CHAPTER 6
LINES AND ANGLES

Some Definitions

- **Point:** It has neither length nor breadth, nor thickness, however, it has a unique position.
- **Line:** A line has neither breadth nor thickness, however it has a sense of length.
- **Plane:** A plane has sense of length and breadth but not of thickness.
- **Line Segment:** A finite portion of a line is called a line segment. A line segment has two end points.
- **Ray:** A portion of a line extended in one direction from a fixed point is called a ray. The fixed point is called the initial point (end point) of the ray.

Note:
(i) A line segment is a part of a line with two end points.
(ii) A ray is a part of a line with one end point.

- **Collinear Points:** Three or more points are said to be collinear if there is a line which contains all of them.
- **Concurrent Lines:** Three or more lines are said to be concurrent if there is a point which lies on all of them.
- **Angle:** An angle is formed by two rays with a common initial point. The common initial point is called the vertex of the angle. The rays forming an angle are called arms or sides of the angle.

Types of Angles:

(i) Acute angle: An angle whose measure lies between 0^0 and 90^0 is called an acute angle.

(ii) Right angle: An angle whose measure is 90^0 is called a right angle.

(iii) Obtuse angle: An angle whose measure is greater than 90^0 but less than 180^0 is called an obtuse angle.

(iv) Straight angle: An angle whose measure is 180^0 is called a straight angle.

(v) Reflex angle: An angle whose measure is greater than 180^0 but less than 360^0 is called a reflex angle.

(vi) Complete angle: An angle whose measure is 360^0 is called a complete angle.

Complementary Angles:
Two angles, the sum of whose measure is 90^0, are called complementary angles. Each of the two complementary angles is called the complement of the other.

Supplementary Angles:
Two angles, the sum of whose measure is 180^0, are called supplementary angles. Each of the two supplementary angles is called the supplement of the other.

➢ **Adjacent Angles:** Two angles are called adjacent angles if

(i) they have the same vertex

(ii) they have a common arm and

(iii) do not overlap

➢ **Linear Pair Angles:** Two adjacent angles are said to form a linear pair angles, if their non-common arms are two opposite rays.

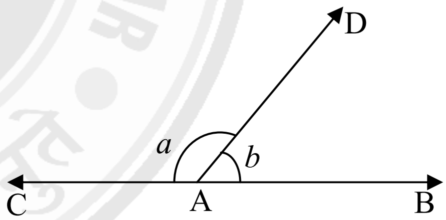
Theorem 6.1: If a ray stands on a line, then the sum of the two adjacent angles so formed is 180^0.

Theorem 6.2: If the sum of two adjacent angles is 180^0, then their non-common arms are two opposite rays.

SOLUTIONS

EXERCISE 6.1

1. In the adjoining figure, AB and AC are opposite rays. If $a - 3b = 20^0$, find a and b.



Solution: We have, $a - 3b = 20^0$

$\Rightarrow a = 20^0 + 3b$

As AB and AC are opposite rays,

$\angle CAD + \angle BAD = 180^0$

$\Rightarrow a + b = 180^0$

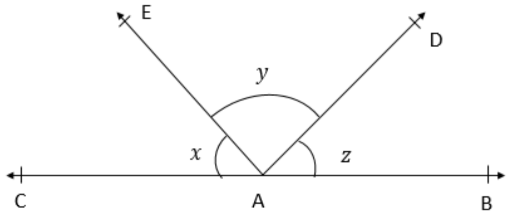
$\Rightarrow 20^0 + 3b + b = 180^0$ [Substituting $a = 20^0 + 3b$]

$\Rightarrow 4b = 160^0$

$\Rightarrow b = \frac{160^0}{4} = 40^0$

$\therefore a = 20^0 + 3 \times 40^0 = 20^0 + 120^0 = 140^0$

2. In the adjoining figure, BAC is a line and $x:y:z = 5:6:7$. Find x, y and z.



Solution: We have, $x:y:z = 5:6:7$

Let $x = 5k, y = 6k$ and $z = 7k$

We know, $x + y + z = 180^0$ [$\because$AD and AE stand on a same side of BC]

$\Rightarrow 5k + 6k + 7k = 180^0$

$\Rightarrow 18k = 180^0$

$\Rightarrow k = 10^0$

$\therefore x = 5 \times 10^0 = 50^0, y = 6 \times 10^0 = 60^0$ and $z = 7 \times 10^0 = 70^0$

3. Two angles of a linear pair in the ratio 6:3. Find the measure of each of these angles.

Solution: Let 6xand 3x (in degrees) be the measures of the angles of the linear pair.

Then, $6x + 3x = 180^0$

$\Rightarrow 9x = 180^0$

$\Rightarrow x = \dfrac{180^0}{9} = 20^0$

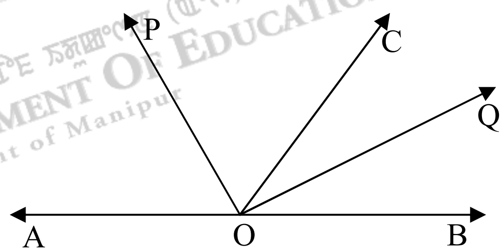
$\therefore$ The measures of the two angles are $6 \times 20^0 = 120^0$and $3 \times 20^0 = 60^0$.

4. Prove that the bisectors of two adjacent supplementary angles include a right angle.

Solution:

Given: $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are the bisectors of two adjacent supplementary angles $\angle AOC$ and $\angle BOC$.

