CHAPTER 8
QUADRILATERAL

Properties of Parallelogram

Theorem: A diagonal of a parallelogram divides it into two congruent triangles.

Theorem: Opposite sides of a parallelogram are equal.

Theorem: A quadrilateral is a parallelogram if its opposite sides are equal.

Theorem: Opposite angles of a parallelogram are equal.

Theorem: A quadrilateral is a parallelogram if its opposite angles are equal.

Theorem: A quadrilateral is a parallelogram if it has a pair of parallel and equal sides.

Theorem: The diagonals of a parallelogram bisect each other.

Theorem: A quadrilateral is a parallelogram if its diagonals bisect each other.

SOLUTIONS

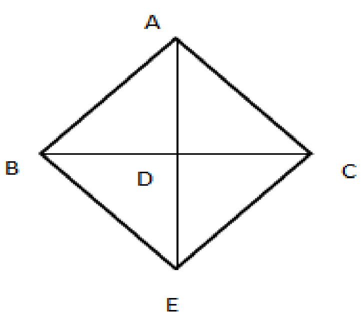
EXERCISE 8.1

1. In a ΔABC, median AD is produced to E such that $AD = BE$. Prove that $ABEC$ is a parallelogram.

Solution:

Given: In ΔABC, the median AD is produced to E such that $AD = DE$.

To Prove: $ABEC$ is a parallelogram.



Proof: As AD is a median in ΔABC,

$\therefore\ BD = CD$

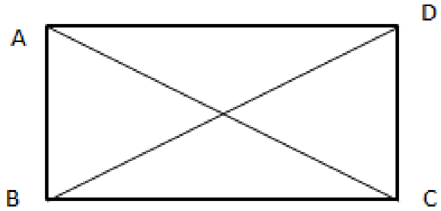
But, $AD = DE$ [Given]

So, AE and BC bisect each other at D.

As the diagonals AE and BC bisect each other at, $ABEC$ is a parallelogram.

2. Show that each angle of a rectangle is a right angle.

Solution:



Given: $ABCD$ is a rectangle.

To Prove: $\angle A = \angle B = \angle C = \angle D = 90°$

Construction: We join AC.

Proof: We know that the diagonals of a rectangle are equal .

$$\therefore \ AC = BD$$

In $\Delta\, ABC$ and $\Delta\, DCB$,

$AB = CD$ [Opposite sides of rectangle]

$AC = BD$

$BC = BC$

$\therefore \ \Delta ABC \cong \Delta DCB$ [SSS]

$\Rightarrow \angle B = \angle C$

Similarly, $\Delta\, BCD \cong \Delta\, ADC$

$\Rightarrow \angle C = \angle D$

And , $\Delta\, BAD \cong \Delta\, CDA$

$\Rightarrow \angle A = \angle D$

$\therefore \ \angle A = \angle B = \angle C = \angle D$

In the quadrilateral $ABCD$,

$$\angle A + \angle B + \angle C + \angle D = 360°$$

$$\Rightarrow \angle A + \angle A + \angle A + \angle A = 360°$$

$$\Rightarrow 4\angle A = 360°$$

$$\Rightarrow 4\angle A = \frac{360°}{4} = 90°$$

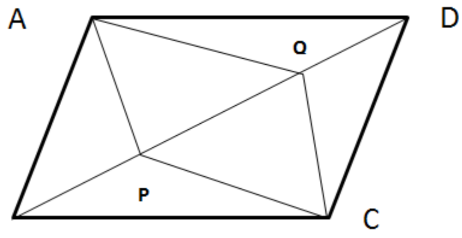
$$\therefore \ \angle A = \angle B = \angle C = \angle D = 90°$$

3. **$ABCD$ is a parallelogram and P, Q are points on the diagonal BD such that $BP = DQ$. Prove that $APCQ$ is a parallelogram.**

Solution:

Given: P, Q are points on the diagonal BD of a parallelogram $ABCD$ such that $BP = DQ$.

Proof: In $\Delta\ APB$ and $\Delta\ CQD$



$AB = CD$

$BP = DQ$ [Given]

$\angle ABP = \angle CDQ$ [Alternate angle]

$\therefore\ \Delta\ APB \cong \Delta\ CQD$ [SAS]

$\Rightarrow AP = CQ$

Similarly, $\Delta\ BPC \cong \Delta\ DQA$

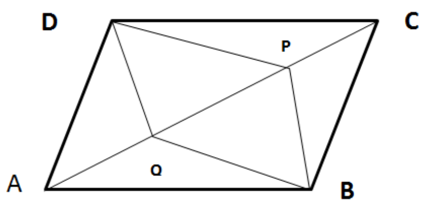
$\Rightarrow CP = AQ$

As the opposite sides of $APCQ$ are equal, $APCQ$ is a parallelogram.

