CHAPTER 12
MENSURATION

- **Area of a triangle** $= \frac{1}{2} \times \text{base} \times \text{altitude}$
- **Heron's Formula**

If a, b, c are the lengths of the three sides of a triangle, then

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \text{semi} - \text{perimeter of the triangle} = \frac{a+b+c}{2}$

SOLUTIONS

EXERCISE 12.1

1. Find the area of the triangle whose sides are

(i) 8 cm, 11 cm and 13 cm

(ii) 10 cm, 16 cm and 20 cm

(iii) **5 cm, 7 cm and 10 cm**

Solution:

(i) Let $a = 8$ cm, $b = 11$ cm and $c = 13$ cm

Then we have, $s = \frac{a+b+c}{2} = \frac{8+11+13}{2}$ cm $= 16$ cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-8)(16-11)(16-13)}$ cm^2

$= \sqrt{16 \times 8 \times 5 \times 3}$ cm^2

$= \sqrt{2^2 \times 2^2 \times 2^2 \times 2 \times 5 \times 3}$ cm^2

$= 2 \times 2 \times 2\sqrt{2 \times 5 \times 3}$ cm^2

$= 8\sqrt{30}$ cm^2

(ii) Let $a = 10$ cm, $b = 16$ cm and $c = 20$ cm

Then we have, $s = \frac{a+b+c}{2} = \frac{10+16+20}{2}$ cm $= \frac{46}{2}$ cm $= 23$ cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{23(23-10)(23-16)(23-20)}$ cm^2

$= \sqrt{23 \times 13 \times 7 \times 3}$ cm^2

$= \sqrt{6279}$ cm^2

(iii) Let $a = 5$ cm, $b = 7$ cm and $c = 10$ cm

Then we have, $s = \frac{a+b+c}{2} = \frac{5+7+10}{2}$ cm $= \frac{22}{2}$ cm $= 11$ cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{11(11-5)(11-7)(11-10)}$ cm^2

$= \sqrt{11 \times 6 \times 4 \times 1}$ cm^2

$= \sqrt{11 \times 6 \times 2^2 \times 1}$ cm^2

$= 2\sqrt{66}$ cm^2

2. Find the area of a triangle, two sides of which are 17 cm and 10 cm and perimeter is 48 cm.

Solution: Here, perimeter of the triangle $(s) = 48$ cm

$a = 17$ cm and $b = 10$ cm

Therefore, third side $c = 48$ cm – (17 + 10) cm = 21 cm

Then we have, $s = \frac{48}{2}$ cm $= 24$ cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-17)(24-10)(24-21)}$ cm^2

$= \sqrt{24 \times 7 \times 14 \times 3}$ cm^2

$= \sqrt{2^2 \times 2 \times 3 \times 7 \times 2 \times 7 \times 3}$ cm^2

$= \sqrt{2^2 \times 2^2 \times 3^2 \times 7^2 \times 3^2}$ cm^2

$= 2 \times 2 \times 3 \times 7$ cm^2

$= 84$ cm^2

3. The sides of a triangle are in the ratio 5:7:8 and its perimeter is 300 cm. Find its area.

Solution: Let the sides of the triangle (in cm) be $5x$, $7x$ and $8x$.

Then by question, we have

$5x + 7x + 8x = 300$

$\Rightarrow 20x = 300$

$\Rightarrow x = 15$

So, the sides of the triangle are 5×15 cm, 7×15 cm and 8×15 cm i.e., 75 cm, 105 cm and 120 cm.

Let $a = 75$ cm, $b = 105$ cm and $c = 120$ cm.

We have, $s = \frac{300}{2}$ cm= 150 cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{150(150-75)(150-105)(150-120)}$ cm^2

$= \sqrt{150 \times 75 \times 45 \times 30}$ cm^2

$= \sqrt{(2 \times 75) \times 75 \times (3 \times 15) \times (2 \times 15)}$ cm^2

$= \sqrt{2^2 \times 75^2 \times 15^2 \times 3}$ cm^2

$= 2 \times 75 \times 15\sqrt{3}$ cm^2

$= 2250\sqrt{3}$ cm^2

4. Perimeter of an equilateral triangle is 36 cm. Find its area using Heron's formula.

Solution: We have $s = \frac{36}{2}$ cm= 18 cm and $a = b = c = \frac{36}{3}$ cm= 12 cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-12)(18-12)(18-12)}$ cm^2

$= \sqrt{18 \times 6 \times 6 \times 6}$ cm^2

$= \sqrt{3 \times 6 \times 6 \times 6 \times 6}$ cm^2

$= \sqrt{6^2 \times 6^2 \times 3}$ cm^2

$= 6 \times 6\sqrt{3}$ cm^2

$= 36\sqrt{3}$ cm^2

5. Find the area of an equilateral triangle whose sides are of 14 cm each using heron's formula.

Solution: We have $a = b = c = 14$ cm

$s = \frac{3 \times 14}{2}$ cm= 21 cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-14)(21-14)(21-14)}$ cm^2

$= \sqrt{(3 \times 7) \times 7 \times 7 \times 7}$ cm^2

$= \sqrt{3 \times 6 \times 6 \times 6 \times 6}$ cm^2

$= \sqrt{7^2 \times 7^2 \times 3}$ cm^2

$= 7 \times 7\sqrt{3}$ cm^2

$= 49\sqrt{3}$ cm^2

6. Find the area of an isosceles triangle whose base is 10 cm and one of its equal sides is 13 cm.

Solution: We have $a = 10$ cm and $b = c = 13$ cm.

$$s = \frac{10+13+13}{2} \text{ cm} = \frac{36}{2} \text{ cm} = 18 \text{ cm}$$

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{18(18-10)(18-13)(18-13)} \text{ cm}^2$$

$$= \sqrt{18 \times 8 \times 5 \times 5} \text{ cm}^2$$

$$= \sqrt{(2 \times 3^2) \times (2 \times 2^2) \times 5 \times 5} \text{ cm}^2$$

$$= \sqrt{2^2 \times 2^2 \times 3^2 \times 5^2} \text{ cm}^2$$

$$= 2 \times 2 \times 3 \times 5 \text{ cm}^2$$

$$= 60 \text{ cm}^2$$

7. Perimeter of an isosceles triangle is 40 cm and each of the equal sides is 15 cm. Find the area of the triangle using Heron's formula.

Solution: Here, perimeter of the triangle $(s) = 40$ cm and $a = b = 15$ cm

Therefore, third side $c = 40$ cm – (15 + 15) cm = 10 cm

Then we have, $s = \frac{40}{2}$ cm = 20 cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{20(20-15)(20-15)(20-10)} \text{ cm}^2$$

$$= \sqrt{20 \times 5 \times 5 \times 10} \text{ cm}^2$$

$$= \sqrt{(2^2 \times 5) \times 5 \times 5 \times (5 \times 2)} \text{ cm}^2$$

$$= \sqrt{2^2 \times 5^2 \times 5^2 \times 2} \text{ cm}^2$$

$$= 2 \times 5 \times 5\sqrt{2} \text{ cm}^2$$

$$= 50\sqrt{2} \text{ cm}^2$$

8. A signboard is in the shape of an equilateral triangle with side 'a'. Find the area of the signboard using Heron's formula. If the perimeter of the board is 210 cm, what will be its area?