To prove: $\angle POQ=90^0$



Proof: Since $\angle AOC$and $\angle BOC$ are adjacent supplementary angles, we have

$$\angle AOC + \angle BOC = 180^0$$

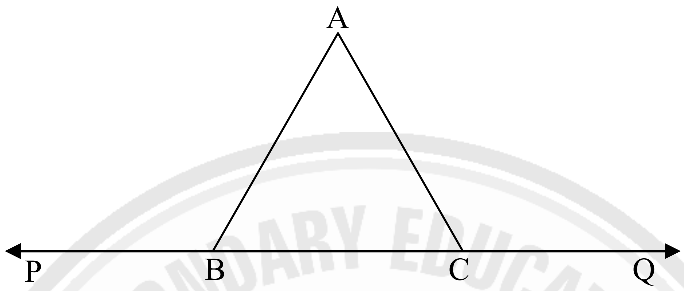
$$\Rightarrow \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = \frac{1}{2} \times 180^0$$

$$\Rightarrow \angle POC + \angle QOC = 90^0$$

$$\therefore \angle POQ = 90^0$$

5. In the ΔABC, $\angle ABC=\angle ACB$, if the side BC is produced both ways, prove that their exterior angles are equal.

Solution:



Given: In ΔABC, $\angle ABC=\angle ACB$. BC is produced both ways to P and Q.

To Prove: $\angle ABP = \angle ACQ$

Proof: We have, AB and AC stand on PQ.

$\therefore \angle ABP+\angle ABC = 180^0$

And $\angle ACQ + \angle ACB = 180^0$

Then, $\angle ABP+\angle ABC = \angle ACQ + \angle ACB$

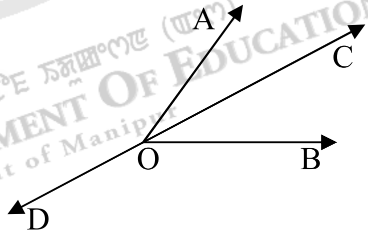
$\therefore \angle ABP = \angle ACQ$ $[\because \angle ABC=\angle ACB]$

6. The $\angle AOB$ is bisected by ray OC and ray OD is opposite to ray OC. Prove that $\angle AOD$ and $\angle BOD$ are equal.

Solution:

Given: $\angle AOB$ is bisected by ray OC and ray OD is opposite to ray OC.

To prove: $\angle AOD=\angle BOD$



Proof: As $\angle AOB$ is bisected by ray OC, $\angle AOC=\angle BOC$

We know, $\angle AOD + \angle AOC=180^0$ $[\because \overrightarrow{OA}$ stands on $\overleftrightarrow{CD}]$

and, $\angle BOD + \angle BOC = 180^0$ $[\because \overrightarrow{OB}$ stands on $\overleftrightarrow{CD}]$

Then, $\angle AOD+\angle AOC = \angle BOD + \angle BOC$

$\therefore \angle AOD = \angle BOD$ $[\because \angle AOC=\angle BOC]$

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- **Vertically opposite angles**

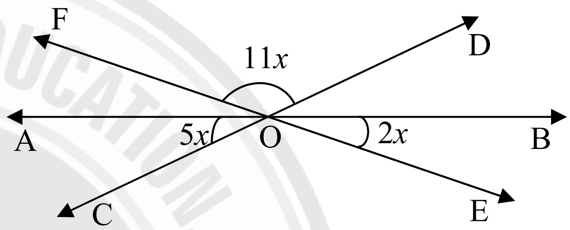
  Two angles are said to be vertically opposite angles, if their arm form two pairs of opposite rays.

Theorem 6.3: If two lines intersect, the vertically opposite angles are equal in measure.

SOLUTIONS

EXERCISE 6.2

1. **In the adjoining figure, fine the value of x.**



Solution: We have, $\angle AOF = \angle BOE$ [vertically opposite angles]

$\therefore \angle AOF = 2x$

CD is a straight line.

$\therefore \angle AOC + \angle AOF + \angle DOF = 180^0$

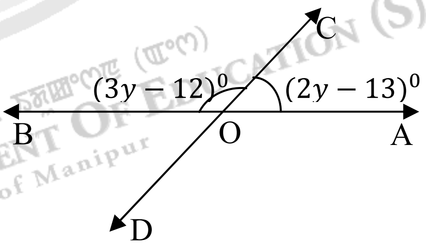
$\Rightarrow 5x + 2x + 11x = 180^0$

$\Rightarrow 18x = 180^0$

$\Rightarrow x = \frac{180^0}{18}$

$\therefore x = 10^0$

2. **In the adjoining figure, $\angle AOC = (2y - 13)^0$ and $\angle BOC = (3y - 12)^0$, find all the four angles.**



Solution: We have, $\angle BOC = \angle AOD$, $\angle AOC = \angle BOD$ [vertically opposite angles]

$\angle BOC = (3y - 12)^0$ and $\angle AOC = (2y - 13)^0$

We know, $\angle BOC + \angle AOC = 180^0$ [$\because$OC stands on AB]

$\Rightarrow (3y - 12)^0 + (2y - 13)^0 = 180^0$

$\Rightarrow 3y + 2y - 25^0 = 180^0$

$\Rightarrow 5y = 205^0$

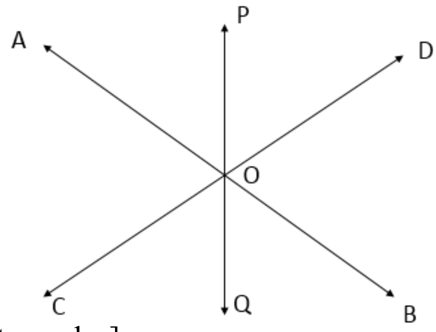
$\Rightarrow y = \frac{205^0}{5} = 41^0$

$\angle AOD = \angle BOC = (3 \times 41 - 12)^0 = (123 - 12)^0 = 111^0$

$\angle AOC = \angle BOD = (2 \times 41 - 13)^0 = (82 - 13)^0 = 69^0$

3. Prove that the bisectors of a pair of vertically opposite angles are in the same straight line.

Given: OP and OQ are bisectors of a pair of vertically opposite angles $\angle AOD$ and $\angle BOC$



To prove: OP and OQ are in the same straight line.

