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To Dope or not to Dope:
Using Game Theory and Statistics to Understand Doping in Cycling's Greatest Race

Note: As this is a very long paper, I have highlighted the most noteworthy sections.

I. INTRODUCTION

A. Abstract

In this paper, I will discuss the topic of doping in the Tour de France. Doping has been an issue in sports since humans played them and it has been particularly well publicized in cycling. Naturally, the most publicity surrounding doping in cycling has centered on its biggest race of the year, the Tour de France. My aim in this paper will be to examine the motivations behind doping in the Tour de France and the regulations that try to stop cyclists from doing it. My hope is that my analysis will produce results that regulatory authorities could practically implement in order to decrease doping rates in the Tour.

The structure of my paper will be as follows. In the first section, I will give an overview of the Tour de France and its history of doping. In the second section, I will carry out a game theoretic analysis of doping behaviors and anti-doping regulations in the Tour de France. In the third section, I will do a statistical analysis of doping and anti-doping regulations in the Tour de France using regression models. In the fourth section, I will compare my results from the second and third sections and conclude.

B. Doping and the Tour de France

The Tour de France is an annual cycling race that takes place in July of every year. It began in 1906 and has been raced almost every year since then.[1] The modern editions of the Tour cover approximately 3600 kilometers (2200 mi) throughout France and its bordering countries. The race is a 23-day event that is broken into 21 stages—during which the cyclists race between 1 and 6 hours a day—and 2 rest days. There are three different kinds of stages: flat, time-trial, and mountainous. The flat and mountainous stages are long routes that usually last over 3 hours while the time-trial stages are 1-hour stages that are meant to be sprints.

Cycling in the Tour de France is a team sport. Usually 20 teams, each with 9 riders, participate in the tour, but the number of cyclists who finish may be lower since individuals and teams may drop out of the race. The strongest cyclist on each team is called the team leader and he tries to win the race. Other team members, called domestiques, help the team leader by pacing him, allowing him to draft, bringing him food or water, and aiding him if he has a mechanical failure.[2]

Cyclists in the Tour de France can win prizes by placing well in a variety of different classifications. The most coveted classification is the general classification (GC), which ranks cyclists based on their total accumulated time throughout the tour.[3] At the end of each stage, the rider with the lowest accumulated time is awarded a yellow jersey to signal his top position in the GC. The overall winner of the Tour de France wins the yellow jersey and a prize of around

1 BBC, "Tour de France 2019: Everything you need to know," (BBC. July 6, 2019.)
[2] Ibid.
[3] Ibid.

€500,000 euros. Cyclists are also ranked in the mountain, points, and youngest rider classification. The mountain classification, designated by the white jersey with red polka dots, ranks riders based on how fast they climb certain mountains. The points classification, designated by a green jersey, ranks cyclists based on points earned at the end of each stage. Higher points are given to cyclists that win time-trials or flat stages, while lower points are awarded to cyclists that finish the mountain stages. The aim with the points classification is to reward sprinters. Finally, the youngest rider classification, designated by a white jersey, ranks riders under 26 years by their accumulated time.[4] By winning in the mountain, points, or young rider classifications, cyclists can earn €25,000. Cyclists can also win around €8,000 for winning the yellow jersey in any given stage.[5]

The Tour de France is probably one of the most grueling modern day sporting events. Cyclists burn over 5000 calories per day, sleep poorly, and have to deal with extreme altitude and climate changes when they move from blistering hot plains and climb 2000 meters to the frigid Alps.[6] Given the extreme nature of the sport and the toll the grand tours take on a cyclist's body, it is not surprising that doping has been widespread in cycling's greatest race.

From 1906 until 1965, doping was not banned in the Tour de France. During that era, doping was a widely accepted means of improving performance and cyclists used a variety of drugs in the Tour, ranging from household stimulants, such as alcohol, cocaine, and chloroform, to prescription medication, such as amphetamines and testosterone.[7] In 1965, the use of performance enhancing drugs was officially outlawed in France and in 1966 the first anti-doping tests were carried out in the Tour. Throughout the 1970s cyclists continued to dope, using a cocktail of drugs that included cortisone and amphetamines. In the late 1970s, effective tests for detecting amphetamine and testosterone were implemented in the Tour de France, so cyclists switched over to erythoropoietin (EPO), which is a drug used to increase red blood cell count in people with anemia. Red blood cells are responsible for circulating oxygen around the body. Thus increasing red blood cell count through EPO can help cyclists increase their Vo2 max, which is the maximum amount of oxygen one's body can use when they exercise. In the 2000s, cyclists also started 'blood doping' by extracting their healthy blood before a race and transfusing it back into themselves during the race in order to give their body a boost in red blood cells. EPO and blood transfusions were the primary doping methods used by cyclists in the 1990s and early 2000s. Most notably, Lance Armstrong won seven Tours between 1998 and 2005 using EPO and blood doping. Since 2005, there has been a variety of high profile doping cases including two major scandals that implicated various cyclists (see the Festina Affair and Operation Puerto) and the 2-year suspension and annulment of Alberto Contador's 2010 Tour de France title.[8]

The organization that oversees cycling and all its anti-doping regulation is called the International Cycling Union (UCI). Whatever anti-doping regulations the UCI implements must be carried out in the Tour de France. The actual testing is carried out by a separate body called the cycling anti-doping federation (CADF). [9] Since 1968, most anti-doping efforts in the Tour have focused on improving testing so that a wider variety of drugs can be detected. Besides this,

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[4] BBC, "Tour de France 2019: Everything you need to know."
[5] MacLeary, John. "Tour de France prize money: How much did the winning riders and teams earn?" (Telegraph, July 29, 2019).
[6] Bien, Louis, "How do cyclists physically survive the Tour de France? We asked a physiologist and former pro rider," (SBNATION, July 10, 2018).
7 Mignon, Patrick, "The Tour de France and the Doping Issue," (The International Journal of History and Sport, 2003), 227-245.
8 Ibid, 227–245.
[9] McMahon, Daniel. "Here's how drug testing works at the Tour de France, the world's greatest race." (Business Insider, 2019).

the UCI has increased the frequency and efficiency of testing both in and out of competition. It has also implemented fines and suspension terms for cyclists found guilty of doping. In forthcoming years, it is likely that the UCI will roll out increasingly innovative methods, such as the athlete biological passport that it implemented in 2008, which tracks a particular cyclists baseline physiological indicators throughout their career in order to easier detect anomalies.

II. GAME THEORETIC ANALYSIS

A. Introduction

Since any competition has players—each of which has strategies and payoffs— most sport events also fall within the set of situations that can be analyzed by game theory. In this section, I will attempt to model the Tour de France as a game and use the tools of game theory to make conclusions about its doping and anti-doping behaviors. I will introduce four games in the following order: (i) a simple single stage game; (i) a complex single stage game; (iii) a repeated game; (iv) a repeated game with imperfect detection. Each game I introduce will have two players and each player will always have two strategies: doping and not doping. Since there are two players and each has two strategies, there will always be four possible payoffs for each player.

In the single stage games, I will introduce the strategic form of the game—including how I calculated the payoffs—and I will then analyze the game to find a Nash equilibrium. The Nash equilibrium in both of these models will be for both players to dope so I will discuss how one could change the payoffs of these two games to force a Nash equilibrium where both players do not dope.

In the repeated games, I will introduce the strategic form of the game and also analyze it for an equilibrium solution. I will spend less time tinkering with the payoffs, and more time discussing an equilibrium concept called grim trigger strategies and how they can lead to cooperation between cyclists.

In my models, I will be using concepts that can be found in most beginner game theory textbooks. All my definitions will closely follow those given by Prajitt Dutta in his textbook *Strategies and Games,* published in 1999.

B. Simple Single Stage Game

B1. Strategic Form Representation

The first model is a Tour de France race that is played once between two players. It has the following strategic form representation:

Players: {i = A, B}

Strategies: {Dope, not Dope}

Payoffs:

$\pi_i(D, D) = P(win|D,D) \times P(not\ caught) \times (prize) - P(win|D,D) \times P(caught) \times (prize + fine) - P(loss|D,D) \times P(caught) \times (fine) + P(loss|D,D) \times P(not\ caught) \times 0$

$\pi_i(D, ND) = P(win|D,ND) \times P(not\ caught) \times (prize) - P(win|D,ND) \times P(caught) \times (prize + fine) - P(loss|D,ND) \times P(caught) \times (fine) + P(loss|D,ND) \times P(not\ caught) \times 0$

$\pi_i(ND, D) = P(win|ND,D) \times (prize) + P(loss|ND,D) \times 0$
$\pi_i(ND, ND) = P(win|ND, ND) \times (prize) + P(loss|ND,ND) \times 0$

Where if *i*=A, then $\pi_i(X, Y)$ is the payoff to A if A plays X and B plays Y.
Where if *i*=B, then $\pi_i(X, Y)$ is the payoff to B if A plays X and B plays Y.

Therefore, $\pi_A(D, D) = \pi_B(D, D)$ and $\pi_A(ND, ND) = \pi_B(ND, ND)$.
Therefore, $\pi_A(D, ND) = \pi_B(ND, D)$ and $\pi_A(ND, D) = \pi_B(D, ND)$.

We can think of the first two payoffs [$\pi_i(D, D)$ and $\pi_i(D, ND)$] as the sum of four mutually exclusive events: winning and being caught, winning and not being caught, losing and being caught, losing and not being caught. Each mutually exclusive event consists in the probability that the event occurs—which is the probability of winning/losing multiplied by the probability of being caught/not being caught—multiplied by the utility the cyclist gets in that scenario. [10] In the last two payoffs [$\pi_i(ND, D)$ and $\pi_i(ND, ND)$], the cyclists do not dope so there is no probability of being caught. The payoffs here are the sum of two mutually exclusive events: winning and losing.

For the first two payoffs, the utility of winning and not being caught is the prize, the utility of winning and being caught is the payment of the prize and fine, the utility of losing and being caught is payment of the fine, and the utility of losing and not being caught is 0.[11] For the last two payoffs, the utility of winning is the prize, and the utility of losing is 0. In the first two payoffs above, I already applied some simplification. Since the utility of getting caught and winning is the payment of the prize and fine, for $\pi_i(D, D)$ and $\pi_i(D, ND)$ I factored out the negative signs attached to prize and fine. This change should not be taken to mean that P(win|D,D) or P(win|D,ND) are actually negative.

