CHAPTER 4
LOGARITHMS

SOLUTIONS

EXERCISE 4.1

1. Rewrite the following using logarithms :

(i) $10^3 = 1000$ **(ii)** $4^{-2} = \frac{1}{16}$ **(iii)** $10^{-3} = 0.001$

(i) Soln. $\log_{10} 1000 = 3$

(ii) Soln. $\log_4 \frac{1}{16} = -2$

(iii) Soln. $\log_{10} 0.001 = -3$

(iv) Soln. $(1000)^{1/3} = 10 \Rightarrow \log_{1000} 10 = 1/3$

2. Rewrite in the exponential form :

(i) $\log_{10}(0.01) = -2$ **(ii)** $\log_2 64 = 6$ **(iii)** $\log_3 243 = 5$

(iv) $\log_m l = p$

Soln:

(i) $\log_{10}(0.01) = -2$
$\Rightarrow 10^{-2} = 0.01$

(ii) $\log_2 64 = 6$
$\Rightarrow 2^6 = 64$

(iii) $\log_3 243 = 5$
$\Rightarrow 3^5 = 243$

(iv) $\log_m l = p$
$\Rightarrow m^p = l$

3. Find the logarithm of

(i) 324 to the base $3\sqrt{2}$ **(ii) 400 to the base $2\sqrt{5}$**

(iii) $\sqrt{3}$ to the base $\sqrt[5]{3}$ **(iv) 0.008 to the base $\sqrt{5}$**

Soln:

(i)

Let $\log_{3\sqrt{2}} 324 = x$

Then, $(3\sqrt{2})^x = 324$

$= 3^4 \times 2^2$

$= (3\sqrt{2})^4$

$\therefore x = 4$

So, $\log_{3\sqrt{2}} 324 = 4$

(ii)

Let $\log_{2\sqrt{5}} 400 = x$

$\Rightarrow (2\sqrt{5})^x = 400$

$= (2\sqrt{5})^4$

$\Rightarrow x = 4$

$\therefore \log_{2\sqrt{5}} 400 = 4$

(iii) Let $\log_{\sqrt[5]{3}} \sqrt{3} = x$

$\Rightarrow \left(\sqrt[5]{3}\right)^x = \sqrt{3}$

$\Rightarrow \left(\sqrt{3}\right)^{2x/5} = \sqrt{3}$

$\Rightarrow x = 5/2$

$\therefore \log_{\sqrt[5]{3}} \sqrt{3} = \dfrac{5}{2}$

(iv)

Let $\log_{\sqrt{5}} 0.008 = x$

$\Rightarrow \left(\sqrt{5}\right)^x = 0.008$

$= \dfrac{\overset{2}{\cancel{8}}}{\underset{250}{\cancel{1000}}} = \dfrac{1}{125} = \dfrac{1}{5^3}$

$= 5^{-3} = \left(\sqrt{5}\right)^{-3\times 2} = \left(\sqrt{5}\right)^{-6}$

$\therefore \log_{\sqrt{5}} 0.008 = -6$

4. Find the base if the logarithm of

(i) 625 is 4 **(ii) 32 is $4/3$** **(iii) 20 is 2**

Soln.

(i) Let m be the base

Then $\log_m 625 = 4$

$\Rightarrow m^4 = 625 = 5^4$

$\Rightarrow m = 5$

$\therefore$The required base is 5.

(ii) Let m be the base.

Then $\log_m 32 = \dfrac{4}{3}$

$\Rightarrow m^{4/3} = 32 = 2^5$

$\Rightarrow m = \sqrt[4]{2^{15}}$

$\therefore$The required base is $\sqrt[4]{2^{15}}$

(iii) Let m be the base .

Then $\log_m 20 = 2$

$\Rightarrow m^2 = 20$

$\Rightarrow m = \sqrt{20}$

$= 2\sqrt{5}$

$\therefore$The required base is $2\sqrt{5}$.

5. Show that $\log(1\times2\times3)=\log1+\log2+\log3$. Is it true for any three positive numbers *m, n, p* (instead of 1, 2, 3)?

Soln. Let $x=\log_{10}1,\ y=\log_{10}2,\ z=\log_{10}3$

$\therefore 10^x=1,\ 10^y=2,\ 10^z=3$

Now, $1\times2\times3=10^x\times10^y\times10^z\ =10^{(x+y+z)}$

$\Rightarrow \log_{10}(1\times2\times3)=x+y+z$

$=\log_{10}1+\log_{10}2+\log_{10}3$

Yes.

