CHAPTER 8
DYNAMICS

SOLUTIONS

KEYNOTES 8.1

1. **Dynamics** is the science of moving bodies which is studied under two heads viz. kinematics and kinetics.

2. **Kinematics** deals with the geometrical aspect of the motion of a body for which entities like mass of the body are irrelevant.

3. **Kinetics** deals with the physical aspect of the motion of a body where mass of the body is indispensable.

4. **The displacement** of a moving particle in any time interval is its change of position during the interval of time.

5. **The velocity** of a moving particle is the rate of its displacement. It is uniform when it always moves along the same straight line in the same sense (i.e. in the same direction) and passes over equal distances in equal intervals of time.

6. **Acceleration** of a moving particle is the rate of change of its velocity. It is uniform when equal change of velocity in the same direction takes in equal interval of time.

7. A negative acceleration is known as **retardation.**

8. **Units of motion:**

   i) Velocity

   ft/sec in FPS system
   m/sec in MKS system
   cm/sec in CGS system

   ii) Acceleration

   ft/sec^2 in FPS system
   m/sec^2 in MKS system
   cm/sec^2 in CGS system

9. For a uniform velocity (v) acquired by a particle, the distance (s) described in t units of time is given by $s = vt$.

10. For a uniformly accelerated motion of a particle, we have

    a) $v = u + ft$

    b) $s = ut + \frac{1}{2} ft^2$

    c) $v^2 = u^2 + 2fs$

    d) The distance described by a particle in the nth second of its motion is given by

    $$s_n = u + \frac{1}{2} f(2n-1)$$

EXERCISE 8.1

1. Convert the following velocities in ft/sec.

i) 60 miles/hr

Soln. i) 60 miles/hr

$= 60 \times 1760 \times$ ft/hr $[\because 1 \text{ mile} = 1760 \times 3 \text{ ft}]$

$= \dfrac{60 \times 1760 \times 3}{60 \times 60}$ ft/sec

= 88 ft/sec

ii) 75 miles/hr

$= \dfrac{75 \times 1760 \times 3}{60 \times 60}$ ft/sec

= 110 ft/sec

iii) 20 yards/min

$= \dfrac{20 \times 3}{60}$ ft/sec $[\because 1 \text{ yard} = 3 \text{ ft}]$

= 1 ft/sec

2. Convert the following velocities in cm/sec.

i) 90 kilometres per hour

= 90 km/hr $= \dfrac{90 \times 1000 \times 100}{60 \times 60}$ cm/sec = 2500 cm/sec

ii) 30 metres per minute

= 30 m/min

$= \dfrac{30 \times 100}{60}$ cm/sec = 50 cm/sec

iii) 10 km/min

$= \dfrac{10 \times 1000 \times 100}{60}$ cm/sec $= \dfrac{50000}{3}$ cm/sec $= 16666\dfrac{2}{3}$ cm/sec

3. Convert the following velocities in m/sec

Soln. i) 80 km/hr

$$= \frac{80 \times 1000}{60 \times 60} \text{ m/sec}$$

$$= \frac{200}{9} \text{ m/sec} = 22\frac{2}{9} \text{ m/Sec}$$

ii) 15 km/min

$$= \frac{15 \times 1000}{60} \text{ m/sec} = 250 \text{ m/sec}$$

iii) 30 cm/sec

$$= \frac{30}{100} \text{ m/sec} = \frac{3}{10} \text{ m/sec}$$

4. Convert the following velocities in km/hr

Soln. i) 100 m/sec

$$= \frac{100 \times 60 \times 60}{1000} \text{ km/hr} = 360 \text{ km/hr}$$

ii) 300 m/min

$$= \frac{300 \times 60}{1000} \text{ km/hr} = 18 \text{ km/hr}$$

iii) 50 m/sec

$$= \frac{50 \times 60 \times 60}{1000} \text{ km/hr} = 180 \text{ km/hr}$$

5. Convert the following acceleration in ft/sec^2.

Soln. i) 1 mile/min^2

$$= \frac{1 \times 1760 \times 3}{60 \times 60} \text{ ft/sec}^2$$

$$= \frac{22}{15} \text{ ft/sec}^2 = 1\frac{7}{15} \text{ ft/sec}^2$$

ii) 5400 miles/hr^2

$$= \frac{5400 \times 1760 \times 3}{60 \times 60 \times 60 \times 60} \text{ ft/sec}^2 = 2.2 \text{ ft/sec}^2$$

iii) 300 yds/min^2

$$= \frac{300\times 3}{60\times 60}\ \text{ft/sec}^2 = \frac{1}{4}\ \text{ft/sec}^2$$

6. Covert the following in m/sec^2

Soln. **i) 980 cm/sec^2**

$$= \frac{980}{100}\ \text{m/sec}^2 = 9.8\ \text{m/sec}^2$$

ii) 1296 km/hr^2

$$= \frac{1296\times 1000}{60\times 60\times 60\times 60}\ \text{m/sec}^2 = 0.1\ \text{m/sec}^2$$

iii) 360 km/min^2

$$= \frac{360\times 1000}{60\times 60}\ \text{m/sec}^2 = 100\ \text{m/sec}^2$$

7. A car is travelling with uniform velocity 50 m/sec. How far it will travel in 10 secs and how long will it take to travel 2 km?

Soln. We have,

i) Uniform velocity, $v = 50$ m/sec

Time taken, $t = 10$ sec

$\therefore$ Distance describe, $s = vt$

$\Rightarrow s = 50\times 10$ m = 500 m

ii) Again, $t = \frac{s}{v} = \frac{2\times 1000}{50}$ sec. = 40 sec.

8. If a train travels 108 kms in 3 hours, what is the average velocity in m/sec?

Soln. Given, $s = 108$ kms $= 108\times 1000$ m

$t = 3$ hours $= 3\times 60\times 60$ sec

$\therefore$ Average velocity $= \frac{108\times 1000}{3\times 60\times 60}$ m/sec = 10 m/sec

9. If the velocity of a train 100 m long is 1 km/min, how many seconds will it take to cross a telegraph post?

Soln. Given,

The distance to be crossed, $s = 100 \text{ m}$

Velocity acquired, $v = 1 \text{ km/min} = \dfrac{1 \times 1000}{60} \text{ m/sec}$

$\therefore$ time taken is given by $t = \dfrac{s}{v} = \dfrac{100}{\dfrac{1000}{60}} \text{ sec} = \dfrac{100 \times 60}{1000} \text{ sec} = 6 \text{ sec.}$

10. A bus starts moving from rest with an acceleration 2m/sec^2. What will be its velocity after 15 seconds and how far it will travel during the time and during the next 5 seconds?

Soln. Given,

Initial velocity, $u = 0$
Uniform velocity, $f = 2 \text{ m/sec}^2$
Time taken, $t = 15 \text{ sec}$
Distance described, $s = ?$
We have,

$$s = ut + \frac{1}{2} ft^2$$

$$= 0 \times 15 + \frac{1}{2} \times 2 \times 15^2 \text{ m}$$

$$= 225 \text{ m}$$

And $v = u + ft$

$= 0 + 2 \times 15 \text{ m/sec} = 30 \text{ m/sec}$

Also, the distance described during the next 5 seconds is given by :

$$s = ut + ft^2$$

$$= 30 \times 5 + \frac{1}{2} \times 2 \times 5 \times 5 \quad \left[\begin{array}{l} \because u = 30 \text{ m/sec after 15 sec.} \\ \quad t = 5 \text{ sec} \end{array} \right]$$

= 150 + 25 m
= 175 m

11. The velocity of a car increases from 20 m/sec to 30 m/sec. while it travels a distance of 25 m. What is its acceleration?

Soln. Given,

u = 20 m/sec
v = 30 m/sec
s = 25 m
f = ?

