CHAPTER 5
TRIGONOMETRIC RATIOS

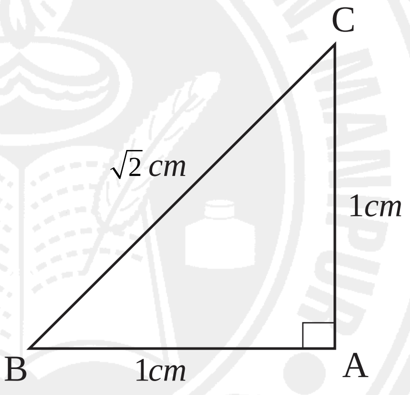
SOLUTIONS

EXERCISE 5.1

In the ΔABC, $\angle A$ is right angle. The lengths of the sides are given in *cm* in each of the following. Find $\sin B, \cos B, \tan B, \sin C, Cos C$ and $\tan C$.

1. $BC = \sqrt{2}$, $AB = AC = 1$

Soln: We have, $BC = \sqrt{2}cm \quad AB = AC = 1$



Then, $\sin B = \dfrac{AC}{BC} = \dfrac{1}{\sqrt{2}}$

$$\cos B = \frac{AB}{BC} = \frac{1}{\sqrt{2}}$$

$$\tan B = \frac{AC}{AB} = \frac{1}{1} = 1$$

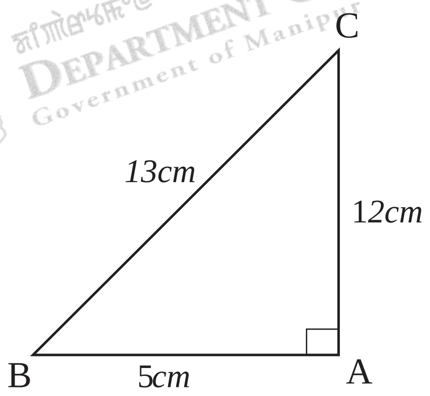
$$\sin C = \frac{AB}{BC} = \frac{1}{\sqrt{2}}$$

$$\cos C = \frac{AC}{BC} = \frac{1}{\sqrt{2}}$$

$$\tan C = \frac{AB}{AC} = \frac{1}{1} = 1$$

2. $AB = 5$, $AC = 12, BC = 13$

Soln: We have, $AB = 5cm$, $AC = 12cm, BC = 13cm$



Then, $\sin B = \dfrac{AC}{BC} = \dfrac{12}{13}$

$$\cos B = \frac{AB}{BC} = \frac{5}{13}$$

$$\tan B = \frac{AC}{AB} = \frac{12}{5}$$

$$\sin C = \frac{AB}{BC} = \frac{5}{13}$$

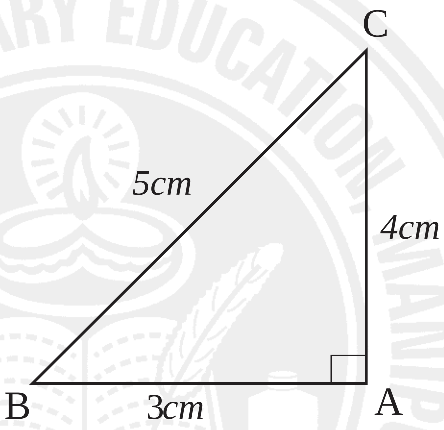
$$\cos C = \frac{AC}{BC} = \frac{12}{13}$$

$$\tan C = \frac{AB}{AC} = \frac{5}{12}$$

3. ***AB* = 3, *AC* = 4, *BC* = 5**

Soln: We have, $AB = 3cm$, $AC = 4cm$, $BC = 5cm$

Then, $\sin B = \frac{AC}{BC} = \frac{4}{5}$



$$\cos B = \frac{AB}{BC} = \frac{3}{5}$$

$$\tan B = \frac{AC}{AB} = \frac{4}{3}$$

$$\sin C = \frac{AB}{BC} = \frac{3}{5}$$

$$\cos C = \frac{AC}{BC} = \frac{4}{5}$$

$$\tan C = \frac{AB}{AC} = \frac{3}{4}$$

4. ***AB* = 20, *AC* = 21, *BC* = 29**

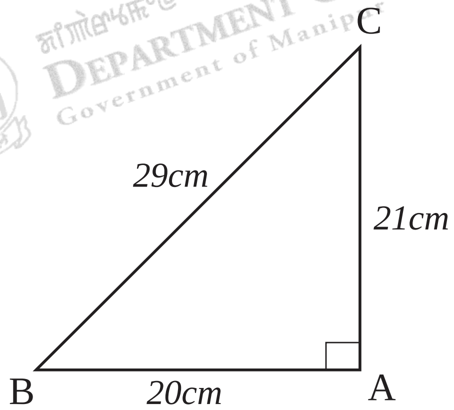
Soln: We have, $AB = 20cm$, $AC = 21\ cm$, $BC = 29\ cm$

