti

$$\int f_X = F_X \Rightarrow \begin{cases} \int_{-\infty}^{x} \frac{1}{2} e^{y} dy & x \leq 0 \\ \int_{-\infty}^{0} \frac{1}{2} e^{y} dy + \int_{0}^{x} \frac{1}{2} e^{-y} dy & x \geq 0 \end{cases}$$

$D(k \cdot f(x)) = k \cdot D f(x)$

$$= \begin{cases} \frac{1}{2} [e^{y}]_{-\infty}^{x} = \frac{1}{2} [e^{x} - \cancel{\tfrac{1}{e^{\infty}}}] = \frac{1}{2} e^{x} & x < 0 \\ \frac{1}{2} [1 - \cancel{\tfrac{1}{e^{\infty}}}] - \frac{1}{2} \underbrace{[e^{-y}]_{0}^{x}}_{[e^{-x} - 1]} \quad \cancel{\tfrac{1}{2}} - \frac{1}{2} e^{-x} + \cancel{\tfrac{1}{2}} + 1 & x \geq 0 \end{cases}$$

$$P(-1 < X < 1) = F_X(1) - F_X(-1) =$$

$$= \left(-\frac{1}{2} e^{-1} \right) - \left(-\frac{1}{2} e \right)$$

$$-\frac{1}{2e} + \frac{1}{2} e$$

$$E[X] = \int_{-\infty}^{+\infty} x f_X(x)\, dx = \int_{-\infty}^{0} x \cdot \frac{1}{2} e^{x} dx + \int$$

$Y = X^3$ => sua densità?

$F_Y(y)$ $P(X^3 \leq y) = P(X \leq \sqrt[3]{y}) = F_X(\sqrt[3]{y}) \Rightarrow$

rivedersele!

derivata di funzione composta!

$$\boxed{f'(g(x)) = g'(x) \cdot f'(g(x))}$$

$$D(F_X(\sqrt[3]{y})) =$$

$$\boxed{\begin{aligned} &(\sqrt[3]{y})' F_X'(\sqrt[3]{y}) = \\ &= (y^{\frac{1}{3}})' f_X(\sqrt[3]{y}) \end{aligned}} =$$

$$\boxed{= \frac{1}{3} \cdot \underbrace{y^{\frac{1}{3}-1}}_{\frac{1}{\sqrt[3]{y^2}}} \cdot f_X(\sqrt[3]{y}) = f_Y(y)}$$

4) $\boxed{RR \quad BB}$

estratta blu => NO reimmessa

——— rossa => reimmessa altra pallina blu

Lucia vince 4b   Stefano vince 0 b

X = numero di palline blu

S

$S = \{0, 1, 2, 3, 4\}$

$X_0 = 2$

$\cancel{X}$	0	1	2	3	4
P_X	0	0	1	0	0

$$
\begin{array}{c|ccccc}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 0 & 0 & 0 \\
1 & \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 \\
2 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\
3 & 0 & 0 & \frac{3}{5} & 0 & \frac{2}{5} \\
4 & 0 & 0 & 0 & 0 & 1
\end{array}
$$

gioco finisce => rimane in quello stato lì

$P(X_{n+1} = 2 \mid X_n = 2) =$

numero di palline rosse (2) è costante

B — −1

R — +1

$X_3 = j \Rightarrow \tilde{\pi}^3_{2j}$

$\tilde{\pi}^3_{20}$
$\tilde{\pi}^3_{21}$
$\vdots$

$$\boxed{P(X_3 = 4)} = \sum_{i=0}^{4} P_{X_0}(i) \cdot \tilde{\pi}^3_{i4} =$$

$$\boxed{= P_{X_0}(2) \cdot \tilde{\pi}^3_{24}}$$

2022-06-20

1)

$D_{90,6}$ $\{(x_1, \dots x_6) \mid x_i \in [1, \dots 90], x_i \neq x_j \forall j \neq i$

2) Prob tutti pari

~~45 · 44 · 43~~

$$\frac{D_{45,6}}{D_{90,6}}$$ $\underbrace{2\ 4\ 6 \dots 24}_{12} \dots 90$

3)

$$P(\leq 24 \mid \text{pari}) = \frac{P(\text{pari} \cap \leq 24)}{P(\text{pari})} = \frac{D_{12,6}}{\cancel{D_{90,6}}} \cdot \frac{\cancel{D_{90,6}}}{D_{45,6}}$$

4) 5 dei sei numeri estratti siano strettamente > 80 e l'altro numero < 25

$$\frac{\binom{6}{1} D_{90-80,5} D_{24,1}}{D_{90,6}} = \frac{D_{10,5} \cdot 24}{D_{90,6}}$$

81 83 23 85 86 88

$$\frac{\overbrace{6 \cdot \cdot 5!}^{6!} \cdot \binom{10}{5} \cdot 24}{6! \cdot \binom{90}{6}}$$

2)

X	Y	2X-Y
0	0	0
0	3	-3
0	4	-4
0	5	-5
2	0	4
2	3	4-3
2	4	4-4
2	5	4-5
6	0	12
6	3	12-3
6	4	12-4
6	5	12-5

no indipendenti

$P_{(X,Y)}(0,0) \neq P_X 0 \cdot P_Y 0$

$$\boxed{P(2X-Y=0) = P_{(X,Y)}(0,0) + P_{(X,Y)}(2,4)}$$

Z

P_Z

$S_Z = \{0, -3, -4, -5, 4, 1, -1$

$$E[X^2Y^3] = \sum_{ij} x_i^2 y_j^3 \, p_{XY}(x_i, y_j)$$

$0 \cdot \ldots + 2^2 \cdot 3^3 \cdot 0 + \ldots$

3)

$$F_X(x) = \boxed{\frac{1}{1+e^{-x}}} = (1+e^{-x})^{-1}$$

$$x^2 \quad 2x^{2-1}$$

$$P(X \geq 0) = 1 - F_X(0) = 1 - \frac{1}{1+1} = \frac{1}{2}$$

$$f_X(x) = F_X'(x) =$$

$$= -(1+e^{-x})^{-2} \cdot e^{-x} \cdot -1 =$$

$$= \boxed{\frac{e^{-x}}{(1+e^{-x})^2}}$$

$$\int 2F \cdot F' = 2\left[\frac{F()^2}{2}\right]$$

$$E\left[\frac{2}{1+e^{-\cancel{x}}}\right] = \int_{-\infty}^{+\infty} \frac{2}{1+e^{-x}} \cdot \frac{e^{-x}}{(1+e^{-x})^2} =$$

$$= \int \frac{-2 \cdot (1+e^{-x}) \cdot e^{-x}}{(1+e^{-x})^2}$$

$$= \left[\ln\left|(1+e^{-x})^2\right|\right]_{-\infty}^{+\infty} =$$

$$= -\left(\ln\left|\left(1+\frac{1}{e^{\infty}}\right)^2\right| - \ln\left|(1+e^{\infty})^2\right|\right)$$

$$= \ln|(\infty) - ..$$

SEMPRE ricondursi a valori che hai già ottenuto

$$Z = 4X + 7 \Rightarrow$$

$$F_Z(\cancel{x}) = \mathbb{P}(4X+7 \leq \cancel{z}) = \mathbb{P}\left(X \leq \frac{\cancel{z}-7}{4}\right) =$$

$$F_X\left(\frac{\cancel{z}-7}{4}\right) =$$ scrivere la F_Z in termini di $F_{\cancel{Z}}$

meglio usare la x

$$f_Z(x) = F_Z'(x) = \frac{1}{4} F_X'\left(\frac{x-7}{4}\right) = \frac{1}{4} f_X\left(\frac{x-7}{4}\right) = \{$$ sostituisce $\frac{x-7}{4}$ al posto di x

già trovata cazzo!

