DEPARTMENT OF EDUCATION (S)
Government of Manipur

CLASS – IX
MATHEMATICS
CHAPTER – 3
COORDINATE GEOMETRY

Cartesian Co-ordinates

Rene Descartes, the great French mathematician and philosopher propounded a system of describing the position of a point in a plane. In honour of Descartes, this system used for describing the position of a point in a plane is known as the Cartesian System of Co-ordinates.

Rectangular Cartesian Co-ordinate System

To fix the position of a point P in a plane, we take two fixed perpendicular lines conventionally one horizontal and other vertical on the plane intersecting at a point. The horizontal line is called X-axis and the vertical line is called Y-axis. The plane with these two co-ordinate axes is known as the Cartesian plane. The point of intersection of the co-ordinate axes is called origin.

Quadrants

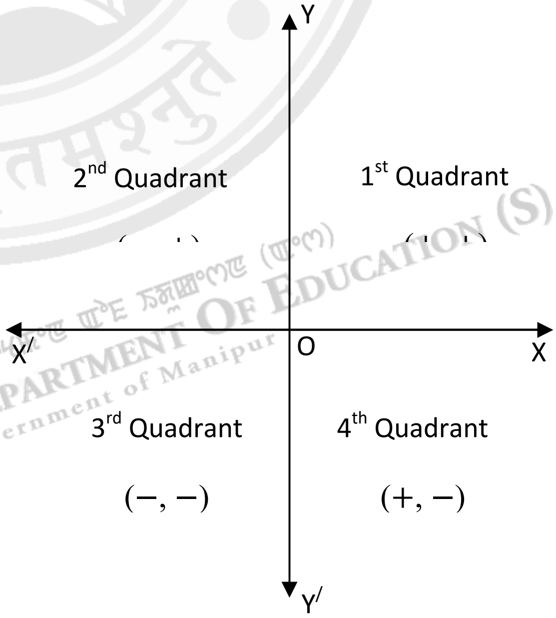
The co-ordinate axes divide the plane into four regions. Each region is called a Quadrant.

i) If a point lies in the 1st quadrant, the signs of its co-ordinates are of the form $(+, +)$.

ii) If a point lies in the 2nd quadrant, then the signs of its co-ordinates are of the form $(-, +)$.

iii) If a point lies in the 3rd quadrant, then the signs of its co-ordinates are of the form $(-, -)$.

iv) If a point lies in the 4th quadrant, then the sings of its co-ordinates are of the form $(+, -)$.



Note: (i) For a point $P(a, b)$ on the Cartesian plane, a is called the x–coordinate or abscissa and b is called the y – coordinate or ordinate of the point P.

(ii) If a point lies on the X-axis, its ordinate is zero and if a point lies on the Y-axis, its abscissa is zero.

Plotting of point on a plane

The steps of locating a point with a given co-ordinates on a plane

Steps 1: We take the co-ordinate axes on the plane so that the origin is at a suitable position preferably at the middle of the plane

Steps 2: We choose the scale on the axes so that the point corresponding to the given co-ordinates may be shown in the plane

Steps 3: We check the sign of the abscissa. If it is positive, we take the required units starting from the origin O along the positive direction of the X-axis. If it is negative, w take required units starting from O along the negative direction of X-axis. If it is zero, it remains at O.

Steps 4: We name the point obtained in step 3, A (say).

Steps 5: We check the sign of the ordinate. If it is positive, we take the required units starting from A along the positive direction of the Y-axis. If it is negative, we take the required units starting from A along the negative direction of the Y-axis. If it is zero, it remains at A.

Steps 6: We name the point obtained in step 5, P (say).

Then P is the required point on the plane with the given co-ordinates.

SOLUTIONS

EXERCISE 3.1

1. In which quadrants, do the following points lie?

(a) $(-2, 7)$

Ans: Second quadrant

(b) $(-2, -3)$

Ans: Third quadrant.

