CHAPTER 6
PERMUTATIONS AND COMBINATIONS

SOLUTIONS

EXERCISE 6.1

1. **Evaluate**

i) $7!$

Soln. $7! = 1\times2\times3\times4\times5\times6\times7 = 5040$

ii) $6! - 5!$

Soln. $6\times5! - 5!$

$= (6-1)\times5! = 5\times1\times2\times3\times4\times5 = 5\times120 = 600$

2. **Complete** $\dfrac{9!}{6!\times3!}$

Soln. $\dfrac{9!}{\cancel{6!}\times3!} = \dfrac{9\times8\times7\times\cancel{6!}}{6!\times1\times2\times3} = 3\times4\times7 = 84$

3. **If** $\dfrac{1}{9!}+\dfrac{1}{10!}=\dfrac{x}{11!}$**, find** x

Soln. We have,

$$\frac{1}{9!}+\frac{1}{10!}=\frac{x}{11!}$$

$$\Rightarrow \frac{10+1}{10!}=\frac{x}{11\times10!}$$

$$\Rightarrow \frac{11\times11}{10!}=\frac{x}{10!}$$

$$\Rightarrow x = 11\times11 = 121$$

4. **Evaluate** ${}^{n}P_{r}$ **when**

i) $n=5,\ r=3$

Soln. ${}^{5}P_{3} = \dfrac{5!}{(5-3)!} = \dfrac{5!}{2!} = 3\times4\times5 = 60$

ii) $n=8,\ r=4$

Soln. ${}^{8}P_{4} = \dfrac{8!}{(8-4)!} = 5\times6\times7\times8 = 1680$

5. **Find *n* if** ${}^{n-1}P_5 : {}^nP_6 = 1:11$

Soln. We have,

$${}^{n-1}P_5 : {}^nP_6 = 1:11$$

$$\Rightarrow \frac{(n-1)!}{(n-1-5)!} : \frac{n!}{(n-6)!} = 1:11$$

$$\Rightarrow \frac{(n-1)!}{(n-6)!} : \frac{n\times(n-1)!}{(n-6)!} = 1:11$$

$$\Rightarrow 1:n = 1:11$$

$$\Rightarrow n = 11$$

6. **Find *r* if**

(i) ${}^5P_r = 2.{}^6P_{r-1}$

Soln.

$${}^5P_r = 2.{}^6P_{r-1}$$

$$\Rightarrow \frac{5!}{(5-r)!} = 2.\frac{6!}{(6-r+1)!}$$

$$\Rightarrow \frac{5!}{(5-r)!} = 2.\frac{6.5!}{(7-r)!}$$

$$\Rightarrow \frac{1}{(5-r)!} = \frac{2\times 6}{(7-r)(6-r)(5-r)!}$$

$$\Rightarrow (7-r)(6-r) = 12$$

$$\Rightarrow 42 - 13r + r^2 = 12$$

$$\Rightarrow r^2 - 13r + 30 = 0$$

$$\Rightarrow (r-3)(r-10) = 0$$

$$\Rightarrow r - 3 = 0 \; or \; r - 10 = 0$$

$$\Rightarrow r = 3 \; or \; r = 10$$

$$\Rightarrow r = 3 [\because r \leq 5]$$

$$\therefore r = 3$$

(ii) ${}^5P_r = {}^6P_{r-1}$

Soln:

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{6!}{(6-r+1)!}$$

$$\Rightarrow \frac{5!}{(5-r)!} = \frac{6\times 5!}{(7-r)(6-r)(5-r)!}$$

$$\Rightarrow (7-r)(6-r) = 6$$

$$\Rightarrow 42 - 13r + r^2 = 6$$

$$\Rightarrow r^2 - 13r + 36 = 0$$

$\Rightarrow (r-4)(r-9)=0$
$\Rightarrow r-4=0 \text{ or } r-9=0$
$\Rightarrow r=4 \text{ or } r=9$
$\Rightarrow r=4 [\because r \le 5]$
$\therefore r=4$

7. **If** ${}^nC_8 = {}^nC_2$, **find** ${}^nC_3$.

