CHAPTER 1
SETS, RELATIONS AND FUNCTIONS

SOLUTIONS

EXERCISE 1.1

Q1. Defined a set. Give example. Is there any collection which is not a set? If so give an example.

Ans :- Set – A well defined collection of distinct objects is called a set.

Example: $A = \{a, e, i, o, u\}$

Yes, A collection of 'lovely flowers ' is not a set .

Q2. What are the different ways to specify a set? Explain with example.

Ans :- A set can be specified in two ways as follows :

(a) Tabular or roaster form - In this form a set is specified by listing all its elements within a pair of curly brackets and the elements are separated by commas

Example: $A = \{1,2,3,4,5\}$

(b) Set builder form - In this form a set is specified by stating a common property which is satisfied by each of the elements and not by any other elements.

e. g a set $\{1,2,3,5,6,\}$ is specified in builder form as

$\{x : x \text{ is a factors of } 30 \text{ less than } 10\}$

Q3. Rewrite the following sets in tabular form:

(i) $\{\boldsymbol{x : x \text{ is a factors of } 20}\}$

(ii) $\{\boldsymbol{x : x \text{ is a letter of the word collection}}\}$

(iii) $\{\boldsymbol{x : x \text{ is a digit in the number } 552327}\}$

(iv) $\{\boldsymbol{x : x \text{ is a prime number lying between } 4 \text{ and } 20}\}$

Soln:- (i) $\{1,2,4,5,10,20\}$ (ii) $\{c, e, i, l, n, o\}$

(iii) $\{2,3,5,7\}$ (iv) $\{5,7,11,13,17,19\}$

Q4. Represent the following sets in set builder form:

(i) $\{\boldsymbol{2, 5, 6, 9}\}$ **(ii)** $\{\boldsymbol{1, 2, 3, 4, 5, 6, 7}\}$

(iii) $\{\boldsymbol{2, 4, 6, 8}\}$ **(iv)** $\{\boldsymbol{3, 6, 9, 12} \ldots\ldots\ldots\ldots..\}$

(v) $\{\boldsymbol{f, a, c, t, o, r}\}$

Soln:- (i) $\{x : x \text{ is a digit of the number } 2569\}$

(ii) $\{x : x \text{ is a natural numbers less than } 8\}$

(iii) $\{x : x \text{ is a positive even integer less than } 10\}$

(iv) $\{x : x = 3n, n \epsilon N\}$

(v) $\{x : x \text{ is a letter in the word 'factor'}\}$

Q5. **State whether the following statements are true or false:**

(i) $\phi = 0$ **(ii)** $0 \epsilon \phi$ **(iii)** $0 \subseteq \phi$ **(iv)** $\phi \subseteq \phi$

(v) $\phi \epsilon \phi$ **(vi)** $\{a\} \subseteq \{a, b, c\}$ **(vii)** a $\subseteq \{a, b, c\}$

(viii) a $\in \{a, b, c\}$ **(ix)** $\{b\} \in \{a, b, c\}$ **(x)** $\{1, 2\} = \{2, 1\}$

(xi) $\{1, 2, 3\} \neq \{2, 1, 3\}$

(xii) $\{2, 5, 6\} = \{x : x\ is\ a\ digit\ in\ 6525\}$ $(xiii)$ **A** $\subseteq$ **P(A)**

(xiv) **A** ϵ **P(A)** **(xv)** $\{A\} \subseteq$ **P(A)** **(xvi)** $\{A\} \epsilon$ **P(A)**

(xvii) Every subset of a finite set is finite.

(xviii) Every subset of an infinite set is infinite

(xix) Every set has a proper subset. (xx) Every set has at least two distinct subsets

(xxi) A non – empty set has at least two subsets:

Soln: (i) $false$ (ii) False (iii) False (iv) True (v) False
(vi) True (vii) False (viii) True (ix) False (x) True
(xi) False (xii) True $(xiii)$ False (xiv) True (xv) True
(xvi) False (xvii) True (xviii) False (xix) False (xx) false
(xxi) True

Q6. Which of the following sets are finite.

(i) The set of days in a week
(ii) The set of insects living on the earth
(iii) The set of integers lessthan 10
(iv) The set of natural numbers greater than 10
(v) The set of prime numbers
(vi) The set of positive integers less than 100
(vii) The set of even primes
(viii) the set of rational numbers lying between 1 and 2

Soln: (i) Finite (ii) Finite (iii) infinite (iv) Infinite (v) Infinite (vi) Finite (vii) Finite (viii) Infinite.

Q7. **Which of the following pairs of sets are equal ?**

(i) $\{x : x\ is\ a\ letter\ in'\ enter'\}$ **and** $\{x : x\ is\ a\ letter\ in\ rent\}$

(ii)$\{x : x\ is\ a\ positive\ integer\ less\ than\ 4\}$ **and** $\{x : x\ is\ a\ digit\ in\ 331212\}$

(iii) $\{x : x \geq 0\ \ and\ \ x \leq 0\}$ **and** ϕ **(iv)** ϕ **and** $\{\phi\}$

Soln: (i) Equal (ii) Equal (iii) unequal (iv) unequal

Q8. **When are two finite sets said to be equivalent? Will all equivalent sets to be equal ? Justify your answer with the help of examples.**

Ans: Two finite sets A and B are said to be equivalent if n(A) = n(B)
No, all equivalent sets mat not be equal . For instance set P = $\{a, b, c, d\}$ and Q = {2,3,5,7} are equivalent sets as n(P) = 4 and n(Q) = 4 , But P $\neq$ Q

Q9. **Which of the following sets are empty:**

(i) $\{x : x\ \neq x\}$ **(ii)** $\{x : x\ is\ a\ real\ number\ whose\ square\ is\ not\ positive\}$

(iii) $\{x : x \in R\ \ and\ \ x^2 < 0\}$ **(iv)** $\{x : x \in N\ and\ \ x^2\ = 0\}$

(v) $\{\ x : x \in R\ \ and\ x^2 = 0\}$

(vi) $\{\ x : x\ is\ a\ prime\ number\ divisible\ by\ 3\}$

(vii) $\{\ x : x\ is\ a\ prime\ number\ divisible\ by\ 6\}$

(viii) $\{\ x : x\ is\ an\ odd\ integer\ \ divisible\ by\ 2\}$

Ans. (i) Empty (ii) Non-empty (iii) Empty (iv) Empty
(v) Non- empty (vi) Non- empty (vii) Empty (viii) Empty

EXERCISE 1.2

Q1. Fill in the blanks with proper words or symbols.

i) If $x \in A \cup B$ then $x \in A$ _____________ $x \in B$.

ii) If $x \in A \cap B$ then $x \in A$ ___________ $x \in B$.

iii) If $x \in A \cap B$ then x ___________ A ____________ $x \in B$.

iv) If $x \in A - B$ then $x \in A$ and x ____ B.

v) If $x \in A' \cap B'$ then x _________ A _________ x _________ B.