Solution: Length of each side of the signboard $= a$

$$\therefore a = b = c \text{ and } s = \frac{a+b+c}{2} = \frac{a+a+a}{2} = \frac{3a}{2}$$

$$\therefore \text{area of the signboard} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{s(s-a)(s-a)(s-a)}$$

$$= \sqrt{s(s-a)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)^3}$$

$$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)^3}$$

$$= \sqrt{3 \times \left(\frac{a}{2}\right)^2 \times \left(\frac{a}{2}\right)^2}$$

$$= \frac{a}{2} \times \frac{a}{2}\sqrt{3}$$

$$= \frac{\sqrt{3}}{4}a^2 \text{ sq. units}$$

Second Part

If the perimeter of the signboard $= 210$ cm, $a = \frac{210}{3}$ cm $= 70$ cm

Then, area of the signboard $= \frac{\sqrt{3}}{4}a^2$ cm^2

$$= \frac{\sqrt{3}}{4} \times 70^2 \text{ cm}^2$$

$$= \frac{\sqrt{3}}{4} \times 70 \times 70 \text{ cm}^2$$

$$= \sqrt{3} \times 35 \times 35 \text{ cm}^2$$

$$= 1225\sqrt{3} \text{ cm}^2$$

9. The sides of a triangular field are 975 m, 1050 m and 1125 m. If this field is sold at the rate of Rs 10 per m^2, find its selling price.

Solution: Let $a = 975$ m, $b = 1050$ m and $c = 1125$ cm

Then, we have $s = \frac{a+b+c}{2} = \frac{975+1050+1125}{2}$ m $= \frac{3150}{2}$ m $= 1575$ m

Area of the triangular field $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{1575(1575-975)(1575-1050)(1575-1125)}$ m^2

$= \sqrt{1575 \times 600 \times 525 \times 450}$ m^2

$= \sqrt{(3 \times 525) \times (2 \times 3 \times 2^2 \times 5^2) \times 525 \times (2 \times 3^2 \times 5^2)}$ m^2

$= \sqrt{2^2 \times 2^2 \times 3^2 \times 3^2 \times 5^2 \times 5^2 \times 525^2}$ m^2

$= 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 525$ m^2

$= 472500$ m^2

$\therefore$ selling price of the triangular field = ₹472500 × 10 = ₹4725000

10. Find the area of an isosceles triangle whose base is 6 cm and perimeter is 16 cm.

Solution: Here, perimeter of the triangle $(s) = 16$ cm and $a = 6$ cm

Therefore, $b = c = \frac{16-6}{2}$ cm$= \frac{10}{2}$ cm $= 5$ cm

Then we have, $s = \frac{16}{2}$ cm $= 8$ cm

$\therefore$ area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(8-6)(8-5)(8-5)}$ cm^2

$= \sqrt{8 \times 2 \times 3 \times 3}$ cm^2

$= \sqrt{(2^2 \times 2) \times 2 \times 3^2}$ cm^2

$= \sqrt{2^2 \times 2^2 \times 3^2}$ cm^2

$= 2 \times 2 \times 3$ cm^2

$= 12$ cm^2

SOLUTIONS

EXERCISE 12.2

1. Find the area of the quadrilateral ABCD in which

(i) AB = 5 cm, BC = 4.5 cm, CD = 3.5 cm, DA = 4 cm and AC = 6.5 cm

(ii) AB = 3 cm, BC = 5 cm, CD = 6 cm, DA = 6 cm and BD = 5 cm

(iii) AB = 3.5 cm, BC = 4.5 cm, CD = 6 cm, DA = 3 cm and BD = 5.5 cm

(iv) AB = 6 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 6 cm

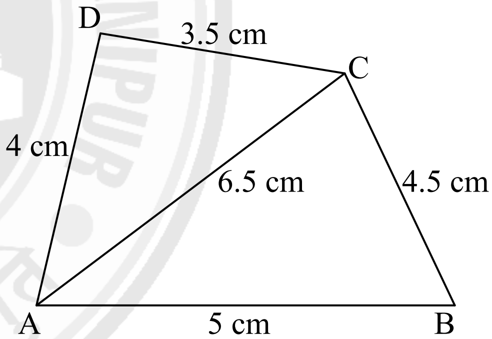
Solution:

(i)

For ΔABC, AB = 5 cm, BC = 4.5 cm and AC = 6.5 cm

$\therefore s = \frac{5+4.5+6.5}{2}$ cm$= \frac{16}{2}$ cm$=$ 8 cm

Then by Heron's formula, we have



Area of $\Delta ABC = \sqrt{8(8-5)(8-4.5)(8-6.5)}$ cm^2

$= \sqrt{8 \times 3 \times 3.5 \times 1.5}$ cm^2

$= \sqrt{126}$ cm^2 $=11.22$ cm^2

For ΔADC, AC = 6.5 cm, CD = 3.5 cm and AD = 4 cm

$\therefore s = \frac{6.5+3.5+4}{2}$ cm$= \frac{14}{2}$ cm$=$ 7 cm

Then by Heron's formula, we have

Area of $\Delta ADC = \sqrt{7(7-6.5)(7-3.5)(7-4)}$ cm^2

$= \sqrt{7 \times 0.5 \times 3.5 \times 3}$ cm^2

$= \sqrt{36.75}$ cm^2 $=6.06$ cm^2

$\therefore$ Area of Quadrilateral ABCD

$=$ Area of ΔABC + Area of ΔADC

$= 11.22$ cm^2 $+ 6.06$ cm^2

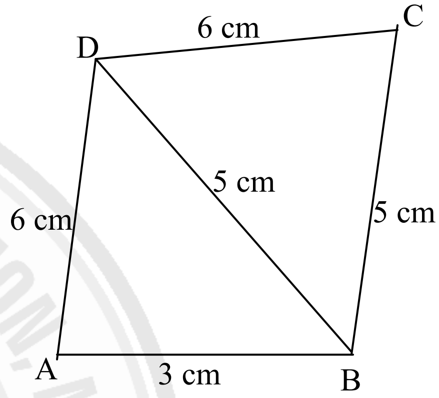
$= 17.26$ cm^2$= 17.3$ cm^2

(ii)

For ΔABD, AB = 3 cm, BD = 5 cm and AD = 6 cm

$\therefore s = \frac{3+5+6}{2}$ cm $= \frac{14}{2}$ cm $= 7$ cm

Then by Heron's formula, we have



Area of ΔABD $= \sqrt{7(7-3)(7-5)(7-6)}$ cm^2

$= \sqrt{7 \times 4 \times 2 \times 1}$ cm^2

$= \sqrt{56}$ cm^2 $=7.5$ cm^2

For ΔBCD, BC = 5 cm, CD = 6 cm and BD = 5 cm

$\therefore s = \frac{5+6+5}{2}$ cm $= \frac{16}{2}$ cm $= 8$ cm

Then by Heron's formula, we have

Area of ΔBCD $= \sqrt{8(8-5)(8-6)(8-5)}$ cm^2

$= \sqrt{(2^2 \times 2) \times 3 \times 2 \times 3}$ cm^2

$= \sqrt{2^2 \times 2^2 \times 3^2}$ cm^2

$= 2 \times 2 \times 3$ cm^2 $=12$ cm^2

$\therefore$ Area of Quadrilateral ABCD

$=$ Area of ΔABD $+$ Area of ΔBCD

$= 7.5$ cm^2 $+ 12$ cm^2

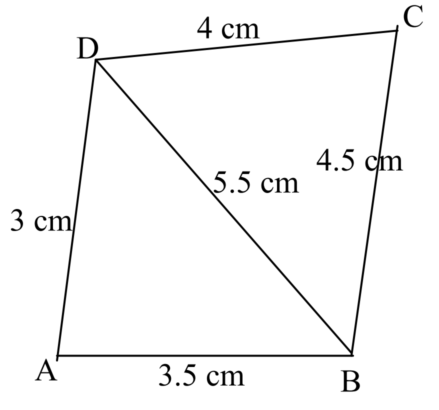
$= 19.5$ cm^2

(iii)

