# CHAPTER 9
# AREA

- **Area:** Any closed curve encloses an amount of surface. That amount of the surface is called the area enclosed by the closed curve.
- **Polygonal Region:** A polygonal region is the region consisting of a polygon and its interior.
  - **Interior of a Triangle:** It is the part of the plane enclosed by the triangle.
  - **Triangular Region:** It is the region consisting of a triangle and its interior.
- **Axioms of Area:** The axioms of area for polygonal regions are given below:

1. Every polygonal region has an area. The area of a polygonal region in square units is a positive real number.
2. Congruent Area Axiom: If $R_1$ and $R_2$ are two polygonal regions such that $R_1 \cong R_2$, then area of $R_1 =$ area of $R_2$.
3. Area Monotone Axiom: If $R_1$ and $R_2$ are two polygonal regions such that $R_1$ is contained in $R_2$, then area of $R_1 <$ area of $R_2$.
4. Area Addition Axiom: If $R_1$ and $R_2$ are two polygonal regions with only a finite number of points or line segments in common and they together form a region $R$, then area of $R =$ area of $R_1 +$ area of $R_2$.
5. Rectangular Area Axiom: For a rectangle ABCD, given that AB = $a$ units and AD = $b$ units, then area of the rectangular region ABCD = $ab$ sq. units.

- **Theorems**

1. A diagonal of a parallelogram divides it into two triangles of equal area.
2. Parallelograms on the same base and between the same parallels are equal in area.
3. The area of a parallelogram is the product of any of its sides and the corresponding altitude.

Corollary: Parallelograms on equal bases and between the same parallels are equal in area.

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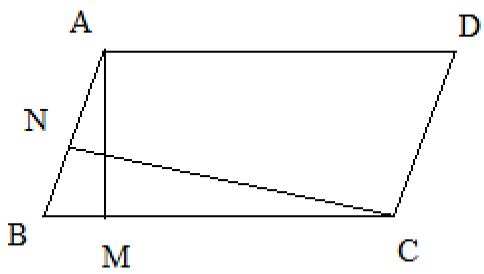
## SOLUTIONS

### EXERCISE 9.1

**1. In a parallelogram ABCD, BC = 10 cm. The altitudes corresponding to the sides BC and AB are respectively 7 cm and 8 cm. Find AB.**

**Solution:** In the parallelogram ABCD, BC = 10 cm. AM ⊥BC and CN ⊥AB, AM= 7cm, CN= 8cm




$$Area\ of\ Par^m\ ABCD = BC \times AM$$

Again, Area of $Par^m\ ABCD = AB \times \mathrm{CN}$

∴ AB× $CN$ = BC× $AM$

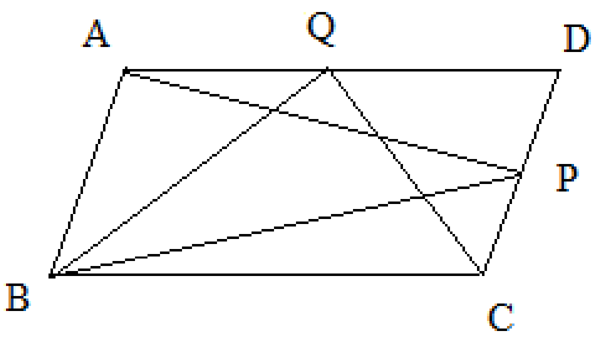
$\Rightarrow AB \times 8 = 10 \times 7$

$$\Rightarrow AB = \frac{10 \times 7}{8} = \frac{35}{4} = 8.75$$

So, AB = 8.75 cm.

**2. P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that area of ΔAPB = area of ΔBQC.**

**Solution:**




**Given:** P and Q are two points lying on the sides DC and AD respectively of a Parallelogram ABCD.

**To prove:** area of ΔAPB = area of ΔBQC

**Proof:** ΔAPB and $Par^m.\,ABCD$ are on the same base AB and between the same parallels.

∴ area of ΔABP $= \frac{1}{2}$ area of $\mathrm{Par^m ABCD}$ ---------------- (1)

ΔBQC and $Par^m.\,ABCD$ are on the same base BC and between the same parallels.

∴ area of ΔBQC $= \frac{1}{2}$ area of $\mathrm{Par^m.\,ABCD}$---------------- (2)

From (1) and (2), we get

Area of Δ APB = Area of ΔBQC.

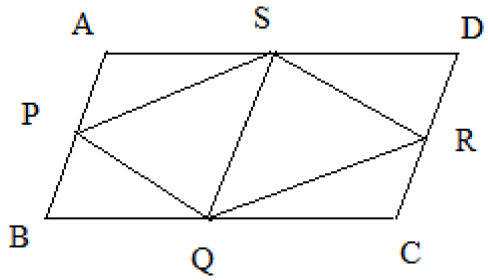
**3. If P, Q, R and S are respectively the mid-points of the sides of a parallelogram ABCD, show that area of quadrilateral PQRS = $\frac{1}{2}$ (area of parallelogram ABCD).**

**Solution:**

**Given:** P, Q, R, S are the mid points of AB, BC, CD and DA of parallelogram ABCD

**To prove:** Area of quadrilateral PQRS= $\frac{1}{2}(Area\ of$ parallelogram $ABCD)$

**Proof**: AD = BC [opposite sides of parallelogram$ABCD$]




$\Rightarrow \quad \frac{1}{2}AD = \frac{1}{2}BC$

$\Rightarrow$AS =BQ

$But \qquad AS \parallel BQ$

As a Pair of opposite sides are equal and parallel, ABQS is a parallelogram.

