Passive Solar Tiny House

James Davis and Marion Madanguit

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1 Background

Passive solar heating is one of the least expensive ways to heat a home. By making use of windows, walls, and floors to efficiently distribute solar energy throughout a space, this heating technique is able to reduce energy consumption significantly. Here is a basic example of a passive solar house:

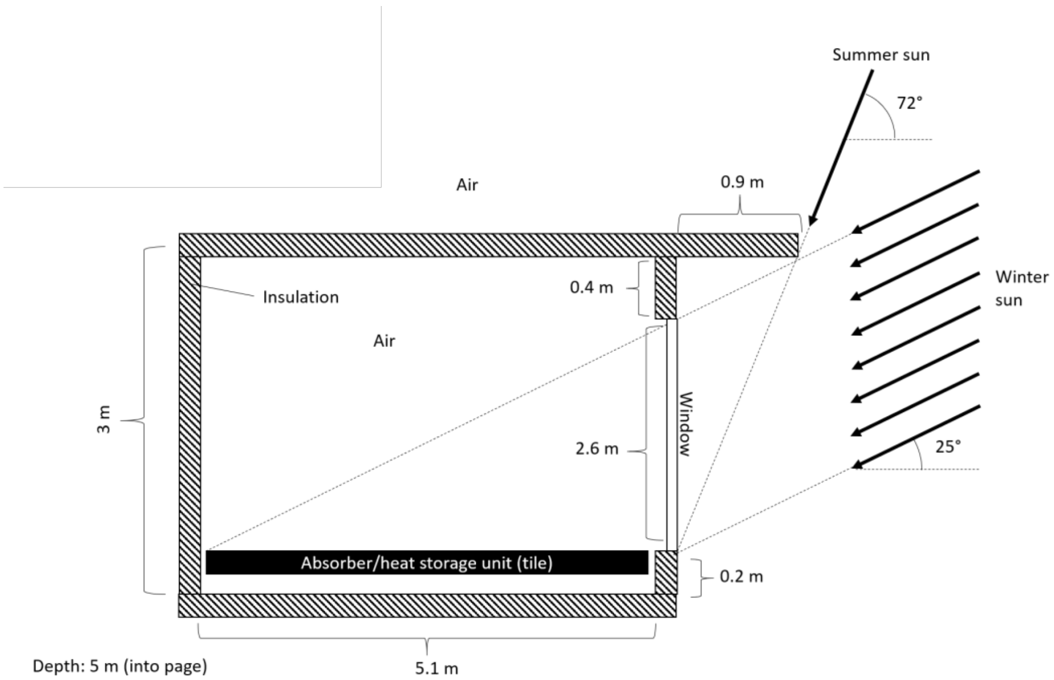

Figure 1: Basic passive solar house design

The House is designed and oriented with respect to the sun as its primary heating source. Large front windows with an overhanging ledge serve to let in low winter sunlight and shade high summer sun. The solar radiation is efficiently absorbed and stored by a tile floor, to be re-radiated past peak insolation. The walls are well-insulated to retain heat throughout cold winters.

Our project goal is to create a simple thermal model of this house and to investigate material choices for wall insulation and floor tile that balance cost and thermal comfort in the varying northeast climate. We will ultimately recommend an insulation material and building specifications that are affordable and able to maintain livable thermal conditions in any season.

2 Modeling

To effectively model the house, we first created a heat flow diagram and resistor network to represent it.