For ΔABD, AB = 3.5 cm, BD = 5.5 cm and AD = 3 cm

$\therefore s = \frac{3.5+5.5+3}{2}$ cm $= \frac{12}{2}$ cm $= 6$ cm

Then by Heron's formula, we have



Area of ΔABD $= \sqrt{6(6-3.5)(6-5.5)(6-3)}$ cm^2

$= \sqrt{6 \times 2.5 \times 0.5 \times 3}$ cm^2

$= \sqrt{22.5}$ cm^2 $=4.7$ cm^2

For ΔBCD, BC = 4.5 cm, CD = 6 cm and BD = 5.5 cm

$\therefore s = \frac{4.5+6+5.5}{2}$ cm $= \frac{16}{2}$ cm $= 8$ cm

Then by Heron's formula, we have

Area of ΔBCD $= \sqrt{8(8-4.5)(8-6)(8-5.5)}$ cm^2

$= \sqrt{8 \times 3.5 \times 2 \times 2.5}$ cm^2

$= \sqrt{140}$ cm^2

$= 11.8$ cm^2

∴ Area of Quadrilateral ABCD

= Area of ΔABD + Area of ΔBCD

$= 4.7$ cm^2 $+ 11.8$ cm^2

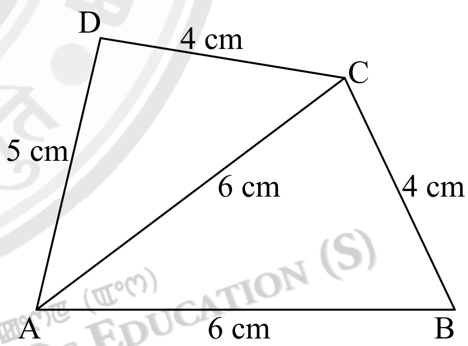
$= 16.5$ cm^2

(iv)

For ΔABC, AB = 6 cm, BC = 4 cm and AC = 6 cm

$\therefore s = \frac{6+4+6}{2}$ cm$= \frac{16}{2}$ cm$= 8$ cm

Then by Heron's formula, we have



Area of ΔABC $= \sqrt{8(8-6)(8-4)(8-6)}$ cm^2

$= \sqrt{8 \times 2 \times 4 \times 2}$ cm^2

$= \sqrt{128}$ cm^2 $=11.3$ cm^2

For ΔADC, AC = 6 cm, CD = 4 cm and AD = 5 cm

$\therefore s = \frac{6+4+5}{2}$ cm$= \frac{15}{2}$ cm$= 7.5$ cm

Then by Heron's formula, we have

Area of ΔADC $= \sqrt{7.5(7.5-6)(7.5-4)(7.5-5)}$ cm^2

$= \sqrt{7.5 \times 1.5 \times 3.5 \times 2.5}$ cm^2

$= \sqrt{98.4375}$ cm^2 $=9.9$ cm^2

∴ Area of Quadrilateral ABCD

= Area of ΔABC + Area of ΔADC

$= 11.3$ cm^2 $+ 9.9$ cm^2

$= 21.2$ cm^2

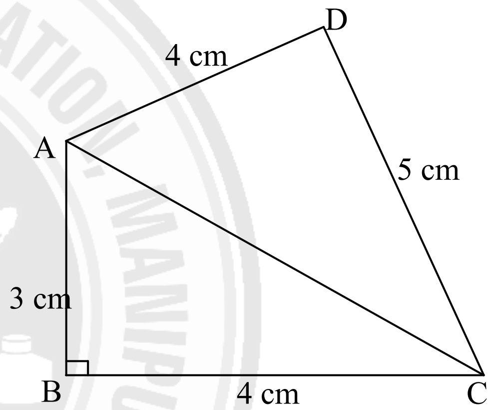
2. Find the area of the quadrilateral ABCD in which

(i) AB = 3 cm, BC = 4 cm, CD = 5 cm, DA = 4 cm and $\angle B = 90^0$

(ii) AB = 6 cm, BC = 4 cm, CD = 3 cm, DA = 5 cm and $\angle C = 90^0$

Solution:

(i)



In the right ΔABC,

$AC^2 = AB^2 + BC^2$ [Pythagoras Theorem]

$\Rightarrow AC^2 = 3^2 + 4^2 = 9 + 16 = 25 = 5^2$

$\Rightarrow AC = 5$ cm

$\therefore$ Area of ΔABC $= \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 4 \times 3$ cm^2 = 6 cm^2

For ΔADC, AC = 5 cm, CD = 5 cm and AD = 4 cm

$\therefore s = \frac{5+5+4}{2}$ cm$= \frac{14}{2}$ cm$= 7$ cm

Then by Heron's formula, we have

Area of ΔADC $= \sqrt{7(7-5)(7-5)(7-4)}$ cm^2

$= \sqrt{7 \times 2 \times 2 \times 3}$ cm^2

$= \sqrt{84}$ cm^2 =9.2 cm^2

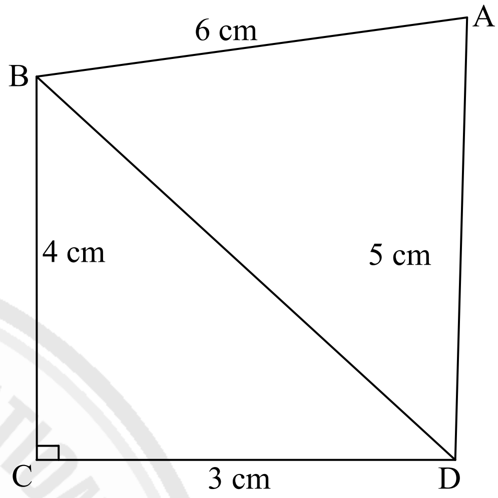
$\therefore$ Area of Quadrilateral ABCD

= Area of ΔABC + Area of ΔADC

= 6 cm^2 + 9.2 cm^2

= 15.2 cm^2

(ii)



In the right ΔBCD,

$BD^2 = BC^2 + CD^2$ [Pythagoras Theorem]

$\Rightarrow BD^2 = 4^2 + 3^2$

$\Rightarrow BD^2 = 16 + 9$

$\Rightarrow BD^2 = 25$

$\Rightarrow BD^2 = 5^2$

$\Rightarrow BD = 5$ cm

∴ Area of ΔBCD $= \frac{1}{2} \times CD \times BC$

$= \frac{1}{2} \times 3 \times 4 \text{ cm}^2$

$= 6 \text{ cm}^2$

For ΔABD, AB = 6 cm, BD = 5 cm and AD = 5 cm

$\therefore s = \frac{6+5+5}{2}$ cm$= \frac{16}{2}$ cm$= 8$ cm

Then by Heron's formula, we have

Area of ΔABD $= \sqrt{8(8-6)(8-5)(8-5)} \text{ cm}^2$

$= \sqrt{(2 \times 2^2) \times 2 \times 3 \times 3} \text{ cm}^2$

$= \sqrt{2^2 \times 2^2 \times 3^2} \text{ cm}^2$

$= 2 \times 2 \times 3 \text{ cm}^2 = 12 \text{ cm}^2$

∴ Area of Quadrilateral ABCD

= Area of ΔBCD + Area of ΔABD

$= 6 \text{ cm}^2 + 12 \text{ cm}^2$

$= 18 \text{ cm}^2$

3. Each side of a rhombus shaped field is 30 m and its longer diagonal is 48 m. Find the area of the field.

Solution:

Let ABCD be the rhombus shaped field in which

$AB = BC = CD = DA = 30$ m and $AC = 48$ m.