6. Prove that

(i) $\log2+16\log\frac{16}{15}+12\log\frac{25}{24}+7\log\frac{81}{80}=1$

Soln.

L.H.S. $=\log2+16\log\frac{16}{15}+12\log\frac{25}{24}+7\log\frac{81}{80}$

$=\log2+16(\log16-\log15)+12(\log25-\log24)+7(\log81-\log80)$

$=\log2+16\{\log2^4-\log(3\times5)\}+12\{\log5^2-\log(3\times2^3)\}+7\{\log3^4-\log(2^4\times5)\}$

$=\log2+16\times4\log2-16\log3-16\log5+24\log5-12\log3-36\log2$
$+28\log3-28\log2-7\log5$

$=\log2+64\log2-36\log2-28\log2-16\log3-12\log3+28\log3$
$+24\log5-16\log5-7\log5$

$=(1+64-36-28)\log2+(28-16-12)\log3+(24-16-7)\log5$

$=\log2+\log5$

$=\log2\times5$

$=\log10$

$=1\ [\because \log_{10}10=1]$

= R.H.S.

Hence proved.

(ii) $\log\frac{14}{15}+\log\frac{28}{27}+\log\frac{405}{196}=\log2$

Soln.

L.H.S. $=\log\frac{14}{15}+\log\frac{28}{27}+\log\frac{405}{196}$

$=\log14-\log15+\log28-\log27+\log405-\log196$

$=\log2+\log7-\log3-\log5+\log2^2+\log7-\log3^3+\log(3^4\times5)-\log(2^2\times7^2)$

$=\log2+\log7-\log3-\cancel{\log}5+2\log2+\log7-3\log3+4\log3+\cancel{\log}5$
$-2\log2-2\log7$

$=(1+2-2)\log2+(4-1-3)\log3+(1+1-2)\log7$

$=\log2=$ R.H.S.

(iii) $4\log 2+3\log 3-2\log 12=\log 3$

Soln.

L.H.S. $=4\cancel{\log}2+3\log 3-2\cancel{\log}12$

$=4\log 2+3\log 3-4\log 2-2\log 3$

$=\log 3=$ R.H.S.

(iv) $\log_2 \dfrac{448}{625}=6+\log_2 7-4\log_2 5$

Soln.

L.H.S. $=\log_2 \dfrac{448}{625}$

$=\log_2 448-\log_2 625$

$=\log_2\left(2^6\times 7\right)-\log_2 5^4$

$=6\log_2 2+\log_2 7-4\log_2 5$

$=6\times 1+\log_2 7-4\log_2 5$

$=6+\log_2 7-4\log_2 5$ = R.H.S.

(v) $\log_2 \log_2 \log_2 16=1$

Soln.

L.H.S. $=\log_2 \log_2 \log_2 16$

$=\log_2 \log_2 \log_2 2^4$

$=\log_2 \log_2 (4\log_2 2)$

$=\log_2 \log_2 4\ [\because \log_2 2=1]$

$=\log_2 \log_2 2^2$

$=\log_2 (2\log_2 2)$

$=\log_2 2$

$=1$ = R.H.S.

(vi) $\log_3 \log_2 \log_2 256=1$

Soln.

L.H.S. $=\log_3 \log_2 \log_2 256$

$=\log_3 \log_2 \log_2 2^8$

$=\log_3 \log_2 (8\log_2 2)\ [\because \log_2 2=1]$

$=\log_3 \log_2 8$

$=\log_3 \log_2 2^3$

$=\log_3 (3\log_2 2)$

$=\log_3 3$

$=1$ = R.H.S.

(vii) $7\log\frac{15}{16}+6\log\frac{8}{3}+5\log\frac{2}{5}+\log\frac{32}{25}=\log 3$

Soln.

L.H.S. $=7\log\frac{15}{16}+6\log\frac{8}{3}+5\log\frac{2}{5}+\log\frac{32}{25}$

$=7(\log 15-\log 16)+6(\log 8-\log 3)+5(\log 2-\log 5)+\log 32-\log 25$

$=7\log(3\times 5)-7\log 2^4+6\log 2^3-6\log 3+5\log 2-5\log 5+\log 2^5-\log 5^2$

$=7\log 3+7\log 5-28\log 2+18\log 2-6\log 3+5\log 2-5\log 5+5\log 2-2\log 5$

$=(-28+18+5+5)\log 2+(7-6)\log 3+(7-5-2)\log 5$

$=\log 3$ = R.H.S.