Ans: i) or ii) and iii) $x \notin A$ and $x \notin B$
iv) $x \notin B$ v) $x \notin A$ and $x \notin B$

Q2. If A = {a,b,c,d,e}, B = {x,y,z} and C = {b,c,p,x,z} find
(i) $A \cup B$ (ii) $B \cap C$ (iii) $C \cup A$ (iv) $A \cap B$ (v) $B \cap C$
(vi) $C \cup A$ (vii) A - B (viii) B - C (ix) C - A (x) B - A
(xi) C - B (xii) A - C (xiii) $(A \cup B) \cup C$ and $A \cup (B \cup C)$
(xiv) $A \cap (B \cap C)$ (xv) $A \cup (B \cap C)$

Soln: (i) $A \cup B$
$= \{a,b,c,d,e,x,y,z\}$

(ii) $B \cap C = \{x,z\}$

(iii) $C \cup A = \{b,c,p,x,z\} \cup \{a,b,c,d,e\}$
$= \{a,b,c,d,e,p,x,z\}$

(iv) $A \cap B = \{\ \}$ or ϕ

(v) $B \cap C = \{x,y,z\} \cap \{b,c,p,x,z\}$
$= \{x,z\}$

(vi) $C \cap A = \{b,c,p,x,z\} \cap \{a,b,c,d,e\}$
$= \{b,c\}$

(vii) $A - B = \{a,b,c,d,e\} - \{x,y,z\}$
$= \{a,b,c,d,e\}$

(viii) $B - C = \{x,y,z\} - \{b,c,p,x,z\}$
$= \{y\}$

(ix) $C - A = \{b,c,p,x,z\} - \{a,b,c,d,e\}$
$= \{p,x,z\}$

(x) $B - A = \{x,y,z\} - \{a,b,c,d,e\}$
$= \{x,y,z\}$

(xi) $C - B = \{b,c,p,x,z\} - \{x,y,z\}$
$= \{b,c,p\}$

(xii) $A - C = \{a,b,c,d,e\} - \{b,c,p,x,z\}$
$= \{a,d,e\}$

(xiii) $(A \cup B) \cup C$ and $A \cup (B \cup C)$

Soln: $(A \cup B) = \{a,b,c,d,e\} \cup \{x,y,z\}$
$= \{a,b,c,d,e,x,y,z\}$

$(A \cup B) \cup C = \{a,b,c,d,e,p,x,y,z\}$

$(B \cup C) = \{b,c,p,x,y,z\}$

$A \cup (B \cup C) = \{a,b,c,d,e,p,x,y,z\}$

(xiv) $A \cap (B \cup C) = \{a,b,c,d,e\} \cap \{b,c,p,x,y,z\} = \{b,c\}$

(xv) $A \cup (B \cap C)$

$B \cap C = \{x,y,z\} \cap \{b,c,p,x,z\}$
$= \{x,z\}$

$A \cup (B \cap C) = \{a,b,c,d,e\} \cup \{x,z\}$
$= \{a,b,c,d,e,x,z\}$

Q3. Let A = {1,2,5,7}, B = {2,5,9}, C = {5,7,8,9}
If the universal set U is the {1,2,3,4,5,6,7,8,9}

Find (i) A' **(ii)** B' **(iii)** C' **(iv)** $A' \cap B$

(v) $B \cap C'$ **(vi)** $A' \cap C'$ **(vii)** $(A \cup B)'$ **(viii)** $(A \cap C)'$

(ix) $B' \cup C'$ **(x)** $A' - B'$ **(xi)** $A' \cup (B \cup C)$

(xii) $A \cap (B' \cup C')$ **(xiii)** $A' U (B' U C')$

(xiv) $(A' \cup B') \cap (A' \bigcup C')$

Soln: (i) A' $= U - A$
$= \{3,4,6,8,9\}$

(ii) B' $= U - B$
$= \{1,3,4,6,7,8\}$

(iii) C' $= U - C$
$= \{1,2,3,4,6\}$

(iv) $A' \cap B$ $= \{3,4,6,8,9\} \cap \{2,5,9\}$
$= \{9\}$

(v) $B \cap C'$ $= \{2,5,9\} \cap \{1,2,3,4,6\}$
$= \{2\}$

(vi) $A' \cap C'$ $= \{3,4,6,8,9\} \cap \{1,2,3,4,6\}$
$= \{3,4,6\}$

(vii) $(A \cup B)'$ $= A' \cap B^1$
$= \{3,4,6,8\}$

(viii) $(A \cap C)'$ $= A' \cup C'$
$= \{1,3,4,6,7,8,9\}$

(ix) $B' \cup C'$ $= \{1,3,4,6\}$

(x) $A' - B'$ $= \{3,4,6,8,9\}$ - 1,3,4,6,7,8}
$= \{9\}$

(xi) $A' \cup (B \cup C) = \{3,4,6,8,9\} \cup \{2,5,7,8,9\}$
$= \{2,3,4,5,6,7,8,9\}$

(xii) $A \cap (B' \cup C') = \{1,2,5,7\} \cap \{1,2,3,4,6,7,8\}$
$= \{1,2,7\}$

(xiii) $A' U (B' U C') = \{3,4,6,8,9\} \cup \{1,2,3,4,6,7,8\}$
$= \{1,2,3,4,6,7,8,9\}$

(xiv) $(A' \cup B') \cap (A' \cup C')$ = $\{1,3,4,6,7,8,9\} \cap \{1,2,3,4,6,8,9\}$
= $\{1,3,4,6,8,9\}$

Q4. Give examples to show their that the following statement are false:

i) A - B = A $\Rightarrow$ B = ϕ
ii) A$\cup$B = A $\Rightarrow$ B = ϕ
iii) A$\cap$B = ϕ $\Rightarrow$ A = ϕ or B = ϕ
iv) (A$\cup$B) - B = A
v) A - B = ϕ $\Rightarrow$ A = B
vi) A$\cup$B = A$\cup$C $\Rightarrow$ B = C
vii) A$\cap$B = A$\cap$C $\Rightarrow$ B = C

Soln: i) A - B = A $\Rightarrow$ B = ϕ

Let A = {1,2,3}, B = {4,5}
A - B = {1,2,3} = A but B $\neq \phi$
$\therefore$ A - B = A $\Rightarrow$ B = ϕ is false.

ii) A$\cup$B = A $\Rightarrow$ B = ϕ

Let A = {1,2,3,4}, B = {2,3}
$\Rightarrow$ A$\cup$B = {1,2,3,4} = A but B ϕ
$\therefore$ A$\cup$B = A $\Rightarrow$ B = ϕ is false.