Soln.

$${}^nC_8 = {}^nC_2$$
$$\Rightarrow {}^nC_{n-8} = {}^nC_2 [\because {}^nC_r = {}^nC_{n-r}]$$
$$\Rightarrow n-8=2$$
$$\Rightarrow n=10$$
$$\therefore {}^nC_3 = {}^{10}C_3$$
$$= \frac{10!}{3!(10-3)!} = \frac{10.9.8.7!}{1.2.3.7!} = 120$$

8. **Determine** ***n*** **if**

i) ${}^{2n}C_4 : {}^nC_4 = 42:1$

Soln.

$${}^{2n}C_4 : {}^nC_4 = 42:1$$
$$\Rightarrow \frac{2n!}{4!(2n-4)!} : \frac{n!}{4!(n-4)!} = 42:1$$
$$\Rightarrow \frac{2n(2n-1)(2n-2)(2n-3)(2n-4)!}{1.2.3.4.(2n-4)!} : \frac{n(n-1)(n-2)(n-3)(n-4)!}{1.2.3.4.(n-4)!} = 42:1$$
$$\Rightarrow \frac{2n(2n-1)(2n-2)(2n-3)}{n((n-1)(n-2)(n-3)} = \frac{42}{1}$$
$$\Rightarrow \frac{2.n.2(n-1)(2n-1)(2n-3)}{n(n-1)(n-2)(n-3)} = \frac{42}{1}$$
$$\Rightarrow \frac{4.(2n-1)(2n-3)}{(n-2)(n-3)} = \frac{42}{1}$$
$$\Rightarrow \frac{2(4n^2-6n-2n+3)}{n^2-5n+6} = \frac{21}{1}$$
$$\Rightarrow 8n^2-16n+6 = 21n^2-105n+126$$
$$\Rightarrow 13n^2-89n+120=0$$
$$\Rightarrow n = \frac{-(-89) \pm \sqrt{(-89)^2-4.13.120}}{2\times 13}$$

$$= \frac{89 \pm \sqrt{7921-6240}}{26} = \frac{89 \pm \sqrt{1681}}{26}$$

$$= \frac{89 \pm 41}{26} = \frac{130}{26} \text{ or } \frac{48}{26}$$

$$= 5 \text{ or } \frac{24}{13}$$

$$= 5 \quad [\because n \text{ cannot be a fraction}]$$

ii) ${}^{2n}C_3 : {}^{n}C_3 = 11:1$

Soln.

$$\Rightarrow \frac{2n!}{3!(2n-3)!} : \frac{n!}{3!(n-3)!} = 11:1$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)(2n-3)!}{3!(2n-3)!} : \frac{n(n-1)(n-2)(n-3)!}{3!(n-3)!} = 11:1$$

$$\Rightarrow \frac{2.n.(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$$

$$\Rightarrow \frac{2.n.2.(n-1)(2n-1)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)}{n-2} = 11$$

$$\Rightarrow 8n - 4 = 11n - 22$$

$$\Rightarrow 3n = 18$$

$$\Rightarrow n = 6$$

9. **Prove that (i)** ${}^{n}C_r + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2} = {}^{n+2}C_r$

L.H.S.

$$= {}^{n}C_r + 2\,{}^{n}C_{r-1} + {}^{n}C_{r-2}$$

$$= ({}^{n}C_r + {}^{n}C_{r-1}) + ({}^{n}C_{r-1} + {}^{n}C_{r-2})$$

$$= {}^{n+1}C_r + {}^{n+1}C_{r-1} \quad [\because {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r]$$

$$= {}^{n+2}C_r = \text{R.H.S.}$$

(ii) ${}^{n}C_r + {}^{n-1}C_r + {}^{n-2}C_r + \ldots\ldots.. + {}^{r}C_r = {}^{n+1}C_{r+1}$

We have ${}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_{r+1}$

$$\therefore {}^{n+1}C_{r+1} - {}^{n}C_{r+1} = {}^{n}C_r \quad \ldots\ldots\ldots\ldots\ldots (1)$$

