CHAPTER 7
VECTORS

SOLUTIONS

EXERCISE 7

Q1. Separate the following entities into scalars and vectors:

Age, mass, time, length, density, pressure, displacement, velocity, force, specific gravity, electric current, temperature, momentum, weight and acceleration.

Soln: **Scalar entities:** Age, mass, time, length, density, pressure, specific gravity, electric current and temperature.

Vector entities: Displacement, velocity, force, momentum, weight and acceleration.

Q2. If $|\vec{a}|=5$ then find the scalar *m* such that $|m\vec{a}|=15$.

Soln: Given $|\vec{a}|=5$

Now, $|m\vec{a}|=15$

$$\Rightarrow \pm m|\vec{a}|=15$$

$$\Rightarrow \pm m\times 5=15$$

$$\Rightarrow m=\pm\frac{15}{5}=\pm 3$$

Q3. If $|\vec{a}|=10$, find $\hat{a}$ and the reciprocal vector of $\vec{a}$.

Soln: We have, $\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{\vec{a}}{10}$

Reciprocal of vector $\vec{a}=\frac{\vec{a}}{|a^2|}=\frac{\vec{a}}{10^2}=\frac{\vec{a}}{100}$

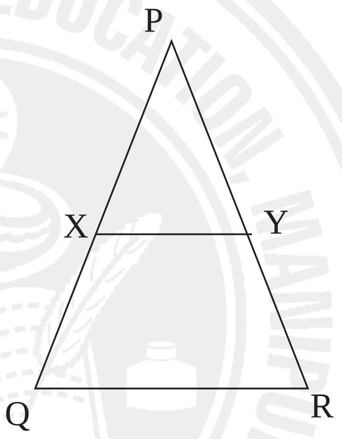
Q4. In a triangle show that the line joining the mid-points of any two sides is parallel to third side and half of its length.

Soln: Let X and Y be the mid-points of the side PQ and PR of the ΔPQR.

Let $\vec{q}$ and $\vec{r}$ be the position vectors of Q and R with P as the origin of reference.

$\therefore$ Position vectors of X and Y are respectively $\frac{\vec{q}}{2}$ and $\frac{\vec{r}}{2}$.

Now, $\overrightarrow{QR} = p.v.$ of $R - p.v.$ of $Q = \left(\vec{r} - \vec{q}\right)$



And,

$$\overrightarrow{XY} = p.v. \text{ of } Y - p.v. \text{ of } X$$

$$= \frac{\vec{r}}{2} - \frac{\vec{q}}{2} = \frac{\vec{r} - \vec{q}}{2}$$

$$= \frac{1}{2}\left(\vec{r} - \vec{q}\right)$$

$$\Rightarrow \overrightarrow{XY} = \frac{1}{2}\overrightarrow{QR}$$

$$\therefore \; XY \parallel QR \text{ and } XY = \frac{1}{2}QR.$$

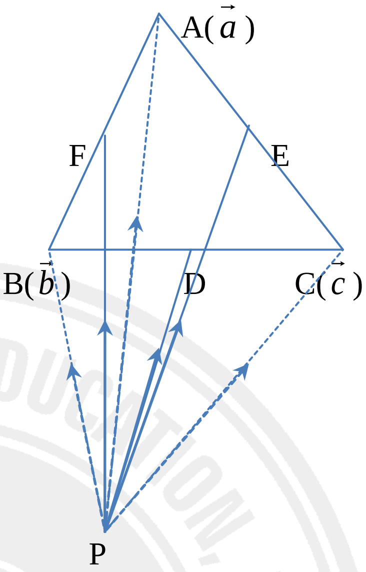
Q5. In a ΔABC, D, E and F are respectively the mid-points of the sides BC, CA and AB. For any arbitrary points P show that $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PD} + \overrightarrow{PC} + \overrightarrow{PF}$.