Similarly, SQCD is a parallelogram.

$\Delta$QPS and parallelogram ABQS are on the same base QS and between the same parallels.

$$\therefore area\ of\ \Delta QPS = \frac{1}{2} area\ of\ \text{parallelogram } ABQS$$

Similarly, $area\ of\ \Delta QRS = \frac{1}{2}\ area\ of$ parallelogram $SQCD$

So, Area of quadrilateral PQRS

$= Area\ of\ \Delta QPS + Area\ of\ \Delta QRS$

$= \frac{1}{2}\ Area\ of$ parallelogram $ABQS + \frac{1}{2}\ Area\ of$ parallelogram $SQCD$

$= \frac{1}{2}(Area\ of$ parallelogram $ABQS + Area\ of$ parallelogram $SQCD)$

$$= \frac{1}{2}\ Area\ of\ \text{parallelogram } ABCD.$$

4. **Prove that the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two parallelograms of equal area.**

**Solution:**

**Given:** P and Q are mid points of the opposite sides AD and BC respectively of a parallelogram ABCD.

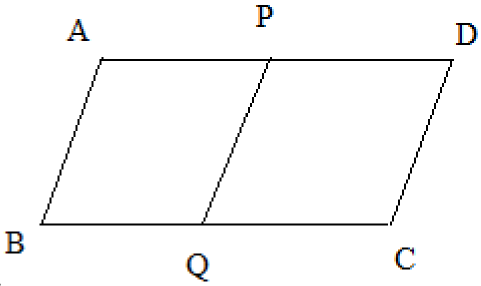
**To Prove:** Area of parallelogram ABQP = area of parallelogram PQCD

**Proof:** We have, AD=BC and AD ∥ BC [being opposite sides of parallelogram]

$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$ and AP ∥ BQ

$\Rightarrow AP = BQ$ and AP ∥ BQ

∴ ABQP is a parallelogram




Similarly, PQCD is also a parallelogram.

Parallelograms ABQP and PQCD are on equal bases BQ and CQ and are between the same parallels.

∴ area of parallelogram ABQP = area of parallelogram PQCD

5. **Prove that three parallelograms formed by joining the mid-points of the three sides of a triangle are equal in area.**

**Solution:**

Given: D, E, F are the mid- point of the sides AB, BC, CA of $\Delta\, ABC$.

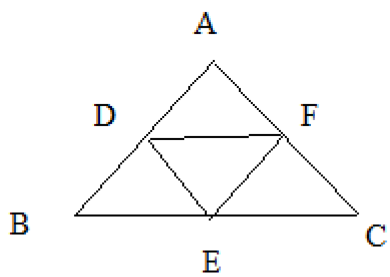
To prove: Area of parallelogram DBEF = area of parallelogram ADEF = area of parallelogram DECF.

Proof: In ΔABC,D and F are mid points of AB and AC.

∴ DF ∥ BC and DF $= \frac{1}{2}$ BC

⇒ DF ∥ BE and DF = BE

∴ DBEF is a parallelogram.




Similarly, DECF and ADEF are parallelograms.

Now, parallelograms DBEF and DECF are on the same base DF and between the parallels DF and BC.

∴ area of parallelogram DBEF = area of parallelogram DECF.

Similarly, area of parallelogram DBEF = area of parallelogram ADEF.

Hence, area of parallelogram DBEF = area of parallelogram ADEF = area of parallelogram DECF.

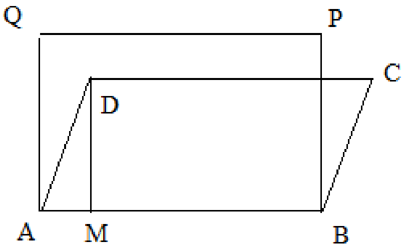
**6. Prove that, of all the parallelograms of given sides, parallelogram which is a rectangle has the greatest area.**

**Solution:**

**Given:** ABCD is a parallelogram and ABPQ is a rectangle in which AD = AQ.

**To prove:** Area of rectangle ABPQ > Area of parallelogram ABCD.

**Construction:** We draw $DM \perp AB$.




**Proof:** In a right ΔADM, we have

AD> DM [∵ in a right triangle, hypotenuse is the longest side]

⇒ AQ> DM

⇒ AB×AQ> AB×DM

∴ Area of rectangle ABPQ > Area of parallelogram ABCD.