iii) A$\cap$B = ϕ A = ϕ or B = ϕ

Let A = {3,4,5}, B = {7,8}
$\Rightarrow$ A$\cap$B = ϕ, but A = ϕ or B = ϕ
$\therefore$ A$\cap$B = ϕ $\Rightarrow$ A = ϕ or B = ϕ is false.

iv) (A$\cup$B) - B = A

Let A = {1,2,3,4}, B = {4,5}
A$\cup$B = {1,2,3,4,5}
(A$\cup$B) - B = {1,2,3} $\neq$ A
$\therefore$ (A$\cup$B) - B = A is false.

v) A - B = ϕ $\Rightarrow$ A = B

Let A = {2,3,4}, B = {1,2,3,4}
A - B = ϕ but A $\neq$ B
$\therefore$ A - B = ϕ $\Rightarrow$ A = B is false.

vi) A$\cup$B = A$\cup$C $\Rightarrow$ B = C

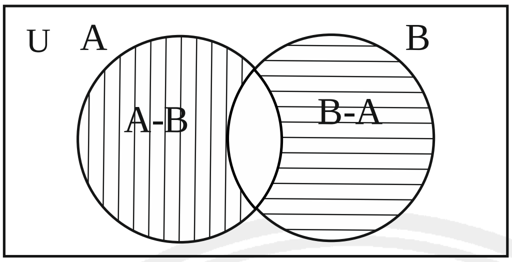
Let A = {1,2,3}, B = {3,4,5}, C = {4,5}
A$\cup$B = {1,2,3,4,5}
A$\cup$C = {1,2,3,4,5}
$\Rightarrow$ A$\cup$B = A$\cup$C but B $\neq$ C
$\therefore$ A$\cup$B = A$\cup$C $\Rightarrow$ B = C is false

vii) A$\cap$B = A$\cap$C $\Rightarrow$ B = C

Let A = {1,2,3,4}, B = {23,5,6}, C = {2,3,7}
A$\cap$B = {2,3}
A$\cap$C = {2,3}
$\Rightarrow$ A$\cap$B = A$\cap$C but B $\neq$ C
$\therefore$ A$\cap$B = A$\cap$C $\Rightarrow$ B = C is false

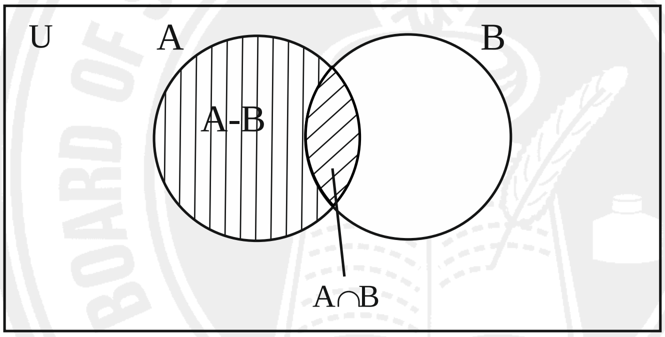
Q5. Prove that the following pairs of sets are disjoint (by Venn diagrams)
(i) A - B and B - A (ii) $A \cap B$ and A - B (iii) $A \cap B$ and B - A
(iv) A - B and B (v) B - A and A

Soln: (i) A - B and B - A



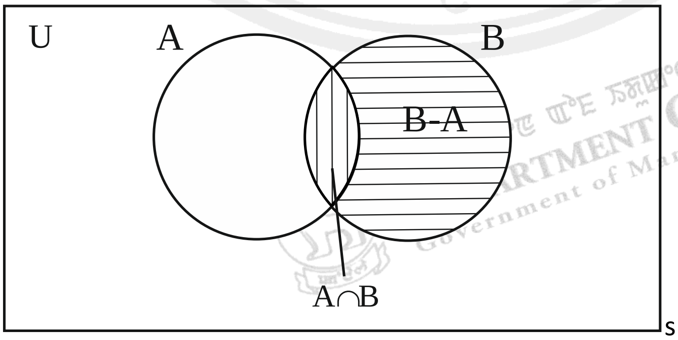
The two shaded regions of A - B and B - A are non-overlapping
$\Rightarrow$ A - B and B - A are disjoint.

(ii) $A \cap B$ and A - B



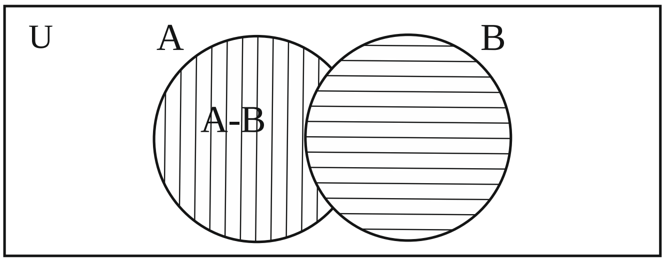
The two shaded regions of $A \cap B$ and A - B are non-overlapping
$\Rightarrow A \cap B$ and A - B are disjoint.

(iii) $A \cap B$ and B - A



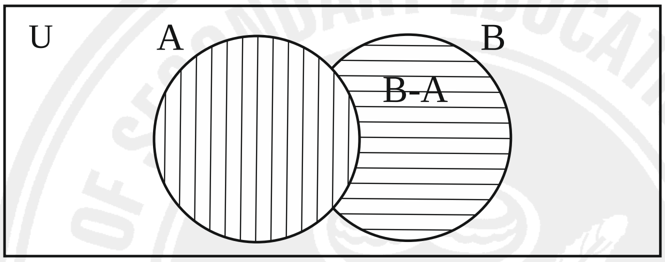
The two shaded regions of $A \cap B$ and B - A are non-overlapping
$\therefore A \cap B$ and B - A are disjoint.

(iv) A - B and B



The two shaded regions of A - B and B are non-overlapping
$\therefore$ A - B and B are disjoint.

(v) B - A and A



The two shaded regions of B - A and A are non-overlapping
B - A and A are disjoint.

Q6. If $A \cup B = \phi$ then prove that $A = \phi$ and $B = \phi$

Soln: Given that $A \cup B = \phi$

As $A \subseteq A \cup B$

$\Rightarrow A \subseteq \phi$ -------------(i)

But ϕ is the subset of every set.