Then, $\sin B = \frac{AC}{BC} = \frac{21}{29}$

$$\cos B = \frac{AB}{BC} = \frac{20}{29}$$

$$\tan B = \frac{AC}{AB} = \frac{21}{20}$$

$$\sin C = \frac{AB}{BC} = \frac{20}{29}$$



$$\cos C = \frac{AC}{BC} = \frac{21}{29}$$

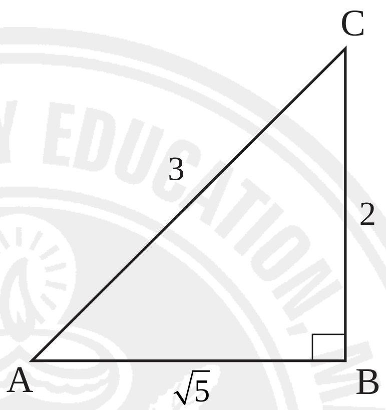
$$\tan C = \frac{AB}{AC} = \frac{20}{21}$$

EXERCISE – 5.2

Q1. In each of the following, one of the three trigonometric ratios (sine, cosine and tangent of an angle) is given. Find the other two ratios.

1. $\sin A = \dfrac{2}{3}$

Soln: In the fig, ABC is a right Δ



Given that $\sin A = \dfrac{2}{3}$

$$\Rightarrow \frac{BC}{AC} = \frac{2}{3}$$

$$\Rightarrow BC = 2, AC = 3$$

Now, by pythogoras theorem,

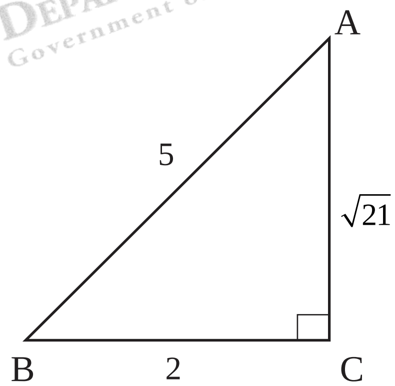
$$AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{3^2 - 2^2}$$
$$= \sqrt{9-4}$$
$$= \sqrt{5}$$

$\therefore$ $\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{5}}{3}$

$\tan A = \dfrac{BC}{AB} = \dfrac{2}{\sqrt{5}}$

2. $\cos B = \dfrac{2}{3}$

Soln: In the fig, ABC is a right Δ



Given that $\cos B = \dfrac{2}{3}$

$$\Rightarrow \frac{BC}{AB} = \frac{2}{5}$$

$$\Rightarrow BC = 2, AB = 5$$

Now, by pythogoras theorem,

$$AC^2 = AB^2 - BC^2$$

$$= 5^2 - 2^2$$

$$= 25 - 4 = 21$$

$$\Rightarrow AC = \sqrt{21}$$

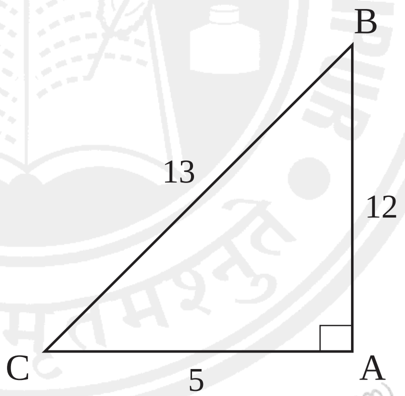
Then, $\sin B = \dfrac{AC}{AB} = \dfrac{\sqrt{21}}{5}$