Proof: As OP bisects $\angle AOD$, $\angle AOP = \angle DOP = \frac{1}{2}\angle AOD$

As OQ bisects $\angle BOC$, $\angle COQ = \angle BOQ = \frac{1}{2}\angle BOC$

We know, $\angle AOD = \angle BOC$ [being vertically opposite angles]

$\Rightarrow \frac{1}{2}\angle AOD = \frac{1}{2}\angle BOC$

$\Rightarrow \angle AOP = \angle DOP = \angle COQ = \angle BOQ$

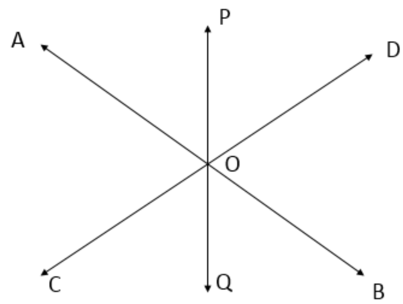
and $\angle COP + \angle DOP = 180^0$ [$\because \overrightarrow{OP}$ stands on $\overleftrightarrow{CD}$]

$\Rightarrow \angle COP + \angle COQ = 180^0$

Hence, $\overrightarrow{OP}$and$\overrightarrow{OQ}$ are in the same straight line.

4. If two straight lines intersect each other, prove that the ray opposite to the bisector of one of the angles thus formed bisects the vertically opposite angles.

Given: Two straight lines AB and CD intersect at O. $\overrightarrow{OP}$ is the bisector of $\angle AOD$ and $\overrightarrow{OQ}$ is drawn opposite to $\overrightarrow{OP}$.



To prove: $\overrightarrow{OQ}$ bisects $\angle BOC$

Proof: As $\overrightarrow{OQ}$ is opposite to $\overrightarrow{OP}$,

$\angle AOP = \angle BOQ$ [vertically opposite angles]

and, $\angle DOP = \angle COQ$ [vertically opposite angles]

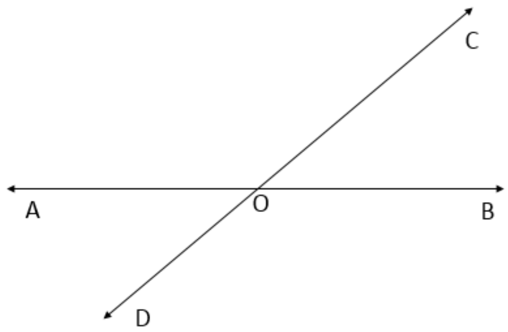
But, $\angle AOP = \angle DOP$ [$\because \overrightarrow{OP}$ bisects $\angle AOD$]

$\therefore \angle BOQ = \angle COQ$

So, $\overrightarrow{OQ}$ bisects$\angle BOC$.

5. From a point O in a line AB, rays OC and OD are drawn on opposite sides of AB such that $\angle BOC = \angle AOD$. Prove that OC and OD are opposite rays.

Given: From a point O on AB, rays OC and OD are drawn on opposite side of AB such that $\angle BOC = \angle AOD$



To prove: OC and OD are opposite rays.

Proof: As OC stands on AB, $\angle AOC + \angle BOC = 180^0$

$$\Rightarrow \angle AOC = 180^0 - \angle BOC$$

As OD stands on AB, $\angle AOD + \angle BOD = 180^0$

$$\Rightarrow \angle BOD = 180^0 - \angle AOD$$

$$\Rightarrow \angle BOD = 180^0 - \angle BOC \qquad [\because \angle AOD = \angle BOC]$$

$$\therefore \angle AOC = \angle BOD$$

As $\angle AOC = \angle BOD$ and $\angle AOD = \angle BOC$,

$\overrightarrow{OC}$ and $\overrightarrow{OD}$ are opposite rays.

Alternative proof

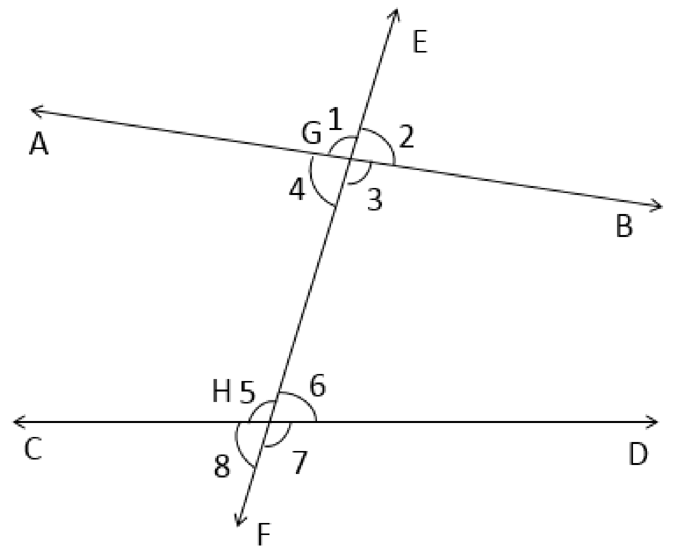
We know, $\angle AOC + \angle BOC = 180^0 \quad [\because \overrightarrow{OC}$ stands on AB]

$$\Rightarrow \angle AOC + \angle AOD = 180^0 \quad [\because \angle AOD = \angle BOC]$$

Then, $\angle AOC$ and $\angle AOD$ form a pair of linear pair angles.

Hence, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ are opposite rays.

- **Angles made by a transversal with two lines:**



A transversal $\overleftrightarrow{EF}$ intersects two lines AB and CD at G and H.

From the above figure,

($\angle 1, \angle 5$), ($\angle 2$, $\angle 6$), ($\angle 3$, $\angle 7$), ($\angle 4$, $\angle 8$) are called pairs of corresponding angles.

($\angle 3$, $\angle 5$), ($\angle 4$, $\angle 6$) are called pairs of alternate interior angles or simply alternate angles.