$$F_Y(x) = \frac{a}{3+e^{-x}}$$

per far sì che sia una funz di ripartizione ⇒

$$\lim_{x \to +\infty} \frac{a}{3+e^{-x}} = \frac{a}{3+\cancel{e^{-\infty}}} = 1 \Rightarrow a = 3$$

$$\lim_{x \to -\infty} = 0$$

monotona crescente + continua (non solo a destra)

4) ad ogni giocata

smette quando 0 o 6

$P(+1) = \frac{3}{4}$ $\quad P(-1) = \frac{1}{4}$

$S = \{0, \dots, 6\}$

	0	1	2	3	4	5	6
0	1						
1	$\frac{1}{4}$		$\frac{3}{4}$				
2		$\frac{1}{4}$		$\frac{3}{4}$			
3			$\frac{1}{4}$		$\frac{3}{4}$		
4				$\frac{1}{4}$		$\frac{3}{4}$	
5					$\frac{1}{4}$		$\frac{3}{4}$
6							1

$$P(X_{n+2} = 5 \mid X_n = 3) = \pi^{(2)}_{3,5}$$

distribuzioni invarianti

$$\overline{\pi}^T = \overline{\pi}^T \Pi \Rightarrow$$

$$\pi_0 = 1\pi_0 + \frac{1}{4}\pi_1$$

$$\pi_1 = \frac{1}{4}\pi_2$$

$\vdots$

$$\pi_0 + \pi_1 + \dots \pi_6 = 1$$

$\Downarrow$

$$\pi_6 = \pi_6$$

$$\pi_0 + \pi_6 = 1$$

$$\Rightarrow p := \pi_0 \Rightarrow$$

$$(p, 0, 0, \dots 1-p)$$

2022-07-11

B/R R/R

1) entrambi lati rossi = 1/2

2) guardando un lato prob che sia rosso

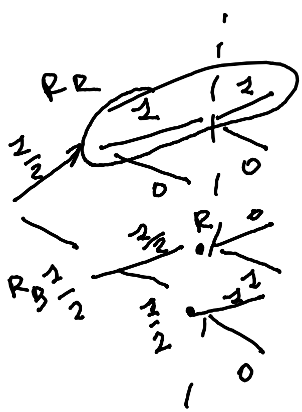
$$\boxed{P(R_1) = P(A \cap R_1) + P(A^c \cap R_1)} = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2}$$

cammini su albero

$$P(R_2 | R_1) = \frac{\overset{1/2}{P(R_1 \cap R_2)}}{\underset{1/2 + \frac{1}{4}}{P(R_1)}} = \cancel{\frac{1}{2}} \cdot \frac{\cancel{4}\,2}{3} = \frac{2}{3}$$

RR = A

A^c = RB



2) $S_X = \{0, \lambda^2\}$

$$\boxed{P_X(0) = \frac{2}{\lambda} \qquad P_X(\lambda^2) = 1 - \frac{2}{\lambda}}$$

sono prob => devono essere comprese fra 0 e 1 !

$$\cancel{\frac{2}{\lambda}} + 1 - \cancel{\frac{2}{\lambda}} = 1 \Rightarrow \forall$$

$$0 \le \frac{2}{\lambda} \le 1 \qquad 2 \le \lambda$$

$$0 \le 1 - \frac{2}{\lambda} \le 1$$

$$\frac{2}{\lambda} \le 1 \qquad \boxed{2 \le \lambda}$$

$$0 \cancel{1} \le \cancel{1} + \frac{2}{\lambda}$$

$$E[X] = 0 \cdot \frac{2}{\lambda} + \lambda^2 \cdot \left(1 - \frac{2}{\lambda}\right) = \boxed{\lambda^2 - 2\lambda}$$

$$\text{Var}(X) = E[X^2] - E[X]^2 =$$

$$E[X^2] = 0 \cdots + \lambda^4\left(1 - \frac{2}{\lambda}\right) =$$

$$= \lambda^4 - 2\lambda^3$$

$$E[X]^2 = (\lambda^2 - 2\lambda)^2 = \lambda^4 - 4\lambda^3 + 4\lambda^2$$

$$\cancel{\lambda^4} - \cancel{2\lambda^3} - \cancel{\lambda^4} + \cancel{4\lambda^3} - 4\lambda^2$$

$$2\lambda^3 - 4\lambda^2$$

$$2\lambda^2(\lambda - 2)$$

Y stessa legge =>

	0	λ^2	
0			$\frac{2}{\lambda}$
λ^2		$\left(1-\frac{2}{\lambda}\right)^2$	$1 - \frac{2}{\lambda}$
	$\frac{2}{\lambda}$	$1 - \frac{2}{\lambda}$	

$$E[XY] = 0 \cdot 0 + 0 \cdot \lambda + \cdots \lambda^2 \cdot 0 + \cdots + \lambda^4 \cdot \left(1 - \frac{2}{\lambda}\right)^2 =$$

$$= \lambda^4 \cdot \left(1 - \frac{4}{\lambda} + \frac{4}{\lambda^2}\right) \qquad \lambda^4\left(1 - \frac{2}{\lambda}\right)\left(1 - \frac{2}{\lambda}\right)$$

$$(\lambda^4 - 2\lambda^3)\left(1 - \frac{2}{\lambda}\right)$$

$$\lambda^4\left(1 - \frac{2}{\lambda}\right)^2$$

$$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) + 2\underbrace{\text{Cov}(X,Y)}_{0}$$

$$\lambda^4 - 2\lambda^3$$

$$-2\lambda^3 + 4\lambda^2$$

$$\lambda^4\left(1 - \frac{4}{\lambda} + \frac{4}{\lambda^2}\right)$$

$$\lambda^4 - 4\lambda^3 + 4\lambda^2$$

$$\lambda^2(\lambda^2 - 4\lambda + 4)$$

$$\lambda^2(\lambda - 2)^2$$

$$f_X(x) = \begin{cases} \frac{2}{3}\left(1-\frac{x}{3}\right) & 0<x<3 \\ 0 & \text{else} \end{cases}$$

$$F_X = \int_0^{x} \frac{2}{3}\left(1-\frac{y}{3}\right)dy$$

x ~~t~~ USA t, j)

$P(X \geq 1) = 1 - F_X(1) = 1 - \left(\frac{1}{3}\underbrace{\left(\frac{6}{3}-\frac{1}{3}\right)}_{\frac{5}{3}}\right)$

$$F_X(t) = \int_{-\infty}^{t} f_X(x)\,dx = \begin{cases} 0 & t \leq 0 \\ \star & 0 \leq t < 3 \\ 1 & t \geq 3 \end{cases}$$

$$\frac{2}{3}\int_0^x 1 - \frac{y}{3} =$$

$$\frac{2}{3}\left(\int_0^x 1 - \frac{1}{3}\int_0^x y\right) =$$

$$= \frac{2}{3}\left(\cancel{x} - \frac{1}{3}\left[\frac{y^2}{2}\right]_0^x\right)$$

$$\frac{2}{3}\left(x - \frac{1}{3}\left[\frac{x^2}{2}\right]\right) = \frac{2}{3}\left(x - \frac{1}{6}x^2\right) = \frac{2}{3}x\left(1 - \frac{1}{6}x\right)$$

$$\frac{2}{3}x - \frac{\cancel{2x^2}}{\cancel{18}} \qquad \boxed{\frac{t}{3}\left(2 - \frac{t}{3}\right)} \ \star$$

