CLASS – IX
MATHEMATICS
CHAPTER – 2
POLYNOMIALS

- **Polynomial:** An algebraic expression involving only non-negative integral powers of a variable is called a polynomial in the variable. Example: $4y^2 - 3y + 2$.
- **Monomial:** A polynomial having only one term is called a monomial. Example: $2x^2, 5x, 3$ etc.
- **Binomial:** A polynomial having only two terms is called a binomial. Example: $x + 2, x^2 - x, y^2 - 9$ etc.
- **Trinomial:** A polynomial having only three terms is called a trinomial. Example: $x^2 - 5x + 6, x^4 + x^2 + 1$ etc.
- **Degree of a polynomial:** The exponent of the variable in a term of a polynomial represents the degree of that term and the highest of the degrees of the term is called the degree of the polynomial.
  Example: In the polynomial, $5x^6 - 3x^4 + 4x^2 + x$, degree = 6
- **General form of a polynomial**

  The most general form of a polynomial of degree n in a single variable x is

$$a_0 + a_1x + a_2x^2 + \dots\dots\dots\dots\dots + a_nx^n$$
$$\text{or } a_nx^n + a_{n-1}x^{n-1} + \dots\dots\dots\dots + a_1x + a_0,$$

where $a_0, a_1, a_2, \dots\dots\dots\dots\dots\dots, a_n$ are constants and $a_n \neq 0$.

Zero Polynomial: A polynomial in which all the coefficients are zero is called a Zero Polynomial and is denoted by 0.

Notes:

(i) Degree of a zero polynomial is not defined.

(ii) A non-zero constant is a polynomial of degree zero.

- **Standard form of a polynomial:** A polynomial is said to be in the standard form when its terms are arranged in ascending order descending powers of the variable.
- **Monic polynomial:** A polynomial in which the coefficient of the highest degree term is 1 is called a Monic polynomial. Example: $x^3 + x^2 + 1, x^4 - 3x^3 + x - 2$ etc.

Note: Monic polynomial of degree zero is 1.

Some special names of polynomials

- **Linear polynomial**: A polynomial of degree one is called a linear polynomial. Example: $x,\ x+3, 4-3x$ etc.
- **Quadratic polynomial**: A polynomial of degree two is called quadratic polynomial. Example: $2x^2+5, x^2+5x-2$ etc.
- **Cubic polynomial**: A polynomial of degree three is called cubic polynomial. Example: $5x^3-1, y^3-5y+2$
- **Biquadratic or quartic polynomial**: A polynomial of degree four is called quadratic polynomial. Example: $5x^4+4x^3+4, 2x^4-3x^3+4x+10$ etc.

SOLUTIONS

EXERCISE 2.1

1. **Examine which of the following expressions are polynomials and which are not. State reasons for your answer.**

i) $\mathbf{4x^2+5x-2}$

Answer: $4x^2+5x-2$

It is a polynomial because the exponents of the variable are non-negative integers.

ii) $\mathbf{x^2-\frac{1}{x}}$

Answer: $x^2-\frac{1}{x}= x^2-x^{-1}$

It is not a polynomial because the exponent of the second term is -1, which is not a non-negative integer.

iii) $\mathbf{3\sqrt{y}+y\sqrt{2}}$

Answer: $3\sqrt{y}+y\sqrt{2} = 3y^{\frac{1}{2}}+y\sqrt{2}$

It is not a polynomial because there is a fractional power $\frac{1}{2}$ of the variable y.

iv) $x^{-3}+4x^{-2}+x^{\frac{1}{3}}+x$

Answer: $x^{-3}+4x^{-2}+x^{\frac{1}{3}}+x$

It is not a polynomial because there are negative and fractional exponents of the variable x.

v) $\sqrt{3}t^5 - \sqrt{2}t^4 + t^3$

Answer: It is a polynomial because the exponents of the variable t are non-negative integers.

vi) $x^3 + 3x^2 + 3x + 1$

Answer: It is a polynomial because the exponents of the variable are non-negative integers.

vii) $x^3 + \frac{1}{x^{-2}} - 10$

Answer: $x^3 + \frac{1}{x^{-2}} - 10 = x^3 + x^2 - 10$

It is a polynomial because the exponents of the variable are non-negative integers.

2. Write the following polynomial in standard form:

i) $x^4 - 5x^6 + x^2 + x^3 + 4$

Solution: $x^4 - 5x^6 + x^2 + x^3 + 4$

$= -5x^6 + x^4 + x^3 + x^2 + 4$, which is in the standard form.

ii) $x^4 + x^2 + \sqrt{2}x^5 - 3x - x^3 + \sqrt{2}$

Solution: $x^4 + x^2 + \sqrt{2}x^5 - 3x - x^3 + \sqrt{2}$

$= \sqrt{2}x^5 + x^4 - x^3 + x^2 - 3x + \sqrt{2}$, which is in the standard form.

iii) $(x - 3)^2 + (x + 2)^3$

Solution: $(x - 3)^2 + (x + 2)^3$

$= x^2 - 2.x.3 + 3^2 + x^3 + 3.x^2.2 + 3.x.2^2 + 2^3$

$= x^2 - 6x + 9 + x^3 + 6x^2 + 12x + 8$

$= x^3 + 7x^2 + 6x + 17$, which is in the standard form.

iv) $(x + 2)(x^2 - 7x + 3)$

Solution: $(x + 2)(x^2 - 7x + 3)$

$= x^3 - 7x^2 + 3x + 2x^2 - 14x + 6$

$= x^3 - 5x^2 - 11x + 6$, which is in the standard form.

v) $(x^2 - 1)^3$

Solution: $(x^2 - 1)^3$

$= (x^2)^3 - 3.(x^2)^2.1 + 3.x^2.1^2 - 1^3$

$= x^6 - 3x^4 + 3x^2 - 1$, which is in the standard form.

3. Write the coefficients of x^2 in each of the following polynomials:

i) $2x + x^2 - 3$

Solution: The coefficient of x^2 is 1.

ii) $2 - x^2 + 3x^3$

Solution: The coefficient of x^2 is -1.

iii) $\frac{1}{2}x^2 + x$

Solution: The coefficient of x^2 is $\frac{1}{2}$.

iv) $x^3 - x + 5$

Solution: $x^3 - x + 5 = x^3 + 0.x^2 - x + 5$

The coefficient of x^2 is 0.

v) $\sqrt{3}x + 1$

Solution: $\sqrt{3}x + 1 = 0.x^2 + \sqrt{3}x + 1$

The coefficient of x^2 is 0.

4. Find the degree of each of the following polynomials.

(i) $x^4 + x^6 + x^2 + 2^7$

Solution: Degree = 6

(ii) $5x^3 + 2x^2 + 3x$

Solution: Degree = 3

(iii) $3 - y^2$

Solution: Degree = 2

(iv) $2x - 7$

Solution: Degree = 1

(v) $3x^2 - 2x + 1$

Solution: Degree = 2

(vi) $x - 6x^2 + 2 + 3x^4$

Solution: Degree = 4

(vii) $x^5 + 10^6$

Solution: Degree = 5

(viii) $(y + 3)(y^3 + y - 2)$

Solution: $(y+3)(y^3+y-2) = y^4 + y^2 - 2y + 3y^3 + 3y - 6$

$= y^4 + 3y^3 + y^2 + y - 6$

Degree = 4

5. Examine, which of the following polynomials are monic or not:

(i) $x^5 + 4x^3 - 3x + 2$

Solution: Coefficient of the highest degree term = 1.

So, the polynomial is monic.

(ii) $1 + 2x + 3x^2 - x^3$

Solution: Coefficient of the highest degree term = $-1 \neq 1$.

So, the polynomial is not a monic.

(iii) $\sqrt{3}x^4 + \sqrt{5}x^2 + \sqrt{7}$

Solution: Coefficient of the highest degree term = $\sqrt{3} \neq 1$.

So, the polynomial is not monic.

(iv) $(y - 3)(y^2 + y + 1)$

Solution: $(y - 3)(y^2 + y + 1) = y^3 + y^2 + y - 3y^2 - 3y - 3$

$= y^3 - 2y^2 - 2y - 3$

Coefficient of the highest degree term = 1.

So, the polynomial is monic.

(v) $2 + \frac{1}{3}x + \frac{2}{5}x^2$

Solution: Coefficient of the highest degree term = $\frac{2}{5} \neq 1$.

So, the polynomial is not monic.

(vi) 1

Solution: $1 = 1.x^0$

Coefficient of the highest degree term = 1.

So, the polynomial is monic.

6. Classify the following as linear, quadratic, cubic and biquadratic polynomials:

(i) $x - x^2$

Ans: Quadratic

(ii) $7x^3$

Ans: Cubic

(iii) $t + t^2 - 3$

Ans: Quadratic

(iv) $2 + 3x$

Ans: Linear

(v) $\boldsymbol{y^4}$

Ans: Bi-quadratic (Quartic)

(vi) $\boldsymbol{3z}$

Ans: Linear

(vii) $\boldsymbol{x - x^3}$

Ans: Cubic

(viii) $\boldsymbol{x + x^2 - x^4 + 3x^3 - 2}$

Ans: Bi-quadratic (Quartic)

7. Show that the product of two monic polynomials is also a monic polynomial.

Solution: Let us consider two monic polynomials $x^2 + 2$ and $x^3 - x + 4$.

Now, $(x^2 + 2)(x^3 - x + 4) = x^5 - x^3 + 4x^2 + 2x^3 - 2x + 8$

$= x^5 + x^3 + 4x^2 - 2x + 8$, which is monic.

Thus, the product of two monic polynomials is also a monic polynomial.

8. Show with suitable example, that the sum of two monic polynomials need not be monic.

Solution: Let us consider two monic polynomials $x^3 + 2x^2 - 7$ and $x^2 + 3x + 4$.

Now $(x^3 + 2x^2 - 7) + (x^2 + 3x + 4) = x^3 + 2x^2 - 7 + x^2 + 3x + 4$

$= x^3 + 3x^2 + 3x - 3$, which is a monic.

Again, let us consider two monic polynomials $x^3 + x^2 - x + 4$ and $x^3 - 4x + 2$.

Now, $(x^3 + x^2 - x + 4) + (x^3 - 4x + 2) = x^3 + x^2 - x + 4 + x^3 - 4x + 2$

$= 2x^3 + x^2 - 5x + 6$, which is not a monic.

Thus, the sum of two monic polynomials need not be monic.

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- **Zero of a polynomial:** A real number 'c' is called a zero a polynomial $p(x)$ if $p(c) = 0$.
- Zero of a polynomial $p(x)$ is obtained by equating it to 0 and solving the resulting equation.
- If c is a zero of the polynomial $p(x)$, then c is called a root of the equation $p(x) = 0$.
- A non-zero constant polynomial has no zero.
- Every linear polynomial in one variable has a unique zero.
- 0 (zero) may be a zero of a polynomial.
- A polynomial can have more than one zeros.

SOLUTIONS

EXERCISE 2.2

1. Find the value of the polynomial $3x^2 - 5x + 4$ at :

i) $x = -1$

Solution: We have, $p(x) = 3x^2 - 5x + 4$

The value of the polynomial $p(x)$ at $x = -1$ is given by

$$p(-1) = 3 \times (-1)^2 - 5 \times (-1) + 4$$
$$= 3 \times 1 + 5 + 4$$
$$= 3 + 5 + 4$$
$$= 12$$

ii) $x = 0$

Solution: We have, $p(x) = 3x^2 - 5x + 4$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = 3 \times 0^2 - 5 \times 0 + 4$$
$$= 0 - 0 + 4$$
$$= 4$$

iii) $x = 2$

Solution: We have, $p(x) = 3x^2 - 5x + 4$

The value of the polynomial $p(x)$ at $x = 2$ is given by

$$p(2) = 3 \times 2^2 - 5 \times 2 + 4$$
$$= 12 - 10 + 4$$
$$= 16 - 10$$
$$= 6$$

iv) $x = \frac{3}{2}$

Solution: We have, $p(x) = 3x^2 - 5x + 4$

The value of the polynomial $p(x)$ at $x = \frac{3}{2}$ is given by

$$p\left(\frac{3}{2}\right) = 3 \times \left(\frac{3}{2}\right)^2 - 5 \times \left(\frac{3}{2}\right) + 4$$
$$= 3 \times \frac{9}{4} - 5 \times \frac{3}{2} + 4$$
$$= \frac{27}{4} - \frac{15}{2} + 4$$

$= \frac{27-30+16}{4}$

$= \frac{43-30}{4}$

$= \frac{13}{4}$

2. Find the value of the polynomial $4x - x^3 + 2x^2 - 5$ at

(i) $x = 1$

Solution: We have, $p(x) = 4x - x^3 + 2x^2 - 5$

The value of the polynomial $p(x)$ at $x = 1$ is given by

$p(1) = 4 \times 1 - 1^3 + 2 \times 1^2 - 5$

$= 4 - 1 + 2 - 5$

$= 6 - 6$

$= 0$

(ii) $x = -2$

Solution: We have, $p(x) = 4x - x^3 + 2x^2 - 5$

The value of the polynomial $p(x)$ at $x = -2$ is given by

$p(-2) = 4 \times (-2) - (-2)^3 + 2 \times (-2)^2 - 5$

$= -8 - (-8) + 2 \times 4 - 5$

$= -8 + 8 + 8 - 5$

$= 3$

(iii) $x = \frac{2}{3}$

Solution: We have, $p(x) = 4x - x^3 + 2x^2 - 5$

The value of the polynomial $p(x)$ at $x = \frac{2}{3}$ is given by

$p\left(\frac{2}{3}\right) = 4 \times \frac{2}{3} - \left(\frac{2}{3}\right)^3 + 2 \times \left(\frac{2}{3}\right)^2 - 5$