We have,

$$v^2 = u^2 + 2fs$$

$$\Rightarrow 30^2 = 20^2 + 2 \times f \times 25$$

$$\Rightarrow 900 = 400 + 50f$$

$$\Rightarrow f = \frac{900 - 400}{50} = \frac{500}{50} \text{ m/sec}^2 = 10 \text{ m/sec}^2$$

12. An arrow can rise 160 m vertically upwards. What is its velocity at the time of shooting? (g = 9.8 m/sec^2)

Soln. At the maximum height,
The final velocity, v = 0
Acceleration due to gravity,

$g = -9.8$ m/sec^2, since it is opposite to the direction of motion of the arrow.

Maximum height, h = 160 m
Initial velocity, u = ?
We have,

$$v^2 = u^2 + 2gh$$

$$\Rightarrow o^2 = u^2 + 2(-9.8) \times 160$$

$$\Rightarrow u^2 = 19.6 \times 160$$

$$\Rightarrow u = \sqrt{196 \times 16} = 14 \times 4 = 56 \text{ m/sec}$$

Hence, the velocity of the arrow at the time of shooting is 56 m/sec.

13. If a body falling freely from the top of a building reaches the ground in 5 seconds. Find the height of the building and the velocity with which it strikes the ground (g = 32 ft/sec^2)

Soln. Given, since the body is falling freely

u = 0
t = 5 sec
g = 32 ft/sec^2
h = ?
v = ?

We have,

$$h = ut + \frac{1}{2}gt^2 = 0\times5+\frac{1}{2}\times32\times5\times5 \text{ ft}$$

$$= \frac{1}{2}\times32\times25 = 16\times25 \text{ ft} = 400 \text{ ft}$$

and $v = u + ft = 0+32\times5 = 160$ ft/sec

14. A body, starting from rest and moving with uniform acceleration, describes 27 m in 3 secs; what is its velocity at the end of 10 seconds? Find also the distance traversed in the 5th second of its motion?

Soln. Given,

$u = 0$
$s = 27$ m
$t = 3$ sec
$f = ?$

We have,

$$s = ut + \frac{1}{2}ft^2$$

$$\Rightarrow 27 = 0\times3 + \frac{1}{2}\times f\times3^2$$

$$\Rightarrow f = \frac{27\times2}{9} = 6 \text{ m/sec}^2$$

The velocity at the end of 10 seconds,

$$v = u + ft$$

$$= 0+6\times10 = 60 \text{ m/sec}$$

and, the distance traversed in the 5th second

$$= (ut + \frac{1}{2}ft_1^2) - (ut_2 + \frac{1}{2}ft_2^2) \quad \begin{bmatrix} t_1 = 5 \\ t_2 = 4 \end{bmatrix}$$

$$= (0\times5+\frac{1}{2}\times6\times5^2) - (0\times4+\frac{1}{2}\times6\times4^2)$$

$$= (3\times25) - (3\times16)$$

$$= 75 - 48 = 27 \text{ m}$$

15. A stone is dropped into a well and reaches the surface of the water in 3 seconds, with what velocity will it reach the surface? Find also the depth of the well. (g = 32 ft/sec^2)

Soln. Given,

$u = 0$
$t = 3$ sec
$g = 32$ ft/sec^2
$v = ?$

We have,

$$v = u + gt$$
$$= 0 + 32 \times 3 \text{ ft/sec}$$
$$= 96 \text{ ft/ sec}$$

and depth of the well, $h = ut + \frac{1}{2}gt^2$

$$= 0 + \frac{1}{2} \times 32 \times 3^2 \text{ ft}$$
$$= 16 \times 9 \text{ ft} = 144 \text{ ft}$$

16. From a balloon ascending with a velocity of 10 ft/sec, a stone is let fall and reaches the ground in 15 seconds. How high was the balloon when the stone was dropped?

Soln. Given, the stone being in motion of the balloon

$u = -10$ ft/sec

$t = 15$ sec

$g = 32$ ft/sec^2

h = ?

Now, the height of the balloon when the stone was dropped,

$$h = ut + \frac{1}{2}gt^2$$
$$= -10 \times 5 + \frac{1}{2} \times 32 \times 15^2$$
$$= -150 + 3600 \text{ ft}$$
$$= 3450 \text{ ft}$$

17. A falling particle in the last second of its fall, passes through 112.7 m. How long did it fall? Find also the height from which it fell. ($g = 9.8$ m/sec^2)

Soln. Given,

n = ?

S_n = 112.7 m

g = 9-8 m/sec^2

u = 0

We have,

$$S_n = u + \frac{1}{2}g(2n-1)$$
$$\Rightarrow 112.7 = 0 + \frac{1}{2} \times 9.8(2n-1)$$
$$\Rightarrow 112.7 = 4.9(2n-1)$$
$$\Rightarrow 112.7 \div 4.9 = 2n - 1$$

$$\Rightarrow 2n-1=\frac{112.7}{4.9}=\frac{1127}{49}=23$$

$$\Rightarrow n=\frac{23+1}{2}=12 \text{ seconds.}$$

$\therefore$ The falling particle falls for 12 sec.

Again, the height from which it fell, $h=un+\frac{1}{2}gn^2$

$$=0\times12+\frac{1}{2}\times9.8\times12^2$$

$$=\frac{1}{2}\times9.8\times144$$

$$=705.6 \text{ m}$$

18. A stone is dropped into a well and the sound of the splash is heard in $7\frac{7}{10}$ seconds. If the velocity of sound be 1120 ft/sec, find the depth of the well. $(g=32 \text{ ft/sec}^2)$

Soln. If t_1 seconds and t_2 seconds be the time of falling of the stone and the time of coming of sound, then we have,

$t_1+t_2=7\frac{7}{10}$ seconds.

Again, $h=v\times t_2=1120\times t_2$ …………. (1)

Also, $h=ut_1+\frac{1}{2}gt_1^2$

$$=0+\frac{1}{2}\times32\times t_1^2=16\times t_1^2 \text{ …………….. (2)}$$

$\therefore$ from (1) & (2), we get,

$$1120\times t_2=16\times t_1^2$$

$$\Rightarrow t_2=\frac{16}{1120}\times t_1^2$$

$$\Rightarrow t_2=\frac{1}{70}\times t_1^2 \text{ …………… (3)}$$

Putting this value in

$t_1+t_2=7\frac{7}{10}$, we get,

$$\Rightarrow t_1+\frac{1}{70}t_1^2=\frac{77}{10}$$

$$\Rightarrow 70t_1+t_1^2=77\times7$$

$$\Rightarrow t_1^2+70t_1-539=0$$

$$\Rightarrow t_1 = \frac{-70 \pm \sqrt{70^2 - 4\times1(-539)}}{2\times1}$$

$$= \frac{-70 \pm \sqrt{4900+2156}}{2}$$

$$= \frac{-70 \pm \sqrt{7056}}{2} = \frac{-70 \pm 84}{2} = \frac{14}{2}$$

$= 7$ [– ve sign is neglected]

$\therefore$ From (2), the depth of the well , $h = 16\times t_1^2 = 16\times 7^2 = 16\times 49 = 784$ ft.

19. A stone is thrown vertically upwards with a velocity of 42 m/sec. Find

i) its velocity at the end of 3 seconds.

ii) the maximum height attained and the tine taken.

Soln. (i) Given,

u = 42 m/sec

g = – 9.8 m/sec^2 [since it is opposite to direction of motion of the stone]

t = 3 sec

v = ?

We have,

$$v = u + gt$$
$$= 42 - 9.8\times3$$
$$= 42 - 29.4$$
$$= 12.6 \text{ m/sec}$$

$\therefore$ Velocity of the stone at the end of 3 seconds is 12.6 m/sec.

(ii) At the maximum height, v = 0

$$\therefore v = u + gt$$
$$\Rightarrow 0 = 42 - 9.8\times t$$
$$\Rightarrow t = \frac{42}{9.8} = \frac{420}{98} = \frac{30}{7} = 4\frac{2}{7} \text{ sec}$$

and $$h = ut + \frac{1}{2}gt^2$$
$$= 42\times\frac{30}{7} + \frac{1}{2}\times(-9.8)\times\left(\frac{30}{7}\right)^2$$
$$= 180 - 90 = 90 \text{ m}$$

$\therefore$ The maximum height attained is 90 m and time taken is $4\frac{2}{7}$ sec.