(c) $(1, 6)$

Ans: First quadrant

(d) $(5, -3)$

Ans: Fourth quadrant

2. **Locate the points given below on the Cartesian plane and also state the quadrants in which they lie.**

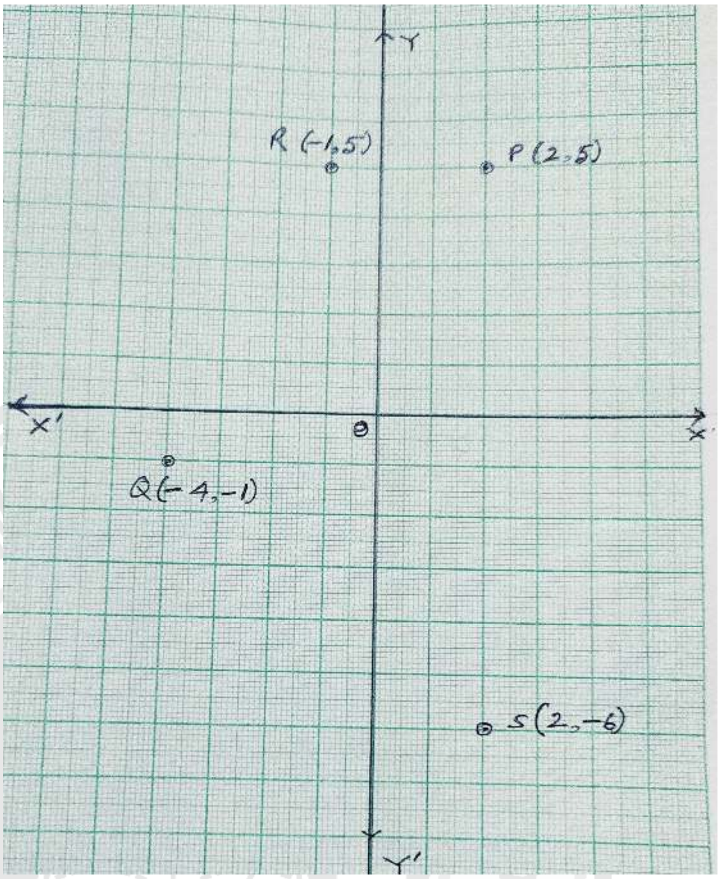
(a) (2, 5)

(b) (−4,−1)

(c) (−1,5)

(d) (2,−6)

Solution:



(a) On the Cartesian plane, P represents (2, 5). It lies in the first quadrant.

(b) On the Cartesian plane, Q represents (−4, −1). It lies in the third quadrant.

(c) On the Cartesian plane, R represents (−1, 5). It lies in the Second quadrant.

(d) On the Cartesian plane, S represents (2,−6). It lies in the fourth quadrant.

3. **On which axis do the following points lie:**

a) (1,0)

Ans: It lies on the X-axis.

b) (0,5)

Ans: It lies on the Y-axis.

c) (−3,0)

Ans: It lies on the X-axis.

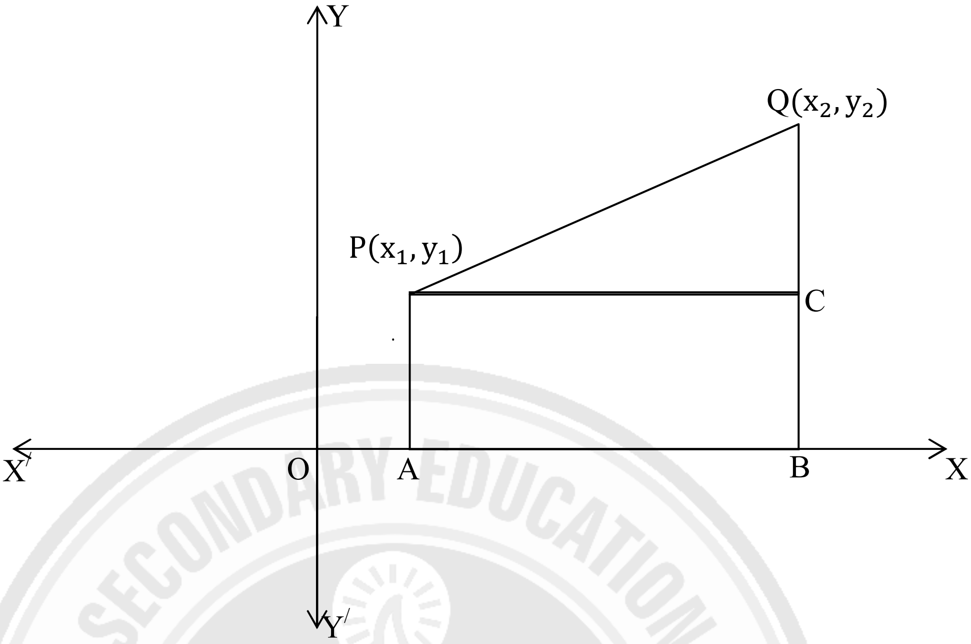
d) (0,−2)

