

### 3.5.1

Suppose  $A$ ,  $B$ , and  $C$  are sets.

**Theorem.**  $A \cap (B \cup C) \subseteq (A \cap B) \cup C$

*Proof.* Let  $x$  be arbitrary and suppose  $x \in A \cap (B \cup C)$ . Thus  $x \in A$  and  $x \in B$  or  $x \in C$ . If  $x \in C$  then  $x \in (A \cap B) \cup C$ . In the case where  $x \in B$  it follows that  $x \in A \cap B$  and therefore  $x \in (A \cap B) \cup C$ . Since  $x$  was arbitrary we can conclude that  $A \cap (B \cup C) \subseteq (A \cap B) \cup C$ .  $\square$

### 3.5.2

Suppose  $A$ ,  $B$ , and  $C$  are sets.

**Theorem.**  $(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$

*Proof.* Let  $x$  be arbitrary and suppose  $x \in (A \cup B) \setminus C$ . Thus  $x \notin C$  and  $x \in A$  or  $x \in B$ . If  $x \in A$  then  $x \in A \cup (B \setminus C)$ . If  $x \in B$  then it follows that  $x \in B \setminus C$  and therefore  $x \in A \cup (B \setminus C)$ . Since  $x$  was arbitrary we can conclude  $(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$ .  $\square$

### 3.5.3

Suppose  $A$  and  $B$  are sets.

**Theorem.**  $A \setminus (A \setminus B) = A \cap B$

*Proof.* Let  $x$  be arbitrary and suppose  $x \in A \setminus (A \setminus B)$ . Then

$$\begin{aligned} x \in A \setminus (A \setminus B) &\text{ iff } x \in A \wedge x \notin A \setminus B \\ &\text{ iff } x \in A \wedge \neg(x \in A \wedge x \notin B) \\ &\text{ iff } x \in A \wedge (x \notin A \vee x \in B) \\ &\text{ iff } (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B) \\ &\text{ iff } x \in A \wedge x \in B \\ &\text{ iff } x \in (A \cap B) \end{aligned}$$

$\square$

### 3.5.4

**Theorem.** If  $A \cap C \subseteq B \cap C$  and  $A \cup C \subseteq B \cup C$  then  $A \subseteq B$ .

*Proof.* Suppose  $A \cap C \subseteq B \cap C$  and  $A \cup C \subseteq B \cup C$ . Let  $x$  be arbitrary and suppose  $x \in A$ . Thus  $x \in A \cup C$  and it follows that  $x \in B \cup C$ . Now if  $x \in B \cup C$  then either  $x \in B$  or  $x \in C$ . If  $x \in B$  then since  $x$  was arbitrary we can conclude  $A \subseteq B$ . In the case that  $x \in C$ , then  $x \in A \cap C$  and it follows that  $x \in B \cap C$ . Therefore  $x \in C$  and  $x \in B$ . Thus, if  $x \in A$  then  $x \in B$  and since  $x$  was arbitrary we can conclude  $A \subseteq B$ .  $\square$

### 3.5.5

Suppose  $A$  and  $B$  are sets.

**Theorem.** If  $A \triangle B \subseteq A$  then  $B \subseteq A$ .

*Proof.* Suppose  $A \triangle B \subseteq A$ . We will prove by contradiction. Let  $x$  be arbitrary and suppose  $x \in B$  and  $x \notin A$ . Since  $x \in B$  and  $x \notin A$  then  $x \in A \triangle B$ . Since  $A \triangle B \subseteq A$ , then  $x \in A$ . But this contradicts  $x \notin A$ . Therefore, if  $x \in B$  then  $x \in A$  and since  $x$  was arbitrary we can conclude that  $B \subseteq A$ .  $\square$

### 3.5.6

Suppose  $A$ ,  $B$ , and  $C$  are sets.

**Theorem.**  $A \cup C \subseteq B \cup C \iff A \setminus C \subseteq B \setminus C$ .

*Proof.* ( $\rightarrow$ ) Suppose  $A$ ,  $B$ , and  $C$  are sets. Suppose  $(A \cup C) \subseteq (B \cup C)$ . Let  $x$  be arbitrary and suppose  $x \in A \setminus C$ , which means  $x \in A$  and  $x \notin C$ . Since  $x \in A$ , then  $x \in A \cup C$  and therefore  $x \in B \cup C$ . This means  $x \in B$  or  $x \in C$  and since  $x \notin C$ , it must be that  $x \in B$ . Now since  $x \in B$  and  $x \notin C$  then  $x \in B \setminus C$ . Therefore, if  $x \in A \setminus C$  then  $x \in B \setminus C$  and since  $x$  was arbitrary we can conclude if  $A \cup C \subseteq B \cup C$  then  $A \setminus C \subseteq B \setminus C$ .

( $\leftarrow$ ) Now suppose  $A \setminus C \subseteq B \setminus C$ . Let  $x$  be arbitrary and suppose  $x \in A \cup C$ , which means  $x \in A$  or  $x \in C$ . If  $x \in C$  then  $x \in B \cup C$  and since  $x$  was arbitrary then  $A \cup C \subseteq B \cup C$ . In the case that  $x \in A$ , since  $A \setminus C \subseteq B \setminus C$  then  $x \in B$ . Therefore,  $x \in B \cup C$  and since  $x$  was arbitrary then  $A \cup C \subseteq B \cup C$ .  $\square$