$\Rightarrow \phi \subseteq A$ -------------(ii)

From (i) and (ii) $A = \phi$

Similarly, $B = \phi$

Q7. If $A \subseteq S$ and $B \subseteq S$, prove that $A \cup B \subseteq S$

Soln: We have, $A \subseteq S$ and $B \subseteq S$

Let $x \in A \cup B$

Then $x \in A$ or $x \in B$

$\Rightarrow x \in S$ or $x \in S$

$\Rightarrow x \in S$

$\Rightarrow A \cup B \subseteq S$

Q8. If $A \subseteq B$ prove that $A \cap B = A$ and $A \cup B = B$

Soln: We have, $A \subseteq B$ $\quad \Rightarrow x \in A \Rightarrow \in xB$

or $x \in B \Rightarrow x \in A$

$A \cap B = \{x: x \in A \text{ and } x \in B\}$

$= \{x: x \in A \text{ and } x \in A\}$

$A \cup B = \{x: x \in A \text{ or } x \in B\}$
$= \{x: x \in B \text{ or } x \in B\}$
$= \{x: x \in B\}$
$= B$

Q9. If $A \cap B =$, prove that $A \subseteq$ and $B \subseteq A'$

Soln: Since $A \cap B = \varphi$

If $x \in A \Rightarrow x \notin B$

then $x \in B'$

$\Rightarrow A \subseteq B'$

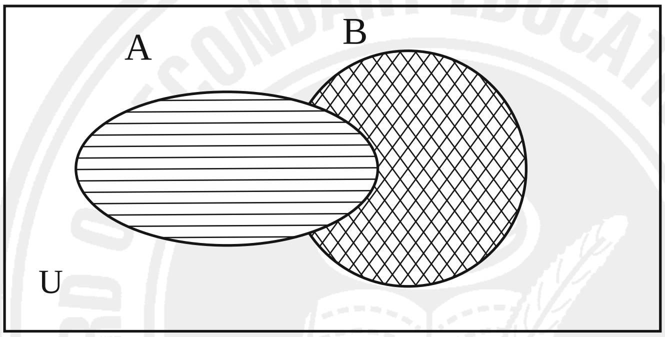
If $x \in B \Rightarrow x \notin A \; x \in A'$

then $x \in$

$\Rightarrow B \subseteq A'$

Q10. If $A \cup B = U$ verify by Venn diagram that $A' \subseteq B$.

Soln:



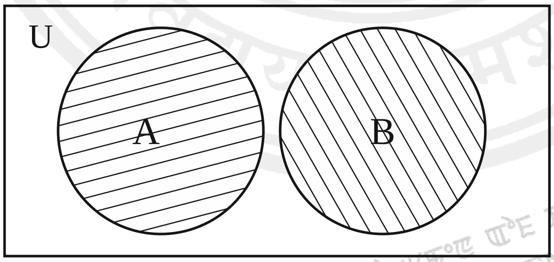
$A \cup B$ is the single shaded region as shown in the fig.

as $A \cup B = U$. A' is represented by double shaded region which is the region of B.

$\therefore A' \subseteq B$

Q11. If $A \cap B = \phi$ verify by Venn diagram that A - B = A, B - A = B.

Soln:



From the above diagram, A - B = A and B - A = B.

$\therefore A \cap B = \phi$

EXERCISE 1.3

Q 1. Out of 50 students in a class 35 can play hockey and 43 can play football. If each students can play at least one ofthe two games. Find the number of students who can play both.

Ans :- Let A – Set of students who can play hockey.

B – Set of students who can play football.

Then , $n(A) = 35$ $n(B) = 43$, $n(A \cup B) = 50$, $n(A \cap B) = ?$

Using , $n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\Rightarrow 50 = 35 + 43 - n(A\cap B)$

$\Rightarrow n(A\cap B) = 78 – 50 = 28$

Hence the number of students who can play both hockey and football = 28

Q 2. In a school there are 20 teachers Out of them only 6 can teach Mathematics and 18 can teach English. If each teacher teaches one or the other subjects, find the number of teachers who can teach both .

Ans :- Let A = set of teachers who can teach mathematics

B = set of teachers who can teach English

Then, $n(A) = 6$ $n(B) = 18$, $n(A\cup B) = 20$ $n(A\cap B) = ?$

By theorem – 1 we have,

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\Rightarrow 20 = 6 + 18 - n(A\cap B)$

$\Rightarrow n(A\cap B) = 24 - 20$

$= 4$

Hence the number of teachers who can teach both = 4

Q 3. In a certain college, 300 students offer Mathematics, 225 offer Physics, 250 offer Chemistry, 200offer both Mathematics and Physics, 115 offer Physics and Chemistry and 172 offer chemistry and Mathematics. If 100 students offer all the three, find the number of students who offer at least one of the three subjects.

Ans:- Let A = students offering Mathematics

B = students offering physics

C = students offering chemistry

Then, $n(A) = 300$ $n(B) = 225$ $n(C) = 250$

$n(A\cap B) = 200$ $n(B\cap C) = 115$ $n(A\cap C) = 172$,

$n(A \cap B \cap C) = 100$

By theorem - 2, we have,

$n(A\cup B\cup C) = n(A) + n(B) + n(C) – n(A\cap B) – n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 300 + 225 + 250 - 200 – 115 – 172 + 100$

$= 875 – 487$

$= 388$

Hence the number of students who offer at least one of the three subjects is 388.

Q 4. There are 50 players in a club each playing at least one of the games badminton, tennis and chess. 30 of them play badminton, 16 play tennis, 8 play both badminton and tennis, 10 play both tennis and chess, 18 play both badminton and chess and 6 play all the three. Find the number of players who play chess.

soln:- Let A = set of players playing badminton

B = set of players playing tennis

C = set of players playing chess

Then, $n(A) = 30$ $n(B) = 16$

$n(A\cap B) = 8$ $n(B\cap C) = 10$ $n(A\cap C) = 18$

$n(A\cup B\cup C) =50$ $n(A\cap B\cap C) = 6$ $n(C) = ?$

By theorem - we have,

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$\Rightarrow 50 = 30 + 16 + n(C) - 8 - 10 - 18 + 6$

$\Rightarrow n(C) = 50 - 52 + 36$

$= 86 - 52$

$= 34$

$\therefore$ the number of players who play chess = 34

Q 5. In a flood relief camp of 128 persons, 25 were men, and the rest women and children. After a week 69 left the camp, 35 of them being children out of those who remained behind 14 were men. How many women left the camp ?

Soln :- Let A = set of women and children who left the camp

B = set of women and children who remained behind.

Then, $n(A\cup B) = 128 - 25 = 103$

$n(B) = 128 - 69 - 14 = 128 - 83 = 45$

$n(A) = ?$ $\quad n(A\cap B) = 0$

We have,

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\Rightarrow 103 = n(A) + 45 - 0$

$\Rightarrow n(A) = 103 - 45$

$= 58$

$\therefore$ the number of women who left the camp = 58 – 35 = 23

Q6. There are 25 pencils on a table, some are blue and other black in colour . 16 of them have eraser at one end. Number of blue pencils is 14 and 5 of the black one do not have eraser. How many blue pencils will have eraser.

