CHAPTER 1
NUMBER SYSTEM

Natural Numbers: 1, 2, 3, ……………………………………

Whole Numbers: 0, 1, 2, 3, ……………………………………

Integers: $\ldots\ldots\ldots\ldots\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\ldots\ldots\ldots\ldots$

- **Rational number**

  It is a number which can be expressed in the form of $\frac{p}{q}$ where p, q are integers and$q \neq 0$.

- Every natural number is a whole number, every whole number is an integer and every integer is a rational number.
- **If a and b are two rational number such that $a<b$, then $\frac{a+b}{2}$ is a rational number lying between a and b.**
- **If $\frac{p}{q}$ and $\frac{r}{s}$ are any two rational numbers and $\frac{p}{q} < \frac{r}{s}$ then $\frac{p}{q} < \frac{p+r}{q+s} < \frac{r}{s}$.**
- **There are an infinite number of rational numbers between two unequal rational numbers.**

SOLUTIONS

EXERCISE 1.1

1. What is a rational number? Is it true that every integer is a rational number?

Ans:- Rational number is a number which can be expressed in the form of $\frac{p}{q}$ where p, q are integers

and $q \neq 0$.

Yes, it is true that every integer is a rational number.

2. If a and b are two unequal rational numbers, show that $\frac{a+b}{2}$ is a rational number lying between a and b.

Soln :- Let us suppose that

$$a < b$$

$\Rightarrow a + b < b + b$ [adding b to both sides]

$\Rightarrow a + b < 2b$

$\Rightarrow \frac{a+b}{2} < \frac{2b}{2}$ [dividing both sides by 2]

$\Rightarrow \frac{a+b}{2} < b$ --------------- (1)

Again, $a < b$

$\Rightarrow a + a < a + b$ [adding a to both sides]

$\Rightarrow 2a < a + b$

$\Rightarrow \frac{2a}{2} < \frac{a+b}{2}$ [dividing both sides by 2]

$\Rightarrow a < \frac{a+b}{2}$ --------------- (2)

Combining (1) and (2), we can write

$a < \frac{a+b}{2} < b$

So, $\frac{a+b}{2}$ is a rational number lying between a and b.

3. Insert four rational numbers between 2 and 3.

Solution :- $2 < 3$

$\Rightarrow 2 < \frac{2+3}{2} < 3$

$\Rightarrow 2 < \frac{5}{2} < 3$

$\Rightarrow 2 < 2.5 < 3$

$\Rightarrow 2 < \frac{2+2.5}{2} < 2.5 < \frac{2.5+3}{2} < 3$

$\Rightarrow 2 < \frac{4.5}{2} < 2.5 < \frac{5.5}{2} < 3$

$\Rightarrow 2 < 2.25 < 2.5 < 2.75 < 3$

$\Rightarrow 2 < \frac{2+2.25}{2} < 2.25 < 2.5 < 2.75 < 3$

$\Rightarrow 2 < \frac{4.25}{2} < 2.25 < 2.5 < 2.75 < 3$

$\Rightarrow 2 < 2.125 < 2.25 < 2.5 < 2.75 < 3$

4. Find five rational numbers between $\frac{1}{4}$ and $\frac{1}{3}$.

Solution:- $\frac{1}{4}<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{1}{2}(\frac{1}{4}+\frac{1}{3})<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{1}{2}(\frac{3+4}{12})<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{7}{24}<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{1}{2}(\frac{1}{4}+\frac{7}{24})<\frac{7}{24}<\frac{1}{2}(\frac{7}{24}+\frac{1}{3})<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{1}{2}(\frac{6+7}{24})<\frac{7}{24}<\frac{1}{2}(\frac{7+8}{24})<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{13}{48}<\frac{7}{24}<\frac{15}{48}<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{1}{2}(\frac{1}{4}+\frac{13}{48})<\frac{13}{48}<\frac{1}{2}(\frac{13}{48}+\frac{7}{24})<\frac{7}{24}<\frac{15}{48}<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{1}{2}(\frac{12+13}{48})<\frac{13}{48}<\frac{1}{2}(\frac{13+14}{48})<\frac{7}{24}<\frac{15}{28}<\frac{1}{3}$

$\Rightarrow \frac{1}{4}<\frac{25}{96}<\frac{13}{48}<\frac{27}{96}<\frac{7}{24}<\frac{15}{48}<\frac{1}{3}$

5. Find six rational numbers between $\frac{1}{3}$ and $\frac{2}{3}$.

Solution:- Using $\frac{p}{q}<\frac{r}{s} \Rightarrow \frac{p}{q}<\frac{p+r}{q+s}<\frac{r}{s}$, we can write

$\frac{1}{3}<\frac{2}{3}$

$\Rightarrow \frac{1}{3}<\frac{1+2}{3+3}<\frac{2}{3}$

$\Rightarrow \frac{1}{3}<\frac{1}{2}<\frac{2}{3}$

$\Rightarrow \frac{1}{3}<\frac{2}{5}<\frac{1}{2}<\frac{3}{5}<\frac{2}{3}$

$\Rightarrow \frac{1}{3}<\frac{3}{8}<\frac{2}{5}<\frac{3}{7}<\frac{1}{2}<\frac{4}{7}<\frac{3}{5}<\frac{5}{8}<\frac{2}{3}$

So, six rational numbers between $\frac{1}{3}$ and $\frac{2}{3}$ are $\frac{3}{8}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{1}{2}$, $\frac{4}{7}$, $\frac{3}{5}$ and $\frac{5}{8}$.

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❖ **Prove that $\sqrt{2}$ is not a rational number.**

Proof :- Let us suppose that $\sqrt{2}$ is a rational number. Then there exists integers p and q such that

$q \neq 0$, p , q are co-prime and $\frac{p}{q} = \sqrt{2}$

$\therefore \left(\frac{p}{q}\right)^2 = (\sqrt{2})^2$ [Squaring both sides]

$\Rightarrow \frac{p^2}{q^2} = 2$

$\Rightarrow \frac{p^2}{q.q} = 2$

$\Rightarrow \frac{p^2}{q} = 2q$ ……………………..(1)

As p and q are co-prime, the left side of (1) is a fraction except for $q = 1$ but the right side is an integer.

If $q = 1$, $\frac{p^2}{1} = 2 \times 1$

$\Rightarrow p^2 = 2$ in which both sides are integers.

It means that the square of p is 2. But there is no integer whose square is 2.[$\because 1^2 < 2 < 2^2$]

So, our supposition that $\sqrt{2}$ is a rational number is contradicted.

Hence $\sqrt{2}$ is not a rational number.

❖ **Dedekind-Cantor Axiom**

"To every real number there corresponds a unique point on the number line and to every point on the number line there corresponds a unique real number."

Irrational numbers:

Irrational numbers are numbers represented on the number line by points other than those representing rational numbers.

Real numbers

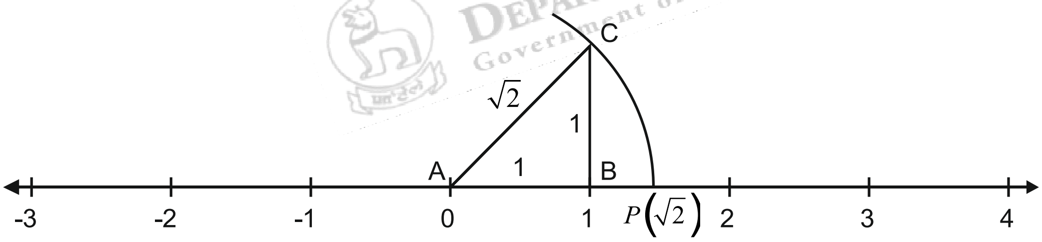
Real numbers are numbers which are either rational or irrational.

SOLUTIONS

EXERCISE 1.2

1. Show that there exist points on the number line not representing rational numbers.

Solution:-



On the number line, the point A represents 0 and B represents 1. We draw BC=1 unit perpendicular to AB and we join AC.

In the rt. △ABC,

$AC^2 = AB^2 + BC^2$ [Pythagoras theorem]

$\Rightarrow AC^2 = 1^2 + 1^2$

$\Rightarrow AC^2 = 1 + 1$

$\Rightarrow AC^2 = 2$

$\Rightarrow AC = \sqrt{2}$

We draw an arc with centre A and radius AC intersecting the number line at P on the right of A. Thus P represents $\sqrt{2}$. But $\sqrt{2}$ is not a rational number. So, there exists points on the number line not representing rational numbers.

2. Prove that there is no rational number whose square is 3.

Solution:- Let if possible x be a rational number such that x^2=3.

Then $x = \sqrt{3}$ is a rational number.

$\Rightarrow$ there exist co-prime integers p and q such that $\sqrt{3} = \frac{p}{q}$.

$\Rightarrow 3q^2 = p^2$ --------------- (1)

$\Rightarrow$ 3 divides p^2

$\Rightarrow$ 3 divides p

$\Rightarrow p = 3p_1$ --------------- (2) where p_1 is some integer.

From (1) and (2),

$3q^2 = 9{p_1}^2$

$\Rightarrow q^2 = 3{p_1}^2$

$\Rightarrow$ 3 divides q

$\Rightarrow q = 3q_1$ --------------- (3) where q_1 is some integer.