For the ΔABC, we have

$a = BC = 30$ m, $b = AC = 48$ m and $c = AB = 30$ m

$\therefore s = \frac{a+b+c}{2} = \frac{30+48+30}{2}$ m $= \frac{108}{2}$ m $= 54$ m

Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-30)(54-48)(54-30)}$ m^2

$= \sqrt{54 \times 24 \times 6 \times 24}$ m^2

$= \sqrt{(3^2 \times 6) \times 24^2 \times 6}$ m^2

$= \sqrt{3^2 \times 6^2 \times 24^2}$ m^2

$= 3 \times 6 \times 24$ m^2

$= 432$ m^2

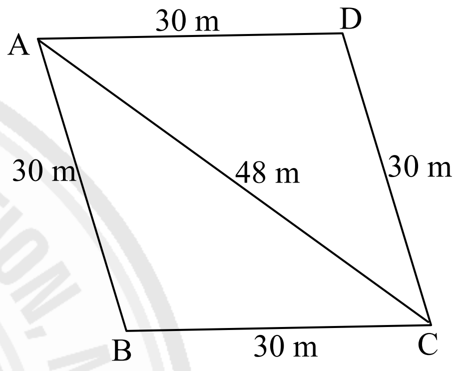
We know, a diagonal of a rhombus divides the rhombus into two congruent triangles.

$\therefore$ Area of the rhombus shaped field ABCD

$= 2 \times$ Area of ΔABC

$= 2 \times 432$ m^2

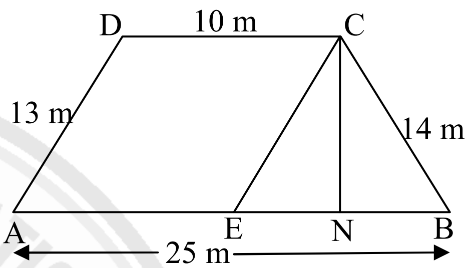
$= 864$ m^2



4. A field is in the shaped of a trapezium whose parallel sides are 25 m and 10 m, and non-parallel sides are 14 m and 13 m. Find the area of the field.

Solution:

Let ABCD be the trapezium shaped field in which AB = 25 m and DC = 10 m are the parallel sides; AD = 13 m and BC = 14 m are the non-parallel sides.



$CE \| DA$ is drawn meeting AB at E to form a parallelogram AECD. CN⊥ AB is also drawn to meet AB at N.

For the ΔBCE, we have

BC = 14 m, CE = 13 m = AD and BE = 25 m – 10 m =15 m.

Semi-perimeter (s) = $\frac{14+13+15}{2}$ m= $\frac{42}{2}$ m= 21 m

Area of ΔBCE= $\sqrt{21(21-14)(21-13)(21-15)}$

$\Rightarrow \frac{1}{2} \times BE \times CN = \sqrt{21 \times 7 \times 8 \times 6}$

$\Rightarrow \frac{1}{2} \times 15 \times CN = \sqrt{(3 \times 7) \times 7 \times (2^2 \times 2) \times (2 \times 3)}$

$\Rightarrow \frac{1}{2} \times 15 \times CN = \sqrt{2^2 \times 2^2 \times 3^2 \times 7^2}$

$\Rightarrow \frac{1}{2} \times 15 \times CN == 2 \times 2 \times 3 \times 7$

$\Rightarrow \frac{1}{2} \times 15 \times CN = 84$

$\Rightarrow CN = \frac{84 \times 2}{15} = \frac{28 \times 2}{5}$

$\Rightarrow CN = \frac{56}{5}$ m

Now, area of the trapezium shaped field ABCD

$= \frac{1}{2}(AB + DC) \times CN$ [∵ Area of trapezium = $\frac{1}{2}$ × sum of the parallel sides × distance between the parallel sides]

$= \frac{1}{2} \times (25 + 10) \times \frac{56}{5}$ m^2

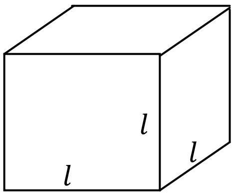
$= \frac{1}{2} \times 35 \times \frac{56}{5}$ m^2

$= 7 \times 28$ m^2

$= 196$ m^2

Surface Area and Volume of Some Basic Solids

1 Cube

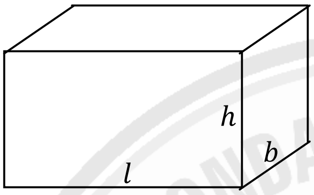


Lateral Surface Area $= 4l^2$

Total Surface Area $= 6l^2$

Volume (Capacity) $= l^3$

2 Cuboid

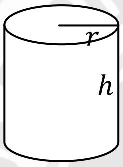


Lateral Surface Area $= 2(l + b)h$

Total Surface Area $= 2(lb + bh + hl)$

Volume (Capacity) $= lbh$

3 Cylinder

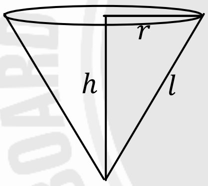


Curve Surface Area $= 2\pi rh$

Total Surface Area $= 2\pi r(r + h)$

Volume (Capacity) $= \pi r^2 h$

4 Cone



Slant Height, $l = \sqrt{r^2 + h^2}$

Curve Surface Area $= \pi rl$

Total Surface Area $= \pi r(r + l)$

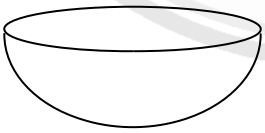
Volume (Capacity) $= \frac{1}{3}\pi r^2 h$

5 Sphere

Curve Surface Area $= 4\pi r^2$

Volume (Capacity) $= \frac{4}{3}\pi r^3$

6 Hemisphere

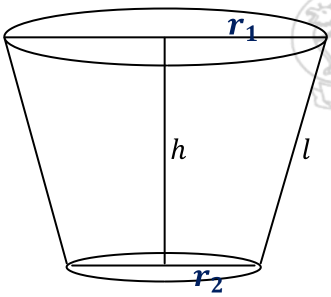


Curve Surface Area $= 2\pi r^2$

Total Surface Area $= 3\pi r^2$

Volume (Capacity) $= \frac{2}{3}\pi r^3$

7 Frustum



Slant Height of a frustum $(l) = \sqrt{(r_1 - r_2)^2 + h^2}$

Curve Surface Area $= \pi(r_1 + r_2)l$

Total Surface Area $= \pi[(r_1 + r_2)l + r_1{}^2 + r_2{}^2]$

Surface Area of a Frustum with larger face open

$$= \pi[(r_1 + r_2)l + r_2{}^2]$$

Volume $= \frac{1}{3}\pi(r_1{}^2 + r_1.r_2 + r_2{}^2)h$

SOLUTIONS

EXERCISE 12.3

(Take $\pi = \frac{22}{7}$ unless otherwise stated)

1. Find the volume and total surface area of a cuboid of edges

(i) 40 cm, 36 cm and 25 cm

(ii) 12 m, 8 m and 7 m

(iii) 20 m, 16 m and 15 m

(iv) 18 cm, 12 cm and 17 cm

Solution:

