3.4.1

Use the methods of this chapter to prove that $\forall x(P(x) \wedge Q(x))$ is equivalent to $\forall x P(x) \wedge \forall x Q(x)$.

We want to prove $\forall x(P(x) \wedge Q(x) \iff \forall x P(x) \wedge \forall x Q(x))$.

Theorem. *The statement* $\forall x(P(x) \wedge Q(x))$ *is equivalent to* $\forall x P(x) \wedge \forall x Q(x)$.

Proof. ($\rightarrow$) Suppose $\forall x(P(x) \wedge Q(x))$. Let y be arbitrary. Since $\forall x(P(x) \wedge Q(x))$ it follows $P(y)$ and $Q(y)$. Since y was arbitrary, we can conclude $\forall x P(x)$ and $\forall x Q(x)$ or $\forall x P(x) \wedge \forall x Q(x)$.

($\leftarrow$) Let y be arbitrary. Since $\forall x P(x)$ and $\forall x Q(x)$ then it follows $P(y)$ and $Q(y)$. Since y was arbitrary we can conclude $\forall x(P(x) \wedge Q(x))$. $\square$