$E[X^p]$, $\forall p \geq 1$

in altri intervalli è pari a 0!

$$E[X^p] = \int_{-\infty}^{+\infty} x^p\left(\frac{2}{3}\cdot\left(1-\frac{x}{3}\right)\right) = \frac{2}{3}\int_0^3 x^p\left(1-\frac{x}{3}\right) =$$

$$\left[\frac{x^{p+1}}{p+1}\left(1-\frac{x}{3}\right)\right]_0^3 + \frac{1}{3(p+1)}\int_0^3 x^{p+1}$$

$$\frac{3^{p+1}}{4}\left(\cancel{1-\frac{3}{3}}\right) - \qquad 0$$

$$= \frac{2}{3}\cdot\frac{1}{3(p+1)}\left[\frac{x^{p+2}}{p+2}\right]_0^3 =$$

$$= \frac{2 \cdot 3^{p+2}}{9(p+1)(p+2)} = \frac{2\cdot\cancel{3^2}\cdot 3^p}{\cancel{9}\cdots}$$

$$F_Y(x) = P(3 - X \leq x) = P_X(X \geq 3-x) =$$

$$1 - F_X(3-x)$$

(sostituisci trasforma nella formula, non sui vincoli)

$1 - F_X$ vincoli

$\Downarrow$

espandi

4) pedina su tavola con cinque caselle 0, .. 4

per i movimenti 4 mon NON truccato

facce = 0, 4

0 X

4 ✓

1 Start => $X_0^* = 1$

x_0	0	1	2	3	4
P_{x_0}	0	1	0	0	0

pedina k = k + 1 lanci

tutti 0 => 0

$\overline{\text{else}}$ $4 \longrightarrow +1$

attuale

$S = \{0, \dots, 4\}$

caselle > lanci

tutti: 0 =>

0

	0	1	2	3	4
0	$\frac{1}{}$	–	–	–	
1	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$		
2	$\frac{1}{8}$		$\frac{6}{8}$	$\frac{1}{8}$	
3	$\frac{1}{16}$			$\frac{14}{16}$	$\frac{1}{16}$
4					1

$$\overrightarrow{P_{x_1}} = \overrightarrow{P_{x_0}} \cdot \overline{\Pi} = \frac{1}{4}, \frac{2}{4}, \frac{1}{4}, 0, 0$$

1 riga

2022-09-19

1) fornito insieme perm $1 \dots n$

A_i = "la permutazione ha come punto fisso i"

• prob che l'iesima pallina estratta sia la numero i?

$$\frac{D_{n-1,n-1}}{D_{n,n}} = \frac{(n-1)!}{n!} = \frac{1}{n}$$

• iesima estratta sia la i e la jesima estratta sia la j, con $i \neq j$

$$P(A_j \cap A_i) = \frac{D_{n-2,n-2}}{D_{n,n}} = \frac{(n-2)!}{n!} = \frac{1}{n \cdot (n-1)}$$

NON PARLA DI "SAPENDO CHE" ⇓ NO COND

ind $\Longleftrightarrow$ $P(A \cap B) = \underbrace{P(A_j)}_{\frac{1}{n}} \cdot \underbrace{P(A_i)}_{\frac{1}{n}} \Rightarrow$ ✗

def di ind

2) no ind $P_{(X,Y)}(0,0) \neq P_X(0) P_Y(0)$

$$E[XY] = \sum_{\substack{i = 0,3,7 \\ j = 0,2,4,6}} i \cdot j \, P_{(X,Y)}(i,j)$$

$$Cov(X,Y) = E[X \cdot Y] - E[X] E[Y]$$

$Z = XY$ => density?

~~0 0~~		~~0~~
~~0 2~~		~~0~~
~~0 4~~		~~0~~
~~0 6~~		~~0~~
~~3 0~~		~~0~~
~~3 2~~		~~6~~	$p = 0$ => X
3 4		12
3 6		18
~~7 0~~		~~0~~
7 2		14
7 4		28
7 6		42

Z	0	
P_Z	0.6	

3) $f_X(x) = \begin{cases} 0 & x<0 \\ 2e^{-2x} & x \geq 0 \end{cases}$

$$F_X(x) = \int_{-\infty}^{x} f_X(t)\,dt = \int_0^x 2e^{-2t}\,dt =$$

$S_Z = \left[\frac{1}{2}, 1\right)$ $\Rightarrow$

$$= -\int^x -2e^{-2t}\,dt$$

$$= -\left[e^{-2t}\right]_0^x =$$

$F_X(x) = \begin{cases} 0 & x \leq 0 \\ \star & x \geq 0 \end{cases}$ $= -\left(e^{-2x} - 1\right) \star$

$1 - e^{-2x}$

$$E[Z] = \int_0^{+\infty} \frac{1}{1+e^{-2x}} \cdot \cancel{2} e^{-2x} = \int_0^{+\infty} \frac{-2e^{-2x}}{1+e^{-2x}} =$$

$$-\left[\ln|1+e^{-2x}|\right]_0^{+\infty} = -(\underbrace{\ln|1+\cancel{e^{-\infty}}|}_{0} - \underbrace{\ln|1+e^0}_{}$$

$$-(-\ln 2) = \boxed{\ln 2}$$

$F_Z(\cancel{t}) = P(Z \leq \cancel{t}) =$

$F_X(t) = P(X \leq t) = P\left(\frac{1}{1+e^{-2x}} \leq \cancel{t}\right)$

$P\left(\frac{1}{\cancel{t}} \leq (1+e^{-2x})\right)$

$\frac{1}{t} - 1 = e^{-2x}$

t non può essere 0 qui

$$\boxed{\frac{-\ln\left(\frac{1}{t}-1\right)}{2} = \cancel{2}x \leq}$$

$$\frac{1}{t} \leq 1 + e^{-2x}$$

$$\frac{1}{t} - 1 \leq e^{-2x}$$

$$\ln(\quad) \leq -2x$$

$S_Z = \left[\frac{1}{2}, 1\right)$ $\Rightarrow$

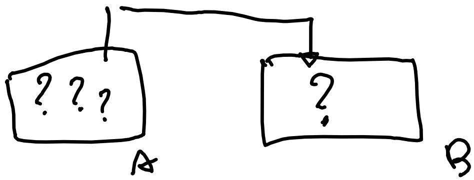
$$\boxed{-\frac{\ln(\,)}{2} \geq x}$$

$$x \leq -\frac{\ln(\,)}{2}$$

$F_Z = \begin{cases} 0 & t \leq \frac{1}{2} \\ 1 & t \geq 1 \end{cases}$

4) 4 pallini

2 urne, A e B



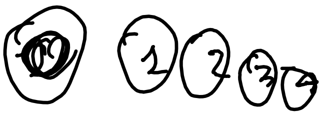
si sceglie un pallino a caso e lo si toglie da un'urna

X_n = numero di pallini presenti nell'urna A dopo l'n-esimo scambio

$X_0 = 3 \Rightarrow$

0	1	2	3	4
0	0	0	1	0

$S = \{0, \dots 4\}$



$$
\begin{array}{c|ccccc}
 & 0 & 1 & 2 & 3 & 4 \\
\hline
0 & 0 & 1 & 0 & 0 & 0 \\
1 & \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 \\
2 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\
3 & 0 & 0 & \frac{3}{4} & 0 & \frac{1}{4} \\
4 & 0 & 0 & 0 & 1 & 0
\end{array}
$$

unica distribuzione inv

$\vec{\pi} = \vec{\pi}\,\Pi$

$$
\begin{cases}
\tilde{\pi}_0 = \frac{1}{4}\pi_1 \\
\hat{\pi}_1 = \hat{\pi}_0 + \frac{1}{2}\hat{\pi}_2 \\
\vdots \\
\tilde{\pi}_0 + \tilde{\pi}_1 + \dots = 1
\end{cases}
$$

introdurre gli eventi

2023-01-10

1)

- $\left(\frac{90}{100}\right)^2$

- $\left(\frac{10}{100}\right)^2$

- ricevuto + =>

$\frac{10}{100} \cdot \frac{10}{100} + \left(\frac{90}{100}\right)^2$

$P(D) = P(A_1^c \cap A_2^c) + P(A_1 \cap A_2)$

- se il segnale viene ricevuto correttamente, prob che il primo canale lo ~~abbia~~ abbia trasmesso correttamente.