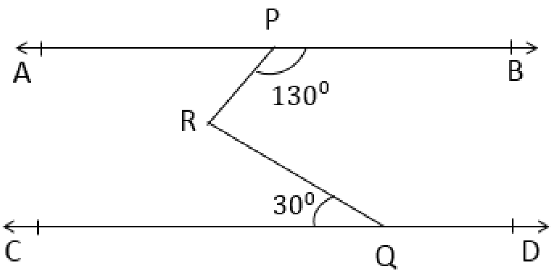
($\angle 3$, $\angle 6$), ($\angle 4$, $\angle 5$) are called pairs of interior angles on the same side of the transversal.

- **Axiom 6.4**: If a transversal intersects two parallel lines, then the angles in each pair of corresponding angles are equal.
- **Axiom 6.5**: If a transversal intersects two lines making a pair of corresponding angles equal, then the lines are parallel.
- **Theorem 6.6:** If a transversal intersects two parallel lines, then alternate angles are equal.
- **Theorem 6.7:** If a transversal intersects two lines in such a way that, a pair of alternate angles are equal, then the two lines are parallel.
- **Theorem 6.8:** If a transversal intersects two parallel lines, then the interior angles on the same side of the transversal are supplementary.
- **Theorem 6.9:** If a transversal intersects two lines in such a way that a pair of interior angles on the same side of the transversal are supplementary, then the two lines are parallel.
- **Theorem 6.10:** Two lines which are parallel to the same line are parallel to one another.

SOLUTIONS

EXERCISE 6.3

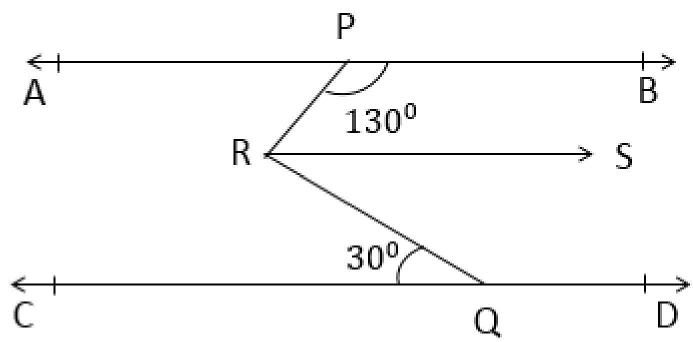
1. **In the adjoining figure, if $AB \parallel CD$, $\angle BPR = 130^0$ and $\angle CQR = 30^0$, find $\angle PRQ$.**



Solution:

AB∥CD, $\angle$BPR=130^0, $\angle$CQR=30^0

We draw RS∥AB and CD.



As AB∥RS,

$\angle$BPR + $\angle$PRS = 180^0

$\Rightarrow 130^0 + \angle PRS = 180^0$

$\Rightarrow \angle PRS = 180^0 - 130^0 = 50^0$

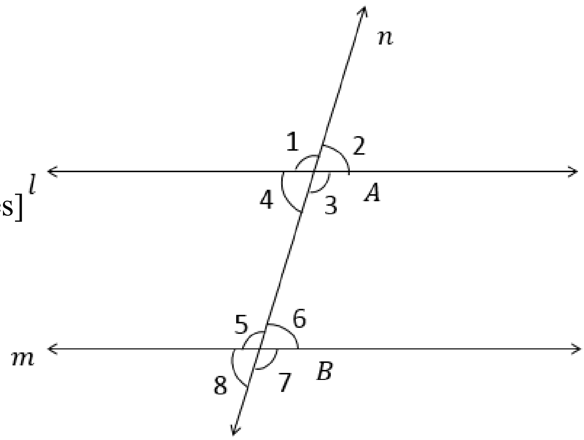
As RS∥CD,

$\angle$QRS = $\angle$CQR [alternate angles]

$\Rightarrow \angle QRS = 30^0$

$\therefore \angle PRQ = \angle PRS + \angle QRS = 50^0 + 30^0 = 80^0$

2. **In the figure, $\ell \parallel$m and transversal n intersects ℓ and m at A and B respectively. If $\angle 1 : \angle 2$ =3:2, determine all the eight angles.**



Solution: $\angle 1 : \angle 2 = 3 : 2$

Let $\angle 1 = 3x, \angle C = 2x$

We know, $\angle 1 + \angle 2 = 180^0$ [linear pair angles]

$\Rightarrow 3x + 2x = 180^0$

$\Rightarrow 5x = 180^0$

$\Rightarrow x = \frac{180^0}{5} = 36^0$

$\therefore \angle 1 = 3 \times 36^0 = 108^0$

$\angle 2 = 2 \times 36^0 = 72^0$

$\angle 1 = \angle 5$ [Corresponding angles]

$\angle 5 = \angle 3$ [Alternate angles]

$\angle 3 = \angle 7$ [Corresponding angles]

$\therefore \angle 1 = \angle 5 = \angle 3 = \angle 7 = 108^0$

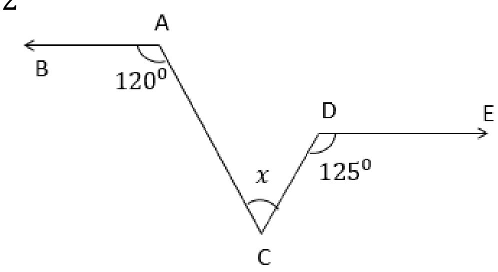
$\angle 2 = \angle 6$ [Corresponding angles]

$\angle 6 = \angle 4$ [Alternate angles]

$\angle 4 = \angle 8$ [Corresponding angles]

$\therefore \angle 2 = \angle 6 = \angle 4 = \angle 8 = 72^0$

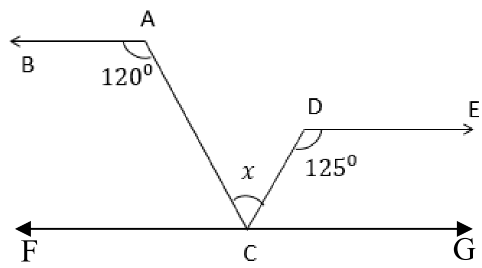
3. In the figure, AB∥DE. Find x.