(i) Here, $l = 40$ cm, $b = 36$ cm and $h = 25$ cm

Volume of the cuboid $= l \times b \times h$

$= 40 \times 36 \times 25 \text{ cm}^3$

$= 36000 \text{ cm}^3$

Total Surface Area $= 2(lb + bh + hl)$

$= 2(40 \times 36 + 36 \times 25 + 25 \times 40) \text{ cm}^2$

$= 2(1440 + 900 + 1000) \text{ cm}^2$

$= 2 \times 3340 \text{ cm}^2$

$= 6680 \text{ cm}^2$

(ii) Here, $l = 12$ m, $b = 8$ m and $h = 7$ m

Volume of the cuboid $= l \times b \times h$

$= 12 \times 8 \times 7 \text{ m}^3$

$= 672 \text{ m}^3$

Total Surface Area $= 2(lb + bh + hl)$

$= 2 \times (12 \times 8 + 8 \times 7 + 7 \times 12) \text{ m}^2$

$= 2 \times (96 + 56 + 84) \text{ m}^2$

$= 2 \times 236 \text{ m}^2$

$= 472 \text{ m}^2$

(iii) Here, $l = 20$ m, $b = 16$ m and $h = 15$ m

Volume of the cuboid $= l \times b \times h$

$= 20 \times 16 \times 15 \text{ m}^3$

$= 4800 \text{ m}^3$

Total Surface Area $= 2(lb + bh + hl)$

$= 2 \times (20 \times 16 + 16 \times 15 + 15 \times 20)$ m^2

$= 2 \times (320 + 240 + 300)$ m^2

$= 2 \times 860$ cm^2

$= 1720$ m^2

(iv) Here, $l = 18$ cm, $b = 12$ cm and $h = 17$ cm

Volume of the cuboid $= l \times b \times h$

$= 18 \times 12 \times 17$ cm^3

$= 3672$ cm^3

Total Surface Area $= 2(lb + bh + hl)$

$= 2 \times (18 \times 12 + 12 \times 17 + 17 \times 18)$ cm^2

$= 2 \times (216 + 204 + 306)$ cm^2

$= 2 \times 726$ cm^2

$= 1452$ cm^2

2. Find the surface area and volume of a cube of side

(i) 12 cm (ii) 19 cm (iii) 5 m (iv) 13 m

Solution:

(i) Length if each side of the cube, $a = 12$ cm

Surface area of the cube $= 6a^2 = 6 \times 12^2$ cm^2 $= 6 \times 144$ cm^2 $= 864$ cm^2

Volume of the cube $= a^3 = 12^3$ cm^3 $= 1728$ cm^3

(ii) Length if each side of the cube, $a = 19$ cm

Surface area of the cube $= 6a^2 = 6 \times 19^2$ cm^2 $= 6 \times 361$ cm^2 $= 2166$ cm^2

Volume of the cube $= a^3 = 19^3$ cm^3 $= 6859$ cm^3

(iii) Length if each side of the cube, $a = 5$ m

Surface area of the cube $= 6a^2 = 6 \times 5^2$ m^2 $= 6 \times 25$ m^2 $= 150$ m^2

Volume of the cube $= a^3 = 5^3$ m^3 $= 125$ m^3

(iv) Length if each side of the cube, $a = 13$ m

Surface area of the cube $= 6a^2 = 6 \times 13^2$ m^2 $= 6 \times 169$ m^2 $= 1014$ m^2

Volume of the cube $= a^3 = 13^3$ m^3 $= 2197$ m^3

3. The volume of a cube is 74088 cm³. Find its edge and surface area.

Solution: Let a (in cm) be the length of each side of the cube.

We have, Volume of the cube = 74088 cm³

$\Rightarrow a^3 = 42^3$

$\Rightarrow a = 42$

∴ length of each edge of the cube is 42 cm.

And Surface Area = $6a^2 = 6 \times 42^2$ cm² = 6×1764 cm² = 10584 cm²

4. The surface area of a cube is 384 cm². Find its edge and volume.

Solution: Let a (in cm) be the length of each side of the cube.

We have, Surface area of the cube = 384 cm²

$\Rightarrow 6a^2 = 384$

$\Rightarrow a^2 = 64$

$\Rightarrow a^2 = 8^2$

$\Rightarrow a = 8$

∴ Length of each edge of the cube is 8 cm.

Volume of the cube = $a^3 = 8^3$ cm³ = 512 cm³

5. The length, breadth and height of a room are 5 m, 4 m and 3.5 m respectively. Find the area of four walls of a room leaving aside 3 windows each of dimensions 2 m by 1 m and 2 doors each of dimensions 2 m by 1.5 m.

Solution: For the room, $l = 5$ m, $b = 4$ m and $h = 3.5$ m

Area of the four walls of the room including 3 windows and 2 doors

$= 2(l + b) \times h$

$= 2(5 + 4) \times 3.5$ m²

$= 7 \times 9$ m²

$= 63$ m²

Area of the 3 widows = $3 \times 2 \times 1$ m² = 6 m²

Area of the 2 doors = $2 \times 2 \times 1.5$ m² = 6 m²

Now, Area of the four walls of the room leaving aside 3 windows and 2 doors

$= (63 - 6 - 6)$ m²

$= 51$ m².

6. Find the curve surface area, total surface area and the volume of a right circular cylinder whose radius r and height h are given by

(i) $r = 7$ cm, $h = 15$ cm **(ii) $r = 3.5$ cm, $h = 12.5$ cm**

(iii) $r = 14$ cm, $h = 35$ cm **(iv) $r = 7$ cm, $h = 25$ cm**

Solution:

(i) We have $r = 7$ cm, $h = 15$ cm

Curve Surface Area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 15 \text{ cm}^2$$

$$= 2 \times 22 \times 15 \text{ cm}^2$$

$$= 660 \text{ cm}^2$$

Total Surface Area $= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 7(7 + 15) \text{ cm}^2$$

$$= 2 \times 22 \times 22 \text{ cm}^2$$

$$= 968 \text{ cm}^2$$

Volume $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 15 \text{ cm}^3$$

$$= 22 \times 7 \times 15 \text{ cm}^3$$

$$= 2310 \text{ cm}^3$$

(ii) We have $r = 3.5$ cm, $h = 12.5$ cm

Curve Surface Area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 3.5 \times 12.5 \text{ cm}^2$$

$$= 2 \times 22 \times 0.5 \times 12.5 \text{ cm}^2$$

$$= 275 \text{ cm}^2$$

Total Surface Area $= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 3.5(3.5 + 12.5) \text{ cm}^2$$

$$= 2 \times 22 \times 0.5 \times 16 \text{ cm}^2$$

$$= 352 \text{ cm}^2$$

Volume $= \pi r^2 h$

$$= \frac{22}{7} \times 3.5 \times 3.5 \times 12.5 \text{ cm}^3$$

$$= 22 \times 0.5 \times 3.5 \times 12.5 \text{ cm}^3$$

$$= 481.25 \text{ cm}^3$$

(iii) We have $r = 14$ cm, $h = 35$ cm

Curve Surface Area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 35$ cm^2

$= 2 \times 22 \times 2 \times 35$ cm^2

$= 3080$ cm^2

Total Surface Area $= 2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 14(14 + 35)$ cm^2

$= 2 \times 22 \times 2 \times 49$ cm^2

$= 4312$ cm^2

Volume $= \pi r^2 h$

$= \frac{22}{7} \times 14 \times 14 \times 35$ cm^3

$= 22 \times 2 \times 14 \times 35$ cm^3

$= 21560$ cm^3

(iv) We have $r = 7$ cm, $h = 25$ cm

Curve Surface Area $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 25$ cm^2

$= 2 \times 22 \times 25$ cm^2

$= 1100$ cm^2

Total Surface Area $= 2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 7(7 + 25)$ cm^2

$= 2 \times 22 \times 32$ cm^2

$= 1408$ cm^2

Volume $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 25$ cm^3

$= 22 \times 7 \times 25$ cm^3

$= 3850$ cm^3

7. The radius of a roller 1.4 m long, is 45 cm. Find the area it sweeps in 75 revolutions.

Solution: For the roller (cylinder),

$$r = 45 \text{ cm} = \frac{45}{100} \text{ m} = 0.45 \text{ m and height, } h = 1.4 \text{ m}$$

Area swept in 1 revolution $=$ C.S.A. of the roller

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 0.45 \times 1.4 \text{ m}^2$

$= 2 \times 22 \times 0.45 \times 0.2 \text{ m}^2$

$= 3.96 \text{ m}^2$

Area swept in 75 revolutions $= 75 \times 3.96 \text{ m}^2 = 297 \text{ m}^2$

8. A garden roller of diameter 1 m is 2.1 m long. Find the area it covers in 100 revolutions.

Solution: For the garden roller (cylinder),

Diameter $= 1$ m

$\therefore$ Radius, $r = \frac{1}{2}$ m

And height, $h = 2.1$ m

Curved Surface Area of the roller $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{1}{2} \times 2.1 \text{ m}^2 = 22 \times 0.3 \text{ m}^2 = 6.6 \text{ m}^2$

$\therefore$ Area covered in 100 revolutions $= 100 \times 6.6 \text{ m}^2 = 660 \text{ m}^2$

9. A cylindrical metal pipe of thickness 1.4 cm and external diameter 56 cm is 14 m long. Find the volume of metal used in the construction of the pipe.