From (2) and (3), p and q have a common factor 3.

It contradicts that p and q are co-prime.

So, x is not a rational number.

Hence there is no rational number whose square is 3.

3. **Prove that $\sqrt{5}$ is an irrational number.**

Proof :- Let us suppose that $\sqrt{5}$ is not an irrational number i.e $\sqrt{5}$ is a rational number. Then there exists integers p and q such that $q \neq 0$, p , q are co-prime and $\frac{p}{q}=\sqrt{5}$

$\therefore (\frac{p}{q})^2=(\sqrt{5})^2$ [Squaring both sides]

$\Rightarrow \frac{p^2}{q^2} = 5$

$\Rightarrow \frac{p^2}{q.q} = 5$

$\Rightarrow \frac{p^2}{q} = 5q$ ……………………..(1)

As p and q are co-prime, the left side of (1) is a fraction except for $q = 1$but the right side is an integer.

If $q = 1$, $\frac{p^2}{1}$=5×1

$\Rightarrow p^2$=5 in which both sides are integers.

It means that the square of p is 5.

But there is no integer whose square is 5.[$\because 2^2 < 5 < 3^2$& no other integer between 2 and 3]

So, our supposition that $\sqrt{5}$ is a rational number is contradicted.

Hence $\sqrt{5}$ is not a rational number i.e. $\sqrt{5}$ is an irrational number.

4. **Prove that $\sqrt{7}$ is not a rational number**.

Proof :- Let us suppose that $\sqrt{5}$ is not a rational number. Then there exists integers p and q such that $q \neq 0$, p , q are co-prime and $\frac{p}{q}=\sqrt{7}$

$\therefore (\frac{p}{q})^2=(\sqrt{7})^2$ [Squaring both sides]

$\Rightarrow \frac{p^2}{q^2} = 7$

$\Rightarrow \frac{p^2}{q.q} = 7$

$\Rightarrow \frac{p^2}{q} = 7q$ ……………………..(1)

As p and q are co-prime, the left side of (1) is a fraction except for $q = 1$but the right side is an integer.

If $q = 1$, $\frac{p^2}{1}$=7×1

$\Rightarrow p^2$=7 in which both sides are integers.

It means that the square of p is 7.

But there is no integer whose square is 7. [$\because 2^2 < 7 < 3^2$ and no other integer between 2 and 3]

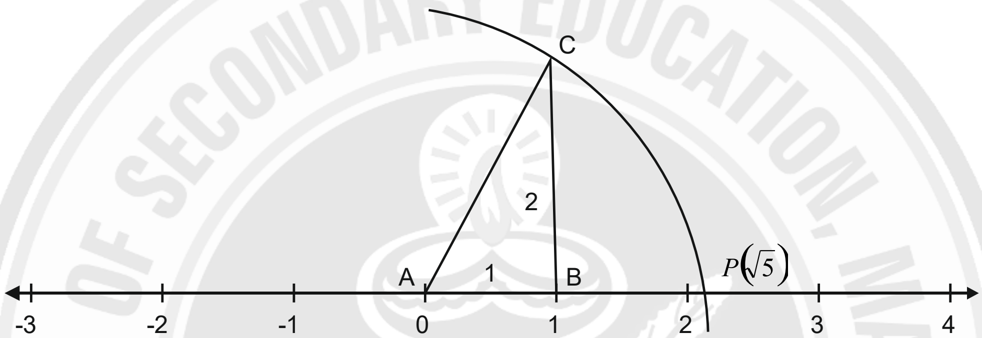
So, our supposition that $\sqrt{7}$ is a rational number is contradicted.

Hence $\sqrt{7}$ is not a rational number.

5. Represent the following number on the number line.

(i) $\sqrt{5}$

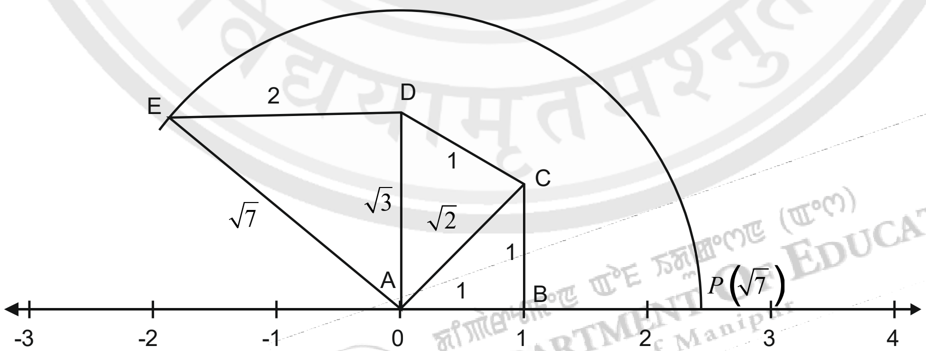
Solution:-



On the number line, A represents 0 and B represents 1. We draw BC=2 units perpendicular to AB and we join AC and AC $= \sqrt{1^2 + 2^2} = \sqrt{5}$. We draw an arc with centre A and radius AC intersecting the number line at P on the right of A. Thus P represents $\sqrt{5}$.

(ii) $\sqrt{7}$

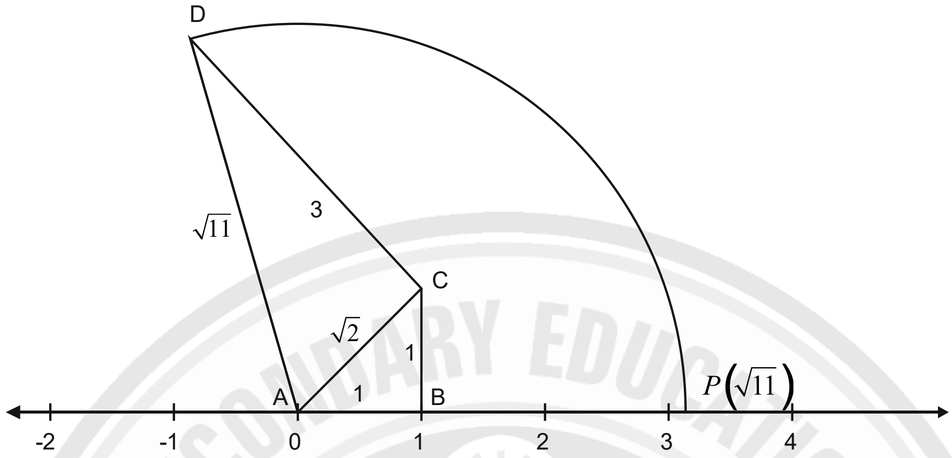
Solution:-



On the number line, A represents 0,B represents 1, we draw BC=1 unit perpendicular to AB and we join AC. We draw CD=1 unit perpendicular to AC and we join AD. We draw DE=2 units perpendicular to AD and we join AE. We draw an arc with centre A and radius $AE = \sqrt{7}$ intersecting the number at P on the right of A. Thus P represents $\sqrt{7}$.

(iii) $\sqrt{11}$

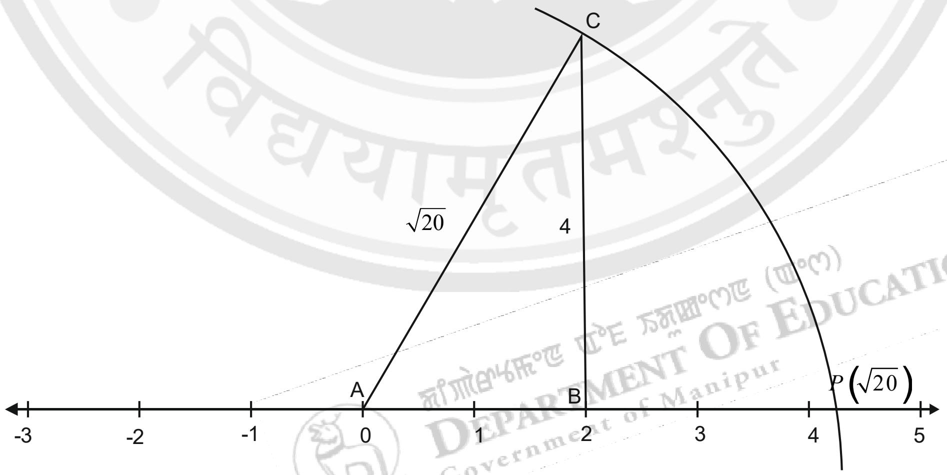
Solution:-



On the number line, A represents 0, B represents 1. We draw BC=1 unit perpendicular to AB and we join AC. We draw CD=3 units perpendicular to AC and we join AD. We draw an arc with centre A and radius AD= $\sqrt{11}$ intersecting the number line at P on the right of A. Thus P represents $\sqrt{11}$.