7. **If $\log(m+n)=\log m+\log n$, express m in terms of n.**

Soln. We have $\log(m+n)=\log m+\log n$

$\Rightarrow \log(m+n)=\log mn$

$\Rightarrow m+n=mn$

$\Rightarrow m-mn=-n$

$\Rightarrow m(n-1)=n$

$\Rightarrow m=\frac{n}{n-1}$

8. **If x be the logarithm of a number to the base $2\sqrt{2}$, show that the logarithm of the number to the base $\sqrt{2}$ is $3x$.**

Soln. Let a be the number.

Given, $\log_{2\sqrt{2}} a=x$

Then, $\left(2\sqrt{2}\right)^x=a$

$\Rightarrow \left(\sqrt{2^3}\right)^x=a$

$\Rightarrow \left(\sqrt{2}\right)^{3x}=a$

$\Rightarrow \log_{\sqrt{2}} a=3x$

Hence proved.

9. **Prove that** $\log_{a^p}(x^p) = \log_a x$ **for any non-zero real number** ***p*****.**

Soln. Let $\log_{a^p}(x^p) = y$

Then, $\left(a^p\right)^y = x^p$

$\Rightarrow a^{py} = x^p$

$\Rightarrow \left(a^y\right)^p = x^p$

$\Rightarrow a^y = x$

$\Rightarrow \log_a x = y$

$\therefore \log_{a^p} x^p = \log_a x$ Hence proved.

10. **Find a, if** $\dfrac{\log(5a-6)}{\log a} = 2$

Soln. We have $\dfrac{\log(5a-6)}{\log a} = 2$

$\Rightarrow \log(5a-6) = 2\log a$

$\Rightarrow \log(5a-6) = \log a^2$

$\Rightarrow 5a-6 = a^2$

$\Rightarrow a^2 - 5a + 6 = 0$

$\Rightarrow a^2 - 2a - 3a + 6 = 0$

$\Rightarrow a(a-2) - 3(a-2) = 0$

$\Rightarrow (a-2)(a-3) = 0$

$\Rightarrow$ either $a = 2$ or 3

11. **Prove that**

(i) $\log_b a \times \log_c b \times \log_a c = 1$

Soln. $\log_b a \times \log_c b \times \log_a c$

$= \log_m a \times \log_b m \times \log_m b \times \log_c m \times \log_m c \times \log_a m$

$= (\log_m a \times \log_a m) \times (\log_b m \times \log_m b) \times (\log_c m \times \log_m c)$

$= (\log_m a \times \dfrac{1}{\log_m a}) \times (\log_b m \times \dfrac{1}{\log_b m}) \times (\log_c m \times \dfrac{1}{\log_c m})$

$= 1$

(ii) $\log_a b \times \log_b c \times \log_c a = 1$

Soln. $\log_a b \times \log_b c \times \log_c a$

$= (\log_a b \times \log_b m) \times \log_m c \times \log_m a \times \log_c m$

$= (\log_a m \times \log_m a) \times (\log_m c \times \log_c m)$

$= 1$

(iii) $\log_b a \times \log_c b \times \log_d c = \log_d a$

Soln. $\log_b a \times \log_c b \times \log_d c$

$= \log_m a \times (\log_b m \times \log_m b) \times (\log_c m \times \log_m c) \times \log_d m$

$= \log_m a \times 1 \times 1 \times \log_d m$

$= \log_m a \times \log_d m$

$= \log_d a$

12. **(i)** **If** $a^2 + b^2 = 7ab$**, show that** $\log \frac{a+b}{3} = \frac{1}{2}(\log a + \log b)$

Soln. We have, $a^2 + b^2 = 7ab$

$\Rightarrow a^2 + b^2 + 2ab = 9ab$

$\Rightarrow (a+b)^2 = 9ab$

$\Rightarrow \left(\frac{a+b}{3}\right)^2 = ab$

$\therefore \log\left(\frac{a+b}{3}\right)^2 = \log(ab)$

$\Rightarrow 2\log\left(\frac{a+b}{3}\right) = \log a + \log b$

$\Rightarrow \log\left(\frac{a+b}{3}\right) = \frac{1}{2}\left(\log a + \log b\right)$

Hence shown.