Solution: For the cylindrical metal pipe, we have

External radius, $R = \frac{56}{2}$ cm $= 28$ cm

Internal radius, $r = 28$ cm $-$ 1.4 cm $= 26.6$ cm

Height, $h = 14$ m $= 1400$ cm

Volume of metal used in the construction of the pipe $= \pi(R^2 - r^2)h$

$= \frac{22}{7} \times (28^2 - 26.6^2) \times 1400 \text{ cm}^3$

$= 22 \times (784 - 707.56) \times 200 \text{ cm}^3$

$= 22 \times 76.44 \times 200 \text{ cm}^3$

$= 336336 \text{ cm}^3$

10. The volume of a right circular cylinder of height 24 cm is 924 cm^3. Find the area of the curve surface of the cylinder.

Solution: For the right circular cylinder,

Height, $h = 24$ cm

Volume $= 924$ cm^3

$$\Rightarrow \pi r^2 h = 924$$

$$\Rightarrow \frac{22}{7} \times r^2 \times 24 = 924$$

$$\Rightarrow r^2 = \frac{7\times 924}{22\times 24}$$

$$\Rightarrow r^2 = \frac{7\times 42}{24} = \frac{7\times 7}{4} = \frac{7^2}{2^2}$$

$$\Rightarrow r = \frac{7}{2}$$

$\therefore$ Curved Surface area of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 24 \text{ cm}^2$$

$$= 2 \times 22 \times 12 \text{ cm}^2$$

$$= 528 \text{ cm}^2$$

11. Find the depth of a well of radius 3.5 m if its capacity is equal to that of a rectangular tank of dimensions 25 m × 11 m × 7 m.

Solution: Radius of the well, $r = 3.5$ m.

Let h (in metre) be the depth of the well.

We know, Volume of the well = Volume of the rectangular tank

$$\Rightarrow \pi r^2 h = 25 \times 11 \times 7$$

$$\Rightarrow \frac{22}{7} \times 3.5 \times 3.5 \times h = 25 \times 11 \times 7$$

$$\Rightarrow h = \frac{7\times 25\times 11 \times 7}{22\times 3.5\times 3.5} = \frac{7\times 25\times 11\times 7\times 100}{22\times 35\times 35} = 50$$

$\therefore$ the required depth of the well is 50 m.

12. A well of diameter 3.5 m is dug 16 m deep. The earth taken out is spread evenly to form a rectangular platform of base 11 m × 7 m. Find the height of the platform.

Solution: For the well (cylinder), we have

Diameter = 3.5 m i.e., $r = \frac{3.5}{2}$ m and height, $h = 16$ m

Volume of the taken out from the well $= \pi r^2 h$

$= \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 16 \text{ m}^3$

$= 22 \times 0.5 \times 3.5 \times 4 \text{ m}^3$

$= 154 \text{ m}^3$

For the rectangular platform, we have

Length, $l = 11$ m and breadth, $b = 7$ m

Area of the base of the platform $= l \times b = 11 \times 7 \text{ m}^2 = 77 \text{ m}^2$

$\therefore$ the required height of the platform $= \frac{\text{Volume of the taken out from the well}}{\text{Area of the base of the platform}} = \frac{154}{77}$ m= 2 m

13. Find the volume, curved surface area and total surface area of a cone, given that

(i) radius of the base = 3.5 m and height = 12 m

(ii) radius of the base = 0.7 m and slant height = 2.5 m

(iii) radius of the base 21 cm and slant height 35 cm

(iv) height = 24 cm and slant height = 25 cm

(v) perimeter of base = 88 cm and height = 48 cm

Solution:

(i) We have, $r = 3.5$ m and $h = 12$ m

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{3.5^2 + 12^2}$ m $= \sqrt{12.25 + 144}$ m $= \sqrt{156.25}$ m= 12.5 m

Now, Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3.5^2 \times 12 \text{ m}^3$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 4 \text{ m}^3$

$= 22 \times 0.5 \times 3.5 \times 4 \text{ m}^3$

$= 154 \text{ m}^3$

C.S.A. $= \pi r l$

$= \frac{22}{7} \times 3.5 \times 12.5 \text{ m}^2$

$= 22 \times 0.5 \times 12.5 \text{ m}^2$

$= 137.5 \text{ m}^2$

T.S.A. $= \pi r(r + l)$

$= \frac{22}{7} \times 3.5(3.5 + 12.5)$ m^2

$= 22 \times 0.5 \times 16$ m^2

$= 176$ m^2

(ii) We have, $r = 0.7$ m $= 0.7 \times 10$ dm$= 7$ dm and $l = 2.5$ m$= 2.5 \times 10$ dm$= 25$ dm

$\therefore h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2}$ dm $= \sqrt{625 - 49}$ dm $= \sqrt{576}$ dm$= 24$ dm

Now, Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$ dm^3

$= \frac{22}{7} \times 7 \times 7 \times 8$ dm^3

$= 22 \times 7 \times 8$ dm^3

$= 1232$ dm^3

C.S.A. $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$ dm^2

$= 22 \times 25$ dm^2

$= 550$ dm^2

T.S.A. $= \pi r(r + l)$

$= \frac{22}{7} \times 7(7 + 25)$ dm^2

$= 22 \times 32$ dm^2

$= 704$ dm^2

(iii) We have, $r = 21$ cm and $l = 35$ cm

$\therefore h = \sqrt{l^2 - r^2} = \sqrt{35^2 - 21^2}$cm $= \sqrt{1225 - 441}$ cm $= \sqrt{784}$ cm$= 28$ cm

Now, Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 21^2 \times 28$ cm^3

$= \frac{1}{3} \times 22 \times 21 \times 21 \times 4$ cm^3

$= 22 \times 7 \times 21 \times 4$ cm^3

$= 12936$ cm^3

C.S.A. $= \pi rl = \frac{22}{7} \times 21 \times 35$ cm^2

$= 22 \times 3 \times 35$ cm^2

$= 2310$ cm^2

T.S.A. $= \pi r(r + l)$

$= \frac{22}{7} \times 21(21 + 35)$ cm^2

$= 22 \times 3 \times 56$ cm^2

$= 3696$ cm^2

(iv) We have, $h = 24$ cm and $l = 25$ cm

$\therefore r = \sqrt{l^2 - h^2} = \sqrt{25^2 - 24^2}$cm $= \sqrt{49}$ cm$= 7$ cm