Soln:- Let A = set of blue pencils with eraser.

B = set of black pencil with eraser

C = set of black pencil without eraser

Then, $n(BUC) = 25 - 14 = 11$

$n(B) = ?$ $\quad n(C) = 5$ $\quad n(B\cap C) = 0$

Now, we have,

$n(BUC) = n(B) + n(C)$

$\Rightarrow 11 = n(B) + 5$

$\Rightarrow n(B) = 11 - 5 = 6$

Here, $n(AUB) = 16$, $\quad n(A) = ?$

We have,

$n(A\cup B) = n(A) + n(B)$

$\Rightarrow 16 = n(A) + 6$

$\Rightarrow n(A) = 16 - 6 = 10$

$\therefore$ no. of blue pencils having eraser = 10

Q7. There are 100 families in a certain locality 50 of them use gas and 80 of them use kerosene forcooking. Find the no. of families using both gas and kerosene for cooking.

Soln:- Let A = set of families using gas

B = set of families using kerosene

Then, $n(A\cup B) = 100$ $\quad n(A) = 50$ $\quad n(B) = 80$ $\quad n(A\cap B) = ?$

We have,

$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

$\Rightarrow$ $100 = 50 + 80 - n(A\cap B)$

$\Rightarrow$ $n(A\cap B) = 130 - 100 = 30$

$\therefore$ the number of families using both gas and kerosene for cooking = 30

Q8. A survey of class X standard boys of a school about whether the play Football, Hockey or Cricket produced the following table :

F	H	C	F∩H	H∩C	C∩F	F∩H∩C
60%	**50%**	**50%**	**30%**	**20%**	**30%**	**10%**

(i) What percentage play Football and Hockey but not Cricket?

(ii) What percentage play non of these games?

Soln:- (i) We have, $n(F\cap H) = 30\%$, $n(F\cap H\cap C) = 10\%$

$\therefore$ percentage of boys playing Football and Hockey only but not Cricket

$= n(F\cap H) - n(F\cap H\cap C)$

$= 30\% - 10\%$

$= 20\%$

(ii) Percentage of boys playing either of these games is

$n(FUHUC) = n(F) + n(H) + n(C) - n(F\cap H) - n(H\cap C) - n(F\cap C) + n(F\cap H\cap C)$

$= (60 + 50 + 50 - 30 - 20 - 30 + 10)\%$

$= (160 - 80 + 10)\%$

$= (170 - 80)\%$

$= 90\%$

Hence no. of boys playing none of these games $= n(U) - n(F \cup H \cup C)$

$= (100 - 90)\%$

$= 10\%$

Q9. S is a finite set of positive integers each of which is divisible by 2 or 5 or 11. Among the elements of S. 195 are multiples of 2, 170 are multiples of 5, 140 are multiples of 11, 80 are multiples of 10, 45 are multiples of 22, 30 are multiples of 55, and 20 are multiples of 110. Find the number of elements in S and find how many of them are divisible by 2 or 5 but not by 11.

Ans:- Let A = set of positive integers divisible by 2

B = set of positive integers divisible by 5

C = set of positive integers divisible by 11

Then, , $n(A) = 195$, $n(B) = 170$, $n(C) = 140$

$n(A\cap B) = 80$, $n(B\cap C) = 30$, $n(A\cap C) = 45$, $n(A\cap B\cap C) = 20$

We have ,

$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 195 + 170 + 140 - 80 - 30 - 45 + 20$

$= 525 - 155$

$= 370$

Number of positive integers in S = 370

Number of positive integers divisible by 2 or 5 or 11 = $n(A\cup B\cup C) = 370$

No. of positive integers divisible by 11 only = $n(C) = 140$

$\therefore$ No. of positive integers divisible by 2 or 5 or but not by 11 = 370 – 140 = 230

EXERCISE – 1.4

Q.1 **Find** $A\times A, A\times B, B\times A$ **and** $B\times B$ **when**

(i) $A=\{1,2\}$ **and** $B=\{2,3,4\}$

Soln. $A=\{1,2\}$, $B=\{2,3,4\}$

$A\times A=\{1,2\}\times\{1,2\}=\{(1,1),(1,2),(2,1)(2,2)\}$

$A\times B=\{1,2\}\times\{2,3,4\}=\{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4)\}$

$B\times A=\{2,3,4\}\times\{1,2\}=\{(2,1),(2,2),(3,1),(3,2),(4,1),(4,2)\}$

$B\times B=\{2,3,4\}\times\{2,3,4\}=\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4),(4,2),(4,3),(4,4)\}$

(ii) $A=\{a,b,c\}$ **and** $B=\{e,f,g\}$

Soln. $A=\{a,b,c\}$, $B=\{e,f,g\}$

$A\times A=\{a,b,c\}\times\{a,b,c\}$

$=\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

$A\times B=\{a,b,c\}\times\{e,f,g\}$

$=\{(a,e),(a,f),(a,g),(b,e),(b,f),(b,g),(c,e),(c,f),(c,g)\}$

$B\times B=\{e,f,g\}\times\{e,f,g\}$

$=\{(e,e),(e,f),(e,g),(f,e),(f,f),(g,g),(g,e),(g,f),(g,g)\}$

(iii) $A=\{a,b,c,d\}$ **and** $B=\{2,5\}$

Soln: $A\times B=\{(a,2),(a,5),(b,2),(b,5),(c,2),(c,5),(d,2),(d,5)\}$

$B\times A=\{(2,a),(2,b),(2,c),(2,d),(5,a),(5,b),(5,c),(5,d)\}$

$A\times A=\{a,b,c,d\}\times\{a,b,c,d\}$

$=\{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d),(c,a),(c,b),(c,c),(c,d),$

$(d,a),(d,b),(d,c),(d,d)\}$

Q.2 **If $A\times B$ has six elements of which three are (1,1), (1,2) and (3,3), find the other three elements of $A\times B$. Also find $B\times A$**

Soln. The possible number of elements in A and B are

i) $n(A) = 1 \; and \; n(B) = 6$

ii) $n(A) = 2 \; and \; n(B) = 3$

iii) $n(A) = 3 \; and \; n(B) = 2$

iv) $n(A) = 6 \; and \; n(B) = 1$

We know that 1, 3 belong to A and 1, 2 and 3 belong to B.

