CHAPTER 3
MATHEMATICAL LOGIC

SOLUTIONS

EXERCISE 3.1

Q1. Which of the following are statements:-

(a) The square of an integer is an even integer.

Ans: It is a statement.

(b) Do you read at night?

Ans: It is not a statement because it is an interrogative sentence.

(c) Come here, Tomba.

Ans: It is not a statement because it is an imperative sentence.

(d) If it rains, then grass grows.

Ans: It is a statement.

(e) 13 is a composite number.

Ans: It is a statement.

(f) A triangle has four sides.

Ans: It is a statement.

Q2. Using the statements

p **: Chaoba is a good teacher.**

q **: Chaoba is a scholar.**

Write the following in symbolic form:

(i) Chaoba is not a good teacher but a scholar.

Ans: Chaoba is not a good teacher: $\sim p$

$\therefore \sim p \wedge q$

(ii) **Chaoba is a good teacher but not a scholar.**

Ans: Chaoba is not a scholar : $\neg q$

$\therefore\ p \wedge \neg q$

(iii) **Chaoba is neither a good teacher nor a scholar.**

Ans: $\neg p \wedge \neg q$

(iv) **Chaoba is a good teacher or he is a scholar and a bad teacher.**

Ans: $p \vee (q \wedge \neg p)$

Q3. Given the truth values of *p, q* and *r* to be *T, F* and *T* respectively. Find the value of :

(i) $(p \vee q) \wedge (q \vee r)$

Soln: Here, the truth table is

p	q	r	$p \vee q$	$q \vee r$	$(q \vee q) \wedge (q \vee r)$
T	F	T	T	T	T

Hence, the truth value of $(p \vee q) \wedge (q \vee r)$ is T.

ii) $(p \Rightarrow q) \Rightarrow (p \wedge \neg q)$

Soln: Here, the truth table is

p	q	r	$p \Rightarrow q$	$\neg q$	$p \wedge \neg q$	$(p \Rightarrow q) \Rightarrow (p \wedge \neg q)$
T	F	T	F	T	T	T

Hence, the truth value of $(p \Rightarrow q) \Rightarrow (p \wedge \neg q)$ is T.

(iii) $(p \Rightarrow q) \wedge (q \Rightarrow r)$

Soln: Here, the truth table is

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$
T	F	T	F	T	F

Hence, the truth value of $(p \Rightarrow q) \wedge (q \Rightarrow r)$ is F.

(iv) $(q \wedge r) \Rightarrow p$

Soln: Here, the truth table is

p	q	r	$q \wedge r$	$(q \wedge r) \Rightarrow p$
T	F	T	F	T

$\therefore$ The truth value of $(q \wedge r) \Rightarrow p$ is T.

(v) $q \vee (r \Rightarrow p)$

Soln: Here, the truth table is

p	q	r	$r \Rightarrow p$	$q \vee (r \Rightarrow p)$
T	F	T	T	T

$\therefore$ The truth value of $q \vee (r \Rightarrow p)$ is T.

Q4. Construct truth table for the following statements:-

(i) $(p \wedge q) \vee \sim r$

Soln: Truth table for $(p \wedge q) \vee \sim r$

p	q	r	$p \wedge q$	$\sim r$	$(p \wedge q) \vee \sim r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

(ii) $(p \Leftrightarrow q) \wedge (\sim r \Rightarrow p)$

Soln: Truth table for $(p \Leftrightarrow q) \wedge (\sim r \Rightarrow p)$ is

p	q	r	$\sim r$	$p \Leftrightarrow q$	$\sim r \Rightarrow p$	$(p \Leftrightarrow q) \wedge (\sim r \Rightarrow p)$
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	T	F	T	F	F	F
F	F	T	F	T	T	T
F	F	F	T	T	F	F

(iii) $(p \vee \sim q) \wedge r$

Soln: The truth table for $(p \vee \sim q) \wedge r$ is

p	q	r	$\sim q$	$p \vee \sim q$	$(p \vee \sim q) \wedge r$
T	T	T	F	T	T
T	T	F	F	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	F

(iv) $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

Soln: The truth table for $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$ is

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Q5. Write the negations of the following?