(iv) $\sqrt{20}$

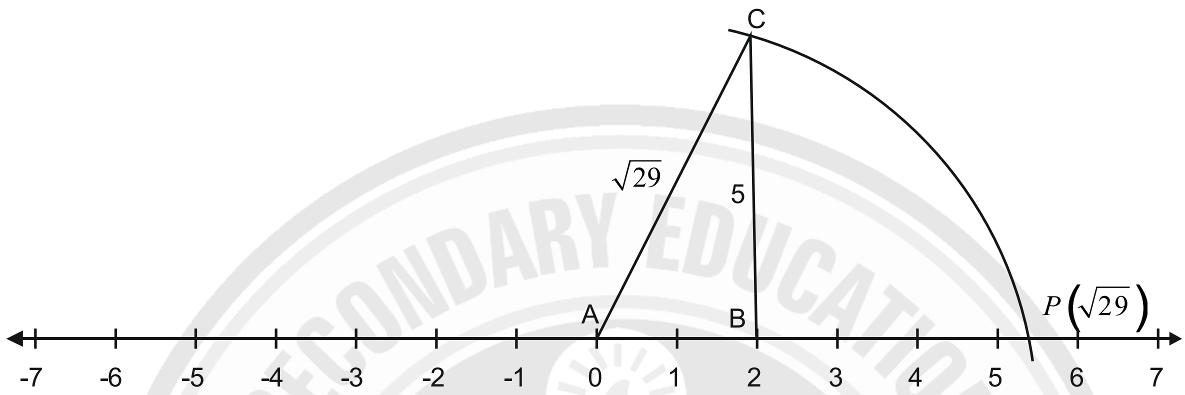
Solution:-



On the number line, A represents 0, B represents 2. We draw BC=4 units perpendicular to AB and we join AC. We draw an arc with centre A and radius AC= $\sqrt{20}$ intersecting the number line at P on the right of A. Thus P represents $\sqrt{20}$.

(v) $\sqrt{29}$

Solution:-

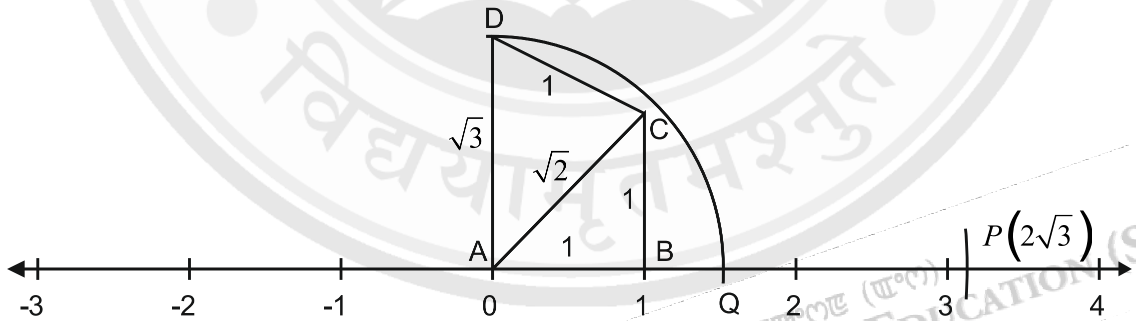


On the number line, A represents 0 and B represents 2. We draw BC=5 units perpendicular to AB and we join AC. We draw an arc with centre A and radius AC= $\sqrt{29}$ intersecting the number line at P on the right of A. Thus P represents $\sqrt{29}$.

6. Locate on the number line the point representing the following number:

(i) $2\sqrt{3}$

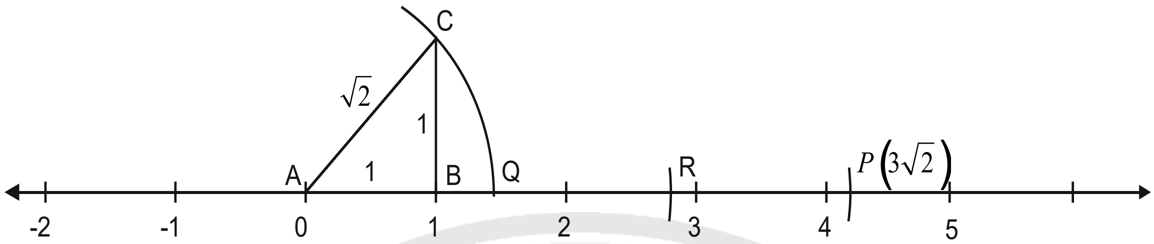
Solution:-



On the number line, A represents 0 and B represents 1. We draw BC=1 unit perpendicular to AB and we join AC. We draw CD=1 unit perpendicular to AC and we join AD. We draw an arc with centre A and radius AD= $\sqrt{3}$ intersecting the number line at Q on the right of A. Again we draw another arc with centre Q and the same radius as before intersecting the number line at P on the right of Q. Thus P represents $2\sqrt{3}$.

(ii) $3\sqrt{2}$

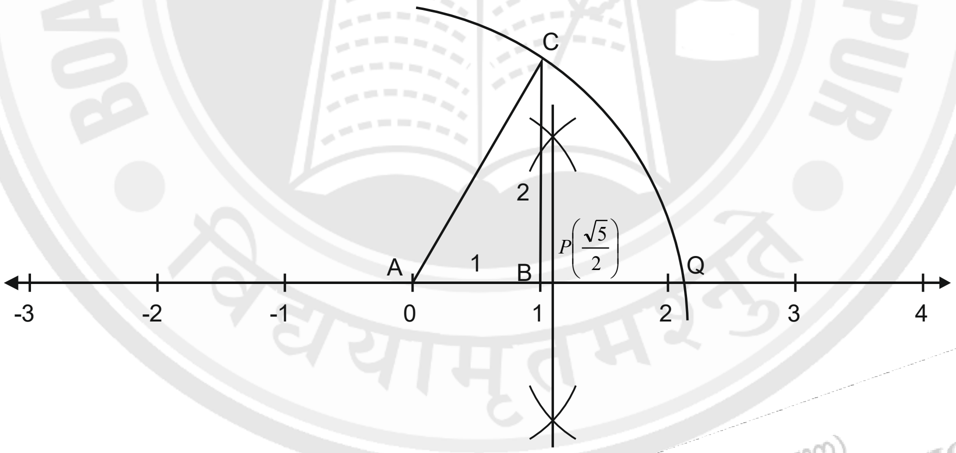
Solution:-



On the number line, A represents 0 and B represents 1. We draw BC=1 unit perpendicular to AB and we join AC. We draw three arcs with centres A,Q,R with AC= $\sqrt{2}$ as radius intersecting theline at Q, R and P respectively on the right of A. Thus P represents $3\sqrt{2}$.

(iii) $\frac{\sqrt{5}}{2}$

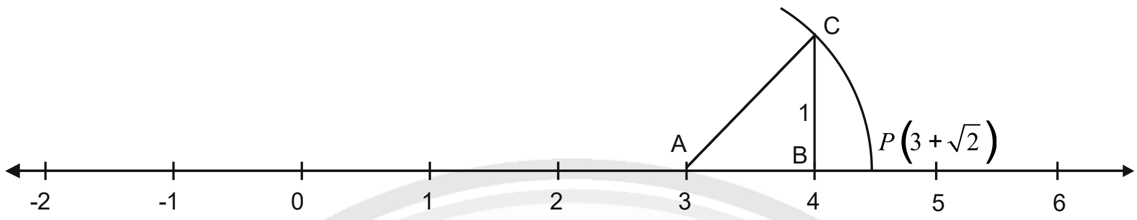
Solution:-



On the number line, A represents 0 and B represents 1. We draw BC=2 units perpendicular to AB and we join AC. We draw an arc with centre A and radius AC= $\sqrt{5}$ intersecting the number line at Q on the right of A. AQ is bisected at P. Thus P represents $\frac{\sqrt{5}}{2}$.

(iv) $\mathbf{3 + \sqrt{2}}$

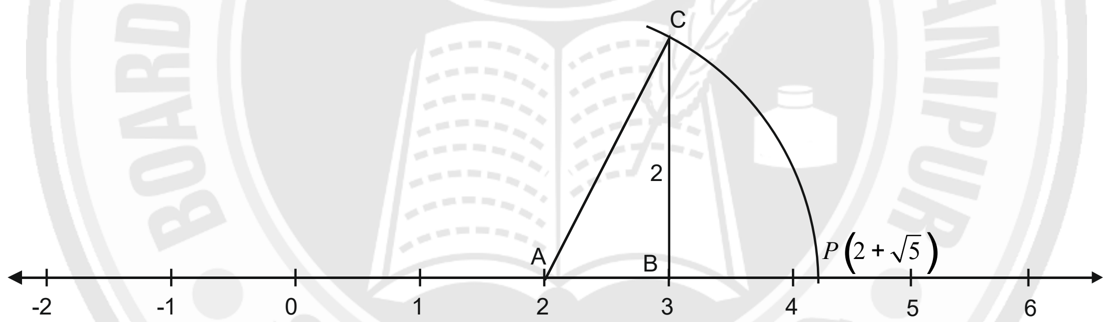
Solution:-



On the number line, A represents 3 and B represents 4. We draw BC=1 unit perpendicular to AB and we join AC.We draw an arc with centre A and radius AC= $\sqrt{2}$ intersecting the number line at P on the right of A. Thus P represents 3 + $\sqrt{2}$