$\frac{\text{prob di cammini che portano + e attraversano A}}{\text{prob di tutti i cammini che portano a +}}$

$\left(\frac{9}{10}\right)^2 \cancel{/} \frac{1}{100} + \frac{81}{100} =$

$\frac{81}{\cancel{100}} \cdot \frac{\cancel{100}}{82} = \frac{81}{82}$

A = 1° canale trasmette correttamente

B = secondo canale trasmette correttamente

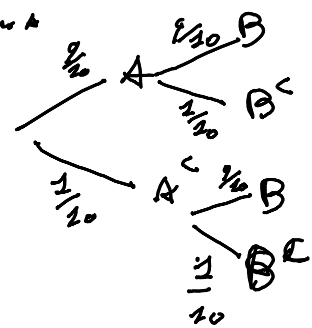
+ − +

− −

+ +

$P(A_1 | D) = \frac{P(A \cap D)}{P(D)}$

$A_1 \cap D = A_1 \cap A_2$



2) 2 dadi 4 facce

X, Y esiti ottenuti

$Z = |X - Y|$

$S_Z = \{0, 1, 2, 3, 4\}$

$X \quad Y$

$\frac{1}{16}$

$\frac{1}{4}$

$\frac{1}{4}$

$\frac{1}{4}$

$\frac{1}{4}$

~~1 1 0~~

1 4 3

~~2 2 0~~

~~3 3 0~~

4 1 3

~~4 4 0~~

Z	0	1	2	3
p_Z	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{2}{16}$

$\frac{1}{4}$

$\frac{1}{4}$

$\frac{1}{4}$

$\frac{1}{4}$

2 4 2

3 1 2

	0	1	2	3	
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
2	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	0	$\frac{1}{4}$
3	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	0	$\frac{1}{4}$
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{4}$
	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{2}{16}$	1

NO IND

$$E[X - 2Z] = E[X] - 2E[Z]$$

LINEARITÀ del valore atteso

livelli viaggio, livelli comunicazione (e.g. gestione dei pacchetti)

router controlli veloci

4)

$$P(1 < X \leq 7/4) = F_X(\tfrac{7}{4}) - F_X(1)$$

$$f_X(x) = F'(x) = \begin{cases} 0 & x < 0 \\ \frac{2}{3}\log(\frac{3}{2}) & 0 \leq x < \frac{3}{2} \\ \frac{1}{x} & \frac{3}{2} \leq x < 2 \\ (1-\log 2)e^{-(x-2)} & x \geq 2 \end{cases}$$

derivata prodotto funzioni?

dove abbiamo imposto arbitrariamente nei punti di disgiunzione...

$$S_Y := \{2, 5\}$$

$$P(Y=2) = P(X \leq 7/4) \underset{\text{def}}{=} F_X(\tfrac{7}{4}) = \log\tfrac{7}{4}$$

per calcolarsi la densità

$$P(Y=5) = 1 - F_X(\tfrac{7}{4})$$

Calcolo densità discreta applicando meramente definizione

$$Z = 3X + 4 \qquad S_Z = [4, +\infty)$$

$$F_Z(t) = P(3X+4 \leq t) = P\left(X \leq \frac{t-4}{3}\right) =$$

$$= F_X\left(\frac{t-4}{3}\right)$$

$$F_Z(x) = \begin{cases} 0 & \frac{x-4}{3} \leq 0 \\ \frac{2}{3}\log(\frac{3}{2})\,\frac{x-4}{3} & 0 \leq \frac{x-4}{3} \leq \frac{3}{2} \\ \end{cases} =$$

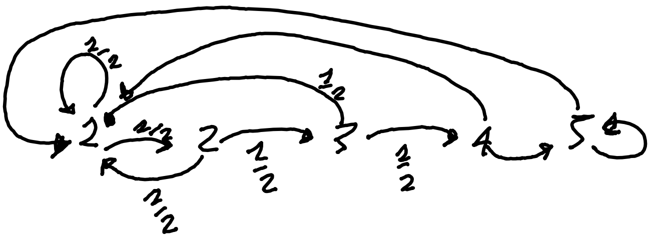
$$= \begin{cases} 0 & x \leq 4 \\ \frac{2}{9}\log\frac{3}{2}\,(x-4) & 4 \leq x \leq \frac{17}{2} \\ \vdots \end{cases}$$

1) 5 scalini

lancio moneta → T +1 (5° = resto)

lancio moneta → C 1

	1	2	3	4	5
1	$\frac{1}{2}$	$\frac{1}{2}$			
2	$\frac{1}{2}$		$\frac{1}{2}$		
3	$\frac{1}{2}$			$\frac{1}{2}$	
4	$\frac{1}{2}$				$\frac{1}{2}$
5	$\frac{1}{2}$				$\frac{1}{2}$



$\{1, 2, 3, 4, 5\}$ è irriducibile

$$\tilde\pi^{(3)}_{3,5} = \frac{1}{2^3}$$

$$\vec{\tilde\pi} = \tilde\pi\,\Pi = \begin{cases} \tilde\pi_1 = \frac{1}{2}\sum \tilde\pi_i \\ \tilde\pi_2 = \frac{1}{2}\tilde\pi_1 \\ \vdots \end{cases}$$

$$\sum \tilde\pi_i = 1$$

2023-01-24

1) 7 palline riposte casualmente in 3 scatole

$\{(x_1, \dots, x_7) \mid x_i \in \{1, 2, 3\}$

$|\Omega| = 3^7$

2) prob che le palle vengano riposte tutte in una stessa scatola

$P(1 \dots 1) + P(2, \dots 2) + P(3, \dots 3) = \frac{3}{3^7} = \frac{1}{3^6}$

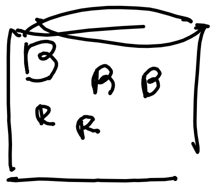
3) prob che due palle siano nella scatola 1 e tre palle siano nella scatola 2

$\boxed{\frac{\binom{5}{3}\binom{7}{2}}{3^7}}$ $\underbrace{1\ 1\ 2\ 2\ 2\ 3\ 3}_{\text{prob} = \frac{1}{3^7}}$

4) prob che le palle di indice dispari siano nella scatola 2

$|\{(2\ x\ 2\ x\ 2\ x\ 2)\}| = 3^3 \Rightarrow \frac{3^3}{3^7} = \frac{1}{3^4}$

2)