Solution:

FG is drawn through parallel to DE.

We have,



$\angle BAC + \angle ACF = 180^0$ [being interior angles on a same side of a transversal]

$\Rightarrow 120^0 + \angle ACF = 180^0$

$\Rightarrow \angle ACF = 60^0$

And $\angle ECD + \angle DCG = 180^0$ [being interior angles on a same side of a transversal]

$\Rightarrow 125^0 + \angle DCG = 180^0$

$\Rightarrow \angle DCG = 55^0$

We know, $\angle ACF + \angle ACD + \angle DCG = 180^0$

$\Rightarrow 60^0 + x + 55^0 = 180^0$

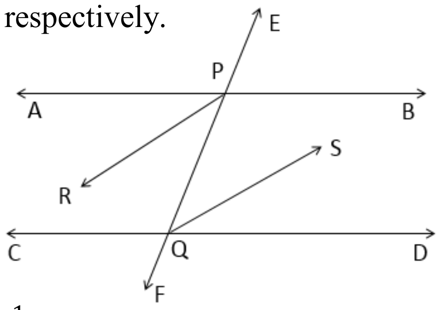
$\Rightarrow x + 115^0 = 180^0$

$\therefore x = 65^0$

4. If a transversal intersects two parallel lines, show that the bisectors of any pair of alternate angles are parallel.

Solution:

Given: A transversal EF intersects two parallel lines AB and CD at P and Q. PR and QS are the bisectors of a pair of alternate angles $\angle APQ$ and $\angle DQP$ respectively.



To prove: PR$\parallel QS$

Proof: As $AB \parallel CD$,

$\angle APQ = \angle DQP$ [alternate angles]

$\Rightarrow \frac{1}{2}\angle APQ = \frac{1}{2}\angle DQP$

$\Rightarrow \angle QPR = \angle PQS$ [$\because \angle QPR = \frac{1}{2}\angle APQ, \angle PQS = \frac{1}{2}\angle DQP$]

But ∠QPR and ∠PQS are alternate angles formed by PR and QS with a transversal EF.

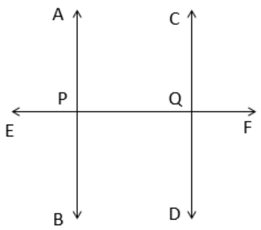
$\therefore$ PR$\parallel QS$

5. Prove that lines which are perpendicular to the same line are parallel to one another.

Solution:

Given: Two lines AB and CD are perpendicular to the same line EF intersecting EF at P and Q.

To prove: $AB \parallel CD$



Proof: As $AB \perp EF,\ \angle APQ = 90^0$

As $CD \perp EF,\ \angle CQF = 90^0$

$\therefore \angle APQ = \angle CQF$

But ∠APQ and ∠CQF are corresponding angles formed by AB and CD with a transversal EF.

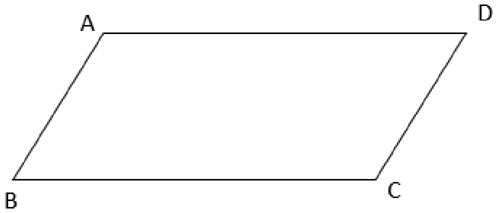
$\therefore AB \parallel CD$

6. Two unequal angles of a parallelogram are in the ratio 4:5. Find all the angles of the parallelogram in degrees.

Solution: Let ABCD be the parallelogram in which $\angle A : \angle B = 4 : 5$

Let $\angle A = 4x, \angle B = 5x$

We know, $\angle A + \angle B = 180^0$ [being interior angles on a same side of a transversal]



$\Rightarrow 4x + 5x = 180^0$

$\Rightarrow 9x = 180^0$

$\Rightarrow x = \frac{180^0}{9} = 20^0$

$\therefore \angle A = 4 \times 20^0 = 80^0$ and $\angle B = 5 \times 20^0 = 100^0$

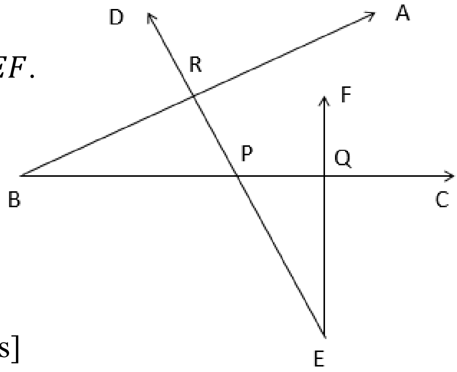
But $\angle A = \angle C$ and $\angle B = \angle D$ [opposite angles of parallelogram are equal]

$\therefore \angle C = 80^0, \angle D = 100^0$

7. Prove that if the two arms of an angle are perpendicular to the two arms of another angle, then the angles are either equal or supplementary.

Solution:

Case I: In two angles $\angle ABC$ and $\angle DEF$, $BA \perp ED$ and $BC \perp EF$.