4. **$ABCD$is a parallelogram in which the perpendiculars BP and DQ are drawn on the diagonal AC from the points B and D respectively. Prove that $BPDQ$ is a parallelogram.**

Solution:

Given: BP and DQ are perpendiculars from B and D on the diagonal AC of a parallelogram$ABCD$.



To Prove: $BPDQ$ is a parallelogram.

Proof: In $\Delta\ AQD$and $\Delta\ CPB$,

$\angle AQD = \angle CPB$ [$= 90°$]

$\angle DAQ = \angle BCP$ [Alternate angles]

$AD = CB$ [Opposite sides of parallelogram]

$\therefore\ \Delta\ AQD \cong \Delta\ CPB$ [AAS congruence]

$\Rightarrow DQ = BP$

But DP and BP are perpendicular to AC.

$\therefore\ DQ \parallel BP$

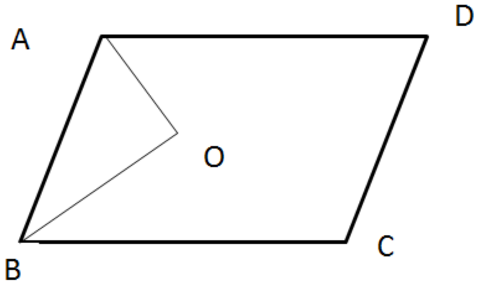
As a pair of opposite sides are equal and parallel, $BPDQ$ is a parallelogram.

5. ***ABCD* is a parallelogram. Bisector of consecutive angles $\angle A$ and $\angle B$ intersect at O. Prove that $\angle AOB = 90^0$.**

Solution:

Given: Bisectors of two consecutive angles $\angle A$ and $\angle B$ of parallelogram $ABCD$ intersect at O.

To Prove: $\angle AOB = 90^0$.



Proof: $\angle A + \angle B = 180°$

$\Rightarrow \frac{1}{2}(\angle A + \angle B) = \frac{1}{2} \times 180°$

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$

$\Rightarrow \angle OAB + \angle OBA = 90°$ $[\because \angle OAB = \frac{1}{2}\angle B, \angle OBA = \frac{1}{2}\angle B]$

In Δ AOB,

$\angle AOB + \angle OAB + \angle OBA = 180°$

$\Rightarrow \angle AOB + 90° = 180°$

$\Rightarrow \angle AOB = 180° - 90° = 90°$

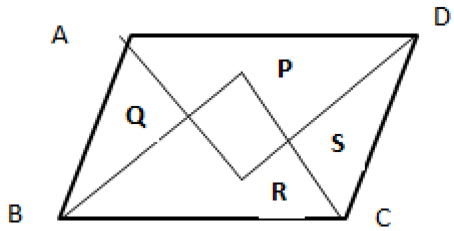
$\therefore \angle AOB = 90°$

6. Show that the bisectors of the angles of a parallelogram form a rectangle.

Solution:

Given: In the parallelogram $ABCD$, the bisectors of $\angle A, \angle B, \angle C, \angle D$ form PQRS.

To Prove: $PQRS$ is a rectangle.



Proof: $\angle A + \angle B = 180°$

$\Rightarrow \frac{1}{2}(\angle A + \angle B) = \frac{1}{2} \times 180°$

$\Rightarrow \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90°$

$\Rightarrow \angle QAB + \angle QBA = 90°$ $[\because \angle QAB = \frac{1}{2}\angle A, \angle QBA = \frac{1}{2}\angle B]$

In Δ AQB,

$\angle AQB + \angle QAB + \angle QBA = 180°$

$\Rightarrow \angle AQB = 180° - 90° = 90°$

$\angle PQR = \angle AQB$ [Vertically opposite angles]

$\therefore \angle PQR = 90°$

Similarly, $\angle QRS = 90°$,

$\angle RSP = 90°$

And $\angle QPS = 90°$

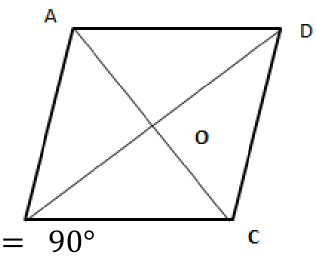
$\therefore\ \angle PQR = \angle QRS = \angle RSP = \angle QPS = 90°$

As all the four angles of $PQRS$ are right angle each, $PQRS$ is a rectangle.