**7. If O is an interior point of a parallelogram ABCD, prove that**

**(i) Area of ΔOAB + area of ΔOCD = $\frac{1}{2}$ (area of parallelogram ABCD)**

**(ii) Area of ΔOBC + area of Δ OAD = area of ΔOAB + area of ΔOCD.**

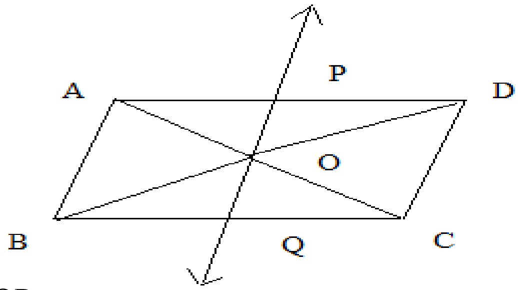
**Solution:**

**Given:** O is an interior point of a parallelogram ABCD,

**To prove:** i) Area of ΔOAB + area of ΔOCD = $\frac{1}{2}$ (area of parallelogram ABCD)

ii) Area of ΔOBC + area of Δ OAD = area of ΔOAB + area of ΔOCD.

**Construction:** We draw a line passing through O parallel to AB intersecting AD and BC at P and Q.




**Proof:** AB∥ PQ and AP ∥ BQ; CD ∥ QP and DP ∥ CQ.

∴ ABQP and PQCD are parallelograms.

i) ΔOAB and parallelogram ABQP are on the base AB and between the parallels AB and PQ.

∴ area of ΔOAB = $\frac{1}{2}$ area of parallelogram ABQP

ΔOCD and parallelogram PQCD are on the base CD and between the parallels DC and PQ.

∴ area of ΔOCD = $\frac{1}{2}$ area of parallelogram PQCD

Now, Area of ΔOAB + area of ΔOCD

$= \frac{1}{2}$ (area of parallelogram ABQP)+ $\frac{1}{2}$ (area of parallelogram PQCD)

$= \frac{1}{2}$ (area of parallelogram ABQP+ area of parallelogram PQCD)

$= \frac{1}{2}$ (area of parallelogram ABCD)

ii) Area of $\Delta$OBC + area of $\Delta$ OAD

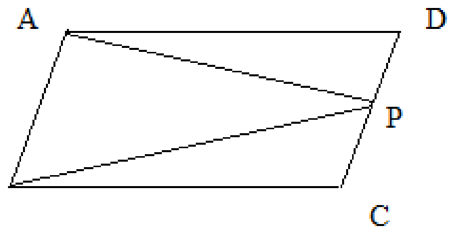
= (Area of parallelogram ABCD) – (Area of $\Delta$OAB + area of $\Delta$OCD)

= (Area of parallelogram ABCD) – $\frac{1}{2}$ (area of parallelogram ABCD)

= $\frac{1}{2}$ (Area of parallelogram ABCD)

= area of $\Delta$OAB + area of $\Delta$OCD

**8. ABCD is a parallelogram and P is any point on the side CD . Prove that area of $\Delta$APD + area of $\Delta$BCP = area of $\Delta$ABP.**




**Solution:**

**Given:** P is a point on the side CD of a $Par^m.ABCD$

**To Prove:** Area of $\Delta APD$ +Area of $\Delta BCP$ =Area of $\Delta ABP$

**Proof:** $\Delta APB$ and $Par^m$. ABCD are on the base AB and between the same parallels.

$\therefore$ area of $\Delta$APB $= \frac{1}{2}$. area of $\text{Par}^{\text{m}}$. ABCD

Area of $\Delta$ $APD$ +Area of $\Delta$ $BCP$ = Area of $Par^m.ABCD$ − Area of $\Delta APB$

= Area of $Par^m.ABCD - \frac{1}{2}$Area of $Par^m.ABCD$

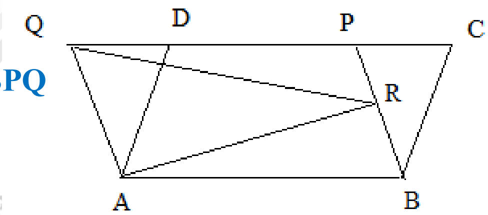
= $\frac{1}{2}$ $Area\ of\ Par^m.ABCD.$

= Area of $\Delta APB$.

**9. ABCD and ABPQ are parallelograms such that the points C, D, P, Q are collinear and R is any point on the side BP. Show that**

**(i) Area of parallelogram ABCD = area of parallelogram ABPQ**

**(ii) Area of $\Delta$ARQ $= \frac{1}{2}$ (area of parallelogram ABCD)**




**Solution:**

**Given:** ABCD and ABPQ are parallelograms such that C, D, P, Q are collinear and R is any point on the side BP.

**To Prove:** i) Area of parallelogram ABCD = area of parallelogram ABPQ

ii) Area of $\Delta$ARQ $= \frac{1}{2}$ (area of parallelogram ABCD)

**Proof :** i) Parallelograms ABCD and ABPQ are on a same base AB and between the same parallels.

$\therefore$ Area of parallelogram ABCD = area of parallelogram ABPQ

ii) $\Delta$ARQ and parallelogram ABPQ are on a same base AQ and between the same parallels.

$\therefore$ Area of $\Delta$ARQ = $\frac{1}{2}$ (area of parallelogram ABPQ)

$= \frac{1}{2}$ (area of parallelogram ABCD)

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## ➢ Theorems

1. Triangles on the same base and between the same parallels are equal in area.

Corollary:

- The area of a triangle is half the product of any of its sides and the corresponding altitude.
- A medium of a triangle divides it into two triangles of equal area.