Note that the probabilities of winning and losing are conditional on the player's choices and that of their opponents, so they change in each payoff. The idea here is that a player's probability of winning, for example, will be higher if they dope and their opponent doesn't than if they both dope. Now let's discuss how I decided the value of the different probabilities and utilities.

[10] I decided to interpret the probability of being caught and the probability of winning/losing as independent probabilities with respect to each other. While some Tour de France officials have commented that sometimes players that win are more likely to be tested for doping, for the most part testing is done randomly in the Tour de France and so, in general, it is not correlated with whether a player wins or loses.
[11] Rules dictate that if a player is caught doping and wins the Tour de France, they must pay back the prize money. Hence why the utility of winning and getting caught is a payment of both the prize and the fine.

Fine: The International Cycling Union's (UCI) 2019 handbook states that if a player is caught doping then they will be fined one year's worth of their net income. A player's net income is defined as 70% of their gross income. [12] A good cyclists gross income—assuming they have not won any grand tours—is around €1,400,000, which means that the fine is about €980,000.[13]

Prize: The prize of the Tour de France has always been around €500,000.[14]

Probabilities of winning/losing: If both players dope or don't dope, then the playing field is 'fair' and since there are two players each has a 50% probability of winning. If one player dopes and the other does not, the one that dopes has a 90% chance of winning, while the player that did not dope has a 10% chance of winning. The rationale for this is as follows: if a player dopes but their opponent does not, clearly this should increase the doper's probability of winning by more than 50%, or else doping does is not advantageous. I chose to set this probability arbitrarily high to start off with. I will later tinker with this probability to see how it changes the NE.

Probabilities of getting caught: The challenge of anti-doping efforts is that each time a foolproof test comes out to expose one kind of drug, everyone moves onto a new one. Until the regulating authority comes up with a new test to detect this new drug, it is pretty easy for players to dope undetected. Most players have been caught only because other cyclists expose them. There is talk of methods that are foolproof against any drug like DNA testing but these tests have yet to hit the market and so for now it is safe to assume that the probability of getting caught if you dope is low. That being said, I set it at 10%.

Given these numbers, we can now calculate the payoffs as numerical values:

$\pi_i(D, D) = 0.5 \times 0.9 \times (500{,}000) - 0.5 \times 0.1 \times (500{,}000+980{,}000) - 0.5 \times 0.1 \times (980{,}000)$
$\pi_i(D, ND) = 0.9 \times 0.9 \times (500{,}000) - 0.9 \times 0.1 \times (500{,}000+980{,}000) - 0.1 \times 0.1 \times (980{,}000)$
$\pi_i(ND, D) = 0.1 \times (500{,}000)$
$\pi_i(ND, ND) = 0.5 \times (500{,}000)$

[12] Union Cycliste Internationale, *Cycling Regulations. Part 14: Anti-doping Rules*, (UCI, 2015), 41.
[13] Lia Harvey, "Cycling salaries: How much do professional cyclists earn?" (Sky Sports, 2015).
14 LeTour, "Sporting stakes/Rules," (LeTour, 2019).

The game can be easily visualized in this 2x2 matrix where player A's payoffs are underlined:

		PLAYER B	
		Dope	not Dope
PLAYER A	Dope	$\underline{102{,}000}$ 102,000	$\underline{262{,}000}$ 50,000
	not Dope	$\underline{50{,}000}$ 262,000	$\underline{250{,}000}$ 250,000

The unique NE of this game is {Dope, Dope}. In fact, doping is also a strictly dominant strategy for both players and thus a dominant strategy solution (DSS). Thus my model predicts that cyclists in the Tour de France will choose to dope.

From the perspective of the UCI this is a suboptimal outcome, so I will now try to figure out how we can change the payoffs in order to force a unique NE where both players choose not to dope. There are four variables that could be realistically changed: (i) prize amount, (ii) fine amount, (iii) probability of getting caught, (iv) probability of winning if one dopes but their opponent does not. For each of these variables I will calculate the ranges in which they must fall in order for the Nash equilibrium to yield an optimal outcome for the UCI.

B2. Payoff Manipulation

1. *Decreasing Prize Amount*

First let's take a look at the general form of the game:

		PLAYER B	
		Dope	not Dope
PLAYER A	Dope	$\underline{\pi_A(D, D)}$ $\pi_B(D, D)$	$\underline{\pi_A(D, ND)}$ $\pi_B(D, ND)$
	not Dope	$\underline{\pi_A(ND, D)}$ $\pi_B(ND, D)$	$\underline{\pi_A(ND, ND)}$ $\pi_B(ND, ND)$

If the UCI wants to decrease the prize amount to force a unique NE located in the bottom right cell, then they need to modify the prize amounts such that the following four inequalities are true:

$$\pi_A(ND, D) > \pi_A(D, D) \qquad \textbf{(A1)}$$
$$\pi_A(ND, ND) > \pi_A(D, ND) \qquad \textbf{(A2)}$$

$$\pi_B(D, ND) > \pi_B(D, D) \qquad \textbf{(A1')}$$
$$\pi_B(ND, ND) > \pi_B(ND, D) \qquad \textbf{(A2')}$$

Notice that (A1) is equal to (A1') and (A2) is equal to (A2'), so we only need to solve (A1) and (A2). For most variables, the solution to (A1) will be higher in absolute value than the solution to (A2), so solving for (A1) is sufficient to solve for (A2). Below I solve for (A1) leaving the prize amount as a variable.

$$0.1\times(\text{Prize}) > 0.5\times0.9\times(\text{Prize}) - 0.5\times0.1\times(\text{Prize} + 980{,}000) - 0.5\times0.1\times(980{,}000) \quad \textbf{(A3)}$$

Solving (A3) leads to the solution that Prize < $\frac{980000}{3}$. So if the UCI wanted to force an optimal outcome, they could decrease the prize amount below €326,667.

2. *Increasing Fine Amount*

To force a NE in the bottom right cell, the UCI needs to solve inequality (A1), leaving the fine amount as a variable. If we setup (A1) with all the values but leave the fine as a variable we get:

$$0.1\times(500{,}000) > 0.5\times0.9\times(500{,}000) - 0.5\times0.1\times(500{,}000+\text{Fine}) - 0.5\times0.1\times(\text{Fine}) \quad \textbf{(A4)}$$

The solution is that Fine > €1, 500, 000. Assuming that the net income of the cyclist is 1,400,000, this means that fine needs to be around 154% the net income of the cyclist or around 108% the gross income of the cyclist.

3. *Increasing Probability of Doping Detection*

The UCI needs to solve inequality (A1), leaving the probability of doping detection as a variable:

$$0.1\times(500{,}000) > 0.5\times(1-P(\text{caught}))\times(500{,}000) - 0.5\times P(\text{caught})\times(500{,}000+980{,}000) - 0.5\times P(\text{caught})\times(980{,}000) \quad \textbf{(A5)}$$

The solution is that P(caught) > $\frac{5}{37}$, which is approximately 13.5%. This means that the UCI would need to increase the probability of getting caught by about 3.5%. What does it mean for the probability of detection to increase by 3.5%? An intuitive explanation would be that this means the UCI would need to introduce some anti-doping measure that makes it 3.5% more likely that the cyclist will get caught. This could be anything from a novel anti-doping test (i.e. a new EPO test) to new testing policies, such as more frequent testing or nighttime testing.

4. *Decreasing Pr(win|D, ND)*

The UCI has virtually no control over the boost that doping gives a cyclist in races, however, in an effort to leave no stone unturned I decided to also tinker with this variable. The inequality to solve for this scenario is (A1), leaving P(win|ND,D) as a variable:

$$P(\text{win}|ND,D))\times(500{,}000) > 0.5\times0.9\times(500{,}000) - 0.5\times0.1\times(500{,}000+980{,}000) - 0.5\times0.1\times(980{,}000) \quad \textbf{(A6)}$$

The solution is P(win|ND,D) > 0.204 and so P(win|D,ND) < 0.796. In order to force an optimal outcome, doping (when the other player does not dope) cannot increase one's probability of winning by more than 79.6% (and cannot decrease the non-doper's probability of winning by more than 20.4%).

B3. Results

Let's summarize the results before moving on. The first game shows that the NE was for both players to dope. In order to change this to a unique NE where both players do not dope, one of four variables needed to change in the following ways: (i) prize money is lowered below €326,666; (ii) fine amount is increased to 154% the net income of a cyclist; (iii) the probability of getting caught is increased to 13.5%; (iv) the probability of winning if one player dopes (but the other one does not) must not exceed 79.6%.

Out of all of these changes, the most feasible seems to be either a change in the fine amount or in the prize money, primarily because these changes can be measured. The modification of fine and prize amounts are, however, very large and cyclists and committees might be resistant to these changes. Another interesting finding is that the probability of doping detection would only need to increase by 3.5% to produce an optimal outcome. This small change seems promising. The issue is that it seems difficult to interpret what this increase would mean and it would be difficult to measure such an increase. Finally, changing the boost that drugs gives a cyclist in a race is not a feasible thing the UCI can do. Nevertheless, it is interesting to see that this probability need only drop by about 10% in order to change the NE.

C. Complex Single Stage Game

C1. Strategic Form Representation

In this next model, we will still be dealing with a single stage game but now an important element will be introduced that was previously left out. Whatever happens in the Tour de France, a cyclist will not just expect immediate payoffs but future one's as well. A player that wins the Tour de France does not just win the prize money immediately, but will also expect a salary increase for the rest of their career. A cyclist that is caught doping in the Tour will not just be immediately fined, but will be suspended for four years and then return to their cycling career with a lower salary than they previously earned. Cyclist's actions have future consequences. That being said, there are a couple of new elements that have been added to the payoff calculations and they are described below:

Salary increase from winning: If a cyclist wins the Tour de France their salary will increase from €980,000 to the average salary of a Tour de France champion—€2,500,000—for the rest of their career.[15]

Salary change from losing: If a cyclist loses the Tour de France, then they will continue to earn the same salary they earned before racing in the Tour—€980,000. Since there is now a utility to losing, the expected payoff of losing is no longer 0.

Salary change from getting caught: If a cyclist is caught doping and it is their first offense, the UCI rules dictate that they be suspended for four years.[16] This consequence is incorporated into my model as an opportunity cost of not winning your standard salary of €980,000 for the next four years. Once a cyclist is caught doping their reputation is often tarnished and if they return to a cycling career, then they will earn a lower salary than they used to earn. I set this at the starting salary for cyclists of €80,000.[17] They will earn this low salary for the rest of their career starting four years after they raced in the tour (because for the first four of those years they were suspended). That being said, being caught doping leads to an opportunity cost of their standard salary (€980,000) for their entire career since they will never earn that salary again. For the first four years (during their suspension), they miss out on their entire standard salary because they are not competing. Once the suspension ends, they miss out on the difference between the standard salary and the lower starting salary (980,000 – 80,000) for the rest of their career.