3 B
2 R

T — 2 sim
C — 3 sim

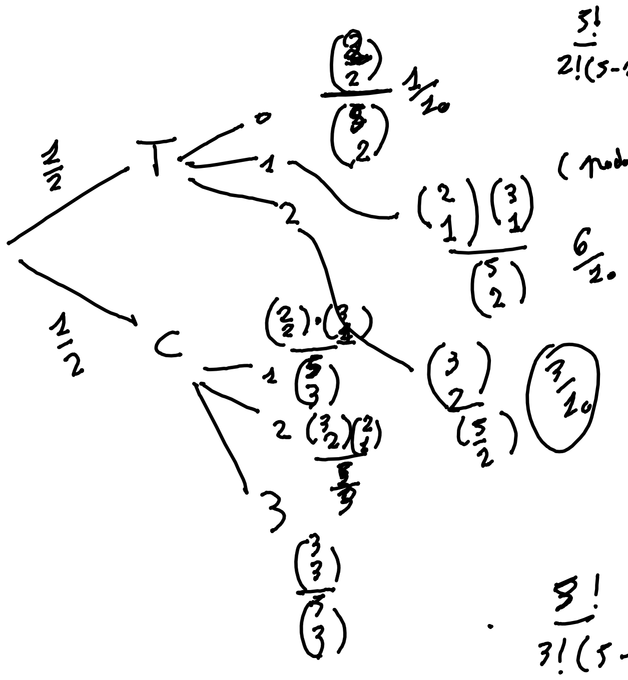
X = "T => 2, C => 3"

Y = "n° palline **blu** estratte dall'urna" # B

X	2	3
P_X	$\frac{1}{2}$	$\frac{1}{2}$

Y	0	1	2	3
P_Y				

$\frac{5!}{2!(5-2)!} = \frac{5!}{2!\,3!}$ $\quad 5 \cdot \frac{4}{2} = 10$



(prodotto composizione tra sottoinsiemi)

$\frac{5!}{3!(5-3)!} \quad 5 \cdot \frac{4}{2!} = 10$

$\frac{3}{10} + \frac{6}{10} + \frac{1}{10} =$

$$\boxed{\begin{array}{cccc} 0 & 1 & 2 & 3 \\ \frac{1}{20} & \frac{9}{20} & \frac{9}{20} & \frac{1}{20} \end{array}}$$

(Prob cammini su albero
$P(X \cap Y) = P(Y|X)P(X)$)

Y \ X	2	3	
0	$\frac{1}{20}$	0	$\frac{1}{20}$
1	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{9}{20}$
2	$\frac{3}{20}$	$\frac{6}{20}$	$\frac{9}{20}$
3	0	$\frac{1}{20}$	$\frac{1}{20}$
	$\frac{1}{2}$	$\frac{1}{2}$	

no ind

$E[X] = 2 \cdot \frac{1}{2} + 3 \cdot \frac{1}{2} = 1 + 1.5 = 2.5$

$$\boxed{P(Y = X) = \sum p(x,y)\, x_i\, y_j}$$

3) $F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x & 0 \le x \le 1 \\ 1 - \frac{1}{2x^2} & x \ge 1 \end{cases}$

$f_X = \begin{cases} 0 & x \le 0 \\ \frac{1}{2} & 0 < x \le 1 \\ \cancel{-\frac{1}{2}}(\cancel{+2}\frac{1}{x^3}) & x > 1 \end{cases}$

$\boxed{P(X \ge 2) =} 1 - F_X(2) = 1 - \left(1 - \frac{1}{2 \cdot 4}\right) =$

$\boxed{= \frac{1}{8}}$

$E[X] = \int_{-\infty}^{+\infty} x f_X = \int_0^1 \frac{1}{2}x + \int_1^{+\infty} \frac{x}{x^3}$

$\frac{1}{x^2}$

$x^n = \frac{x^{n+1}}{n+1}$ $\quad x^{-2} = \frac{x^{-2+1}}{-2+1}$

$S_X = [0, +\infty)$

$\frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 + \left[\frac{1}{x}\right]_1^{\infty} =$

$= \frac{1}{4} + 1 = \frac{5}{4}$ $\quad S_Y = (1, +\infty)$

$1 -$

$\boxed{F_Y(x) =} P(X + 1 < x) = P(X < x - 1) \boxed{= F_X(x-1)}$

$x > 1$

$\boxed{\cancel{P}(Y \le 3) =} F_Y(3) = F_X(3-1) = F_X(2) = 1 - \frac{1}{2 \cdot 2^2} =$

$1 - \frac{1}{8} \boxed{= \frac{7}{8}}$

$P(Y \le 3 \cap X > 1) \Rightarrow F_X(2) = \frac{7}{8} \cdot \frac{1}{2}$

$P(\underline{X+1} \le 3 \cap X > 1) = P(1 < X < 2)$

SOSTITUIRE LA Y!

$P(Y \le 3 \cap X > 1) =$

$\underbrace{P(Y \le 3 \mid X > 1)}_{\text{"sapendo che"...}} P(X > 1)$

$P(X + 1 \le 3 \mid X > 1)\, P(X > 1)$

$P(X \le 2 \mid X > 1)\, P(X > 1)$

$\frac{P(X \le 2 \cap X > 1)}{\cancel{P(X > 1)}} \cancel{P(X > 1)}$

1)

2021-09-14

1-90 + 5 estratti + 3 palline 3' 3'' 3'''

$\boxed{1\ 2\ 3\ 3'\ 3''\ 3'''\ 4\ 5\ \dots\ 90} \Rightarrow$ insieme combinazione semplici

$\{(x_1, x_2, \dots, x_5) \mid \sum \delta_3(\{x_i\}) \le 3 \land x_i \in \{1, \dots, 90\} \land (x_i \ne 3 \land x_j \ne 3 \Rightarrow x_i \ne x_j)\}$

$|\Omega| = \underbrace{93 \cdot 92 \cdot \ldots \cdot 89}_{} = D_{93,5}$

A = "viene estratto 2 volte il numero 3"

come dove quali altri

$$\frac{2! \cdot \binom{5}{2} \cdot \binom{3}{2} \cdot D_{93-4,3}}{|\Omega|}$$

quali delle tre palline

$\cancel{2!}\,\frac{5 \cdot 4}{\cancel{2!}(5-2)!} \quad 3!$

$5!$

P(almeno due volte) = $1 - P(0) - P(1)$

$P(0) = \frac{D_{93-4,5}}{|\Omega|}$ $\qquad P(1) = \frac{\binom{5}{1}\binom{4}{1} D_{93-4,4}}{|\Omega|}$

ordine

$$\frac{\binom{4}{1}\binom{2}{1} \cdot \boxed{D_{89,3}} \cdot \binom{3}{1}}{D_{93,5}}$$

quale prima pallina numero 3

$89,3$

$C_{89,3} \cdot 3!$

$A_{4}^{?} \cdot 3! =$

$4!$

$3!$

$\boxed{5!}$

27) ± 1 sulle facce di una moneta

X, Y ris ottenuti 2 lanci

	-1	$+1$
-1	$\frac{1}{4}$	$\frac{1}{4}$
$+1$	$\frac{1}{4}$	$\frac{1}{4}$

X	Y	S		D		T	
-1	-1	-2	$\frac{1}{4}$	0	$\frac{1}{2}$	1	$\frac{1}{2}$
-1	$+1$	0	$\frac{1}{2}$	-2	$\frac{1}{4}$	-1	$\frac{1}{2}$
$+1$	-1	0		2	$\frac{1}{4}$	-1	
$+1$	$+1$	2	$\frac{1}{4}$	0		1	

$$P_{S,T}(-2, 1) = \frac{1}{4} \neq P_S(-2)\, P_T(1)$$

$$P(X=-1 \cap T=1) = P(T=1 \mid X=-1)\, P(X=-1)$$

$$= P(XY=1 \mid X=-1)\, \frac{1}{2} =$$

$$= P(Y=-1)\, \frac{1}{2} = \frac{1}{4}$$

2) $\lim_{x \to -\infty} F(x) = 0$

$\lim_{x \to +\infty} 1 - a e^{-(x-2)} = 1 \Rightarrow$

$1 - \underbrace{a \cancel{e}^{-\infty+\cancel{2}}}_{0} = 1$

$\cancel{1} - \frac{1}{2} = \cancel{1} - a \underbrace{e^{-(2-2)}}_{1} \Rightarrow a = \frac{1}{2}$ così è continua e monotona