(a) 5 is a rational number.

Ans: 5 is not a rational number.

(b) 3 is not a prime.

Ans: 3 is a prime.

(c) All integers are rational numbers.

Ans: Some integers are not rational numbers.

(d) There are natural numbers which are not integers.

Ans: All natural numbers are integers.

(e) A triangle has four sides.

Ans: It is not true that a triangle has four sides.

(f) Man is mortal.

Ans: Man is immortal.

(g) If water is cold, then fire is hot.

Ans: Water is cold but fire is not hot.

(h) Kumar and Kanta are intelligent.

Ans: Kumar or Kanta is not intelligent.

(i) Some students never read.

Ans: There is no student who never reads.

(j) Every student is honest.

Ans: some students are dishonest.

(k) An integer is either positive or negative.

Ans: It is not the case that every integer is either positive or negative.

(l) If there is a will, then there is a way.

Ans: There is a will and there is not a way.

Q6. Find the negation of

(i) $(p \vee q) \wedge r$

Ans: The negation of $(p \vee q) \wedge r$ is $(\sim p \wedge \sim q) \vee \sim r$.

The truth table is given below:

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$\sim p \wedge \sim q$	$(\sim p \wedge \sim q) \vee \sim r$
T	T	T	T	T	F	F
T	F	T	T	T	F	F
T	T	F	T	F	F	T
T	F	F	T	F	F	T
F	T	T	T	T	F	F
F	F	T	F	F	T	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

(ii) $p \wedge q \Rightarrow r$

Soln: The negation of $p \wedge q \Rightarrow r$ is $(p \wedge q) \wedge \sim r$

p	q	r	$p \wedge q$	$(p \wedge q) \Rightarrow r$	$\sim r$	$(p \wedge q) \wedge \sim r$
T	T	T	T	T	F	F
T	T	F	T	F	T	T
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	F	T	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F
F	F	F	F	T	T	F

(iii) $p \wedge (q \Rightarrow r)$

Soln: The negation of $p \wedge (q \Rightarrow r)$ is $\sim p \vee (q \wedge \sim r)$

p	q	r	$q \Rightarrow r$	$p \wedge (q \Rightarrow r)$	$\sim p$	$q \wedge \sim r$	$\sim p \vee (q \wedge \sim r)$
T	T	T	T	T	F	F	F
T	T	F	F	F	F	T	T
T	F	T	T	T	F	F	F
T	F	F	T	T	F	F	F
F	T	T	T	F	T	F	T
F	T	F	F	F	T	T	T
F	F	T	T	F	T	F	T
F	F	F	T	F	T	F	T

Q7. Examine whether the following statements are tautologies or contradictions.

a) $p \Rightarrow p \vee q$

Soln: The truth table for $p \Rightarrow p \vee q$ is

p	q	$p \vee q$	$p \Rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

The truth values of the statement $p \Rightarrow p \vee q$ are all true. Hence the statement is a tautology.

(b) $p \wedge q \Rightarrow q$

Soln: The truth table for $p \wedge q \Rightarrow q$ is

p	q	$p \wedge q$	$p \wedge q \Rightarrow q$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Since the truth value of the statement $p \wedge q \Rightarrow q$ are all true. Hence the given statement is a tautology.

(c) $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$

Soln: The truth table of the statement $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ is

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$	$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	F	F	T
F	F	F	T	T	T

Since the truth values of the statement are all true.

$\therefore$ The given statement is a tautology.

(d) $(p \wedge q) \wedge \neg(p \vee q)$

Soln: The truth table of the statement $(p \wedge q) \wedge \neg(p \vee q)$ is

p	q	$p \wedge q$	$p \vee q$	$\neg(p \vee q)$	$(p \wedge q) \wedge \neg(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

Since the truth values of the statement are all false.

$\therefore$ The given statement is a contradiction.

(e) $(p \vee q) \wedge r \Rightarrow (q \vee r)$

Soln: The truth table of the given statement is

p	q	r	$(p \vee q)$	$(p \vee q) \wedge r$	$q \vee r$	$(p \vee q) \wedge r \Rightarrow (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	F	F	T	T
F	F	F	F	F	F	T

Here, the truth values of the given statement are all true.