Career length: The average career length of a good professional cyclist is around 15 years. It takes a cyclist a couple of years to make it to the point where they are racing as a team leader in the Tour de France. I assumed this takes five years. That means that after the Tour each cyclist has 10 years left in their career. For simplicity, this variable is independent of whether the player wins, loses, or gets caught doping.

Discount Factor: I will model future payoffs by discounting them in order to take account of the time value of money. The discount factor, which is based on the current 3-month T-Bill rate, is 98.5%.[18] When I display the payoffs below, I will keep the discount factor as a general variable δ in order to make it easier to read.

Time Frame: I will assume that all future payoffs begin 1 year after the cyclist raced in the tour. So if the tour takes place in year T=0, all future payoffs start at T=1. Starting future payoffs at T=1 just means that every player is paid their new salary at the *end* of a year of racing, where each year of racing starts and ends with the Tour de France. For example, if a cyclist wins the Tour in 2019, then they will start to earn their increased salary before the next Tour in 2020.

The payoff calculations are similar to those in the last game. The first two payoffs consist in a sum over four mutually exclusive events, while the last two are a sum over two mutually exclusive events. The probabilities of winning, losing, being caught, and not being caught remain the same, but the utilities for each expected payoff change. Added to the utility of winning is a

[15] Harvey, "Cycling salaries: How much do professional cyclists earn?" 2015.
[16] UCI, *UCI Cycling Regulations: Part 14: Anti-doping Rules*, 34.
[17] Harvey, "Cycling salaries: How much do professional cyclists earn?" 2015.
[18] YCharts. *3 Month Treasury Bill Rate*. (YCharts, December 26, 2019).

salary increase for the rest of one's career. Added to the utility of being caught are the 4-year suspension and the decrease in salary upon returning to cycling. Added to the utility of losing is the payment of the standard salary for the rest of one's career. The strategic form representation of the game is as follows:

Players: $\{i$ = A, B$\}$

Strategies: {Dope, not Dope}

Payoffs:

$$\pi_i(D, D) = 0.5\times0.9\times(500{,}000 + \textstyle\sum_{i=1}^{t=10} 2{,}500{,}000\delta^t) - 0.5\times0.1\times(500{,}000 + 980{,}000 + \sum_{i=1}^{t=10} 980{,}000\delta^t - \sum_{i=5}^{t=10} 80{,}000\delta^t) + 0.5\times0.9\times(\sum_{i=1}^{t=10} 980{,}000\delta^t) - 0.5\times0.1\times(980{,}000 + \sum_{i=1}^{t=10} 980{,}000\delta^t - \sum_{i=5}^{t=10} 80{,}000\delta^t)$$

$$\pi_i(D, ND) = 0.9\times0.9\times(500{,}000 + \textstyle\sum_{i=1}^{t=10} 2{,}500{,}000\delta^t) - 0.9\times0.1\times(500{,}000 + 980{,}000 + \sum_{i=1}^{t=10} 980{,}000\delta^t - \sum_{i=5}^{t=10} 80{,}000\delta^t) + 0.1\times0.9\times(\sum_{i=1}^{t=10} 980{,}000\delta^t) - 0.1\times0.1\times(980{,}000 + \sum_{i=1}^{t=10} 980{,}000\delta^t - \sum_{i=5}^{t=10} 80{,}000\delta^t)$$

$$\pi_i(ND, D) = 0.10\times(500{,}000 + \textstyle\sum_{i=1}^{t=10} 2{,}500{,}000\delta^t) + 0.9\times(\sum_{i=1}^{t=10} 980{,}000\delta^t)$$

$$\pi_i(ND, ND) = 0.50\times(500{,}000 + \textstyle\sum_{i=1}^{t=10} 2{,}500{,}000\delta^t) + 0.5\times(\sum_{i=1}^{t=10} 980{,}000\delta^t)$$

		PLAYER B	
		Dope	not Dope
PLAYER A	Dope	13.7m 13.7m	22.1m 10m
	not Dope	10m 22.1m	16m 16m

The payoffs in the 2x2 matrix have been rounded to one decimal place and "m" is used to denote a million. As with the previous model, the NE and DSS are {Dope, Dope}. Again, we have a suboptimal outcome from the perspective of the UCI, so let's take a look at how we can shift this to a unique NE of {not Dope, not Dope}. As with the previous model, in order to change the NE we will always need to solve the same four inequalities:

$\pi_A(ND, D) > \pi_A(D, D)$ **(A1)**
$\pi_A(ND, ND) > \pi_A(D, ND)$ **(A2)**
$\pi_B(D, ND) > \pi_A(D, D)$ **(A1')**

$\pi_B(ND, ND) > \pi_A(ND, D)$ **(A2')**

As with our previous model, (A1) is equal to (A1') and (A2) is equal to (A2'). Here (A2) gives a solution with a higher absolute value bound than (A1), so we will use (A2) in all our calculations.

C2. Payoff Manipulation

1. *Increase suspension time*

If we solve for (A2), keeping the length of suspension as a variable, there is no such solution. The reason for this becomes clear if we simplify (A2), leaving the suspension time as a variable (x):

$$16{,}277{,}199 > 18{,}824{,}076 + 8{,}000 \sum_{i=x}^{t=10} 0.985\text{^}t \qquad \textbf{(A7)}$$

Increasing the suspension time in this equation decreases the amount of time during which racers can earn their lower salary once they return to cycling. As suspension time increases, the discounted cash flows from the cyclists' post-suspension career decreases but since it remains positive throughout, the RHS will always be greater than the LHS of (A7). The only way to change this would be for the summation on the RHS to be negative, which is impossible given the positive discount factor. While it seems reasonable that there would be a noticeable effect from increasing the suspension time in this model, the positive payoffs from winning the tour are so large that they cannot be outweighed by the negative payoff of suspension time. This is probably one of the most unrealistic conclusions that came out of my models. After all, it seems likely that if the UCI changed the suspension time to 10 years then it would have a huge impact on doping because if a player is caught their career is basically over. That being said, more work is needed to change the way suspension time or related variables are modeled.

2. *Increase the discount factor*

There is no solution for $0 \leq \delta \leq 1$. The reason for this is clear if we simplify (A2), leaving our discount factor as a variable (δ):

$$250{,}000 > 262{,}000 + 275{,}200 \sum_{i=1}^{t=10} \delta^t + 8000 \sum_{i=5}^{t=10} \delta^t \qquad \textbf{(A8)}$$

The discount factor was initially set at 0.985. Increasing it will only increase the RHS of the equation. Decreasing the discount factor will decrease the RHS but not enough to make it lower than the LHS. It is clear that even if the discount factor is 0 (A8) will not be true.
It seems that changing the discount factor will not change the outcome because this variable affects both punishments (i.e. opportunity cost of suspension) and rewards (i.e. increase in salary) similarly. Thus increasing it or decreasing it will change the punishments and rewards in the same direction and by a similar magnitude. Intuitively, the discount factor should not change anything because unlike fines or prizes this variable does not affect different scenarios disproportionately (i.e. caught v.s. not caught) so it cannot change cyclist behavior.

3. *Decrease Tour de France champion salary*

Could the UCI put a cap on the salary that Tour de France champions win? The intuition behind this is that if Tour de France champion's salary is limited by a ceiling set by the regulating authority, then the payoff of winning the Tour falls but so does the payoff of doping. If we solve (A2) keeping champion salary as a variable, we find that this salary cannot be higher than approximately €1,590,000.

4. *Increase the fine*

Solving for (A2) gets us that the fine amount would need to increase to above approximately €26,880,000. This is because the immediate punishment of the fine is dwarfed by the future payoffs of winning the Tour—or even losing the Tour and not getting caught. Consequently the fine amount needs to be substantially raised in order to make a difference.

5. *Increase the probability of getting caught*

As with the previous model, we can increase the probability of getting caught such that the NE is optimal for the UCI. Solving (A2), we get that the probability must be greater than 0.181. This is higher than the probability of 0.135 that was the result to this question in the first game because now the difference between π_A(ND, ND) and π_A(D, ND) has increased and so the probability required to make (A2) true is higher.

6. *Decrease the prize amount*

Solving for (A2) gets us that the prize amount would need to decrease below approximately –€11,270,000. As with the findings on fine amounts, the immediate payoff of the prize amount is dwarfed by the future payoffs of winning or losing the Tour and consequently the prize amount needs to be substantially lowered in order to make a difference.

C3. Results

Results: Enriching our model by including future payoffs leads to interesting results. In terms of reasonable results, we found that the UCI could put a cap of about €1.6 million on Tour de France champion salaries and increase the probability of catching a cyclist doping to above 18.1% in order to force an optimal outcome. In addition, I found that changing the discount factor would have no effect because it affects rewards and punishments equally. My findings related to fines and prize amounts is that they would need to be raised in absolute value to absurdly high amounts in order to make a difference, primarily because the immediate payoffs of both are so much smaller than the future cash flows that were introduced in this model. Intuitively it makes sense that immediate payoffs are less valuable due to future cash flows but the values found seem unreasonably high. This, along with the model's result that increasing the suspension time cannot force an optimal outcome, are the primary weaknesses of this second model. The main takeaway from this second model is that future payoffs, such as a salary increase, matter more than immediate payoffs, such as fines or prizes. If this game approximates reality better than the first one, it suggests that the most successful anti-doping measures will be those that affect a cyclist's future payoffs, rather than their immediate payoffs.

D. Repeated Game

D1. Strategic Form Representation and Grim Trigger Strategies

This next game takes us back to the first one we looked at—but now the game is repeated infinitely. Repeating the game makes sense since many top cyclists race in the Tour more than once in their life. But why is it infinitely repeated if a cyclist's career is always finite? The rationale for making this an *infinite* repeated game requires us to think of this model as modeling any cycling race and not just the Tour de France. Most cyclists participate in hundreds of races throughout their career and in many of them it makes sense to dope. A payoff that the cyclist gets in the hundredth race will actually be close to the payoff they get in the infinitely repeated game because the future payoff is just the sum of a constant and (with a discount factor between 0 and 1), as the number of summations increases, the future payoff approaches a limit. In addition, modeling our game as infinite leads to the possibility of cyclists cooperating.

