

CHAPTER 5 INTRODUCTION TO EUCLID'S GEOMETRY

Study of Geometry

To study Geometry in a systematic way Euclid made a scheme. This consists of

- i) giving some entities in the form of definitions.
- enunciating nine general axioms. ii)
- iii) stating five postulates which are axioms related to Geometry.

Definition of Some Entities

- A Point: A point in a plane is that whose position is known but having no length, no breadth and no thickness. In other words a point has no dimension.
- A Line: A line straight or curved an entity having length but no breadth and no thickness. A line is said to have one dimension. The ends of a line are points.
- A Surface: A surface is that entity having both length and breadth but no thickness. A surface has two dimensions. The edges of surfaces are lines.
- A Solid: A solid is a material body having length, breadth and thickness. A solid has three dimensions. A solid is bounded by surfaces. EDUCATION (S)

Nine General Axioms

Axioms are self evident statements which need no proofs

Axiom 1: Things which are equal to the same thing are equal to one another.

Axiom 2: Addition

If equals are added to equals, the sums are equal.

Axiom 3: Subtraction

If equal are taken away from equals, the remainders are equal.

Axiom 4: Multiplication

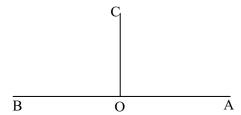
Things which are multiples of equals are equal to one another.

Axiom 5: Division

Things which are equal parts of the same are equal to one another.



Axiom 6:



If O is a point in a straight line AB, then a line OC, which turns about O from the position OA to the position OB must pass through one and only one position in which it is perpendicular to AB

Axiom 7: Every straight line of finite length i.e. a line segment has one and only one point of bisection.

Axiom 8: Every angle has one and only one o internal bisector.

Axiom 9: Axioms of Superposition

Magnitudes (entities) which can be made to coincide with one another are equal.

Euclid's Five Postulates

Postulate 1:

- i) A straight line may be drawn from point to another point.
- Given two distinct points, there is one and only one line through them i.e. a unique line passes ii) through two distinct points.

Postulate 2: A terminated line can be produced indefinitely.

A terminated line is now called a line segment.

Postulate 3: A circle can be drawn with any centre and any radius.

Postulate 4: All right angles are equal to one another.

Postulate 5: If a straight line falling on two straight lines makes the interior angles on the same side taken together less than two right angles, then the two straight lines, if produced indefinitely, meet on the same side on which the sum of the angles is less than two right nment of Manipul PARTMEN angles.

Another form of this postulate is as follows:

Given a straight line and fixed point on the line, there is only one line passing through the point and parallel to the given line.

Two distinct intersecting lines cannot be parallel to the same line.

Theorem: Two distinct straight lines cannot have more than one point in common.

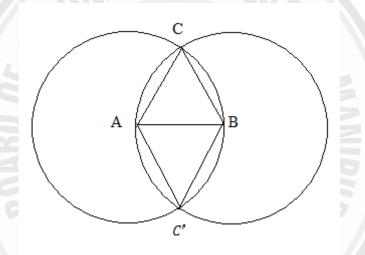


SOLUTIONS

EXERCISE 5.1

1. Using the postulate of drawing a circle, show that an equilateral triangle can be drawn with a given line segment as a side.

Solution:



By postulate of drawing a circle, we know that a circle can be drawn with any centre and any radius.

AB is a line segment. We can draw a circle with centre A and radius AB. We can draw another circle with centre Band radius BA intersecting the first Circle at C and C'. We join AC, AC', BC and BC'.

$$AB = AC = AC'$$
 [same radii of circle with centre A]

and BA = BC = BC' [same radii of circle with centre B]

$$AB = AC = AC' = BC = BC'$$

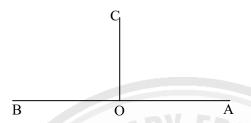
So, \triangle ABC and \triangle ABC' are equilateral triangles.

Thus, an equilateral triangle can be drawn with a given line segment as a side.



2. Given a line segment AB and a point C not on it, using appropriate axiom show that there is only one perpendicular from C on AB.

Solution:



By Euclid's axiom 6 we know that if O is a point in a straight line AB, then a line OC, which turns about O from the position OA to the position OB must pass thorough one and only one position in which is perpendicular to AB.

If C is the one and only position, out of the line segments that can be drawn from C on AB, OC is the only line segment which is perpendicular to AB.

Thus, there is only one perpendicular from C on AB.

3. Using Euclid's Postulate 5, show that there exists a line parallel to a given line.

Solution:

By Euclid's postulate 5, we know that if a straight line falling in two straight lines makes the interior angles on the same side taken together less than two right angles, if produced indefinitely, meet on the same side on which the sum of the angle is less than two right angles. If the sum of the two interior angles on the same side is equal to two right angles, the two lines if produced definitely will never meet at any point. So, the two lines will be parallel.

Thus there exists a line parallel a given line