(v) $\mathbf{2 + \sqrt{5}}$

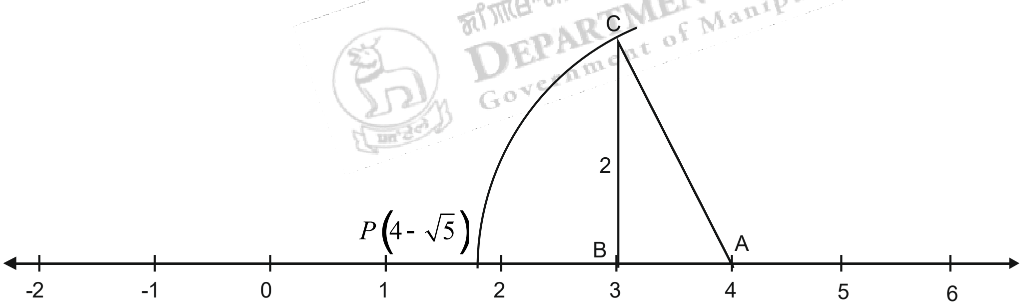
Solution:-



On the number line, A represents 2 and B represents 3. We draw BC=2 unit perpendicular to AB and we join AC. We draw an arc with centre A and radius AC= $\sqrt{5}$ intersecting the number line at P on the right of A. Thus P represents 2 + $\sqrt{5}$

(vi) $\mathbf{4 - \sqrt{5}}$

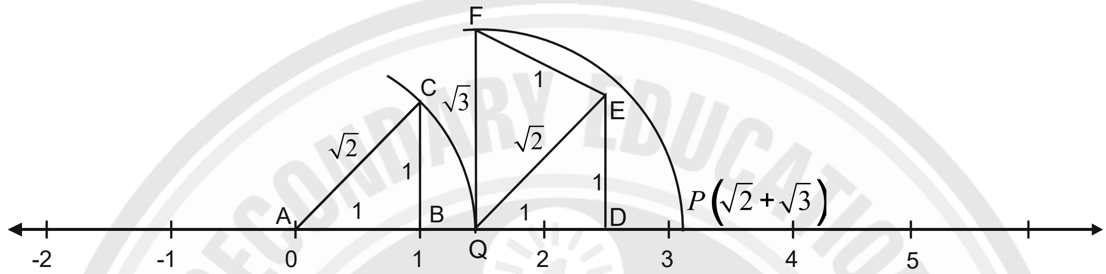
Solution:-



On the number line A represents 4 and B represents 3. We draw BC=2 unit perpendicular to AB and we join AC.We draw and arc with centre A and radius AC= $\sqrt{5}$ intersecting the number line at P on the left of A. Thus P represents $4 - \sqrt{5}$.

vii) $\sqrt{2} + \sqrt{3}$

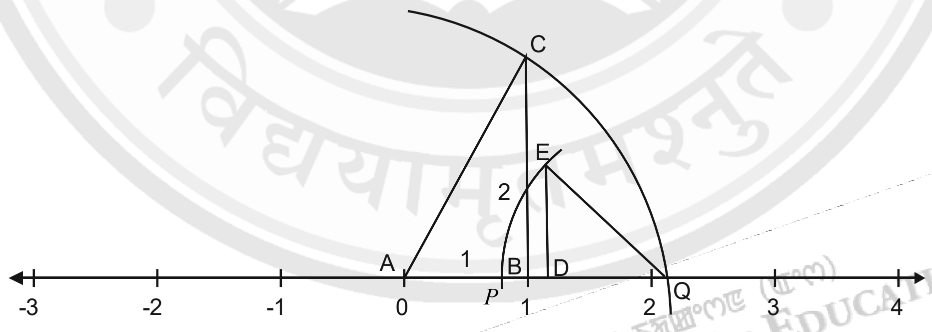
Solution:-



On the number line, Q represents $\sqrt{2}$ and D is taken on the right of Q such that QD=1 unit. We draw DE=1 unit perpendicular to QD and we join QE. We draw EF=1 unit perpendicular to QE and we join QF. We draw an arc with centre Q and radius QF intersecting the number line at P on the right of Q. Thus P represents $\sqrt{2} + \sqrt{3}$.

viii). $\sqrt{5} - \sqrt{2}$

Solution:-



On the number line, Q represents $\sqrt{5}$and we take a point D on the left of Q such that QD=1 unit. We draw DE=1 unit perpendicular to QD and we join QE. We draw an arc with centre Q and radius QE intersecting the number line at P on the left of Q. Thus P represents $\sqrt{5} - \sqrt{2}$.

7. **Write any three rational numbers and any three irrational numbers lying between 2.1 and 2.2**.

Ans:- Three rational numbers between 2.1 and 2.2 are 2.11, 2.12, 2.13.
Three irrational numbers between 2.1 and 2.2 are 2.11010010001…., 2.12020020002…, 2.13030030003…

8. Write any four irrational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Ans :- $\frac{1}{3} = 0.33..$

$\frac{1}{2} = 0.5$

Four irrational numbers lying between $\frac{1}{3}$ and $\frac{1}{2}$ are

0.34040040004……, 0.353353335….., 0.4040040004……, 0.4242242224……...

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❖ **Laws of exponents:-** If m, n are integers and x, y are non-zero rational numbers, then

i) $x^m \times x^n = x^{m+n}$

ii) $\frac{x^m}{x^n} = x^{m-n}$

iii) $(xy)^m = x^m y^m$

iv) $(x^m)^n = x^{mn}$

The above Laws of exponents hold good when bases are positive real numbers and exponents are rational numbers, positive or negative.

❖ **Show that x^0=1 for any non zero rational number x.**

Solution:- We know that

$\frac{x^m}{x^n} = x^{m-n}$

If $m = n$,

$\frac{x^n}{x^n} = x^{n-n}$

$\Rightarrow 1 = x^0 \quad \therefore x^0 = 1$

❖ **Show that $x^{-n} = \frac{1}{x^n}$ for any non –zero rational number x.**

Solution :- We know that,

$\frac{x^m}{x^n} = x^{m-n}$

If $m = 0$,

$\frac{x^0}{x^n} = x^{o-n}$

$\frac{1}{x^n} = x^{-n}$

$\therefore x^{-n} = \frac{1}{x^n}$

❖ **Rationalising factor**

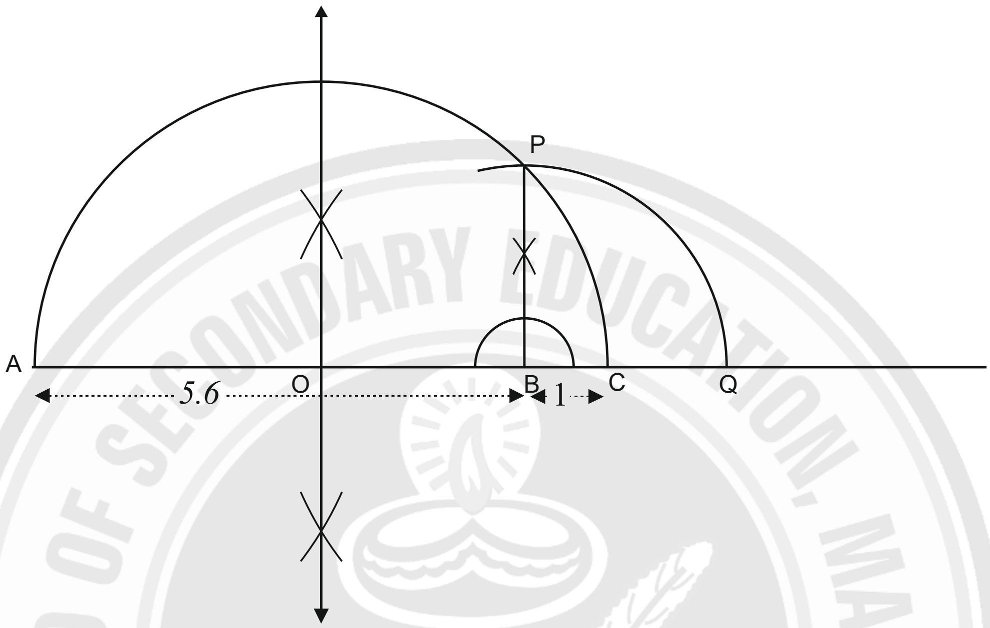
If the product of two irrational numbers is a rational number then each is called a rationalising factor of the other.