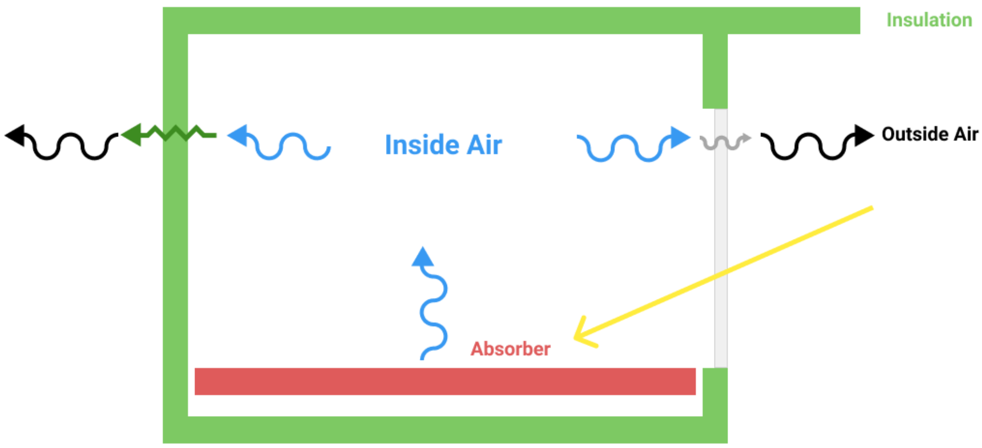

Figure 2: Passive solar house with components significant to thermal modeling labeled

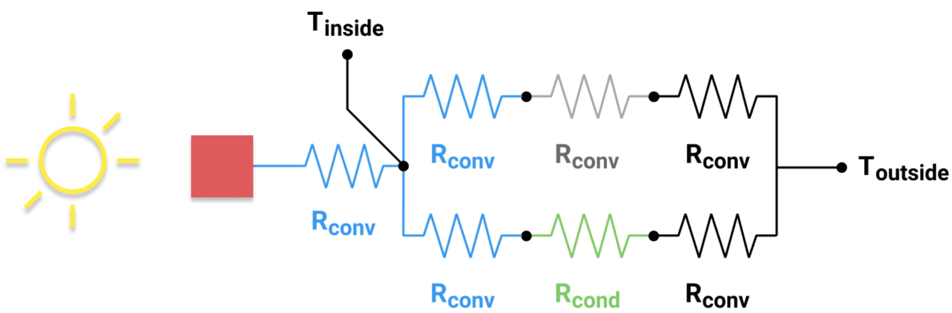

Figure 3: Resistor network representing the thermal resistances as the heat flows from the sun to the absorber to the inside air and back out

We assumed that heat transfer through the double-paned window could be modeled by a single convection term. We also assumed that there was no conduction between the absorber thermal mass and the walls (making it effectively floating). In addition, we chose to neglect radiation outside of the solar radiation coming in through the window and assumed that all solar radiation hitting the window was absorbed by our heat storage unit, which is at uniform temperature. This simplifies the resistor network and systems of thermal equations significantly and aids in producing meaningful solutions.

To solve for the inside air's temperature, two separate equations are used to represent heat flow in the two heat storage components: the absorber (equation 1) and the inside air (equation

2).

$$m_{Absorber}c_{absorber}\frac{dT_{inside}}{dt} = \dot{Q}_{sun} - \frac{T_{absorber} - T_{inside}}{R_1} \tag{1}$$

$$m_{inside}c_{inside}\frac{dT_{inside}}{dt} = \frac{T_{absorber} - T_{inside}}{R_1} - \frac{T_{inside} - T_{outside}}{R_2} \tag{2}$$

To find individual resistances, we used two equations for resistance due to convection (equation 3) and conduction (equation 4).

Variable	Significance	Unit
L	Thickness of Material	m
k	Thermal Conductivity	$\frac{w}{mk}$
A	Surface Area	m^2
h	Heat Transfer Coefficient	$\frac{w}{m^2k}$

$$R_{convection} = \frac{L}{kA} \tag{3}$$

$$R_{conduction} = \frac{1}{hA} \tag{4}$$

We substituted physical constants for the thermal conductivity of the insulation, and calculated the surface area of the absorber and walls from the house dimensions of the design discussed in our Background section. To evalutate the overall resistance of each side of the resistance network, we used equations 5 and 6.

$$R_1 = R_{absorber\ convection} \tag{5}$$

$$R_2 = \left(\frac{1}{R_{w\ conv\ in} + R_{w\ internal\ conv} + R_{w\ conv\ out}} + \frac{1}{R_{i\ conv\ in} + R_{i\ cond} + R_{i\ conv\ out}}\right)^{-1} \tag{6}$$

Energy from the sun, $\dot{Q}_{sun}$, was modeled using a sunosoidal function (equation 7).