The strategic form representation of the third game is exactly the same as the simple single stage model, except that now it repeats infinitely. A strategic form representation and the 2x2 matrix of the single stage game are given below:

Players: $\{i = \text{A, B}\}$

Strategies: {Dope, not Dope}

Payoffs:
$\pi_i(\text{D, D}) = 0.5\times0.9\times(500{,}000) – 0.5\times0.1\times(500{,}000+980{,}000) –0.5\times0.1\times(980{,}000)$
$\pi_i(\text{D, ND}) = 0.9\times0.9\times(500{,}000) – 0.9\times0.1\times(500{,}000+980{,}000) –0.1\times0.1\times(980{,}000)$
$\pi_i(\text{ND, D}) = 0.1 \times (500{,}000)$
$\pi_i(\text{ND, ND}) = 0.5 \times (500{,}000)$

PLAYER B

PLAYER A		Dope	not Dope
	Dope	102,000 102,000	262,000 50,000
	not Dope	50,000 262,000	250,000 250,000

In infinite games, we use the solution concept of sub-game perfect Nash equilibrium (SPNE). An SPNE is an NE that occurs in every subgame of the infinite game. [19] It has been proven that if a game has a unique NE in its single stage form, then this NE is also the SPNE of the game.[20] Since the single stage NE was {not Dope, not Dope}, it is also the SPNE of the infinitely repeated game. However, since this is an infinite game it is actually possible to have an equilibrium of {not Dope, not Dope}, without changing any other variables associated with the payoffs, given a high enough discount factor.

We do this by using a grim trigger strategy. Informally, a grim trigger strategy is a set up where players decide to cooperate by playing an agreed-upon strategy. If any player deviates from the agreed-upon strategy, then the other player punishes them by playing another strategy—usually one that leads to a suboptimal outcome—for an infinite amount of time. Formally the grim trigger strategy is defined as follows:

> Given a set of players $S_i = \{S_1, S_2, S_3, \ldots, S_N\}$, all players agree to cooperate by playing a *norm* strategy whereby all players play S_i^*. For any particular player s_i, if in the previous round, all players played the *norm* strategy, then s_i continues playing s_i^*. If not, s_i plays a *punishment* strategy by playing $s_i^\#$ forever.[21]

For this game, the *norm* strategy is to play {not Dope} and the *punishment* strategy will be for a player to play {Dope}. In addition to this, since the feasibility of a grim trigger strategy depends on the discount factor, I will leave this as a general variable for now so we can figure out the range for which we can sustain an NE of {not Dope, not Dope}. There are two important payoffs to calculate. First there is the players' payoff if they play the norm strategy for the entire game. Secondly, there is the players' payoff if they deviate in a round and are punished for the rest of the game. Without loss of generality, let us assume that the player deviates in the first round. The payoffs for cooperating and deviating are as follows:

$$\text{Cooperate: } \sum_{i=0}^{\infty} 250{,}000 \times \delta^i = \left(\frac{250{,}000}{1-\delta}\right) \qquad \textbf{(A9)}$$

$$\text{Deviate: } 262{,}000 + \sum_{i=1}^{\infty} 102{,}000 \times \delta^i = 262{,}000 + \delta \times \left(\frac{102{,}000}{1-\delta}\right) \qquad \textbf{(A10)}$$

The grim trigger is sustainable as long as three conditions are true (i) no player will want to deviate from cooperating; (ii) no player will want to deviate from punishing; (iii) no player will want to deviate from being punished. Obviously, any player who is doping will not want to deviate if the other player is also doping. This takes care of condition (ii) and (iii). To figure out condition (i) we need to solve the following equation:

$$\left(\frac{250{,}000}{1-\delta}\right) > 262{,}000 + \delta \times \left(\frac{102{,}000}{1-\delta}\right) \qquad \textbf{(A11)}$$

The solution to (A11) is that $\delta > 0.075$. So as long as the cyclists value the future by a discount factor higher than 7.5%, the NE of {not Dope, not Dope} is sustainable via a grim trigger

[19] The definition of a subgame is provided in the appendix to Section II.
[20] Dutta, Prajitt. *Strategies and Games: Theory and Practice*, (Cambridge, MIT, 1999), 198.
[21] This is my own definition.

strategy. If we use the discount factor of 98.5% we have been using all along, this is more than enough to sustain the grim trigger equilibrium in the infinite game. Cooperation seems to be a very feasible outcome in this game.

D2. Payoff Manipulation

Since the possibility of an NE of {not Dope, not Dope} is now based on cooperation, let's look at what changing some of the payoffs does to (A11). For this analysis, the following simplification of (A11) will be useful:

$$\delta > \frac{262{,}000 - 250{,}000}{262{,}000 - 102{,}000} \qquad \textbf{(A12)}$$

which, in general form for Player A, is:

$$\delta > \frac{\pi_A(\mathrm{D,ND}) - \pi_A(\mathrm{ND,ND})}{\pi_A(\mathrm{D,ND}) - \pi_A(\mathrm{D,D})} \qquad \textbf{(A13)}$$

Decrease Prize Amount: Decreasing the prize amount decreases π_i(ND, ND), π_i(D, D), and π_i(D, ND). Looking at (A13), if the prize amount approaches 0, then the lower bound for the discount factor will decrease.

Increase the Fine: Increasing the fine amount decreases π_i(D, D) and π_i(D, ND). If we take the limit of (A13) as the fine approaches infinity, we get negative infinity. So the lower bound of the discount factor decreases. Intuitively, since the payoff of deviation decreases, it is easier to cooperate.

Increase probability of being caught doping: Increasing this variable also lowers π_i(D, D) and π_i(D, ND). Therefore, as with the fine amount, increasing the probability of getting caught doping also decreases the lower bound for the discount factor, making cooperation easier.

Decrease P(win|D, ND): Decreasing this variable will lower π_i(D, ND). This means that (A13) gets smaller, so the lower bound for the discount factor decreases.

D3. Folk Theorem

More interesting, however, is finding the set of equilibrium we can sustain using grim trigger strategies. The answer to this question is that virtually any strategy can be sustained given a high enough discount factor. This result was formally proven by Eric Maskin and Drew Fudenberg (1986). It is called the folk theorem. I will give their informal definition of the theorem but not their proof:

> "When either there are only two players or a 'full dimensionality' condition holds, any individually rational payoff vector of a one-shot game of complete information can arise in a perfect equilibrium of the infinitely-repeated game if players are sufficiently patient (533)."

This means that in our two-player, repeated game, any individually rational payoff vector can arise if the discount factor is high enough.[22] An individually rational payoff is "an outcome that Pareto dominates the minimax (Fudenberg and Maskin 533)." A minimax point of any game is the outcome when both players try to play a minmax strategy. The minimax strategy is to minimize the maximum payoffs of your opponent. An outcome is Pareto dominated if some other outcome would make at least one player better off without hurting any other players.

What does the folk theorem say about our repeated game? Suppose we keep the general structure of the game the same, so that π_A(D, ND)>π_A(ND, ND)>π_A(D, D)>π_A(ND, D) and π_B(ND, D)>π_B(ND, ND)> π_B(D, D)>π_B(D, ND).[23] Let's also suppose that we are trying to find a grim trigger strategy where the norm strategy is always for players to not dope. This means that the deviation possible for player A in any stage of the game is π_A(D, ND) and for player B it is π_B(ND, D). It also follows that the punishment strategy of this grim trigger must be π_{AorB}(D, D) because any other punishment strategy would lead to deviation by at least one of the two players.[24] I'll call this the *socially optimal* grim trigger strategy.

If we keep these assumptions in mind, it follows that the minimax point in this game is always π_{AorB}(D, D)—it is always what would be the 'punishment payoff' in the socially optimal grim trigger strategy. So for this particular repeated game, the folk theorem claims that the socially optimal grim trigger strategy can be sustained with a high enough discount factor as long as its norm payoff strictly dominates the punishment payoff for one player and makes no player worst off. In other words, as long as π_{AorB}(ND, ND)> π_{AorB}(D, D). This theorem can be proven with respect to my particular game if we start by writing (A11) in its general form:

$$\left(\frac{\pi_i(\mathrm{ND,ND})}{1-\delta}\right) > \pi_i(\mathrm{D,ND}) + \frac{\delta\times(\pi_i(\mathrm{D,D}))}{1-\delta} \qquad \textbf{(A14)}$$

If we redefine the variables such that D = π_A(D, ND) = π_B(ND, D); C = π_A(ND, ND) = π_B(ND, ND); P = π_A(D, D) = π_B(D, D):

$$\frac{\mathrm{C}}{1-\delta} > D + \delta \times \left(\frac{\mathrm{P}}{1-\delta}\right) \qquad \textbf{(A15)}$$

This can be simplified to:

$$\delta > \frac{D-C}{D-P} \qquad \textbf{(A16)}$$

Since 0≤δ≤1,

$$\mathrm{C} > \mathrm{P} \qquad \textbf{(A17)}$$

[22] A game of complete information is one where each player is aware of all the sequences, strategies, and payoffs throughout gameplay. This is the case in our repeated game.

[23] This general structure of payoffs is called a prisoner's dilemma structure. It makes sense to depict cycling races in this fashion because the 'dilemma' is that although the payoff to playing 'fair' is higher than that of playing 'unfair', the structure of the game leads to an equilibrium where both players play unfair (i.e. dope).

[24] If the punishment strategy were π_i(D, ND), then player $-i$ would have an incentive to deviate from the punishment phase. If the punishment strategy were π_i(ND, D), then player *i* would have an incentive to deviate from the punishment phase. If the punishment strategy were π_i(ND, ND), then both players would have an incentive to deviate in the punishment phase.

D4. Results

In the infinitely repeated game, one of the PSNE was for players to dope. But using a grim trigger strategy, I was able to show that it was possible for players to cooperate by not doping, given a discount factor greater than 7.5%. I also showed that this discount factor would decrease if prize amounts or Pr(win|D, ND) decreased, and if fine amounts or the probability of being caught increased. Additionally, according to the folk theorem players can cooperate in this game by not doping only as long as the payoff to cooperating is strictly greater than the payoff to being punished. In a game where punishing gave a higher payoff than cooperating, a grim trigger strategy of the kind I have looked at would not be possible—even if the discount factor were very high.