20. A point starts from rest and moves with a uniform acceleration of 2 m/sec^2; find the time taken by it to traverse the first, second and third metre respectively.

Soln. Given,

$$u = 0$$
$$s_1 = 1 \text{ m}$$
$$f = 2\text{m/sec}^2$$
$$t_1 = ?$$

We have,

$$s_1 = ut_1 + \frac{1}{2} f t_1^2$$
$$\Rightarrow 1 = 0 + \frac{1}{2} \times 2 \times t_1^2$$
$$\Rightarrow t_1^2 = 1$$
$$\Rightarrow t_1 = 1 \text{ second}$$

For $s = 2$ m, $s = ut_2 + \frac{1}{2} f t_2^2$

$$\Rightarrow 2 = 0 + \frac{1}{2} \times 2 \times t_2^2$$
$$\Rightarrow t_2 = \sqrt{2}$$

$\therefore$ Time taken for the second metre $=$ time taken for 2 m $-$ time taken for 1m

$$= \left(\sqrt{2} - 1\right) \text{sec.}$$

For $s = 3$ m, $s = ut_3 + \frac{1}{2} f t_3^2$

$$\Rightarrow 3 = 0 + \frac{1}{2} \times 2 \times t_3^2$$
$$\Rightarrow t_3 = \sqrt{3}$$

$\therefore$ Time taken for the third metre $=$ time taken for 3 m $-$ time taken for 2m

$$= \left(\sqrt{3} - \sqrt{2}\right) \text{sec.}$$

21. A particle is moving with uniform retardation, in the third and eight second after starting it moves through 255 cm and 225 cm respectively, find its initial velocity and its retardation.

Soln. Let u cm/sec be the initial velocity of the particle.

Then, the distance described in the third second (s_3) is given by

$$s_3 = u + \frac{1}{2}(-f)(2 \times 3 - 1)$$

$$\Rightarrow 255 = u + \frac{1}{2}(-f) \times 5$$

$$\Rightarrow 255 = u - \frac{5}{2} f \quad \ldots\ldots\ldots\ldots (1)$$

Similarly,

$$s_8 = u + \frac{1}{2}(-f)(2 \times 8 - 1)$$

$$\Rightarrow 225 = u - \frac{15}{2} f$$

Subtracting (2) from (1), we get,

$$255 - 225 = -\frac{5}{2} f + \frac{15}{2} f$$

$$\Rightarrow 30 = \frac{-5+15}{2} f$$

$$\Rightarrow 30 = 5f \Rightarrow f = \frac{30}{5} = 6 \text{ cm/sec}^2$$

From (1),

$$225 = u - \frac{5}{2} \times 6$$

$$\Rightarrow u = 255 + 15 = 270 \text{ cm/sec.}$$

$\therefore$ Initial velocity and retardation of the particle are 270 cm/sec and 6 cm/sec^2 respectively.

22. A body starting from rest and moving with uniform acceleration describes 999 cm in the fifth second; find its acceleration.

Soln. We have,

$$s_5 = u + \frac{1}{2} f (2 \times 5 - 1)$$

$$\Rightarrow 999 = 0 + \frac{1}{2} f (10 - 1)$$

$$\Rightarrow f = \frac{999 \times 2}{9} = 222 \text{ cm/sec}^2$$

23. A point starts with a velocity of 100 cm/sec and moves with a retardation of 10 cm/sec^2. When will its velocity be zero and how far will it have gone?

Soln. Given,

Initial velocity, $u = 100$ cm/sec^2
Retardation, $f = -10$ cm/sec^2
Final velocity, $v = 0$

Time taken, $t = ?$

Distance described, $s = ?$

We have,

$$v = u + ft$$

$$\Rightarrow 0 = 100 + (-10) \times t$$

$$\Rightarrow t = \frac{-100}{-10} = 10 \text{ sec}$$

Also, $s = ut + \frac{1}{2} ft^2$

$$= 100 \times 10 + \frac{1}{2}(-10) \times 10^2$$

$$= 1000 - 500 = 500 \text{ cm}$$

24. A body starts with a velocity of 5 cm/sec and moves with a uniform acceleration of 5 cm/sec^2. How far it will move after 4 seconds?

Soln. Given,

$$u = 5 \text{ cm/sec}$$

$$f = 5 \text{ cm/sec}^2$$

$$t = 4 \text{ sec}$$

$$s = ?$$

We have,

$$s = ut + \frac{1}{2} ft^2$$

$$= 5 \times 4 + \frac{1}{2} \times 5 \times 4^2$$

$$= 20 + 40 = 60 \text{ cm.}$$

25. A cyclist driving with uniform velocity of 20 ft/sec is 84 ft behind an engine which is just starting from rest with uniform acceleration 2 ft/sec^2. When will the cyclist meet the engine? Explain the double answer.

Soln. Let x ft be the distance described by the engine when the cyclist meets the engine at the first time.

Then, the time taken by cyclist to describe (x+84) ft. is equal to the time taken by the engine to describe x ft.

We have,

$$x + 84 = 20 \times t \text{ .... (1)}$$

Also,

$$x = ut + \frac{1}{2} ft^2$$

i.e. $x = 0 + \frac{1}{2} \times 2 \times t^2 = t^2$ ………….. (2)

Substituting the value of *x* from (2) in equation (1) we get,

$$t^2 + 84 = 20t$$
$$\Rightarrow t^2 - 20t + 84 = 0$$
$$\Rightarrow t = \frac{20 \pm \sqrt{400 - 336}}{2}$$
$$= \frac{20 \pm \sqrt{64}}{2} = \frac{20 \pm 8}{2} = \frac{28}{2} \text{ or } \frac{8}{2} = 14 \text{ sec or } 6 \text{ sec}$$

Hence, the cyclist meets the engine after 6 seconds and 14 seconds.

The reason for double answer is that the cyclist meet the engine in 6 sec. After 6 seconds the cyclist will move forward and the engine will follow him with an acceleration of 2 ft/sec^2 and then the engine will meet the cyclist after 16 sec. After the second meeting, they will never meet again.

26. A car moving in a straight path, with uniform acceleration passes over 7 metres in the 2nd second and 11 metres in the 4th second of its motion. Find the initial velocity and acceleration.

Soln. Let *u* m/sec be the initial velocity and *f* m/sec^2 be the uniform acceleration produced by the car respectively.

$$s_2 = u + \frac{1}{2} f(2 \times 2 - 1)$$
$$\Rightarrow 7 = u + \frac{3}{2} f \quad \ldots\ldots\ldots\ldots\ldots (1)$$

And $$s_4 = u + \frac{1}{2} f(2 \times 4 - 1)$$
$$\Rightarrow 11 = u + \frac{7}{2} f \quad \ldots\ldots\ldots\ldots\ldots (2)$$

Subtracting (1) from (2) we get,

$$4 = \frac{7}{2} f - \frac{3}{2} f$$
$$\Rightarrow 4 = \frac{7-3}{2} f$$
$$\Rightarrow f = 2 \text{ m/sec}^2$$

Putting the value of f in equation(1), we have

$$7 = u + \frac{3}{2} \times 2$$
$$\Rightarrow u = 4 \text{ m/sec}$$

$\therefore$ The initial velocity and acceleration of the car are 4 m/sec and 2 m/sec^2 respectively.

27. A particle starts with an initial velocity *u* and passes successively over the two halves of a given distance with acceleration f_1 and f_2 respectively. Show that the final velocity is the same as if the whole distance were traversed with uniform acceleration $\frac{1}{2}(f_1+f_2)$.

Soln. Let v_1 be the final acquired velocity by the particle for the first half of the distance covered.