(ii) **If** $a^2 + b^2 = 27ab$ **and** $a > b$**, show that** $\log \frac{a-b}{5} = \frac{1}{2}\left(\log a + \log b\right)$

Soln. We have, $a^2 + b^2 = 27ab$

$\Rightarrow a^2 + b^2 - 2ab = 25ab$

$\Rightarrow (a-b)^2 = 25ab$

$\Rightarrow \left(\frac{a-b}{5}\right)^2 = ab$

$\Rightarrow \log\left(\frac{a-b}{5}\right)^2 = \log ab$

$\Rightarrow 2\log\left(\frac{a-b}{5}\right) = \log a + \log b$

$\Rightarrow \log\left(\frac{a-b}{5}\right) = \frac{1}{2}\left(\log a + \log b\right)$

13. **If** $a^{2-x}b^{3x} = a^{x+3}b^{x}$**, show that** $x = \dfrac{\log a}{2(\log b - \log a)}$

Soln. We have, $a^{2-x}b^{3x} = a^{x+3}b^{x}$

$$\Rightarrow \frac{b^{3x}}{b^{x}} = \frac{a^{x+3}}{a^{2-x}}$$

$$\Rightarrow b^{2x} = a^{2x+1}$$

$$\Rightarrow b^{2x} = a^{2x}.a$$

$$\Rightarrow \left(\frac{b}{a}\right)^{2x} = a$$

$$\therefore \log\left(\frac{b}{a}\right)^{2x} = \log a$$

$$\Rightarrow 2x\log\left(\frac{b}{a}\right) = \log a$$

$$\Rightarrow x = \frac{\log a}{2\log(\frac{b}{a})}$$

$$= \frac{\log a}{2(\log b - \log a)}$$

Hence shown.

14. **If** $\log(x^2y^3) = a$ **and** $\log\dfrac{x}{y} = b$**, find** $\log x$ **and** $\log y$ **in terms of *a* and *b*.**

Soln. We have, $\log(x^2y^3) = a$ and $\log\left(\dfrac{x}{y}\right) = b$

Then, $2\log x + 3\log y = a$ ………. (1)

And, $\log x - \log y = b$ ……….. (2)

Multiplying both sides of (2) by 3 and adding with (1), we get

$$2\log x + 3\log y + 3\log x - 3\log y = a + 3b$$

$$\Rightarrow 5\log x = a + 3b$$

$$\Rightarrow \log x = \frac{1}{5}(a + 3b)$$

Again, multiplying both sides of (2) by 2 and subtracting from (1), we get

$$2\log x + 3\log y - 2\log x + 2\log y = a - 2b$$

$$\Rightarrow 5\log y = a - 2b$$

$$\Rightarrow \log y = \frac{1}{5}(a - 2b)$$

15. **Prove that** $\log_a x \times \log_b y = \log_b x \times \log_a y$

Soln. L.H.S. $= \log_a x \times \log_b y$

$= \log_b x \times \log_a b \times \log_a y \times \log_b a$

$= \log_b x \times \cancel{\log_a} b \times \log_a y \times \dfrac{1}{\cancel{\log_a} b}$

$= \log_b x \times \log_a y$ = R.H.S. Hence proved.

16. **If** $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b}$**, show that**

(i) $abc = 1$ **(ii)** $a^a b^b c^c = 1$

Soln.

(i) Let $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b} = \lambda$

$\Rightarrow \log a = \lambda(b-c)$, $\log b = \lambda(c-a)$, $\log c = \lambda(a-b)$

$\Rightarrow 10^{\lambda(b-c)} = a$, $10^{\lambda(c-a)} = b$, $10^{\lambda(a-b)} = c$

Now, $abc = 10^{\lambda(b-c)}.10^{\lambda(c-a)}.10^{\lambda(a-b)} = c$

$= 10^{\lambda b - \lambda c + \lambda c - \lambda a + \lambda a - \lambda b}$

$= 10^{\circ}$

$= 1$

(ii) Let $\dfrac{\log a}{b-c} = \dfrac{\log b}{c-a} = \dfrac{\log c}{a-b} = \lambda$

$\Rightarrow \log a = \lambda(b-c)$, $\log b = \lambda(c-a)$, $\log c = \lambda(a-b)$

Now, $a\log a + b\log b + c\log c = \lambda a(b-c) + \lambda b(c-a) + \lambda c(a-b)$