Soln: Here D, E and F are respectively the mid-points of the sides BC, CA and AB. Let $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of A, B, C with P as the origin of reference i.e. $\overrightarrow{PA} = \vec{a}$, $\overrightarrow{PB} = \vec{b}$ and $\overrightarrow{PC} = \vec{c}$

Now, position vectors of D, $\overrightarrow{PD} = \dfrac{\vec{b}+\vec{c}}{2}$

Position vectors of E, $\overrightarrow{PE} = \dfrac{\vec{a}+\vec{c}}{2}$

Position vectors of F, $\overrightarrow{PF} = \dfrac{\vec{a}+\vec{b}}{2}$



$$\therefore\ \overrightarrow{PD}+\overrightarrow{PE}+\overrightarrow{PF} = \frac{\vec{b}+\vec{c}}{2}+\frac{\vec{a}+\vec{c}}{2}+\frac{\vec{a}+\vec{b}}{2}$$

$$= \frac{2\vec{a}+2\vec{b}+2\vec{c}}{2}$$

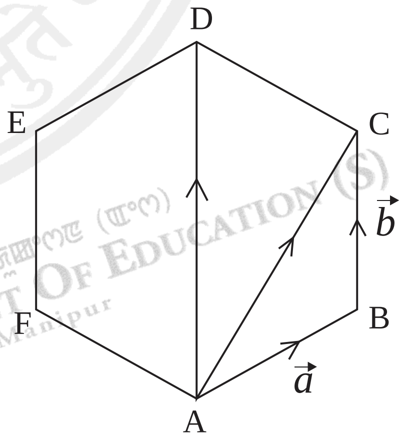
$$= \frac{2\left(\vec{a}+\vec{b}+\vec{c}\right)}{2}$$

$$= \vec{a}+\vec{b}+\vec{c}$$

$= \overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ Hence proved.

Q6. If $\vec{a}$ and $\vec{b}$ are the adjacent sides of a regular hexagon taken in order, find the vectors determined by the other sides of the hexagon taken in the same order.

Soln: Let $ABCDEF$ be the given regular hexagon.



Let $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{BC}=\vec{b}$ be the two adjacent sides.

$\Rightarrow \overrightarrow{AD} \parallel \overrightarrow{BC}$ and $\overrightarrow{AD}=2\overrightarrow{BC}$

$\Rightarrow \overrightarrow{AD}=2\vec{b}$

In ΔABC, $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\vec{a}+\vec{b}$

In ΔACD, $\overrightarrow{CD}=\overrightarrow{CA}+\overrightarrow{AD}=-\overrightarrow{AC}+\overrightarrow{AD}=-\left(\vec{a}+\vec{b}\right)+2\vec{b}$

$= \vec{b}-\vec{a}$.

$\therefore\ DE \parallel BA$ and $DE = BA$

$\therefore\ \overrightarrow{DE}=\overrightarrow{BA}=-\overrightarrow{AB}=-\vec{a}$

$EF \parallel CB$ and $EF = CB$

$\therefore \; \overrightarrow{EF} = \overrightarrow{CB} = -\overrightarrow{BC} = -\vec{b}$

Similarly $\overrightarrow{FA} = \overrightarrow{DC} = -\overrightarrow{CD} = -\left(\vec{b} - \vec{a}\right)$

$= \vec{a} - \vec{b}$.

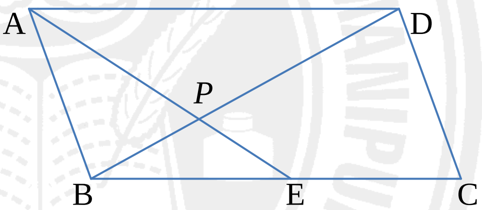
Q7. *ABCD* is a parallelogram and *E* is the midpoint of *BC*. Show that *AE* and *BD* trisect each other.

Soln: Let P be the point of intersection of *AE* and *BD*.

Let $\vec{a}, \vec{c}$ and $\vec{d}$ be the *P.V.* of *B, C* and *D* with *A* as origin of reference.