$= \frac{8}{3} - \frac{8}{27} + \frac{8}{9} - 5$

$= \frac{72-8+24-135}{27}$

$= \frac{96-143}{27}$

$= \frac{-47}{27}$

(iv) $x = 5$

Solution: We have, $p(x) = 4x - x^3 + 2x^2 - 5$

The value of the polynomial $p(x)$ at $x = 5$ is given by

$$p(5) = 4 \times 5 - 5^3 + 2 \times 5^2 - 5$$
$$= 20 - 125 + 50 - 5$$
$$= 70 - 130$$
$$= -60$$

3. Find the value of each of the following polynomials at the points $x = 0, -3$ and $\frac{1}{2}$:

(i) $2x^2 + x + 1$

Solution: We have, $p(x) = 2x^2 + x + 1$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = 2 \times 0^2 + 0 + 1$$
$$= 0 + 0 + 1$$
$$= 1$$

The value of the polynomial $p(x)$ at $x = -3$ is given by

$$p(-3) = 2 \times (-3)^2 - 3 + 1$$
$$= 2 \times 9 - 3 + 1$$
$$= 18 - 3 + 1$$
$$= 16$$

The value of the polynomial $p(x)$ at $x = \frac{1}{2}$ is given by

$$p\left(\frac{1}{2}\right) = 2 \times \left(\frac{1}{2}\right)^2 + \frac{1}{2} + 1$$
$$= 2 \times \frac{1}{4} + \frac{1}{2} + 1$$
$$= \frac{1}{2} + \frac{1}{2} + 1$$
$$= 2$$

(ii) $\boldsymbol{x^3 - 2x^2 + 3x - 4}$

Solution: We have, $p(x) = \boldsymbol{x^3 - 2x^2 + 3x - 4}$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = \boldsymbol{0^3 - 2 \times 0^2 + 3 \times 0 - 4}$$
$$= -4$$

The value of the polynomial $p(x)$ at $x = -3$ is given by

$$p(-3) = (-3)^3 - 2 \times (-3)^2 + 3 \times (-3) - 4$$
$$= -27 - 2 \times 9 - 9 - 4$$
$$= -27 - 18 - 9 - 4$$
$$= -58$$

The value of the polynomial $p(x)$ at $x = \frac{1}{2}$ is given by

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{2}\right) - 4$$

$$= \frac{1}{8} - 2 \times \frac{1}{4} + \frac{3}{2} - 4$$

$$= \frac{1}{8} - \frac{1}{2} + \frac{3}{2} - 4$$

$$= \frac{1-4+12-32}{8}$$

$$= \frac{13-36}{8}$$

$$= \frac{-23}{8}$$

(iii) $\mathbf{5 - 6x + x^2}$

Solution: We have, $p(x) = 5 - \mathbf{6x} + \boldsymbol{x^2}$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = \mathbf{5} - \mathbf{6} \times 0 + \mathbf{0^2} = 5$$

The value of the polynomial $p(x)$ at $x = -3$ is given by

$$p(-3) = \mathbf{5} - \mathbf{6} \times (-3) + (-\mathbf{3})^2$$

$$= \mathbf{5} + \mathbf{18} + 9$$

$$= \mathbf{32}$$

The value of the polynomial $p(x)$ at $x = \frac{1}{2}$ is given by

$$p\left(\frac{1}{2}\right) = \mathbf{5} - \mathbf{6} \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

$$= \mathbf{5} - \mathbf{3} + \frac{1}{4}$$

$$= \mathbf{2} + \frac{1}{4}$$

$$= \frac{9}{4}$$

(iv) $\mathbf{3x + 2}$

Solution: We have, $p(x) = \mathbf{3x + 2}$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = \mathbf{3} \times 0 + 2 = 0 + 2$$

$$= 2$$

The value of the polynomial $p(x)$ at $x = -3$ is given by

$$p(-3) = \mathbf{3} \times (-3) + 2$$

$$= -9 + 2$$

$$= -7$$

The value of the polynomial $p(x)$ at $x = \frac{1}{2}$ is given by

$$p\left(\frac{1}{2}\right) = 3 \times \frac{1}{2} + 2$$
$$= \frac{3}{2} + 2$$
$$= \frac{3+4}{2}$$
$$= \frac{7}{2}$$

(v) $\boldsymbol{x\,(x+1)(\,x-3)}$

Solution: We have, $p(x) = x(x+1)(x-3)$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = 0(0+1)(0-3)$$
$$= 0$$

The value of the polynomial $p(x)$ at $x = -3$ is given by

$$p(-3) = -3(-3+1)(-3-3)$$
$$= -3 \times (-2) \times (-6)$$
$$= -36$$

The value of the polynomial $p(x)$ at $x = \frac{1}{2}$ is given by

$$p\left(\frac{1}{2}\right) = \frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}-3\right)$$
$$= \frac{1}{2} \times \frac{3}{2} \times \left(-\frac{5}{2}\right)$$
$$= -\frac{15}{8}$$

(vi) $\boldsymbol{(2x-1)(\,x+3)}$

Solution: We have, $p(x) = (2x-1)(x+3)$

The value of the polynomial $p(x)$ at $x = 0$ is given by

$$p(0) = (2 \times 0 - 1)(0+3)$$
$$= -1 \times 3$$
$$= -3$$

The value of the polynomial $p(x)$ at $x = -3$ is given by

$$p(-3) = \{2 \times (-3) - 1\}(-3+3)$$
$$= -7 \times 0$$
$$= 0$$

The value of the polynomial $p(x)$ at $x = \frac{1}{2}$ is given by

$$p\left(\frac{1}{2}\right) = \left(2 \times \frac{1}{2} - 1\right)\left(\frac{1}{2} + 3\right)$$
$$= (1 - 1) \times \frac{7}{2}$$
$$= 0 \times \frac{7}{2}$$
$$= 0$$

4. Find $p(1), p(-1)$ and $p(2)$ in each case:

(i) $p(x) = 5x + 1$

Solution: $p(x) = 5x + 1$

$$p(1) = 5 \times 1 + 1 = 5 + 1 = 6$$
$$p(-1) = 5 \times (-1) + 1 = -5 + 1 = -4$$
$$p(2) = 5 \times 2 + 1 = 10 + 1 = 11$$

(ii) $p(y) = y^2 - y + 1$

Solution: $p(y) = y^2 - y + 1$

$$p(1) = 1^2 - 1 + 1 = 1 - 1 + 1 = 1$$
$$p(-1) = (-1)^2 - (-1) + 1 = 1 + 1 + 1 = 3$$
$$p(2) = 2^2 - 2 + 1 = 4 - 2 + 1 = 3$$

(iii) $p(z) = 1 + 2z + 3z^2 - z^3$

Solution: $p(z) = 1 + 2z + 3z^2 - z^3$

$$p(1) = 1 + 2 \times 1 + 3 \times 1^2 - 1^3$$
$$= 1 + 2 + 3 - 1$$
$$= 5$$
$$p(-1) = 1 + 2 \times (-1) + 3 \times (-1)^2 - (-1)^3$$
$$= 1 - 2 + 3 \times 1 - (-1)$$
$$= 1 - 2 + 3 + 1$$
$$= 5 - 2$$
$$= 3$$
$$p(2) = 1 + 2 \times 2 + 3 \times 2^2 - 2^3$$
$$= 1 + 4 + 12 - 8$$
$$= 17 - 8$$
$$= 9$$

(iv) $\boldsymbol{p(t) = (t+1)(t+2)(t+3)}$

Solution: $p(t) = (t+1)(t+2)(t+3)$

$p(1) = (1+1)(1+2)(1+3)$

$= 2 \times 3 \times 4$

$= 24$

$p(-1) = (-1+1)(-1+2)(-1+3)$

$= 0 \times 1 \times 2$

$= 0$

$p(2) = (2+1)(2+2)(2+3)$

$= 3 \times 4 \times 5$

$= 60$

(v) $\boldsymbol{p(s) = s^3 + 2s - 3}$

Solution: $p(s) = s^3 + 2s - 3$

$p(1) = 1^3 + 2 \times 1 - 3$

$= 1 + 2 - 3$

$= 3 - 3$

$= 0$

$p(-1) = (-1)^3 + 2 \times (-1) - 3$

$= -1 - 2 - 3$

$= -6$

$p(2) = 2^3 + 2 \times 2 - 3$

$= 8 + 4 - 3$

$= 12 - 3$

$= 9$

(vi) $p(u) = 2u(u-1)(u+5)$

Solution: $p(u) = 2u(u-1)(u+5)$

$p(1) = 2 \times 1(1-1)(1+5)$

$= 2 \times 0 \times 6$

$= 0$

$p(-1) = 2 \times (-1)(-1-1)(-1+5)$

$= -2 \times (-2) \times 4$

$= 16$

$p(2) = 2 \times 2(2 - 1)(2 + 5)$

$= 4 \times 1 \times 7$

$= 28$

5. Check whether the following are zeros of the polynomials indicated against them:

(i) $\boldsymbol{x = \frac{1}{2}, 2x - 1}$

Solution: Let $p(x) = 2x - 1$

Then, $p\left(\frac{1}{2}\right) = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$

$\therefore x = \frac{1}{2}$ is zero of the polynomial $p(x) = 2x - 1$.

(ii) $\boldsymbol{x = 0;\ x^3}$

Solution: Let $p(x) = x^3$

Then, $p(0) = 0^3 = 0$

$\therefore x = 0$ is zero of the polynomial $p(x) = x^3$.

(iii) $\boldsymbol{x = 2, -2;\ x^2 - 4}$

Solution: Let $p(x) = x^2 - 4$

$p(2) = 2^2 - 4 = 4 - 4 = 0$

$\therefore x = 2$ is zero of the polynomial $p(x) = x^2 - 4$.

And $p(-2) = (-2)^2 - 4 = 4 - 4 = 0$

$\therefore x = -2$ is also zero of the polynomial $p(x) = x^2 - 4$.

(iv) $\boldsymbol{x = 1, 2;\ x^2 - x - 2}$

Solution: Let $p(x) = x^2 - x - 2$

$p(1) = 1^2 - 1 - 2 = 1 - 1 - 2 = -2 \neq 0$

$\therefore x = 1$ is not zero of the polynomial $p(x) = x^2 - x - 2$.

And $p(2) = 2^2 - 2 - 2 = 4 - 4 = 0$

$\therefore x = 2$ is zero of the polynomial $p(x) = x^2 - x - 2$.

(v) $\boldsymbol{x = 1, -2, 3;\ (x - 1)(x - 2)(x - 3)}$

Solution: Let $p(x) = (x - 1)(x - 2)(x - 3)$

$p(1) = (1 - 1)(1 - 2)(1 - 3) = 0 \times (-1) \times (-2) = 0$

$\therefore x = 1$ is zero of the polynomial $p(x) = (x - 1)(x - 2)(x - 3)$.

$p(-2) = (-2 - 1)(-2 - 2)(-2 - 3) = (-3) \times (-4) \times (-5) = -60 \neq 0$

$\therefore x = -2$ is not zero of the polynomial $p(x) = (x - 1)(x - 2)(x - 3)$.

And $p(3) = (3 - 1)(3 - 2)(3 - 3) = 2 \times 1 \times 0 = 0$

$\therefore x = 3$ is zero of the polynomial $p(x) = (x - 1)(x - 2)(x - 3)$.

(vi) $\boldsymbol{x = \frac{2}{\sqrt{3}}, -\frac{1}{\sqrt{3}}; 3x^2 - 1}$

Solution: Let $p(x) = 3x^2 - 1$

$$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{4}{3} - 1 = 4 - 1 = 3 \neq 0$$

$\therefore x = \frac{2}{\sqrt{3}}$ is not zero of the polynomial $p(x) = 3x^2 - 1$.

$$p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3 \times \frac{1}{3} - 1 = 1 - 1 = 0$$

$\therefore x = -\frac{1}{\sqrt{3}}$ is zero of the polynomial $p(x) = 3x^2 - 1$.

(vii) $\boldsymbol{x = \sqrt{5}, -\sqrt{5};\ x^2 + 5}$

Solution: Let $p(x) = x^2 + 5$

$$p(\sqrt{5}) = \sqrt{5}^2 + 5 = 5 + 5 = 10 \neq 0$$

$\therefore x = \sqrt{5}$ is not zero of the polynomial $p(x) = x^2 + 5$.

$$p(-\sqrt{5}) = \left(-\sqrt{5}\right)^2 + 5 = 5 + 5 = 10 \neq 0$$

$\therefore x = -\sqrt{5}$ is not zero of the polynomial $p(x) = x^2 + 5$.

(viii) $\boldsymbol{x = \frac{1}{3}, 2; 3x^2 - 5x - 2}$

Solution: Let $p(x) = 3x^2 - 5x - 2$

$$p\left(\frac{1}{3}\right) = 3 \times \left(\frac{1}{3}\right)^2 - 5 \times \frac{1}{3} - 2$$
$$= 3 \times \frac{1}{9} - \frac{5}{3} - 2$$
$$= \frac{1}{3} - \frac{5}{3} - 2$$
$$= \frac{1-5-6}{3}$$
$$= \frac{-10}{3} \neq 0$$

$\therefore x = \frac{1}{3}$ is not zero of the polynomial $p(x) = 3x^2 - 5x - 2$.

$$p(2) = 3 \times (2)^2 - 5 \times 2 - 2 = 12 - 10 - 2 = 0$$

$\therefore x = 2$ is zero of the polynomial $p(x) = 3x^2 - 5x - 2$.

6. Find the zero of each of the following polynomials:

(i) $\boldsymbol{x + 7}$

Solution: The zero of the polynomial is given by

$$x + 7 = 0$$
$$\Rightarrow x = -7$$

$\therefore -7$ is the zero of the polynomial $x + 7$.