As $BA \perp ED, \angle BRP = 90^0$

As $BC \perp EF, \angle EQP = 90^0$

$\therefore \angle BRP = \angle EQP$

and $\angle BPR = \angle EPQ$ [vertically opposite angles]

In $\Delta BPR, \angle BPR + \angle BRP + \angle RBP = 180^0$

In $\Delta EPQ, \angle EPQ + \angle EQP + \angle PEQ = 180^0$

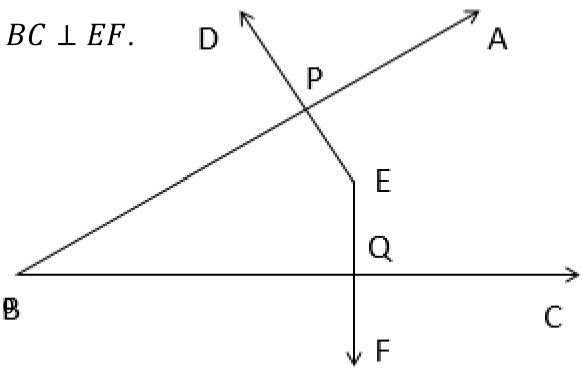
$\therefore \angle BPR + \angle BRP + \angle RBP = EPQ + \angle EQP + \angle PEQ$

$\Rightarrow \angle RBP = \angle PEQ$ [$\because \angle BPR = \angle EPQ, \angle BRP = \angle EQP$]

$\Rightarrow \angle ABC = \angle DEF$

So, the two angles are equal.

Case II: In two angles $\angle ABC$ and $\angle DEF$, $BA \perp ED$ and $BC \perp EF$.



As $AB \perp ED, \ \angle BPE = 90^0$

As $BC \perp EF, \angle BQE = 90^0$

In the quadrilateral PBQE,

$\angle BPE + \angle PBQ + \angle BQE + \angle PEQ = 360^0$

$\Rightarrow \ 90^0 + \angle ABC + 90^0 + \angle DEF = 360^0$

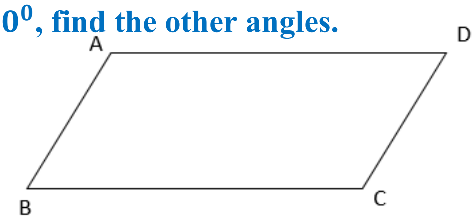
$\Rightarrow \angle ABC + \angle DEF = 360^0 - 180^0$

$\Rightarrow \angle ABC + \angle DEF = 180^0$

So, the angles are supplementary.

Hence, if the two arms of an angle are perpendicular to the two arms of another angle, then the angles are either equal or supplementary.

8. If one angle of a parallelogram in which $\angle B = 60^0$, find the other angles.



Solution: In the parallelogram ABCD, we have

$\angle B = 60^0$

and $\angle A + \angle B = 180^0$ [being interior angles on a same side of a transversal]

$\Rightarrow \angle A + 60 = 180^0$

$\Rightarrow \angle A = 120^0$

But $\angle A = \angle C, \angle B = \angle D$ [opposite angles of parallelogram are equal]

$\therefore \angle C = 120^0, \angle D = 60^0$.

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Triangle and its Angles

- **Theorem 6.11 (Angle Sum Property of Triangle):** The sum of the measures of three angles of a triangle is 180^0.

Exterior angles of a triangle:

- **Theorem 6.12:** If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

SOLUTIONS

EXERCISE 6.4

1. **If the angles of a triangle are in the ratio 1: 2: 6, determine the three angles.**

Solution: Let $x, 2x, 6x$ be the measures of the three angles of the triangle.

$\therefore x + 2x + 6x = 180^0$

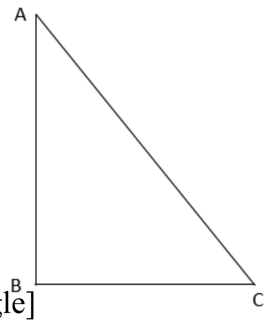
$\Rightarrow 9x = 180^0$

$\Rightarrow x = \dfrac{180^0}{9} = 20^0$

$\therefore$ the measure of the three angles of the triangle are $20^0, 2 \times 20^0$ and 6×20^0

i.e. 20^0, 40^0 and 120^0.

2. If one angle of a triangle is equal to the sum of the other two, show that the triangle is a right triangle.



Solution:

Given: In ΔABC, $\angle B = \angle A + \angle C$.

To prove: ABC is a right triangle

Proof: In ΔABC, we have

$\angle A + \angle B + \angle C = 180^0$ [by Angle Sum Property of Triangle]

$$\Rightarrow \angle B + (\angle A + \angle C) = 180^0$$

$$\Rightarrow \angle B + \angle B = 180^0$$

$$\Rightarrow 2\angle B = 180^0$$

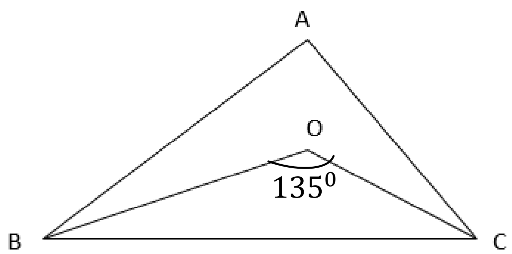
$$\Rightarrow \angle B = \frac{180^0}{2} = 90^0$$

Hence, ABC is a right triangle.

3. If the bisectors of the base angles of a triangle enclose an angle of 135^0, prove that the triangle is a right angle.

Solution:

Given: In ΔABC, the bisectors of the base angles $\angle B$ and $\angle C$ intersect at O such that $\angle BOC = 135^0$.



To prove: ABC is a right triangle.

Proof: In ΔBOC, we have

$$\angle BOC + \angle OBC + \angle OCB = 180^0$$

$$\Rightarrow 135^0 + \tfrac{1}{2}\angle B + \tfrac{1}{2}\angle C = 180^0 \qquad [\because \angle OBC = \tfrac{1}{2}\angle B, \angle OCB = \tfrac{1}{2}\angle C]$$

$$\Rightarrow \frac{1}{2}(\angle B + \angle C) = 180^0 - 135^0 = 45^0$$

$$\Rightarrow \angle B + \angle C = \frac{1}{2} \times 45^0$$

$$\Rightarrow \angle B + \angle C = 90^0$$

In ΔABC, we have

$$\angle A + \angle B + \angle C = 180^0$$

$$\Rightarrow \angle A + 90^0 = 180^0$$

$$\Rightarrow \angle A = 90^0$$

$\therefore$ ABC is a right triangle.

