3.5.1

Suppose A, B, and C are sets.

Theorem. $A \cap (B \cup C) \subseteq (A \cap B) \cup C$

Proof. Let x be arbitrary and suppose $x \in A \cap (B \cup C)$. Thus $x \in A$ and $x \in B$ or $x \in C$. If $x \in C$ then $x \in (A \cap B) \cup C$. In the case where $x \in B$ it follows that $x \in A \cap B$ and therefore $x \in (A \cap B) \cup C$. Since x was arbitrary we can conclude that $A \cap (B \cup C) \subseteq (A \cap B) \cup C$. $\square$

3.5.2

Suppose A, B, and C are sets.

Theorem. $(A \cup B) \setminus C \subseteq A \cup (B \setminus C)$

Proof. Let x be arbitrary and suppose $x \in (A \cup B) \setminus C$. Thus $x \notin C$ and $x \in A$ or $x \in B$. If $x \in A$ then $x \in A \cup (B \setminus C)$. If $x \in B$ then if follows that $x \in B \setminus C$ and therefore $x \in A \cup (B \setminus C)$. Since x was arbitrary we can conclude $A \cap (B \cup C) \subseteq (A \cap B) \cup C$. $\square$

3.5.3

Suppose A and B are sets.

Theorem. $A \setminus (A \setminus B) = A \cap B$

Proof. Let x be arbitrary and suppose $x \in A \setminus (A \setminus B)$. Then

$$\begin{aligned}
x \in A \setminus (A \setminus B) &\text{ iff } x \in A \land x \notin A \setminus B \\
&\text{ iff } x \in A \land \neg(x \in A \land x \notin B) \\
&\text{ iff } x \in A \land (x \notin A \lor x \in B) \\
&\text{ iff } (x \in A \land x \notin A) \lor (x \in A \land x \in B) \\
&\text{ iff } x \in A \land x \in B \\
&\text{ iff } x \in (A \cap B)
\end{aligned}$$

$\square$