(ii) $5x - 2$

Solution: The zero of the polynomial is given by

$$5x - 2 = 0$$
$$\Rightarrow 5x = 2$$
$$\Rightarrow x = \frac{2}{5}$$

$\therefore \frac{2}{5}$ is the zero of the polynomial $5x - 2$.

(iii) $2x$

Solution: The zero of the polynomial is given by

$$2x = 0$$
$$\Rightarrow x = \frac{0}{2}$$
$$\Rightarrow x = 0$$

$\therefore$ 0 is the zero of the polynomial $2x$.

(iv) $2x + 5$

Solution: The zero of the polynomial is given by

$$2x + 5 = 0$$
$$\Rightarrow 2x = -5$$
$$\Rightarrow x = \frac{-5}{2}$$

$\therefore \frac{-5}{2}$ is the zero of the polynomial $2x + 5$.

(v) $ax, (a \neq 0)$

Solution: The zero of the polynomial is given by

$$ax = 0$$
$$\Rightarrow x = \frac{0}{a}$$
$$\Rightarrow x = 0$$

$\therefore$ 0 is the zero of the polynomial $ax, (a \neq 0)$.

(vi) $cx + d, (c$ and d are constants, $c \neq 0)$

Solution: The zero of the polynomial is given by

$$cx + d = 0$$
$$\Rightarrow cx = -d$$
$$\Rightarrow x = \frac{-d}{c}$$

$\therefore \frac{-d}{c}$ is the zero of the polynomial $cx + d$.

****

- **Factorisation**: The process of expressing a given polynomial as the product of its prime factors is called factorisation.
- **Factorisation of $x^2 + bx + c$, where $a \neq 0$, by splitting the middle term**

Let $px + q$ and $rx + s$ be the factors of $ax^2 + bx + c$.

Then $ax^2 + bx + c = (px + q)(rx + s)$

$= prx^2 + (ps + qr)x + qs$

Comparing the coefficients of like terms, we get

$a = pr, b = ps + qr$ and $c = qs$

We see that b is the sum of two numbers ps and qrwhose product is

$(ps)(qr) = (pr)(qs) = ac$

In particular, to factorise $ax^2 + bx + c$ where a, b, c are integers and $a \neq 0$, we have to write b as the sum of two numbers whose product is $a\,c$.

- **Some algebraic identities**

i) $(\boldsymbol{a} + \boldsymbol{b})^2 = a^2 + 2ab + b^2$

ii) $(\boldsymbol{a} - \boldsymbol{b})^2 = a^2 - 2ab + b^2$

iii) $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b}) = a^2 - b^2$

iv) $(\boldsymbol{a} + \boldsymbol{b})^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + b^3 + 3ab\,(a + b)$

v) $(\boldsymbol{a} - \boldsymbol{b})^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$= a^3 - b^3 - 3ab\,(a - b)$

vi) $\boldsymbol{a}^3 + \boldsymbol{a}^3 = (a + b)(\,a^2 - ab + \,a^2)$

vii) $\boldsymbol{a}^3 - \boldsymbol{a}^3 = (a - b)(a^2 + ab + a^2)$

SOLUTIONS

EXERCISE 2.3

1. Factorise the following by splitting the middle term:

i) $x^2 + 3x + 2$

Solution: $x^2 + 3x + 2 = x^2 + (2 + 1)x + 2$

$= x^2 + 2x + x + 2$

$= x(x + 2) + 1(x + 2)$

$= (x + 2)(x + 1)$

ii) $x^2 + 5x + 6$

Solution: $x^2 + 5x + 6 = x^2 + (3 + 2)x + 6$

$= x^2 + 3x + 2x + 6$

$= x(x + 3) + 2(x + 3)$

$= (x + 3)(x + 2)$

iii) $x^2 - 6x + 8$

Solution: $x^2 - 6x + 8 = x^2 - (4 + 2)x + 8$

$= x^2 - 4x - 2x + 8$

$= x(x - 4) - 2(x - 4)$

$= (x - 4)(x - 2)$

iv) $x^2 + 3x - 28$

Solution: $x^2 + 3x - 28 = x^2 + (7 - 4)x + 28$

$= x^2 + 7x - 4x + 28$

$= x(x + 7) - 4(x + 7)$

$= (x + 7)(x - 4)$

v) $x^2 - 7x + 12$

Solution: $x^2 - 7x + 12 = x^2 - (4 + 3)x + 12$

$= x^2 - 4x - 3x + 12$

$= x(x - 4) - 3(x - 4)$

$= (x - 4)(x - 3)$

vi) $\boldsymbol{x^2 - 3x - 18}$

Solution: $x^2 - 3x - 18 = x^2 - (6 - 3)x - 18$

$= x^2 - 6x + 3x - 18$

$= x(x - 6) + 3(x - 6)$

$= (x - 6)(x + 3)$

vii) $\boldsymbol{3x^2 - x - 2}$

Solution: $3x^2 - x - 2 = 3x^2 - (3 - 2)x - 2$

$= 3x^2 - 3x + 2x - 2$

$= 3x(x - 1) + 2(x - 1)$

$= (x - 1)(3x + 2)$

viii) $\boldsymbol{6x^2 + x - 2}$

Solution: $6x^2 + x - 2 = 6x^2 + (4 - 3)x - 2$

$= 6x^2 + 4x - 3x - 2$

$= 2x(3x + 2) - 1(3x + 2)$

$= (3x + 2)(2x - 1)$

ix) $\boldsymbol{x^2 - 30x + 216}$

Solution: $x^2 - 30x + 216 = x^2 - (18 + 12)x + 216$

$= x^2 - 18x - 12x + 216$

$= x(x - 18) - 12(x + 18)$

$= (x - 18)(x - 12)$

x) $\boldsymbol{8y^2 + 2y - 15}$

Solution: $8y^2 + 2y - 15 = 8y^2 + (12 - 10)y - 15$

$= 8y^2 + 12y - 10y - 15$

$= 4y(2y + 3) - 5(2y + 3)$

$= (2y + 3)(4y - 5)$

xi) $\boldsymbol{4t^2 - 16t + 7}$

Solution: $4t^2 - 16t + 7 = 4t^2 - (14 + 2)t + 7$

$= 4t^2 - 14t - 2t + 7$

$= 2t(2t - 7) - 1(2t - 7)$

$= (2t - 7)(2t - 1)$

xii) $\boldsymbol{a^2 - 7a - 120}$

Solution: $a^2 - 7a - 120 = a^2 - (15 - 8)a - 120$

$= a^2 - 15a + 8a - 120$

$= a(a - 15) + 8(a - 15)$

$= (a - 15)(a + 8)$

xiii) $\boldsymbol{3x^2 + 60x - 288}$

Solution: $3x^2 + 60x - 288 = 3(x^2 + 20x - 96)$

$= 3\{x^2 + (24 - 4)x - 96\}$

$= 3(x^2 + 24x - 4x - 96)$

$= 3\{x(x + 24) - 4(x + 24)\}$

$= 3(x - 4)(x + 24)$

xiv) $\boldsymbol{2x^2 + 6x - 56}$

Solution: $2x^2 + 6x - 56 = 2(x^2 + 3x - 28)$

$= 2\{x^2 + (7 - 4)x - 28\}$

$= 2(x^2 + 7x - 4x - 28)$

$= 2\{x(x + 7) - 4(x + 7)\}$

$= 2(x + 7)(x - 4)$

xv) $\boldsymbol{30x^2 + 25x - 30}$

Solution: $30x^2 + 25x - 30 = 5(6x^2 + 5x - 6)$

$= 5\{6x^2 + (9 - 4)x - 6\}$

$= 5(6x^2 + 9x - 4x - 6)$

$= 5\{3x(2x + 3) - 2(2x + 3)\}$

$= 5(2x + 3)(3x - 2)$

xvi) $\frac{1}{4}x^2 - 2x + 3$

Solution: $\frac{1}{4}x^2 - 2x + 3 = \frac{1}{4}(x^2 - 8x + 12)$

$= \frac{1}{4}\{x^2 - (6+2)x + 12\}$

$= \frac{1}{4}(x^2 - 6x - 2x + 12)$

$= \frac{1}{4}\{x(x-6) - 2(x-6)\}$

$= \frac{1}{4}(x-6)(x-2)$

xvii) $\sqrt{2}\,x^2 + 5x + 3\sqrt{2}$

Solution: $\sqrt{2}\,x^2 + 5x + 3\sqrt{2} = \sqrt{2}\,x^2 + (3+2)x + 3\sqrt{2})$

$= \sqrt{2}\,x^2 + 3x + 2x + 3\sqrt{2})$

$= x(\sqrt{2}\,x + 3) + \sqrt{2}\,(\sqrt{2}\,x + 3)$

$= (\sqrt{2}\,x + 3)(x + \sqrt{2}\,)$

xviii) $2\sqrt{3}x^2 + 7x - 3\sqrt{3}$

Solution: $2\sqrt{3}x^2 + 7x - 3\sqrt{3} = 2\sqrt{3}x^2 + (9-2)x - 3\sqrt{3})$

$= 2\sqrt{3}x^2 + 9x - 2x - 3\sqrt{3})$

$= \sqrt{3}x(2x + 3\sqrt{3}) - 1(2x + 3\sqrt{3})$

$= (2x + 3\sqrt{3})(\sqrt{3}x - 1)$

xix) $\frac{\sqrt{3}}{2}x^2 - x - 48\sqrt{3}$

Solution: $\frac{\sqrt{3}}{2}x^2 - x - 48\sqrt{3} = \frac{1}{2}(\sqrt{3}x^2 - 2x - 96\sqrt{3})$

$= \frac{1}{2}\{\sqrt{3}x^2 - (18-16)x - 96\sqrt{3}\}$

$= \frac{1}{2}(\sqrt{3}x^2 - 18x + 16x - 96\sqrt{3})$

$= \frac{1}{2}\{\sqrt{3}x(x - 6\sqrt{3}) + 16(x - 6\sqrt{3})\}$

$= \frac{1}{2}(x - 6\sqrt{3})(\sqrt{3}x + 16)$

xx) $5\sqrt{2}a^2 - 7a - 12\sqrt{12}$

Solution: $5\sqrt{2}a^2 - 7a - 12\sqrt{12} = 5\sqrt{2}a^2 - (15-8)a - 12\sqrt{12}$

$= 5\sqrt{2}a^2 - 15a + 8a - 12\sqrt{12}$

$= 5a(\sqrt{2}\,a - 3) + 5\sqrt{2}\,(\sqrt{2}\,a - 3)$

$= (\sqrt{2}\,a - 3)(5a + 4\sqrt{2}\,)$

xxi) $\boldsymbol{x^2 - \frac{5c}{6} + \frac{1}{6}}$

Solution: $x^2 - \frac{5c}{6} + \frac{1}{6} = \frac{1}{6}(6x^2 - 5x + 1)$

$= \frac{1}{6}\{6x^2 - (3 + 2)x + 1\}$

$= \frac{1}{6}(6x^2 - 3x - 2x + 1)$

$= \frac{1}{6}\{3x(2x - 1) - 1(2x - 1)\}$

$= \frac{1}{6}(2x - 1)(3x - 1)$

xxii) $\boldsymbol{24 + 5x - x^2}$

Solution: $24 + 5x - x^2 = -x^2 + 5x + 24$

$= -(x^2 - 5x - 24)$

$= -\{x^2 - (8 - 3)x - 24\}$

$= -(x^2 - 8x + 3x - 24)$

$= -\{x(x - 8) + 3(x - 8)\}$

$= -(x - 8)(x + 3)$

OR

$24 + 5x - x^2 = 24 + (8 - 3)x - x^2$

$= 24 + 8x - 3x - x^2$

$= 8(3 + x) - x(3 + x)$

$= (x + 3)(8 - x)$

xxiii) $\boldsymbol{\sqrt{3}x^2 + (3\sqrt{2} - 2)x - 6}$

Solution: $\sqrt{3}x^2 + (3\sqrt{2} - 2)x - 6 = \sqrt{3}x^2 + 3\sqrt{3}x - 2x - 6$

$= \sqrt{3}x(x + 3) - 2(x + 3)$

$= (x + 3)(\sqrt{3}x - 2)$

xxiv) $\boldsymbol{\frac{2}{3} - \frac{x}{3} - x^2}$

Solution: $\frac{2}{3} - \frac{x}{3} - x^2 = \frac{1}{3}(2 - x - 3x^2)$

$= -\frac{1}{3}(3x^2 + x - 2)$

$= -\frac{1}{3}(3x^2 + (3 - 2)x - 2)$

$= -\frac{1}{3}(3x^2 + 3x - 2x - 2)$

$= -\frac{1}{3}\{3x(x + 1) - 2(x + 1)\}$

$= -\frac{1}{3}(x + 1)(3x - 2)$

2. Factorise the following using suitable identities:

i) $\mathbf{16x^2 + 8x + 1}$

Solution: $16x^2 + 8x + 1 = (4x)^2 + 2.4x.1 + 1^2$

$= (4x + 1)^2$ [using $a^2 + 2ab + b^2 = (a + b)^2$]

ii) $\mathbf{4y^2 - 12y + 9}$

Solution: $4y^2 - 12y + 9 = (2y)^2 - 2.2y.3 + 3^2$

$= (2y - 3)^2$ [using $a^2 - 2ab + b^2 = (a - b)^2$]

iii) $\mathbf{25x^2 - 36}$

Solution: $25x^2 - 36 = (5x)^2 - 6^2$

$= (5x + 6)(5x - 6)$ [using $a^2 - b^2 = (a + b)(a - b)$]

iv) $\mathbf{4x^2 - 81y^2}$

Solution: $4x^2 - 81y^2 = (2x)^2 - (9y)^2$

$= (2x + 9y)(2x - 9y)$ [using $a^2 - b^2 = (a + b)(a - b)$]

v) $\mathbf{a^2x^2 - 9b^2y^2}$

Solution: $a^2x^2 - 9\,b^2y^2 = (ax)^2 - (3by)^2$

$= (ax + 3by)(ax - 3by)$ [using $a^2 - b^2 = (a + b)(a - b)$]

vi) $\mathbf{\frac{x^2}{9} - \frac{y^2}{16}}$

Solution: $\frac{x^2}{9} - \frac{y^2}{16} = \left(\frac{x}{3}\right)^2 - \left(\frac{y}{4}\right)^2$

