CHAPTER 11
CONSTRUCTIONS

- **In geometrical construction as far as practicable, only two geometrical instruments, namely a ruler and a compass will be used.**
- **The analysis part which reveals all the necessary clues for the construction problem, is not demanded part to be shown.**

➢ How to deal with a problem of geometrical construction

A problem of geometrical construction requires

(a) skill to use an ungraduated ruler i.e. a straight edge and a compass and

(b) reasoning, base on axioms and propositions related to the figure to be constructed.

For a geometrical construction we usually follow a process consisting of the stages given below:

I. We examine the given data and the required conditions of the problem.

II. We analyse the problem by drawing a rough figure as required by the problem.

From this rough figure we examine various possible ways to construct the required figure using known basic constructions.

III. We then write the steps of actual construction in accordance with the analysis in stage II.

IV. We then prove that the constructed figure satisfies all the required conditions.

➢ Constructions to be studied in this chapter:

- **Construction of a triangle given its base, sum of the other two sides and one base angle.**
- **Construction of a triangle given its base, difference of the other two sides and one base angle.**
- **Construction of triangle given its perimeter and base angles.**
- **Construction of circumcircle of a triangle.**
- **Construction of the incircle of a triangle.**

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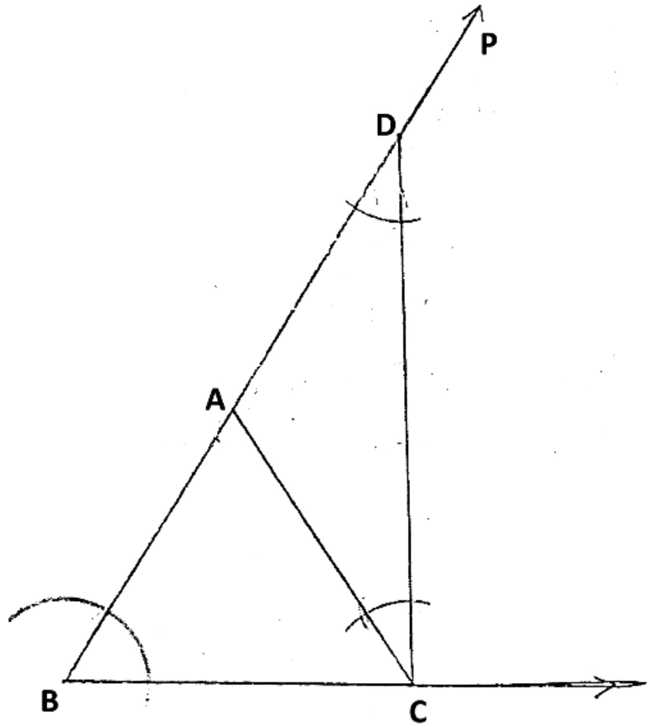
SOLUTIONS

EXERCISE 11.1

1. Construct a triangle ABC in which BC = 6 cm, ∠B = 60^0 and AB + AC = 11 cm.

Solution:



Given: In a ΔABC, BC = 6 cm, ∠B = 60^0 and AB + AC = 11 cm

Required: To construct ΔABC

Steps of construction:

(i) Two rays BP and BQ are drawn such that ∠PBQ = 60^0.

(ii) Two points D and C are marked on BP and BQ such that BD = 11 cm and BC = 6 cm.

(iii) CD is joined and a point A is marked on BD such that ∠ADC = ∠ACD.

Thus, we get the required ΔABC.

Proof: By construction, BC = 6 cm and ∠B = 60^0.