$\therefore \; n(A) \geq 2 and \; n(B) \geq 3$

So, out of the possibilities, we must have ii) i.e. $n(A) = 2 \; and \; n(B) = 3$

$\therefore \; A = \{1,3\} and \; B = \{1,2,3\}$

and $A \times B = \{(1,1),(1,2),(1,3),(3,1)(3,2),(3,3)\}$

$\therefore$ The other three elements are $(1,2),(3,1),(3,2)$

Also $B \times A = \{(1,1), 1,3),(2,1),(2,3),(3,1)(3,3)\}$

Q.3 **If $A\times B$ had eight elements five of which are (*a,a*),(*a,c*),(*b,b*),(*b,c*) and (*b,d*) find other three elements.**

Soln. Possible number of elements in A and B are given by

i) $n(A) = 1 \; and \; n(B) = 8$
ii) $n(A) = 2 \; and \; n(B) = 4$
iii) $n(A) = 4 \; and \; n(B) = 2$
iv) $n(A) = 8 \; and \; n(B) = 1$
But from the order pairs, we know that
$a, b \in A \; and \; a, b, c, d \in B$
$\therefore n(A) \geq 2 and \; n(B) \geq 4$
So, out of the above possibilities we must have (ii)
i.e. $n(A) = 2 \; and \; n(B) = 4$
$\therefore \; A = \{a, b\} and \; B = \{a, b, c, d\}$
$and \; A \times B = \{(a,a),(a,b),(a,c),(a,d),(b,a),(b,b),(b,c),(b,d)\}$
Hence the other three elements of $A \times B \; are \; (a,b),(b,a),(a,d)$.

Q.4 Define a relation from a set to set, its domain and range.

Ans. The relation *R* from a non-empty set *A* to a non-empty set *B* is defined as the subset of $A \times B$

i.e. $R = \{(x, y) : x \in A \text{ and } y \in B\}$

Domain of *R* = The set of all the first components of the elements of *R* is called domain of *R*.

Range of relation of *R* = The set of the second components of the elements of *R* is called range of *R*.

Q.5 Write down the elements of the relations R is the set of N given by

$R = \{(x, y) : x, y \in N, 2x + y = 12\}$**. Find its domain and range**

Soln. We have, $R = \{(x, y) : x, y \in N, 2x + y = 12\}$

$= \{(1,10), (2,8), (3,6), (4,4)(5,2)\}$

Domain of $R = \{1, 2, 3, 4, 5\}$

Range of $R = \{2, 4, 6, 8, 10\}$

Q.6 When is a relation *R* in a set *A* said to be

(i) reflexive (ii) symmetric (iii) transitive (iv) anti-symmetric?

Ans. (i) Reflexive: A relation *R* is said to be reflexive if $(a, a) \in R$ for all $a \in A$, i.e. if each elements of *A* is related to itself.

(ii) Symmetric: A relation *R* is said to be symmetric if whenever a is related to *b*, *b* is related to *a* i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for any $a, b, \in A$

(iii) Transitive: A relation *R* is said to be transitive if *a* is related to *b*, and *b* is related to *c*

$\Rightarrow a$ is related to *c*

i.e. if $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$

(iv) Anti-symmetric: A relation *R* on a set *A* is said to be anti-symmetric if $(a,b) \in R$ and

$(b,a) \in R \Rightarrow a = b$

Q.7 Can a relation *R* is a set *A* be symmetric as well as anti-symmetric? Justify your answer by means of examples.

Soln. Yes, a relation *R* in a set *A* be symmetric as well as anti-symmetric
Consider the relation *R* is a set $A = \{1, 2, 3\}$

Given by $R = \{(1,1), (2,2), (3,3)\}$ which is symmetric as well as anti-symmetric.

Q.8 Show that the relation 'less than or equal to denoted by $\leq$ in N the set of natural numbers is reflexive, anti-symmetric and transitive.

Soln. Reflexivity: For any natural number x

$x = x \Rightarrow xRx$

$\therefore R$ is reflexive.

Anti-symmetric: For any $x, y \in N$ if $x \leq y$ and $y \leq x$

i.e. xRy and $yRx \Rightarrow x = y$

$\therefore R$ is anti-symmetric.

Transitive: For any $x, y, z \in N$

xRy and yRz

i.e. $x \leq y$ and $y \leq z$

$\Rightarrow x \leq z$

$\Rightarrow xRz$

$\therefore R$ is transitive.

Q.9 Give an example of Relation which is (a) symmetric, and reflexive but not transitive.

Ans. Let $A = \{a, b, c\}$ and $R = \{(a,a)(b,b),(c,c)(a,c),(c,a),(b,c),(c,b)\}$

Here R is reflexive and symmetric but not transitive.

(b) Symmetric and transitive but not reflexive.

$R = \{(1,3),(3,1),(1,1),(3,3)\}$ on $A = \{1,2,3\}$

(c) Reflexive and but not symmetric

$R = \{(1,1),(2,2),(3,3),(1,3)\}$ on $A = \{1,2,3\}$

(d) Reflexive but neither symmetric not transitive

Ans. $R = \{(1,1), (2,2), (3,3), (2,3), (3,1)\}$on $A = \{1,2,3\}$

(e) Symmetric but neither reflexive nor transitive

Ans. $R = \{(1,1),(1,3),(3,1),(2,3),(3,2)\}$ on $A = \{1,2,3\}$

(f) Transitive but neither reflexive nor symmetric

Ans. $R = \{(1,3),(3,2),(1,2)\}$ on $A = \{1,2,3\}$

Q.10 **Let $A=\{1,2,3\}$ and $R=\{(1,1),(2,2)(2,3),(3,2),(3,3)\}$. Prove that *R* is an equivalence relation in *A*.**

Soln. We have, $R=\{(1,1),(2,2)(2,3),(3,2),(3,3)\}$ on $A=\{1,2,3\}$

$(1,1),(2,2),(3,3)\in R \Rightarrow R$ is reflexive.

$(2,3)\in R \Rightarrow (3,2)\in R \Rightarrow R$ is symmetric

$(2,3) \in R, (3,2) \in R\ and\ (2,2) \in R$

$\therefore R$ is transitive

Here *R* is reflexive, symmetric and transitive so *R* is an equivalence relation.

Q.11 **Let *A* be the set of all triangles in a plane and *R* be the relation in *A* defined by xRy if *x* is congruent to $y(x, y\in A)$. Prove that the *R* is an equivalence relation.**

Ans. We have A = set of all triangles in a plane.

$R=\{x,y):x$ is congruent to *y*,: *x*, *y* ∈ *A*}

For any triangle *x*, *x* is congruent to

$\Rightarrow xRx \Rightarrow R$ is reflexive.

If $x\cong y \Rightarrow y\cong x$

$\Rightarrow xRy \Rightarrow yRx \therefore R$ is symmetric.

For any three triangle $x,y,z\in A$.

$x\cong y$ and $y\cong z$

$\Rightarrow x\cong z$

$\Rightarrow xRz \therefore R$ is transitive

Hence *R* is an equivalence relation.