$= \left(\frac{x}{3} + \frac{y}{4}\right)\left(\frac{x}{3} - \frac{y}{4}\right)$ [using $a^2 - b^2 = (a + b)(a - b)$]

vii) $\mathbf{(a + b)^2 - c^2}$

Solution: $(a + b)^2 - c^2 = \{(a + b) + c\}\{(a + b) - c\}$

[using $a^2 - b^2 = (a + b)(a - b)$]

$= (a + b + c)(a + b - c)$

viii) $\mathbf{4x^2 - 81y^2}$

Solution: $4x^2 - 81y^2 = (2x)^2 - (9y)^2$

$= (2x + 9y)(2x - 9y)$ [using $a^2 - b^2 = (a + b)(a - b)$]

ix) $\boldsymbol{a^2 - (2b + 3c)^2}$

Solution: $a^2 - (2b + 3c)^2 = \{a + (2b + 3c)\}\{a - (2b + 3c)\}$

[using $a^2 - b^2 = (a + b)(a - b)$]

$= (a + 2b + 3c)(a - 2b - 3c)$

x) $\boldsymbol{(4a + 7b)^2 - (3a + 8b)^2}$

Solution: $(4a + 7b)^2 - (3a + 8b)^2$

$= \{(4a + 7b) + (3a + 8b)\}\{(4a + 7b) - (3a + 8b)\}$

[using $a^2 - b^2 = (a + b)(a - b)$]

$= (4a + 7b + 3a + 8b)(4a + 7b - 3a - 8b)$

$= (7a + 15b)(a - b)$

xi) $\boldsymbol{\frac{a^2}{121} - \frac{256^2}{144}}$

Solution: $\frac{a^2}{121} - \frac{25^2}{144} = \left(\frac{a}{11}\right)^2 - \left(\frac{5b}{12}\right)^2$

$= \left(\frac{a}{11} + \frac{5b}{12}\right)\left(\frac{a}{11} - \frac{5b}{12}\right)$ [using $a^2 - b^2 = (a + b)(a - b)$]

xii) $\boldsymbol{(a + b + c)^2 - (a + b - c)^2}$

Solution: $(a + b + c)^2 - (a + b - c)^2$

$= \{(a + b + c) + (a + b - c)\}\{(a + b + c) - (a + b - c)$

[using $a^2 - b^2 = (a + b)(a - b)$]

$= (a + b + c + a + b - c)(a + b + c - a - b + c)$

$= (2a + 2b) \times 2c$

$= 2(a + b) \times 2c$

$= 4(a + b)c$

xiii) $\boldsymbol{25x^2 + 5x + \frac{1}{4}}$

Solution: $25x^2 + 5x + \frac{1}{4} = (5x)^2 + 2 \times 5x \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

$= \left(5x + \frac{1}{2}\right)^2$ [using $a^2 + 2ab + b^2 = (a + b)^2$]

xiv) $\mathbf{243x^2 + 54x + 3}$

Solution: $243x^2 + 54x + 3 = 3(81x^2 + 18x + 1)$

$= 3\{(9x)^2 + 2 \times 9x \times 1 + 1^2\}$

$= 3(9x+1)^2$ [using $a^2 + 2ab + b^2 = (a+b)^2$]

xv) $\mathbf{18a^2 + 8 - 24a}$

Solution: $18a^2 + 8 - 24a = 2(9a^2 + 4 - 12a)$

$= 2(9a^2 - 12a + 4)$

$= 2\{(3a)^2 - 2.3a.2 + 2^2\}$

$= 2(3a-2)^2$ [using $a^2 - 2ab + b^2 = (a-b)^2$]

xvi) $\mathbf{49a^2 + 70ab + 25b^2}$

Solution: $49a^2 + 70ab + 25b^2 = (7a)^2 + 2.7a.5b + (5b)^2$

$= (7a+5b)^2$ [using $a^2 + 2ab + b^2 = (a+b)^2$]

xvii) $\mathbf{8x^3 + 12x^2 + 6x + 1}$

Solution: $8x^3 + 12x^2 + 6x + 1$

$= (2x)^3 + 3.(2x)^2.1 + 3.2x.1^2 + 1^3$

$= (2x+1)^3$ [using $a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$]

xviii) $\mathbf{27a^3 - 27a^2b + 9ab^2 - b^3}$

Solution: $27a^3 - 27a^2b + 9ab^2 - b^3$

$= (3a)^3 - 3.(3a)^2.b + 3.3a.b^2 - b^3$

$= (3a-b)^3$ [using $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$]

xix) $\mathbf{64x^3 + 144x^2y + 108xy^2 + 27y^3}$

Solution: $64x^3 + 144x^2y + 108xy^2 + 27y^3$

$= (4x)^3 + 3.(4x)^2.3y + 3.4x.(3y)^2 + (3y)^3$

$= (4x+3y)^3$ [using $a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$]

xx) $\mathbf{x^6 - 8y^3 - 6x^4y + 12x^2y^2}$

Solution: $x^6 - 8y^3 - 6x^4y + 12x^2y^2$

$= (x^2)^3 - (2y)^3 - 3.(x^2)^2.2y + 3.x^2.(2y)^2$

$= (x^2)^3 - (2y)^3 - 3.x^2.2y(x^2 - 2y)$

$= (x^2 - 2y)^3$ [using $a^3 - b^3 - 3ab(a-b) = (a-b)^3$]

xxi) $\frac{a^3}{8}+\frac{8b^3}{27}+\frac{a^2b}{2}+\frac{2ab^2}{3}$

Solution: $\frac{a^3}{8}+\frac{8b^3}{27}+\frac{a^2b}{2}+\frac{2ab^2}{3}$

$=\left(\frac{a}{2}\right)^3+\left(\frac{2b}{3}\right)^3+3.\left(\frac{a}{2}\right)^2.\frac{2b}{3}+3.\frac{a}{2}.\left(\frac{2b}{3}\right)^2$

$=\left(\frac{a}{2}\right)^3+3.\left(\frac{a}{2}\right)^2.\frac{2b}{3}+3.\frac{a}{2}.\left(\frac{2b}{3}\right)^2+\left(\frac{2b}{3}\right)^3$

$=\left(\frac{a}{2}+\frac{2b}{3}\right)^3$ [using $a^3+3a^2b+3ab^2+b^3=(a+b)^3$]

xxii) $27x^3+9x^2+x+\frac{1}{27}$

Solution: $27x^3+9x^2+x+\frac{1}{27}$

$=(3x)^3+3(3x)^2.\frac{1}{3}+3.3x.\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3$

$=\left(3x+\frac{1}{3}\right)^3$ [using $a^3+3a^2b+3ab^2+b^3=(a+b)^3$]

xxiii) $27-125a^3-135a+225a^2$

Solution: $27-125a^3-135a+225a^2$

$=3^3-(5a)^3-3.3^2.5a+3.3(5a)^2$

$=3^3-3.3^2.5a+3.3(5a)^2-(5a)^3$

$=(3-5a)^3$ [using $a^3-3a^2b+3ab^2-b^3=(a-b)^3$]

xxiv) x^3+27

Solution: $x^3+27=x^3+3^3$

$=(x+3)(x^2-x.3+3^2)$ [using$a^3+b^3=(a+b)(a^2-ab+b^2$]

$=(x+3)(x^2-3x+9)$

xxv) $1-8a^3$

Solution: $1-8a^3$

$=1^3-(2a)^3$

$=(1-2a)\{1^2+1\times 2a+(2a)^2\}$ [using$a^3-b^3=(a-b)(a^2+ab+b^2)$]

$=(1-2a)(1+2a+4a^2)$

xxvi) $8x^3+125y^3$

Solution: $8x^3+125y^3$

$=(2x)^3+(5y)^3$

$=(2x+5y)\{(2x)^2-2x.5y+(5y)^2\}$[using$a^3+b^3=(a+b)(a^2-ab+b^2)$]

$=(2x+5y)(4x^2-10xy+25y^2)$

xxvii) $\boldsymbol{64a^3 - 8b^3}$

Solution: $64a^3 - 8b^3$

$= (4a)^3 - (2b)^3$

$= (4a - 2b)\{(4a)^2 + 4a.2b + (2b)^2\}$[using$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]

$= (4a - 2b)(16a^2 + 8ab + 4b^2)$

xxviii) $\boldsymbol{\frac{x^3}{8} + 216y^3}$

Solution: $\frac{x^3}{8} + 216y^3$

$= \left(\frac{x}{2}\right)^3 + (6y)^3$

$= \left(\frac{x}{2} + 6y\right)\left\{\left(\frac{x}{2}\right)^2 - \frac{x}{2}.6y + (6y)^2\right\}$ [using$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$]

$= \left(\frac{x}{2} + 6y\right)\left(\frac{x^2}{4} - 3xy + 36y^2\right)$

xxix) $\boldsymbol{3a^4b - 24ab^4}$

Solution: $3a^4b - 24ab^4$

$= 3ab(a^3 - 8b^3)$

$= 3ab\{a^3 - (2b)^3\}$

$= 3ab(a - 2b)\{a^2 + a.2b + (2b)^2\}$ [using$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]

$= 3ab(a - 2b)(a^2 + 2ab + 4b^2)$

xxx) $\boldsymbol{54x^6y + 2x^3y^4}$

Solution: $54x^6y + 2x^3y^4$

$= 2x^3y(27x^3 + y^3)$

$= 2x^3\, y\{(3x)^3 + y^3\}$

$= 2x^3y(3x + y)\{(3x)^2 - 3x.y + y^2\}$ [using$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$]

$= 2x^3y(3x + y)(9x^2 - 3xy + y^2)$

3. Resolve into factors:

i) $\boldsymbol{x^4 - y^4}$

Solution: $x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2 + y^2)(x^2 - y^2)$ [using $a^2 - b^2 = (a+b)(a-b)$]

$= (x^2 + y^2)(x + y)(x - y)$

ii) $\boldsymbol{x^4 + x^2 + 1}$

Solution: $x^4 + x^2 + 1 = (x^2)^2 + 2x^2 - x^2 + 1$

$= (x^2)^2 + 2x^2.1 + 1^2 - x^2$

$= (x^2 + 1)^2 - x^2$ [using $(a^2 + 2ab + b^2) = (a + b)^2$]

$= (x^2 + 1 + x)(x^2 + 1 - x)$ [using $a^2 - b^2 = (a + b)(a - b)$]

$= (x^2 + x + 1)(x^2 - x + 1)$

iii) $\boldsymbol{8(a + b)^3 - (a - b)^3}$

Solution: $8(a + b)^3 - (a - b)^3$

$= \{2(a + b)\}^3 - (a - b)^3$

$= \{2(a + b) - (a - b)\}[\{2(a + b)\}^2 + 2(a + b)(a - b) + (a - b)^2]$

[using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$= (2a + 2b - a + b)\{4(a + b)^2 + 2(a^2 - b^2) + a^2 - 2ab + b^2\}$

$=(a + 3b)\{4(a^2 + 2ab + b^2) + 2a^2 - 2b^2 + a^2 - 2ab + b^2\}$

$= (a + 3b)(4a^2 + 8ab + 4b^2 + 2a^2 - 2b^2 + a^2 - 2ab + b^2)$

$= (3a + 3b)(7a^2 + 6ab + 3b^2)$

iv) $\boldsymbol{(x + 2)^3 + (x - 2)^3}$

Solution: $(x + 2)^3 + (x - 2)^3$

$= (x^3 + 3.x^2.2 + 3.x.2^2 + 2^3) + (x^3 - 3.x^2.2 + 3.x.2^2 - 2^3)$

$= x^3 + 6x^2 + 12x + 8 + x^3 - 6x^2 + 12x - 8$

$= 2x^3 + 24x$

$= 2x\ (x^2 + 12)$

v) $\boldsymbol{a^3 + b^3 + a + b}$

Solution: $\boldsymbol{a^3 + b^3 + a + b}$ $= (a + b)(a^2 - ab + b^2) + (a + b)$

$= (a + b)(a^2 - ab + b^2 + 1)$

vi) $\boldsymbol{a^6 - b^6}$

Solution: $a^6 - b^6 = (a^3)^2 - (b^3)^2$

$= (a^3 + b^3)(a^3 - b^3)$ [using $a^2 - b^2 = (a + b)(a - b)$]

$= (a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2)$

$= (a + b)(a - b)(a^2 + ab + b^2)(a^2 - ab + b^2)$

vii) $\boldsymbol{a^3 + b^3 + 3ab\,(a+b) - 8}$

Solution: $a^3 + b^3 + 3ab\,(a+b) - 8$

$= (a+b)^3 - 2^3$

$= (a+b-2)\{(a+b)^2 + (a+b).2 + 2^2\}$

[using $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]

$= (a+b-2)\{(a+b)^2 + 2(a+b) + 4\}$

viii) $\boldsymbol{x^3 - 3x^2 + 3x + 7}$

Solution: $x^3 - 3x^2 + 3x + 7$

$= (x^3 - 3x^2.1 + 3x.1^2 - 1^3) + 1^3 + 7$

$= (x-1)^3 + 8$ [using $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$]

$= (x-1)^3 + 2^3$

$= (x-1+2)\{(x-1)^2 - (x-1).2 + 2^2\}$

[using $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$]