2. Converse of Theorem 4: Two triangles having equal areas and standing on the same base and on the same side of it lie between the same parallels.

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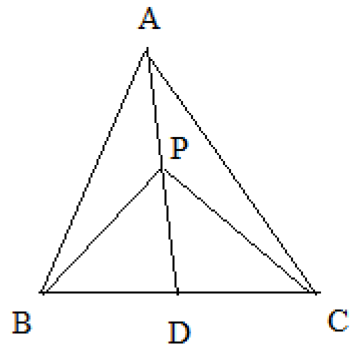
## SOLUTIONS

### EXERCISE 9.2

**1. AD is a medium of triangle ABC and P is any point on AD. Show that Area of $\Delta ABP$ =Area of $\Delta ACP$**

**Solution:**




**Given:** P is a point on the median AD of $\Delta ABC$.

**To Prove:** Area of $\Delta ABP$ = Area of $\Delta ACP$

**Proof:** As AD is a median of $\Delta ABC$,

Area of $\Delta ABD$ = Area of $\Delta ACD$ ------------ (1)

As PD is a median of $\Delta BPC$

Area of $\Delta PBD$ = Area of $\Delta PCD$ ------------ (2)

Subtracting (2) from (1)

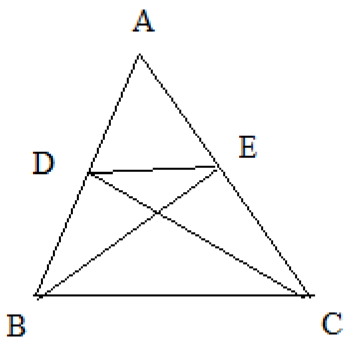
Area of $\Delta ABD$ − Area of $\Delta PBD$ = Area of $\Delta ACD$ −Area of $\Delta PCD$

∴ Area of $\Delta ABP$ = Area of $\Delta ACP$

**2. ABC is triangle and DE is drawn parallel to BC, cutting the other sides at D and E. Join BE and CD. Prove that**

**(i) Area of $\Delta DBC$ = Area of $\Delta EBC$**

**(ii) Area of $\Delta\ BDE$ = Area of $\Delta CDE$**




**Solution:**

**Given:** In ΔABC, DE ∥ BC is drawn cutting AB and AC at D and E.

**To Prove:** (i) Area of $\Delta DBC$ = Area of $\Delta EBC$

(ii) Area of $\Delta\ BDE$ = Area of $\Delta CDE$

**Proof:**

(i) ΔDBC and ΔEBC are on the same base BC and between the same parallels

∴ area of ΔDBC = area of ΔDBC

(ii) ΔBDE and ΔCDE are on the same base DE and between the same parallels

∴ area of ΔBDE = area of ΔCDE

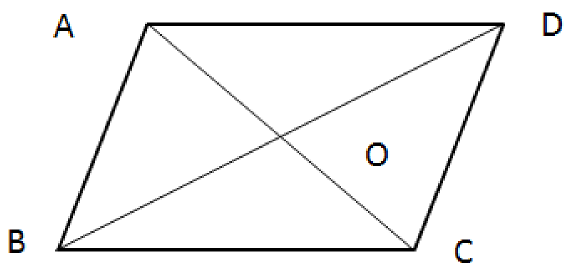
**3. Show that the diagonals of a parallelogram divide it into four triangles of equal area.**

**Solution:**

**Given:** In the parallelogram ABCD, the diagonals AC and BD intersect at O.

**To Prove:** Area of ΔAOB = Area of ΔBOC = Area of ΔCOD = Area of ΔDOA

**Proof:** We know that the diagonal AC and BD of Par$^{m}$. ABCD bisect each other at O.




OA is a medium of ΔABD.

$\therefore$ Area of $\Delta$ AOB = Area of $\Delta$ DOA

OB is a medium of ΔABC.

$\therefore$ Area of $\Delta$ AOB = Area of $\Delta$ BOC

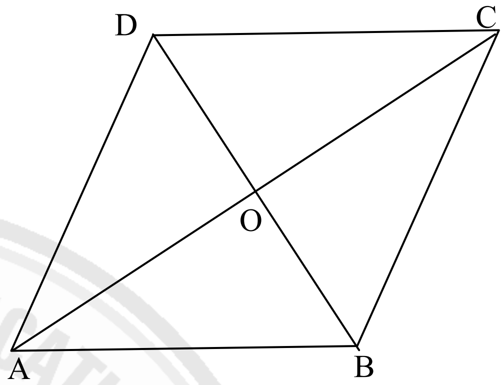
OC is a medium of ΔBCD.

