3.4.1

Use the methods of this chapter to prove that $\forall x(P(x) \wedge Q(x))$ is equivalent to $\forall x P(x) \wedge \forall x Q(x)$.

We want to prove $\forall x(P(x) \wedge Q(x) \iff \forall x P(x) \wedge \forall x Q(x))$.

Theorem. *The statement $\forall x(P(x) \wedge Q(x))$ is equivalent to $\forall x P(x) \wedge \forall x Q(x)$.*

Proof. ($\rightarrow$) Suppose $\forall x(P(x) \wedge Q(x))$. Let y be arbitrary. Since $\forall x(P(x) \wedge Q(x))$ it follows $P(y)$ and $Q(y)$. Since y was arbitrary, we can conclude $\forall x P(x)$ and $\forall x Q(x)$ or $\forall x P(x) \wedge \forall x Q(x)$.

($\leftarrow$) Let y be arbitrary. Since $\forall x P(x)$ and $\forall x Q(x)$ then it follows $P(y)$ and $Q(y)$. Since y was arbitrary we can conclude $\forall x(P(x) \wedge Q(x))$. $\qquad\square$

3.4.2

Prove that if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$.

Theorem. *If $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$.*

Proof. Let x be arbitrary and suppose $x \in A$. Since $A \subseteq B$ then $x \in B$ and since $A \subseteq C$ then $x \in C$ or $x \in B \cap C$. Therefore, if $x \in A$ then $x \in B \cap C$ and since x was arbitrary we can conclude $A \subseteq B \cap C$. $\qquad\square$

3.4.3

Suppose $A \subseteq B$. Prove that for every set C, $C \setminus B \subseteq C \setminus A$.

Theorem. *Suppose $A \subseteq B$, then for every set C, $C \setminus B \subseteq C \setminus A$.*

Proof. Suppose $A \subseteq B$ and C is an arbitrary set. Let x be arbitrary and suppose $x \in C \setminus B$, which means $x \in C$ and $x \notin B$. Since $x \notin B$ and $A \subseteq B$, then $x \notin A$, which means that $x \in C \setminus A$. Therefore, if $x \in C \setminus B$ then $x \in C \setminus A$ and since x and C were arbitrary, we can conclude $\forall C(C \setminus B \subseteq C \setminus A)$. $\qquad\square$

3.4.5

Prove that if $A \subseteq B \setminus C$ and $A \neq \varnothing$ then $B \not\subseteq C$.

Theorem. *If $A \subseteq B \setminus C$ and $A \neq \varnothing$ then $B \not\subseteq C$.*

Proof. Let x be arbitrary and suppose $x \in A$. Since $A \subseteq B \setminus C$ then $x \in B$ and $x \notin C$. Since x was arbitrary we can conclude $B \not\subseteq C$. $\qquad\square$