$= (x+1)(x^2 - 2x + 1 - 2x + 2 + 4)$

$= (x+1)(x^2 - 4x + 7)$

ix) $\boldsymbol{x^3 + 3x^2 + 3x - 7}$

Solution: $x^3 + 3x^2 + 3x - 7$

$= (x^3 + 3x^2.1 + 3x.1^2 + 1^3) - 1^3 - 7$

$= (x+1)^3 - 8$ [using $a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$]

$= (x+1)^3 - 2^3$

$= (x+1-2)\{(x+1)^2 + (x+1).2 + 2^2\}$

[using $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$]

$= (x-1)(x^2 + 2x + 1 + 2x + 2 + 4)$

$= (x-1)(x^2 + 4x + 7)$

x) $\boldsymbol{2(x+y)^2 - 9(x+y) - 5}$

Solution: $2(x+y)^2 - 9(x+y) - 5$

$= 2(x+y)^2 - (10-1)(x+y) - 5$

$= 2(x+y)^2 - 10(x+y) + (x+y) - 5$

$= 2(x+y)(x+y-5) + 1(x+y-5)$

$= (x+y-5)\{2(x+y) + 1\}$

$= (x+y-5)(2x+2y+1)$

xi) $\mathbf{8(a+1)^2+2(a+1)(b+2)-15(b+2)^2}$

Solution: Putting $a+1=x, b+2=y$, we have

$$8(a+1)^2+2(a+1)(b+2)-15(b+2)^2$$
$$= 8x^2+2xy-15y^2$$
$$= 8x^2+(12-10)xy-15y^2$$
$$= 8x^2+12xy-10xy-15y^2$$
$$= 4x(2x+3y)-5y(2x+3y)$$
$$= (2x+3y)(4x-5y)$$
$$= \{2(a+1)+3(b+2)\}\{4(a+1)-5(b+2)\}$$
$$= (2a+2+3b+6)(4a+4-5b-10)$$
$$= (2a+3b+8)(4a-5b-6)$$

xii) $\mathbf{x^{12}-y^{12}}$

Solution: $x^{12}-y^{12}$

$$=(x^6)^2-(y^6)^2$$
$$=(x^6+y^6)(x^6-y^6)$$
$$=\{(x^2)^3+(y^2)^3\}\{(x^2)^3-(y^2)^3\}$$
$$=(x^2+y^2)\{(x^2)^2-x^2y^2+(y^2)^2\}(x^2-y^2)\{(x^2)^2+x^2y^2+(y^2)^2\}$$
$$=(x^2+y^2)(x^4-x^2y^2+y^4)(x+y)(x-y)(x^4+x^2y^2+y^4)$$
$$=(x-y)(x+y)(x^2+y^2)(x^4-x^2y^2+y^4)(x^4+x^2y^2+y^4)$$
$$=(x-y)(x+y)(x^2+y^2)(x^4-x^2y^2+y^4)\{(x^4+2x^2y^2+y^4)-x^2y^2\}$$
$$=(x-y)(x+y)(x^2+y^2)(x^4-x^2y^2+y^4)[\{(x^2)^2+2x^2y^2+(y^2)^2\}-x^2y^2]$$
$$=(x-y)(x+y)(x^2+y^2)(x^4-x^2y^2+y^4)\{(x^2+y^2)^2-(xy)^2\}$$
$$=(x-y)(x+y)(x^2+y^2)(x^4-x^2y^2+y^4)(x^2+y^2+xy)(x^2+y^2-xy)$$
$$=(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)(x^2-xy+y^2)(x^4-x^2y^2+y^4)$$

xiii) $\mathbf{x^4+4}$

Solution: x^4+4

$$= (x^2)^2+2.x^2.2+2^2-4x^2$$
$$= (x^2+2)^2-(2x)^2$$
$$= (x^2+2+2x)(x^2+2-2x)$$
$$= (x^2+2x+2)(x^2-2x+2)$$

xiv) $\boldsymbol{x(x+z)-y(y+z)}$

Solution: $x(x+z)-y(y+z)$

$= x^2+zx-y^2-yz$

$= (x^2-y^2)+(zx-yz)$

$= (x+y)(x-y)+z(x-y)$

$= (x-y)(x+y+z)$

xv) $\boldsymbol{x(x-2)(x-4)+4x-8}$

Solution: $x(x-2)(x-4)+4x-8$

$= x(x-2)(x-4)+4(x-2)$

$= (x-2)\{x(x-4)+4\}$

$= (x-2)(x^2-4x+4)$

$= (x-2)(x^2-2x.2+2^2)$

$= (x-2)(x-2)^2$

$= (x-2)^3$

xvi) $\boldsymbol{5x^3-30x^2+45x}$

Solution: $5x^3-30x^2+45x$

$= 5x(x^2-6x\ +9)$

$= 5x(x^2-2.x.3+3^2)$

$= 5x(x-3)^2$

xvii) $\boldsymbol{a^2+b^2+2(ab+bc+ca)}$

Solution: $a^2+b^2+2(ab+bc+ca)$

$= a^2+b^2+2ab+2(bc+ca)$

$= (a+b)^2+2c(a+b)$

$= (a+b)(a+b+2c)$

xviii) $\boldsymbol{(x+2)(x^2+25)-10x^2-20x}$

Solution: $(x+2)(x^2+25)-10x^2-20x$

$= (x+2)(x^2+25)-10x(x+2)$

$= (x+2)(x^2+25\ -10x)$

$= (x+2)(x^2-10x+25\ )$

$= (x+2)(x^2-2x.5+5^2)$

$= (x+2)(x-5)^2$

xix) $x(x^3 - y^3) - 3x^2y(x - y)$

Solution:- $x(x^3 - y^3) - 3x^2y(x - y)$

$= x\{x^3 - y^3 - 3xy(x - y)\}$

$= x(x - y)^3$

xx) $x^4 - 5x^2 + 4$

Solution: $x^4 - 5x^2 + 4$

$= (x^2)^2 - (4 + 1)x^2 + 4$

$= (x^2)^2 - 4x^2 - x^2 + 4$

$= x^2(x^2 - 4) - 1(x^2 - 4)$

$= (x^2 - 4)(x^2 - 1)$

$= (x^2 - 2^2)(x^2 - 1^2)$

$= (x + 2)(x - 2)(x + 1)(x - 1)$

$= (x + 1)(x + 2)(x - 1)(x - 2)$

xxi) $x^4 + x^2 - 2$

Solution: $x^4 + x^2 - 2$

$= x^4 + (2 - 1)x^2 - 2$

$= x^4 + 2x^2 - x^2 - 2$

$= x^2(x^2 + 2) - 1(x^2 + 2)$

$= (x^2 + 2)(x^2 - 1^2)$

$= (x^2 + 2)(x + 1)(x - 1)$

$= (x - 1)(x + 1)(x^2 + 2)$

xxii) $x^4 + 5x^2 + 9$

Solution: $x^4 + 5x^2 + 9$

$= (x^2)^2 + 2.x^2.3 + 3^2 - x^2$

$= (x^2 + 3)^2 - x^2$

$= (x^2 + 3 + x)(x^2 + 3 - x)$

$= (x^2 + x + 3)(x^2 - x + 3)$

xxiii) $(x + y)^3(a + b) - (x + y)(a + b)^3$

Solution: $(x + y)^3(a + b) - (x + y)(a + b)^3$

$= (x + y)(a + b)\{(x + y)^2 - (a + b)^2\}$

$= (x + y)(a + b)\{(x + y) + (a + b)\}\{(x + y) - (a + b)\}$

$= (x + y)(a + b)(x + y + a + b)(x + y - a - b)$

****

- **H.C.F. of polynomials:** The H.C.F. of two or more polynomials is defined as the polynomial of highest degree with the greatest leading coefficient, which is a factor of each of the given polynomials.
- **L.C.M. of polynomials:** The L.C.M. of two or more polynomials is defined as the polynomial of lowest degree and smallest leading coefficient which is exactly divisible by each of the given polynomials.
- **H.C.F. of polynomials by factorisation**

To find the H.C.F. of two or more polynomials by the method of factorisation, we may proceed as follows:

i) Resolve each of the given polynomials into irreducible or prime factors.

ii) Find the H.C.F. of the leading coefficients of the polynomials.

iii) Find the product of the H.C.F of the leading coefficients and factors with their highest powers common to all polynomials.

- **L.C.M. of the polynomials by Factorisation**

To find the L.C.M. of two or more polynomials by the method of factorisation, we may proceed as follows:

i) Resolve each of the given polynomials into its prime factors.

ii) Find the L.C.M. of the leading coefficients of the polynomials.

iii) Find the product of the L.C.M. of the leading coefficients and the factors with their highest powers involved in either of the polynomials.

SOLUTIONS

EXERCISE 2.4

1. Find the H.C.F. of the following polynomials.

i) $\boldsymbol{a^2b^3 \text{ and } a^3b^2}$

Solution: a^2b^3 and a^3b^2

H.C.F. $= a^2b^2$

ii) $\boldsymbol{4xy^2 \text{ and } 6x^2y^4}$

Solution: $4xy^2 = 2^2xy^2$

$6x^2y^4 = 2 \times 3x^2y^4$

$\mathrm{H.C.F.} = 2xy^2$

iii) $\mathbf{9x^3y^4}$**and**$\mathbf{24x^2y^2z^3}$

Solution: $9x^3y^4 = 3^2 \times x^3y^4$

$24x^2y^2z^3 = 2^3 \times 3 \times x^2y^2z^3$

H.C.F. = $3x^2y^2$

iv) $\mathbf{36x^2a^4c^5}$ **,**$\mathbf{24xy^2a^3b^4}$and $\mathbf{60y^3a^6b^2c}$

Solution: $36x^2a^4c^5 = 2^2 \times 3^2 \times x^2a^4c^5$

$24xy^2a^3b^4 = 2^3 \times 3 \times xy^2a^3b^4$

$60y^3a^6b^2c = 2^2 \times 3 \times 5 \times y^3a^6b^2c$

H.C.F. = $2^2 \times 3 \times a^3$ = $12a^3$

v) $\mathbf{4a^2b^3}$**and**$\mathbf{18b^2c^3(a+b)}$

Solution: $4a^2b^3 = 2^2 \times a^2b^3$

$18b^2c^3(a+b) = 2 \times 3^2 \times b^2c^2(a+b)$

$\mathrm{H.C.F.} = 2b^2$

vi) $\mathbf{x^2 - 9}$ **and** $\mathbf{x^3 + 27}$

Solution: $x^2 - 9 = x^2 - 3^2$

$= (x+3)(x-3)$

$x^3 + 27 = x^3 + 3^3$

$= (x+3)(x^2 - x.3 + 3^2)$

$= (x+3)(x^2 - 3x + 9)$

H.C.F. $= x + 3$

vii) $\mathbf{2a^2x - a^2}$**and** $\mathbf{4ax^2 - a}$

Solution: $2a^2x - a^2 = a^2(2x-1)$

$4ax^2 - a = a(4x^2 - 1)$

$= a\{(2x)^2 - 1^2\}$

$= a(2x+1)(2x-1)$

H.C.F. $= a(2x-1)$

viii) $\mathbf{a^3 - ab^2}$**and** $\mathbf{a^4 + 2a^3b + a^2\,b^2}$

Solution: $a^3 - ab^2 = a(a^2 - b^2)$

$= a(a+b)(a-b)$

$a^4 + 2a^3b + a^2\,b^2 = a^2(a^2 + 2ab + b^2)$

$= a^2(a+b)^2$

H.C.F. = $a(a+b)$

ix) $\mathbf{12(a^6 - a^2b^2c^2)}$ **and** $\mathbf{20(a^4b^2c^2 + a^2b^3c^3)}$

Solution: $12(a^6 - a^2b^2c^2) = 2^2 \times 3a^2(a^4 - b^2c^2)$

$= 2^2 \times 3a^2\{(a^2)^2 - (bc)^2\}$

$= 2^2 \times 3a^2(a^2 + bc)(a^2 - bc)$

$20(a^4b^2c^2 + a^2b^3c^3) = 2^2 \times 5 \times a^2b^2c^2(a^2 + bc)$

H.C.F. $= 2^2 \times a^2(a^2 + bc)$

$= 4a^2(a^2 + bc)$

x) $\mathbf{12x^3 - 48x}$ **and** $\mathbf{9x^5 - 72x^2}$

Solution: $12x^3 - 48x = 12x(x^2 - 4)$

$= 2^2 \times 3x(x^2 - 2^2)$

$= 2^2 \times 3x(x + 2)(x - 2)$

$9x^5 - 72x^2 = 9x^2(x^3 - 8)$

$= 3^2.x^2(x^3 - 2^3)$

$= 3^2.x^2(x - 2)(x^2 + x.2 + 2^2)$

$= 3^2.x^2(x - 2)(x^2 + 2x + 4)$

H.C.F. $= 3x(x - 2)$

xi) $\mathbf{6(x-1)^2}$**and**$\mathbf{9(x^2 - 3x + 2)}$

Solution: $6(x-1)^2 = 2 \times 3(x-1)^2$

$9(x^2 - 3x + 2) = 3^2\{x^2 - (2 + 1)x + 2\}$

$= 3^2(x^2 - 2x - x + 2)$

$= 3^2\{x(x - 2) - 1(x - 2)\}$

$= 3^2(x - 2)(x - 1)$

H.C.F. $= 3(x - 1)$

xii) $\mathbf{x^2 - 8x + 15}$ **and** $\mathbf{x^2 - 2x - 15}$

Solution: $x^2 - 8x + 15 = x^2 - (5 + 3)x + 15$

$= x^2 - 5x - 3x + 15$

$= x(x - 5) - 3(x - 5)$

$= (x - 5)(x - 3)$

$x^2 - 2x - 15 = x^2 - (5 + 3)x + 15$

$= x^2 - 5x - 3x + 15$

$= x(9 - 5) - 3(x - 5)$

$= (x - 5)(x + 3)$

H.C.F. $= x - 5$

xiii) $\boldsymbol{a^2 + a - 42}$ **and** $\boldsymbol{a^2 - 2a - 24}$

Solution: $a^2 + a - 42 = a^2 + (7-6)a - 42$

$= a^2 + 7a - 6a - 42$

$= a(a+7) - 6(a+7)$

$= (a+7)(a-6)$

$a^2 - 2a - 24 = a^2 - (6-4)a - 24$

$= a^2 - 6a + 4a - 24$

$= a(a-6) + 4(a-6)$

$= (a-6)(a+4)$

H.C.F. $= a - 6$

xiv) $\boldsymbol{4x^3 + 12x^2 + 9x}$ **and** $\boldsymbol{4x^2 - 2x - 12}$

Solution: $4x^3 + 12x^2 + 9x = x(4x^2 + 12x + 9)$

$= x\{(2x)^2 + 2.2x.3 + 3^2\}$

$= x(2x+3)^2$

$4x^2 - 2x - 12 = 2(2x^2 - x - 6)$

$= 2\{2x^2 - (4-3)x - 6\}$

$= 2(2x^2 - 4x + 3x - 6)$

$= 2\{2x(x-2) + 3(x-2)\}$

$= 2(x-2)(2x+3)$

H.C.F. $= 2x + 3$

xv) $\boldsymbol{a^2 - b^2, (a+b)^2}$ **and** $\boldsymbol{a^3 + b^3}$

Solution: $a^2 - b^2 = (a+b)(a-b)$

$(a+b)^2 = (a+b)(a+b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

H.C.F. $= (a+b)$

xvi) $\boldsymbol{(x+2)^2(x+3)^3(x+5)^4}$**and**$\boldsymbol{(x+3)^2(x+4)^3(x+5)}$