SOLUTIONS

EXERCISE 1.3

1. Construct a line segment of length $\sqrt{5.6}$.

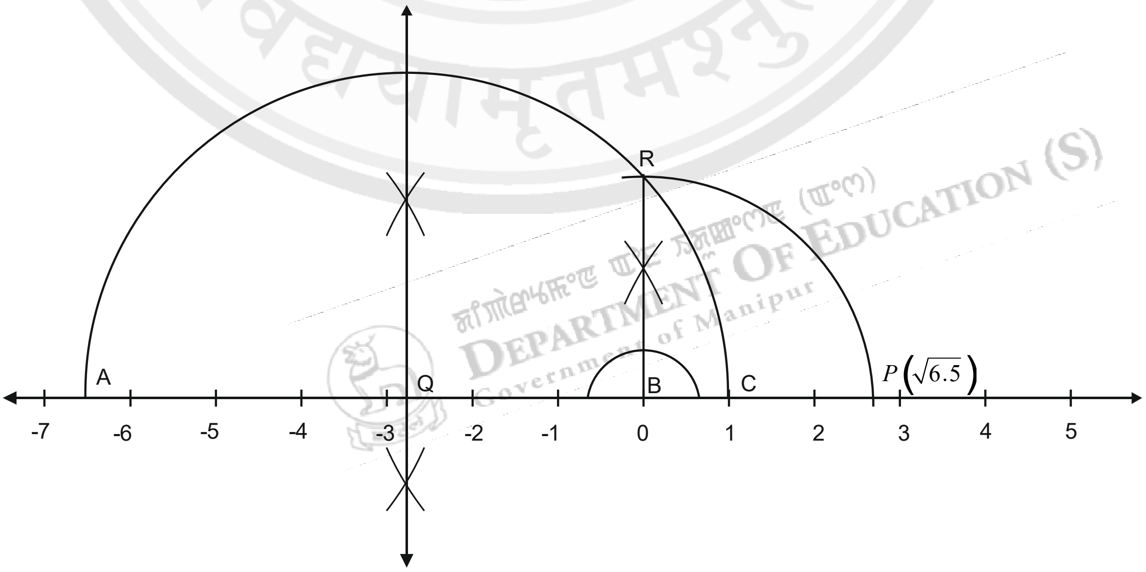
Solution:-



We draw a line segment AB=5.6 units and we produce it to C such that BC=1 unit. We bisect AC at O and we draw a semi-circle with centre O and radius OA. We draw a line perpendicular to AC at B intersecting the semi circle at P. Again an arc is drawn with centre B and radius BP to intersect the BC produced at Q.Thus BQ=$\sqrt{5.6}$.

2. Represent $\sqrt{6.5}$ on the number line.

Solution:-



On the number line, B represents 0 and we take a point A on the left of B such that BA=6.5 units. C represents 1. We bisect AC at Q and we draw a semi circle with centre Q and radius QA. We draw a perpendicular at B with AC intersecting the semi circle at R. We draw an arc with centre B and radius BR intersecting the number line at P on the right of B. Thus P represents $\sqrt{6.5}$.

3. Classify the following numbers as rational or irrational:

(i) $\mathbf{3+\sqrt{2}}$

Ans:- Irrational

(ii) $\mathbf{3-\sqrt{2}}$

Ans :- Irrational

(iii)$\mathbf{2\sqrt{5}}$

Ans :- Irrational

(iv)$\mathbf{\frac{1}{\sqrt{7}}}$

Ans :- Irrational

(v) $\mathbf{(3+\sqrt{3})-\sqrt{3}}$

Ans:- $(3+\sqrt{3})-\sqrt{3}$

$= 3+\sqrt{3}-\sqrt{3}$

$= 3$, which is a rational.

(vi) $(13+\sqrt{7})+(4-\sqrt{7})$

Ans:- $(13+\sqrt{7})+(4-\sqrt{7})$

$= 13+\sqrt{7}+4-\sqrt{7}$

$= 13+4$

$= 17$, which is a rational.

(vii) $\mathbf{\frac{4\sqrt{21}}{5\sqrt{21}}}$

Ans:- $\frac{4\sqrt{21}}{5\sqrt{21}} = \frac{4}{5}$, which is a rational.

(viii) $\mathbf{(\sqrt{2}+\sqrt{3})(\sqrt{3}-\sqrt{2})}$

Ans:- $(\sqrt{2}+\sqrt{3})(\sqrt{3}-\sqrt{2})$

$= (\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$

$= (\sqrt{3})^2-(\sqrt{2})^2$

$= 3-2 = 1$, which is a rational.

4. Simplify:-

(i) $\sqrt{\mathbf{18}} + \sqrt{\mathbf{64}}$

Solution :- $\sqrt{18} + \sqrt{64} = \sqrt{3^2 \times 2} + \sqrt{2^2 \times 2^2 \times 2^2}$

$= 3\sqrt{2} + 2 \times 2 \times 2$

$= 3\sqrt{2} + 8$

$= 8 + 3\sqrt{2}$

(ii) $\sqrt{\mathbf{48}} + \sqrt{\mathbf{12}}$

Solution:- $\sqrt{48} + \sqrt{12} = \sqrt{2^2 \times 2^2 \times 3} + \sqrt{2^2 \times 3}$

$= 2 \times 2\sqrt{3} + 2\sqrt{3}$

$= 4\sqrt{3} + 2\sqrt{3}$

$= 6\sqrt{3}$

(iii) $\sqrt{\mathbf{5}} + \sqrt{\mathbf{80}}$

Solution:- $\sqrt{5} + \sqrt{80} = \sqrt{5} + \sqrt{2^2 \times 2^2 \times 5}$

$= \sqrt{5} + 2 \times 2\sqrt{5}$

$= \sqrt{5} + 4\sqrt{5}$

$= 5\sqrt{5}$

(iv) $\sqrt{\mathbf{50}} - \sqrt{\mathbf{32}}$

Solution:- $\sqrt{50} - \sqrt{32} = \sqrt{5^2 \times 2} - \sqrt{2^2 \times 2^2 \times 2}$

$= 5\sqrt{2} - 2 \times 2\sqrt{2}$

$= 5\sqrt{2} - 4\sqrt{2}$

$= \sqrt{2}$

(v) $\sqrt{\mathbf{363}} - \sqrt{\mathbf{147}}$

Solution:- $\sqrt{363} - \sqrt{147} = \sqrt{11^2 \times 3} - \sqrt{7^2 \times 3}$

$= 11\sqrt{3} - 7\sqrt{3}$

$= 4\sqrt{3}$

(vi) $\mathbf{2\sqrt{45} - 5\sqrt{20} + \sqrt{80}}$

Solution:- $2\sqrt{45} - 5\sqrt{20} + \sqrt{80} = 2\sqrt{3^2 \times 5} - 5\sqrt{2^2 \times 5} + \sqrt{2^2 \times 2^2 \times 5}$

$= 2 \times 3\sqrt{5} - 5 \times 2\sqrt{5} + 2 \times 2\sqrt{5}$

$= 6\sqrt{5} - 10\sqrt{5} + 4\sqrt{5}$

$= 10\sqrt{5} - 10\sqrt{5}$

$= 0$

(vii) $2\sqrt{12}+\sqrt{75}-7\sqrt{3}$

Solution:- $2\sqrt{12}+\sqrt{75}-7\sqrt{3} = 2\sqrt{2^2\times 3}+\sqrt{5^2\times 3}-7\sqrt{3}$

$= 2\times 2\sqrt{3}+5\sqrt{3}-7\sqrt{3}$

$= 4\sqrt{3}+5\sqrt{3}-7\sqrt{3}$

$= 9\sqrt{3}-7\sqrt{3}$

$= 2\sqrt{3}$

(viii) $\sqrt{7}\times\sqrt{14}$

Solution :- $\sqrt{7}\times\sqrt{14} = \sqrt{7\times 14}$

$= \sqrt{7\times 2\times 7}$

$= \sqrt{2\times 7^2} = 7\sqrt{2}$

(ix) $2\sqrt{3}\times 3\sqrt{2}$

Solution:- $2\sqrt{3}\times 3\sqrt{2} = 2\times 3\sqrt{3\times 2} = 6\sqrt{6}$

(x) $8\sqrt{12}\times 3\sqrt{24}$

Solution:- $8\sqrt{12}\times 3\sqrt{24} = 8\sqrt{2^2\times 3}\times 3\sqrt{2^2\times 2\times 3}$

$= 8\times 2\sqrt{3}\times 3\times 2\sqrt{2\times 3}$

$= 8\times 2\times 3\times 2\sqrt{3\times 2\times 3}$

$= 96\sqrt{3^2\times 2}$

$= 96\times 3\sqrt{2}$

$= 288\sqrt{2}$

(xi) $3\sqrt{10}\div 4\sqrt{15}$

Solution :- $3\sqrt{10}\div 4\sqrt{15} = \frac{3\sqrt{10}}{4\sqrt{15}}$

$= \frac{3\sqrt{10}\times\sqrt{15}}{4\sqrt{15}\times\sqrt{15}}$

$= \frac{3\sqrt{10\times 15}}{4\times 15}$

$= \frac{3\sqrt{2\times 5\times 3\times 5}}{4\times 3\times 5}$

$= \frac{\sqrt{2\times 3\times 5^2}}{4\times 5}$

$= \frac{5\sqrt{2\times 3}}{4\times 5}$

$= \frac{\sqrt{6}}{4}$

(xii) $6\sqrt{12} \div 3\sqrt{27}$

Solution:- $6\sqrt{12} \div 3\sqrt{27} = \frac{6\sqrt{12}}{3\sqrt{27}}$

$$= \frac{6\sqrt{12}}{3\sqrt{27}}$$

$$= \frac{2\sqrt{12}}{\sqrt{27}}$$

$$= \frac{2\sqrt{2^2 \times 3}}{\sqrt{3^2 \times 3}}$$

$$= \frac{2 \times 2\sqrt{3}}{3\sqrt{3}}$$

$$= \frac{4}{3}$$

5. Multiply:-

(i) $3\sqrt{5} + 2\sqrt{3}$ **by** $4\sqrt{5} - 5\sqrt{3}$

Solution:- $(3\sqrt{5} + 2\sqrt{3})(4\sqrt{5} - 5\sqrt{3})$

$= 3\sqrt{5} \times 4\sqrt{5} - 3\sqrt{5} \times 5\sqrt{3} + 2\sqrt{3} \times 4\sqrt{5} - 2\sqrt{3} \times 5\sqrt{3}$