Ans: It lies on the Y-axis.

e) (0,0)

Ans: It lies on both the X and Y axes.

Distance between two points



Let XOX/ and YOY/ be the two co-ordinate axes. P (x_1, y_1) and Q(x_2, y_2) be any two points in the Cartesian plane. PA and QB are drawn perpendicular to the X- axis. PC is also drawn perpendicular to QB meeting QB at C.

We have $BC = AP = y_1$

$PC = AB = OB - OA = x_2 - x_1$

and $QC = QB - BC = QB - PA = y_2 - y_1$

In ΔPCQ, ∠PCQ= 90^0,

∴ by Pythgoras theorm, we have

$$PQ^2 = PC^2 + QC^2$$

$$\Rightarrow PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\Rightarrow PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

∴The distance between any two points P (x_1, y_1) and Q(x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note:

1. The distance between any two points P (x_1, y_1) and Q(x_2, y_2) can also be taken as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
2. The distance of any point (x, y) from the origin O is $\sqrt{x^2 + y^2}$.

SOLUTIONS

EXERCISE 3.2

1. Find the distance of the following points from the origin:

(a) (2, 5)

Solution: The distance of the point (2, 5) from the origin $= \sqrt{2^2 + 5^2}$

$= \sqrt{4 + 25}$

$= \sqrt{29}$ units

(b) (6, −10)

Solution: The distance of the point (6, −10) from the origin $= \sqrt{6^2 + (-10)^2}$

$= \sqrt{36 + 100}$

$= \sqrt{136} = \sqrt{2^2 \times 2 \times 17}$

$= 2\sqrt{34}$ units

(c) (−6, 12)

Solution: The distance of the point (−6, 12) from the origin $= \sqrt{(-6)^2 + 12^2}$

$= \sqrt{36 + 144}$

$= \sqrt{180}$

$= \sqrt{2^2 \times 3^2 \times 5}$

$= 2 \times 3\sqrt{5}$

$= 6\sqrt{5}$ units

2. Find the distance between the pairs of points

(a) (1, 3), (7, 2)

Solution: The distance between (1, 3) and (7, 2) $= \sqrt{(1-7)^2 + (3-2)^2}$

$= \sqrt{(-6)^2 + 1^2}$

$= \sqrt{36 + 1}$

$= \sqrt{37}$ units

(b) (7, −2), (3, −1)

Solution: The distance between (7, −2) and (3, −1) $= \sqrt{(7-3)^2 + \{-2-(-1)\}^2}$

$= \sqrt{4^2 + (-2+1)^2}$

$= \sqrt{16 + (-1)^2}$

$= \sqrt{16 + 1}$

$= \sqrt{17}$ units

(c) (10, 4), (−1, −2)

Solution: The distance between (10, 4) and (−1, −2) $=\sqrt{\{10-(-1)\}^2+\{4-(-2)\}^2}$

$=\sqrt{(10+1)^2+(4+2)^2}$

$=\sqrt{11^2+6^2}$

$=\sqrt{121+36}$

$=\sqrt{157}$ units

(d) (−1, 3), (4, −2)

Solution: The distance between (−1, 3) and (4, −2) $=\sqrt{(-1-4)^2+\{3-(-2)\}^2}$