Then, $v_1^2 = u^2 + 2f_1\frac{s}{2}$

$$\Rightarrow v_1^2 = u^2 + f_1 s \quad \ldots\ldots\ldots\ldots (1)$$

And for the second half of the distance covered $\frac{s}{2}$,

$$v_2^2 = v_1^2 + 2f_2\frac{s}{2}$$

$$\Rightarrow v_2^2 = v_1^2 + f_2 s$$

$$= u^2 + f_1 s + f_2 s \quad [\because \text{ of } (1)\,]$$

$$= u^2 + (f_1 + f_2)s$$

i.e. $v_2^2 = u^2 + 2.\frac{1}{2}(f_1+f_2)s$

This relation shows that the final velocity is the same as if the whole distance were traversed with uniform acceleration $\frac{1}{2}(f_1+f_2)$.

28. If *s* is the space described in *t* seconds and s' during the next t' seconds by a particle moving in a straight line with uniform acceleration *f*, show that

$$f = \frac{2\left(\frac{s'}{t'} - \frac{s}{t}\right)}{t+t'}$$

Soln. Let *u* be the initial velocity acquired by the particle, v_1 be the final velocity acquired by the particle for the first distance *s*.

Then, $v_1 = u + ft \quad \ldots\ldots\ldots\ldots\ldots (1)$

And $s = ut + \frac{1}{2}ft^2$

$$\Rightarrow s = t(u + \frac{1}{2}ft)$$

$$\Rightarrow \frac{s}{t} = u + \frac{1}{2} ft \; \ldots\ldots\ldots\ldots. (2)$$

And also,

$$s' = v_1 t' + \frac{1}{2} f t'^2$$

$$\Rightarrow \frac{s'}{t'} = v_1 + \frac{1}{2} f t' \; \ldots\ldots\ldots\ldots\ldots (3)$$

From (2) & (3), we get,

$$\frac{s'}{t'} - \frac{s}{t} = v_1 + \frac{1}{2} f t' - u - \frac{1}{2} ft$$

$$= u + ft + \frac{1}{2} f t' - u - \frac{1}{2} ft \quad \text{[using (1)]}$$

$$= \frac{1}{2} ft + \frac{1}{2} f t'$$

$$= \frac{1}{2} f (t + t')$$

$$\Rightarrow f = \frac{2\left(\frac{s'}{t'} - \frac{s}{t}\right)}{t + t'}$$

29. **A particle moving with constant acceleration from *A* to *B* in a straight line *AB* has velocities *u* and *v* at *A* and *B* respectively. Prove that its velocity at *C*, the mid-point of *AB*, is $\sqrt{\frac{u^2+v^2}{2}}$.**

Soln. Let f be the constant acceleration, $AB = s$ and v_1 be the velocity at the mid-point C of AB. Then,

$$AC = CB = \frac{s}{2}$$

Now, $v_1^2 = u^2 + 2.f.\frac{s}{2}$

$$\Rightarrow v_1^2 = u^2 + fs \; \ldots\ldots\ldots\ldots. (1)$$

And $v^2 = u^2 + 2fs$

$$\Rightarrow fs = \frac{v^2 - u^2}{2} \; \ldots\ldots\ldots\ldots\ldots(2)$$

From (1) and (2) we get

$$v_1^2 = u^2 + \frac{v^2 - u^2}{2}$$

$$= \frac{2u^2 + v^2 - u^2}{2}$$

$$= \frac{u^2 + v^2}{2}$$

$$\Rightarrow v_1 = \sqrt{\frac{u^2 + v^2}{2}}$$

$\therefore$ The velocity at C is $\sqrt{\frac{u^2 + v^2}{2}}$.

KEYNOTES 8.2

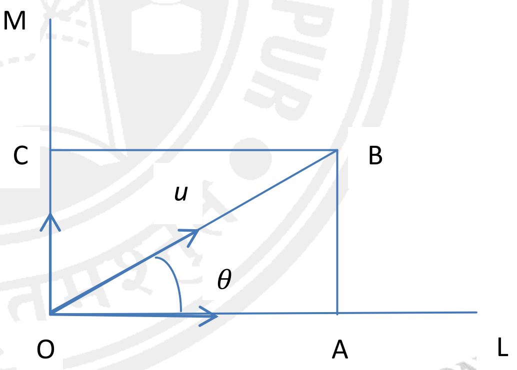
1. **Simultaneous velocities** - A body may have more than one velocity at a time. These velocities are called simultaneous velocities.

2. **Parallelogram law of velocities** - If a body possesses two simultaneous velocities which can be represented in magnitude and direction by two adjacent sides of a parallelogram drawn from one of its angular points, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the angular point.

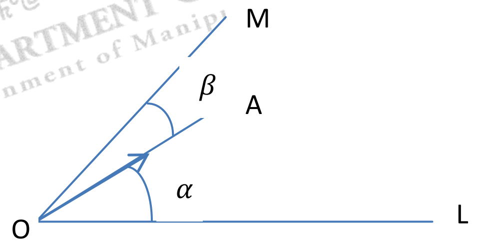
3. **Resolution of Velocities** – The process of splitting a given velocity into mutually perpendicular components is called resolution of the velocity.

Notes:

1) The components of u along two mutually perpendicular OL and OM are $u.cos\theta$ and $u.sin\theta$ respectively. These mutually perpendicular components of the velocity are called the resolved parts or the resolutes along these direction.

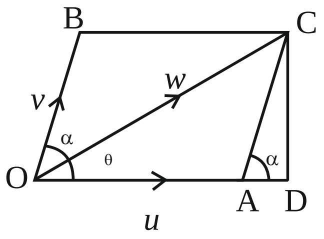


2) Let OL and OM be two arbitrary directions making angles α and β respectively with u .Then the components of u along OL and OM are $\frac{u.\sin\beta}{\sin(\alpha+\beta)}$ and $\frac{u.\sin\alpha}{\sin(\alpha+\beta)}$ respectively.



4. **Resultant velocity:** The single velocity which is equivalent to two or more velocities is called their resultant velocity.

5. **Resultant of two velocities:** If $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent the velocities u and v so that $\angle AOB = \alpha$ .Completing the parallelogram $OACB$, then the diagonal $\overrightarrow{OC}$ represents the resultant velocity w of u and v.



Thus, $w^2 = u^2 + v^2 + 2uv\cos\alpha$ which gives the magnitude of the resultant velocity and if θ is the angle between u and w, then $\tan\theta = \dfrac{v\sin\alpha}{u + v\cos\alpha}$ which gives the direction of the resultant velocity.

Corollary:

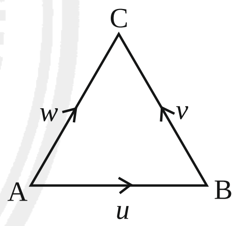
i) If u and v are perpendicular (i.e. $\alpha = 90^0$),, then $w^2 = u^2 + v^2$ and $\tan\theta = \dfrac{v}{u}$.

ii) If u and v have the same direction (i.e. $\alpha = 0$), then $w = u + v$ and direction of the resultant velocity is along the common direction(i.e. $\theta = 0$).

iii) If u and v have the opposite direction (i.e. $\alpha = 180^0$), then $w = |u - v|$ and direction of the resultant velocity is along u or v according as $u > v$ or $v > u$.

6. **Triangle law of velocities**

If a body has two simultaneous velocities u and v which can respectively represented in magnitude and direction by the sides AB and BC of a triangle ABC, then their resultant is represented by the third side AC in magnitude and direction.



Thus, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

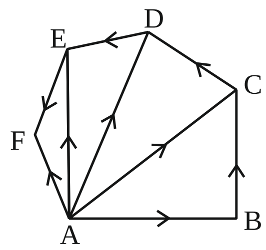
$\Rightarrow \vec{u} + \vec{v} = \vec{w}$

Or,

If a body has three simultaneous velocities which can be represented in magnitude and direction by the three sides of a triangle taken in order, then the resultant velocity is zero thereby keeping the body at rest.