$P\left(\frac{3}{2} \leq x \leq 2\right) = F_X(2) - F_X\left(\frac{3}{2}\right)$

$$\begin{cases} 0 & x \leq 1 \\ 1 - \frac{1}{x} & 1 \leq x \leq 2 \\ 1 - \frac{1}{2} e^{-(x-2)} \end{cases} \Rightarrow$$

$\Downarrow$

$$f_X(x) = \begin{cases} 0 & x < 1 \\ -\frac{1}{x^2} & 1 \leq x \leq 2 \\ \frac{1}{2} e^{-x+2} & x > 2 \end{cases}$$

$S_X = \{1, \dots\}$

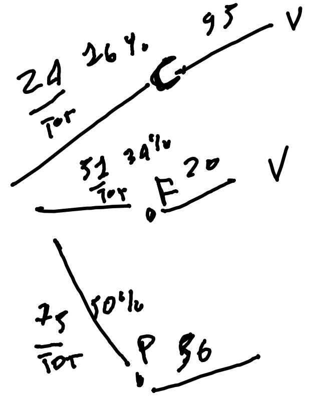
$$Y = \begin{cases} 0 & \text{se } x \leq 2 \\ 1 & x > 2 \end{cases} \Rightarrow \text{è una Bernoulliana} \Rightarrow$$

$\Rightarrow P(Y=1) = P(X>2) = 1 - F_X(2) = \frac{1}{2}$

$$F_Y = \begin{cases} 0 & y \leq 0 \\ 1-p & 0 \leq y \leq 1 \\ 1 & y \geq 1 \end{cases}$$

$\boxed{Y \sim B\left(\frac{1}{2}\right)}$

2021-07-13



Prob(verità) =

$$\frac{24}{TOT}\cdot\frac{95}{100}+\frac{51}{TOT}\cdot\frac{20}{100}+\dots$$

$\boxed{P(\text{peggior} \mid \text{verità}) =}$

è nota $P(P)$
$P(A|P)$,

$$= \frac{P(\text{peggior} \cap \text{verità})}{P(\text{verità})} = \boxed{\frac{P(\text{verità} \mid \text{peggior})\, P(\text{peggior})}{P(\text{verità})}}$$

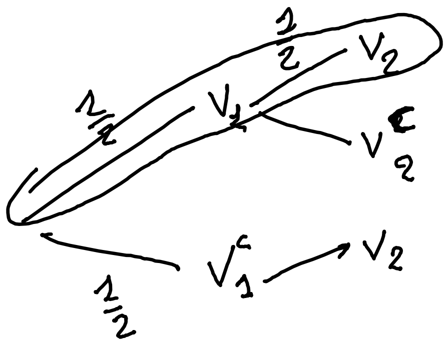
$$\frac{\text{Persone che dicono verità}}{TOT}\cdot\frac{\text{persone che dicono verità}-1}{TOT-1}$$

~~22~~ ~~4+21~~

$$\frac{\cancel{24}\,4}{\cancel{12}\,6}\cdot\frac{\cancel{25}\,19}{\cancel{100}\,\cancel{50}\,\cancel{25}\,5}$$

$$\frac{6\cdot 19}{3\cdot 5}=\frac{114}{5}$$

$19 \cdot 6 = 114$



2)

$X = \{1, 2, 3\}$ $\quad X \sim unif$

Y = numero della pallina estratta $= \{1, \dots, 6\}$

trovi tutte le possibili coppie => sommi

X	Y		$Z = \cancel{Y} - X$	(x_i, z_i)
1	1	1/6	0	(1,0)
1	2	1/6	1	(1,1)
1	3	0		
1	4	0		
1	5	0		
1	6	0		
2	1	1/12	-1	(1,0)
2	2	.	0	
2	3	.	1	
2	4	1/12	2	
2	5		3	
2	6	0	4	
3	1	1/18	-2	
3	2		-1	
3	3		0	
3	4		1	
3	5		2	
3	6	1/18	3	

$(1,0) = P(X=1 \cap Y=1) + P(X=2 \cap Y=1)$

PROB 0 => non fanno parte di S_Z

$$\boxed{P(Z=0 \mid X=3)P(X=3) = P(Y=3 \mid X=3)P(X=3) = \frac{P(Y=3 \cap X=3)}{\cancel{P(X=3)}}\cancel{P(X=3)} = \frac{1}{18}}$$

	1	2	3
1	1/6	1/12	1/18
2	1/6	1/12	.
3	0	1/12	.
4	0	1/12	.
5	0	0	
6	0	0	1/18
	1/3	1/3	1/3

non ind

$P(Z=0 \mid X=2)P(X=2)$

$P(Y=2 \mid X=2)P(X=2)$

$=$

$P(Y=2 \cap X=2) =$

$\frac{1}{12}$

3)

$$f_X(x) = \begin{cases} \frac{3\alpha^3}{x^4} & x \ge 2 \\ 0 & \text{else} \end{cases} \qquad 3 \cdot 8 = 24$$

$$x^{-4} \Rightarrow \frac{x^{-4+1}}{-4+1}$$

$$\Rightarrow 3\alpha^3 \int_2^{+\infty} \frac{1}{x^4} = 1 \Rightarrow \frac{-\cancel{3}\alpha^3}{\cancel{3}} \left[\frac{1}{x^3} \right]_2^{+\infty} = 1$$

$$\frac{\alpha^3}{8} = 1 \Rightarrow \alpha = 2 \qquad \boxed{f_X(x) \ge 0 \ \forall x}$$

prodotto derivate

$$D(f(x)g(x)) = f(x)g'(x) + f(x)' g(x)$$

$$\int_2^{+\infty} \cancel{x} \frac{24}{x^{\cancel{4}3}} = -\frac{24}{2} \left[\frac{1}{x^2} \right]_2^{\infty} = \frac{24}{8} = 3$$

$$F_X = 24 \int_{-\infty}^{x} \frac{1}{y^4} \, dy = 24 \int_2^x \frac{1}{y^4} \, dy = -8 \left[\frac{1}{y^3} \right]_2^x =$$

$$= -\frac{8}{x^3} + 1 =$$

$$1 - \frac{8}{x^3}$$

⇓

$$F_X(x) = \begin{cases} 1 - \frac{8}{x^3} & x \ge 2 \\ 0 & \text{else} \end{cases}$$

inizia ad accumularsi solo quando $x \ge 2$

$F_Y(x) =$

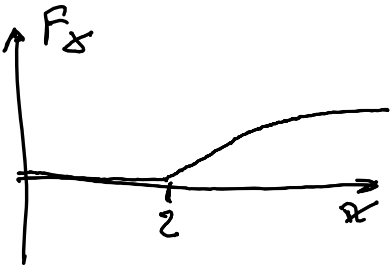
$= P(Y \le x) =$

$= P(X^5 \le x) =$

$= P(X \le \sqrt[5]{x}) =$

$F_X(\sqrt[5]{x})$

⇓



1) 2021-06-22

$\{(x_1, \dots, x_9) \mid x_i \in \{1,2,3\}\}$

$|\Omega| = 3^9$

$\cancel{A}$ ≐ "sul primo vagone salgono tre persone"

B = su ogni vagone salgono tre persone

C ≐ " su un vagone salgono due persone, su un altro tre, sul rimanente quattro

$P(A) = \frac{2^{9-3}}{3^9} \cdot \binom{9}{3}$

1 1 1 [2|3] [2|3] ...