$$\dot{Q} = A_{window}(-361 \cdot cos(\frac{pi \cdot t}{12 \cdot 3600}) + 224 \cdot cos(\frac{pi \cdot t}{6 \cdot 3600}) + 210) \tag{7}$$

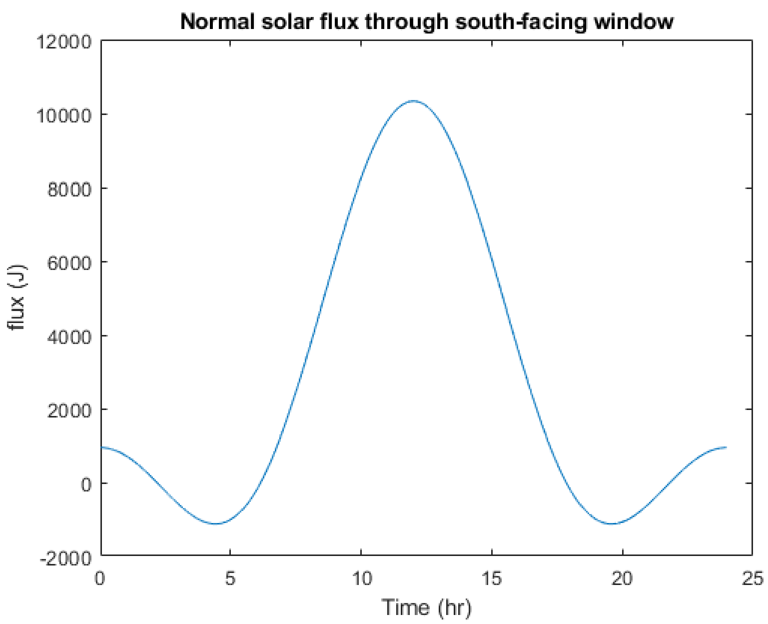

Figure 4: Plot of equation 7

Outside temperature, $T_{outside}$, was modeled using a sunosiodal function (equation 8).

$$T_{outside} = -3 + 6 \cdot sin(\frac{2 \cdot pi \cdot t}{24 \cdot 60 \cdot 60} + \frac{3 \cdot pi}{4}) \tag{8}$$

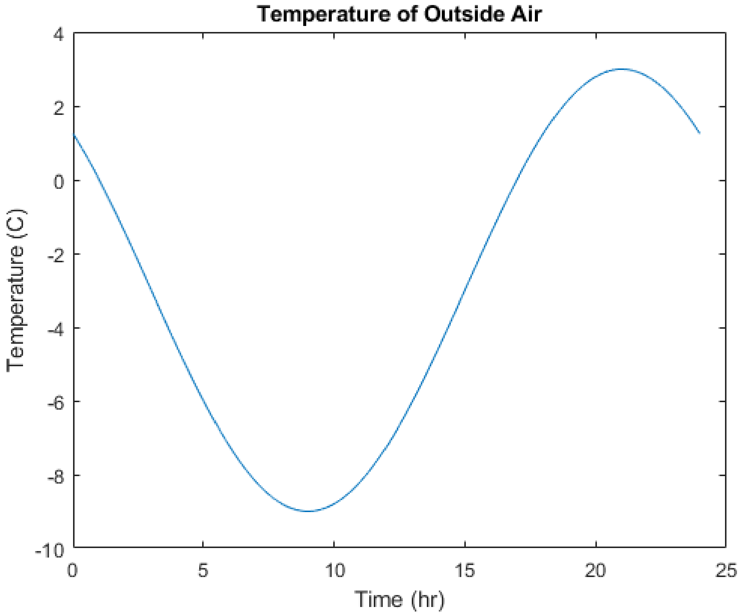

Figure 5: Plot of equation 8

Additionally, we also tried out real world temperature data from the NOAA Climate Extremes Index. figues NOAA weather station in Boston for all of 2019.