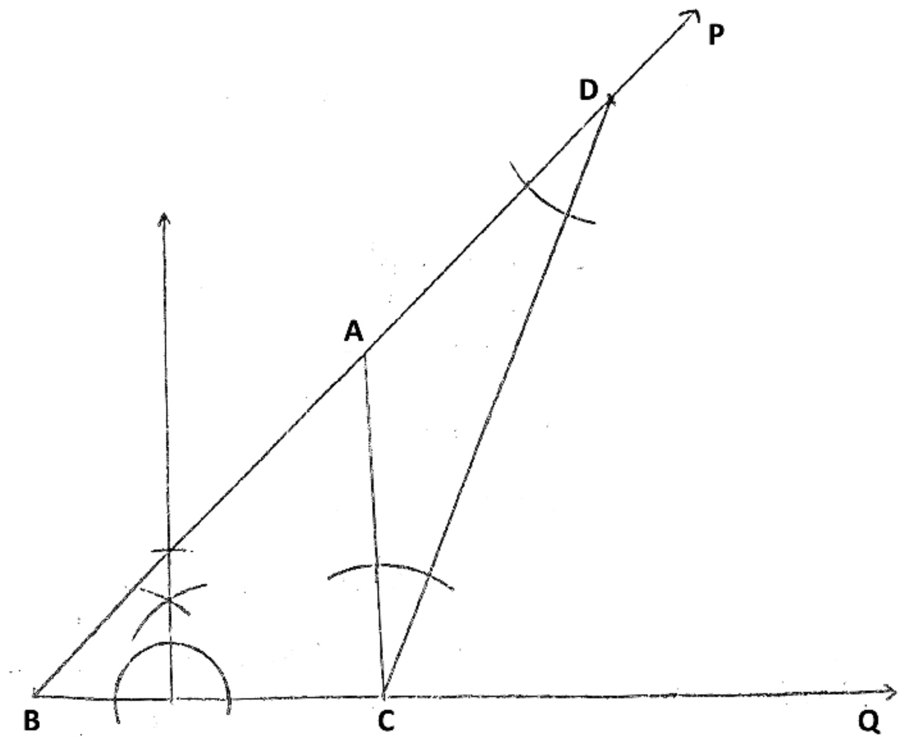
In ΔACD, ∠ACD = ∠ADC

∴ AD = AC

AB + AC = AB + AD = BD = 11 cm

2. Construct a triangle ABC, given that BC = 5 cm, ∠B = 45^0 and AB + AC = 12 cm.

Solution:



Given: In a ΔABC, BC = 5 cm, ∠B = 45^0 and AB + AC = 12 cm

Required: To construct ΔABC

Steps of construction:

(i) Two rays BP and BQ are drawn such that ∠PBQ = 45^0.

(ii) Two points D and C are marked on BP and BQ such that BD = 12 cm and BC = 5 cm.

(iii) CD is joined and a point A is marked on BD such that ∠ADC = ∠ACD.

Thus, we get the required ΔABC.

Proof: By construction, BC = 5 cm and ∠B = 45^0.

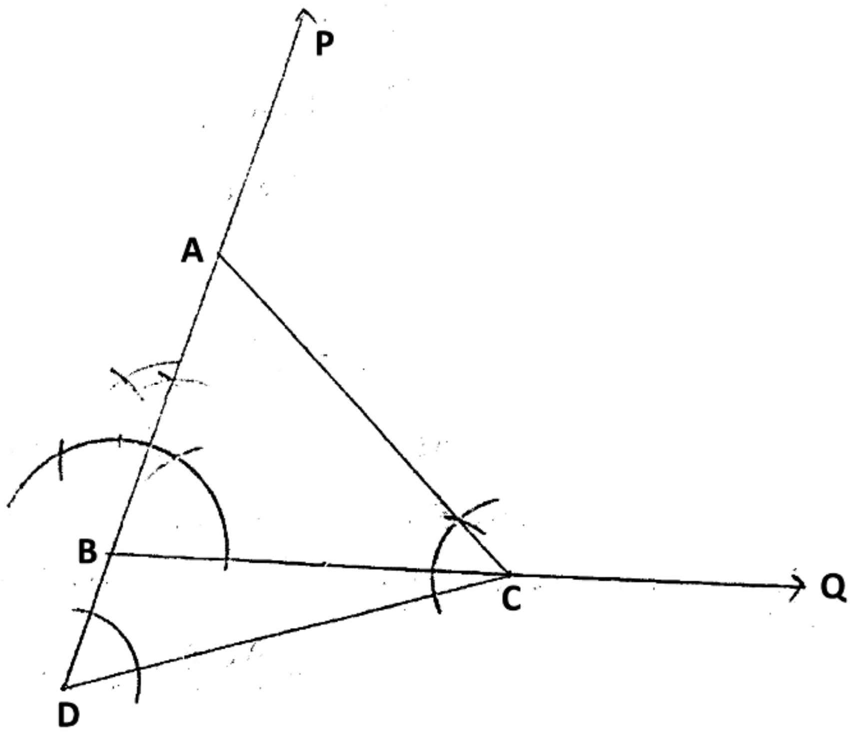
In ΔACD, ∠ACD = ∠ADC

∴ AD = AC

AB + AC = AB + AD = BD = 12 cm

3. Construct a triangle ABC in which BC = 5.5 cm, ∠B = 75⁰ and AC − AB = 2 cm.

Solution:



