3.6.2

Theorem. *There is a unique $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, $xy + x - 4 = 4y$.*

Proof. First we prove existence. Let $x = 4$ and suppose y is an arbitrary real number. Then we have $4y+4-4 = 4y+0 = 4y$, as desired. To prove uniqueness suppose a and b are arbitrary real numbers and that $ay + a - 4 = 4y$ and that $by + b - 4 = 4b$. For $ay + a - 4 = 4y$, let $y = b$ and we have $ab + a - 4 = 4b$. For $by + b - 4 = 4y$, let $y = a$ and we have $ba + b - 4 = 4a$. Now subtracting both sides of $ab + a - 4 = 4b$ from $ba + b - 4 = 4a$ we have

$$\begin{aligned}
ba + b - 4 - (ab + a - 4) &= 4a - 4b \\
ba + b - 4 - ab - a + 4 &= 4a - 4b \\
b - a &= 4a - 4b \\
b + 4b &= 4a + a \\
5b &= 5a \\
b &= a
\end{aligned}$$

Therefore, if $ay + a - 4 = 4y$ and $by + b - 4 = 4y$, then $a = b$.

□