Now, Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 24$ cm^3

$= \frac{22}{7} \times 7 \times 7 \times 8$ cm^3

$= 22 \times 7 \times 8$ cm^3

$= 1232$ cm^3

C.S.A. $= \pi rl$

$= \frac{22}{7} \times 7 \times 25$ cm^2

$= 22 \times 25$ cm^2

$= 550$ cm^2

T.S.A. $= \pi r(r + l)$

$= \frac{22}{7} \times 7(7 + 25)$ cm^2

$= 22 \times 32$ cm^2

$= 704$ cm^2

(v) We have, $h = 48$ cm

And perimeter of the base $= 88$ cm

$\Rightarrow 2\pi r = 88$ cm

$\Rightarrow 2 \times \frac{22}{7} \times r = 88$ cm

$\Rightarrow r = \frac{88 \times 7}{2 \times 22}$ cm

$\Rightarrow r = 14$ cm

$\therefore l = \sqrt{r^2 + h^2}$

$= \sqrt{14^2 + 48^2}$cm

$= \sqrt{196 + 2304}$ cm

$= \sqrt{2500}$ cm$= 50$ cm

Now, Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 48$ cm^3

$= \frac{22}{7} \times 14 \times 14 \times 16$ cm^3

$= 22 \times 2 \times 14 \times 16$ cm^3

$= 9856$ cm^3

C.S.A. $= \pi r l$

$= \frac{22}{7} \times 14 \times 50$ cm^2

$= 22 \times 2 \times 50$ cm^2

$= 2200$ cm^2

T.S.A. $= \pi r(r + l)$

$= \frac{22}{7} \times 14(14 + 50)$ cm^2

$= 22 \times 2 \times 64$ cm^2

$= 2816$ cm^2

14. The curved surface area of a cone of slant height 50 cm is 2200 cm^2. Find the volume of the cone.

Solution: For the cone, we have

Slant height, $l = 50$ cm

C.S.A. $= 2200$ cm^2

$\Rightarrow \pi r l = 2200$

$\Rightarrow \frac{22}{7} \times r \times 50 = 2200$

$\Rightarrow r = \frac{7\times2200}{50\times22}$

$\Rightarrow r = 14$ cm

Then, $h = \sqrt{l^2 - r^2} = \sqrt{50^2 - 14^2}$ cm $= \sqrt{2500 - 196}$ cm$= \sqrt{2304}$ cm$= 48$ cm

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 14^2 \times 48$ cm^3

$= \frac{22}{7} \times 14 \times 14 \times 16$ cm^3

$= 22 \times 2 \times 14 \times 16$ cm^3

$= 9856$ cm^3

15. The volume of a cone of height 24 cm is 1232 cm³. Find the slant height and total surface area of the cone.

Solution: For the cone, we have

Height, $h = 24$ cm

Volume $= 1232$ cm³

$\Rightarrow \frac{1}{3}\pi r^2 h = 1232$

$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 1232$

$\Rightarrow \frac{22}{7} \times r^2 \times 8 = 1232$

$\Rightarrow r^2 = \frac{1232 \times 7}{22 \times 8} = 7 \times 7 = 7^2$

$\Rightarrow r = 7$

$\therefore$ slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 24^2}$ cm

$= \sqrt{49 + 576}$ cm

$= \sqrt{625}$ cm

$= \sqrt{25^2}$ cm

$= 25$ cm

And T.S.A. of the cone $= \pi r(r + l)$

$= \frac{22}{7} \times 7 \times (7 + 25)$ cm²

$= 22 \times 32$ cm²

$= 704$ cm²

16. A conical tent of height 24 m is made of canvas. If the circumference of the base is 44 m, find the volume of air inside the tent and the cost of canvas used, at Rs 150 per square meter.

Solution: For the conical tent, we have

$h = 24$ m

Circumference of the base $= 44$ m

$\Rightarrow 2\pi r = 44$

$\Rightarrow 2 \times \frac{22}{7} r = 44$

$\Rightarrow r = \frac{44 \times 7}{2 \times 22}$

$\Rightarrow r = 7$ m

$\therefore l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$ m $= \sqrt{49 + 576}$ m $= \sqrt{625}$ m $= \sqrt{25^2}$ m $= 25$ m

Now, Volume of air inside the tent $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24 \text{ m}^3$

$= 22 \times 7 \times 8 \text{ m}^3$

$= 1232 \text{ m}^3$

And area of the canvas used $= \pi r l = \frac{22}{7} \times 7 \times 25 \text{ m}^2 = 22 \times 25 \text{ m}^2 = 550 \text{ m}^2$

$\therefore$ Cost of the canvas used = Rs 550 × 150 = Rs 82500

17. A circus tent is cylindrical upto a height of 15 m and conical above it. If the radius of the base is 28 m and the slant height of the conical part is 35 m find the total area of canvas used in making the tent and volume of air inside the tent.

Solution: Radius of the cylindrical part = Radius of the conical part = $r = 28$ m

Height of the cylindrical part, $H = 15$ m

For the conical part,

Slant height, $l = 35$ m

$\therefore$ Height, $h = \sqrt{l^2 - r^2} = \sqrt{35^2 - 28^2}$ m

$= \sqrt{(35 - 28) \times (35 + 28)}$ m

$= \sqrt{7 \times 63}$ m $= \sqrt{3^2 \times 7^2}$ m

$= 21$ m

Now, Area of the canvas used = C.S.A. of the cylindrical part + C.S.A. of the conical part

$= 2\pi r H + \pi r l$

$= \pi r(2H + l)$

$= \frac{22}{7} \times 28(2 \times 15 + 35) \text{ m}^2$

$= 22 \times 4 \times 65 \text{ m}^2$

$= 5720 \text{ m}^2$

Volume of air inside the tent = Volume of the cylindrical part + Volume of the conical part

$= \pi r^2 H + \frac{1}{3}\pi r^2 h$

$= \pi r^2 \left(H + \frac{1}{3}h\right)$

$= \frac{22}{7} \times 28 \times 28 \left(15 + \frac{1}{3} \times 21\right) \text{ m}^3$

$= 22 \times 4 \times 28(15 + 7) \text{ m}^3$

$= 22 \times 4 \times 28 \times 22 \text{ m}^3$

$= 54208 \text{ m}^3$

18. The radii of the top and bottom of a bucket are 21 cm and 14 cm. Determine the capacity and curved surface area of the bucket if its height is 24 cm.

Solution: We have, $r_1 = 21$ cm, $r_2 = 14$ cm and $h = 24$ cm.

$$\therefore l = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$= \sqrt{(21-14)^2 + 24^2} \text{ cm}$$

$$= \sqrt{49 + 576} \text{ cm}$$

$$= \sqrt{625} \text{ cm}$$

$$= \sqrt{25^2} \text{ cm}$$

$$= 25 \text{ cm}$$

Now, Capacity of the bucket $= \frac{1}{3}\pi({r_1}^2 + r_1.r_2 + {r_2}^2)h$

$$= \frac{1}{3} \times \frac{22}{7}(21^2 + 21 \times 14 + 14^2) \times 24 \text{ cm}^3$$

$$= \frac{22}{7}(441 + 294 + 196) \times 8 \text{ cm}^3$$

$$= \frac{22}{7} \times 931 \times 8 \text{ cm}^3$$

$$= 22 \times 133 \times 8 \text{ cm}^3$$

$$= 23408 \text{ cm}^3$$

And, C.S.A. of the bucket $= \pi(r_1 + r_2)l$

$$= \frac{22}{7} \times (21 + 14) \times 25 \text{ cm}^2$$

$$= \frac{22}{7} \times 35 \times 25 \text{ cm}^2$$

$$= 22 \times 5 \times 25 \text{ cm}^2$$

$$= 2750 \text{ cm}^2$$

19. Rain water from a horizontal roof 11 m × 6 m drains into a vessel in the form of a frustum of a cone. The height of the vessel is 1.4 m and the radii of its top and bottom are 0.9 m and 0.6 m. If the vessel is just full, find the rainfall in centimeters.