Given: In a ΔABC, BC = 5.5 cm, ∠B = 75^0 and AC − AB = 2 cm

Required: To construct ΔABC

Steps of construction:

(i) Two rays BP and BQ are drawn such that ∠PBQ = 75^0.

(ii) PB is produced to D so that BD = 2 cm.

(iii) A point C is marked on BQ so that BC = 5.5 cm.

(iv) CD is joined and a point A is marked on BP such that ∠ADC = ∠ACD.

Thus, we get the required ΔABC.

Proof: By construction, BC = 5.5 cm and ∠B = 75^0.

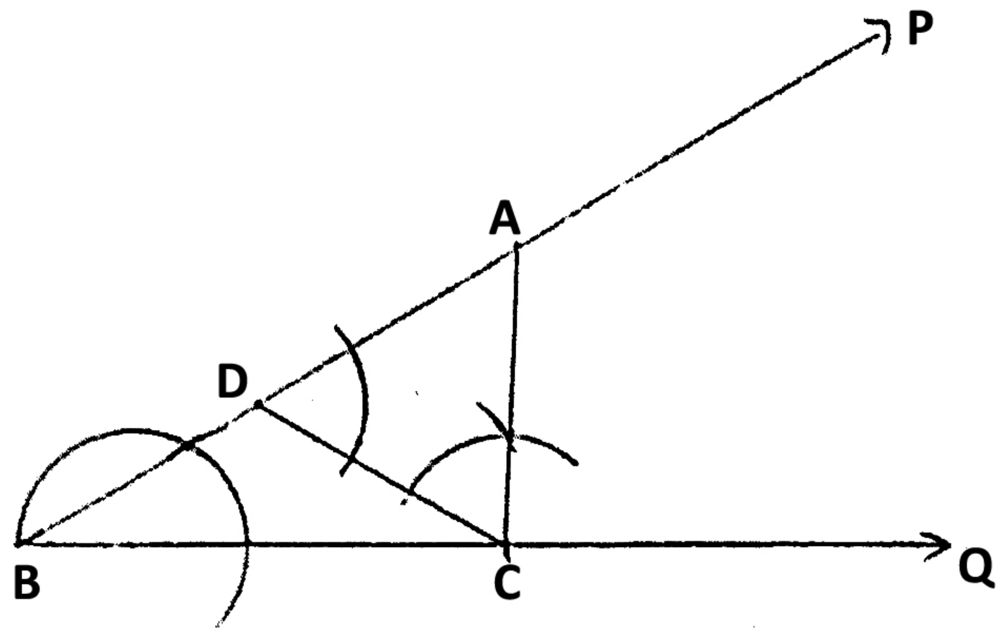
In ΔACD, ∠ACD = ∠ADC

∴ AD = AC

AC − AB = AD − AB = BD = 2 cm

4. Construct a triangle ABC in which BC = 4.5 cm, $\angle B = 30^0$ and AB − AC = 2.5 cm.

Solution:



Given: In a ΔABC, BC = 4.5 cm, $\angle B = 30^0$ and AB − AC = 2.5 cm

Required: To construct ΔABC

Steps of construction:

(i) Two rays BP and BQ are drawn such that $\angle PBQ = 30^0$.

(ii) A point D is marked on BP so that BD = 2.5 cm.

(iii) A point C is marked on BQ so that BC = 4.5 cm.

(iv) CD is joined and a point A is marked on BP such that $\angle ADC = \angle ACD$.

Thus, we get the required ΔABC.

Proof: By construction, BC = 4.5 cm and $\angle B = 30^0$.