$\Rightarrow \log a^a + \log b^b + \log c^c = \lambda(ab - ac + bc - ab + ac - bc)$

$\Rightarrow \log(a^a b^b c^c) = 0$

$\Rightarrow a^a b^b c^c = 1$

17. **If** $\log_3 15 = p$**, show that** $\log_5 675 = \dfrac{2p+1}{p-1}$

Soln. We have, $\log_3 15 = p$

$\Rightarrow \log_3(3\times 5) = p$

$\Rightarrow \log_3 3 + \log_3 5 = p$

$\Rightarrow \log_3 5 = p - 1$

$\Rightarrow \log_5 3 = \dfrac{1}{p-1}$ ……… (1)

Now, $\log_5 675 = \log_5(5^2 \times 3^3)$

$= 2\log_5 5 + 3\log_5 3$

$$= 2 + 3 \times \frac{1}{p-1} \qquad \text{[from (1)]}$$

$$= \frac{2p-2+3}{p-1}$$

$$= \frac{2p+1}{p-1}$$

18. **Prove that** $\log_5 3 < \frac{8}{5}$

Soln. We know that,

$3^5 < 5^8$

Taking logarithms on both sides to the base 5, we have

$\Rightarrow \log_5 3^5 < \log_5 5^8$

$\Rightarrow 5\log_5 3 < 8\log_5 5$

$\Rightarrow 5\log_5 3 < 8 \qquad [\because \log_5 5 = 1]$

$\Rightarrow \log_5 3 < \frac{8}{5}$

Hence proved.

19. **Prove that** $\log_{75} 135 = \frac{2p-1}{3-p}$ **where** $p = \log_{15} 45$

Soln. We have,

$$p = \log_{15} 45$$

$$= \log_{15}(15 \times 3)$$

$$= \log_{15} 15 + \log_{15} 3$$

$$= 1 + \log_{15} 3$$

$\Rightarrow \log_{15} 3 = p - 1$ ……………(1)

Again, $p = \log_{15} 45$

$$= \log_{15}(5 \times 3^2)$$

$$= \log_{15} 5 + 2\log_{15} 3$$

$$= \log_{15} 5 + 2(p-1)$$

$$= \log_{15} 5 + 2p - 2$$

$\Rightarrow \log_{15} 5 = 2 - p$ ……………. (2)

Now, $\log_{75} 135 = \frac{\log_{15} 135}{\log_{15} 75}$

$$= \frac{\log_{15}(15 \times 9)}{\log_{15}(15 \times 5)}$$

$$= \frac{\log_{15} 15 + \log_{15} 9}{\log_{15} 15 + \log_{15} 5}$$

$$= \frac{1 + 2\log_{15} 3}{1 + \log_{15} 5}$$

$$= \frac{1 + 2(p-1)}{1 + (2-p)} \quad \text{[from (1) \& (2)]}$$

$$= \frac{2p-1}{3-p}. \quad \text{Hence proved.}$$

20. **Solve :**

(i) $\log_{10} x + \log_{10}(x-15) = 2$

Soln. $\log_{10} x + \log_{10}(x-15) = 2$

$\Rightarrow \log_{10} x(x-15) = 2$

$\Rightarrow 10^2 = x(x-15)$

$\Rightarrow 100 = x^2 - 15x$

$\Rightarrow x^2 - 15x - 100 = 0$

$\Rightarrow x^2 + 5x - 20x - 100 = 0$

$= (x+5)(x-20) = 0$

$\Rightarrow$ either $x = -5$ or $x = 20$

Since x cannot be negative, so $x = 20$

(ii) $\log_2 x + \log_2 \frac{x}{16} = \log_2 \frac{x}{64}$

Soln. $\log_2 x + \log_2 \frac{x}{16} = \log_2 \frac{x}{64}$

$\Rightarrow \log_2 (x.\frac{x}{16}) = \log_2 \frac{x}{64}$

$\Rightarrow \frac{x^2}{\cancel{16}_{1}} = \frac{x}{\cancel{64}_{4}}$

$\Rightarrow 4x^2 = x$

$\Rightarrow x(4x-1) = 0$

$\Rightarrow$ either $x = 0$ or $x = \frac{1}{4}$

$\Rightarrow x = \frac{1}{4}$ [$\because$ x cannot be 0]

$\therefore x = \frac{1}{4}$

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