7. **Polygon of Velocities:**

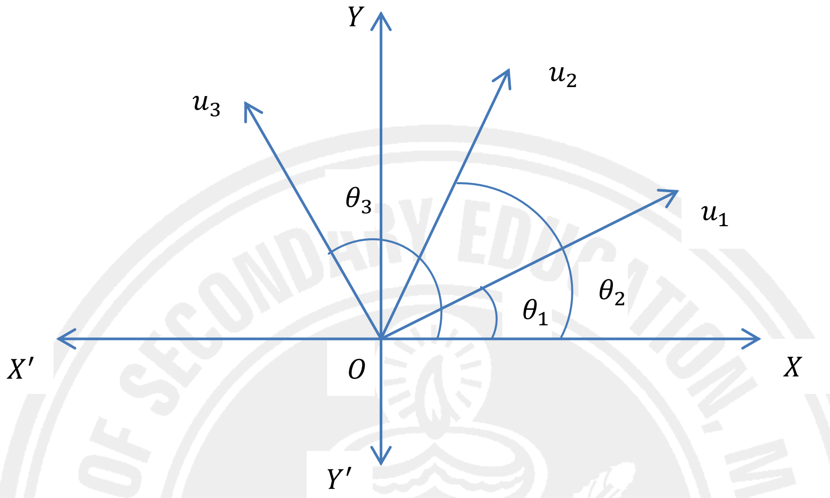
Let $ABCDEF$ be a polygon and there be a particle having simultaneous velocities which can be represented in magnitude and direction by the sides AB, BC, CD, DE and EF. Then, the resultant velocity is represented in magnitude and direction by the side AF.



Thus, $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EF} = \overrightarrow{AF}$

8. **The Resultant of a number of simultaneous co-planar velocities of a particle:**

Let a particle at O has velocities $u_1, u_2, u_3, \ldots u_n$ making respectively angles $\theta_1, \theta_2, \theta_3, \ldots \theta_n$ with a suitably chosen direction OX in the plane. And let OY be perpendicular to OX.



If X and Y respectively denote the algebraic sums of all components along OX and OY,

Then, $X = u_1 \cos\theta_1 + u_2 \cos\theta_2 + u_3 \cos\theta_3 + \ldots + u_n \cos\theta_n$

And, $Y = u_1 \sin\theta_1 + u_2 \sin\theta_2 + u_3 \sin\theta_3 + \ldots + u_n \sin\theta_n$

Then, the resultant velocity w is given by the relation $w = \sqrt{X^2 + Y^2}$ ----- (i)

And, the angle θ made by w with OX is given by $\tan\theta = \dfrac{Y}{X}$ ------ (ii)

Equations (i) and (ii) give respectively the magnitude and direction of the resultant velocity.

EXERCISE 8.2

1. Find the magnitude of the resultant of each of the following pairs of simultaneous velocities with the angle between them given therewith –

i) 30 cm/sec and 40 cm/sec, 90°

Soln. If w cm/sec be the magnitude of the resultant velocity of the two simultaneous velocities 30 cm/sec and 40 cm/sec, then we have

$$w^2 = 30^2 + 40^2 + 2.30.40\cos 90°$$
$$= 900 + 1600 + 0$$
$$= 2500$$
$$\Rightarrow w = \sqrt{2500} = 50 \text{ cm/sec}$$

(ii) 60 m/sec and 80 m/sec, 60°

We have

$$\text{Resultant velocity, } w = \sqrt{60^2 + 80^2 + 2.60.80.\cos 60°}$$
$$= \sqrt{3600 + 6400 + 9600 \times \frac{1}{2}}$$
$$= \sqrt{3600 + 6400 + 4800}$$
$$= \sqrt{14800}$$
$$\Rightarrow w = \sqrt{400 \times 37} = 20\sqrt{37} \text{ m/sec}$$

iii) 5 m/sec and 10 m/sec, 45°

We have

$$\text{Resultant velocity, } w = \sqrt{5^2 + 10^2 + 2.5.10.\cos 45°}$$
$$= \sqrt{25 + 100 + 100 \times \frac{1}{\sqrt{2}}}$$
$$= \sqrt{125 + 50\sqrt{2}}$$
$$\Rightarrow w = \sqrt{25(5 + 2\sqrt{2})}$$
$$= 5\sqrt{(5 + 2\sqrt{2})} \text{ m/sec}$$

iv) 6 m/sec and 10 m/sec, 120°

We have

$$\text{Resultant velocity, } w = \sqrt{6^2 + 10^2 + 2.6.10\cos 120°}$$
$$= \sqrt{36 + 100 + 120\left(-\frac{1}{2}\right)} \quad \left[\because \cos 120° = -\frac{1}{2}\right]$$
$$= \sqrt{136 - 60}$$
$$\Rightarrow w = \sqrt{76} = \sqrt{4 \times 19} = 2\sqrt{19} \text{ m/sec}$$

v) 10 m/sec and 30 m/sec, 135°

We have,

$$\text{Resultant velocity, } w=\sqrt{10^2+30^2+2.10.30.\cos 135^\circ}$$
$$=\sqrt{100+900+600(-\sin 45^\circ)}$$
$$=\sqrt{1000+600\times\left(-\frac{1}{\sqrt{2}}\right)}$$
$$=\sqrt{1000-600\times\frac{\sqrt{2}}{2}}$$
$$=\sqrt{1000-300\sqrt{2}}$$
$$=\sqrt{100(10-3\sqrt{2})}$$
$$\Rightarrow w=10\sqrt{10-3\sqrt{2}}\ \text{ m/sec}$$

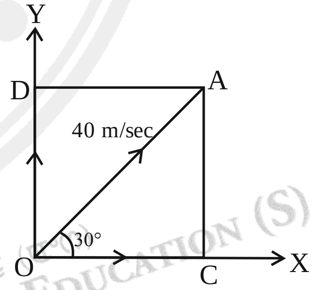
2. Find the resolved parts of each of the following velocities whose inclination to one of the resolved parts is also given alongside

(i) 40 m/sec, 30° (ii) 70 m/sec, 60° (iii) 60 m/sec, 45°
(iv) 10 m/sec, 120° (v) $20m/\sec$, 135^0
Interprete the cases in (iv) and (v).

(i) 40 m/sec, 30°

We have,

The resolved parts of 40 m/sec along $\overline{OX}$ and $\overline{OY}$ are $\overline{OC}$ and $\overline{OD}$



Then $\overline{OC}=OA\cos 30^\circ$
$$=40\times\frac{\sqrt{3}}{2}$$
$$=20\sqrt{3}$$
$$\overline{OD}=OA\sin 30^\circ\ =40\times\frac{1}{2}=20\ \text{ m/sec}$$

(ii) 70 m/sec, 60°

The resolved parts are

$$70\times\cos 60^\circ\ =70\times\frac{1}{2}\ =35\text{ m/sec}$$
$$\text{And, } 70\times\sin 60^\circ\ =70\times\frac{\sqrt{3}}{2}=35\sqrt{3}\ \text{ m/sec}$$

(iii) 60 m/sec, 45°

Resolved parts are

$$60\times\cos 45^\circ = 60\times\frac{1}{\sqrt{2}} = 30\sqrt{2}\ \text{m/sec}$$

$$\text{And, } 60\times\sin 45^\circ = 60\times\frac{1}{\sqrt{2}} = 30\sqrt{2}\ \text{m/sec}$$

(iv) 10 m/sec, 120°

Resolved parts are

$$10\times\cos 120^\circ = 10\times\left(-\frac{1}{2}\right) = -5\ \text{m/sec}$$

$$\text{And , } 10\times\sin 120^\circ = 10\times\frac{\sqrt{3}}{2} = 5\sqrt{3}\ \text{m/sec}$$

The negative sign of the first resolved part shows that its direction is opposite to (+ve) direction of X-axis i.e. the given direction.