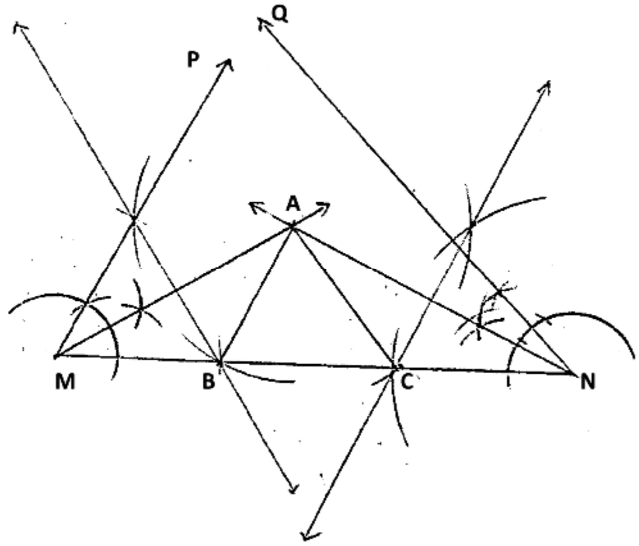
In ΔACD, $\angle ACD = \angle ADC$

$\therefore$ AD = AC

AB − AC = AB − AD = BD = 2.5 cm

5. Construct a triangle ABC, given that AB + BC + CA = 9.5 cm, ∠B = 60^0 and ∠C = 45^0 cm.

Solution:



Given: In a ΔABC, AB + BC + CA = 9.5 cm, ∠B = 60^0 and ∠C = 45^0 cm

Required: To construct ΔABC

Steps of construction:

(i) A line segment MN = 9.5 cm is drawn.

(ii) Two rays MP and NQ are drawn such that ∠PMN = 60^0 and ∠QNM = 45^0.

(iii) Bisectors of ∠PMN and ∠QNM are drawn to intersect each other at A.

(iv) The perpendicular bisectors of AM and AN are drawn to intersect MN at B and C respectively.

(v) AB and AC are joined.

Thus, we get the required ΔABC.

Proof: B is on the perpendicular bisector of AM.

∴ AB = MB

⇒ ∠BMA = ∠BAM

⇒ ∠AMP = ∠BAM

∴ PM ∥ AB

⇒ ∠PMN = ∠ABC [corresponding angles]

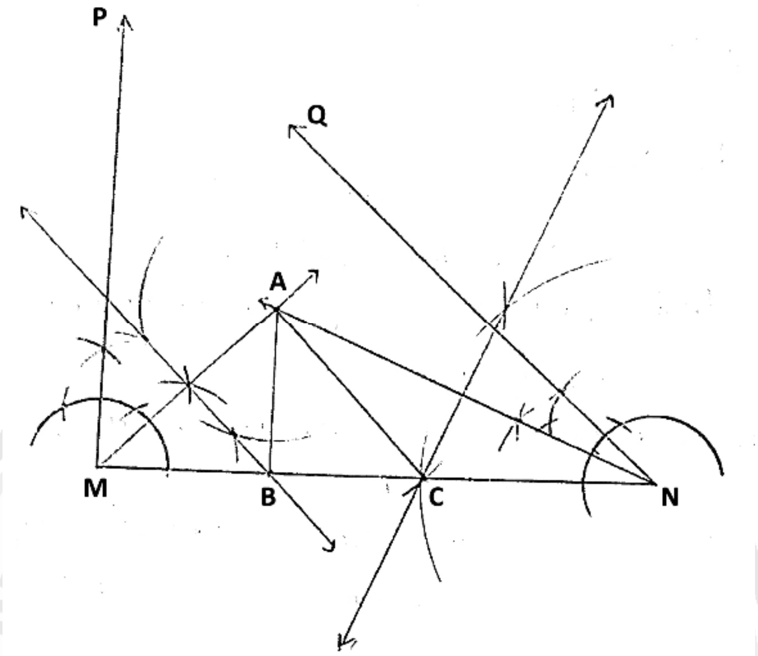
∴ ∠ABC = 60^0

Similarly, ∠ACB = 45^0

AB + BC + CA = BM + BC + CN = MN = 9.5 cm

6. Construct a triangle ABC in which AB + BC + CA = 10 cm, $\angle B = 90^0$ and $\angle C = 45^0$ cm.

Solution:



Given: In a ΔABC, AB + BC + CA = 10 cm, $\angle B = 90^0$ and $\angle C = 45^0$ cm.