E. Repeated Game with Imperfect Detection

E1. Strategic Form Representation

In the final model, I present a repeated game with imperfect detection. A game of imperfect detection is one where player's actions are not observable. In these games, grim trigger strategies are still possible but they need to be modified to account for this complication. Players can use what is called a threshold trigger strategy. This is one where both players cooperate, but if at any point any player's payoff drops below a threshold, then they enter a punishment phase. It is formally defined in Dutta (1999) as follows:

> "Suppose the threshold is a number m. Players start by cooperating and continue to do so as long as both players' payoffs are above m in every stage. If at any point, any player's payoff drops below m, players play the punishment phase for T stages before restarting the strategy."[25]

One element I added here was the idea of a "forgiving trigger." Informally, this is a grim trigger strategy where the punishment phase is finite, and after that both players restart the strategy. The idea is that an infinite punishment is sometimes not necessary to sustain a grim trigger strategy. Sometimes it is enough to just make the length of the punishment long enough.

In the context of my paper, the notion of imperfect detection is interpreted as the repeated game I previously presented, but without the probability of getting caught. Since there is no chance of being caught doping, players cannot know whether their opponents doped and the game becomes one of imperfect detection. Finding a threshold grim trigger is akin to asking whether cooperation is possible among cyclists even when no regulatory body exists to catch or punish them.

The way the game works is that payoffs are no longer fixed but are distributed around an average in some way. Players still observe their own and their opponent's payoffs but they are not sure whether this payoff means that their opponent doped or did not. The way they try to figure this out is by observing whether their payoff falls below the threshold 'm', which is

[25] Dutta, *Strategies and Games,* 235.

π_A(ND, D) for player A and π_B(D, ND) for player B. If it does, it is likely that their opponent doped, giving them a much lower payoff than normal.

I chose to model the game as follows. Suppose the threshold is €50,000 and the payoffs are no longer fixed but are normally distributed around some average, where the average is the fixed payoffs we got in the first repeated game—but now without the probability of being caught. Since the threshold grim trigger strategy will only punish a player if the playoffs fall below the threshold, we need to calculate the probabilities with which (i) a player that cooperates will cause payoffs to fall below 'm' and (ii) a player that deviates will cause payoffs to fall below 'm'. These are denoted as P_C and P_D, respectively. These probabilities can be calculated from the standard deviations by finding their z-scores. P_C is approximately 0.2 and P_D is approximately 0.8. In these games, the probability of detection if you deviate should always be higher. So $P_D > P_C$ will always be true. An in-depth explanation of how I determined P_C and P_D is given in the appendix to this section.

I will now display the strategic form representation of the game. The players and strategies are the same, but now the payoffs are normal distributions. For ease of understanding, I also included how the average of each payoff was calculated. The 2x2 matrix representation of the single stage game without imperfect detection is also displayed because it will be important for the remainder of this section.

Players: {i = A, B}

Strategies: {Dope, not Dope}

Payoff Mean Calculation:

π_i(D, D) = 0.5 × (500, 000)
π_i(D, ND) = 0.9 × (500, 000)
π_i(ND, D) = 0.1 × (500, 000)
π_i(ND, ND) = 0.5 × (500, 000)

Payoffs:

π_i(D, D) = N~(250,000, 235,000)
π_i(D, ND) = N~(450,000,423,000)
π_i(ND, D) = N~(50,000, 47,000)
π_i(ND, ND) = N~(250,000, 235,000)

Probabilities:

$P_C = 0.2$
$P_D = 0.8$

PLAYER B

		Dope	not Dope
PLAYER A	Dope	250,000 250,000	450,000 50,000
	not Dope	50,000 450,000	250,000 250,000

Since this is a game of imperfect detection, the Folk Theorem must be modified. The condition for the existence of a threshold trigger strategy is that $\pi_{AorB}(ND, ND)$ must be greater than $\pi_{AorB}(D, D)$ minus some value. The result of the proof is given below and the entire proof is given in Appendix A.

$$C > P - \frac{C-D}{P_D - P_C} \times \frac{1-\delta}{\delta} \qquad \textbf{(A18)}$$

Where $C = \pi_A(ND, ND) = \pi_B(ND, ND)$; $P = \pi_A(D, D) = \pi_B(D, D)$; $D = \pi_A(D, ND) = \pi_B(ND, D)$; P_C and P_D are defined above. Note that the value subtracted from P is a negative ratio because D>C and $P_D > P_C$.

Given that this is true, we know that the game I just derived has no threshold grim trigger equilibrium since C = P. Thus we need to decrease P by the factor given in (A18). Let's make P 200,000. The new payoffs and probabilities for this version of the repeated game with imperfect detection are:

Payoff Mean Calculation:

$\pi_i(D, D) = 0.5 \times (500, 000) - 50,000$
$\pi_i(D, ND) = 0.9 \times (500, 000)$
$\pi_i(ND, D) = 0.1 \times (500, 000)$
$\pi_i(ND, ND) = 0.5 \times (500, 000)$

Payoffs:

$\pi_i(D, D) = N\sim(200,000, 188,000)$[26]
$\pi_i(D, ND) = N\sim(450,000,423,000)$.
$\pi_i(ND, D) = N\sim(50,000, 47,000)$.
$\pi_i(ND, ND) = N\sim(250,000, 235,000)$

[26] Standard deviation is still 94% of the average. This means that P_C will decrease from 80% to about 79%.

Probabilities:

$P_C = 0.21$
$P_D = 0.79$

E2. Grim Trigger Equilibrium

Suppose we want to find the length of the punishment ('T') for which there exists a socially optimal grim trigger strategy. There are two primary components to the equation we need to solve:

$$V_C = C + \delta[(1 - P_C)v + P_C(P \sum_0^T \delta^t + \delta^T v)] \quad \textbf{(A19)}$$

$$V_D = D + \delta[(1 - P_D)v + P_D(P \sum_0^T \delta^t + \delta^T v)] \quad \textbf{(A20)}$$

Where V_C is the payoff of cooperating and V_D is the payoff of deviating.

V_C is calculated as follows. First there is the immediate payoff from cooperating, the next stages are discounted, and there are two possible outcomes. With a probability of $1-P_C$, the payoff for player *i* will not fall below 'm,' and they will get the future payoffs from continued cooperation. With a probability of P_C, the payoff for player *i* will fall below 'm', and they will be punished for T stages before the game returns to cooperation for the rest of the stages. V_D is calculated similarly. The inequality that needs to be solved is:

$$V_C > V_D \quad \textbf{(A21)}$$

Using the values from the second version of the repeated game with imperfect detection, one gets that the T needed to sustain the threshold grim trigger strategy with a discount factor of 0.985 is T > 12. T is so high because players can deviate without being detected and so the payoff to deviation increases. Self-policing between cyclists is certainly possible, but condition (A18) must be true for the game. T will have to be high, but it is certainly possible for it to be smaller if P is lowered even more. If, for example, P is lowered to 150,000 the solution to (A21) becomes T>5.

E3. Payoff Manipulation

How does manipulating the payoffs change the length of punishment? Intuitively, as P or D increases, the length of punishment must increase in order to compensate for the higher payoff of being punished or deviating. In addition, as P_C increases or P_D decreases, the length of the punishment must also increase because the probability of being detected if you deviate has fallen and so the payoff of deviation has increased. As the discount factor increases, the length of the punishment can decrease because the players value the future more and the lower payoff they get from punishments (in relation to cooperating) is increased.

The same results we found with the previous repeated game (without imperfect detection) all remain the same, except that now the magnitude of the changes in payoffs related to winning (i.e. prize amount) and those related to doping (i.e. fines) need to increase in absolute value relative to the previous model because deviation has become more tempting due to the fact that it may not even be detected.

F. Conclusion

My game theoretic analysis produced some interesting results. For the simple single stage game, it was possible to change the NE to an optimal outcome by decreasing prize amounts, increasing fines, or increasing the probability of doping detection by reasonable amounts. Adding future payoffs to this model led to the complex single stage game, which produced a variety of interesting results, suggesting that immediate payoffs have less impact than future payoffs and that the UCI could have more success if it focuses on policies that affect future payoffs, such as salary caps on Tour de France champion salaries. The repeated game led to the result that the discount factor would need to be higher than 7.5% to sustain a grim trigger strategy—which seems reasonable. Introducing the folk theorem also led to the result that the payoff to cooperation must always be higher than the payoff to punishment in order to sustain the socially optimal grim trigger. Finally, the repeated game with imperfect detection lead to the result that it is possible for cyclists to cooperate even in the presence of imperfect detection, given that the difference between the payoffs to cooperation and detection is high enough.

There are still areas for improvement in all my models, but my hope is that as they improve they will better approximate reality and lead more realistic results. One obvious next step would be to try and create a model that incorporates more than two players. This was a serious drawback in all my models, given that the Tour has hundreds of competitors. Some other possible avenues for future investigation would be dynamic games and behavioral game theory. Dynamic games are ones where the payoffs of the game can change due to events in previous stages of the game. This would allow for the possibility of incorporating changes to the game if, for example, a cyclist dopes, is caught, and their future payoffs to winning now change. Creating games that incorporate behavioral game theory would take account of the fact that sometimes sports players do not make the rational decisions they are supposed to make in traditional game theory.

III. ECONOMETRIC ANALYSIS

A. Introduction

In part III of my paper, I will complement the theoretical analysis I did in part II with a statistical analysis, which uses regression models to study the relationship between doping regulations and doping behavior. I will first describe and analyze the dataset. Then I will construct an OLS and fractional logistic regression and discuss the results. Thirdly, I will perform a LASSO regression and discuss the results. Finally, I will compare all three models and conclude.

B. Dataset

All the data is from 1968 and 2017 inclusive. The data starts in 1968 because that was when the first riders tested positive for doping (testing started in 1966). It ends in 2017 because I considered the data from 2018 and onwards to be too new and subject to change. In total there were 16 different variables for each year. One of them was the dependent variable and the other 15 were independent variables. Five of the independent variables were ones of interest while the other 10 were control variables.