$=\sqrt{(-5)^2+(3+2)^2}$

$=\sqrt{25+5^2}$

$=\sqrt{25+25}$

$=\sqrt{2\times 25}$

$=\sqrt{2\times 5^2}$

$=5\sqrt{2}$ units

3. The Co-ordinates of A are (-4, 8) and those of B are $(x, 3)$. Find x of AB=13.

Solution: We have, AB = 13

$\Rightarrow \sqrt{(-4-x)^2+(8-3)^2}=13$

$\Rightarrow \{-(x+4)\}^2+(8-3)^2=13^2$

$\Rightarrow (x+4)^2+5^2=169$

$\Rightarrow x^2+8x+16+25=169$

$\Rightarrow x^2+8x+41=169$

$\Rightarrow x^2+8x-128=0$

$\Rightarrow x^2+(16-8)x-128=0$

$\Rightarrow x^2+16x-8x-128=0$

$\Rightarrow x(x+16)-8(x+16)=0$

$\Rightarrow (x+16)(x-8)=0$

$\Rightarrow$ Either $x+16=0$ or $x-8=0$

$\Rightarrow x=-16$ or $x=8$

$\therefore\ x=-16$ or 8

4. **Show that the point A (2,7) , B (3,0), C (-4,-1) are vertices of an isosceles triangle and find the length of the base**.

Solution: Here,

$$AB = \sqrt{(2-3)^2 + (7-0)^2}$$
$$= \sqrt{(-1)^2 + 7^2}$$
$$= \sqrt{1+49}$$
$$= \sqrt{50}$$
$$= \sqrt{2\times 5^2}$$
$$= 5\sqrt{2}$$

$$BC = \sqrt{\{3-(-4)\}^2 + \{0-(-1)\}^2}$$
$$= \sqrt{(3+4)^2 + (0+1)^2}$$
$$= \sqrt{7^2 + 1^2}$$
$$= \sqrt{49+1}$$
$$= \sqrt{50}$$
$$= \sqrt{2\times 5^2}$$
$$= 5\sqrt{2}$$

$$AC = \sqrt{\{2-(-4)\}^2 + \{7-(-1)\}^2}$$
$$= \sqrt{(2+4)^2 + (7+1)^2}$$
$$= \sqrt{6^2 + 8^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100}$$
$$= 10$$

As, AB= BC ≠ AC, the points are vertices of an isosceles triangle and the length of the base AC is 10 units.

5. **Show that the points A (4,3) , B(1,2) , C(1,0), D (4,1) are vertices of a parallelogram and find the length of its diagonals**.

Solution: Here,

$$AB = \sqrt{(4-1)^2 + (3-2)^2}$$
$$= \sqrt{3^2 + 1^2}$$
$$= \sqrt{9+1}$$
$$= \sqrt{10}$$

BC $= \sqrt{(1-1)^2+(2-0)^2}$

$= \sqrt{2^2}$

$= 2$

CD $= \sqrt{(1-4)^2+(0-1)^2}$

$= \sqrt{(-3)^2+(-1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

and DA $= \sqrt{(4-4)^2+(1-3)^2}$

$= \sqrt{0^2+(-2)^2}$

$= \sqrt{2^2}$

$= 2$

Thus AB = CD and BC = DA.

So, the opposite sides of the quadrilateral ABCD are equal in length.

Hence, the points are the vertices of parallelogram.

Here, AC $= \sqrt{(4-1)^2+(3-0)^2} = \sqrt{3^2+3^2} = \sqrt{2\times 3^2} = 3\sqrt{2}$

BD $= \sqrt{(1-4)^2+(2-1)^2} = \sqrt{(-3)^2+1^2} = \sqrt{9+1} = \sqrt{10}$

∴ the lengths of the diagonals are $3\sqrt{2}$ units and $\sqrt{10}$ units.

6. Show that (2.1) is the centre of the circum- circle of the triangle whose vertices are (-3,-9), (13, -1) and (-9,3).

Solution: The distance of (2,1) from $(-3,-9)$ $= \sqrt{(-3-2)^2+(-9-1)^2}$

$= \sqrt{(-5)^2+(-10)^2}$

$= \sqrt{25+100}$

$= \sqrt{125}$

$= 5\sqrt{5}$

The distance of (2, 1) from $(13,-1)$ $= \sqrt{(13-2)^2+(-1-1)^2}$

$= \sqrt{11^2+(-2)^2}$

$= \sqrt{121+4}$

$= \sqrt{125}$

$= 5\sqrt{5}$

The distance of (2,1)from (−9, 3) $= \sqrt{(-9-2)^2+(3-1)^2}$

$= \sqrt{(-11)^2+2^2}$

$= \sqrt{121+4}$

$= \sqrt{125}$

$= 5\sqrt{5}$

Now, we have (2.1) is equidistant from (−3,−9), (13,−1) and (−9,3).