(v) 20 m/sec, 135°

The resolved parts are

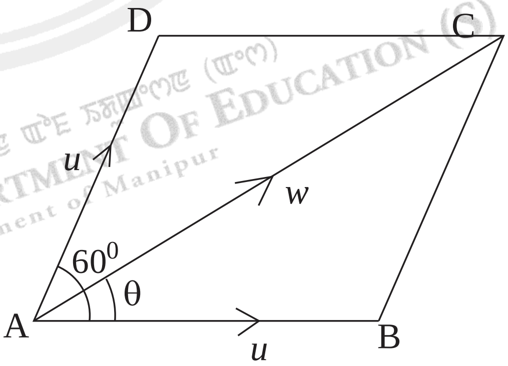
$$20\cos 135^0 = 20(-\sin 45^0) = 20\times\left(-\frac{1}{\sqrt{2}}\right) = -10\sqrt{2}m/\sec$$

$$\text{And , } 20\sin 135^0 = 20\times\frac{1}{\sqrt{2}} = 10\sqrt{2}m/\sec .$$

The negative sign of the first resolved part shows that its direction is opposite to (+ve) direction of X-axis i.e. the given direction.

3. A body has two equal (in magnitude) simultaneous velocities with 60^0 as the angle between them. Determine the resultant velocity and show that it is equally inclined to the velocities.

Soln: Let the two equal velocities u and u be represented by $\overrightarrow{AB}$ and $\overrightarrow{AD}$ as shown in figure.



Then $\overrightarrow{AC}$ represents the resultant velocity of the two velocities.

$$\therefore\ w^2 = u^2 + u^2 + 2u.u.\cos 60^0$$

$$= 2u^2 + 2u^2.\frac{1}{2}$$

$$= 3u^2$$

$$\therefore\ w = \sqrt{3u^2} = u\sqrt{3}$$

If θ be the angle made by the resultant with $\overrightarrow{AB}$, then

$$\tan\theta = \frac{u\sin 60^0}{u+u\cos 60^0} = \frac{\sin 60^0}{1+\cos 60^0}$$

$$= \frac{\frac{\sqrt{3}}{2}}{1+\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{2+1}{2}} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

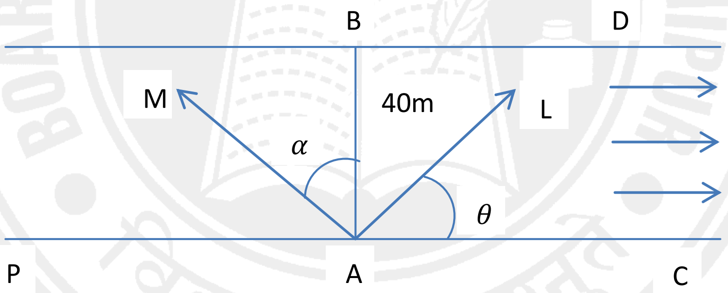
$$\Rightarrow \theta = 30^0$$

Thus, the resultant velocity makes 30^0 with $\overrightarrow{AB}$ and also 30^0 with $\overrightarrow{AD}$.

$\therefore$ The resultant velocity is equally inclined to the two velocities.

4. A man who can swim in still water at the rate of 8m/sec wishes to cross a river 40 metre wide and whose current is flowing at the rate of 4m/sec. Find the difference between the time taken by the man in crossing the river by the shortest time and shortest distance.

Soln: Let *A* be the position of the man on one bank of the river from where he is to swim and *B* is the point on the other bank just opposite to *A* so that $AB = 40m$.



Let $\overrightarrow{AC}$ be the direction of the flow of water and $\overrightarrow{AL}$ be the direction of the man who attempts to swim making an angle θ with AC. Then, the resolved part along AC = $8\cos\theta$ *m/sec* and resolved part along $AB = 8\sin\theta$ *m/sec.*

We have, $8\sin\theta$ *m/sec* is the only velocity to carry the man across the river.

Hence, the time taken to cross the river (*t*) is given by

$$40 = 8\sin\theta \times t$$

i.e. $t = \dfrac{40}{8\sin\theta} = \dfrac{5}{\sin\theta}$

For minimum value of time, the value of $\sin\theta$ should be maximum.

$\therefore \sin\theta = 1$ when $\theta = 90^0$

Thus minimum time, $t = \dfrac{5}{1} = 5\sec$.

But the man will not reach the other bank at the point B but somewhere at the point D of the other bank as he will be carried down by the current.

In order to cross the river directly along AB, let the man swim in upstream direction $\overrightarrow{AM}$ where $\angle BAM = \alpha$. Then,

the resolved part along AB = $8\cos\alpha$ *m/sec.*

and, the resolved part along $AP = 8\sin\alpha$ *m/sec.*

Hence, the components along $\overline{AC}$ and $\overline{AP}$ must neutralize each other.

$\therefore\ 8\sin\alpha = 4$

$\Rightarrow \sin\alpha = \frac{4}{8} = \frac{1}{2}$

$\Rightarrow \alpha = 30^0$

Hence, the time taken by the man in crossing the shortest distance.

$$= \frac{AB}{8\cos\alpha}$$

$$= \frac{40}{8\cos 30^0} = \frac{5}{\frac{\sqrt{3}}{2}}$$

$$= \frac{10\sqrt{3}}{3}\sec s$$

$$= 10\times\frac{1.732}{3}\sec$$

$$= \frac{17.32}{3}\sec = 5.774\sec.$$

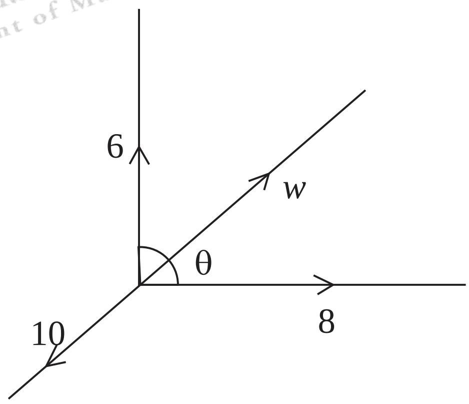
Thus, difference between the two times

$= (5.774-5)$ secs.

= 0.774 secs.

5. A body has two simultaneous velocities of 6 and 8 units along two different directions. When a third velocity of 10 units is applied to the body, it comes to rest. Find the angle between the first two velocities.

Soln: Since the body comes to rest, the resultant of 6 units and 8 units must be opposite to 10 units in a straight line. Now, if the angle between two simultaneous velocities 6 units and 8 units is θ, then we have



$$w^2 = 6^2 + 8^2 + 2.6.8.\cos\theta$$

$\Rightarrow (-10)^2 = 36 + 64 + 96\cos\theta$

$\Rightarrow 100 - 100 = 96\cos\theta$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90^0$

Hence, the angle between the two velocities is 90^0.

6. Find the maximum and minimum resultants of three simultaneous velocities of 5, 12 and 13 units possessed by a body.

Soln: We know that the three velocities must have the maximum velocity when they are along the same line and the same direction.

Hence, maximum resultant = 5+12+13=30 units.

Also, the resultant of 5 unit and 12 units if they are perpendicular to each other is given by

$$w^2 = 5^2 + 12^2 = 169$$
$$\Rightarrow w = \sqrt{169} = 13 \text{ units}$$

If the third velocity 13 units is just opposite to this resultant, they cancel each other.
Hence, the minimum resultant = 0.

7. Can the three simultaneous velocities of 2, 3 and 7 units keep a body at rest? Give reason for your answer.

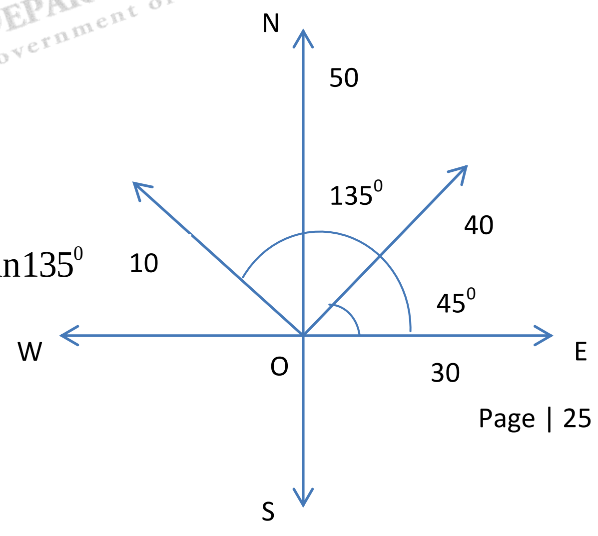
Soln: No, because neither the given three velocities can't represent three sides of a triangle nor the resultant velocity of 2 and 3 units along the same line and same direction = (3+2) = 5 units which cannot balance with the third velocity of 7 units.