L.H.S.

$$= {}^{n}C_r + {}^{n-1}C_r + {}^{n-2}C_r + \ldots\ldots.. + {}^{r+1}C_r + {}^{r+1}C_{r+1} \quad [\text{Putting } {}^{r}C_r = {}^{n+1}C_{r+1}]$$

$$= \left({}^{n+1}C_{r+1} - {}^{n}C_{r+1}\right) + \left({}^{n}C_{r+1} - {}^{n-1}C_{r+1}\right) + \left({}^{n-1}C_{r+1} - {}^{n-2}C_{r+1}\right) +$$

$$\ldots\ldots\ldots\ldots\ldots\ldots + \left({}^{r+2}C_{r+1} - {}^{r+1}C_{r+1}\right) + {}^{r+1}C_{r+1} \quad [\because \text{ of } (1)]$$

$$= {}^{n+1}C_{r+1} \quad [\text{cancelling like terms of opposite signs}]$$

= R.H.S.

10. **Prove that** $\dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\dfrac{(4n-1)(4n-3)..........5.3.1}{\{(2n-1)(2n-3)...........5.3.1\}^2}$

Soln.

L.H.S. $$=\frac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\frac{\dfrac{(4n)!}{(4n-2n)!(2n)!}}{\dfrac{(2n)!}{(2n-n)!n!}}$$

$$=\frac{(4n)!}{(2n)!(2n)!}\times\frac{n!n!}{(2n)!}$$

$$=\frac{1.2.3.4.....(4n-1)(4n)}{1.2.3.4.......(2n-1)(2n)}\times\left\{\frac{1.2.3.4.............(n-1).n}{1.2.3.4.........(2n-1)(2n)}\right\}^2$$

$$=\frac{1.3.5.7.......(4n-1)\{1.2.3......(2n-1).2n\}.2^{2n}}{1.2.3.4......(2n-1)(2n)}\times\left[\frac{1.2.3........(n-1).n}{1.3.5.7....(2n-1)\{1.2.3.....(n-1)n\}2^n}\right]^2$$

$$=\{1.3.5.7............(4n-1)\}2^{2n}\times\frac{1}{\{1.3.5.7...........(2n-1)\}^2.2^{2n}}$$

$$=\frac{(4n-1)(4n-3)..........5.3.1}{\{(2n-1)(2n-3).........5.3.1\}^2}$$ = R.H.S.

11. **How many 3-digit even numbers can be formed from the digits 1,2,3,4,5,6 if the digits can be repeated?**

Soln. Here, the even digits are 2,4,6.

$\therefore$ The digits in one's place can be selected in ${}^3C_1$ ways since digits can be repeated.

The digit in tens place can be selected in ${}^6C_1$ ways and the digit in hundreds place can be selected in ${}^6C_1$ ways.

$\therefore$ Required total number of 3-digits even numbers

$$={}^3C_1\times{}^6C_1\times{}^3C_1$$
$$=3\times6\times6=108$$

12. **How many 6-digits telephone numbers can be constructed using the digits 0 to 9 if each number starts with 24 and no digit appears more than once?**

Soln. We have, digits are 0,1,2,3,4,5,6,7,8,9

24 is reserved for starting the 6-digits telephone numbers.

So, the remaining digits are 0,1,3,5,6,7,8,9 (8 digits) which can be arranged in ${}^8P_4$ ways.

$\therefore$ Total number of 6-digit telephone numbers

$$={}^8P_4=\frac{8!}{(8-4)!}=\frac{8!}{4!}=8\times7\times6\times5=1680$$

13. How many words with or without meaning can be formed using all the letters of the word EQUATION using each letter exactly once?

Soln. In the word EQUATION, there are 8 different letters and these 8 letters can be arranged taking all at time in ${}^{8}P_{8}$ ways.

Hence, the required number of words formed

$= {}^{8}P_{8} = 8! = 1\times2\times3\times4\times5\times6\times7\times8$

$= 40320$

14. In how many ways can the letters of the word PERMUTATIONS be arranged if

i) the words starts with *P* and end with *S*

ii) the vowels are all together

iii) there are always 4 letters between P and S?