4. In any regular polygon of n sides, prove that each of its angles measures $\frac{n-2}{n} \times 180^0$.

Solution: If we join a vertex of a regular polygon of n sides to the other vertices, the number of triangles formed is $n-2$.

If θ be the measure of each of the angles, then

$$n \times \theta = (n-2) \times 180^0$$

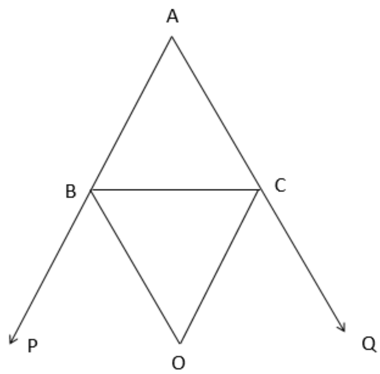
$$\Rightarrow \theta = \frac{n-2}{n} \times 180^0$$

$\therefore$ the measure of each of the angles of the regular polygon of n sides is $\frac{n-2}{2} \times 180^0$.

5. The sides AB and AC of the ΔABC are produced to P and Q respectively. The bisectors of $\angle PBC$ and $\angle QCB$ intersect at O. Prove that $\angle BOC = 90^0 - \frac{1}{2}\angle BAC$.

Solution:

Given: In ΔABC, AB and AC are produced to P and Q. The bisectors of $\angle PBC$ and $\angle QCB$ intersect at O.



To prove: $\angle BOC = 90^0 - \frac{1}{2}\angle BAC$

Proof: As CB stands on AP,

$$\angle PBC + \angle ABC = 180^0$$

$$\Rightarrow \angle PBC = 180^0 - \angle ABC$$

$$\Rightarrow \frac{1}{2}\angle PBC = \frac{1}{2}(180^0 - \angle ABC)$$

$\Rightarrow \angle OBC = 90^0 - \frac{1}{2}\angle ABC$ ----------------------(1) [$\therefore \angle OBC = \frac{1}{2}\angle PBC$]

Similarly, $\angle OCB = 90^0 - \frac{1}{2}\angle ACB$ -------------------------(2)

Adding (1) and (2),

$$\angle OBC + \angle OCB = 90^0 - \frac{1}{2}\angle ABC + 90^0 - \frac{1}{2}\angle ACB$$

$\Rightarrow 180^0 - \angle BOC = 180^0 - \frac{1}{2}(\angle ABC + \angle ACB)$ [$\because \angle BOC + \angle OBC + \angle OCB = 180^0$]

$\Rightarrow \angle BOC = \frac{1}{2}(\angle ABC + \angle ACB)$

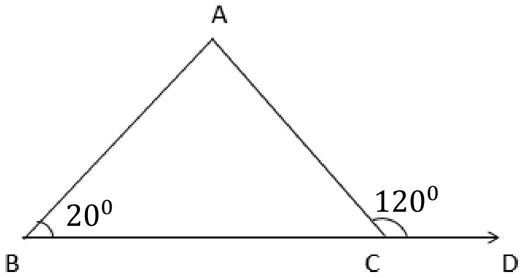
$\Rightarrow \angle BOC = \frac{1}{2}(180^0 - \angle BAC)$ [$\therefore \angle BAC + \angle ABC + \angle ACB = 180^0$]

$\Rightarrow \angle BOC = \frac{1}{2} \times 180^0 - \frac{1}{2}\angle BAC$

$\therefore \angle BOC = 90^0 - \frac{1}{2}\angle BAC$

6. An exterior angle of a triangle is 120^0 and one of the interior opposite angles is 20^0. Find the other two angles of the triangle.

Solution:



Let ABC be the triangle. BC is produced to D such that ∠ACD = 120^0 and ∠ABC = 20^0.

We know, ∠BAC + ∠ABC = ∠ACD

⇒ ∠BAC + 20^0 = 120^0

⇒ ∠BAC = 100^0

In ΔABC, we know

$$\angle BAC + \angle ABC + \angle ACB = 180^0 \text{ [by angle sum property of triangle]}$$

$$\Rightarrow 100^0 + 20^0 + \angle ACB = 180^0$$

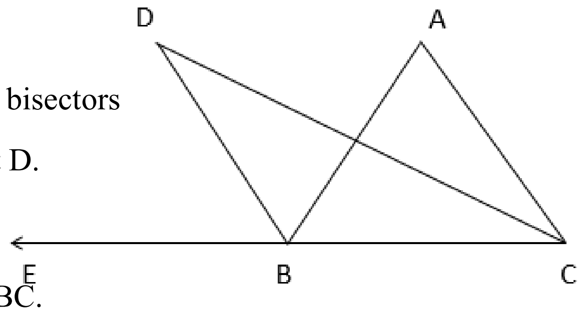
$$\Rightarrow 120^0 + \angle ACB = 180^0$$

$$\Rightarrow \angle ACB = 60^0$$

So, the other two angles of the triangle are 100^0 and 60^0.

7. ABC is a triangle. The bisector of the exterior angle at B and the bisector of $\angle C$ intersect each other at D. Prove that $\angle BDC = \frac{1}{2}\angle A$.

Solution:



Given: ∠ABE is an exterior angle of a ΔABC. The bisectors of ∠ABE and ∠ACB intersect each other at D.