8. Find the magnitude and direction of the resultant of the following four simultaneous velocities of a body : 30m/sec due East, 40m/sec due North-East, 50m/sec due North and 10m/sec due North-West.

Soln: From the figure, resolving the velocities along OE and ON we have

$$X = 30\cos 0^0 + 40\cos 45^0 + 50\cos 90^0 + 10\cos 135^0$$
$$= 30\times 1 + 40\times\frac{1}{\sqrt{2}} + 50\times 0 + 10\times\left(-\frac{1}{\sqrt{2}}\right)$$
$$= 30 + 20\sqrt{2} - 5\sqrt{2}$$
$$= 30 + 15\sqrt{2}$$
$$= 15\left(2+\sqrt{2}\right) \text{ m/sec.}$$

$$Y = 30\sin 0^0 + 40\sin 45^0 + 50\sin 90^0 + 10\sin 135^0$$



$$= 30\times 0 + 40\times\frac{1}{\sqrt{2}} + 50\times 1 + 10\times\frac{1}{\sqrt{2}}$$
$$= 20\sqrt{2} + 50 + 5\sqrt{2}$$
$$= 25\sqrt{2} + 50$$
$$= 25\left(\sqrt{2}+2\right) \text{ m/sec.}$$

$$\therefore \text{Resultant velocity, } w = \sqrt{\left\{15\left(2+\sqrt{2}\right)\right\}^2 + \left\{25\left(2+\sqrt{2}\right)\right\}^2}$$
$$= \sqrt{\left(15^2+25^2\right)\left(2+\sqrt{2}\right)^2}$$
$$= \left(2+\sqrt{2}\right)\sqrt{225+625}$$
$$= \left(2+\sqrt{2}\right)\sqrt{850} = \left(2+\sqrt{2}\right)5\sqrt{2}.\sqrt{17}$$
$$= \sqrt{2}\left(\sqrt{2}+1\right)5.\sqrt{2}.\sqrt{17}$$
$$= 10\sqrt{17}\left(1+\sqrt{2}\right) \text{ m/sec.}$$

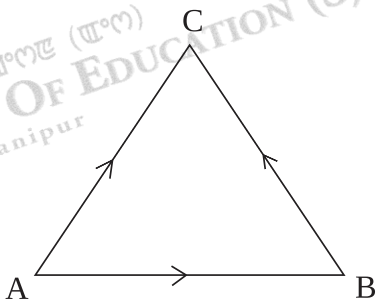
Let θ be the angle made by the resultant with $\overline{OE}$, then we have,

$$\tan\theta = \frac{Y}{X} = \frac{25\left(2+\sqrt{2}\right)}{15\left(2+\sqrt{2}\right)} = \frac{5}{3}$$

i.e. the resultant make an angle θ north of east such that $\tan\theta = \frac{5}{3}$.

Q9. Find the resultant of three simultaneous velocities all equal in magnitude when the angle between each two of them is 120°.

Soln: Since the three simultaneous velocities are all equal in magnitude and the angle between each two of them is 120°, the three velocities can be represented by the sides AB, BC, CA of the equilateral triangle ABC taken in order.



Thus $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$

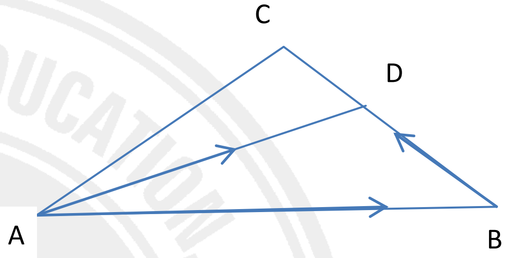
i.e. the resultant velocity is zero.

10. A body has two simultaneous velocities which can be represented in magnitude and direction by 3AB and 2BC of a triangle ABC. Find the position of the point D on BC such that *k*AD represents the resultant of the two velocities, where *k* is a constant. State the value of *k*.

Soln: Here the two simultaneous velocities are represented by 3AB and 2BC of a ΔABC.

Now by triangle law of vectors,

$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$



$\Rightarrow 3\overrightarrow{AB} = 3\overrightarrow{AD} + 3\overrightarrow{DB}$……………….(i)

Also, $2\overrightarrow{BC} = 2(\overrightarrow{BD} + \overrightarrow{DC})$…………………(ii)

Adding (i) and (ii), we get

$3\overrightarrow{AB} + 2\overrightarrow{BC} = 3\overrightarrow{AD} + 3\overrightarrow{DB} + 2\overrightarrow{BD} + 2\overrightarrow{DC}$

$= 3\overrightarrow{AD} - 3\overrightarrow{BD} + 2\overrightarrow{BD} + 2\overrightarrow{DC}$

$\Rightarrow k\overrightarrow{AD} = 3\overrightarrow{AD} + \left(2\overrightarrow{DC} - \overrightarrow{BD}\right)$

$\Rightarrow k = 3 \; and \; 2\overrightarrow{DC} - \overrightarrow{BD} = 0$

$\Rightarrow \left|2\overrightarrow{DC}\right| = \left|\overrightarrow{BD}\right|$

$\Rightarrow \frac{BD}{DC} = \frac{2}{1}$

∴ D divides BC in the ratio 2:1 and the value of *k* is 3.

11. Giving reason state if a body having three simultaneous velocities 2*u*, 3*u* and 6*u* can have their resultant velocity zero.

Soln: No, because neither the given three velocities can't represent three sides of a triangle nor the resultant velocity of *2u* and *3u* along the same line and same direction = *(3u+2u) = 5u* which cannot balance with the third velocity of *6u* .

KEYNOTE 8.3

1. **Momentum:** When a body is in motion, it acquires a certain property by virtue of its motion. This property which is depending on both amount of the mass and velocity of the body is called the momentum of the body.
   $\therefore$ Momentum = mass × velocity.

2. **Force:** A force is an external agent that changes or tends to change the state of rest or state of uniform motion along a straight line of a body.

3. **Newton's laws of motion:**
   a) First law: Everybody continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled by external forces to change that state.
   b) Second law: The rate of change of momentum is proportional to the motive force and takes place in the direction of the straight line in which the force acts.
   We have, $p = mf$
   c) Third law: To every action, there is an equal and opposite reaction.

4. **Weight:** The weight of a body is the resultant force with which the body is attracted towards the centre of the Earth.
   The weight W of a body of mass m, $W = mg$

Notes:

(i) The equation $P = mf$ is sometimes called the ***kinetic equation.***

(ii) Units of force:
In the FPS system, the unit of force is poundal.
In the CGS system, the unit of force is dyne.
In the MKS system, the unit of force is Newton.
($1N=10^5$ dynes, 1 megadyne = 10^6 dynes = 10N)

(iii) Gravitational unit of force: The unit of force expressed in terms of the gravitational pull on the mass is called the gravitational or statical unit of force.
e.g. A body of mass 1 lb on the surface of the earth is attracted to the centre of the earth by a force called 1 pound weight denoted by 1 lb.wt.
$\therefore$ Using $P = mf$, this force, 1 lb.wt. $= 1 \times 32 = 32\ poundals$.
Similarly,

1 gm . wt	=	981 dynes
1 kg . wt	=	9.81 Newtons

EXERCISE 8.3

1. Find the acceleration produced by a constant force of 10 newtons on a mass of 5 kgs. Also find the velocity acquired by the body when the force acts for 10 seconds if initially the body was moving with a uniform velocity of $5m/\sec$.

