CHAPTER 13
TRIGONOMETRY

Trigonometry is the branch of Mathematics which originally dealt with the angles and sides of a triangle and relations between them.

Branches of Trigonometry

Trigonometry is broadly divided into two branches. They are

(i) Plane Trigonometry and

(ii) Spherical Trigonometry

Angle in Trigonometry

In Trigonometry, an angle is formed by the revolution of a line about a fixed point in it keeping itself always in the same plane.

The fixed point which is the centre of revolution is called the origin. The revolving line forms positive or negative angles according to the revolution is in the anti-clockwise or clockwise direction.

Note: In plane geometry an angle is said to be formed by two rays having a common origin or initial point. The measure of an angle is always between 0^0 and 360^0. Further the sign of an angle has no relevance. In Trigonometry the magnitude of an angle has no restriction.

Measurement of Angles

Angles are measured in three systems.

They are (i) The Sexagesimal System

(ii) The centesimal system

(iii) The circular system.

- **The Sexagesimal System**

In this system a right angle is divided into 90 equal parts and each one of them is called one degree denoted by 1^0. Each degree is divided into 60 equal parts, each one is called a minute denoted by $1^/$ and again one minute is divided into 60 equal parts each one called one second denoted by $1^{//}$.

Thus, we write

1 right angle = 90^0

1^0 = $60^/$

$1^/$ = $60^{//}$

- **The Centesimal System**

In this system a right angle is divided into 100 equal parts and each one of them is called one grade denoted by 1^g. Each grade is divided into 100 equal parts, each one is called a centesimal minute denoted by $1^\backslash$ and again one centesimal minute is divided into 100 equal parts each one called one centesimal second denoted by $1^{\backslash\backslash}$.

Thus, we write

1 right angle = 100^g

1^g = $100^\backslash$

$1^\backslash$ = $60^{\backslash\backslash}$

- **The Circular System**

In this system, the measure of an angle is expressed in radians.

A radian is an angle subtended at the centre of any circle by an arc whose length is equal to the radius of the circle. It is denoted by 1^c.

Theorems

1. The circumference of a circle bears a constant ratio to its diameter.
2. A radian is a constant angle.

- **Relation among the three System of Measurement of Angles**

$$180^0 = 200^g = \pi^c = 2 \text{ right angles}$$

If x^0, y^g and z^c are the measures of an angle in the three systems, then

$$\frac{x}{180} = \frac{y}{200} = \frac{z}{\pi}.$$

Note: 1. When a straight angle is divided into 180 equal parts one part is one degree.

2. When a straight angle is divided into 200 equal parts one part is one grade.

- **If an arc of length (s) subtends an angle θ (in radian) at the centre of a circle with r as radius, then $s = r\theta$.**

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SOLUTIONS

EXERCISE 13.1

1. Express the following into grades $60^0, 54^0 30', 120^0 20' 30''$.

Solution:

$$60^0 = \left(60 \times \frac{10}{9}\right)^g = \left(\frac{200}{3}\right)^g = 66.6667^g = 66^g\, 66^{\backslash} 67^{\backslash\backslash}$$

$$54^0 30' = \left(54 + \frac{30}{60}\right)^0 = \left(54 + \frac{1}{2}\right)^0 = \left(\frac{108+1}{2}\right)^0$$

$$= \left(\frac{109}{2} \times \frac{10}{9}\right)^g = \left(\frac{109 \times 5}{9}\right)^g = \left(\frac{545}{9}\right)^g$$

$$= 60.5556^g = 60^g\, 55^{\backslash} 56^{\backslash\backslash}$$

$$120^0 20' 30'' = \left(120 + \frac{20}{60} + \frac{30}{60 \times 60}\right)^0 = \left(120 + \frac{1}{3} + \frac{1}{120}\right)^0 = \left(\frac{14400+40+1}{120}\right)^0 = \left(\frac{14441}{120}\right)^0$$

$$= \left(\frac{14441}{120} \times \frac{10}{9}\right)^g = \left(\frac{14441}{12} \times \frac{1}{9}\right)^g = \left(\frac{14441}{108}\right)^g$$

$$= 133.7129^g = 133^g\, 71^{\backslash} 29^{\backslash\backslash}$$

2. Express the following into circular measures:

45^0, 60^0, 120^0, $60^0 45^{//}$.

Solution:

$$45^0 = \left(45 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{4}$$

$$60^0 = \left(60 \times \frac{\pi}{180}\right)^c = \frac{\pi^c}{3}$$

$$120^0 = \left(120 \times \frac{\pi}{180}\right)^c = \left(\frac{2\pi}{3}\right)^c$$

$$60^0 45' = \left(60 + \frac{45}{60}\right)^0 = \left(60 + \frac{3}{4}\right)^0 = \left(\frac{240+3}{4}\right)^0 = \left(\frac{243}{4} \times \frac{\pi}{180}\right)^c = \left(\frac{27}{4} \times \frac{\pi}{20}\right)^c = \frac{27\pi^c}{80}$$

3. Express the following into degrees:

2^c, $4\frac{1}{2}^c$, $\frac{3\pi^c}{4}$, $\frac{2}{3}\pi$

Solution:

$$2^C = \left(2 \times \frac{180}{\pi}\right)^0 = \left(\frac{360}{\pi}\right)^0$$

$$4\frac{1}{2}^C = \frac{9^c}{2} = \left(\frac{9}{2} \times \frac{180}{\pi}\right)^0 = \left(\frac{810}{\pi}\right)^0$$

$$\frac{\pi^C}{4} = \left(\frac{\pi}{4} \times \frac{180}{\pi}\right)^0 = 45^0$$

$$\frac{3\pi^C}{4} = \left(\frac{3\pi}{4} \times \frac{180}{\pi}\right)^0 = 135^0$$

$$\frac{2}{3}\pi^C = \left(\frac{2}{3}\pi \times \frac{180}{\pi}\right)^0 = 120^0$$

4. Find the angle between the hour and minute hands of a clock when it is 15 minute past 3 a.m. expressing it in all the 3 systems of measurement.