Solution: For the vessel (frustum),

Height, $h = 1.4 \text{ m} = 1.4 \times 100 \text{ cm} = 140 \text{ cm}$

Radius of the top, $r_1 = 0.9 \text{ m} = 0.9 \times 100 \text{ cm} = 90 \text{ cm}$

Radius of the bottom, $r_2 = 0.6 \text{ m} = 0.6 \times 100 \text{ cm} = 60 \text{ cm}$

Volume of water in the vessel $= \frac{1}{3}\pi({r_1}^2 + r_1.r_2 + {r_2}^2)h$

$$= \frac{1}{3} \times \frac{22}{7}(90^2 + 90 \times 60 + 60^2) \times 140 \text{ cm}^3$$

$$= \frac{1}{3} \times 22(8100 + 5400 + 3600) \times 20 \text{ cm}^3$$

$$= \frac{1}{3} \times 22 \times 17100 \times 20 \text{ cm}^3$$

$$= 22 \times 5700 \times 20 \text{ cm}^3$$

$$= 2508000 \text{ cm}^3$$

For the horizontal roof, we have

Length, $l = 11 \text{ m} = 11 \times 100 \text{ cm} = 1100 \text{ cm}$

Breadth, $b = 6 \text{ m} = 6 \times 100 \text{ cm} = 600 \text{ cm}$

Area of the roof $= l \times b = 1100 \times 600 \text{ cm}^2 = 660000 \text{ cm}^2$

$\therefore$ the required rainfall $= \frac{\text{Volume of water in the vessel}}{\text{Area of the roof}}$

$$= \frac{2508000}{660000} \text{ cm}$$

$$= \frac{2508}{660} \text{ cm} = 3.8 \text{ cm}$$

20. A solid cone of height 16.8 cm and base radius 4.2 cm is melted and recast into a solid sphere. Determine the radius of the sphere.

Solution: For the cone, we have

Radius, $R = 4.2$ cm and height, $h = 16.8$ cm

Let r be the radius of the solid sphere.

We know, Volume of the sphere = Volume of the cone

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 h$

$\Rightarrow 4r^3 = R^2 h$

$\Rightarrow 4r^3 = 4.2 \times 4.2 \times 16.8$

$\Rightarrow r^3 = 4.2 \times 4.2 \times 4.2 = 4.2^3$

$\Rightarrow r = 4.2$

$\therefore$ radius of the sphere is 4.2 cm.

21. A solid sphere is melted and recast into a cone whose (base) radius is the same as that of the sphere. Show that the slant height of the cone bears a ratio $\sqrt{17}:4$ to its height.

Solution: We know,

Volume of the cone = Volume of the sphere

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow h = 4r$

$\Rightarrow r = \frac{h}{4}$

And Slant height, $l = \sqrt{r^2 + h^2}$

$\Rightarrow l = \sqrt{\left(\frac{h}{4}\right)^2 + h^2}$

$\Rightarrow l = \sqrt{\frac{h^2}{16} + h^2}$

$\Rightarrow l = \sqrt{\left(\frac{1}{16} + 1\right) h^2}$

$\Rightarrow l = \sqrt{\frac{17}{16}}h$

$\Rightarrow \frac{l}{h} = \sqrt{\frac{17}{4^2}}$

$\Rightarrow \frac{l}{h} = \frac{\sqrt{17}}{4}$

$\Rightarrow l : h = \sqrt{17} : 4$

Alternatively,

Volume of the cone = Volume of the sphere

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow h = 4r$

And $l = \sqrt{r^2 + h^2} = \sqrt{r^2 + (4r)^2}$

$= \sqrt{r^2 + 16r^2}$

$= \sqrt{17r^2}$

$= \sqrt{17}r$

Now, $l : h = \frac{\sqrt{17}r}{4r} = \frac{\sqrt{17}}{4} = \sqrt{17} : 4$

22. A solid is in the form of a cone surmounted on a hemisphere of the same radius. If the height of the cone is 24 cm and radius of the hemisphere is 7 cm, find the volume and surface area of the solid.

Solution: We have, $r = 7$ cm

For the cone,

$h = 24$ cm

and $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2}$ cm $= \sqrt{49 + 576}$ cm$= \sqrt{625}$ cm$= \sqrt{25^2}$ = cm 25 cm

Volume of the solid = volume of the conical part + volume of the hemispherical part

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7(24 + 2 \times 7)$ cm^3$= \frac{1}{3} \times 22 \times 7 \times 38$ cm^3

$= \frac{5852}{3}$ cm^3$= 1950\frac{2}{3}$ cm^3

Surface area of the solid = C.S.A. of the conical part + C.S.A of the hemispherical part

$$= \pi r l + 2\pi r^2$$

$$= \pi r(l + 2r)$$

$$= \frac{22}{7} \times 7(25 + 2 \times 7)\ \text{cm}^2$$

$$= 22 \times 39\ \text{cm}^2$$

$$= 858\ \text{cm}^2$$

23. How many lead shots, each of radius 6 cm can be made out of a rectangular slab of dimensions $44\ cm \times 36\ cm \times 24\ cm$?

Solution: Radius of each lead shot (sphere), $r = 6$ cm

$$\text{Number of lead shots} = \frac{\text{volume of the rectangular slab}}{\text{volume of a lead chot}}$$

$$= \frac{l.b.h}{\frac{4}{3}\pi r^3}$$

$$= \frac{44 \times 36 \times 24}{\frac{4}{3}\times\frac{22}{7}\times 6\times 6\times 6}$$

$$= \frac{44 \times 36 \times 24\times 3\times 7}{4\times 22\times 6\times 6\times 6}$$

$$= 42$$

24. Find the surface area and volume of a solid in the form of a right circular cylinder with hemispherical ends if the whole length is 22 cm and radius of the cylinder is 3 cm. (Take $\pi = 3.14$)

Solution: We have, $r = 3$ cm

Height of the cylindrical part, $h = 22$ cm $- 2\times 3$ cm $= 16$ cm

$$= 28.26 \times 20\ \text{cm}^2$$

Volume of the solid = Volumes of the two hemi-spherical ends+ Volume of the cylinder

$$= 2 \times \frac{2}{3}\pi r^3 + \pi r^2 h$$

$$= \pi r^2\left(\frac{4}{3}r + h\right)$$

$$= 3.14 \times 3 \times 3\left(\frac{4}{3} \times 3 + 16\right)\ \text{cm}^3$$

$$= 28.26 \times (4 + 16)\ \text{cm}^3$$

$$= 28.26 \times 20\ \text{cm}^3$$

$$= 565.2\ \text{cm}^3$$

Surface Area of the solid $= $ C.S.A. of the two hemispherical ends $+$ C.S.A. of the cylinder

$= 2 \times 2\pi r^2 + 2\pi rh$

$= 2\pi r(2r + h)$

$= 2 \times 3.14 \times 3(2 \times 3 + 16)\ \text{cm}^2$

$= 18.84 \times 22\ \text{cm}^2$

$= 414.48\ \text{cm}^2$

25. A metallic right circular cylinder of radius 6 cm and height 14 cm is melted and recast into three spheres. If the radii of two of these spheres are 2 cm and 3 cm, find the radius of the third.

Solution: For the right circular cylinder,

$r = 6$ cm and $h = 14$ cm

Let r_1, r_2 and r_3 respectively be the radii of the first, second and the third spheres.

Then, $r_1 = 2$ cm and $r_2 = 3$ cm

Sum of the volumes of the three spheres = volume of the right circular cylinder

$\Rightarrow \frac{4}{3}\pi({r_1}^3 + {r_2}^3 + {r_3}^3) = \pi r^2 h$

$\Rightarrow \frac{4}{3}(2^3 + 3^3 + {r_3}^3) = 6 \times 6 \times 14$

$\Rightarrow (8 + 27 + {r_3}^3) = 6 \times 6 \times 14 \times \frac{3}{4}$

$\Rightarrow 35 + {r_3}^3 = 378$

$\Rightarrow {r_3}^3 = 378 - 35 = 343$

$\Rightarrow {r_3}^3 = 7^3$

$\Rightarrow r_3 = 7$

$\therefore$ radius of the third sphere is 7 cm.