[2|3] 1 1 1 [2|3] ...

1 1 1 2 2 2 3 3 3

shuffled

$P(B)$

$\binom{9}{3}\binom{6}{3}\frac{1}{3^9}$

A B C D E F G H I

$\frac{\frac{9!}{3!(9-3)!} \cdot \frac{6!}{3!(6-3)!}}{3^9}$

3 amici

4 bar

Prob che 2 si incontrino nello stesso bar? ~~su 2 bar~~

$\frac{\frac{\cancel{3}\cancel{9}4 \cdot \cancel{8} \cdot 7}{\cancel{3} \cdot \cancel{2}} \cdot \frac{\cancel{6} \cdot 5 \cdot 4}{\cancel{3 \cdot 2}}}{3^9}$

$\frac{4 \cdot 3 \cdot \binom{3}{2}}{4^3}$ bar in comune, altro bar, tra due amici, $\binom{3}{2} = \binom{3}{1}$

A A B

$\frac{\cancel{3} \cdot 4 \cdot 7 \cdot 5 \cdot 4}{\cancel{3} \cdot 3^8}$

$16 \cdot 35$

e.g. 2 bar

3 amici

A A A

B B B

C C C

D D D

A A B

B B A

A B A

B A B

B A A

A B B

1 1 2 2 2 3 3 3 3

2 2 1 1 1 3 3 3 3

2 1 1 1 2

$3 \cdot 2 \cdot \binom{9}{2}\binom{7}{3} / 3^9$

2) due dadi, 1 è regolare, l'altro $P(1)=\frac{1}{2}$

X, Y ris dei due dadi

$P(X=1 \cap Y=1) \overset{ind}{=} P(X=1)\,P(Y=1)$

$Z = XY$

X	Y		Z	U max	V min
1	1	1/6	1	~~1~~	~~1~~
1	2	1/12	2	2	1
1	3	1/12	3	~~3~~	~~1~~
2	1	1/6	2	2	1
2	2	1/12	4	2	2
2	3	1/12	6	3	2
3	1	1/6	3	~~3~~	~~1~~
3	2	1/12	6	3	2
3	3	1/12	9	~~3~~	~~3~~

$\frac{1}{12}\cdot 4 = \frac{1}{2}$

$P(3,1) = \frac{1}{6} + \frac{1}{12} = \frac{1}{4}$

X\Y	1	2	3
1	1/6	1/12	1/12
2	1/6	1/12	1/12
3	1/6	1/12	1/2

OPPURE ricordando che sono ind...

$E[(X-2)(Y-2)] =$

$= \cancel{(1-2)(1-2)}\,\frac{1}{6} + \cancel{(1-2)(2-2)} +$

$+ \cancel{(1-2)(3-2)}\cdot\frac{1}{12} +$

$\cancel{(3-2)(1-2)}\cdot\frac{1}{6} + \cancel{(3-2)(3-2)}\,\frac{1}{12}$

U\V	1	2	3	
1	1/6	0	0	1/6
2	1/4			
3	1/4		1/12	

3)

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ \left(1 - \frac{1}{e}\right) x & 0 \le x \le 1 \\ 1 - e^{-x} & x \ge 1 \end{cases}$$

$$f_X = \begin{cases} 0 & x < 0 \\ 1 - \frac{1}{e} & 0 \le x \le 1 \\ e^{-x} & x > 1 \end{cases}$$

$$P(X \ge 2) = 1 - F_X(2) = 1 - (1 - e^{-2}) = \frac{1}{e^2}$$

$Y = 3X$

$$E[Y] = 3E[X] =$$

$$E[X] = \underbrace{\int_0^1 x\left(1 - \frac{1}{e}\right)}_{\left(1-\frac{1}{e}\right)\left(\frac{1}{2}\right)} + \int_1^{+\infty} x e^{-x}$$

$$\left(1 - \frac{1}{e}\right)\left(\frac{1}{2}\right) - \underbrace{\int_1^{+\infty} x \cdot - e^{-x}}$$

$$= \underbrace{\left[e^{-x} x\right]_1^{+\infty}}_{-\frac{1}{e}} + \underbrace{\int_1^{+\infty} - e^{-x}}_{\underbrace{\left[e^{-x}\right]_1^{+\infty}}_{-\frac{1}{e}}} = \boxed{-\frac{2}{e}}$$

$$\frac{1}{2} \underbrace{- \frac{1}{2e} + \frac{4}{2e}}$$

$$\frac{3}{2} + \frac{9}{2e}$$

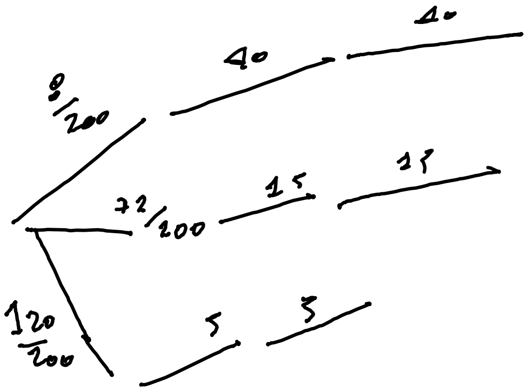
$$F_Y(x) = P(Y \le x) = P(3X \le x) = P\left(X \le \frac{x}{3}\right) = F_X\left(\frac{x}{3}\right)$$

quindi

sostituisco ok

1)

2021-05-27



$$P(\text{principiante} \cap \text{centro } 1 \cap \text{centro } 2) =$$

$$\frac{120}{200} \cdot \frac{5}{100} \cdot \frac{5}{100}$$

$$P(\text{centro } 1 \cap \text{centro } 2) =$$

$$\frac{8}{100} \cdot \left(\frac{40}{100}\right)^2 + \frac{72}{100} \cdot \left(\frac{15}{100}\right)^2 + \frac{120}{200} \left(\frac{5}{100}\right)^2$$

$$P(\text{principiante} \mid \text{centro } 1 \cap \text{centro } 2) =$$

$$= \frac{P(\text{princi} \cap \, :. \, \cap \,)}{P(\text{centro } 1 \cap \text{centro } 2)} =$$

2) dado truccato 6 facce

~~prob~~ numeri pari è doppia rispetto a quelli di dispari

X = risultato lancio

$S_x = \{1, \dots, 6\}$

$1 = P(\text{pari}) + P(\text{dispari})$

$\cancel{1} = 2\,P(\text{dispari}) + P(\text{dispari}) =$

$= 1 = 3\,P(\text{dispari})$

$P(\text{dispari}) = \frac{1}{3}$

$P(\{1, 3, 5\}) = \left(\frac{1}{3}\right)$

$\frac{2}{6} : 3$

$\frac{2}{3} = P(\{2, 4, 6\})$

1	2	3	4	5	6
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

$E[|X-3|] = 2 \cdot \frac{1}{9} + 1 \cdot \frac{2}{9} + 1 \cdot \frac{2}{9} + 2 \cdot \frac{1}{9} + 3 \cdot \frac{2}{9}$