7. If the diagonals of a quadrilateral bisect each other at right angles, show that it is a rhombus.

Solution:

Given: $ABCD$ is a quadrilateral in which the diagonals AC and BD bisect each other at right angles at O.



To Prove: $ABCD$ is a rombus.

Proof: As AC and BD bisect at right angles at O.

$OA = OC$, $OB = OD$

$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

In $\Delta\ AOD$ and $\Delta\ COB$,

$OA = OC$

$OD = OB$

$\angle AOD = \angle COB$ [$= 90°$]

$\therefore\ \Delta\ AOD \cong \Delta\ COB$ [SAS congruence]

$\Rightarrow\ \angle OAD = \angle OCB$

$\Rightarrow\ \angle CAD = \angle ACB$

As a pair of alternate angles are equal,

$AD \parallel BC$

Similarly, $AB \parallel CD$

$\therefore$ $ABCD$ is a parallelogram.

In $\Delta\ AOB$ and $\Delta\ COB$,

$OA = OC$

$OB = OB$

$\angle AOB = \angle COB$ [$= 90°$]

$\therefore\ \Delta\ AOB \cong \Delta\ COB$ [SAS congruence]

$\Rightarrow\ AB = BC$

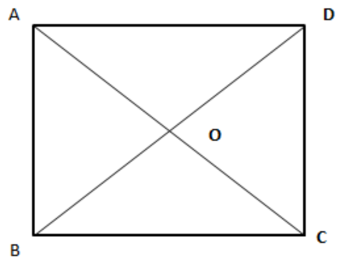
As two consecutive sides of parallelogram $ABCD$ are equal, $ABCD$ is a rhombus.

8. Show that the diagonals of a square are equal and bisect each other at right angles.

Solution:

Given: $ABCD$ is a square in which the diagonals AC and BD intersect at O.

To Prove: AC and BD bisect each other at right angles.



Proof: In $\Delta\ ABC$ and $\Delta\ DCB$,

$AB = DC$ [Sides of square]

$BC = CB$

$\angle ABC = \angle DCB$ [$= 90°$]

$\therefore\ \Delta ABC \cong \Delta DCB$ [SAS]

$\Rightarrow AC = BD$

In ΔAOD and ΔCOB,

$AD = CB$

$\angle OAD = \angle OCB$ [Alternate angles]

$\angle ODA = \angle OBC$

$\therefore\ \Delta AOD \cong \Delta COB$ [ASA congruence]

$\Rightarrow OA = OC$ and $OD = OB$

In ΔAOB and ΔCOB,

$OA = OC$

$OB = OB$

$AB = CB$

$\therefore\ \Delta AOB \cong \Delta COB$ [SSS congruence]

$\Rightarrow \angle AOB = \angle COB$

But, $\angle AOB + \angle COB = 180°$ [Linear pair angles]

$\Rightarrow \angle AOB + \angle AOB = 180°$

$\Rightarrow 2\angle AOB = 180°$

$\Rightarrow \angle AOB = \dfrac{180°}{2} = 90°$

$\therefore AC \perp BD$

Hence, AC and BD bisect each other at right angles.

9. If the diagonals of a quadrilateral are equal and bisect each other at right angles, show that it is a square.

Solution:

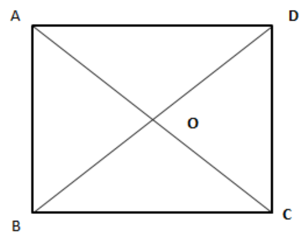
Given: $ABCD$ is a quadrilateral in which the diagonals AC and BD are equal and bisect each other at right angles at O.

To Prove: $ABCD$ is a square.

Proof: As AC and BD are equal and bisect each other at right angles at O,

$$AC = BD, OA = OC, OB = OD$$

$$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$$



In ΔAOB and ΔCOB,

$OA = OC$

$OB = OB$

$\angle AOB = \angle COB$ [= 90°]

$\therefore \Delta AOB \cong \Delta COB$ [SAS congruence]

$\Rightarrow AB = BC$

Similarly $BC = CD$ and $CD = DA$

$\therefore AB = BC = CD = DA$

In ΔABC and ΔDCB,

$AB = DC$

$AC = DB$ [Given]

$BC = CD$

$\therefore \Delta ABC \cong \Delta DCB$ [SSS congruence]

$\Rightarrow \angle B = \angle C$

Similarly, $\angle C = \angle D$ and $\angle D = \angle A$

$\therefore \angle A = \angle B = \angle C = \angle D$

In the quadrilateral $ABCD$,

$$\angle A + \angle B + \angle C + \angle D = 360°$$

$$\Rightarrow \angle A + \angle A + \angle C + \angle D = 360°$$

$$\Rightarrow 4\angle A = 360°$$

$$\Rightarrow \angle A = \frac{(360°)}{4} = 90°$$

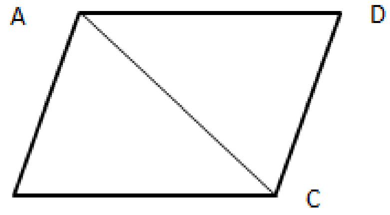
As all the sides are equal in length and all the angles are right angles each, $ABCD$ is a square.