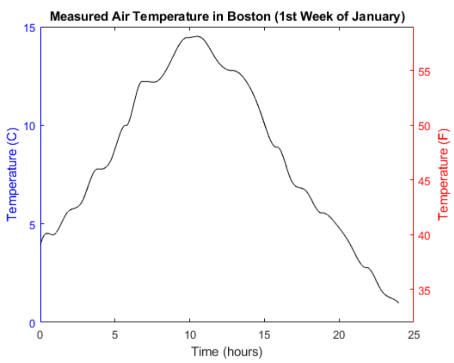

Figure 6: Plot of equation 8

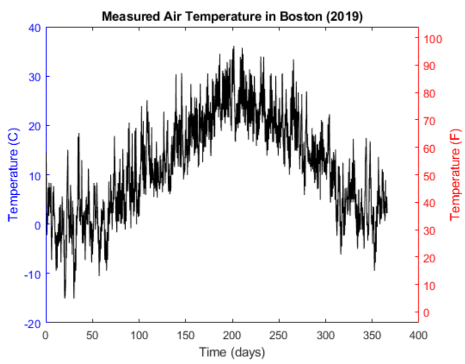

Figure 7: Plot of equation 8

3 Optimization

We experimented with three parameters in our house design: the thickness of the floor tile, the thickness of the wall insulation, and the material of the wall insulation.

3.1 Thickness of the Floor Tile and Wall Insulation

We evaluated the thickness of the floor tile and wall insulation with respect to thermal comfort.

Our ideal house would be able to maintain a temperature of 20°C over a period of 7 days. Obviously, no model of a passive solar house is able to perfectly maintain a single temperature for such a long period of time. The house's reliance on a noncontinuous heat source means that its temperature will naturally fluctuate daily. This is evident in figure 8.

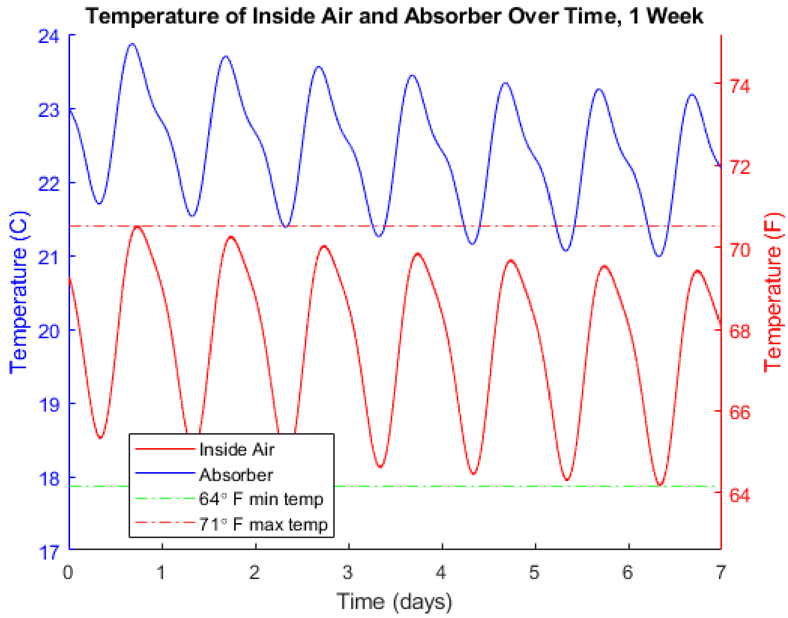

Figure 8: Natural fluctuation in the temperature of the inside air

To account for these oscillations in temperature, we decided to calculate the average temperature of the inside air over the week-long period. We wanted this average temperature as close as possible to our ideal temperature of 20°C. To accomplish this, we had our optimization function choose material thicknesses that would minimize the difference between our average indoor temperature and 20°C.

One special case that we had to consider was when the indoor temperatures failed to normalize within the 7-day period. This was due to the long time it took for tiles with large heat capacities to reach a steady state temperature. We found that there were some instances where the average temperature would be close to 20°C, but over the course of a week, the daily average temperature would slowly rise or sink beyond habitual levels. This is demonstrated in figure 9.