$\therefore Area\ of\ \Delta\ BOC = Area\ of\ \Delta\ COD$

Hence, Area of ΔAOB = Area of ΔBOC = Area of ΔCOD = Area of ΔDOA

**4. Show that area of a rhombus is half the product of the length of its diagonals.**

**Solution:**




**Given:** ABCD is a rhombus. Diagonals AC and BD intersect each other at O.

**To prove:** Area of the rhombus $= \frac{1}{2} \times AC \times BD$

**Proof:** We know, the diagonals of a rhombus bisect each other at right angles.

∴ AC⊥BD

Area of rhombus ABCD

= Area of ΔABC + Area of ΔACD

$= \frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$

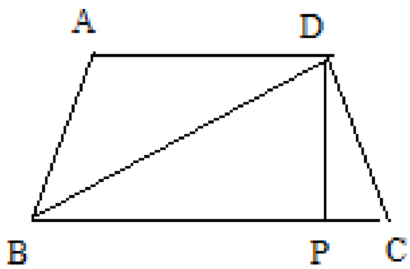
$= \frac{1}{2} \times AC \times (OB + OD)$

$= \frac{1}{2} \times AC \times BD$

**5. Prove that the area of trapezium is half the product of the sum of the length of the parallel sides and distance between them.**

**Solution:**

**Given:** ABCD is a trapezium in which $AD \parallel BC$ and $DP$ is the perpendicular distance between AD and BC




**To prove:** Area of Trapezium$ABCD$ $= \frac{1}{2} \times (AD + BC) \times DP$

**Proof:** Area of Δ ABD $= \frac{1}{2} \times AD \times DP$

Area of Δ DBC $= \frac{1}{2} \times BC \times DP$

Area of Trapezium $ABCD$ = Area of ΔABD + Area of ΔDBC

$$= \frac{1}{2} \times AD \times DP + \frac{1}{2} \times BC \times DP$$

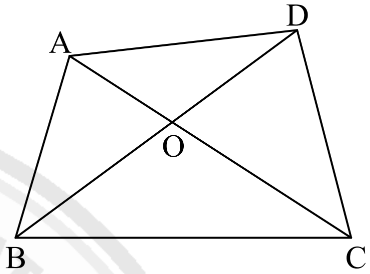
$$= \frac{1}{2} \times (AD + BC) \times DP$$

∴ Area of Trapezium$ABCD$ $= \frac{1}{2} \times (AD + BC) \times DP$

6. **The diagonals AC and BD of quadrilateral ABCD intersect at O. If BO=OD, prove that Area of $\Delta ABC =$ Area of $\Delta ADC$.**

**Solution:**

Given: Diagonal AC and BD of a quadrilateral ABCD intersect at O such that OB = OD.




To prove: Area of ΔABC = Area of ΔADC

Proof: We have, OB = OD

∴ OA and OC are medians of ΔABD and ΔBCD.

Then, Area of ΔAOB = Area of ΔAOD

And Area of ΔBOC = Area of ΔCOD

Now, Area of ΔABC = Area of ΔAOB + Area of ΔBOC

= Area of ΔAOD + Area of ΔCOD

= Area of ΔADC

7. **D,E,F are the mid-point of the sides BC,CA,AB respectively of triangle ABC, prove that BDEF is parallelogram whose area is half that of $\Delta ABC$ and area of $\triangle DEF = \frac{1}{4}(\text{area of } \triangle ABC)$.**

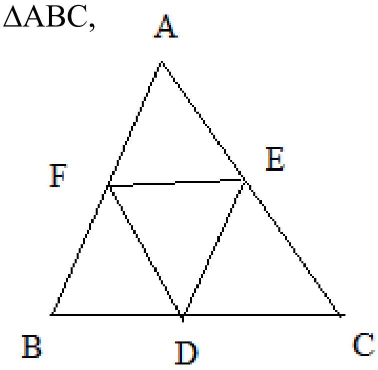
**Solution:**

**Given:** D, E, F are mid- points of the sides BC, AC and AB respectively of ΔABC.

**To prove:** $Area\ of\ Par^m.BDEF = \frac{1}{2}\ Area\ of\ \Delta ABC$

$Area\ of\ \Delta DEF = \frac{1}{4}(\ Area\ of\ \Delta ABC)$.

**Proof:** Since F and E are the mid points of AB and AC of ΔABC,




$FE \parallel BC$ and FE = $\frac{1}{2}$ BC

⇒ $FE \parallel BD$ and FE = BD

∴ BDEF is a parallelogram.

DF is a diagonal of the parallelogram BDEF.

∴ Area of ΔDEF = Area of ΔBDF

Similarly, Area of ΔDEF = Area of ΔDCE

And Area of ΔDEF = Area of ΔAEF

∴ Area of ΔDEF = Area of ΔBDF = Area of ΔDCE = Area of ΔAEF

But, Area of ΔDEF + Area of ΔBDF + Area of ΔDCE + Area of ΔAEF = Area of ΔABC

⇒Area of ΔDEF + Area of ΔDEF + Area of ΔDEF + Area of ΔDEF = Area of ΔABC

⇒ 4×Area of ΔDEF = Area of ΔABC

⇒ Area of ΔDEF = $\frac{1}{4}$ × Area of ΔABC

And, area of parallelogram BDEF = Area of ΔBDF + Area of ΔDEF

= Area of ΔDEF + Area of ΔDEF

= 2 (Area of ΔDEF)

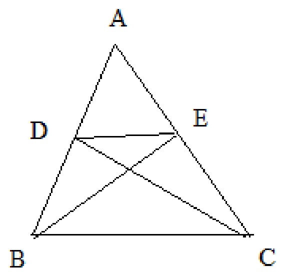
$= 2 \times \frac{1}{4}$ (Area of ΔABC)

$= \frac{1}{2}$ (Area of ΔABC)

**8. Prove that the straight line joining the mid-points of two sides of a triangle is parallel to the third side.**

**Solution:**

**Given:** D and E are mid points of AB and AC of a ΔABC.




**To Prove:** DE$\parallel BC$

**Construction:** CD, BE are joined.

**Proof:** As D is mid-point of AB, CD is a medium of ΔABC.

∴ Area of ΔDBC = $\frac{1}{2}$ Area of ΔABC

As E is mid-point of AC, BE is a medium of ΔABC.