10. If the diagonal AC of a parallelogram bisects $\angle A$, Show that $ABCD$ is rhombus.

Solution:

Given: In the parallelogram $ABCD$, the diagonal AC bisects $\angle A$.

To Prove: $ABCD$ is a rhombus.



Proof: As AC bisects $\angle A$,

$$\angle BAC = \angle DAC$$

As $AD \parallel BC$

$$\angle DAC = \angle ACB \quad \text{[Alternate Angles]}$$

$\therefore \angle BAC = \angle ACB$

As $\angle BAC = \angle ACB$ in ΔABC,

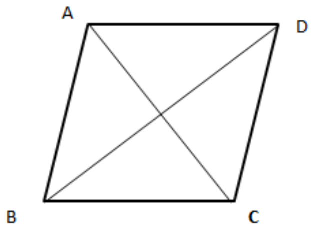
$$BC = AB$$

$\therefore$ Constructive sides of parallelogram $ABCD$ are equal.

So, $ABCD$ is a rhombus.

11. $ABCD$ is a rhombus. Show that the diagonal AC bisects $\angle A$ as well as $\angle C$ and the diagonal BD bisect $\angle B$ as well as $\angle D$.

Solution:



Given: $ABCD$ is a rhombus in which AC and BD are diagonals.

To Prove: AC bisects $\angle A$ as well as $\angle C$

BD bisects $\angle B$ as well as $\angle D$.

Proof: In ΔADC and ΔABC,

$$AD = AB, DC = BC, AC = AC$$

$\therefore \Delta ADC \cong \Delta ABC$ [SSS congruence]

$$\Rightarrow \angle DAC = \angle BAC \text{ and } \angle DCA = \angle BCA$$

So, AC bisects $\angle A$ as well as $\angle C$.

In ΔABD and ΔCBD,

$$AB = AD, CB = CD, BD = BD$$

$\therefore \Delta ABD \cong \Delta CBD$ [SSS congruence]

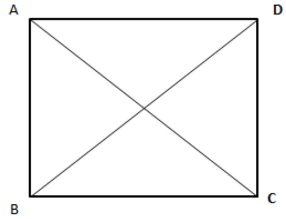
$$\Rightarrow \angle ABD = \angle CBD \text{ and } \angle ADB = \angle CDB$$

So, BD bisects $\angle B$ as well as $\angle D$.

12. **$ABCD$ is a rectangle in which the diagonal AC bisects $\angle A$ as well as $\angle C$. Show that**

i) $ABCD$ is a square.

ii) Diagonal BD bisects $\angle B$ and $\angle D$.



Solution:

Given: $ABCD$ is a rectangle in which the diagonal AC bisects $\angle A$ as well as $\angle C$.

To Prove: i) $ABCD$ is a square.

ii) Diagonal BD bisects $\angle B$ and $\angle D$.

Proof: i) We have, $\angle A = \angle C$ ($= 90^0$) (being angles of a rectangle)

$\Rightarrow \frac{1}{2} \angle A = \frac{1}{2} \angle C$ (as $\angle A = \angle C$, being angles of a rectangle)

$\Rightarrow \angle BAC = \angle BCA$

$\therefore BC = AB$

As two consecutive sides of rectangle $ABCD$ are equal, $ABCD$ is a square.

ii) In $\Delta\ ABD$ and $\Delta\ CBD$,

$AB = CB, AD = CD, BD = BD$

$\therefore\ \Delta\ ABD \cong \Delta\ CBD$ [SSS congruence]

$\Rightarrow \angle ADB = \angle CDB$ and $\angle ABD = \angle CBD$

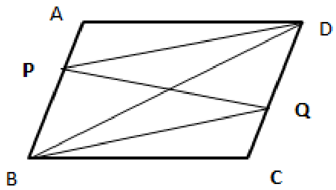
So, BD bisects $\angle B$ as well as $\angle D$.

13. **$ABCD$ is a parallelogram. P and Q are points on AB and DC respectively such that $AP = CQ$. Prove that PQ and BD bisect each other.**

Solution:

Given: P and Q are points on AB and DC respectively such that $AP = CQ$.