The dependent variable I am testing is the percentage of cyclists that tested positive for performance enhancing drugs (ped_tot). It was calculated as the ratio of riders that tested positive for doping in a given race over the total number of entrants in that race. Riders were considered to have tested positive for PED's if at least one of the following criteria were met: (i) refused to submit to a test, (ii) tested for hematocrit levels greater than 50%, (iii) confessed to doping, or (iv) were sanctioned by some organization for doping. It is important to note that as revelations surface and extra tests are carried out, the percentage of riders that tested positive for PED's in a given race may change. The data from the past ten years is especially susceptible to change, but I still decided to keep it in my regression because some interesting anti-doping measures were implemented in the early 2000s. The data on PED testing was taken from a French website on doping in cycling that meticulously collected its data through a variety of media.[27]

Five of the independent variables are variables of interest (i.e. not control variables) and they are all dummy variables that state whether an anti-doping policy was in effect in a given year. If it was in effect, then this variable is equal to 1 and it is equal to 0 otherwise. The data for these variables was taken from a variety of sources. One important assumption I made is that once a test has been put in place, it will be implemented in all future races. A brief description of each of these variables is given below:

> amph_test*:* Denotes whether there were anti-amphetamine tests being carried out in a given year. The first reliable anti-amphetamine test was administered in the Tour de France in 1974.[28]
>
> anti_epo_test*:* Denotes whether there were anti-EPO tests being carried out in a given year. The first anti–EPO test was administered in the 2001 Tour de France.[29]

[27] Stéphane Huby. "Tour de France: les vrais chiffres du dopage." (Cyclisme-dopage. 2019).
[28] UK Anti-Doping. "Story of Anti-Doping." (UKAD. 2019).
[29] Verbruggen, Hein. "UCI introduces the EPO test." (Hein Verbruggen, 2019).

bio_passport*:* Denotes when the biological passport was implemented for Tour de France riders, which started in 2008. The biological passport is an anti-doping method whereby each rider's physiological indicators are tracked throughout their career, so that it can be easier for authorities to detect when a specific rider's baseline indicators are abnormal, which could indicate the presence of PED's.[30]

night_test*:* Denotes when nighttime tests were carried out in the Tour de France, which started in 2015. Prior to that, riders were only tested during the day.[31]

ooct*:* Denotes when out of competition testing was carried out in relation to the Tour de France. Out of competition testing is testing that occurs before or after the competition starts or ends. It started in 2001 but a report by the Cycling Independent Reform Commission indicates that it was not prioritized until about 2006. In fact, between 2002 and 2005 only 2.5% of doping tests carried out by the UCI were out of competition. For that reason, this variable takes effect starting in 2006 in my model.[32]

10 independent variables were control variables. These were all taken from the official Tour de France website that publishes statistics on the race since its inception in 1903.[33] A brief description of each variable is included below:

num_stages*:* Number of stages in a given Tour de France race. Usually there are 21 stages and a shorter race at the beginning called a prologue, which counts as 0.5 of a stage.

tot_length*:* The total length of the race measured in kilometers.

avg_speed*:* Average speed of racers measured in km/h.

num_entrants*:* The number of cyclists that entered the race.

num_finishers*:* The number of cyclists that finished the race.

first_prize_amount*:* The prize amount awarded to the cyclist that came in first. Measured in 2018 euros.

total_prize_amount*:* The total amount of money designated as prize money in a given race. Measured in 2018 euros.

tot_time_winner*:* The total number of hours it took for the winner of the Tour to finish the race.

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[30] Fotheringham, Alasdair. "Cycling: Riders to be banned from Tour de France without clean 'passport.' " (Independent*.* October 26, 2007).
[31] Daniel Benson, ""Tour de France: night-time doping tests done before race." (Cycling News*.* July 4, 2015).
[32] Cycling Independent Reform Commission. Report to the President of the Union Cycliste Internationale. (CIRC, 2015), 127.
[33] LeTour. "The History of the Tour de France." (LeTour, 2019).

second_lag_time*:* The difference between the total time of the first and second place cyclist measured in hours.

year: the year in which the race took place

B. Dataset Diagnostics

There are two aspects of this dataset that are concerning. Firstly, there is severe *imperfect* multicollinearity between predictor variables. Imperfect multicollinearity on some variable X_1 will cause its estimated coefficient B_1 to have a large sampling variance, and B_1 will be imprecisely estimated. This is less of an issue if it occurs solely with control variables, but if multicollinearity occurs with the variables of interest (i.e., the dummy variables), then we cannot be confident that the coefficient estimates on these variables are precise.

The second issue with this dataset is that there are many predictor variables (15) relative to the sample size (50 observations). This will likely lead to overfitting, meaning that the coefficient estimates on the variables of interest will not generalize to out of sample observations. It is unclear what the ideal number of predictors would in the case of this specific dataset. Harrell[34] argues that a good rule of thumb is that there should be between *n/10* and *n/20* predictor variables, where *n* is the sample size. However, other studies have argued that such 'rules of thumb' are too simplistic and have proposed more nuanced criteria.[35] Due to lack of space, I will not discuss different overfitting criteria. I think one can at least agree that overfitting is at least a possibility with this dataset. Hence, if we find an opportunity to drop predictor variables without leading to more widespread issues (e.g., omitted variables bias) then we should do so.

Given that multicollinearity and overfitting are the two hurdles to estimating the effect of our variables of interest on the response variable, I will attempt to lessen these dual issues by specifying a regression that uses a *subset* of predictor variables that suffers from less multicollinearity. Predictor variables will be *excluded* from the subset of final predictor variables based on multicollinearity considerations. There are a myriad of ways to engage in variable selection. The most basic option is to *manually* select predictor variables based on subject area knowledge. There are also *automated* variable selection methods that will select predictor variables according to some algorithm. The most basic of these methods is stepwise regression. There are also methods that use penalization terms, such as ridge, LASSO, and elastic net regressions. In terms of the automated selection methods, Harrell[36] argues convincingly that stepwise regression is in general not a good approach to variable selection because this method selects variables based on their estimated regression coefficients rather than their true values. Hence, if a coefficient is overestimated rather than underestimated it may be erroneously retained. Due to these considerations, one of the penalization-based methods will be preferred. I will opt for a LASSO regression because it drops predictor variables from a regression, whereas a ridge regression merely shrinks them towards zero. I will not include an elastic net regression because it gave a similar result as the LASSO regression. Elastic net is best understood as a combination of LASSO and ridge regressions; however, for the purposes of this essay understanding its exact specification is unimportant.

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[34] Harrell, *Regression Modelling Strategies*, 72-75.
[35] Riley et al., "Minimum sample size for developing a multivariable prediction model: PART II - binary and time-to-event outcomes," 2018.
[36] Harrell, *Regression Modelling Strategies*, 67-72.

In what follows, I will perform three regressions to estimate the effects of anti-doping policies in the Tour de France. First, I will manually select a subset of predictor variables to remove multicollinearity and reduce the possibility of overfitting. Then I will run both an OLS and fractional logistic regression on this subset of predictor variables. Secondly, I will use LASSO to automatically select a subset of predictor variables and then estimate their coefficients. I will conclude by comparing the results of these different approaches.

D. OLS Regression: Manual Feature Selection

The first method will be to manually select a subset of predictor variables and then perform a regression on the subset of variables. Variable selection will be based on multicollinearity between variables. I will assess multicollinearity between variables based on a Pearson correlation matrix and Variance Inflation Factors (VIF). I will consider a correlation above |0.7| to be severe and worthy of further investigation. A Pearson correlation matrix of all variables in this dataset can be found in the appendix to this essay. A table of the VIF for each predictor variable based on an unrestricted OLS regression (e.g., a regression of ped_tot on all predictor variables) can be found below.

Table 1.1: VIF by Variable Based on Unrestricted OLS Regression

Variable	VIF	Variable	VIF
amph_test	6.8	year	61
epo_test	12	num_stages	5.5
bio_passport	7.7	tot_length	45
night_test	2.7	avg_speed	45
ooct	7.6	num_entrants	19
num_finishers	9.9	tot_prize_money	16
first_prize_money	18	tot_time_winner	120
second_lag_time	2.2		

All values rounded to two significant figures.

In practice a VIF greater than 10 is typically a sign of severe multicollinearity that merits further investigation. Based on this criterion, the following variables merit further inspection: epo_test, first_prize_money, year, tot_length, avg_speed, num_entrants, tot_prize_money, tot_time_winner. Immediately, there are at least two pairs of variables where either of the variables in each pair may be 'redundant'. The variables for total prize money (tot_prize_money) and first place prize money (first_prize_money) have a correlation of 0.92, and the variables for the number of entrants (num_entrants) and number of finishers (num_finishers) have a correlation of 0.88. It makes sense that these pairs of variables are so severely correlated. Intuitively, the first-place prize each year is just a subset of the total prize money, and the same with the number of finishers relative to the number of entrants. Now they are not perfectly correlated because there may not be a constant ratio of first place prize money to total prize money and number of finishers to number of entrants. Nevertheless, I think these variables share a high enough correlation and are intuitively related, such that dropping one of them is warranted. I will choose to drop the variable in this pair with the highest VIF value. In doing so,we stand to also lessen multicollinearity that existed between the removed variable and other variables. Hence, we drop first_prize_money and num_entrants.

The next variable I want to focus on is the year variable. While it is important to control for time-fixed effects, this should be balanced with multicollinearity considerations. In that regard, it seems that the year variable exhibits a high correlation with some of the variables of interest. In particular, year has a correlation of 0.82 and 0.74 with epo_test and ooct, respectively. This correlation would affect the estimation of coefficients on the variables of interest, so I remove the year variable. One issue with this move is that it may lead to omitted variable bias. However, I think it is worth assuming this risk given the severe multicollinearity with the variables of interest. Finally, I also remove the avg_speed variable because it is unclear how it was measured by Tour de France officials. Hence, it is in some respect uninterpretable. This doesn't mean it may not be useful to control for omitted variable bias but given that it also has a high VIF and its information is captured by other variables (e.g., total length, total time of winner), I choose to drop it to get rid of multicollinearity. Again, I think the risk of omitted variable bias is worth taking to lessen some of the multicollinearity in this data set.

Based on these moves we are left with the following subset of variables and their respective VIFs (based on a regression of ped_tot on this restricted set of variables):

Table 1.2: VIF by Variable Based on Restricted OLS Regression

Variable	VIF	Variable	VIF
amph_test	2.3	num_stages	2.9
epo_test	4.9	tot_length	3.0
bio_passport	5.7	tot_prize_money	5.5
night_test	1.6	ooct	6.9
second_lag_time	1.7	num_finishers	3.6

All values rounded to two significant figures.

These VIF values are much lower than before, which suggests that we got rid of some multicollinearity. Note, however, that there is still multicollinearity present between some of the variables of interest. We have a correlation of 0.78 between epo_test and ooct and 0.89 between bio_passport and ooct. Resolving how to account for these remaining multicollinear relations goes beyond the scope of this paper, so I just want to note that we need to keep this in mind when reading off the estimates of these variables.