$= 12 \times 5 - 15\sqrt{15} + 8\sqrt{15} - 10 \times 3$

$= 60 - 7\sqrt{15} - 30$

$= 30 - 7\sqrt{15}$

(ii) $\sqrt{2} + \sqrt{5} + \sqrt{7}$ **by** $\sqrt{2} + \sqrt{5} - \sqrt{7}$

Solution:- $(\sqrt{2} + \sqrt{5} + \sqrt{7})(\sqrt{2} + \sqrt{5} - \sqrt{7})$

$= \{(\sqrt{2} + \sqrt{5}) + \sqrt{7}\}\{(\sqrt{2} + \sqrt{5}) - \sqrt{7}\}$

$= (\sqrt{2} + \sqrt{5})^2 - \sqrt{7}^2$

$= (\sqrt{2})^2 + 2.\sqrt{2}.\sqrt{5} + (\sqrt{5})^2 - 7$

$= 2 + 2\sqrt{10} + 5 - 7$

$= 2\sqrt{10} + 7 - 7$

$= 2\sqrt{10}$

(iii) $\sqrt{3}-\sqrt{7}+2\sqrt{5}$ **by** $\sqrt{3}+\sqrt{7}-2\sqrt{5}$

Solution:- $(\sqrt{3}-\sqrt{7}+2\sqrt{5})(\sqrt{3}+\sqrt{7}-2\sqrt{5})$

$= \{\sqrt{3}-(\sqrt{7}-2\sqrt{5})\}\{\sqrt{3}+(\sqrt{7}-2\sqrt{5})\}$

$= (\sqrt{3})^2-(\sqrt{7}-2\sqrt{5})^2$

$= 3-\{(\sqrt{7})^2-2.\sqrt{7}.2\sqrt{5}+(2\sqrt{5})^2\}$

$= 3-(7-4\sqrt{35}+4\times 5)$

$= 3-7+4\sqrt{35}-20$

$= 3-27+4\sqrt{35}$

$= -24+4\sqrt{35}$

6. Express the following avoiding fractional or negative exponents:-

(i) $3^{\frac{2}{5}}$

Solution:- $3^{\frac{2}{5}} = \sqrt[5]{3^2}$

(ii) $5^{-\frac{3}{4}}$

Solution:- $5^{-\frac{3}{4}} = \frac{1}{5^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{5^3}}$

(iii) $\frac{5}{2^{-\frac{4}{3}}}$

Solution:- $\frac{5}{2^{-\frac{4}{3}}} = 5\times 2^{\frac{4}{3}} = 5\sqrt[3]{2^4}$

(iv) $3^{-\frac{2}{3}}\times 3^{-\frac{1}{2}}$

Solution:- $3^{-\frac{2}{3}}\times 3^{-\frac{1}{2}} = 3^{-\frac{2}{3}+\left(-\frac{1}{2}\right)}$

$= 3^{-\frac{2}{3}-\frac{1}{2}} = 3^{\frac{-4-3}{6}}$

$= 3^{-\frac{7}{6}} = \frac{1}{3^{\frac{7}{6}}} = \frac{1}{\sqrt[6]{3^7}}$

(v) $5^{-\frac{2}{3}}\div 2\sqrt{5^{-3}}$

Solution:- $\frac{5^{-\frac{2}{3}}}{2\sqrt{5^{-3}}} = \frac{1}{2}\times\frac{5^{-\frac{2}{3}}}{5^{-\frac{3}{2}}}$

$= \frac{1}{2}\times 5^{-\frac{2}{3}-(-\frac{3}{2})}$

$= \frac{1}{2}\times 5^{-\frac{2}{3}+\frac{3}{2}}$

$= \frac{1}{2} \times 5^{\frac{-4+9}{6}}$

$= \frac{1}{2} \times 5^{\frac{5}{6}}$

$= \frac{1}{2} \times \sqrt[6]{5^5}$

7. Express the following avoiding radical signs and negative exponents:-

(i) $\left(\sqrt{3}\right)^5$

Solution:- $\left(\sqrt{3}\right)^5 = 3^{\frac{5}{2}}$

(ii) $\left(\sqrt[3]{5}\right)^{-4}$

Solution:- $\left(\sqrt[3]{5}\right)^{-4} = 5^{-\frac{4}{3}} = \frac{1}{5^{\frac{4}{3}}}$

(iii) $\frac{1}{\sqrt[5]{7^{-3}}}$

Solution:- $\frac{1}{\sqrt[5]{7^{-3}}} = \frac{1}{7^{-\frac{3}{5}}} = 7^{\frac{3}{5}}$

(iv) $\sqrt[3]{2^4} \times \left(\sqrt[6]{2}\right)^{-2}$

Solution:- $\sqrt[3]{2^4} \times \left(\sqrt[6]{2}\right)^{-2} = 2^{\frac{4}{3}} \times 2^{\frac{-2}{6}}$

$= 2^{\frac{4}{3}} \times 2^{\frac{-1}{3}}$

$= 2^{\frac{4}{3}-\frac{1}{3}}$

$= 2^{\frac{4-1}{3}}$

$= 2$

(v) $\sqrt[4]{3^{-5}} \div \left(\sqrt[6]{3}\right)^{-9}$

Solution:- $\sqrt[4]{3^{-5}} \div \left(\sqrt[6]{3}\right)^{-9} = 3^{-\frac{5}{4}} \div 3^{-\frac{9}{6}}$

$= 3^{\frac{-5}{4}-\left(\frac{-9}{6}\right)}$

$= 3^{\frac{-5}{4}+\frac{3}{2}}$

$= 3^{\frac{-5+6}{4}}$

$= 3^{\frac{1}{4}}$

8. Find the value of

(i) $\mathbf{16^{-\frac{3}{4}}}$

Solution:- $16^{-\frac{3}{4}} = (2^4)^{-\frac{3}{4}}$

$= 2^{4\times\left(-\frac{3}{4}\right)}$

$= 2^{-3}$

$= \frac{1}{2^3}$

$= \frac{1}{8}$

(ii) $\mathbf{\sqrt[5]{32^3}}$

Solution:-$\sqrt[5]{32^3} = 32^{\frac{3}{5}}$

$= (2^5)^{\frac{3}{5}}$

$= 2^{5\times\frac{3}{5}}$

$= 2^3$

$= 8$

(iii) $\mathbf{\sqrt{9^5}}$

Solution:-$\sqrt{9^5} = 9^{\frac{5}{2}}$

$= (3^2)^{\frac{5}{2}}$

$= 3^{2\times\frac{5}{2}}$

$= 3^5$

$= 243$

(iv) $\mathbf{125^{-\frac{2}{3}}}$

Solution:- $125^{-\frac{2}{3}} = (5^3)^{-\frac{2}{3}}$

$= 5^{3\times\left(\frac{-2}{3}\right)}$

$= 5^{-2}$

$= \frac{1}{5^2}$

$= \frac{1}{25}$

(v) $\left(\frac{1}{343}\right)^{-\frac{2}{3}}$

Solution:- $\left(\frac{1}{343}\right)^{-\frac{2}{3}} = \left(\frac{1}{7^3}\right)^{-\frac{2}{3}}$

$= \left\{\left(\frac{1}{7}\right)^3\right\}^{-\frac{2}{3}}$

$= \left(\frac{1}{7}\right)^{3\times\left(\frac{-2}{3}\right)}$

$= \left(\frac{1}{7}\right)^{-2}$

$= 7^2 = 49$

9. Express with rational denominator.

(i) $\frac{\sqrt{3}+1}{4+3\sqrt{3}}$

Solution :- $\frac{\sqrt{3}+1}{4+3\sqrt{3}} = \frac{(\sqrt{3}+1)(4-3\sqrt{3})}{(4+3\sqrt{3})(4-3\sqrt{3})}$

$= \frac{4\sqrt{3}-9+4-3\sqrt{3}}{4^2-(3\sqrt{3})^2}$

$= \frac{\sqrt{3}-5}{16-27}$

$= \frac{\sqrt{3}-5}{-11}$

$= \frac{5-\sqrt{3}}{11}$

(ii) $\frac{3+4\sqrt{2}}{5-3\sqrt{2}}$

Solution:- $\frac{3+4\sqrt{2}}{5-3\sqrt{2}} = \frac{(3+4\sqrt{2})(5+3\sqrt{2})}{(5-3\sqrt{2})(5+3\sqrt{2})}$

$= \frac{15+9\sqrt{2}+20\sqrt{2}+24}{5^2-(3\sqrt{2})^2}$

$= \frac{39+29\sqrt{2}}{25-18}$

$= \frac{39+29\sqrt{2}}{7}$

(iii) $\frac{\sqrt{5}+2\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

Solution:- $\frac{\sqrt{5}+2\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}+2\sqrt{3})(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