To Prove: PQ and BD bisect each other.



Proof: $AP = CQ$

and $AB = CD$ [Opposite sides of Parallelogram]

$\therefore$ $AB - AP = CD - CQ$

$\Rightarrow BP = DQ$

and $BP \parallel DQ$ [$\because AB \parallel CD$

As a pair of opposite sides are equal and parallel, $PBQD$ is a parallelogram.

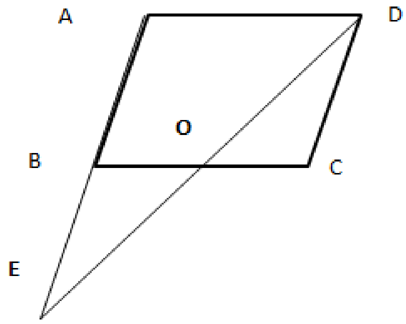
As the diagonals of parallelogram bisect each other, the diagonals PQ and BD of parallelogram $PBQD$ bisect each other.

14. $ABCD$ is a parallelogram. The side AB is produced to E such that $AB = BE$. Prove that ED bisects BC.

Solution:

Given: The side AB of parallelogram $ABCD$ is produced to E such that $AB = BE$. We join DE intersecting BC at O.

To Prove: $OB = OC$.



Proof: $AB = BE$ [Given]

But, $AB = CD$ [Sides of Parallelogram]

$\therefore BE = CD$

In ΔBOE and ΔCOD,

$BE = CD$

$BE = CD$

$\angle BOE = \angle COD$ [Vertically Opposite Angles]

$\angle OBE = \angle OCD$ [Alternate Angles]

$\therefore \Delta BOE \cong \Delta COD$ [AAS congruence]

$\Rightarrow OB = OC$

15. ABC and DEF are two triangles such that AB and BC are respectively equal and parallel to DE and EF. Vertices A, B and C are joined to vertices D, E and F. Show that :

i) $ABED$ is a parallelogram.

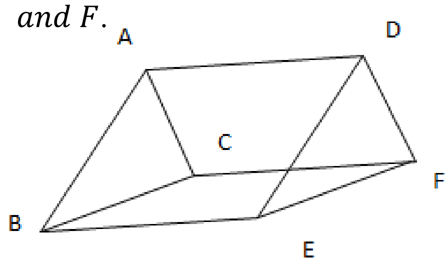
ii) $BEFC$ is a parallelogram.

iii) $ACFD$ is a parallelogram.

iv) $\Delta ABC \cong \Delta DEF$

Solution:

Given: In ΔABC and ΔDEF, AB and BC are equal and parallel to DE and EF. Vertices A, B and C are joined to vertices D, E and F.



To Prove: i) $ABED$ is a parallelogram.

i) $BEFC$ is a parallelogram.

ii) $ACFD$ is a parallelogram.

iii) $\Delta ABC \cong \Delta DEF$

Proof : i) $AB = DE$ and $AB \parallel DE$

So, a pair of opposite sides of $ABED$ are equal and parallel,

$\therefore ABED$ is a parallelogram.

ii) $BC = EF$ and $BC \parallel EF$

So, a pair of opposite sides of $BEFC$ are equal and parallel,

$\therefore$ $BEFC$ is a parallelogram.

iii) As $ABED$ is a parallelogram.

AD $= BE$ and $AD \parallel BE$

As $BEFC$ is a parallelogram.

$\therefore$ $BE = CF$ and $BE \parallel CF$

So, a pair of opposite sides of $ACFD$ are equal and parallel,

$\therefore$ $ACFD$ is a parallelogram.

iv) In ΔABC and ΔDEF,

$AB = DE$

$BC = EF$

$AC = DF$ [Opposite sides of Parallelogram ACFD]

$\therefore$ $\Delta ABC \cong \Delta DEF$ [SSS congruence]

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Mid – Point Theorem:

The line segment joining the mid-points of two sides of a triangle is parallel to the third side and is half of it in length.

Converse of Mid-Point Theorem

The line drawn through the mid- point of one side of a triangle parallel to another side bisects the third side.

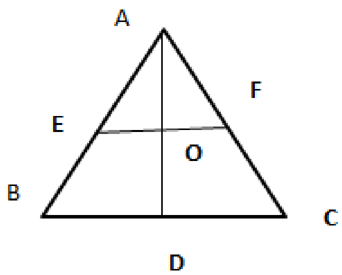
SOLUTIONS

EXERCISE 8.2

1. Prove that any line segment drawn from a vertex of a triangle to a point on the opposite side is bisected by the segment joining the middle points of the other two sides.