Solution: Angle moved by the minute hand in 15 minutes = 90^0.

When the minute hand moves 360^0, angle moved by the hour hand = $\left(\frac{360}{12}\right)^0 = 30^0$

When the minute hand moves 1^0, angle moved by the hour hand = $\left(\frac{30}{360}\right)^0 = \left(\frac{1}{12}\right)^0$

When the minute hand moves 90^0, angle moved by the hour hand = $\left(\frac{1}{12} \times 90\right)^0 = \left(\frac{15}{2}\right)^0 = 7\frac{1}{2}^0$

$$= \left(\frac{15}{2} \times \frac{10}{9}\right)^g = \left(\frac{5\times5}{3}\right)^g = \left(\frac{25}{3}\right)^g = 8\frac{1}{3}^g$$

$$= \left(\frac{25}{3} \times \frac{\pi}{200}\right)^c = \left(\frac{1}{3} \times \frac{\pi}{8}\right)^c = \frac{\pi}{24}^c$$

5. When the angles of a triangle are measured in the sexagesimal system their magnitudes are in the ration 2: 3: 4. Find the angles in the radians.

Solution: Let $2x, 3x,\ 4x$ (in radians) be the measures of the three angles of the triangle.

Then, $2x + 3x + 4x = \pi$ $\quad [\because 180^0 = \pi^c]$

$\Rightarrow 9x = \pi$

$\Rightarrow x = \frac{\pi}{9}$

$\therefore$ The required measures of the angles are $2 \times \frac{\pi}{9}$, $3 \times \frac{\pi}{9}$ and $4 \times \frac{\pi}{9}$ i.e. $\frac{2\pi}{9}$, $\frac{\pi}{3}$ and $\frac{4\pi}{9}$.

6. Find the magnitude of an internal angle of a regular polygon of 20 sides. Express it in degrees and radians.

Solution: The magnitude of an internal angle of a regular polygon of n sides is $\frac{n-2}{n} \times 180^0$.

$\therefore$ The magnitude of an internal angle of a regular polygon of 20 sides $= \frac{20-2}{20} \times 180^0$

$= 18 \times 9^0$

$= 162^0$

$= 162 \times \frac{\pi}{180}^c$

$= \frac{9\pi}{10}^c$

7. A circular building subtends an angle $14'$ at a point on the ground $2700m$ away from the building. Find approximately the height of the building.

Solution: The angle subtended by the building at the point, $\theta = 14' = \left(\frac{14}{60}\right)^0 = \left(\frac{7}{30}\right)^0 = \left(\frac{7}{30} \times \frac{\pi}{180}\right)^c$

The distance of the point from the building, $r = 2700$ m

$\therefore$ The height of the building, $(s) = r\theta$

$= 2700 \times \frac{7}{30} \times \frac{\pi}{180}$ m

$= 27 \times \frac{7}{3} \times \frac{22}{7} \times \frac{1}{18}$ m

$= 11$m

8. Find the ratio of the radii of two circles at the centres of which two arcs of the same length subtend 15^0 and 24^0.

Solution: Let r_1 and r_2 respectively be the radii of the first and the second circles.

For the first circle,

$$\theta_1 = 15^0 = \left(15 \times \frac{\pi}{180}\right)^c$$

Arc length $(s) = r_1.\theta_1 = r_1 \times 15 \times \frac{\pi}{180}$

For the second circle,

$$\theta_2 = 24^0 = \left(24 \times \frac{\pi}{180}\right)^c$$

Arc length $(s) = r_2.\theta_2 = r_2 \times 24 \times \frac{\pi}{180}$

Then, $r_1 \times 15 \times \frac{\pi}{180} = r_2 \times 24 \times \frac{\pi}{180} \qquad (= s)$

$$\Rightarrow 15r_1 = 24r_2$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{24}{15} = \frac{8}{5}$$

The ratio of the radii of the two circles is 8 : 5.

9. The minute hand of a clock is $21cm$ long. Find the distance described by its tip in 20 *minutes*.

Solution: We have,

r = Length of the minute hand = 21 cm

θ = Angle moved by the minute hand in 20 minutes

$$= \left(\frac{360}{60} \times 20\right)^0 = 120^0 = \left(120 \times \frac{\pi}{180}\right)^c = \frac{2}{3}\pi$$

∴ The distance described by the tip of the minute hand, $S = r\theta$

$$= 21 \times \frac{2}{3}\pi \text{ cm}$$

$$= 7 \times 2 \times \frac{22}{7} \text{ cm}$$

$$= 44 \text{ cm}$$

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