$= \frac{5-\sqrt{15}+2\sqrt{15}-6}{(\sqrt{5})^2-(\sqrt{3})^2}$

$= \frac{-1+\sqrt{15}}{5-3}$

$= \frac{-1+\sqrt{15}}{2}$

(iv) $\frac{\sqrt{5}+\sqrt{3}}{4-\sqrt{15}}$

Solution:- $\frac{\sqrt{5}+\sqrt{3}}{4-\sqrt{15}} = \frac{(\sqrt{5}+\sqrt{3})(4+\sqrt{15})}{(4-\sqrt{15})(4+\sqrt{15})}$

$= \frac{4\sqrt{5}+\sqrt{75}+4\sqrt{3}+\sqrt{45}}{4^2-(\sqrt{15})^2}$

$= \frac{4\sqrt{5}+5\sqrt{3}+4\sqrt{3}+3\sqrt{5}}{16-15}$

$= 7\sqrt{5}+9\sqrt{3}$

(v) $\frac{1}{1+\sqrt{6}-\sqrt{7}}$

Solution:- $\frac{1}{1+\sqrt{6}-\sqrt{7}} = \frac{1\{(1+\sqrt{6})+\sqrt{7}\}}{\{(1+\sqrt{6})-\sqrt{7}\}\{(1+\sqrt{6})+\sqrt{7}\}}$

$= \frac{1+\sqrt{6}+\sqrt{7}}{(1+\sqrt{6})^2-\sqrt{7}^2}$

$= \frac{1+\sqrt{6}+\sqrt{7}}{1^2+2.1.\sqrt{6}+\sqrt{6}^2-7}$

$= \frac{1+\sqrt{6}+\sqrt{7}}{1+2\sqrt{6}+6-7}$

$= \frac{(1+\sqrt{6}+\sqrt{7})\sqrt{6}}{2\sqrt{6}\times\sqrt{6}}$

$= \frac{\sqrt{6}+6+\sqrt{42}}{2\times 6}$

$= \frac{6+\sqrt{6}+\sqrt{42}}{12}$

(vi) $\frac{4}{1+\sqrt{2}+\sqrt{3}}$

Solution:- $\frac{4}{1+\sqrt{2}+\sqrt{3}} = \frac{4\{(1+\sqrt{2})-\sqrt{3}\}}{\{(1+\sqrt{2})+\sqrt{3}\}\{(1+\sqrt{2})-\sqrt{3}\}}$

$= \frac{4(1+\sqrt{2}-\sqrt{3})}{(1+\sqrt{2})^2-(\sqrt{3})^2}$

$= \frac{4(1+\sqrt{2}-\sqrt{3})}{1^2+2.1.\sqrt{2}+\sqrt{2}^2-3}$

$= \frac{4(1+\sqrt{2}-\sqrt{3})}{1+2\sqrt{2}+2-3}$

$= \frac{4(1+\sqrt{2}-\sqrt{3})}{3+2\sqrt{2}-3}$

$= \frac{4(1+\sqrt{2}-\sqrt{3})}{2\sqrt{2}}$

$= \frac{2(1+\sqrt{2}-\sqrt{3})}{\sqrt{2}}$

$= \frac{2(1+\sqrt{2}-\sqrt{3})\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$

$$= \frac{2(\sqrt{2}+2-\sqrt{6})}{2}$$

$$= 2 + \sqrt{2} - \sqrt{6}$$

(vii) $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$

Solution:- $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = \frac{1\,\{(\sqrt{2}+\sqrt{3})-\sqrt{5}\}}{\{(\sqrt{2}+\sqrt{3})+\sqrt{5}\}\{(\sqrt{2}+\sqrt{3})-\sqrt{5}\}}$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-\sqrt{5}^2}$$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2})^2+2.\sqrt{2}.\sqrt{3}+(\sqrt{3})^2-5}$$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+2\sqrt{6}+3-5}$$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{5+2\sqrt{6}-5}$$

$$= \frac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}}$$

$$= \frac{(\sqrt{2}+\sqrt{3}-\sqrt{5})\times\sqrt{6}}{2\sqrt{6}\times\sqrt{6}}$$

$$= \frac{(\sqrt{12}+\sqrt{18}-\sqrt{30}}{2\times 6}$$

$$= \frac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$$

10. Simplify :-

(i) $\frac{\sqrt{18}+2\sqrt{27}}{\sqrt{75}-\sqrt{48}-\sqrt{32}+\sqrt{50}}$

Solution:- $\frac{\sqrt{18}+2\sqrt{27}}{\sqrt{75}-\sqrt{48}-\sqrt{32}+\sqrt{50}} = \frac{3\sqrt{2}+3\sqrt{3}}{5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}}$

$$= \frac{3(\sqrt{2}+\sqrt{3})}{\sqrt{2}+\sqrt{3}}$$

$$= 3$$

(ii) $\frac{\sqrt{18}}{\sqrt{3}+\sqrt{6}} - \frac{\sqrt{48}}{\sqrt{2}+\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

Solution:- $\frac{\sqrt{18}}{\sqrt{3}+\sqrt{6}} - \frac{\sqrt{48}}{\sqrt{2}+\sqrt{6}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}} = \frac{3\sqrt{2}(\sqrt{3}-\sqrt{6})}{(\sqrt{3}+\sqrt{6})(\sqrt{3}-\sqrt{6})} - \frac{4\sqrt{3}(\sqrt{2}-\sqrt{6})}{(\sqrt{2}+\sqrt{6})(\sqrt{2}-\sqrt{6})} + \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$

$$= \frac{3\sqrt{2}(\sqrt{3}-\sqrt{6})}{(\sqrt{3})^2-(\sqrt{6})^2} - \frac{4\sqrt{3}(\sqrt{2}-\sqrt{6})}{(\sqrt{2})^2-(\sqrt{6})^2} + \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{3\sqrt{2}(\sqrt{3}-\sqrt{6})}{3-6} - \frac{4\sqrt{3}(\sqrt{2}-\sqrt{6})}{2-6} + \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{2-3}$$

$$= \frac{3\sqrt{2}(\sqrt{3}-\sqrt{6})}{-3} - \frac{4\sqrt{3}(\sqrt{2}-\sqrt{6})}{-4} + \frac{\sqrt{6}(\sqrt{2}-\sqrt{3})}{-1}$$

$$= \frac{3\sqrt{2}(\sqrt{6}-\sqrt{3})}{3} - \frac{4\sqrt{3}(\sqrt{6}-\sqrt{2})}{4} + \sqrt{6}(\sqrt{3}-\sqrt{2})$$

$$= \sqrt{2}(\sqrt{6}-\sqrt{3}) - \sqrt{3}(\sqrt{6}-\sqrt{2}) + \sqrt{6}(\sqrt{3}-\sqrt{2})$$

$$= \sqrt{12} - \sqrt{6} - \sqrt{18} + \sqrt{6} + \sqrt{18} - \sqrt{12}$$

$$= 0$$

(iii) $\mathbf{\frac{1}{\sqrt{5}-\sqrt{3}} - \frac{1}{\sqrt{5}+\sqrt{3}}}$

Solution:- $\frac{1}{\sqrt{5}-\sqrt{3}} - \frac{1}{\sqrt{5}+\sqrt{3}} = \frac{(\sqrt{5}+\sqrt{3})-(\sqrt{5}-\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$

$$= \frac{\sqrt{5}+\sqrt{3}-\sqrt{5}+\sqrt{3}}{\sqrt{5}^2-\sqrt{3}^2}$$

$$= \frac{2\sqrt{3}}{5-3}$$

$$= \frac{2\sqrt{3}}{2}$$

$$= \sqrt{3}$$

(iv) $\mathbf{\frac{2}{4+3\sqrt{2}} + \frac{7}{3-\sqrt{2}} - \frac{31}{1+4\sqrt{2}}}$

Solution:- $\frac{2}{4+3\sqrt{2}} + \frac{7}{3-\sqrt{2}} - \frac{31}{1+4\sqrt{2}} = \frac{2(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})} + \frac{7(3+\sqrt{2})}{(3-\sqrt{2})(3-\sqrt{2})} - \frac{31(1-4\sqrt{2})}{(1+4\sqrt{2})(1-4\sqrt{2})}$

$$= \frac{2(4-3\sqrt{2})}{4^2-(3\sqrt{2})^2} + \frac{7(3+\sqrt{2})}{3^2-\sqrt{2}^2} - \frac{31(1-4\sqrt{2})}{1^2-(4\sqrt{2})^2}$$

$$= \frac{2(4-3\sqrt{2})}{16-18} + \frac{7(3+\sqrt{2})}{9-2} - \frac{31(1-4\sqrt{2})}{1-32}$$

$$= \frac{2(4-3\sqrt{2})}{-2} + \frac{7(3+\sqrt{2})}{7} - \frac{31(1-4\sqrt{2})}{-31}$$