∴ Area of ΔEBC = $\frac{1}{2}$ Area of ΔABC

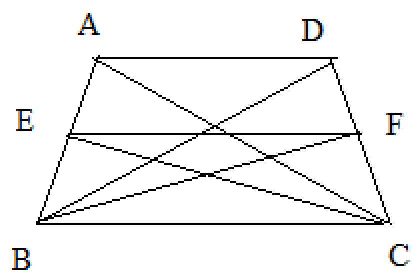
So, Area of ΔDBC = Area of ΔEBC

But ΔDBC and ΔEBC are on the same base BC and on the same side of BC.

$\therefore DE \parallel BC$

**9. Prove the straight line joining the mid-points of the oblique sides of a trapezium is parallel to each of the parallel sides.**

**Solution:**




**Given:** ABCD is a trapezium in which $AD \parallel BC$. E and F are the mid-points of the oblique sides AB and CD.

**To Prove:** $EF \parallel BC$.

**Construction:** AC, BD, BF, CE are joined.

**Proof:** ΔABC and ΔDBC are on the same base BC and between the same parallels AD and BC.

∴ Area of ΔABC = Area of ΔDBC

As E is mid-point of AB, CE is a median of ΔABC.

∴ Area of ΔEBC = $\frac{1}{2}$ Area of ΔABC

As F is the mid-point of CD, BF is a median of ΔDBC.

$\therefore$ Area of ΔFBC = $\frac{1}{2}$ Area of ΔDBC

So, Area of ΔEBC = Area of ΔFBC

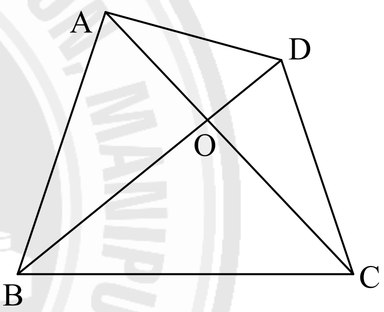
But ΔEBC and ΔFBC are on the same base BC and on the same side of BC.

$\therefore EF \parallel BC$

**10. If the diagonals AC and BD of a quadrilateral ABCD are perpendicular to one another, prove that area of the quadrilateral = $\frac{1}{2} \times AC \times BD$.**

**Solution:**

**Given:** In the quadrilateral ABCD, the diagonals AC and BD are perpendicular to each other intersecting each other at O.




**To prove:** Area of the quadrilateral ABCD = $\frac{1}{2}$ ×AC× BD

**Proof:** Area of ΔABC = $\frac{1}{2}$ ×AC× OB

Area of ΔADC = $\frac{1}{2}$ ×AC× OD

$\therefore$ Area of the quadrilateral ABCD = Area of ΔABC + Area of ΔADC

$= \frac{1}{2} \times AC \times OB + \frac{1}{2} \times AC \times OD$

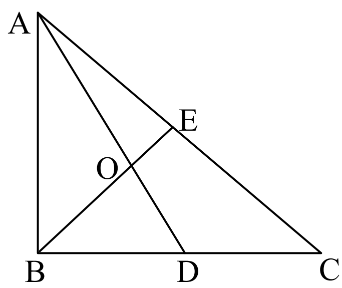
$= \frac{1}{2} \times AC \times (OB + OD)$

$= \frac{1}{2} \times AC \times BD$

**11.** If the medians AD and BE of a triangle ABC intersect at O, prove that $Area$ of $\Delta\ AOB$ = Area of quadrilateral $CDOE$.

Solution:

Given: The medians AD and BE of ΔABC intersect at O.




To prove: Area of ΔAOB = Area of quadrilateral CDOE

Proof: Since AD and BE are medians of ΔABC,

Area of ΔABD = Area of ΔBCE (= $\frac{1}{2}$ Area of ΔABC)

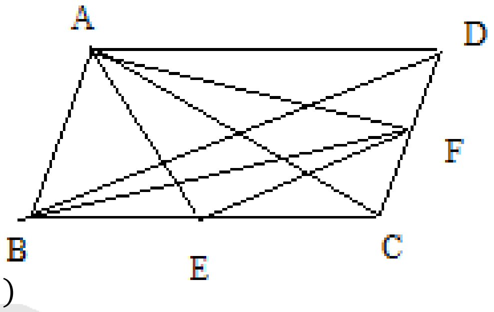
⇒ Area of ΔABD − Area of ΔBOD

= Area of ΔBCE − Area of ΔBOD

⇒ Area of ΔAOB = Area of quadrilateral CDOE

**12. ABCD is a Parallelogram, E and F are the mid-points of the sides BC and CD.**

**Prove that Area of AEF $= \frac{3}{8}$(Area of Par$^{m}$. ABCD).**




**Solution:**

**Given:** E and F are the mid-points of the sides BC and CD of Parallelogram ABCD.

**To prove:** $Area\ of\ \triangle AEF = \frac{3}{8}(\,area\ of\ Par^{m}.ABCD)$

**Construction:** AC, BD, BF, EF are joined.

**Proof:** Area of $\triangle$ ECF $= \frac{1}{2}$Area of$\triangle$ BCF  [ $\because$EF is median in $\triangle$ BCF]

$= \frac{1}{2}.\frac{1}{2}$ Area of $\triangle$ BCD  [ $\because$BF is median in $\triangle$ BCD]