Solution: First polynomial $= (x+2)^2(x+3)^3(x+5)^4$

Second polynomial $= (x+3)^2(x+4)^3(x+5)$

H.C.F. $= (x+3)^2(x+5)$

xvii) $(\boldsymbol{x^2-4})(\boldsymbol{x^2+5x+6})$**and**$(\boldsymbol{x+3})^2(\boldsymbol{3x^2+3x-6})$

Solution: First polynomial $= (x^2-4)(x^2+5x+6)$

$= (x^2-2^2)\{x^2+(3+2)x+6\}$

$= (x^2-2^2)(x^2+3x+2x+6)$

$= (x+2)(x-2)\{x(x+3)+2(x+3)\}$

$= (x+2)(x-2)(x+3)(x+2)$

$= (x+2)^2(x-2)(x+3)$

Second polynomial $= (x+3)^2(3x^2+3x-6)$

$= (x+3)^2\times 3(x^2+x-2)$

$= 3(x+3)^2\{x^2+(2-1)x-2\}$

$= 3(x+3)^2(x^2+2x-x-2)$

$= 3(x+3)\{x(x+2)-1(x+2)\}$

$= 3(x+3)(x+2)(x-1)$

H.C.F. $= (x+2)(x+3)$

xviii) $\boldsymbol{2x^2-8}$ **,** $\boldsymbol{4x+8}$ **and** $\boldsymbol{8(x^2+4x+4)}$

Solution: First polynomial $= 2x^2-8$

$= 2(x^2-4)$

$= 2(x^2-2^2)$

$= 2(x+2)(x-2)$

Second polynomial $= 4x+8$

$= 4(x+2)$

$= 2^2(x+2)$

Third polynomial $= 8(x^2+4x+4)$

$= 2^3(x^2+2.x.2+2^2)$

$= 2^3(x+2)^2$

$\therefore$ H.C.F. $= 2(x+2)$

xix) $\boldsymbol{x^2 - 5x - 14,\ 3x^2 + 5x - 2}$ **and** $\boldsymbol{x^3 + 8}$

Solution: First polynomial $= x^2 - 5x - 14$

$= x^2 - (7-2)x - 14$

$= x^2 - 7x + 2x - 14$

$= x(x-7) + 2(x-7)$

$= (x-7)(x+2)$

Second polynomial $= 3x^2 + 5x - 2$

$= 3x^2 + (6-1)x - 2$

$= 3x^2 + 6x - x - 2$

$= 3x(x+2) - 1(x+2)$

$= (x+2)(3x-1)$

Third polynomial $= x^3 + 8$

$= x^3 + 2^3$

$= (x+2)(x^2 - x.2 + 2^2)$

$= (x+2)(x-2)^2$

H.C.F. $= x + 2$

xx) $\boldsymbol{x^2 - 9,\ (x+3)^2,\ x^2 + x - 6}$ **and** $\boldsymbol{2x^2 + 5x - 3}$

Solution: First polynomial $= x^2 - 9$

$= x^2 - 3^2$

$= (x+3)(x-3)$

Second polynomial $= (x+3)^2$

Third polynomial $= x^2 + x - 6$

$= x^2 + (3-2)x - 6$

$= x^2 + 3x - 2x - 6$

$= x(x+3) - 2(x+3)$

$= (x+3)(x-2)$

Fourth polynomial $= 2x^2 + 5x - 3$

$= 2x^2 + (6-1)x - 3$

$= 2x^2 + 6x - x - 3$

$= 2x(x+3) - 1(x+3)$

$= (x+3)(2x-1)$

H.C.F. $= x + 3$

xxi) $8(16x^2 - 24x + 9)$ and $20(3 + 17x - 28x^2)$

Solution:

$$8(16x^2 - 24x + 9) = 2^3\{(4x)^3 - 2.4x.3 + 3^2\}$$
$$= 2^3(4x-3)^2$$
$$20(3 + 17x - 28x^2) = -2^2 \times 5(28x^2 - 17x - 3)$$
$$= -2^2 \times 5\{28x^2 - (21-4)x - 3\}$$
$$= -2^2 \times 5(28x^2 - 21x + 4x - 3)$$
$$= -2^2 \times 5\{7x(4x-3) + 1(4x-3)\}$$
$$= -2^2 \times 5(4x-3)(7x+1)$$
$$\text{H.C.F.} = 2^2(4x-3)$$
$$= 4(4x-3)$$

2. Find the L.C.M. of the following polynomials.

i) a^3b^2, and a^2bc

Solution: First polynomial $= a^3b^2$

Second polynomial $= a^2bc$

L.C.M. $= a^3b^2c$

ii) $4x^3y^2$and$10x^2y^4$

Solution: First polynomial $= 4x^3y^2 = 2^2.x^3.y^2$

Second polynomial $= 10x^2y^4 = 2 \times 5.x^2y^4$

L.C.M. $= 2^2 \times 5x^3.y^4$

$= 20\,x^3y^4$

iii) $8a^2b^2c^3$and $12a^3b^4c^2$

Solution: First polynomial $= 8a^2b^2c^3 = 2^3.a^2b^2c^3$

Second polynomial $= 12a^3b^4c^2 = 2^2 \times 3a^3b^4c^2$

L.C.M. $= 2^3 \times 3\,a^3b^4c^3$

$= 24\,a^3b^4c^3$

iv) $35xyza^2$, $28x^2y^2zb$ and $42xabc$

Solution: First polynomial $= 35xyza^2 = 5 \times 7xyza^2$

Second polynomial $= 28x^2y^2zb = 2^2 \times 7x^2y^2zb$

Third polynomial $= 42xabc = 2 \times 3 \times 7xabc$

L.C.M. $= 2^2 \times 3 \times 5 \times 7\,x^2y^2za^2bc$

$= 420\,x^2y^2z\,a^2bc$

v) $\boldsymbol{16a^2b^3}$ **and** $\boldsymbol{24\ ab^4(a-b)}$

Solution: First Polynomial = $16a^2b^3 = 2^4.a^2b^3$

Second polynomial = $24\ ab^4(a-b) = 2^3 \times 3\ ab^4(a-b)$

L.C.M. $= 2^4 \times 3a^2b^4(a-b)$

$= 48\ a^2b^4(a-b)$

vi) $\boldsymbol{a^3b - ab^3}$ **and** $\boldsymbol{a^3b^2 + a^2b^3}$

Solution: $a^3b - ab^3 = ab(a^2 - b^2)$

$= ab(a+b)(a-b)$

$a^3b^2 + a^2b^3 = a^2b^2(a+b)$

L.C.M. $= a^2b^2(a+b)(a-b)$

vii) $\boldsymbol{(x+1)^2(x+2)^3(x+3)}$**and**$\boldsymbol{(x+1)^3(x+2)(x+3)^2}$

Solution: First Polynomial $= (x+1)^2(\boldsymbol{x+2})^3(x+3)$

Second polynomial $= (x+1)^3(x+2)(x+3)^2$

L.C.M. $= (x+1)^3(\boldsymbol{x+2})^3(x+3)^2$

viii) $\boldsymbol{x^3 - 8}$ **and** $\boldsymbol{x^2 - 5x + 6}$

Solution: First Polynomial $= x^3 - 8$

$= x^3 - 2^3$

$= (x-2)(x^2 + x.2 + 2^2)$

$= (x-2)(x^2 + 2x + 4)$

Second Polynomial $= x^2 - 5x + 6$

$= x^2 - (3+2)x + 6$

$= x^2 - 3x - 2x + 6$

$= x(x-3) - 2(x-3)$

$= (x-3)(x-2)$

L.C.M. $= (x-2)(x^2 + 2x + 4)(x-3)$

$= (x^3 - 8)(x-3)$

ix) $\boldsymbol{a^2(a^2-4)}$ **and** $\boldsymbol{a^4 + 2a^3 - 8a^2}$

Solution: First polynomial $= a^2(a^2 - 4)$

$= a^2(a^2 - 2^2)$

$= a^2(a+2)(a-2)$

Second polynomial = $a^4 + 2a^3 - 8a^2$

$= a^2(a^2 + 2a - 8)$

$= a^2\{a^2 + (4 - 2) - 8\}$

$= a^2(a^2 + 4a - 2a - 8)$

$= a^2\{a(a + 4) - 2(a + 4)\}$

$= a^2(a + 4)(a - 2)$

L.C.M. $= a^2(a - 2)(a + 2)(a + 4)$

$= a^2(a + 4)(a^2 - 4)$

x) $\boldsymbol{x^2 - 4x + 3}$ **and** $\boldsymbol{x^2 - 5x + 6}$

Solution: First polynomial $= x^2 - 4x + 3$

$= x^2 - (3 + 1)x + 3$

$= x^2 - 3x - x + 3$

$= x(x - 3) - 1(x - 3)$

$= (x - 3)(x - 1)$

Second polynomial $= x^2 - 5x + 6$

$= x^2 - (3 + 2)x + 6$

$= x^2 - 3x - 2x + 6$

$= x(x - 3) - 2(x - 3)$

$= (x - 3)(x - 2)$

L.C.M. $= (x - 1)(x - 2)(x - 3)$

xi) $\boldsymbol{x^2 + 8x + 15}$ **and** $\boldsymbol{x^3 + 10x^2 + 25x}$

Solution: First polynomial $= x^2 + 8x + 15$

$= x^2 + (5 + 3)x + 15$

$= x^2 + 5x + 3x + 15$

$= x(x + 5) + 3(x + 5)$

$= (x + 5)(x + 3)$

Second polynomial $= x^3 + 10x^2 + 25x$

$= x(x^2 + 10x + 25)$

$= x(x^2 + 2.x.5 + 5^2)$

$= x(x + 5)^2$

L.C.M. $= x(x + 3)(x + 5)^2$

xii) $\boldsymbol{4a^2x^2, 2x(x^2-a^2)}$ **and** $\boldsymbol{6a^3x(x^3+a^3)}$

Solution: First polynomial $= 4a^2x^2$

$= 2^2a^2x^2$

Second polynomial $= 2x(x^2-a^2)$

$= 2x(x+a)(x-a)$

Third polynomial $= 6a^3x(x^3+a^3)$

$= 2\times 3\,a^3x(x+a)(x^2-ax+a^2)$

L.C.M. $= 2^2\times 3a^3x^2(x+a)(x-a)(x^2-ax+a^2)$

$= 12a^3x^2(x-a)(x^3+a^3)$

xiii) $\boldsymbol{x^3+y^3,\ (x+y)^2}$ **and** $\boldsymbol{x^2-y^2}$

Solution: First polynomial $= x^3+y^3$

$= (x+y)(x^2-xy+y^2)$

Second polynomial $= (x+y)^2$

Third polynomial $= x^2-y^2$

$= (x+y)(x-y)$

L.C.M. $= (x+y)^2(x-y)(x^2-xy+y^2)$

$= (x+y)(x-y)(x+y)(x^2-xy+y^2)$

$= (x^2-y^2)(x^3+y^3)$

xiv) $\boldsymbol{4x^2(x+a)^2, 6a^2x(x^2-a^2)}$ **and** $\boldsymbol{9x^3(x^3-a^3)}$