$$\tan B = \frac{AC}{BC} = \frac{\sqrt{21}}{2}$$

3. $\tan C = \dfrac{12}{5}$

Soln: In the fig, ABC is a right Δ

& $\tan C = \dfrac{12}{5}$



$$\Rightarrow \frac{AB}{AC} = \frac{12}{5}$$

$$\Rightarrow \frac{AB}{AC} = \frac{12}{5}$$

$$\Rightarrow AB = 12, AC = 6$$

Now, by pythogoras theorem,

$$BC^2 = AB^2 + AC^2$$

$$= 12^2 + 5^2$$

$$= 144 + 25$$

$$\Rightarrow BC = \sqrt{169} = 13$$

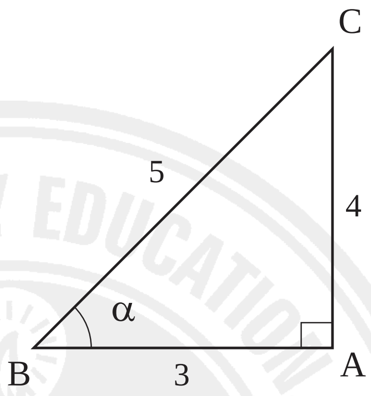
Then, $\sin C = \dfrac{AB}{BC} = \dfrac{12}{13}$

$$\cos C = \frac{CA}{BC} = \frac{5}{13}$$

4. $\sin\alpha = \frac{4}{5}$

Soln: In the fig, ABC is a right Δ in which $\angle B = \alpha$

& $\sin\alpha = \frac{4}{5}$



$$\Rightarrow \frac{AC}{BC} = \frac{4}{5}$$

$$\therefore AC = 4, BC = 3$$

Now, by pythogoras theorem,

$$AB^2 = BC^2 - AC^2$$

$$= 5^2 - 4^2$$

$$= 25 - 16$$

$$\Rightarrow AB = \sqrt{9} = 3$$

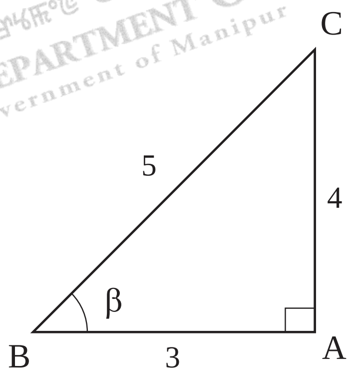
Then, $\cos\alpha = \frac{AB}{BC} = \frac{3}{5}$

$$\tan\alpha = \frac{AC}{AB} = \frac{4}{3}$$

5. $\cos\beta = \frac{3}{5}$

Soln: In the fig, ABC is a right Δ in which $\angle B = \beta$

& $\cos\beta = \frac{3}{5}$



$$\Rightarrow \frac{AB}{BC} = \frac{3}{5}$$

$$\therefore AB = 3, BC = 5$$

Now, by pythogoras theorem,

$$AC^2 = BC^2 - AB^2$$

$$= 5^2 - 3^2$$

$$= 25 - 9$$

$$\Rightarrow AC = \sqrt{16} = 4$$

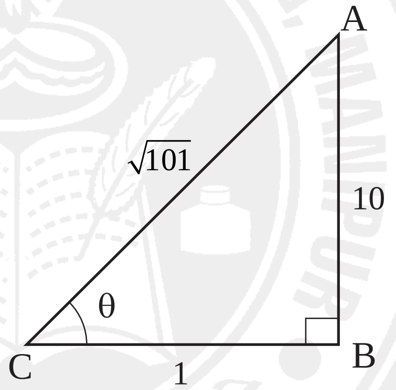
Then, $\sin\beta = \dfrac{AC}{BC} = \dfrac{4}{5}$

$$\tan\beta = \frac{AC}{AB} = \frac{4}{3}$$

6. $\tan\theta = 10$

Soln: In the fig, ABC is a right Δ in which $\angle C = \theta$

& $\tan\theta = 10$



$$\Rightarrow \frac{AB}{BC} = \frac{10}{1}$$

$$\therefore AB = 10, BC = 1$$

Now, by pythogoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 10^2 + 1^2$$

$$= 100 + 1$$

$$\Rightarrow AC = \sqrt{101}$$

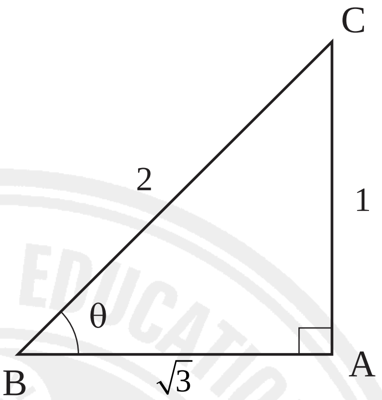
$\therefore$ $\sin\theta = \dfrac{AB}{BC} = \dfrac{10}{\sqrt{101}}$

$$cos\theta = \frac{BC}{AC} = \frac{1}{\sqrt{101}}$$

EXERCISE – 5.3

Q1. **If $\operatorname{cosec}\theta = 2$, find the other five trigonometric ratios of θ.**