$$= -(4-3\sqrt{2}) + (3+\sqrt{2}) + (1-4\sqrt{2})$$

$$= -4 + 3\sqrt{2} + 3 + \sqrt{2} + 1 - 4\sqrt{2}$$

$$= 4 - 4 + 4\sqrt{2} - 4\sqrt{2}$$

$$= 0$$

(v) $\mathbf{\frac{1}{1+\sqrt{2}-\sqrt{3}} - \frac{1}{1+\sqrt{2}+\sqrt{3}}}$

Solution:- $\frac{1}{1+\sqrt{2}-\sqrt{3}} - \frac{1}{1+\sqrt{2}+\sqrt{3}} = \frac{\{(1+\sqrt{2})+\sqrt{3}\}-\{(1+\sqrt{2})-\sqrt{3}\}}{\{(1+\sqrt{2})-\sqrt{3}\}\{(1+\sqrt{2})+\sqrt{3}\}}$

$$= \frac{1+\sqrt{2}+\sqrt{3}-1-\sqrt{2}+\sqrt{3}}{(1+\sqrt{2})^2-(\sqrt{3})^2}$$

$$= \frac{2\sqrt{3}}{1+2\sqrt{2}+\sqrt{2}^2-3}$$

$$= \frac{2\sqrt{3}}{1+2\sqrt{2}+2-3}$$

$$= \frac{2\sqrt{3}}{3+2\sqrt{2}-3}$$

$$= \frac{2\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}$$

$$= \frac{\sqrt{3}\times\sqrt{2}}{\sqrt{2}^{2}}$$

$$= \frac{\sqrt{6}}{2}$$

(vi) $\frac{11}{1-\sqrt{3}+\sqrt{5}} - \frac{11}{1+\sqrt{3}+\sqrt{5}}$

Solution:- $\frac{11}{1-\sqrt{3}+\sqrt{5}} - \frac{11}{1+\sqrt{3}+\sqrt{5}} = \frac{11(1+\sqrt{3}+\sqrt{5})-11(1-\sqrt{3}+\sqrt{5})}{(1-\sqrt{3}+\sqrt{5})(1+\sqrt{3}+\sqrt{5})}$

$$= \frac{11\{(1+\sqrt{3}+\sqrt{5})-(1-\sqrt{3}+\sqrt{5})\}}{(1+\sqrt{5}-\sqrt{3})(1+\sqrt{5}+\sqrt{3})}$$

$$= \frac{11(1+\sqrt{3}+\sqrt{5}-1+\sqrt{3}-\sqrt{5})}{(1+\sqrt{5})^{2}-\sqrt{3}^{2}}$$

$$= \frac{11\times2\sqrt{3}}{1+2\sqrt{5}+\sqrt{5}^{2}-3}$$

$$= \frac{11\times2\sqrt{3}}{1+2\sqrt{5}+5-3}$$

$$= \frac{11\times2\sqrt{3}}{2\sqrt{5}+3}$$

$$= \frac{11\times2\sqrt{3}(2\sqrt{5}-3)}{(2\sqrt{5}+3)(2\sqrt{5}-3)}$$

$$= \frac{11\times2(2\sqrt{15}-3\sqrt{3})}{(2\sqrt{5})^{2}-3^{2}}$$

$$= \frac{11\times2(2\sqrt{15}-3\sqrt{3})}{4\times5-9}$$

$$= \frac{11\times2(2\sqrt{15}-3\sqrt{3})}{11}$$

$$= 4\sqrt{15} - 6\sqrt{3}$$

(vii) $3\sqrt{35} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} - \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$

Solution:- $3\sqrt{35} + \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} - \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}} = 3\sqrt{35} + \frac{(\sqrt{7}-\sqrt{5})(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})} - \frac{(\sqrt{7}+\sqrt{5})(\sqrt{7}+\sqrt{5})}{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})}$

$$=3\sqrt{35} + \frac{(\sqrt{7}-\sqrt{5})^{2}}{(\sqrt{7})^{2}-(\sqrt{5})^{2}} - \frac{(\sqrt{7}+\sqrt{5})^{2}}{(\sqrt{7})^{2}-(\sqrt{5})^{2}}$$

$$=3\sqrt{35} + \frac{(\sqrt{7})^{2}-2.\sqrt{7}.\sqrt{5}+(\sqrt{5})^{2}}{7-5} - \frac{(\sqrt{7})^{2}+2.\sqrt{7}.\sqrt{5}+(\sqrt{5})^{2}}{7-5}$$

$$=3\sqrt{35} + \frac{7-2\sqrt{35}+5}{2} - \frac{7+2\sqrt{35}+5}{2}$$

$=3\sqrt{35}+\frac{12-2\sqrt{35}}{2}-\frac{12+2\sqrt{35}}{2}$

$=3\sqrt{35}+\frac{2(6-\sqrt{35})}{2}-\frac{2(6+\sqrt{35})}{2}$

$=3\sqrt{35}+(6-\sqrt{35})-(6+\sqrt{35})$

$=3\sqrt{35}+6-\sqrt{35}-6-\sqrt{35}$

$=3\sqrt{35}-2\sqrt{35}$

$=\sqrt{35}$

11. Show that the following are rational.

(i) $\frac{(\sqrt{3}-\sqrt{2})(5+2\sqrt{6})}{\sqrt{3}+\sqrt{2}}$

Solution:- $\frac{(\sqrt{3}-\sqrt{2})(5+2\sqrt{6})}{\sqrt{3}+\sqrt{2}}=\frac{(\sqrt{3}-\sqrt{2})(5+2\sqrt{6})(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$=\frac{(\sqrt{3}-\sqrt{2})^2(5+2\sqrt{6})}{(\sqrt{3})^2-(\sqrt{2})^2}$

$=\frac{\{(\sqrt{3})^2-2.\sqrt{3}.\sqrt{2}+(\sqrt{2})^2\}(5+2\sqrt{6})}{3-2}$

$=\frac{(3-2.\sqrt{6}+2)(5+2\sqrt{6})}{1}$

$=(5-2\sqrt{6})\,(5+2\sqrt{6})$

$=5^2-(2\sqrt{6})^2$

$=25-4\times 6$

$=25-24$

$=1$ which is a rational.

(ii) $\frac{5-\sqrt{5}}{3+\sqrt{5}}+2\sqrt{5}$

Solution:- $\frac{5-\sqrt{5}}{3+\sqrt{5}}+2\sqrt{5}=\frac{(5-\sqrt{5})(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}+2\sqrt{5}$

$=\frac{15-5\sqrt{5}-3\sqrt{5}+5}{3^2-(\sqrt{5})^2}+2\sqrt{5}$

$=\frac{20-8\sqrt{5}}{9-5}+2\sqrt{5}$

$=\frac{4(5-2\sqrt{5})}{4}+2\sqrt{5}$

$=5-2\sqrt{5}+2\sqrt{5}$

$=5$ which is a rational.

(iii) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}}$

Solution:- $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} = \frac{1-\sqrt{2}}{(1+\sqrt{2})(1-\sqrt{2})} + \frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} + \frac{\sqrt{3}-\sqrt{4}}{(\sqrt{3}+\sqrt{4})(\sqrt{3}-\sqrt{4})}$

$= \frac{1-\sqrt{2}}{1^2-\sqrt{2}^2} + \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2} + \frac{\sqrt{3}-\sqrt{4}}{\sqrt{3}^2-\sqrt{4}^2}$

$= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \frac{\sqrt{3}-\sqrt{4}}{3-4}$

$= \frac{1-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} + \frac{\sqrt{3}-\sqrt{4}}{-1}$

$= -(1-\sqrt{2}) - (\sqrt{2}-\sqrt{3}) - (\sqrt{3}-\sqrt{4})$

$= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + \sqrt{4}$

$= -1 + \sqrt{2}^2$

$= -1 + 2$

$= 1$ which is a rational.

(iv) $x^2 - xy + y^2$ **where** $x = \frac{\sqrt{2}-1}{\sqrt{2}+1}$ **and** $y = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

Solution:- $x = \frac{\sqrt{2}-1}{\sqrt{2}+1}$

$= \frac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}$

$= \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2 - 1^2}$

$= \frac{(\sqrt{2})^2 - 2.\sqrt{2}.1 + 1^2}{2-1}$

$= \frac{2-2\sqrt{2}+1}{1}$

$= 3 - 2\sqrt{2}$

$y = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$= \frac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2 - 1^2}$

$= \frac{(\sqrt{2})^2 + 2.\sqrt{2}.1 + 1^2}{2-1}$

$= \frac{2+2.\sqrt{2}+1}{2-1}$

$= \frac{2+2.\sqrt{2}+1}{1}$

$= 3 + 2\sqrt{2}$

$$x^2 - xy + y^2 = \left(3 - 2\sqrt{2}\right)^2 - (3 - 2\sqrt{2})(3 + 2\sqrt{2}) + \left(3 + 2\sqrt{2}\right)^2$$

$$= 3^2 - 2.3.\,2\sqrt{2} + \left(2\sqrt{2}\right)^2 - \{3^2 - \left(2\sqrt{2}\right)^2\} + 3^2 + 2.3.2\sqrt{2} + \left(2\sqrt{2}\right)^2$$

$$= \left(9 - 12\sqrt{2} + 8\right) - (9 - 8) + \left(9 + 12\sqrt{2} + 8\right)$$

$$= 17 - 12\sqrt{2} - 1 + 17 + 12\sqrt{2}$$

$=34 - 1$

$=33$ which is a rational.

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