Solution: First polynomial $= 4x^2(x+a)^2$

$= 2^2.x^2(x+a)^2$

Second polynomial $= 6a^2x(x^2-a^2)$

$= 2\times 3a^2x(x-a)(x+a)$

Third polynomial $= 9x^3(x^3-a^3)$

$= 3^2.x^3(x-a)(x^2+ax+a^2)$

L.C.M. $= 2^2\times\ 3^2a^2x^3(x+a)^2(x-a)(x^2+ax+a^2)$

$= 36a^2x^3(x+a)^2(x^3-a^3)$

xv) $\mathbf{6x^2 - x - 1, 3x^2 + 7x + 2}$ **and** $\mathbf{2x^2 + 3x - 2}$

Solution:
$$
\begin{aligned}
6x^2 - x - 1 &= 6x^2 - x - 1 \\
&= 6x^2 - (3-2)x - 1 \\
&= 6x^2 - 3x + 2x - 1 \\
&= 3x(2x-1) + 1(2x-1) \\
&= (2x-1)(3x+1) \\
3x^2 + 7x + 2 &= 3x^2 + (6+1)x + 2 \\
&= 3x^2 + 6x + x + 2 \\
&= 3x(x+2) + 1(x+2) \\
&= (x+2)(3x+1) \\
2x^2 + 3x - 2 &= 2x^2 + (4-1)x - 2 \\
&= 2x^2 + 4x - x - 2 \\
&= 2x(x+2) - 1(x+2) \\
&= (x+2)(2x-1) \\
\text{L.C.M} &= (x+2)(2x-1)(3x+1)
\end{aligned}
$$

xvi) $\mathbf{x^2 + 2x - 15, x^2 + 9x + 20}$ **and** $\mathbf{x^2 + 4x - 21}$

Solution:
$$
\begin{aligned}
x^2 + 2x - 15 &= x^2 + (5-3)x - 15 \\
&= x^2 + 5x - 3x - 15 \\
&= x(x+5) - 3(x+5) \\
&= (x+5)(x-3) \\
x^2 + 9x + 20 &= x^2 + (5+4)x + 20 \\
&= x^2 + 5x + 4x + 20 \\
&= x(x+5) + 4(x+5) \\
&= (x+5)(x+4) \\
x^2 + 4x - 21 &= x^2 + (7-3)x - 21 \\
&= x^2 + 7x - 3x - 21 \\
&= x(x+7) - 3(x+7) \\
&= (x+7)(x-3) \\
\text{L.C.M.} &= x-3)(x+4)(x+5)((x+7)
\end{aligned}
$$

xvii) $\mathbf{4(x^2 - y^2), 6(x^3 - y^3)}$ **and** $\mathbf{9(x^3 + y^3)}$

Solution: $4(x^2 - y^2) = 2^2(x+y)(x-y)$

$6(x^3 - y^3) = 2 \times 3\,(x-y)(x^2+xy+y^2)$

$9(x^3 + y^3) = 3^2(x+y)(x^2-xy+y^2)$

L.C.M . = $2^2 \times 3^2(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$

$= 36(x+y)(x^2-xy+y^2)(x-y)(x^2+xy+y^2)$

$= 36(x^3+y^3)(x^3-y^3)$

$= 36\{(x^3)^2 - (y^3)^2\}$

$= 36(x^6 - y^6)$

xviii) $\mathbf{5(x-y)^2, 10(x^2-y^2)}$ **and** $\mathbf{15(x+y)^2}$

Solution: $5(x-y)^2 = 5(x-y)^2$

$10(x^2-y^2) = 2 \times 5(x+y)(x-y)$

$15(x+y)^2 = 3 \times 5(x+y)^2$

L.C.M. $= 5 \times 2 \times 3(x+y)^2(x-y)^2$

$= 30(x+y)^2(x-y)^2$

3. Find the H.C.F. and L.C.M. of the following polynomials:

i) $\mathbf{2x^2 - 8}$ **and** $\mathbf{5x^2 - 20x + 20}$

Solution: First polynomial $= 2x^2 - 8$

$= 2(x^2 - 4)$

$= 2(x^2 - 2^2)$

$= 2(x+2)(x-2)$

Second polynomial $= 5x^2 - 20x + 20$

$= 5(x^2 - 4x + 4)$

$= 5(x^2 - 2.x.2 + 2^2)$

$= 5(x-2)^2$

H.C.F. $= x - 2$

L.C.M. $= 2 \times 5(x+2)(x-2)^2$

$= 10(x+2)(x-2)^2$

ii) $\mathbf{(x^2-4)(x^2+5x+6), (x+3)^2(2x^2+x-6)}$ **and** $\mathbf{4x^2-12x+9}$

Solution: First polynomial $= (x^2-4)(x^2+5x+6)$

$= (x^2-2^2)\{x^2+(3+2)x+6\}$

$= (x+2)(x-2)(x^2+3x+2x+6)$

$= (x+2)(x-2)\{x(x+3)+2(x+3)\}$

$= (x+2)(x-2)(x+3)(x+2)$

$= (x+2)^2(x-2)(x+3)$

Second polynomial $= (x+3)^2(2x^2+x-6)$

$= (x+3)^2\{2x^2+(4-3)x-6\}$

$= (x+3)^2(2x^2+4x-3x-6)$

$= (x+3)^2\{2x(x+2)-3(x+2)\}$

$= (x+3)^2(x+2)(2x-3)$

Third polynomial $= 4x^2-12x+9$

$= (2x)^2-2.2x.3+3^2$

$= (2x-3)^2$

H.C.F. = 1

L.C.M.= $(x+2)^2(x-2)(x+3)^2(2x+3)^2$

iii) $\boldsymbol{a^2-ab-2b^2}$**and**$\boldsymbol{a^3-a^2b-4ab^2+4b^3}$

Solution: First polynomial $= a^2-ab-2b^2$

$= a^2-(2-1)ab-2b^2$

$= a^2-2ab+ab-2b^2$

$= a(a-2b)+b(a-2b)$

$= (a-2b)(a+b)$

Second polynomial $= a^3-a^2b-4ab^2+4b^3$

$= a^2(a-b)-4b^2(a-b)$

$= (a-b)(a^2-4b^2)$

$= (a-b)\{a^2-(2b)^2\}$

$= (a-b)(a+2b)(a-2b)$

∴ H.C.F. $= (a-2b)$

and L.C.M. $= (a-2b)(a+b)(a-b)(a+2b)$

$= (a+b)(a-b)(a-2b)(a+2b)$

$= (a^2-b^2)(a^2-4b^2)$

iv) $\boldsymbol{(a^3-b^3)(a+b)^2, a^4-b^4}$ **and** $\boldsymbol{3a^4+2a^3b-5a^2b^2}$

Solution: First polynomial $= (a^3-b^3)(a+b)^2$

$= (a-b)(a^2+ab+b^2)(a+b)^2$

Second polynomial = $a^4 - b^4$

$= (a^2)^2 - (b^2)^2$

$= (a^2 + b^2)(a^2 - b^2)$

$= (a^2 + b^2)(a + b)(a - b)$

Third polynomial $= 3a^4 + 2a^3b - 5a^2b^2$

$= a^2(3a^2 + 2ab - 5b^2)$

$= a^2(3a^2 + (5 - 3)ab - 5b^2)$

$= a^2(3a^2 + 5ab - 3ab - 5b^2)$

$= a^2\{a(3a + 5b) - b(3a + 5)\}$

$= a^2(a - b)(3a + 5b)$

$\therefore$ H.C.F. $= a - b$

and L.C.M = $a^2(a - b)(a + b)^2(a^2 + b^2)(a^2 + ab + b^2)(3a + 5b)$

$= a^2(a + b)^2(a^2 + b^2)(a^3 - b^3)(3a + 5b)$

v) $\mathbf{3(x^2 + 2x - 8)}$, $\mathbf{6(x^2 + 9x + 20)}$ **and** $\mathbf{15(x^2 + x - 12)}$

Solution: $3(x^2 + 2x - 8) = 3(x^2 + (4 - 2)x - 8)$

$= 3(x^2 + 4x - 2x - 8)$

$= 3\{x(x + 4) - 2(x + 4)\}$

$= 3(x + 4)(x - 2)$

$6(x^2 + 9x + 20) = 2 \times 3\{x^2 + (5 + 4)x + 20\}$

$= 2 \times 3(x^2 + 5x + 4x + 20)$

$= 2 \times 3\{x(x + 5) + 4(x + 5)\}$

$= 2 \times 3(x + 5)(x + 4)$

$15(x^2 + x - 12) = 3 \times 5\{x^2 + (4 - 3)x - 12\}$

$= 3 \times 5(x^2 + 4x - 3x - 12)$

$= 3 \times 5\{x(x + 4) - 3(x + 4)\}$

$= 3 \times 5(x + 4)(x - 3)$

$\therefore$ H.C.F. $= 3(x + 4)$

and L.C.M $= 3 \times 2 \times 5(x + 4)(x - 2)(x + 5)(x - 3)$

$= 30(x - 2)(x - 3)(x + 4)(x + 5)$

vi) $(\mathbf{2x-3})^{\mathbf{2}}(\mathbf{x^2+x-2})$ **,** $\mathbf{4x^2-x-18}$ **and** $\mathbf{2x^2-9x+9}$

Solution: $(2x-3)^2(x^2+x-2) = (2x-3)^2\{(x^2+(2-1)x-2\}$

$= (2x-3)^2(x^2+2x-x-2)$

$= (2x-3)^2\{x(x+2)-1(x+2)\}$

$= (2x-3)^2(x+2)(x-1)$

$4x^2-x-18 = 4x^2-(9-8)x-18$

$= 4x^2-9x+8x-18$

$= x(4x-9)+2(4x-9)$

$= (x+2)(4x-9)$

$2x^2-9x+9 = 2x^2-(6+3)x+9$

$= 2x^2-6x-3x+9$

$= 2x(x-3)-3(x-3)$

$= (x-3)(2x-3)$

$\therefore$ H.C.F $= 1$

and L.C.M. $= (x-1)(x+2)(x-3)(4x-9)(2x-3)^2$

4. Show with suitable example, that the product of two polynomials is equal to the product of their H.C.F. and L.C.M.

Solution: Let x^2-4x+3 and x^2-5x+6 be two polynomials.

$x^2-4x+3 = x^2-(3+1)x+3$

$= x^2-3x-x+3$

$= x(x-3)-1(x-3)$

$= (x-3)(x-1)$

$x^2-5x+6 = x^2-(3+2)x+6$

$= x^2-3x-2x+6$

$= x(x-3)-2(x-2)$

$= (x-3)(x-2)$

$\therefore$ H.C.F. $= x-3$

and L.C.M. $= (x-1)(x-2)(x-3)$

$\text{H.C.F} \times \text{L.C.M.} = (x-3)\times(x-1)(x-2)(x-3)$

$= \{(x-3)(x-1)\}\times\{(x-2)(x-3)\}$

$= (x^2-4x+3)\times(x^2-5x+6)$

So, the product of two polynomials is equal to the product of their H.C.F. and L.C.M.

****

- **Common Zero(s) of polynomials**

  A number which is a zero of each of two or more polynomials is said to be a common zero of the polynomials.

Notes: 1. The common zeros of the polynomials are given by the zeros of the H.C.F. of the polynomials.

2. If the H.C.F. of the polynomials is a constant polynomial, they have no common zero as a constant polynomial has no zero.

SOLUTIONS

EXERCISE 2.5

1. Examine whether the following polynomials have common zero(s) or not:

i) $2x^2 - 8$ and $4(x^2 - 2x - 15)$

Solution: $2x^2 - 8 = 2(x^2 - 4)$

$= 2(x^2 - 2^2)$

$= 2(x+2)(x-2)$

$4(x^2 - 2x - 15) = 4\{(x^2 - (5-3)x - 15\}$

$= 4(x^2 - 5x + 3x - 15)$

$= 4\{x(x-5) + 3(x-5)\}$

$= 2^2.(x+3)(x-5)$

∴ H.C.F. = 2, which is a constant polynomial.

Hence the given polynomials have no common zero.

ii) $x^2 - 5x + 6$ and $x^2 - 3x + 2$

Solution: $x^2 - 5x + 6 = x^2 - (3+2)x + 6$

$= x^2 - 3x - 2x + 6$

$= x(x-3) - 2(x-3)$

$= (x-3)(x-2)$

$x^2 - 3x + 2 = x^2 - (2+1)x + 2$

$= x^2 - 2x - x + 2$

$= x(x-2) - 1(x-2)$

$= (x-2)(x-1)$

∴ H.C.F. = $x - 2$, which is not a constant polynomial.

Hence the given polynomials have common zero.

iii) $\mathbf{12(x^2-9)}$ **and** $\mathbf{18(x^2+2x-15)}$

Solution: $12(x^2-9) = 2^2 \times 3(x^2-3^2)$

$= 2^2 \times 3(x-3)(x+3)$

$18(x^2+2x-15) = 2 \times 3^2\{x^2+(5-3)x-15\}$

$= 2 \times 3^2(x^2+5x-3x-15)$

$= 2 \times 3^2\{x(x+5)-3(x+5)\}$

$= 2 \times 3^2(x+5)(x-3)$

$\therefore$ H.C.F. $= 2 \times 3(x-3)$

$= 6(x-3)$, which is not a constant polynomial.

Hence the given polynomials have common zero.

iv) $\mathbf{(x+2)^2(x-1)}$ **and** $\mathbf{25(x^2+3x-10)}$

Solution: First polynomial $= (x+2)^2(x-1)$

Second polynomial $= 25(x^2+3x-10)$

$= 5^2(x^2+(5-2)x-10)$

$= 5^2(x^2+5x-2x-10)$

$= 5^2\{x(x+5)-2(x+5)\}$

$= 5^2(x+5)(x-2)$

$\therefore$ H.C.F $= 1$, which is a constant polynomial.

Hence the given polynomials have no common zero.

v) $\mathbf{3(x+2)^2}$, $\mathbf{x^2-4x-12}$ **and** $\mathbf{5x(x^2-4)}$

Solution: First polynomial $= 3(x+2)^2$

Second polynomial $= x^2-4x-12$

$= x^2-(6-2)x-12$

$= x^2-6x+2x-12$

$= x(x-6)+2(x-6)$

$= (x-6)(x+2)$

Third polynomial $= 5x(x^2-4)$

$= 5x(x^2-2^2)$

$= 5x(x+2)(x-2)$

$\therefore$ H.C.F. $= x+2$, which is not a constant polynomial.