Soln: Since $\operatorname{cosec}\theta = 2$



$$\therefore \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{2}$$

Here, ABC is a right triangle in which $\angle B = \theta$

Then, $AC = 1, BC = 2$

Now, $BC^2 = AC^2 + AB^2$

$$\Rightarrow 2^2 = 1^2 + AB^2$$

$$\Rightarrow AB^2 = 4 - 1 = 3$$

$$\Rightarrow AB = \sqrt{3}$$

$$\therefore \cos\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{2}$$

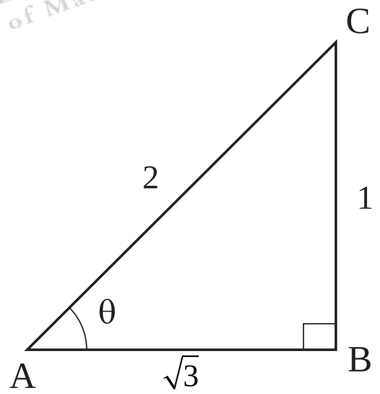
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$$\tan\theta = \frac{AC}{AB} = \frac{1}{\sqrt{3}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

Q2. **If $\sec\theta = \frac{2}{\sqrt{3}}$, find the other five trigonometric ratios of θ.**

Soln: Here, ABC is a right Δ in which $\angle A = \theta$



And $\sec\theta = \frac{2}{\sqrt{3}}$

$$\Rightarrow \frac{AC}{AB} = \frac{2}{\sqrt{3}}$$

$$\therefore\ AC = 2, AB = \sqrt{3}$$

Now, $BC^2 = \sqrt{AC^2 - AB^2}$

$$= \sqrt{2^2 - (\sqrt{3})^2}$$

$$= \sqrt{4-3} = 1$$

$$\therefore\ \sin\theta = \frac{BC}{AC} = \frac{1}{2}$$

$$\cos\theta = \frac{AB}{AC} == \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \sqrt{3}$$

$$\cos ec\theta = \frac{1}{1/2} = 2$$

Q3. In a ΔABC rt. angled at C, if $\tan A = \sqrt{3}$ and $\tan B = \frac{1}{\sqrt{3}}$, show that

$$\sin A\cos B + \cos A\sin B = 1$$

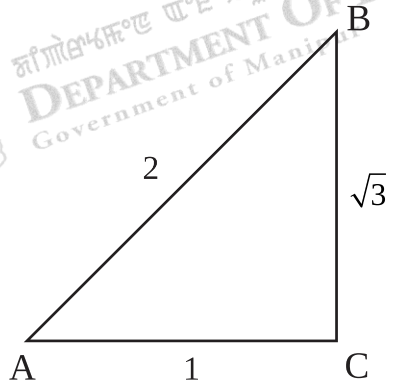
Soln: Here, $\tan A = \sqrt{3}$

$$\Rightarrow \frac{BC}{AC} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow BC = \sqrt{3}, AC = 1$$

$$\therefore\quad AB = \sqrt{AC^2 + AB^2}$$

$$= \sqrt{1+3}$$



$$= \sqrt{4} = 2$$

$$\sin A = \frac{BC}{AB} = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{AC}{AB} = \frac{1}{2}$$

Again, $\tan B = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{AC}{BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow AC = 1, BC = \sqrt{3}$$

$$\sin B = \frac{AC}{AB} = \frac{1}{2}$$

$$\cos B = \frac{BC}{AB} = \frac{\sqrt{3}}{2}$$

$$\therefore \sin A\cos B + \cos A\sin B = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} + \frac{1}{2}\times\frac{1}{2}$$

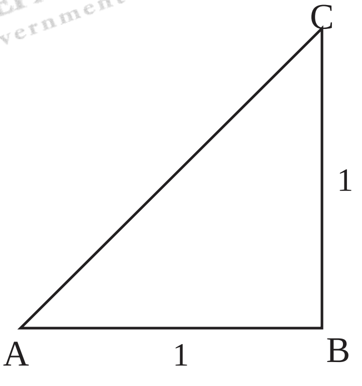
$$= \frac{3}{4} + \frac{1}{4}$$

$$= \frac{4}{4} = 1$$

Hence, $\sin A\cos B + \cos A\sin B = 1$

Q4. If $\tan A = 1$ and $\tan B = \sqrt{3}$, evaluate $\cos A\cos B - \sin A\sin B$ [Here, A and B are not the angles of the same triangle.]