Solution:

Given: AD is a line segment joining the vertex A of $\Delta\, ABC$ to a point D on BC. E and F are the mid-points of AB and AC. EF intersects AD at O.



To Prove: $OA = OD$

Proof: As E and F are the mid-points of AB and AC in $\Delta\, ABC$,

$$EF \parallel BC$$

$$\Rightarrow EO \parallel BD$$

In $\Delta\ ABD$, E is the mid- point of AB and $EO \parallel BD$.

$\therefore$ O is the mid –point of AD.

$$\Rightarrow OA = OD$$

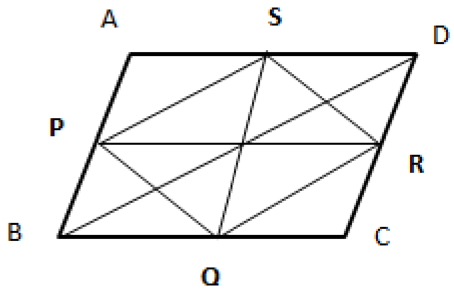
So, $OA = OD$.

2. Prove that the quadrilateral formed by joining the mid-points of the sides of a rhombus is a rectangle.

Solution:

Given: P, Q, R, S are the mid-points of the sides AB, BC, CD, and DA of a rhombus ABCD.

To prove: PQRS is a rectangle.



Construction: BD, PR and QS are joined.

Proof: In ΔABD,

P and S are mid-points of AB and AD.

$\therefore PS \parallel BD$ and $PS = \frac{1}{2}BD$.

In ΔBCD,

Q and R are the mid-points of BC and CD.

$\therefore QR \parallel BD$ and $QR = \frac{1}{2}BD$

So, $PS \parallel QR$ and $PS = QR$

As a pair of opposite sides of PQRS are equal and parallel,

PQRS is a parallelogram.

$AD \parallel BC \Rightarrow AS \parallel BQ$

And $AD = BC \Rightarrow \frac{1}{2}AD = \frac{1}{2}BC \Rightarrow AS = BQ$

As a pair of opposite sides of ABQS are equal and parallel,

ABQS is a parallelogram.

$\Rightarrow AB = QS$

Similarly, PBCR is a parallelogram.

$\Rightarrow BC = PR$

But, $AB = BC$ [sides of rhombus ABCD]

$\therefore QS = PR$

As the diagonals QS and PR of parallelogram PQRS are equal,

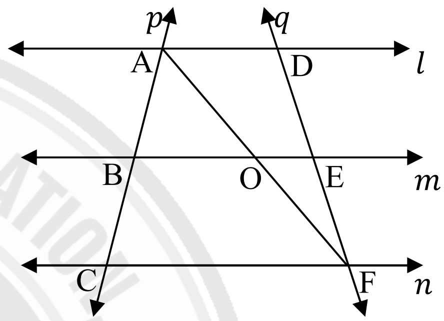
PQRS is a rectangle.

3. If there are three parallel lines and the intercepts made by them on a transversal are equal in length, prove that the intercepts on any other transversal are also equal in length.

Solution:

Given: A transversal p intersects three parallel lines l, m, n at A, B, C such that $AB = BC$. Another transversal q intersects l, m, n at D, E, F.

To prove: $DE = EF$



Construction: We join AF intersecting BE at O.

Proof: In ΔACF,

B is the mid-point of AC and $BO \parallel CF$.

$\therefore O$ is the mid-point of AF.

In ΔADF,

O is the mid-point of AF and $OE \parallel AD$.

$\therefore E$ is the mid-point of DF

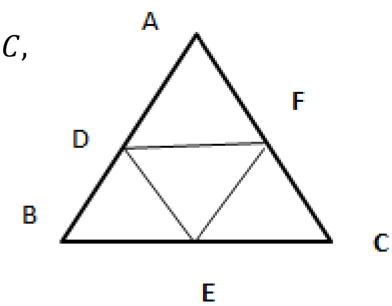
So, $DE = EF$.

4. In $\Delta ABC, D, E$ and F are mid-points of the sides AB, BC and CA respectively. Show that ΔABC is divided into four congruent triangles by joining D, E and F.

Solution:

Given: D, E and F are the mid-points of the sides AB, BC and CA of ΔABC.

To prove: D, E, F divide ΔABC into four congruent triangles.



Proof: As D and F are the mid-points of AB and AC in ΔABC,

$DF \parallel BC \Rightarrow DF \parallel BE$

As E and F are the mid-points of BC and AC in ΔABC,

$EF \parallel AB$

$\Rightarrow EF \parallel BD$.