D. OLS and Fractional Logistic Regression: Results

The final model uses the following 10 predictor variables: amph_test, epo_test, bio_passport, night_test, ooct, num_stages, tot_length, num_finishers, tot_prize_money, second_lag_time. The response variable is ped_tot. The final model and results are presented below. Note that the standard errors are robust to heteroskedasticity. I will explain why later on.

Table 2: Restricted OLS Regression Results

Variable	Estimate	Standard Error	T Value	Significance level
intercept	0.95	0.15	6.5	***
amph_test	0.084	0.028	3.0	**

epo_test	– 0.042	0.021	-2.0	
bio_passport	– 0.082	0.019	-4.2	***
night_test	– 0.025	0.019	-1.3	
ooct	– 0.082	0.020	-4.2	***
num_stages	– 0.024	0.0078	-3.1	**
tot_length	2.9×10^{-5}	3.6×10^{-5}	0.81	
num_finishers	– 0.0023	3.0×10^{-4}	-7.7	***
tot_prize_money	4.2×10^{-8}	9.5×10^{-9}	4.4	***
second_lag_time	0.41	0.12	3.5	**

Multiple R^2: 93.1% Adjusted R^2: 91.4%

*'***' =0.001 significance level; **= 0.01 significance level; * = 0.05 significance level*

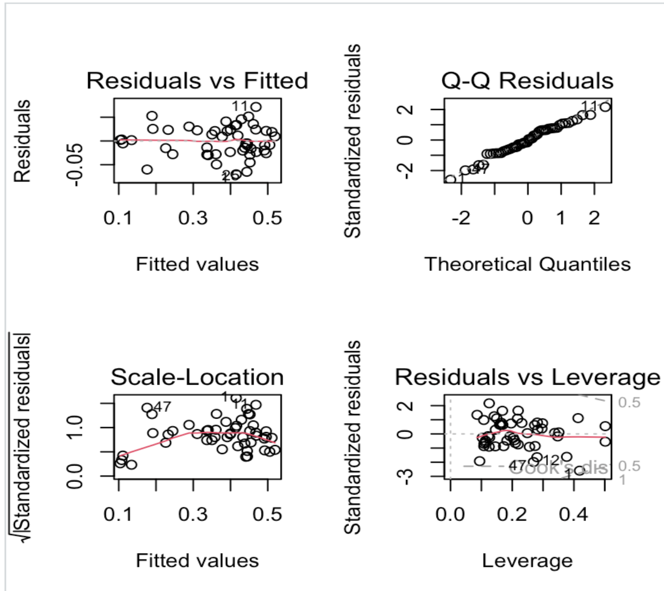
All values are rounded to two significant figures.

The multiple and adjusted R-squared are both above 90%, which suggests that the model is a good fit. This could, however, be due to overfitting, since there are still 10 explanatory variables for 50 observations, when according to Harrell's rule of thumb (see footnote 34 above), there should be at least 100. The magnitudes and signs of the coefficients for the anti-doping tests are reasonable for the most part. Remember that the response variable is a percentage, so the coefficient for the biological passport is saying that an implementation of this policy decreases ped_tot by 0.082%. One issue is that the sign on the amph_test coefficient is positive, which does not make sense. This could be due to the presence of multicollinearity, but I am unsure. It merits further investigation.

As for the control variables, it is interesting that num_stages is negative. This could be because increasing the number of stages increases the possibility of being tested each day, thereby inducing players to dope less out of fear of getting caught. The total prize money also had a surprisingly small effect on ped_tot. Finally, it was odd to me that the sign on the coefficient for second_lag_time is positive. Intuitively, a shorter lag between 1st and 2nd place would induce more competition, which in turn could induce a higher temptation to dope.

The following variables were not significant: epo_test, night_test, and tot_length. It is not surprising that the epo_test is not significant. While it was implemented in 2001, Lance Armstrong went on to win 4 Tour de France races between 2001 and 2005 using EPO. That night_test was not significant either suggests that perhaps it is not effective or it has not been prioritized— this was the case with out of competition testing between 2002 and 2005, when it constituted only 2.5% of overall anti-doping testing. Finally, most surprising is that tot_length was not significant. One would think that longer races mean that cyclists are more tired and the temptation to use PEDs is greater. Finally, let's run some model diagnostics.

Restricted OLS Regression Model Diagnostics



On the top left, the flat horizontal line on the residuals vs. fitted values plot indicates that the relationship between dependent and independent variables is likely linear. The linear relationship on the top right Q-Q plot indicates that residuals are likely normally distributed. The bottom left plot has a non-horizontal line, which indicates that the residuals exhibit *heteroskedasticity*. The residuals vs. leverage plot on the bottom right is used to identify outliers that may affect the regression. It appears there were no such outliers. Hence, most of the assumptions for OLS appear to be met. However, the presence of heteroskedastic errors makes the estimation of coefficient standard errors erroneous. To correct for this, we need to estimate heteroskedastic robust errors for the coefficients. This was done using the HC1 method in the 'sandwich' package.[37] The reported standard errors in the model results above are the heteroskedastic robust ones.

Note that this OLS regression is not well-suited for prediction. That is because although OLS regressions are typically used to model continuous dependent variables, it cannot account for *bounded* continuous dependent variables. The result is that if we were to use this model to make predictions of ped_tot, we might get results below 0 or above 1, which obviously doesn't make sense. This is less of a problem if the dependent variable does not have many values near the boundary of 0 and 1, which is the case with our dependent variable, ped_tot : its minimum is 0.12, maximum is 0.54, and the middle 50% of the data (Q1-Q3) is between 0.32 and 0.45. However, if we want to ensure better predictions and guard against errors in the estimation of coefficient standard errors (and hence significance tests), then we should use other methods, such as a fractional logistic or beta regression.[38] For completeness sake, I ran a fractional logistic regression, which is basically a logistic regression where the dependent variables are fractions of 1 (as opposed to the usually binary 1 and 0).[39] The regression was run on the same restricted set

[37] See documentation for 'sandwich' at https://sandwich.r-forge.r-project.org/reference/vcovCL.html for more information on HC1 estimator for heteroskedastic errors.
[38] See Kubinec, "What To Do (And Not to Do) with Modeling Proportions/Fractional Outcomes," 2022 (Kubinec, 2022)
[39] The fractional logistic regression was run using the glm package with the method quasibinomial('logit')

of variables on which the OLS regression was run, and the standard errors are heteroskedastic robust errors calculated using the same method as the OLS regression.

Table 3: Fractional Logistic Regression Results

Variable	Estimate	Standard Error	T Value	Significance level
intercept	1.9	0.63	2.9	**
amph_test	0.34	0.12	2.7	**
epo_test	– 0.19	0.091	-2.0	*
bio_passport	– 0.48	0.11	-4.5	***
night_test	– 0.42	0.12	-3.4	***
ooct	– 0.35	0.084	-4.2	***
num_stages	– 0.096	0.035	-2.7	**
tot_length	1.0×10^{-4}	1.6×10^{-4}	0.62	
num_finishers	– 0.010	1.4×10^{-3}	-7.2	***
tot_prize_money	1.9×10^{-7}	4.9×10^{-8}	4.0	***
second_lag_time	1.7	0.50	3.5	***

*'***' =0.001 significance level; **= 0.01 significance level; * = 0.05 significance level*
All values are rounded to two significant figures.

Compared to the OLS regression, the coefficients for most of the variables increase by approximately one-tenths place. Furthermore, whereas night_test and epo_test were insignificant in the OLS regression, they are deemed significant in the fractional logistic regression. Finally, now the tests that have the strongest effect on ped_tot are bio_passport and night_test, whereas under the OLS regression they were bio_passport and ooct. Overall, the fractional logistic regression differs from the OLS regression by deeming more anti-doping policies significant. Given more space, I would engage in a deeper discussion on whether fractional logistic regression was the appropriate method to use or whether, for example, a beta regression would have been more well suited.

E. LASSO Regression

LASSO (least absolute shrinkage and selection operation) is a regression method that shrinks the value of independent variables towards a central point. It uses a method called L1 regularization, which shrinks coefficients according to squared error *and* the absolute value of their magnitude (called the 'penalty term'). The variable λ is a parameter that controls the strength of the penalty term and it differs depending on the data set. As λ approaches infinity, all the predictor variables approach 0. A representation of the LASSO regression is depicted below:

$$\sum_{i=1}^{n}(y_i - \sum_{j} x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{p} |\beta_j|$$

The LASSO regression is well suited to correct for overfitting because it selects only a certain number of explanatory variables to include in the final regression. However, it is not robust to multicollinearity because when two or more variables are correlated, it tends to arbitrarily select one to drop. This is a weakness of LASSO that is important to keep in mind moving forward. Although all the coefficients have been shrunk, they are still interpretable in the same sense as an OLS regression.[40]

The LASSO regression model was built in R using the cv method in the 'glmnet' package to select the optimal lambda. Cv.glmnet performs an n-fold cross-validation. Cross-validation is a process whereby a dataset is first split into a training and validation subset. Then the training set is split into n-folds. The model is trained on n-1 folds and its mean-squared error (MSE) is calculated on the remaining fold. The process is repeated n times, each time selecting a distinct fold to calculate MSE. Finally, an average MSE is calculated based on the MSE for each fold iteration. Cross-validation is done for a sequence of lambdas and the one yielding the lowest average MSE is selected and trained on the entire initial dataset. I performed a 5-fold cross validation where cv.glmnet automatically selected a sequence of lambdas.[41] Since this dataset was so small, I used the entire dataset to train across 5-folds and did not separate out a validation set.

Typically, we estimate the coefficients based on the lambda that resulted in the lowest MSE (lambda.min). However, we can also use the lambda that results in the lowest MSE plus 1 standard error (lambda.1se). In this case, if we estimate a model based on lambda.min, we get 11 coefficients, whereas estimating a model based on lambda.1se gives 9 coefficients. So lambda.1se gives us the model that has a lesser probability of overfitting. The results of the model estimated based on lambda.1se are given below.

Table 4: LASSO Regression Results

Variable	**Estimate**
intercept	0.87
amph_test	0.025
epo_test	–0.015
bio_passport	–0.10
night_test	–0.047
ooct	–0.080
num_stages	–0.015
num_finishers	–0.015
tot_prize_money	1.5×10^{-8}
second_lag_time	0.16

All values are rounded to two significant figures.