Soln: Here, $\tan A = 1$



$$\Rightarrow \frac{BC}{AB} = \frac{1}{1}$$

$$\Rightarrow BC = 1, AB = 1$$

$$\therefore \quad AC = \sqrt{AC^2 + AB^2}$$

$$= \sqrt{1^2 + 1^2} = \sqrt{2}$$

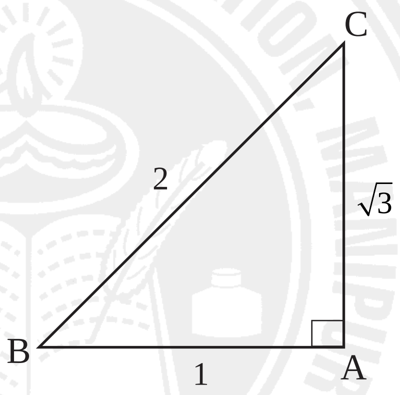
$$\therefore \quad \cos A = \frac{AB}{AC} = \frac{1}{\sqrt{2}}$$

$$\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{2}}$$

Again, $\tan B = \sqrt{3}$

$$\Rightarrow \frac{AC}{AB} = \frac{\sqrt{3}}{1}$$

$$\Rightarrow AC = \sqrt{3}, AB = 1$$



Now, $BC = \sqrt{AB^2 + AC^2}$

$$= \sqrt{1^2 + (\sqrt{3})^2}$$

$$= \sqrt{1+3} = \sqrt{4} = 2$$

$$\therefore \quad \sin B = \frac{AC}{BC} = \frac{\sqrt{3}}{2}$$

$$\cos B = \frac{AB}{BC} = \frac{1}{2}$$

$\cos A \cos B - \sin A \sin B$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$$

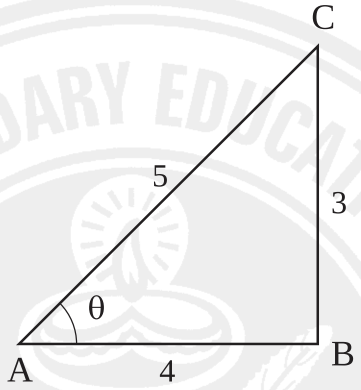
$$= \frac{1-\sqrt{3}}{2\sqrt{2}}$$

Q5. **If** $\sec\theta = \frac{5}{4}$**, verify that** $\tan\theta = \sin\theta.\sec\theta$

Soln: Here, $\sec\theta = \frac{5}{4}$

$$\Rightarrow \frac{AC}{AB} = \frac{5}{4}$$

$$\Rightarrow BC = 5, AB = 4$$



$\therefore$ $$BC = \sqrt{5^2 - 4^2}$$

$$= \sqrt{9} = 3$$

and, $$\sin\theta = \frac{BC}{AC} = \frac{3}{5}$$

$$\tan\theta = \frac{BC}{AB} = \frac{3}{4}$$

Now, $$\sin\theta.\sec\theta = \frac{3}{5} \times \frac{5}{4}$$

$$= \frac{3}{4}$$

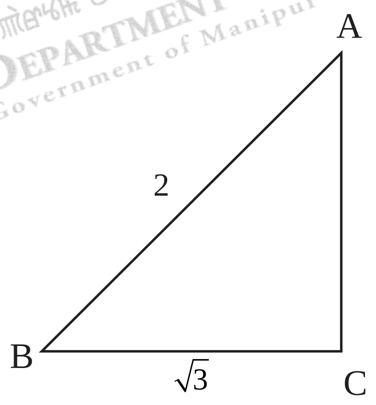
$$= \tan\theta$$

Hence, $\tan\theta = \sin\theta.\sec\theta$

Q6. **If** $\cos B = \frac{\sqrt{3}}{2}$**, show that** $3\sin B - 4\sin^3 B = 1$**.**

Soln: Here, $\cos B = \frac{\sqrt{3}}{2}$

$$\Rightarrow \frac{BC}{AB} = \frac{\sqrt{3}}{2}$$



$\therefore$ $$\Rightarrow BC = \sqrt{3}, AB = 2$$

$\therefore$ $$AC = \sqrt{AB^2 - BC^2}$$

$$= \sqrt{2^2 - (\sqrt{3})^2}$$

$= \sqrt{4-3}$

$= \sqrt{1} = 1$

$\sin B = \frac{AC}{AB} = \frac{1}{2}$

Now, $3\sin B - 4\sin^3 B = 3.\frac{1}{2} - 4\left(\frac{1}{2}\right)^3$

$= \frac{3}{2} - 4 \times \frac{1}{8}$

$= \frac{3}{2} - \frac{1}{2}$

$= \frac{3-1}{2}$

$= \frac{2}{2} = 1$

Hence proved.