Soln. i) the word PERMUTATIONS has 12 letters of which 2 are T's and letters P, S are reserved for beginning and end of the words.

$\therefore$ remaining 10 letters can be arranged in $\frac{10!}{2!}$ ways

as T is repeated twice.

$\therefore$ Total number of ways $= \frac{10!}{2!}$

ii) PERMUTATIONS

There are 5 vowels and 7 remaining letters of which T is repeated twice.

$\therefore$ Taking 5 vowels as a single letter and 7 remaining letters = 8 letters in total, the words can be arranged in $\frac{8!}{2!}$ ways.

Again, 5 vowels can be arranged amongst themselves in $5!$ ways.

$\therefore$ Total number of ways $= \frac{8!}{2!}\times 5!$

iii) PERMUTATIONS – 12 letters where T occurs twice and excluding P and S, the remaining 10 letters can be arranged in $\frac{10!}{2!}$ ways.

Since there are always 4 letters between P and S then P or S can be placed in 7 ways by always keeping 4 letters between them.

1. P in 1st place and S in 6th place.
2. P in 2nd place and S in 7th place.
3. P in 3rd place and S in 8th place.
4. P in 4th place and S in 9th place.
5. P in 5th place and S in 10th place.

6. P in 6th place and S in 11th place.
7. P in 7th place and S in 12th place.

Again, the positions of P and S can be arranged in 2! Ways.

$\therefore$ The required number of ways

$$=\frac{10!}{2!}\times 7\times 2!=25401600$$

15. How many chords can be drawn through 25 points on a circle?

Soln. For drawing a chord, any two points can be selected.

$\therefore$ These two points can be selected in ${}^{25}C_2$ ways

$$=\frac{25!}{2!\times 23!}=300 \text{ ways}$$

Thus, the number of chords drawn = 300.

16. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in the combination.

Soln. Since exactly one ace is to be included in the 5 card combinations, 1 ace can be selected from 4 aces in ${}^4C_1$ ways. Again, 4 cards from the remaining (52-4) = 48 cards can be selected in ${}^{48}C_4$ ways.

$\therefore$ The number of ways of selecting 5 card combinations containing exactly one ace

$$={}^{48}C_4\times {}^4C_1$$

$$=\frac{48!}{4!(48-4)!}\times\frac{4!}{1!(4-1)!}$$

$$=\frac{48.47.46.45}{1.2.3.4}\times 4=778320$$

17. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl, if each cricket team of 11 must include exactly 4 bowlers?

Soln. Since, there are only 5 bowlers, the number of ways of selecting 4 bowlers can be done in ${}^5C_4$ ways and 7 other players can be selected out of 12 remaining players in ${}^{12}C_7$ ways.

$\therefore$ The number of ways of selecting a cricket team

$$={}^5C_4\times {}^{12}C_7=5\times\frac{12\times 11\times 10\times 9\times 8\times 7\times 6}{1\times 2\times 3\times 4\times 5\times 6\times 7}$$

$$=3960$$

18. Two political parties having respectively 15 and 23 MLAs form a government. In how many ways can a cabinet with 7 ministers holding different portfolios be formed such that 3 are from the party with fewer numbers of MLAs.

Soln. 3 ministers can be selected out of 15 MLAs in ${}^{15}C_3$ ways.

4 ministers can be selected out of 23 MLAs in ${}^{23}C_4$ ways.

7 ministers holding different portfolios can be arranged amongst themselves in 7! ways.

$\therefore$ the number of cabinet with 7 ministers

$= {}^{15}C_3 \times {}^{23}C_4 \times 7!$

19. A photo of ten students, 5 Indians, 3 Americans and 2 Chinese is to be taken so that all the Indians are together and so are the Americans and the Chinese. How many photographs are possible?

Soln. We consider 5 Indians, 3 Americans and 2 Chinese as one group each, then we have 3 groups. These 3 groups can be arranged in 3! Ways.