Hence, (2,1) is the centre of the circum-circle of the triangle whose vertices are (−3,−9), (13,−1) and (−9,3).

7. Show that the points (4, 4), (5,−1), (−6, 2) are the vertices of a right triangle.

Solution: Let A(4,4) , B(5,−1), C(−6,2) be the three points.

Then, AB $= \sqrt{(4-5)^2+\{4-(-1)\}^2}$

$= \sqrt{(-1)^2+(4+1)^2}$

$= \sqrt{1+5^2}$

$= \sqrt{1+25}$

$= \sqrt{26}$

BC $= \sqrt{\{5-(-6)\}^2+(-1-2)^2}$

$= \sqrt{(5+6)^2+(-3)^2}$

$= \sqrt{121^2+9}$

$= \sqrt{121+9}$

$= \sqrt{130}$

and AC $= \sqrt{\{4-(-6)\}^2+(4-2)^2}$

$= \sqrt{(4+6)^2+2^2}$

$= \sqrt{10^2+4}$

$= \sqrt{100+4}$

$= \sqrt{104}$

Now, $BC^2 = (\sqrt{130})^2 = 130$

and $AB^2 + AC^2 = (\sqrt{26})^2 + (\sqrt{104})^2 = 26+104 = 130$

As $BC^2 = AB^2 + AC^2$, by converse of Pythagoras Theorem, ABC is a right triangle right angled at A.

Hence, the points (4, 4), (5,−1), (−6, 2) are the vertices of a right triangle.

8. Show that the points (p, p), $(-p,-p)$, $(p\sqrt{3}, -p\sqrt{3})$ are the vertices of an equilateral triangle.

Solution: Let A(p,p), B $(-p,-p)$, C$(p\sqrt{3}, -p\sqrt{3})$ be the three points.

$$\begin{aligned}
\text{Then, AB} &= \sqrt{\{p-(-p)\}^2+\{p-(-p)\}^2} \\
&= \sqrt{(p+p)^2+(p+p)^2} \\
&= \sqrt{(2p)^2+(2p)^2} \\
&= \sqrt{2\times(2p)^2} \\
&= 2\sqrt{2}p
\end{aligned}$$

$$\begin{aligned}
\text{BC} &= \sqrt{(-p-p\sqrt{3})^2+\{-p-(-p\sqrt{3})\}^2} \\
&= \sqrt{\{-(p+p\sqrt{3})\}^2+(-p+p\sqrt{3})^2} \\
&= \sqrt{(p+p\sqrt{3})^2+(p\sqrt{3}-p)^2} \\
&= \sqrt{\left\{p^2+2p.p\sqrt{3}+(p\sqrt{3})^2\right\}+\left\{(p\sqrt{3})^2-2p\sqrt{3}.p+p^2\right\}} \\
&= \sqrt{p^2+2\sqrt{3}p^2+3p^2+3p^2-2\sqrt{3}p^2+p^2} \\
&= \sqrt{p^2+3p^2+3p^2+p^2} \\
&= \sqrt{8p^2} \\
&= 2\sqrt{2}p
\end{aligned}$$

$$\begin{aligned}
\text{And AC} &= \sqrt{(p-p\sqrt{3})^2+\{p-(-p\sqrt{3})\}^2} \\
&= \sqrt{(p-p\sqrt{3})^2+(p+p\sqrt{3})^2} \\
&= \sqrt{p^2-2p.p\sqrt{3}+(p\sqrt{3})^2+p^2+2p.p\sqrt{3}+(p\sqrt{3})^2} \\
&= \sqrt{p^2+3p^2+p^2+3p^2} \\
&= \sqrt{8p^2} \\
&= 2\sqrt{2}p
\end{aligned}$$

Here, AB = BC = AC

Hence, the points A(p,p), B $(-p,-p)$ and C$(p\sqrt{3}, -p\sqrt{3})$ are the vertices of an equilateral triangle.

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