KEY NOTES:

1. Three identities:

(i) $\sin^2\theta+\cos^2\theta=1$

(ii) $1+\tan^2\theta=\sec^2\theta$

(iii) $1+\cot^2\theta=\operatorname{cosec}^2\theta$

2. Quotient relations:

(i) $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

(ii) $\cot\theta=\dfrac{\cos\theta}{\sin\theta}$

EXERCISE-5.4

Prove that (Q. No. 1 – 13)

1. $\cos^2\theta-\sin^2\theta=1-2\sin^2\theta$

Soln: LHS $=\cos^2\theta-\sin^2\theta$

$=(1-\sin^2\theta)-\sin^2\theta \quad [\because \sin^2\theta+\cos^2\theta=1]$

$=1-2\sin^2\theta$

= R.H.S.

Hence proved.

2. $1-\sin^2\theta = \dfrac{1}{1+\tan^2\theta}$

Soln: R.H.S. $= \dfrac{1}{1+\tan^2\theta} = \dfrac{1}{\sec^2\theta}$

$= \dfrac{1}{\frac{1}{\cos^2\theta}} = \cos^2\theta$

$= 1-\sin^2\theta$

$=$ LHS

Hence proved.

3. $2\cos^2\theta - 1 = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

Soln: R.H.S. $= \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$

$= \dfrac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\times\cos^2\theta$

$= \cos^2\theta-\sin^2\theta$

$= \cos^2\theta-(1-\cos^2\theta) \quad [\because \sin^2\theta+\cos^2\theta=1]$

$= \cos^2\theta-1+\cos^2\theta$

$= 2\cos^2\theta-1 = \text{L.H.S.}$

Hence proved.

4. $\sin^4\theta - \cos^4\theta$

Soln: L.H.S. $= \sin^4\theta - \cos^4\theta$

$= \left(\sin^2\theta\right)^2 - \left(\cos^2\theta\right)^2$

$= \left(\sin^2\theta - \cos^2\theta\right)\left(\sin^2\theta + \cos^2\theta\right)$

$= \left(\sin^2\theta - \cos\theta\right)\times 1$

$= \sin^2\theta - \cos^2\theta$

= R.H.S.

Hence proved.

5. $(1-\cos^2\theta)(1+\cot^2\theta) = 1$

Soln: L.H.S. $=(1-\cos^2\theta)(1+\cot^2\theta)$

$= \left(1-\cos^2\theta\right)\cos ec^2\theta$

$= \sin^2\theta \times \frac{1}{\sin^2\theta}$

= 1 = R.H.S.

Hence proved.

6. $\sqrt{\sec^2\theta - 1} = \sin\theta\sec\theta$

Soln: L.H.S. $= \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta}$

$= \tan\theta = \frac{\sin\theta}{\cos\theta} = \sin\theta.\sec\theta \quad \left[\because \frac{1}{\cos\theta} = \sec\theta\right]$

= R.H.S.

Hence proved.

7. $\sec^4\theta - \tan^4\theta = \sec^2\theta + \tan^2\theta$

Soln: L.H.S. $= \sec^4\theta - \tan^4\theta$

$= \left(\sec^2\theta\right)^2 - \left(\tan^2\theta\right)^2$

$= \left(\sec^2\theta - \tan^2\theta\right)\left(\sec^2\theta + \tan^2\theta\right)$

$= 1\times\left(\sec^2\theta + \tan^2\theta\right) \quad \left[\because 1+\tan^2\theta = \sec^2\theta\right]$

$= \sec^2\theta + \tan^2\theta$

= R.H.S.

Hence proved.

8. $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta.\operatorname{cosec}^2\theta$

Soln: L.H.S. $= \sec^2\theta + \operatorname{cosec}^2\theta$

$= \dfrac{1}{\cos^2\theta} + \dfrac{1}{\sin^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta.\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta.\sin^2\theta}$

$= \dfrac{1}{\cos^2\theta} \times \dfrac{1}{\sin^2\theta}$

$= \sec^2\theta.\operatorname{cosec}^2\theta$ = R.H.S.

Hence proved.