All the anti-doping tests were included by the model, but the amph_test coefficient was positive, which suggests that multicolinearity may still be present. As for the control variables, it is interesting that num_finishers had a negative sign. One would think that increasing the number of finishers makes the race more competitive, thereby increasing the probability of doping. It was

[40] Although LASSO standardizes variables before shrinking them. It returns coefficients on the *unstandardized* values when using the package glmnet. So, the coefficients are still interpretable.

[41] The small size of the dataset meant that the cross-validation policy produced different optimal lambdas on different iterations of the function. To solve this, I iterated cv.glmnet 100 times with a loop.

interesting that tot_prize_money has such a low coefficient. One would think that this is one of the strongest motivators for doping. The R-squared of the model was 88.7%, which suggests that the model was a good fit. But this model still has a lot of explanatory variables relative to the sample size, so this could be due to overfitting.

G. Conclusion

The OLS and LASSO regressions estimated negative coefficients of similar magnitudes for the anti-doping tests—except in the case of bio_passport where LASSO estimated a much higher coefficient relative to OLS (i.e., –0.1 vs. – 0.082). The fractional logistic regression estimated coefficients that were about one-tenths of a place higher than with their LASSO and OLS counterparts. In terms of the variables of interest, epo_test and night_test were deemed insignificant in the OLS model but were included in the LASSO and fractional logistic regression. The most effective tests, according to the LASSO and OLS models, were the biological passport and out of competition testing, while it was the biological passport and night test for the fractional logistic regression. One issue with all three models was that the sign on amph_test was positive. This could be due to the fact that neither approach was able to eliminate multicolinearity. In all models, the coefficient on total prize money was very small. In addition to this, the coefficients on num_stages and num_finishers were negative when one would think that they should be positive. If we suppose Harrell's rule of thumb that there must be *n/10* to *n/20* predictors for some sample size *n*, then all three models may have suffered from overfitting (see fn34 above).

It is unclear to me which of these models is the 'best'. If we are interested in prediction, then this would likely be the fractional logistic regression (since OLS cannot account for bounded continuous variables) or LASSO regression. Furthermore, the fractional logistic regression and LASSO regression deem all anti-doping policies to be significant, while the OLS regression deems some insignificant. This discrepancy might be a point in favor of the LASSO and fractional logistic regression over the OLS one. However, the estimates on coefficients are more similar in magnitude for the OLS and LASSO regression. I am unsure why this is the case, but it would be interesting to know whether this discrepancy could inform our choice of models.

The final conclusions, according to all models, is that the most effective tests are the biological passport, out of competition, and (according to the fractional logistic model) night testing. These findings suggest that the UCI would have more success in its anti-doping efforts if it focused less on tests to combat specific drugs and more on tests that test for a broader set of drugs (i.e., biological passport) or more expansive testing procedures (i.e., out of competition testing / night testing).

IV. CONCLUSION

Let's summarize the results of this paper. In section II, the single stage models suggest that if one limits the game to immediate payoffs, then the optimal outcome (i.e., both players not doping) can be enforced by changing prize amounts, fines, or the probability of doping detection by reasonable amounts. Adding future payoffs leads to the result that future payoffs matter more than immediate payoffs. Finally, the last two repeated games show that—given a high enough discount factor or punishment length 'T'—cooperation is possible both when there is a regulatory authority and when there is no regulatory authority. In section III, the statistical models suggest that the athlete biological passport, out of competition, and night testing are the most effective anti-doping measures to be rolled out by the UCI thus far.

What can we say in conclusion about doping in the Tour de France? First I would like to note that the comparison of the findings from sections II and III are limited by the fact that many of the variables in the game theoretic models were not included in the regression models. In particular, fines and suspension times were missing in the regression models (unfortunately I could not find the data anywhere). In comparing the two sections, we are limited mostly to anti-doping tests and prize money.

With respect to the anti-doping tests, the single stage game theoretic models dealt with this as the probability of getting caught if you dope. It was initially 10%. In the first model the result was that it would need to increase to 13.5% to force an optimal outcome, while in the second model it needed to increase to 18.1%. Whether it's 13.5 % or 18.1%, let's call this the 'optimal detection value.' All regression models agreed that the biological passport and out of competition testing were statistically significant anti-doping tests. This suggests that perhaps those two tests get us to (or close to) the optimal detection value found in the game theoretic models. Perhaps the tests that were not significant in some models, such as the EPO test in the OLS model, also moved us closer to the optimal detection value as well—but not close enough. It would be interesting to see whether there is a relationship between this 'optimal detection value' and coefficient significance in the regressive models.

There is only one variable not related to anti-doping testing that is common across both sections: prize amount. In the first single stage model in section II, it was possible to change prize money to force an optimal NE. In the second single stage model with future payoffs it was possible—although unreasonable because the amount was so high. The results from the regressive models were that total prize money is significant, but not first place prize money. Perhaps future game theoretic models need to take into account the fact that total prize money could matter more for the payoffs of players than first prize money—but I am not sure how that is possible yet.

Some room for improvement would be to augment the regressive models with more variables in order to have more room for comparison with the game theoretic models. Some key variables to include would be fines, suspension times, and the salary of Tour de France champions. The problem is that this could just exacerbate the issue of over fitting we found in the regressive models. But perhaps we could find a way to remove more predictor variables from the dataset based on more analysis, meaning that we could have a more parsimonious model with less risk of overfitting (however, this may introduce the risk of omitted variable bias).

There are still a lot of questions. In particular, more work needs to be done to understand the impact of prize money on doping. Does it actually have any effect on an athlete's decision to dope? Or is it such a small factor when compared to the future payoffs of an increased salary or the reputation that comes with winning—as we concluded in the complex single stage game? In addition, why does total prize money matter more than first place prize money in our regressive models? Perhaps this is because the amount of money up for grabs increases not just for first place but also for any of the other prizes. There is still a lot of work to do, but my hope is that this study can convince organizations that game theory and econometrics have the potential to be very useful in making decisions about anti-doping regulation in sports.

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APPENDIX to Part II: GAME THEORY

Normal Distributions of (ND, ND) and (ND, D)

Denote the normal distributions of (ND, ND) and (ND, D) as N_1 and N_2, respectively. The averages of N_1 and N_2 are the payoffs one would get in the repeated game (without imperfect detection) if there were no probability of getting caught. So we have N1~(250,000, σ_1) and N2~(50,000, σ_2).

Now we have to decide the values of σ_1 and σ_2. The only restriction was that $P_D > P_C$. I decided to set P_C to 0.2. This is equal to a z-score of about 0.85, so σ_1 solves the following equation $0.85 = \frac{250{,}000 - 50{,}000}{\sigma}$. So (ND, ND) is normally distributed as N_1~(250,000, 235,000). The ratio of the mean to the standard deviation for N_1 is 0.94 and so I chose to apply this same ratio for the normal distribution of (ND, D), giving a standard deviation of (0.94 × 50,000) 47,000, so N_2~(50,000, 47,000).

Proof of (A18)

If the norm strategy is to not dope and the punishment is to dope, C is the immediate payoff to cooperating, which is π_A(ND, ND) = π_B(ND, ND). P is the payoff from being punished, which is π_A(D, D) = π_B(D, D). D is the payoff from deviating, which is π_A(D, ND) = π_B(ND, D). Also v denotes the infinite payoffs to cooperating starting from T = 0 and $0 < \delta < 1$. The payoff to cooperating (V_C) and deviating (V_D) are:

$$V_C = C + \delta[(1 - P_C)v + P_C(P(\sum_{t=0}^{T} \delta^t) + \delta^T v)]$$

$$V_D = D + \delta[(1 - P_D)v + P_D(P(\sum_{t=0}^{T} \delta^t) + \delta^T v)]$$

These two equations can be simplified to:

$$V_C = C + \delta v + P_C(\delta P(\sum_{t=0}^{T} \delta^t) + \delta^{T+1} v - \delta v)$$

$$V_D = D + \delta v + P_D(\delta P(\sum_{t=0}^{T} \delta^t) + \delta^{T+1} v - \delta v)$$

Now we state the inequality and also set $(\delta P(\sum_{t=0}^{T} \delta^t) + \delta^{T+1} v - \delta v) = K$:

$$C + \delta v + P_C K > D + \delta v + P_D K$$

This can be simplified to:

$$\frac{C - D}{P_D - P_C} > K$$

Note that D > C and $P_D > P_C$ so the LHS is a negative ratio. Now we substitute the original equation back in for K:

$$\frac{C-D}{P_D-P_C} > \delta P(\sum_{t=0}^{T}\delta^t) + \delta^{T+1}v - \delta v$$

Now we bring the LHS term to the RHS and δv to the LHS:

$$\delta v > \delta P(\sum_{t=0}^{T}\delta^t) + \delta^{T+1}v - \frac{C-D}{P_D-P_C}$$

Now suppose P = C – n. Where "n" is some integer subtracted from C.

$$\delta v > \delta(C-n)(\sum_{t=0}^{T}\delta^t) + \delta^{T+1}v - \frac{C-D}{P_D-P_C}$$

My goal is to find the conditions on C, P, and T under which this inequality is true. Another way to put this is that we are trying to find the highest possible P such that there is T that would still make the inequality true. If we find that value of P, it follows that all values of P below it must have a T that makes the inequality true, while any value of P above it has no value of T (even an infinite one) that could make this inequality true. Thus we can take the limit of T as it approaches infinity and substitute in $v = \sum_{t=0}^{\infty} C\delta^t$.

$$\lim_{T\to\infty} \quad \delta\sum_{t=0}^{\infty} C\delta^t > \delta(C-n)(\sum_{t=0}^{T}\delta^t) + \delta^{T+1}(\sum_{t=0}^{\infty} C\delta^t) - \frac{C-D}{P_D-P_C}$$

This is equal to:

$$\sum_{t=1}^{\infty} C\delta^t > \sum_{t=1}^{\infty} C\delta^t - \sum_{t=1}^{\infty} n\delta^t + 0 - \frac{C-D}{P_D-P_C}$$

Which simplifies to:

$$\frac{\delta C}{1-\delta} > \frac{\delta C}{1-\delta} - \frac{\delta n}{1-\delta} - \frac{C-D}{P_D-P_C}$$

Isolate n. This is the value by which P must be greater than C in order for the inequality to be true:

$$n > -\frac{C-D}{P_D-P_C} \times \frac{1-\delta}{\delta}$$

Substitute n = C – P and simplify:

$$C > P - \frac{C - D}{P_D - P_C} \times \frac{1 - \delta}{\delta}$$

APPENDIX to Part III: ECONOMETRICS

(*Pearson*) *Correlation Matrix for all Variables*