Required: To construct the ΔABC

Steps of Construction:

(i) A line segment MN = 10 cm is drawn.

(ii) Two rays MP and NQ are drawn such that $\angle PMN = 90^0$ and $\angle QNM = 45^0$.

(iii) Bisectors of $\angle PMN$ and $\angle QNM$ are drawn to intersect each other at A.

(iv) The perpendicular bisectors of AM and AN are drawn to intersect MN at B and C respectively.

(v) AB and AC are joined.

Thus, we get the required ΔABC.

Proof: B is on the perpendicular bisector of AM.

$\therefore AB = MB$

$\Rightarrow \angle BMA = \angle BAM$

$\Rightarrow \angle AMP = \angle BAM$

$\therefore PM \parallel AB$

$\Rightarrow \angle PMN = \angle ABC$ [corresponding angles]

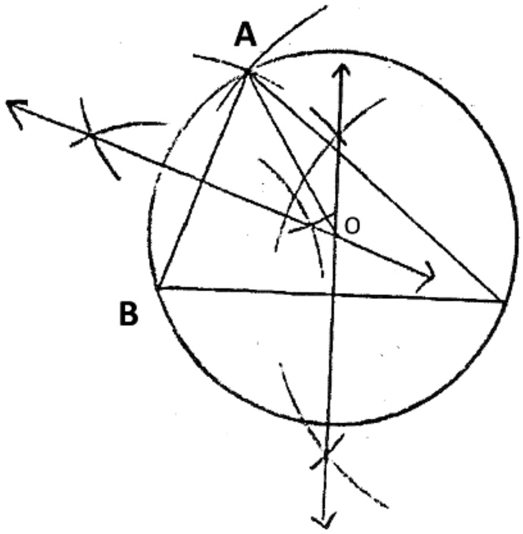
$\therefore \angle ABC = 90^0$

Similarly, $\angle ACB = 45^0$

AB + BC + CA = BM + BC + CN = MN = 10 cm

7. In ΔABC, AB + BC + CA = 12 cm and AB : BC : CA = 2 : 3 : 3. Construct the circumcircle of the triangle.

Solution:



Given: In ΔABC, AB + BC + CA = 12 cm and AB : BC : CA = 2 : 3 : 3.

Required: To construct the ΔABC.

[Analysis: We have, AB + BC + CA = 12 cm and AB : BC : CA = 2 : 3 : 3.

$$\therefore AB = \frac{2}{2+3+3} \times 12 \text{ cm} = \frac{2}{8} \times 12 \text{ cm} = 3 \text{ cm}$$

$$BC = CA = \frac{3}{2+3+3} \times 12 \text{ cm} = \frac{3}{8} \times 12 \text{ cm} = \frac{3}{2} \times 3 \text{ cm} = \frac{9}{2} \text{ cm} = 4.5 \text{ cm.}]$$

Steps of construction:

(i) ΔABC in which AB = 3 cm and BC = CA = 4.5 cm is drawn.

(ii) Perpendicular bisectors of AB and BC are drawn intersecting each other at O.

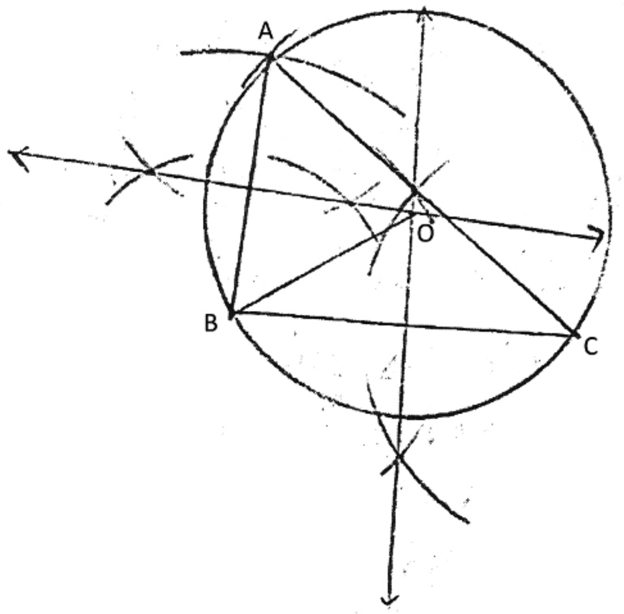
(iii) A circle is drawn with centre O and radius OA, which is the required circumcircle of the ΔABC.