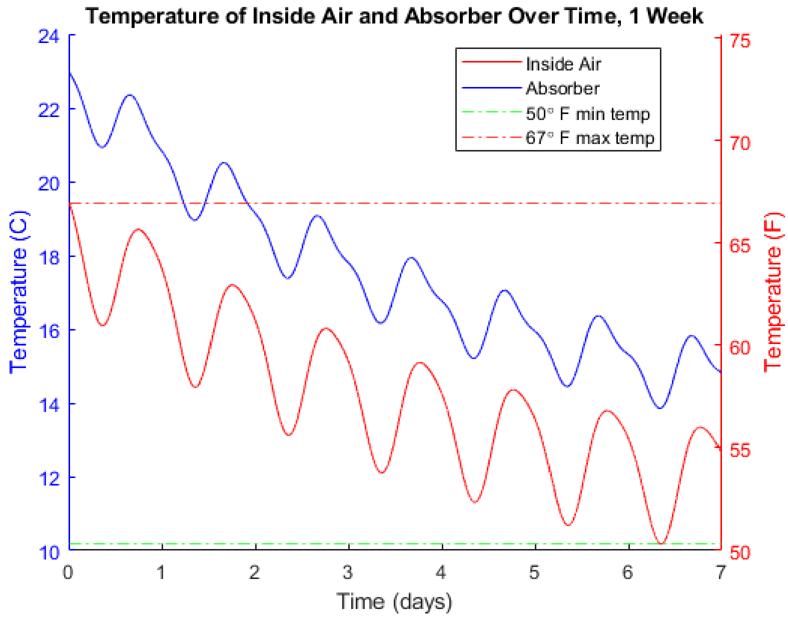

Figure 9: Case where average temperature is ideal but fluctuation has yet to normalize

A house that is unable to reach a stable temperature within 7 days is unreasonable. To eliminate these cases from our optimization function, we made sure that the inside temperature on day 1 was within 1°C of the inside temperature on day 7.

From our equations, we know that thicker floor tiles result in slower rates of heat gain. This is because it takes more time to heat a heavier floor tile than it does a lighter one. The longer it takes to heat the floor tile, the longer it takes to heat the inside air, since the temperature of the inside air relies on heat convection off of the the floor tile. On the other hand, thicker walls result in slower rates of heat loss. This is because it takes more time for heat to conduct through thicker walls than it does for thinner walls. The longer it takes for heat to leave the inside, the longer it takes to cool down the house.

Finding the optimal thicknesses of the two materials is a matter of balancing the two rates of heat gain and heat loss to achieve a stable temperature around 20°C. As demonstrated in our plot of average temperature vs material thicknesses below, this balance is achieved with a thick floor tile and thin walls of insulation. This allows for enough heat to leave the house during the day, while also allowing for enough heat to warm the house during the night.