Q.12 **Let *R* be a relation in *Z* defined by aRb iff *a* is a multiple of *b*. Prove that *R* is reflexive, anti-symmetric and transitive.**

Soln. For any integer *a*, *a* is a multiple of a itself.

$\Rightarrow aRa \Rightarrow R$ is reflexive

If *a* is a multiple of *b* and *b* is a multiple of *a*

$\Rightarrow a = b$

i.e. aRb and $bRa \Rightarrow a = b$

$\therefore R$ is anti-symmetric.

If aRb and bRc then$a$ is a multiple of b and b is a multiple of c

$\Rightarrow a = bp$ for some integer p. ---------(i)

And $b = cq$ for some integer q. ------------(ii)

From (i) and (ii)

$\Rightarrow ab = bc(pq)$

$\Rightarrow a = c(pq) = cm$ where $m = pq$

$\Rightarrow a$ is a multiple of c.$\Longrightarrow aRc$

$\therefore R$ is transitive

Q.13 Define an equivalent relation. If *R* is a relation in the set *Z* of all integers defined by the open statement "$x - y$ is divisible by 7", that is $R = \{(x, y) : (x, y \in Z, x - y$ is divisible by 7}. Prove that *R* is an equivalent relation.

Ans. A relation R is a set A is said to be equivalent if R is reflexive, symmetric and transitive.

For any $x \in Z$, $x - x = 0$ which is divisible by 7.

$\Rightarrow (x, x) \in R \Rightarrow R$ is reflexive.

Suppose $x - y$ is divisible by 7

then - $(y - x)$ is also divisible by 7

$\Rightarrow xRy \Rightarrow yRx$

$\therefore R$ is symmetric.

Suppose, xRy and yRz

i.e. $x - y$ is divisible by 7

and $y - z$ is also divisible by 7

Now $x - z = x - y + y - z$ which is divisible by 7

$\therefore xRy, yRz \text{ then } xRz$

$\therefore R$ is transitive

Hence R is an equivalent relation.

Q.14 **If *R* is an equivalent relation in a set *A*. Prove that the inverse relation R^{-1} is also an equivalent relation in *A*.**

Soln. As R is an equivalence relation in set A. for any $a \in A$ $(a,a) \in R \Rightarrow (a,a) \in R^{-1}$

$\therefore R^{-1}$ is reflexive.

For any $a,b \in A, (a,b) \in R \Rightarrow (b,a) \in R$

$\Rightarrow (b,a) \in R^{-1} \Rightarrow (a,b) \in R^{-1}$

$\therefore R^{-1}$ is symmetric.

For any $a,b,c \in A$

$\Rightarrow (a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a,c) \in R$

$\therefore (c,b) \in R^{-1}$ and $(b,a) \in R^{-1}$

$\Rightarrow (c,a) \in R^{-1}$

$\therefore R^{-1}$ is transitive

Hence R is an equivalence relation.

Q.15 **Examine if the relation *R* defined by *Q* by xRy if $|x-y|<5$ is an equivalence relation or not.**

Soln. For any $x \in Q$

$|x-x| = |0| < 5$

$\Rightarrow xRx \therefore R$ is reflexive.

For any $x, y \in Q$

If $|x-y|<5$

$\Rightarrow |-(y-x)| < 5$

$\Rightarrow |y-x| < 5$

$\Rightarrow xRy \Rightarrow yRx \therefore R$ is symmetric.

We know that $|2-6|<5 \Rightarrow 2R6$

$|6-10|<5 \Rightarrow 6R10$

$\Rightarrow |2-10| \not< 5 \Rightarrow 2 \not R\, 10$

$\Rightarrow R$ is not transitive

Hence R is not an equivalence relation.

EXERCISE 1.5

1. Which of the following collection define a function? State domain and range in the case of a function.

(i) $\{(1, a), (2, b), (3, c), (4, a)\}$

Solution: It is a function because no two different ordered pairs have the same first component.

Domain $=\{1, 2, 3, 4\}$ and Range $=\{a, b, c\}$

(ii) $\{(a, 1), (b, 2), (c, 3), (a, 4)\}$

Solution: It is not a function because two different ordered pairs have the same first component a.

(iii) $\{(1, 1), (2, 3), (3, 2), (4, 1), (5, 2), (6, 3)\}$

Solution: It is a function because no two different ordered pairs have the same first component.

Domain $=\{1, 2, 3, 4, 5, 6\}$ and Range $=\{1, 2, 3\}$.

(iv) $\{(1, 1), (2, 2), (3, 1), (4, 2), (2, 3), (1, 2)\}$

Solution: It is not a function because two different ordered pairs have the same first component.

2. A function $f: R \to R$ is given by

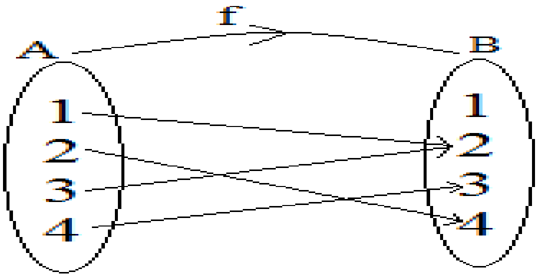
$$f(\mathrm{x}) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ 0, \text{ if } x \text{ is irrational} \end{cases}$$

Define f in words and find the values of

$f(1), f(2), f(\frac{1}{3}), f(\sqrt{2})$ and $f(2+\sqrt{3})$.

Solution: f maps every rational number to 1 and every irrational number to 0. And $f(1)=1, f(2)=1, f(\frac{1}{3})=1, f(\sqrt{2})=0$ and $f(2+\sqrt{3})=0$.

3. A function f is described by the diagram:



Find the domain, co-domain and range of f. Is the function (i) one-one (ii) onto ?

Solution: Domain ={ 1, 2, 3, 4}

Co-domain= {1, 2, 3, 4} and

Range = { 2, 3, 4}

(i) f is not one- one as the elements 1 and 3 have the same f - image 2.

(ii) f is not onto as Co-domain $\neq$ Range.

4. Let A= {1, 2} and B ={ a, b}, write down all possible functions from A to B as set of ordered pairs. Identify the bijective ones.

Solution: The possible functions are given as :

f_1 = { (1, a), (2, a)}, f_2 = {(1, b), (2, b)}, f_3 ={ (1, a), (2, b)}, f_4 = {(1, b), (2, a)}.

f_3 and f_4 are bijective.

5. Let $A = \{1, 2, 3, 4, 5\}$ and B be the set of positive integers less than 10. A function $f : A \to B$ is given by $f(1) = 1, f(2) = 3, f(3) = 5, f(4) = 7, f(5) = 9$. Give a general formula which can describe f. Examine if f is injective, surjective or bijective.