Proof: As O lies on the perpendicular bisectors of AB and BC, O is equidistant from A, B and C.

So, the circle drawn with O as centre and OA as radius will pass through all the three vertices A, B and C of the ΔABC.

8. Construct the circumcircle of a triangle whose sides are 3.5 cm, 4.5 cm, 5.5 cm in length.

Solution:



Given: In ΔABC, AB = 3.5 cm, BC = 4.5 cm and CA = 5.5 cm.

Required: To construct the circumcircle of ΔABC.

Steps of construction:

(i) ΔABC in which AB = 3.5 cm and BC = 4.5 cm and CA = 5.5 cm is drawn.

(ii) Perpendicular bisectors of AB and BC are drawn intersecting each other at O.

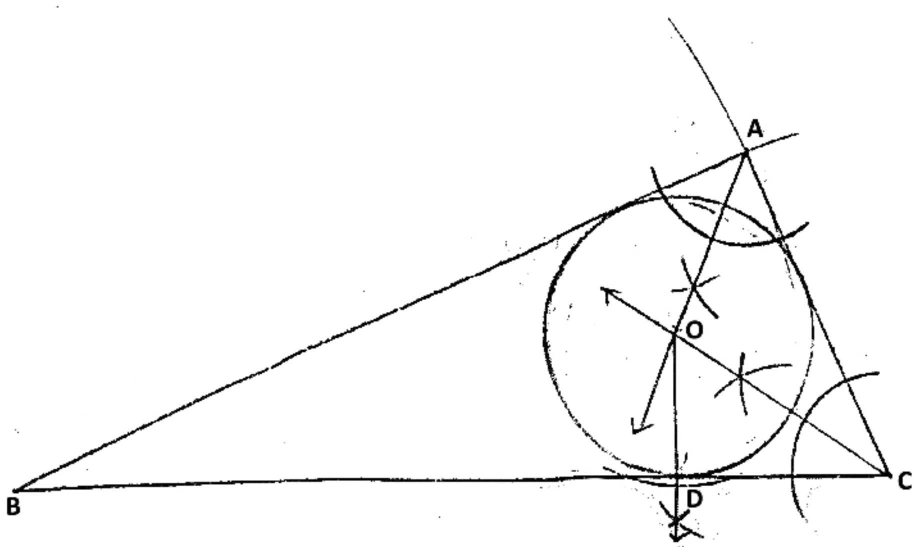
(iii) A circle is drawn with centre O and radius OA, which is the required circumcircle of the ΔABC.

Proof: As O lies on the perpendicular bisectors of AB and BC, O is equidistant from A, B and C.

So, the circle drawn with O as centre and OA as radius will pass through all the three vertices A, B and C of the ΔABC.

9. Construct a triangle ABC and its incircle when AB = 12 cm, BC = 13 cm and CA = 5 cm.

Solution:



Given: In a ΔABC, AB = 12 cm, BC = 13 cm and CA = 5 cm.

Required: To construct the ΔABC and its circumcircle.

Steps of construction:

(i) A line segment BC = 13 cm is drawn.

(ii) Two arcs are drawn with B and C as centres and radii 12 cm and 5 cm respectively intersecting each other at A.

(iii) AB and AC are joined and we get the ΔABC.

(iv) The bisectors of $\angle A$ and $\angle C$ are drawn intersecting each other at O.

(v) $OD \perp BC$ is drawn.

(vi) A circle is drawn with O as centre and OD as radius

This circle is the required incircle of the ΔABC.