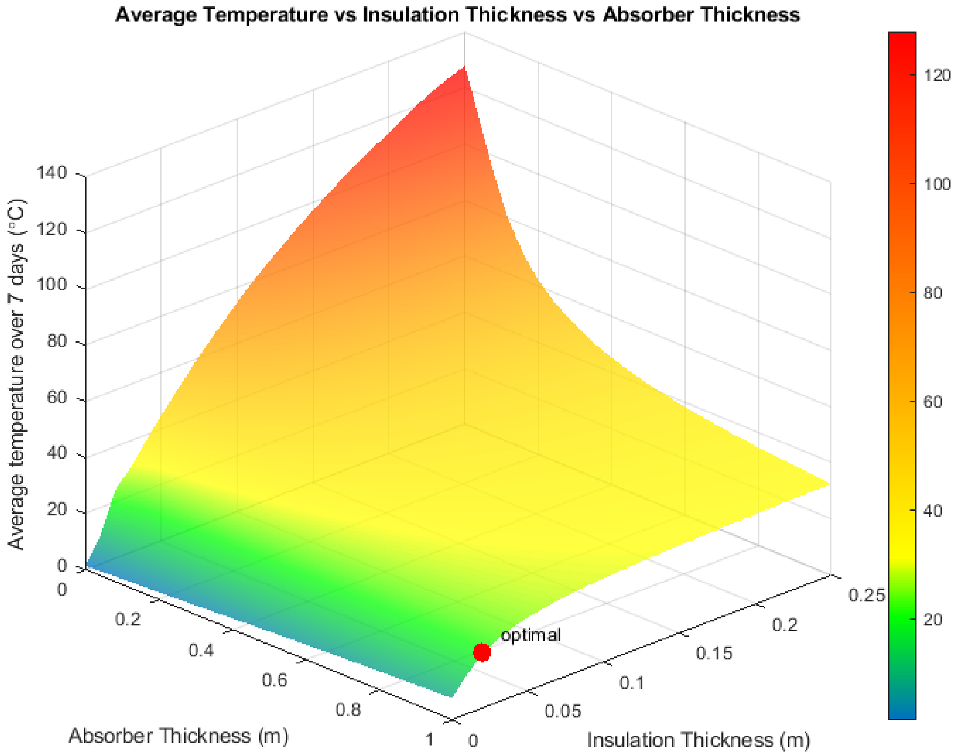

Figure 10: Average temperature vs material thicknesses

3.2 Material of the Wall Insulation

We evaluated five common material choices for wall insulation with respect to cost. These materials are listed in the table below (see [6] [2] [4] [7] [5] [3] for material data)

Material	Thermal Conductivity ($\frac{W}{mk}$)	Ideal Thickness (m)	Total Cost
Aerogel	.018	0.0096	$62928
Cellulose	0.035	0.02	$102
Fiberglass	0.04	0.02	$63
Sheep's Wool	0.038	0.02	$264
Wood Fibre	0.038	0.02	$182

Given each material's k-value, we were able to calculate the optimal thickness and total cost. Ultimately, we found that fiberglass is the most affordable option. However, if cost was no object, and paper thin walls seem appealing, aerosol would be a choice material. Cellulose insulation, while slightly more expensive, uses over 85% recycled material [1], and performs similarly to fiberglass at identical thickness. Balancing cost, comfort, ease of building, and environmental impact, cellulose seems like the ideal choice.

4 Results and Discussion

Going forward with cellulose insulation, our optimization function found that a 2cm thick application of insulation coupled with a large .85m thick thermal mass resulted in very comfortable house temperature. 11

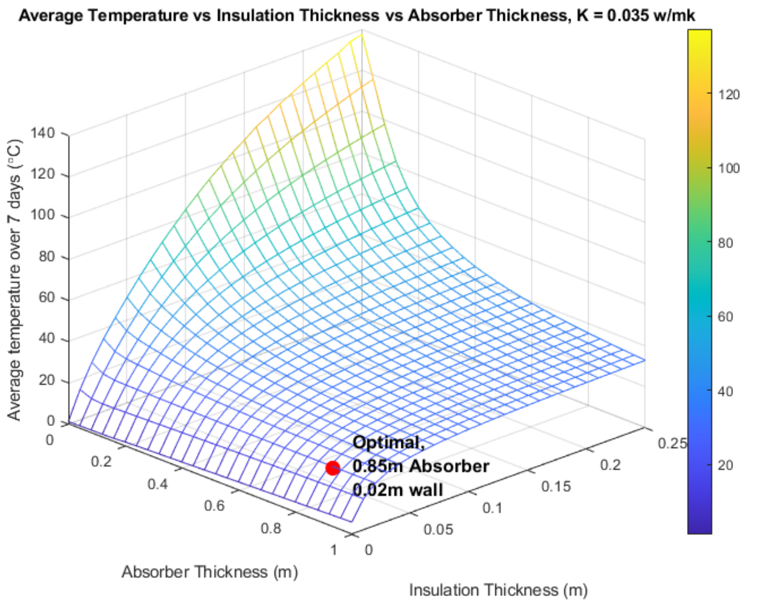

Figure 11: 3d plot of temperature vs thickness variables with the k value of cellulose.

Over a period of 10 days, the average temperature rises by only a degree, and levels off.