9. $\tan^2\theta - \sin^2\theta = \tan^2\theta.\sin^2\theta$

Soln: L.H.S. $= \tan^2\theta - \sin^2\theta$

$= \dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

$= \dfrac{\sin^2\theta - \sin^2\theta.\cos^2\theta}{\cos^2\theta}$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta}(1-\cos^2\theta)$$

$$= \tan^2\theta.\sin^2\theta$$

= R.H.S.

Hence proved.

10. $\dfrac{1-\sin x}{1+\sin x} = (\sec x - \tan x)^2$

Soln: L.H.S. $= \dfrac{1-\sin x}{1+\sin x} = \dfrac{(1-\sin x)(1-\sin x)}{(1+\sin x)(1-\sin x)}$

$$= \frac{(1-\sin x)^2}{1-\sin^2 x} = \left(\frac{1-\sin x}{\cos x}\right)^2$$

$$= \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right)^2$$

$= (\sec x - \tan x)^2$ = R.H.S.

Hence proved.

11. $\dfrac{\sin\theta}{1-\cos\theta} = \dfrac{1+\cos\theta}{\sin\theta}$

Soln: L.H.S. $= \dfrac{\sin\theta}{1-\cos\theta} = \dfrac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$= \dfrac{1+\cos\theta}{\sin\theta}$ = R.H.S.

Hence proved.

12. $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\dfrac{1+\sin\theta}{\cos\theta}$

Soln: L.H.S. $=\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}$

$$=\frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}\qquad\left[\because 1+\tan^2\theta=\sec^2\theta\right]$$

$$=\frac{(\tan\theta+\sec\theta)+(\tan^2\theta-\sec^2\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)(1+\tan\theta-\sec\theta)}{(1+\tan\theta-\sec\theta)}$$

$$=\tan\theta+\sec\theta=\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}$$

$$=\frac{\sin\theta+1}{\cos\theta}=\frac{1+\sin\theta}{\cos\theta}=\text{R.H.S.}$$

Hence proved.

14. $\dfrac{1+\sin\theta}{\cos\theta}=\dfrac{\cos\theta}{1-\sin\theta}$

Soln: L.H.S. $=\dfrac{1+\sin\theta}{\cos\theta}=\dfrac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}$

$$=\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}$$

$$=\frac{\cos\theta}{\cos\theta}\times\frac{\cos\theta}{1-\sin\theta}$$

$$=\frac{\cos\theta}{1-\sin\theta}=\text{R.H.S.}$$

Q14. Eliminate θ from the equation:-

(i) $x=r\cos\theta$, $y=r\sin\theta$

Soln: Here, $x=r\cos\theta$

$$\Rightarrow x^2=r^2\cos^2\theta \quad - \quad \text{(i)}$$

& $y = r\sin\theta$

$$\Rightarrow y^2 = r^2\sin^2\theta \qquad - \qquad \text{(ii)}$$

Adding (i) & (ii), we get

$$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta$$

$$= r^2(\cos^2\theta + \sin^2\theta)$$

$$= r^2 \times 1$$

$\therefore\ x^2 + y^2 = r^2$

(ii) $x = a\sec\theta$, $y = b\tan\theta$

Soln: Here, $x = a\sec\theta$

$$\Rightarrow \sec\theta = \frac{x}{a}$$

$$\Rightarrow \sec^2\theta = \frac{x^2}{a^2} \qquad - \qquad \text{(i)}$$

& $y = b\tan\theta$

$$\Rightarrow \tan^2\theta = \frac{y^2}{b^2} \qquad - \qquad \text{(ii)}$$

Subtracting (i) from (ii), we get

$$\sec^2\theta - \tan^2\theta = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\Rightarrow 1 = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(iii) $x = a\cos ec\theta$, $y = b\cot\theta$

Soln: Here, $x = a\cos ec\theta$

$$\Rightarrow \cos ec\theta = \frac{x}{a}$$

$$\Rightarrow \cos\sec^2\theta = \frac{x^2}{a^2} \quad - \quad \text{(i)}$$

& $\quad y = b\cot\theta$

$$\Rightarrow \cot\theta = \frac{y}{b}$$

$$\Rightarrow \cot^2\theta = \frac{y^2}{b^2} \quad - \quad \text{(ii)}$$

Subtracting (ii) from (i), we get

$$\cos ec^2\theta - \cot^2\theta = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\Rightarrow 1 = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\Rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