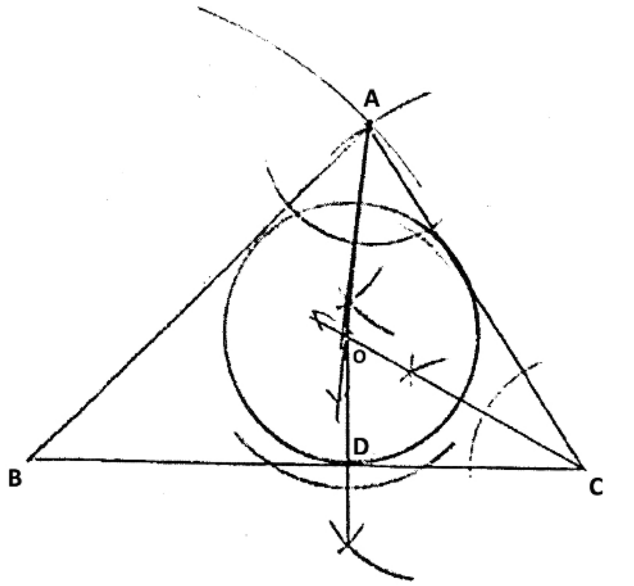
Proof: As O lies on the bisector of $\angle A$, O is equidistant from AB and AC. Again, since O lies on the bisector of $\angle C$, it is equidistant from AC and BC. So, O is equidistant from the three sides AB, BC and CA of the ΔABC. As such, the circle drawn with centre O and radius OD will touch all the three sides.

10. The sides AB, BC and CA of a ΔABC are 6 cm, 7 cm and 5 cm in length.

Draw (i) the incircle (ii) the circumcircle of the triangle.

Solution:

(i)



Given: For a ΔABC, AB = 6 cm, BC = 7 cm and CA = 5 cm.

Required: To construct the incircle of the ΔABC.

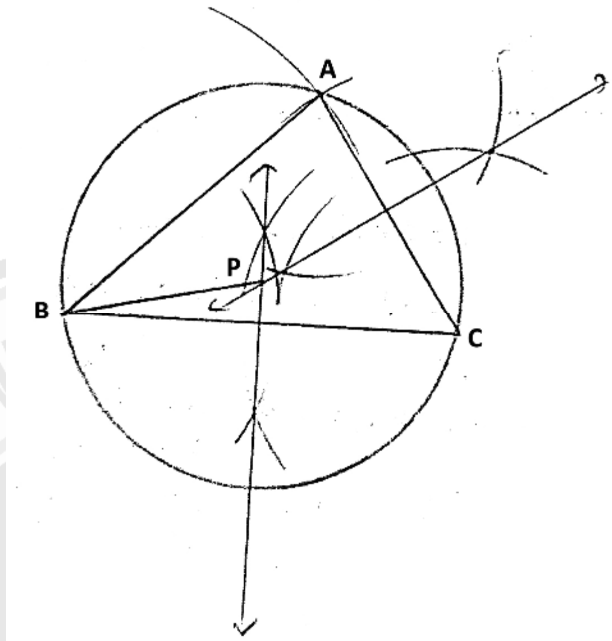
Steps of construction:

1. The ΔABC is constructed such that AB = 6 cm, BC = 7 cm and CA = 5 cm.
2. Bisectors of ∠A and ∠C are drawn intersecting each other at O.
3. OD ⊥ BC is drawn meeting BC at D.
4. With O as centre and OD as radius a circle is drawn.

   This is the required incircle of the ΔABC.

Proof: As O lies on the bisector of ∠A, it is equidistant from the sides AB and AC. Again, since O lies on the bisector of ∠C, it is equidistant from the sides CB and CA. So, O is equidistant from the three sides AB, BC, CA of the ΔABC. As such, the circle drawn with centre O and radius OD will touch all the three sides.

(ii)



Given: For a ΔABC, AB = 6 cm, BC = 7 cm and CA = 5 cm.

Required: To construct the circumcircle of the ΔABC.

Steps of construction:

1. The ΔABC is constructed such that AB = 6 cm, BC = 7 cm and CA = 5 cm.
2. Perpendicular bisectors of the sides BC and CA are drawn intersecting each other at P.
3. With P as centre and PA as radius, a circle is drawn.

   This is the required ciecumcircle.

Proof: By construction, P is equidistant from A, B and C. So, the circle drawn with P as centre and PA as radius will pass through all the three vertices A, B, C of the ΔABC.

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