3.5.1

Suppose A, B, and C are sets.

Theorem. $A \cap (B \cup C) \subseteq (A \cap B) \cup C$

Proof. Let x be arbitrary and suppose $x \in A \cap (B \cup C)$. Thus $x \in A$ and $x \in B$ or $x \in C$. If $x \in C$ then $x \in (A \cap B) \cup C$. In the case where $x \in B$ it follows that $x \in A \cap B$ and therefore $x \in (A \cap B) \cup C$. Since x was arbitrary we can conclude that $A \cap (B \cup C) \subseteq (A \cap B) \cup C$. □

3.5.2

Suppose A, B, and C are sets.

Theorem. $(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$

Proof. Let x be arbitrary and suppose $x \in (A \cup B) \setminus C$. Thus $x \notin C$ and $x \in A$ or $x \in B$. If $x \in A$ then $x \in A \cup (B \setminus C)$. If $x \in B$ then if follows that $x \in B \setminus C$ and therefore $x \in A \cup (B \setminus C)$. Since x was arbitrary we can conclude $A \cap (B \cup C) \subseteq (A \cap B) \cup C$. □

3.5.3

Suppose A and B are sets.

Theorem. $A \setminus (A \setminus B) = A \cap B$

Proof. Let x be arbitrary and suppose $x \in A \setminus (A \setminus B)$. Then

$$\begin{aligned} x \in A \setminus (A \setminus B) &\text{ iff } x \in A \wedge x \notin A \setminus B \\ &\text{ iff } x \in A \wedge \neg(x \in A \wedge x \notin B) \\ &\text{ iff } x \in A \wedge (x \notin A \vee x \in B) \\ &\text{ iff } (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B) \\ &\text{ iff } x \in A \wedge x \in B \\ &\text{ iff } x \in (A \cap B) \end{aligned}$$

□

3.5.4

Theorem. *If* $A \cap C \subseteq B \cap C$ *and* $A \cup C \subseteq B \cup C$ *then* $A \subseteq B$.

Proof. Suppose $A \cap C \subseteq B \cap C$ and $A \cup C \subseteq B \cup C$. Let x be arbitrary and suppose $x \in A$. Thus $x \in A \cup C$ and it follows that $x \in B \cup C$. Now if $x \in B \cup C$ then either $x \in B$ or $x \in C$. If $x \in B$ then since x was arbitrary we can conclude $A \subseteq B$. In the case that $x \in C$, then $x \in A \cap C$ and it follows that $x \in B \cap C$. Therefore $x \in C$ and $x \in B$. Thus, if $x \in A$ then $x \in B$ and since x was arbitrary we can conclude $A \subseteq B$. □

3.5.5

Suppose A and B are sets.

Theorem. *If $A \triangle B \subseteq A$ then $B \subseteq A$.*

Proof. Suppose $A \triangle B \subseteq A$. We will prove by contradiction. Let x be arbitrary and suppose $x \in B$ and $x \notin A$. Since $x \in B$ and $x \notin A$ then $x \in A \triangle B$. Since $A \triangle B \subseteq A$, then $x \in A$. But this contradicts $x \notin A$. Therefore, if $x \in B$ then $x \in A$ and since x was arbitrary we can conclude that $B \subseteq A$. $\square$