$\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{6}{9} \quad \frac{14}{9}$

$Y = |X-3|$

2, 1, 0, 3

$\frac{1}{9} + \frac{2}{9}$ $\frac{4}{9}$ $\frac{1}{9}$ $2/9$

P_Y	$\frac{1}{9}$	$4/9$	$2/9$	$2/9$
Y	0	1	2	3

$P(Y = 2X) =$

$= P(|X-3| = 2X) =$

$= P(X - 3 = -2X \cup X - 3 = 2X) =$

$= \underbrace{P(X=1)}_{1/9} \cup P(X=-3)$

3)

$$f_X(x) \begin{cases} 0 & \text{else} \\ x & 0 \le x \le 1 \\ a - x & 1 < x \le 2 \end{cases}$$

$$1 = \underbrace{\int_0^1 x}_{} + \int_1^2 a - x =$$

$$\frac{1}{2} + a - \int_1^2 x$$

$$\frac{1}{2} = a - \left[\frac{x^2}{2}\right]_1^2$$

$$\frac{4}{2} - \frac{1}{2}$$

$2 - x \geq 0 \ \forall_x \geq 1 \leq 2$

$$\cancel{\frac{1}{2}} = a - 2 + \cancel{\frac{1}{2}} \qquad a = 2$$

$$\int_0^x y \, dy = \left[\frac{y^2}{2}\right]_0^x = \frac{x^2}{2}$$

$$\underset{1<x<2}{F_X(x)} = \int_0^1 y \, dy + \int_1^x 2 - y \, dy$$

$$\frac{1}{2} + 2\int_1^x 1 \, dy - \int_1^x y \, dy$$

$$\cancel{\frac{1}{2}} + 2x \cancel{-2} - \frac{x^2}{2} + \cancel{\frac{1}{2}}$$

$$\cancel{2} - 2$$

$$F_Y(x) = P(Y \le x) = P(X^3 < x) = P(X < \sqrt[3]{x}$$

$$P\left(X \geq \frac{3}{2}\right) = 1 - F_X\left(\frac{3}{2}\right) = 1 - \left(\cancel{2} \cdot \frac{3}{\cancel{2}} - \left(\frac{3}{2}\right)^2 \cdot \frac{1}{2} - 1\right)$$

ragione sempre prima sugli estremi del supporto!

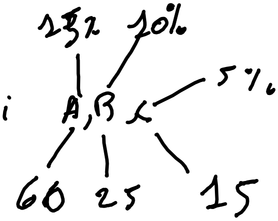
$$1 - \left(3 - \frac{9}{8} - 1\right)$$

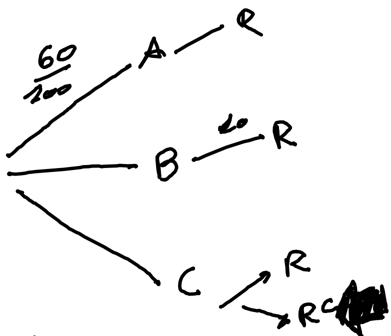
$$1 - \left(2 - \frac{9}{8}\right)$$

$$-\frac{8}{8} + \frac{9}{8} = \frac{1}{8}$$

2021-02-07

1) 100 p, 3 treni





a) 10 % (sapendo B, prob ritardo)

b) $\frac{25}{100} \cdot \frac{10}{100}$

c) prob ritardo =

$\frac{15}{100} \cdot \frac{60}{100} + \dots + \dots$

d) $P(B \mid \text{ritardo}) = \frac{P(B \cap \text{ritardo})}{P(\text{ritardo})} = \frac{P(\text{ritardo} \mid B)\, P(B)}{P(\text{ritardo})}$

2)

X	0	1	2	3	4	5
P_X	$\frac{a}{2}$	b	$\frac{1}{4}$	$\frac{3}{20}$	$\frac{1}{20}$	$\frac{3}{20}$
	$\downarrow \frac{1}{3}$	$\downarrow \frac{1}{3}$				

$P(X=1 \mid X \leq 3) = \frac{1}{4} =$

$$= \frac{P(X=1 \cap X \leq 3)}{P(X \leq 3)} = \frac{1}{4}$$

$4\,P(X=1) = P(X \leq 3) =$

$4\,P(X=1) = 1 - P(X>3)$

$4b = 1 - \frac{4}{20} = \frac{16}{20} = 4b \qquad \frac{4}{20} = b \qquad \boxed{\frac{1}{5} = b}$

$\frac{10a}{20} + \cancel{\frac{4}{20} + \frac{5}{20}} + \frac{3}{20} + \frac{3}{20} + \frac{1}{20} + \frac{3}{20}$

$\frac{10a + \cancel{10}}{\cancel{20}} = \cancel{20} \quad \frac{4}{10} \quad \boxed{\frac{2}{5} = a}$

$E[X^2] = \cancel{\frac{4}{20}} + \frac{\cancel{20}}{20} + \frac{27}{20} + \frac{16}{20} + \frac{75}{20} = \frac{142}{20} = \frac{71}{10}$

$Y = \frac{1}{3}X^3 \Rightarrow S_Y = \{0, \frac{1}{3}, \frac{8}{3}, 9, \frac{64}{3}, \frac{125}{3}\}$

$P(Y=x) =$

$P(\frac{1}{3}X^3 = x) = \cancel{P(X=\dots)} + P(X=0) = \frac{1}{3}$

$X^3 = 3x \Rightarrow x=0$

3)

$$F_X(x) = \begin{cases} 0 & 0 \\ \frac{1}{2}x & 0 \quad 1 \\ 1 - \frac{1}{2x^2} & 1 \end{cases}$$

x^{-2-1}

$$f = \begin{cases} 0 \\ \frac{1}{2} \\ \cancel{-\frac{1}{2}} \cdot \cancel{-2} \cdot \frac{1}{x^3} \end{cases}$$

$x^2 = 2x^{3-1}$

$$P(X \geq 2) = 1 - F_X(2) =$$

$$1 - \left(1 - \frac{1}{8}\right) = \frac{1}{8}$$

$$\int_0^{+\infty} x f_x = \frac{1}{2}\int_0^1 x + \int_1^{+\infty} \frac{x}{x^{3}}$$

$$\frac{1}{x^2} = \frac{x^{-2+1}}{-2+1}$$

$$\frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{1}{x}\right]_1^{+\infty}$$

$$\frac{1}{2}\cdot\frac{1}{2} + 1 = \frac{5}{4}$$

$$Y = \begin{cases} 1 & x \leq 1 \\ x+1 & x > 1 \end{cases}$$

$$P(Y \leq 3 \cap X > 1) = P(Y \leq 3 \mid X > 1)P(X > 1) =$$

$$= \frac{P(X+1 \leq 3 \cap X > 1)}{\cancel{P(X>1)}} \cancel{P(X>1)}$$

$$= P(X > 1 \cap X \leq 2) =$$

$$P(1 \leq X \leq 2)$$

$$= F_X(2) - F_X(1)$$

$$\{Y \leq 3\} = \{X \leq 1\} \cup \{X+1 \leq 3 \cap X > 1\}$$

identificare!

? ? ?