Parametrization of Logistic Maturity Function

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I was working with some maturity-at-age/length data that had a different parameterization that I was used to and so I wanted to convert from this new parametrization to the one I was familiar with.

In the new parametrization I came across, the proportion of mature individuals at a given age x (or size) is a function $p : \mathbb{R} \to \mathbb{R}$ defined as

$$p(x) = \frac{e^{(a+xb)}}{1 + e^{(a+xb)}} \tag{1}$$

where $a, b \in \mathbb{R}$ are estimated parameters. By dividing through by e^{a+xb} in the numerator and denominator, this function can be equivalently defined as

$$p(x) = \frac{1}{1 + e^{-(a+xb)}}, \tag{2}$$

which is the form I will use throughout this document. In the parametrization I am familiar with, the proportion of mature individuals at a given age x (or size) is a function $p' : \mathbb{R} \to \mathbb{R}$ defined as

$$p'(x) = \frac{1}{1 + e^{-k(x+x_0)}} \tag{3}$$

where $k \in \mathbb{R}$ is the logistic growth rate or steepeness of the curve and $x_0 \in \mathbb{R}$ is the age when $p = 0.5$. To see that $p = p'$, let z be arbitrary and suppose $z \in \mathbb{R}$. Now let $x_0 = a/b$ and $k = b$. Then

$$p'(z) = \frac{1}{1 + e^{-k(z+x_0)}} = \frac{1}{1 + e^{-b(z+(a/b))}} = \frac{1}{1 + e^{-(a+bz)}} = p(z). \tag{4}$$