Hence the given polynomials have common zero.

vi) $\mathbf{16(x+1)^2, 24(x^2-3x-18)}$ **and** $\mathbf{36(x^2-6x+8)}$**.**

Solution: First polynomial $= 16(x+1)^2$

$= 2^4(x+1)^2$

Second polynomial $= 24(x^2-3x-18)$

$= 2^3 \times 3\{x^2-(6-3)x-18\}$

$= 2^3 \times 3(x^2-6x+3x-18)$

$= 2^3 \times 3\{x(x-6)+3(x-6)\}$

$= 2^3 \times 3(x-6)(x+3)$

Third polynomial $= 36(x^2-6x+8)$

$= 2^2 \times 3^2\{x^2-(4+2)x+8\}$

$= 2^2 \times 3^2(x^2-4x-2x+8)$

$= 2^2 \times 3^2\{x(x-4)-2(x-4)\}$

$= 2^2 \times 3^2(x-4)(x-2)$

$\therefore$ H.C.F. $= 2^2 = 4$, which is a constant polynomial.

Hence the given polynomials have no common zero.

2. Find the common zero(s) of the following polynomials.

i) $\mathbf{x^2-x}$ **and** $\mathbf{x^2-3x+2}$

Solution: $x^2-x = x(x-1)$

$x^2-3x+2 = x^2-(2+1)x+2$

$= x^2-2x-x+2$

$= x(x-2)-1(x-2)$

$= (x-2)(x-1)$

$\therefore$ H.C.F. $= x-1$

Now, $x-1 = 0$

$\Rightarrow x = 1$

Hence, 1 is the only the common zero of the given polynomials.

ii) $\mathbf{x^2-9}$ **and** $\mathbf{x^2-5x+6}$

Solution: $x^2-9 = x^2-3^2$

$= (x+3)(x-3)$

$x^2-5x+6 = x^2-(3+2)x+6$

$= x^2-3x-2x+6$

$= x(x-3)-2(x-3)$

$= (x-3)(x-2)$

$\therefore$ H.C.F. $= x - 3$

Now, $x - 3 = 0$

$\Rightarrow x = 3$

Hence, 3 is the only common zero of the given polynomials.

iii) $\boldsymbol{x^2 + 6x + 8}$ **and** $\boldsymbol{x^2 - 4x - 12}$

Solution:

$$\begin{aligned} x^2 + 6x + 8 &= x^2 + (4 + 2)x + 8 \\ &= x^2 + 4x + 2x + 8 \\ &= x(x + 4) + 2(x + 4) \\ &= (x + 4)(x + 2) \\ x^2 - 4x - 12 &= x^2 - (6 - 2)x - 12 \\ &= x^2 - 6x + 2x - 12 \\ &= x(x - 6) + 2(x - 6) \\ &= (x - 6)(x + 2) \end{aligned}$$

$\therefore$ H.C.F. $= x + 2$

Now, $x + 2 = 0$

$\Rightarrow x = -2$

So, -2 is the only common zero of the given polynomials.

iv) $\boldsymbol{2x^2 + 7x + 3}$**and**$\boldsymbol{x^2 + x - 6}$

Solution:

$$\begin{aligned} 2x^2 + 7x + 3 &= 2x^2 + (6 + 1)x + 3 \\ &= 2x^2 + 6x + x + 3 \\ &= 2x(x + 3) + 1(x + 3) \\ &= (x + 3)(2x + 1) \\ x^2 + x - 6 &= x^2 + (3 - 2)x - 6 \\ &= x^2 + 3x - 2x - 6 \\ &= x(x + 3) - 2(x + 3) \\ &= (x + 3)(x - 2) \end{aligned}$$

$\therefore$ H.C.F. $= x + 3$

Now, $x + 3 = 0$

$\Rightarrow x = -3$

Hence, -3 is the only common zero of the given polynomials.

v) $\boldsymbol{x^3 + 2x^2}$ **and** $\boldsymbol{x^3 + x^2 - 2x}$

Solution: $x^3 + 2x^2 = x^2(x+2)$

$x^3 + x^2 - 2x = x(x^2 + x - 2)$

$= x\{x^2 + (2-1)x - 2\}$

$= x\{x^2 + 2x - x - 2\}$

$= x\{x(x+2) - 1(x+2)\}$

$= x(x+2)(x-1)$

$\therefore$ H.C.F. $= x(x+2)$

Equating each of the factor of the H.C.F. to 0, we get $x = 0$ and $x = -2$

Hence, the common zeros of the given polynomials are 0 and -2.

vi) $\boldsymbol{(x-1)^2(x+3)}$ **and** $\boldsymbol{x^3 + 2x^2 - 3x}$

Solution: First polynomial $= (x-1)^2(x+3)$

Second polynomial $= x^3 + 2x^2 - 3x$

$= x(x^2 + 2x - 3)$

$= x\{x^2 + (3-1)x - 3\}$

$= x(x^2 + 3x - x - 3)$

$= x\{x(x+3) - 1(x+3)\}$

$= x(x+3)(x-1)$

$\therefore$ H.C.F. $= (x-1)(x+3)$

Equating each of the factor of the H.C.F. to 0, we get $x - 1 = 0$ and $x + 3 = 0$

i.e. $x = 1$ and $x = -3$

Hence, the common zeros of the given polynomials are 1 and -3.

vii) $\boldsymbol{4x^3 - 9x}$ **and** $\boldsymbol{2x^2 - x - 3}$

Solution: $4x^3 - 9x = x(4x^2 - 9)$

$= x\{(2x)^2 - 3^2\}$

$= x(2x+3)(2x-3)$

$2x^2 - x - 3 = 2x^2 - (3-2)x - 3$

$= 2x^2 - 3x + 2x - 3$

$= x(2x-3) + 1(2x-3)$

$= (2x-3)(x+1)$

$\therefore$ H.C.F. $= 2x - 3$

Now, $2x - 3 = 0$

$$\Rightarrow x = \frac{3}{2}$$

Hence, $\frac{3}{2}$ is only common zero of the given polynomials.

viii) $\boldsymbol{x^4 - x^3 - 12x^2}$ **and** $\boldsymbol{x^3 - 9x}$

Solution: $x^4 - x^3 - 12x^2 = x^2(x^2 - x - 12)$

$= x^2\{x^2 - (4-3)x - 12\}$

$= x^2(x^2 - 4x + 3x - 12)$

$= x^2\{x(x-4) + 3(x-4)\}$

$= x^2(x-4)(x+3)$

$x^3 - 9x = x(x^2 - 9)$

$= x(x^2 - 3^2)$

$= x(x+3)(x-3)$

H.C.F. $= x(x+3)$

Equating each of the factor of the H.C.F. to 0, we get $x = 0$ and $x + 3 = 0$

i.e. $x = 0$ and $x = -3$

Hence, the common zeros of the given polynomials are 0 and -3.

ix) $\boldsymbol{x^2 + 2x}$ **and** $\boldsymbol{x^2 - 3x - 10}$

Solution: $x^2 + 2x = x(x+2)$

$x^2 - 3x - 10 = x^2 - (5-2)x - 10$

$= x^2 - 5x + 2x - 10$

$= x(x-5) + 2(x-5)$

$= (x-5)(x+2)$

H.C.F. $= x + 2$

Now, $x + 2 = 0$

$\Rightarrow x = -2$

Hence, -2 is the only common zero of the given polynomials.

x) $\boldsymbol{3x^3 + 5x^2 - 2x}$ **and** $\boldsymbol{9x^4 - 6x^3 + x^2}$

Solution: $3x^3 + 5x^2 - 2x = x(3x^2 + 5x - 2)$

$= x\{3x^2 + (6-1)x - 2\}$

$= x(3x^2 + 6x - x - 2)$

$= x\{3x(x+2) - 1(x+2)\}$

$= x(x+2)(3x-1)$

$$
\begin{aligned}
9x^4 - 6x^3 + x^2 &= x^2(9x^2 - 6x + 1) \\
&= x^2\{(3x)^2 - 2.3x.1 + 1^2\} \\
&= x^2(3x-1)^2 \\
\therefore \text{H.C.F.} &= x(3x-1)
\end{aligned}
$$

Equating each of the factor of the H.C.F. to 0, we get $x = 0$ and $3x - 1 = 0$

i.e. $x = 0$ and $x = \frac{1}{3}$

Hence, the common zeros of the given polynomials are 0 and $\frac{1}{3}$.

3. Find the common zero(s) of the following polynomials.

i) $x^2 - 3x, x^2 - 4x + 3$ and $x^3 - 27$

Solution:

$$
\begin{aligned}
\text{First polynomial} &= x^2 - 3x \\
&= x(x-3) \\
\text{Second polynomial} &= x^2 - 4x + 3 \\
&= x^2 - (3+1)x + 3 \\
&= x^2 - 3x - x + 3 \\
&= x(x-3) - 1(x-3) \\
&= (x-3)(x-1) \\
\text{Third polynomial} &= x^3 - 27 \\
&= x^3 - 3^3 \\
&= (x-3)(x^2 + x.3 + 3^2) \\
&= (x-3)(x^2 + 3x + 9) \\
\therefore \text{H.C.F.} &= x - 3
\end{aligned}
$$

Now, $x - 3 = 0$

$\Rightarrow x = 3$

Hence, 3 is the only common zero of the given polynomials.

ii) $(2x-3)^2(x^2 - x - 2), 2x^2 - 7x + 6$ and $(x-2)^2(2x-3)$

Solution:

$$
\begin{aligned}
\text{First polynomial} &= (2x-3)^2(x^2 - x - 2) \\
&= (2x-3)^2\{x^2 - (2-1)x - 2\} \\
&= (2x-3)^2(x^2 - 2x + x - 2) \\
&= (2x-3)^2\{x(x-2) + 1(x-2)\} \\
&= (2x-3)^2(x-2)(x+1)
\end{aligned}
$$

Second polynomial $= 2x^2 - 7x + 6$

$= 2x^2 - (4+3)x + 6$

$= 2x^2 - 4x - 3x + 6$

$= 2x(x-2) - 3(x-2)$

$= (x-2)(2x-3)$

Third polynomial $= (x-2)^2(2x-3)$

$\therefore$ H.C.F. $= (x-2)(2x-3)$

Equating each of the factor of the H.C.F. to 0, we get $x - 2 = 0$ and $2x - 3 = 0$

i.e. $x = 2$ and $x = \frac{3}{2}$.

Hence, the common zeros of the given polynomials are 2 and $\frac{3}{2}$.

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iii) $\boldsymbol{x^4 - 4x^2, x^4 + 2x^3 - 8x^2}$ **and** $\boldsymbol{x^3 - 4x^2 + 4x}$

Solution: First polynomial $= x^4 - 4x^2$

$= x^2(x^2 - 4)$

$= x^2(x^2 - 2^2)$

$= x^2(x+2)(x-2)$

Second polynomial $= x^4 + 2x^3 - 8x^2$

$= x^2(x^2 + 2x - 8)$

$= x^2\{x^2 + (4-2)x - 8\}$

$= x^2(x^2 + 4x - 2x - 8)$

$= x^2\{x(x+4) - 2(x+4)\}$

$= x^2(x+4)(x-2)$

Third polynomial $= x^3 - 4x^2 + 4x$

$= x(x^2 - 4x + 4)$

$= x(x^2 - 2.x.2 + 2^2)$

$= x(x-2)^2$

$\therefore$ H.C.F. $= x(x-2)$

Equating each of the factor of the H.C.F. to 0, we get $x = 0$ and $x - 2 = 0$

i.e. $x = 0$ and $x = 2$

Hence, the common zeros of the given polynomials are 0 and 2.

iv) $\mathbf{24(x^3-9x), 18(x^4+x^3-6x^2)}$ **and** $\mathbf{30x^3(x+3)^2}$

Solution: First polynomial $= 24(x^3-9x)$

$= 2^3 \times 3x(x^2-9)$

$= 2^3 \times 3x(x^2-3^2)$

$= 2^3 \times 3x(x+3)(x-3)$

Second polynomial $= 18(x^4+x^3-6x^2)$

$= 2 \times 3^2.x^2\ (x^2+x-6)$

$= 2 \times 3^2.x^2(x^2+3x-2x-6)$

$= 2 \times 3^2.x^2\{x(x+3)-2(x+3)\}$

$= 2 \times 3^2.x^2(x+3)(x-2)$

Third polynomial $= 30x^3(x+3)^2$

$= 2 \times 3 \times 5x^3(x+3)^3$

$\therefore$ H.C.F. $= 2 \times 3x(x+3) = 6x(x+3)$

Equating each of the factor of the H.C.F. to 0, we get $x=0$ and $x+3=0$ i.e. $x=0$ and $x=-3$.

Hence, the common zeros of the given polynomials are 0 and -3.

v) $\mathbf{8(x-1)^2(x^2-5x+6)}$ **,** $\mathbf{12(x-2)^2(x^2-4x+3), 40(x-3)^2(x^2-3x+2)}$

Solution: $8(x-1)^2(x^2-5x+6) = 2^3(x-1)^2(x^2-3x-2x+6)$

$= 2^3(x-1)^2\{x(x-3)-2(x-3)\}$

$= 2^3(x-1)^2(x-3)(x-2)$

$12(x-2)^2(x^2-4x+3) = 2^2 \times (x-2)^2(x^2-3x-x+3)$

$= 2^2 \times (x-2)^2\{x(x-3)-1(x-3)\}$

$= 2^2 \times 3(x-2)^2(x-3)(x-1)$

$40(x-3)^2(x^2-3x+2) = 2^3 \times 5(x-3)^2(x^2-2x-x+2)$

$= 2^3 \times 5(x-3)^2\{x(x-2)-1(x-2)\}$

$= 2^3 \times 5(x-3)(x-2)(x-1)$

$\therefore$ H.C.F. $= 2^2(x-1)(x-2)(x-3)$

$= 4(x-1)(x-2)(x-3)$

Equating each of the factor of the H.C.F. to 0, we get $x-1=0, x-2=0$ and $x-3=0$ i.e. $x=1, x=2$ and $x=3$.

Hence, the common zeros of the given polynomials are 1, 2 and 3.

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