As the opposite sides are parallel, $\Delta DBEF$ is a parallelogram.

As DE is diagonal in parallelogram $DBEF$, $\Delta DEF \cong \Delta DBE$

Similarly, $\Delta DEF \cong \Delta ADF$

And $\Delta DEF \cong \Delta ECF$

$\therefore \Delta DEF \cong \Delta DBE \cong \Delta ADF \cong \Delta ECF$

So, D, E, F divide ΔABC into four congruent triangles.

5. **$ABCD$ is a rectangle and P, Q, R, S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral $PQRS$ is a rhombus.**

Solution:

Given: P, Q, R, S are mid-points of the sides AB, BC, CD, DA of a rectangle $ABCD$.

To prove: $PQRS$ is a rhombus.

Proof: In ΔPAS and ΔPBQ,

$AP = BP$ [$\because$ P is the mid-point of AB]

$AS = BQ$ [$\because \frac{AD}{2} = \frac{BC}{2}$]

$\angle PAS = \angle PBQ$ [$= 90°$]

$\therefore \Delta PAS \cong \Delta PBQ$ [SAS congruence]

$\Rightarrow PS = PQ$.

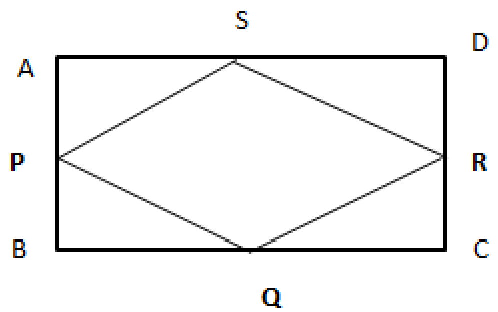
Similarly, $\Delta QBP \cong \Delta QCR$

$\Rightarrow PQ = QR$

And, $\Delta RCQ \cong \Delta RDS$

$\Rightarrow QR = RS$

$\therefore PS = PQ = QR = RS$



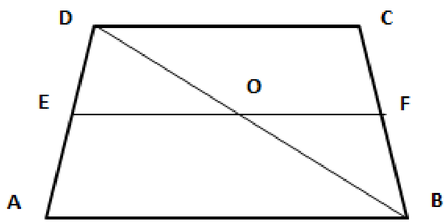
As all the four sides are equal, $PQRS$ is a rhombus.

6. **$ABCD$is a trapezium in which $AB \parallel DC$, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is the mid-point of BC.**

Solution:

Given: In the trapezium $ABCD$, $AB \parallel DC$ and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting the diagonal BD at O and the side BC at F.

To prove: $BF = CF$



Proof: In ΔABD, E is the mid-point of AD and $OE \parallel AB$,

$\therefore$ $O$is the mid-point of BD.

In ΔBCD, O is the mid-point of BD and $OF \parallel DC$.

$\therefore$ F is the mid-point of BC.

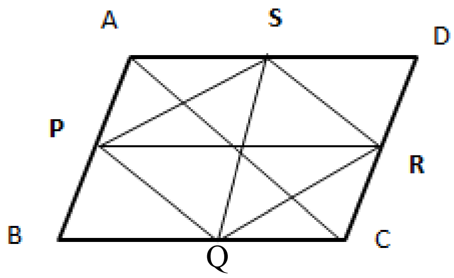
So, $BF = CF$.

7. Show that the line segment joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution:

Given: P, Q, R, S are mid-points of the sides BC, CD, DA of a quadrilateral $ABCD$.

To prove: PR and QS bisect each other.



Construction: We join AC.

Proof: In $\Delta ABC, P$ and Q are the mid-points of AB and BC.

$\therefore PQ \parallel AC \text{and} PQ = \frac{1}{2}AC$ ----------------(1)

In $\Delta ADC, S$ and R are the mid-points of AD and CD.

$\therefore SR \parallel AC \text{and} SR = \frac{1}{2}AC$ ------------------(2)

From (1) and (2),

$PQ \parallel SR \text{and} PQ = SR$

As a pair of opposite sides are equal and parallel, $PQRS$ is a parallelogram. PRandQS are the diagonals of parallelogram $PQRS$.

So, PR and QS bisect each other.

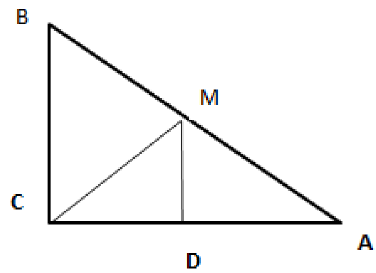
8. $ABC$is a triangle right angled at C. A line through the mid-point M of the hypotenuse AB and parallel to BC intersects AC at D.