$= \frac{1}{2}.\frac{1}{2}.\frac{1}{2}$ Area ofPar$^{m}$. ABCD  [ $\because$BD is a diagonal in Par$^{m}$. ABCD]

$= \frac{1}{8}$ Area of Par$^{m}$. ABCD

Area of $\triangle$ ABE $= \frac{1}{2}$Area of $\triangle$ ABC  [ $\because$ AE is median in $\triangle$ ABC]

$= \frac{1}{2}.\frac{1}{2}$Area of Par$^{m}$. ABCD  [ $\because$AC is diagonal inPar$^{m}$. ABCD]

$= \frac{1}{4}$Area of Par$^{m}$. ABCD

Area of $\triangle\, ADF = \frac{1}{2}$Area of $\triangle\, ADC$  [ $\because$ AF is median in $\triangle\, ADC$]

$= \frac{1}{2}\,.\frac{1}{2}$ Area of $Par^{m}.ABCD$ [ $\because$ AC is diagonal in $Par^{m}.ABCD$]

$= \frac{1}{4}$ Area of $Par^{m}.ABCD$

$\therefore$Area of $\triangle\, AEF =$  Area of $Par^{m}.ABCD$ $-$Area of $\triangle\, ECF$ $-$Area of $\triangle\, ABE$ $-$Area of$\triangle\, ADF$

$=$ Area of $Par^{m}.ABCD - \frac{1}{8}$ Area of$Par^{m}.ABCD - \frac{1}{4}$ Area of$Par^{m}.ABCD$ $-\frac{1}{4}$ Area of $Par^{m}.ABCD$

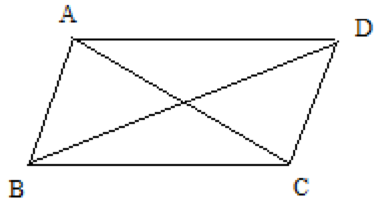
$= \left(1 - \frac{1}{8} - \frac{1}{4} - \frac{1}{4}\right)$ Area of $Par^{m}.ABCD$

$= \left(\frac{8-1-2-2}{8}\right) Area\ of\ Par^{m}.ABCD$

$= \frac{3}{8}(Area\ of\ Par^{m}.ABCD)$

**13. If each diagonal of a quadrilateral divides it into two triangles of equal area, show that the quadrilateral is a parallelogram.**

**Solution:**




**Given:** Each of the diagonals AC and BD of the quadrilateral ABCD divides it into two triangles of equal area.

**To prove:** ABCD is a parallelogram.

**Proof :** As AC divides quadrilateral ABCD into two triangles of equal area,

$$\text{Area of } \triangle ABC = \frac{1}{2} \text{ Area of quad.} ABCD.$$

As BD divides quadrilateral ABCD into two triangles of equal area,

$$\text{Area of} \triangle DCB = \frac{1}{2} \text{ Area of quad.} ABCD.$$

$$\therefore \text{Area of } \triangle ABC = \text{Area of } \triangle DCB$$

But ΔABC and ΔDCB are on a same base BC and on a same side of BC.

$$AD \parallel BC$$

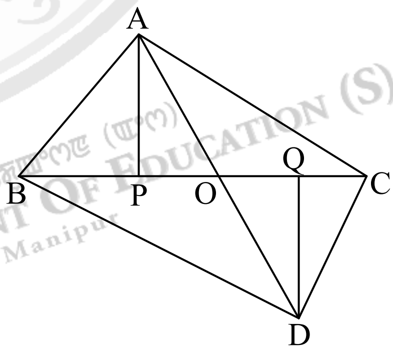
Similarly, $AB \parallel CD$

As the opposite sides of the quad. ABCD are parallel, ABCD is a parallelogram.

**14. Triangles ABC and DBC are on the same base BC and on the opposite sides of BC such that area of $\triangle ABC = area\ of\ \triangle DBC$ . Show that BC bisect AD.**

**Solution:**

**Given:** ΔABC and ΔDBC are on the same base BC and on the opposite sides of BC such that Area of ΔABC = Area of ΔDBC. AD intersects BC at O.




