Parametrization of Logistic Selectivity Function in Stock Synthesis

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Short outline for now.

Parameterization of logistic selectivity function in Stock Synthesis is different than a typical logistic function. (See logisitic function for a detailed description of typical logistic function.) The typical logistic function is

$$f(x) = \frac{L}{1 + e^{-k(x-x_0)}} \tag{1}$$

where k is the logistic growth rate or steepnes of curve, L is the maximum value, and x_0 is the midpoint.

In Stock Synthesis, f is used to describe selectivity at age x (or could insert size here instead) and so $L = 1$ and x_0 is the age at which $f(x) = 0.5$.

In Stock Synthesis, the logistic selectivity function[1] describes selectivity at age x as

$$s(x) = \frac{1}{1 + e^{-ln(19)(x-p1)/p2}} \tag{2}$$

where $p1$, $p2$ are parameters. In this parameterization, the $p1$ parameter is the same as the x_0 paremeter from above. So when x is the age when selectivity is 0.5 (i.e., x_0 or $p1$) then

$$s(x_0) = \frac{1}{1 + e^{-ln(19)(p1-p1)/p2}} = \frac{1}{1 + e^0} = \frac{1}{1+1} = \frac{1}{2} = 0.5 \tag{3}$$

The logistic selectivity function can also be written as[2]

$$s(x) = (1 + e^{-ln(19)\frac{x-x_{50}}{x_{95}-x_{50}}})^{-1} \tag{4}$$

where x_{50} is the same as $p1$ and x_0 above, and x_{95} is the age when selectivity is 0.95 ($s(x) = 0.95$). So when $x = x_{95}$ then

$$\frac{x - x_{50}}{x_{95} - x_{50}} = 1 \tag{5}$$

and so

$$s(x_{95}) = (1 + e^{-ln(19)})^{-1} = 0.95. \tag{6}$$

To get from the Stock Synthesis parameterization to the typical parameterization of the logistic function let $k = log(19)/p2$ so that

[1] See https://nmfs-stock-synthesis.github.io/doc/SS330_User_Manual.html#selectivity-pattern-details

[2] see https://maiakapur.netlify.app/post/ln19/why-ln19/

$$f(x) = \frac{1}{1+e^{-k(x-x_0)}} = \frac{1}{1+e^{-\left(\frac{log(19)}{p2}\right)(x-x_0)}} = \frac{1}{1+e^{-log(19)\left(\frac{x-x_0}{p2}\right)}}. \tag{7}$$

Also we can see that $p2 = x_{95} - x_{50}$.

Why the 19? (still trying to understand this better but rough idea so far)

The reason for the $log(19)$ has to do with the odds ratio and ensuring that when $x = x_{95}$ the selectivity will equal 0.95. The odds ratio is the probability of an event occuring divided by the probability of that event not occurring. So the odds that selectivity is equal to 0.95 will be 0.95/0.05, which is the same as $(19/20)/(1/20) = 19$.

Also, see Why do we use ln(19)? for a good explanation.