Show that

i) D is the mid-point of AC.
ii) $MD \perp AC$
iii) $CM = MA = \frac{1}{2}AB$.

Solution:

Given: ABC is a right triangle right angled at C. M is the mid-point of the hypotenuse AB and $MD \parallel BC$ intersects AC at D.



To prove: i) $D$is mid-point of AC.
ii) $MD \perp AC$.
iii) $CM = MA = \frac{1}{2}AB$.

Proof: i) In $\Delta ABC, M$ is the mid-point of AB and $MD \parallel BC$.

$\therefore D$ is the mid-point of AC.

ii) As $MD \parallel BC$

$\angle ACB = \angle ADM$ [corresponding angles]

But, $\angle ACB = 90°$

$\therefore \angle ADM = 90°$

$\Rightarrow MD \perp AC.$

iii) As $\angle ACB = 90°$,

The circle drawn with AB as diameter will pass through C.

$\therefore CM = AM = BM$ [radii of a same circle]

But, $AM = \frac{AB}{2}$

So, $CM = MA = \frac{1}{2}AB.$

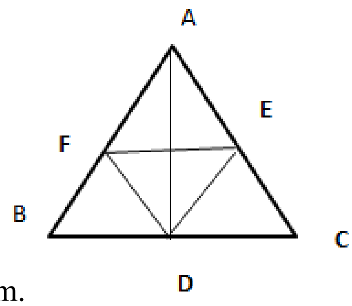
9. ABC is an isosceles triangle in which $AB = AC$ and D, E, F are mid-points of the sides BC, CA, AB respectively. Prove that AD and EF bisect one another at right angles.

Solution:

Given: In the isosceles $\Delta ABC, AB = AC, D, E, F$ are mid-points of BC, AC and AB.

To prove: AD and EF bisect one another at right angles.

Proof: As D and E are mid-points of BC and AC in ΔABC,



$DE \parallel AB \Rightarrow DE \parallel AF$

As D and F are mid-points of BC and AB in ΔABC,

$DF \parallel AC \Rightarrow DF \parallel AE$

As the opposite sides are parallel, $AFDE$ is a parallelogram.

$AB = AC$

$\Rightarrow \frac{1}{2}AB = \frac{1}{2}AC$

$\Rightarrow AF = AE$

Two consecutive sides AF and AE of parallelogram $AFDE$ are equal.

$\therefore AFDE$ is a rhombus.

We know that the diagonals of a rhombus bisect one another at right angles.

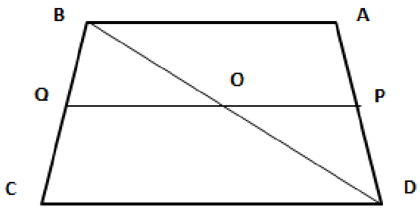
$\therefore AD$ and EF, the diagonals of rhombus $AFDE$ bisect one another at right angle.

10. ***ABCD* is a trapezium in which $AB \parallel CD$. If P and Q are mid-points of AD and BC respectively. Prove that $PQ \parallel AB$ and $PQ = \frac{1}{2}(AB + CD)$.**

Solution:

Given: In the trapezium $ABCD$, $AB \parallel CD$. P and Q are mid-points of AD and BC.

To prove: $PQ \parallel AB$ and $PQ = \frac{1}{2}(AB + CD)$.



Construction: We join BD intersecting PQ at O.

Proof: As P is mid-point of AD,

$$AP = DP$$

$$\Rightarrow \frac{AP}{DP} = 1$$

As Q is the mid-point of BC,

$$BQ = CQ$$

$$\Rightarrow \frac{BQ}{CQ} = 1$$

$$\therefore \quad \frac{AP}{DP} = \frac{BQ}{CQ}$$

As $\frac{AP}{DP} = \frac{BQ}{CQ}$ and $AB \parallel CD$,

$$PQ \parallel AB \parallel CD$$

Now, in ΔBCD, Q is the mid-point of BC and QO ∥ CD.

∴ O is mid-point of BD.

In ΔBCD, Q and O are mid-points of BC and BD.

$$\therefore \quad OQ = \frac{1}{2}CD$$

In ΔDAB, P and O are mid-points of AD and BD

$$\therefore \quad OP = \frac{1}{2}AB.$$

So, $PQ = OP + OQ$

$$= \frac{1}{2}AB + \frac{1}{2}CD$$

$$= \frac{1}{2}(AB + CD).$$

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