**To prove:** OA = OD i.e. BC bisects AD.

**Construction:** AP and DQ are drawn perpendicular to BC.

**Proof:** We have,

Area of ΔABC = Area of ΔDBC

$$\Rightarrow \frac{1}{2} \times BC \times AP = \frac{1}{2} \times BC \times DQ$$

$$\Rightarrow AP = DQ$$

Now, in ΔAOP and ΔDOQ, we have

∠APO = ∠DQO (= $90^0$)

∠AOP = ∠DO [vertically opposite angles]

and AP = DQ

∴ ΔAOP ≅ ΔDOQ [by AAS congruence]

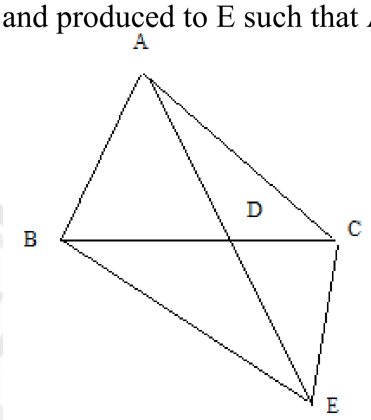
∴ OA = OD i.e. BC bisects AD.

**15. Any point D is taken in the base BC of $\triangle ABC$ and AD is product to E such that $AD = DE$. Show that area of $\triangle BCE$ = area of $\triangle ABC$.**

**Solution:**

**Given:** D is any point on BC of ΔABC and AD is joined and produced to E such that AD=DE.

**To Prove:** Area of Δ BCE =Area of Δ ABC.




**Proof:** We have, AD = DE

∴ BD and CD are medians of ΔABE and ΔACE.

As BD is a median in ΔABE,

Area of ΔABD = Area of ΔEBD

As CD is a median in Δ ACE,

Area of ΔACD = Area of ΔECD

Now, Area of ΔBCE = Area of ΔEBD + Area of ΔECD

= Area of ΔABD + Area of ΔACD

= Area of ΔABC

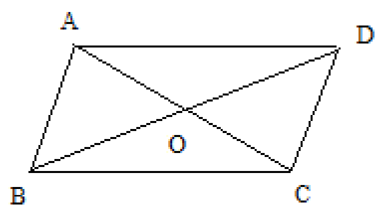
**16. The diagonals AC and BD of quadrilateral ABCD intersect at O and divide the quadrilateral ABCD into four triangle of equal area. Show that ABCD is a parallelogram.**

Solution:

**Given:** The diagonals AC and BD of the quadrilateral ABCD intersect at O such that

Area of ΔAOB = area of ΔBOC = area of Δ COD

= area of ΔDOA.




**To Prove:** ABCD is a parallelogram.

**Proof:** We have, Area of ΔAOB = Area of ΔCOD

⇒ Area of Δ AOB + Area of Δ BOC = Area of Δ COD + Area of Δ BOC

⇒ Area of Δ ABC = Area of Δ DBC.

But ΔABC and ΔDBC are on a same base BC and on a same side of BC.

$\therefore AD \parallel BC$

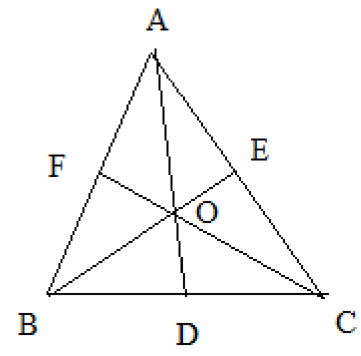
Similarly, AB ∥ DC

Hence, ABCD is a parallelogram.

**17. If the medians of a $\triangle ABC$ intersect at O, prove that**

**area of $\triangle AOB$ = area of $\triangle BOC$ = area of $\triangle$ COA = $\frac{1}{3}(\boldsymbol{area\ of}\ \triangle ABC)$.**

**Solution:**




**Given:** The median AD, BE and CF of $\triangle ABC$ intersect at O.

**To prove:** Area of $\triangle$ AOB = Area of $\triangle$ BOC = area of $\triangle$COA = $\frac{1}{3}(\ area\ of\ \ \triangle ABC)$

**Proof:** As AD is a median in $\triangle ABC$,

Area of $\triangle$ ABD = Area of $\triangle$ ACD --------------- (1)

As OD is a median in $\triangle OBC$,

Area of $\triangle OBD$ = Area of $\triangle OCD$ --------------- (2)

Subtracting (2) from (1),

Area of$\triangle$ ABD − Area of $\triangle$ OBD = Area of$\triangle$ ACD − Area of$\triangle$ OCD.

$\Rightarrow Area\ \ \triangle AOB = Area$ of $\triangle$ AOC

Similarly,

Area of$\triangle$ BOC=Area of $\triangle$ COA

$\therefore$Area of$\triangle$ AOB = Area of $\triangle$ BOC = Area of$\triangle$ COA

But, Area of$\triangle$ AOB +Area of $\triangle$ BOC +Area of $\triangle$ COA = Area of $\triangle$ ABC

$\Rightarrow$Area of $\triangle$ AOB +Area of$\triangle$ AOB +Area of$\triangle$AOB = Area of $\triangle$ ABC

$\Rightarrow$3 Area of$\triangle$ AOB = Area of $\triangle$ ABC

$\Rightarrow$Area of$\triangle$ AOB = $\frac{1}{3}$ Area of $\triangle$ ABC

Hence, Area of $\triangle$ AOB = Area of $\triangle$ BOC = area of $\triangle$COA = $\frac{1}{3}(\ area\ of\ \ \triangle ABC)$

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