Here, $\overrightarrow{AD} = \overrightarrow{BC}$ | opposite side of $\|^{gm}$ ABCD.

$\Rightarrow \vec{d} = \vec{c} - \vec{b}$



Since, *E* is the mid-point of *BC*

$\therefore$ P.V. of $E = \dfrac{\vec{b} + \vec{c}}{2}$

Let *AP*:*PE*=*k*:1 and *BP*:*PD*=*m*:1

$$\therefore \text{ Position vector of } P = \frac{\dfrac{k\left(\vec{b}+\vec{c}\right)}{2} + 1.\vec{0}}{k+1} = \frac{m\vec{d} + 1.\vec{b}}{m+1}$$

$$\Rightarrow \frac{k\left(\vec{b}+\vec{c}\right)}{2(k+1)} = \frac{m\left(\vec{c}-\vec{b}\right)+\vec{b}}{m+1}$$

$$\Rightarrow \frac{k}{2(k+1)}\vec{b} + \frac{k}{2(k+1)}\vec{c} = \frac{m}{m+1}\vec{c} + \frac{(1-m)}{m+1}\vec{b}$$

$$\Rightarrow \frac{k}{2(k+1)} = \frac{1-m}{m+1} \qquad - \qquad \text{(i)}$$

$$\Rightarrow \frac{k}{2(k+1)} = \frac{m}{m+1} \qquad - \qquad \text{(ii)}$$

Subtracting (i) from (ii), we get

$$\Rightarrow 0=\frac{m}{m+1}-\frac{1-m}{m+1}$$

$$\Rightarrow \frac{m-1+m}{m+1}=0$$

$$\Rightarrow 2m-1=0$$

$$\Rightarrow m=\frac{1}{2}$$

Using the value of m in eqn. (i), we have

$$\frac{k}{2(k+1)}=\frac{1-\frac{1}{2}}{\frac{1}{2}+1}$$

$$\Rightarrow \frac{k}{2(k+1)}=\frac{\frac{1}{2}}{\frac{3}{2}}$$

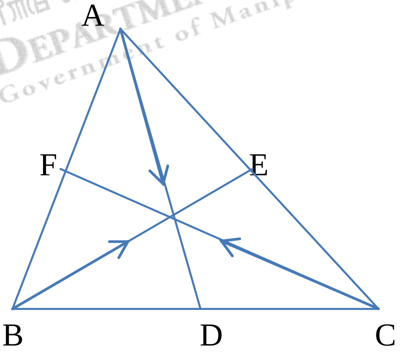
$$\Rightarrow 3k=2k+2$$

$$\Rightarrow k=2$$

$\therefore$ AE and BD trisect each other at P.

Q8. Show that the sum of the vectors determined by the medians of a triangle directed from the vertices is zero.

Soln: Let AD, BE and CF be the medians of the ΔABC.



Let $\vec{b}$ and $\vec{c}$ be the position vectors of B and C with A as origin of reference.

Position vectors of D, E and F are respectively

$\frac{\vec{b}+\vec{c}}{2}$, $\frac{\vec{c}}{2}$ and $\frac{\vec{b}}{2}$

Now, $\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}=\left(\frac{\vec{b}+\vec{c}}{2}\right)+\left(\frac{\vec{c}}{2}-\vec{b}\right)+\left(\frac{\vec{b}}{2}-\vec{c}\right)$

$$=\left(\frac{\vec{b}+\vec{c}}{2}\right)+\left(\frac{\vec{c}-2\vec{b}}{2}\right)+\left(\frac{\vec{b}-2\vec{c}}{2}\right)$$

$$=\frac{1}{2}\left[\vec{b}+\vec{c}+\vec{c}-2\vec{b}+\vec{b}-2\vec{c}\right]$$

$$=\frac{1}{2}\left[2\vec{b}-2\vec{b}+2\vec{c}-2\vec{c}\right]$$

$$=\frac{1}{2}\times 0 = 0$$

$\therefore$ The sum of the vectors determined by the medians of a triangle directed from the vertices is zero.

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