Soln: Given,

Force, $P = 10N$
Mass of the body, $m = 5kg$.
Acceleration, f= ?
By the formula
$p = mf$, we have
$\Rightarrow 10 = 5\times f$
$\Rightarrow f = 2m/\sec^2$.
Again,
initial velocity, $u = 5m/\sec$
time taken, $t = 10\sec$
acceleration, $f = 2m/\sec^2$
final velocity, $v = ?$
We have
$v = u + ft = 5 + 2\times 10 = 25m/\sec$

2. A heavy body of 80 kgs is being raised from the bottom of a pit 109m deep by a uniform force of 120kg-wt. Find the time taken by the body to reach the top of the pit.

Soln: Given, mass $m = 80kg$
Force, $F = 120kg$ wt = 120×9.81N
Initial velocity, u = 0 , h= 109 m.
Then, the net force producing motion
= $F - mg$
= 120×9.81-80×9.81
= 392.4
$= \frac{3924}{10}$
$= \frac{1962}{5}\ N$
By Kinetic equation, we get,
$F - mg = mf$
$\Rightarrow \frac{1962}{5} = 80\times f$

$$\Rightarrow f = \frac{1962}{5\times 80} = \frac{981}{200} m/\sec^2$$

Hence, from the equation

$$h = ut + \frac{1}{2} ft^2$$

$$\Rightarrow 109 = 0\times t + \frac{1}{2}\times\frac{981}{200}\times t^2$$

$$\Rightarrow t = \sqrt{\frac{2\times 200\times 109}{981}} = \sqrt{\frac{400}{9}} = \frac{20}{3} = 6\frac{2}{3}\sec$$

3. Find the magnitude of the uniform force which increases the velocity of a body of mass 5 kgs. from 20m/sec to 30m/sec in 10 minutes.

Soln: Given,

$$u = 20m/\sec$$
$$v = 30m/\sec$$
$$t = 10\min = 10\times 60\sec = 600\sec$$
$$m = 5kg$$
$$f = ?,\ F = ?$$

We have,

$$v = u + ft$$

$$\Rightarrow 30 = 20 + f\times 600$$

$$\Rightarrow f = \frac{10}{600} = \frac{1}{60} m/\sec^2$$

$\therefore$ by Kinetic equation, we get,

$$F = mf = 5\times\frac{1}{60} = \frac{1}{12} N.$$

4. A mass of 4 kgs falls freely for 200m and then brought to rest by penetrating 2m in the mud. Find the average thrust of the mud on the body.

Soln: Given,

$$u = 0$$
$$m = 4kg.$$
$$h = 200m$$
$$g = 9.81m/\sec^2$$

We have,

$$v^2 = u^2 + 2gh,$$ where v is the final velocity just before penetration,

$$= o^2 + 2 \times 9.81 \times 200$$
$$= 3924$$
$$\Rightarrow v = \sqrt{3924}$$

If $f\ m/\sec^2$ be the retardation produced in penetrating the mud and v_1 be the final velocity after penetration, then we get,

$$v_1^2 = v^2 - 2fh$$
$$\Rightarrow 0^2 = 3924 - 2 \times f \times 2$$
$$\Rightarrow f = \frac{3924}{4} = 981 m/\sec^2$$

Let R Newton be the average thrust of the mud on the body which acts vertically upward and also the wt. of the body $=4g$ Newton which acts vertically downward.

Since, $P = mg$

$$\Rightarrow R - 4f = mg$$
$$\Rightarrow R = 4 \times 981 + 4 \times 9.81$$
$$= 3924 + 39.24$$
$$= 3963.24N$$
$$= \frac{3963.24}{9.81} kg.wt.$$
$$= 404 kg.wt \quad [1 \text{ kg.wt} = 9.81 \text{ N}]$$

5. A truck of mass 5 (metric) tons and moving at a uniform rate of 60m/sec is brought to rest in 10 seconds by the uniform application of brakes. Find the distance covered by the truck during this time and also calculate the magnitude of the force of resistance developed by the brakes in megadynes.

Soln: Given,

$$m = 5(metric)tons = 5000kg$$
$$u = 60m/\sec$$
$$v = 0$$
$$t = 10\sec onds$$
$$F = ?, \quad f = ?, \quad s = ?$$

We have,

$$v = u + ft$$
$$\Rightarrow 0 = 60 + f \times 10$$
$$\Rightarrow f = \frac{-60}{10} = -6m/\sec^2$$

And $s = ut + \frac{1}{2} ft^2$

$= 60 \times 10 + \frac{1}{2} \times (-6) \times 10^2$

$= 600 - 300 = 300m$

$\therefore$ Distance covered by the truck during this time is 300 m.

Also, $F = mf$

$= 5000 \times (-6) N$

= - 30000N

$= \frac{-30000}{10}$ megadynes [1 megadyne= 10 N]

$= -3000$ megadynes.

Hence, the resistance developed by the brakes is 3000 megadynes

6. A body falls freely through a distance of 10m from rest. It is then brought to rest in 1 second by a vertical force. A similar second force can bring the body to rest in 2 seconds. Show that the first force is $\frac{17}{12}$ times the second force. (Take $g = 980 cm/\sec^2$)

Soln: We have,

The final velocity of the body in falling 10m is given by

$V^2 = u^2 + 2gh$

$\Rightarrow V^2 = 0^2 + 2 \times 9.8 \times 10$

$\Rightarrow V = \sqrt{196} m/\sec$

$= 14 m/\sec$

For the first case, the force resisting motion = $mg - R_1$

$\therefore V_1 = V + ft$

$\Rightarrow 0 = 14 + f \times 1$

$\Rightarrow f = -14 m/\sec^2$

By Kinetic equation, we get,

$mg - R_1 = mf$

$\Rightarrow R_1 = m(g - f) = m(9.8 + 14) N = m \times 23.8 N$ ………………(i)

and for second case, the force resisting motion

$= mg - R_2$

$\therefore V_1 = V + ft$

$\Rightarrow 0 = 14 + f \times 2$

$\Rightarrow f = -7 m/\sec^2$

$\therefore$ By Kinetic equation, we get

$$mg - R_2 = mf$$

$$\Rightarrow R_2 = m(g-f) = m(9.8+7)N$$

$$= m \times 16.8N \quad \text{........................(ii)}$$

Now, from (i) and (ii)

$$\frac{R_1}{R_2} = \frac{m \times 23.8}{m \times 16.8} = \frac{238}{168} = \frac{17}{12}$$

i.e. $R_1 = \frac{17}{12} R_2$.

This means that the first force is $\frac{17}{12}$ times the second force.

7. A thin glass plate can just support a weight of 20 kgs. A body is placed on it and the plate is raised with the body on it with gradual increasing acceleration. It is observed that the plate just breaks when the acceleration is 6.54 m/sec^2. Find the mass of the body.

Soln: Given,

Motive force $= F - mg$

$= (20 \times 9.81 - m \times 9.81)N$

Mass of the body, $m = ?$

Acceleration produced, $f = 6.54 m/\sec^2$

$$F - mg = mf$$

$$\Rightarrow 20 \times 9.81 - m \times 9.81 = m \times 6.54$$

$$\Rightarrow 20 \times 9.81 = m(9.81 + 6.54)$$

$$\Rightarrow m = \frac{20 \times 9.81}{16.35} = 12kg.$$

8. A body of mass *m* is put on another body of mass *M* and they are falling freely. Assuming that the air offers no resistance, show that one body does not give any pressure on the other.

Soln: If R be the reaction of the body of mass M on the body of mass m. Then the net force producing motion is $(mg - R)$.

Hence, from the Kinetic equation, we get,

$$mg - R = mf$$

$$\Rightarrow R = mg - mf = m(g-f)$$

But the two bodies fall freely.

Hence, $f = g$.

Thus, $R = m(g - f) = m(g - g) = 0$

i.e. there is no reaction of the larger body on the smaller body. This means that according to Newton's third law of motion, no action is given to the larger body i.e. the smaller body does not give any pressure on the larger body.

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