Again, 5 Indians can be arranged amongst themselves in 5! ways.

3 Americans can be arranged amongst themselves in 3! ways.

& 2 Chinese can be arranged amongst themselves in 2! ways.

$\therefore$ possible number of photographs to be taken = $5! \times 3! \times 2! \times 3!$.

20. In how many ways can 10 boys and 7 girls can be arranged in a row so that no two girls are together?

Soln. Le the 10 boys be put in a row as shown below – (B represents boys)

\- B - B - B - B - B - B - B - B - B - B –

Since no two girls are together, the girls can only be put in places marked as - . The number of such places is 11. So 7 girls can be arranged in ${}^{11}P_7$ ways to occupy the positions of the girls. Again, among 10 boys, they can be arranged in ${}^{10}P_{10}$ or 10! ways.

$\therefore$ The number of ways of arrangements $= 10! \times {}^{11}P_7$.

21. Numbers are formed by using all the digits 2,5,6,8,9. How many of the numbers so formed are divisible by 2 or 5? (Digits not repeated)

Soln. The even digits are 2,6,8.

So, the numbers in which digits in ones place for divisibility by 2 can be arranged in following ways.

The number in which 2 lies in one place can be formed in 4! ways.

The number in which 6 lies in one place can be formed in 4! ways.

The number in which 8 lies in one place can be formed in 4! ways.

and for divisibility by 5, the number in which 5 lies in ones place can be formed in 4! ways.

$\therefore$ the total number of ways in forming the 5-digit number $= 4! + 4! + 4! + 4!$

$= 4 \times 4!$

22. A row of 5 chairs is to be occupied by 3 boys and 2 girls. There are 7 boys and 4 girls present. In how many ways can the line be formed, if the two end chairs have to be occupied by 2 boys?

Soln. Out of 7 boys, 3 boys can be selected in $^7C_3$ ways and out of 4 girls, 2 girls can be selected in $^4C_2$ ways.

Again, since two ends chairs must be occupied by 2 boys, this can be done in $3\times2=6$ ways.

Also, the inner 3 chairs (occupied by 2 girls and 1 boys) can be arranged in 3! ways

$\therefore$ Total number of arrangement $= {}^7C_3 \times {}^4C_2 \times 6 \times 3!$

$$= \frac{7.6.5}{1.2.3} \times \frac{4.3}{1.2} \times 6 \times (3.2.1)$$
$$= 210 \times 36$$
$$= 7560$$

23. A rectangle is cut by 6 lines parallel and 8 line perpendicular to the base. Find the total number of rectangles thus formed.

Soln. Since 6 parallel lines to the base are drawn, hence there are 8 parallel lines. Out of these 8 parallel lines, 2 lines can be selected to form rectangles in $^8C_2$ ways.

Similarly, out of 10 parallel lines (which are perpendicular to the base) can be selected in $^{10}C_2$ ways.

$\therefore$ Total number of rectangles formed

$$= {}^8C_2 \times {}^{10}C_2 = \frac{8.7}{1.2} \times \frac{10.9}{1.2}$$
$$= 1260$$

24. From 10 teacher and 4 students, a committee of 6 is to be formed. In how many ways can this be done?

(a) when the committee contains exactly 2 students.

(b) at least 2 students.

Soln. (a) Since, the committee contains exactly 2 students, 2 students can be selected out of 4 students in $^4C_2$ ways and remaining 4 members can be selected out of 10 teachers in $^{10}C_4$ ways.

$\therefore$ Numbers of ways of forming the committee

$= {}^4C_2 \times {}^{10}C_4$ ways.

(b) Since the committee contains at least 2 students, the number of way of forming the committee is given by –

i) 2 student and 4 teachers $= {}^4C_2 \times {}^{10}C_4 = 1260$

ii) 3 student and 3 teachers $= {}^4C_3 \times {}^{10}C_3 = 480$

iii) 4 student and 2 teachers $= {}^4C_4 \times {}^{10}C_2 = 45$

$\therefore$ Total number of ways of forming the committee

$= 1260 + 480 + 45$

$= 1785$

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