Hence, the given statement is a tautology.

(f) $(p \wedge \sim q) \wedge q \Leftrightarrow p \wedge q$

Soln: The truth table the statement $(p \wedge \sim q) \wedge q \Leftrightarrow p \wedge q$ is

p	q	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \wedge q$	$p \wedge q$	$(p \wedge \sim q) \wedge q \Leftrightarrow p \wedge q$
T	T	F	F	F	T	F
T	F	T	T	F	F	T
F	T	F	F	F	F	T
F	F	T	F	F	F	T

The truth values are not all true nor false.

$\therefore$ The given statement $(p \wedge \sim q) \wedge q \Leftrightarrow p \wedge q$ is neither a tautology nor a contradiction.

(g) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Soln: The truth table of the given statement is

p	q	r	$q \vee r$	$p \wedge (q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$(p \wedge q) \vee (p \wedge r)$	$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	F	F	F	F	T
F	T	F	T	F	F	F	F	T
F	F	T	T	F	F	F	F	T
F	F	F	F	F	F	F	F	T

The truth values of the given statement are all true.

$\therefore$ The given statement is a tautology.

(h) $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$

Soln: The truth table of the statement $(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ is

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$\sim q \Rightarrow \sim p$	$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

The truth values of the given statement are all true.

$\therefore$ The given statement is a tautology.

(i) $(p \Leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$

Soln: The truth table of the given statement is

p	q	$p \Leftrightarrow q$	$p \wedge q$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$	$(p \Leftrightarrow q) \Leftrightarrow (p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	T	T	F	F	F	T	T
T	F	F	F	F	T	F	F	T
F	T	F	F	T	F	F	F	T
F	F	T	F	T	T	T	T	T

The truth values of the given statement are all true.

$\therefore$ The given statement is a tautology.

Q8. The negation of the statement "3>5 and 5 – 3 =2" is

A. $3 \ngtr 5$ and $5 - 3 \neq 2$

B. $3 > 5$ and $5 - 3 \neq 2$

C. $3 \ngtr 5$ or $5 - 3 \neq 2$

D. $3 \ngtr 5$ and $5 - 3 \neq 2$

Ans: **C.** **$3 > 5$ or $5 - 3 \neq 2$**

Q9. The negation of 'If it rains, then grass is green' is

A. If it rains, then grass is not green.

B. If it does not rain, then grass is green.

C. It does not rain but grass is green.

D. It rains but grass is not green.

Ans: **D.** **It rains but grass is not green.**

Q10. The converse of the statement "If a whole number is even, then it is divisible by 2' is

A. An even whole number is not divisible by 2.

B. If a whole number is divisible by 2, then it is even.

C. If a whole number is even, then it is not divisible by 2.

D. For a whole number to be even it is sufficient that it is divisible by 2.

Ans: **B.** **If a whole number is divisible by 2, then it is even.**

Q11. State whether the following statements are atomic or compound:

1. All natural numbers are integers.

Ans: It is an atomic statement because it cannot be broken up into two or more statements.

2. If the mountain is high, then the sea is deep.

Ans: It is a compound statement because it is a combination of two simple statements.

3. Integers are not rational numbers.

Ans: It is an atomic statement.

4. An integer is called a prime if it has no proper factor.

Ans: It is a compound statement.

5. An integer having proper factors is said to be composite.

Ans: It is an atomic statement.

Q12. Write each sentence in the conditional form: (using 'if – then')

(a) All rational numbers are real numbers.

Ans: If a number is rational then it is a real number.

(b) Freezing water expands.

Ans: If water freezes, then it expands.

(c) A positive integer having no proper divisor is a prime.

Ans: If a positive integer has no proper divisor, then it is a prime.

(d) Two sides of isosceles triangle are equal.

Ans: If a triangle is isosceles, then two of its sides are equal.

Q13. When it does not rain but grass grows, what is the truth value of the statement "If it rains, then grass grows"?

Ans: Here, both the statements 'it rains' and 'grass grows', have the truth value T. So, the truth value of the given statement is T.

****