To prove: $\angle BDC = \frac{1}{2}\angle A$

Proof: We have, ∠ABE is an exterior angle of ΔABC.

$$\therefore \angle ABE = \angle BAC + \angle ACB$$

Since ∠ABE is an exterior angle of a ΔABC,

$$\angle BDC + \angle BCD = \angle DBE$$

$$\Rightarrow \angle BDC + \frac{1}{2}\angle ACB = \frac{1}{2}\angle ABE$$

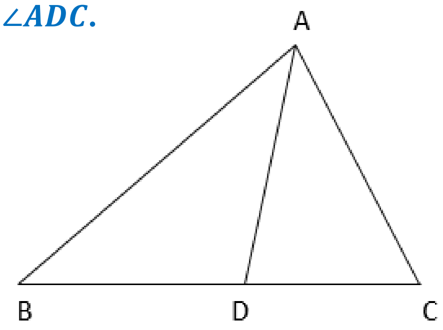
$$\Rightarrow \angle BDC + \frac{1}{2}\angle ACB = \frac{1}{2}(\angle BAC + \angle ACB)$$

$$\Rightarrow \angle BDC + \frac{1}{2}\angle ACB = \frac{1}{2}\angle BAC + \frac{1}{2}\angle ACB$$

$$\Rightarrow \angle BDC = \frac{1}{2}\angle BAC$$

$$\therefore \angle BDC = \frac{1}{2}\angle A$$

8. In ΔABC, AD bisects $\angle A$ and $\angle C > \angle B$. Prove that $\angle ADB > \angle ADC$.



Solution:

Given: In ΔABC, AD bisects $\angle A$ and $\angle C > \angle B$.

To prove: $\angle ADB > \angle ADC$.

Proof: As AD bisects $\angle A, \angle BAD = \angle CAD = \frac{1}{2}\angle BAC$

As $\angle ADC$ is an exterior angle of ΔABD,

$$\angle ADC = \angle ABD + \angle BAD$$

$$\Rightarrow \angle ADC = \angle B + \frac{1}{2}\angle BAC \text{ --------------- (1)}$$

As $\angle ADB$ is an exterior angle of ΔACD,

$$\angle ADB = \angle ACD + \angle CAD$$

$$\Rightarrow \angle ADB = \angle C + \frac{1}{2}\angle BAC \text{ --------------- (2)}$$

We have, $\angle C > \angle B$ (given)

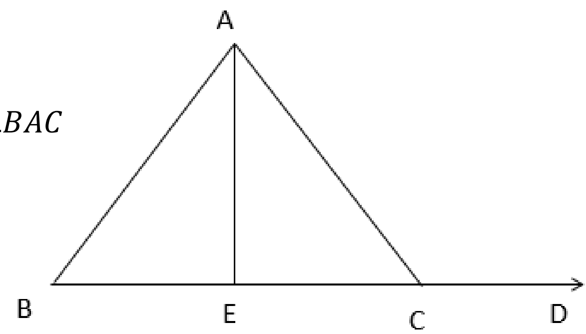
$$\Rightarrow \angle C + \frac{1}{2}\angle BAC > \angle B + \frac{1}{2}\angle BAC$$

$\therefore \angle ADB > \angle ADC$. [using (1) and (2)]

9. The side BC of a triangle ΔABC is produced to D to form the exterior angle $\angle ACD$.The bisector of $\angle BAC$ intersects BC at E. Prove that $\angle ABC + \angle ACD = 2\angle AEC$.

Solution:

Given: The side BC of ΔABC is produced to D. The bisector of $\angle BAC$ intersects BC at E.



To prove: $\angle ABC + \angle ACD = 2\angle AEC$

Proof: As AE bisects $\angle A$, $\angle BAE = \angle CAE = \frac{1}{2}\angle BAC$

In ΔABE, $\angle AEC$ is an exterior angle,

$$\therefore \angle ABE + \angle BAE = \angle AEC$$

$$\Rightarrow \angle ABE = \angle AEC - \angle BAE$$

$$\Rightarrow \angle ABC = \angle AEC - \frac{1}{2}\angle BAC \text{ --------------- (1)}$$

In ΔACE, $\angle ACD$ is an exterior angle,

$$\therefore \angle ACD = \angle AEC + \angle CAE$$

$$\Rightarrow \angle ACD = \angle AEC + \frac{1}{2}\angle BAC \text{ --------------- } (2)$$

Adding (1) and (2), we get

$$\angle ABC + \angle ACD = \angle AEC - \frac{1}{2}\angle BAC + \angle AEC + \frac{1}{2}\angle BAC$$

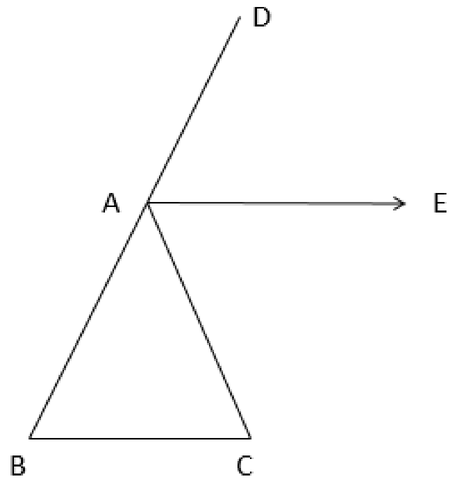
$$\therefore \angle ABC + \angle ACD = 2\angle AEC$$

10. ABC is an isosceles triangle in which AB=AC and AE bisects the exterior angle CAD at A. Prove that $AE \parallel BC$.

Solution:

Given: In the isoscelesΔABC, AB=AC. AE bisects the exterior $\angle CAD$ at A.

To prove: $AE \parallel BC$



Proof: As AB=AC in ΔABC, $\angle ACB = \angle ABC$

As AE bisects $\angle CAD$, $\angle CAE = \angle DAE$

As $\angle CAD$ is exterior angle of ΔABC,

$$\angle CAD = \angle ABC + \angle ACB$$

$$\Rightarrow \angle CAE + \angle DAE = \angle ABC + \angle ABC$$

$$\Rightarrow \angle DAE + \angle DAE = 2\angle ABC$$

$$\Rightarrow 2\angle DAE = 2\angle ABC$$

$$\Rightarrow \angle DAE = \angle ABC$$

But ∠DAE and ∠ABC are corresponding angles formed by AE and BC with BD as transversal.

$$\therefore AE \parallel BC.$$

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