Solution: It is observed that different elements of A have different f - image in B, So, f is injective.

Since, co-domain $\neq$ range, f is not surjective.

Hence, f is not bijective.

6. Show that the mapping $f : N \to Q$ defined by $f(n) = \frac{1}{n}, n \in N$ is one-one but not onto.

Solution: Let $x, y \in N$ such that $f(x) = f(y)$.

Then, $f(x) = f(y)$

$\Rightarrow \frac{1}{x}=\frac{1}{y}$

$\Rightarrow x=y$

$\therefore f$ is one –one.

Let $y \in N$ (co-domain) be such that $f(x)=y$.

Then, $f(x)=y$

$\Rightarrow \frac{1}{x}=y$

$\Rightarrow x=\frac{1}{y}$

It is observed that when $y=2, x=\frac{1}{2} \notin N$.

Thus, f is not onto.

7. Show that the function $\phi: Z \to Z$ defined by $\phi(x)=2x+3 (x \in Z)$ is one –one but not onto. Find the range of ϕ.

Solution: Let $x, y \in Z$ be such that $\phi(x)=\phi(y)$.

Then, $\phi(x)=\phi(y)$

$\Rightarrow 2x+3=2y+3$

$\Rightarrow 2x=2y$

$\Rightarrow x=y$

$\therefore \phi$ is one-one.

Let $y \in Z(co-domain)$ be arbitrary element such that $\phi(x)=y$.

Then, $\phi(x)=y$

$\Rightarrow 2x+3 = y$

$\Rightarrow x=\frac{y-3}{2}$

When $y=0, x=\frac{-3}{2} \notin Z$. So, ϕ is not onto.

To find the Range of ϕ:

Clearly , $x=\dfrac{y-3}{2}$ is defined when y is the set of all odd integers.

So, the range of ϕ is the set of all odd integers.

8. **Show that the function $\phi : Q \to Q$ defined by $\phi(x)=2x+3 (x \in Q)$ is a bijection. Find ϕ-image of 5 and pre-image of 5.**

Solution: Let $x, y \in Q$ be such that $\phi(x)=\phi(y)$.

Then, $\phi(x)=\phi(y)$

$\Rightarrow 2x+3=2y+3$
$\Rightarrow 2x=2y$
$\Rightarrow x=y$

$\therefore$ ϕ is one-one.

Let $y \in Q$ (co-domain) be arbitrary element such that $\phi(x)=y$.

Then, $\phi(x)=y$

$\Rightarrow 2x+3 = y$

$\Rightarrow x=\dfrac{y-3}{2} \in Q$

Thus, for all $y \in Q$ (co-domain), there exist an pre-image $x=\dfrac{y-3}{2} \in Q$

such that $\phi(x)=y$.

$\therefore$ ϕ is onto.

Since, ϕ is one-one and onto, it is a bijection.

Also, $\phi(5)=2\times 5+3=13$ and Pre-image of5= 1 as $\phi(x)=5 \Rightarrow 2x+3=5 \Rightarrow x=1$

9. **Examine whether the following functions are one-one or many-one:**

(i) $f : N \to N$ **defined by** $f(n)=n^2$**.**

Solution: It is one-one as different elements have different images.

(ii) $f: Z \to Z$ defined by $f(a) = a^2$.

Solution: It is not one-one for , $f(1) = 1 = f(-1)$ but $1 \neq -1$. So, it is many-one.

(iii) $f: Z \to Z$ defined by $f(a) = a^3$.

Solution: It is one-one, for $f(a) = f(b) \Rightarrow a^3 = b^3 \Rightarrow a = b$.

(iv) $f: R \to R$ defined by $f(x) = 3x + 5$.

Solution: $f(x) = f(y) \Rightarrow 3x + 5 = 3y + 5$

$\Rightarrow 3x = 3y \Rightarrow x = y$.

$\therefore f$ is one-one.

(v) $f: R^+ \to R^+$ defined by $f(x) = \frac{1}{x}$.

Solution: $f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$.

$\therefore f$ is one-one.

10. Examine whether the following functions are onto or into:

(i) $f: N \to N$ defined by $f(n) = n + 5$.

Solution: Let $f(n) = m$. Then,

$f(n) = m$

$\Rightarrow n + 5 = m$

$\Rightarrow n = m - 5$

When $m = 1, = -4 \notin N . \therefore there\ is\ no\ pre\_image\ of\ 1\ in\ domain$.

Thus, f is not onto and hence into.

(ii) $f: Z \to Z$ defined by $f(a) = a + 5$.

Solution: Let $f(a) = b$. Then

$f(a)=b$
$\Rightarrow a+5 \;=b$
$\Rightarrow a=b-5\in Z.$
$\Rightarrow$ Every element of co-domain have f-pre-image in domain.
Thus, f is onto .

(iii) $f:R^{+}\to R^{+}$ defined by $f(x)=x^{2}+5.$

Solution: Let $f(x)=y$. Then,

$f(x)=y$
$\Rightarrow x^{2}+5 \;=y$
$\Rightarrow x=\sqrt{y-5}.$
When $y=1, x=\sqrt{-4}\notin R^{+}.$
$\therefore$ There is no pre-image of 1 in domain
Thus, f is not onto and hence into .

(iv) $f:R\to R$ defined by $f(x)=3x+5.$

Solution: Let $f(x)=y$. Then,

$f(x)=y$
$\Rightarrow 3x+5 \;=y$
$\Rightarrow x=\dfrac{y-5}{3}\in R.$
$\Rightarrow$ all the elements of co-domain are the f-image of at least one element in domain.
Thus, f is onto.

(v) $f:R^{+}\to R^{+}$ defined by $f(x)=\dfrac{1}{x}.$

Solution: Let $f(x)=y$. Then,

$f(x)=y$
$\Rightarrow \dfrac{1}{x} \;=y$
$\Rightarrow x=\dfrac{1}{y}\in R^{+}.$
Thus, f is onto.

11. Find the range of the function $f : N \to Q$ defined by $f(n) = \begin{cases} n^2, \; if \;\; n \text{ is odd} \\ \frac{1}{n}, if \;\; n \text{ is even} \end{cases}$.

Is the function one-one or many-one? Give reason for your answer. Find the value of

$f(3),\ f(4),\ f(5)\ and\ f(16).$

Solution: The given function is one-one as different elements of N have different images in Q i.e.

$Range = \{1^2,\ 3^2,\ 5^2, \ldots\} \cup \{\frac{1}{2},\ \frac{1}{4},\ \frac{1}{6}, \ldots\}.$

And, $f(3) = 3^2 = 9,\ f(4) = \frac{1}{4},\ f(5) = 5^2 = 25\ and\ f(16) = \frac{1}{16}.$

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