(iv) $\sin\theta + \cos\theta = a$

$\tan\theta + \cot\theta = b$

Soln: We have, $\sin\theta + \cos\theta = a \quad - \quad \text{(i)}$

$\tan\theta + \cot\theta = b \quad - \quad \text{(ii)}$

Squaring both sides of (i), we get

$$(\sin\theta + \cos\theta)^2 = a^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = a^2$$

$$\Rightarrow 1 + 2\sin\theta\cos\theta = a^2$$

$$\Rightarrow 2\sin\theta\cos\theta = a^2 - 1$$

From (ii), we get

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = b$$

$$\Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = b$$

$$\Rightarrow \frac{1}{\frac{a^2-1}{2}} = b \qquad \text{[by using (iii)]}$$

$$\Rightarrow \frac{2}{a^2-1} = b$$

$$\Rightarrow 2 = b(a^2-1)$$

$$\Rightarrow b(a^2-1) = 2$$

(v) $x\cos\theta + y\sin\theta = 3$

$y\cos\theta - x\sin\theta = 4$

Soln: Here, $x\cos\theta + y\sin\theta = 3$

$$\Rightarrow (x\cos\theta + y\sin\theta)^2 = 3^2$$

$$\Rightarrow x^2\cos^2\theta + y^2\sin^2\theta + 2xy\sin\theta\cos\theta = 9 \qquad - \qquad \text{(i)}$$

& $y\cos\theta - x\sin\theta = 4$

$$\Rightarrow (y\cos\theta - x\sin\theta)^2 = 4^2$$

$$\Rightarrow y^2\cos^2\theta + x^2\sin^2\theta - 2xy\sin\theta\cos\theta = 16 \qquad - \qquad \text{(ii)}$$

Adding (i) & (ii), we get

$$x^2(\cos^2\theta + \sin^2\theta) + y^2(\sin^2\theta + \cos^2\theta) = 25$$

$$\Rightarrow x^2 + y^2 = 25$$

Q15. **If** $\tan\theta + \sec\theta = x$, **show that** $\sin\theta = \dfrac{x^2 - 1}{x^2 + 1}$

Soln: Here, $\tan\theta + \sec\theta = x$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = x$$

$$\Rightarrow \frac{1+\sin\theta}{\cos\theta} = x$$

$$\Rightarrow \left(\frac{1+\sin\theta}{\cos\theta}\right)^2 = x^2 \quad \text{[squaring both sides]}$$

$$\Rightarrow \frac{(1+\sin\theta)^2}{\cos^2\theta} = x^2$$

$$\Rightarrow \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = x^2$$

$$\Rightarrow \frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)} = x^2$$

$$\Rightarrow \frac{1+\sin\theta}{1-\sin\theta} = x^2$$

$$\Rightarrow \frac{(1+\sin\theta)-(1-\sin\theta)}{(1+\sin\theta)+(1-\sin\theta)} = \frac{x^2-1}{x^2+1} \quad \text{[By dividendo \& componendo]}$$

$$\Rightarrow \frac{1+\sin\theta-1+\sin\theta}{1+\sin\theta+1-\sin\theta} = \frac{x^2-1}{x^2+1}$$

$$\Rightarrow \frac{2\sin\theta}{2} = \frac{x^2-1}{x^2+1}$$

$$\Rightarrow \sin\theta = \frac{x^2-1}{x^2+1}$$

Q16. **If** $\tan\theta = \frac{a}{b}$, **show that** $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2}$

Soln: here, $\tan\theta = \frac{a}{b}$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$$

$$\Rightarrow \frac{a\sin\theta}{b\cos\theta} = \frac{a^2}{b^2} \qquad \text{[Multiplying both sides by } \frac{a}{b}\text{]}$$

$$\Rightarrow \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2 - b^2}{a^2 + b^2} \quad \text{[by dividendo \& componendo]}$$

Q17. **If** $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, **prove that**

$\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

Soln: We have, $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$$\Rightarrow (\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2 \quad \text{[squaring both sides)}$$

$$\Rightarrow \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$\Rightarrow 1 + 2\cos\theta\sin\theta = 2(1 - \sin^2\theta)$$

$$\Rightarrow 1 + 2\cos\theta.\sin\theta = 2 - 2\sin^2\theta$$

$$\Rightarrow 2\sin^2\theta = 2 - 1 - 2\cos\theta\sin\theta$$

$$= 1 - 2\cos\theta\sin\theta$$

$$= \sin^2\theta + \cos^2\theta - 2\cos\theta\sin\theta$$

$$= (\cos\theta - \sin\theta)^2$$

$$\cos\theta - \sin\theta = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta$$

Hence proved.

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