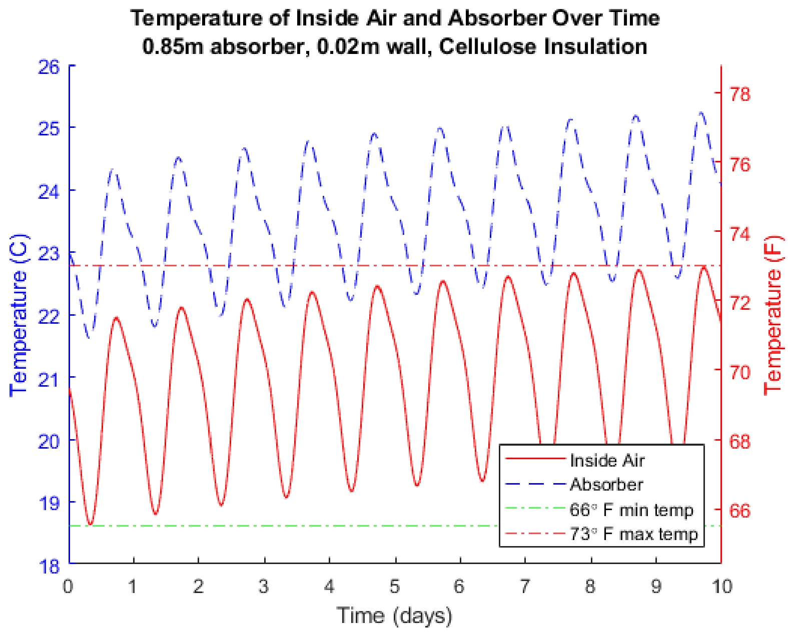

Figure 12: plot of optimal house temperatures over a long time span

after normalizing, a house built with these parameters reaches a comfortable max of 72°F and a nice min of 66°F, perfect for sleeping (fig: 13). The floor was a bit warmer, but a warm floor is always comforting to cold feet in the winter.

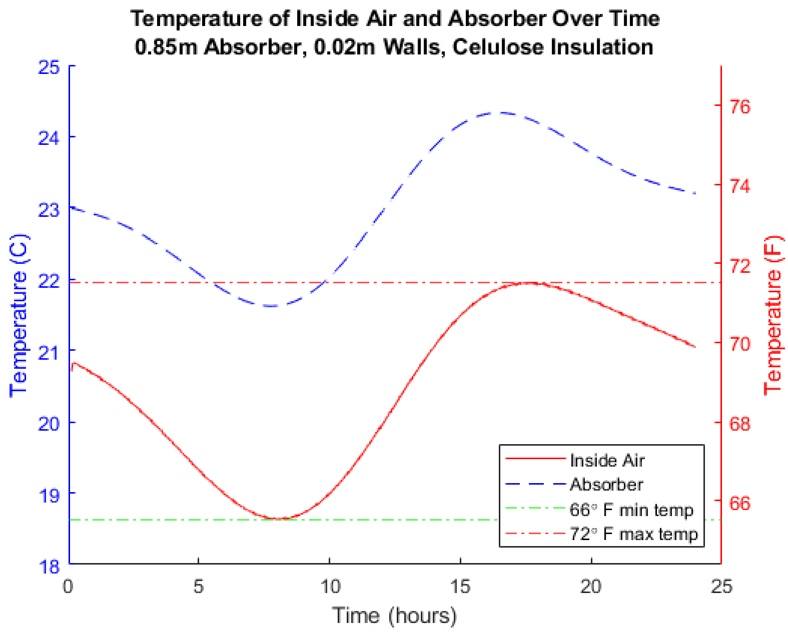

Figure 13: plot of optimal house temperatures over a short time span.

References

[1] *environmental considerations of building insulation*. National Park Service, 2000.

[2] *Greenfiber Low Dust Cellulose Blown-In Insulation 19 lbs.-INS541LD*. URL: `http://www.homedepot.com/p/Greenfiber-Low-Dust-Cellulose-Blown-In-Insulation-19-lbs-INS541LD/100318635`.

[3] *Insulation materials and their thermal properties*. URL: `http://www.greenspec.co.uk/building-design/insulation-materials-thermal-properties`.

[4] *Owens Corning R-13 EcoTouch PINK Kraft Faced Insulation Fiberglass Insulation Batt 15 in. x 93 in.-BF10*. URL: `http://www.homedepot.com/p/Owens-Corning-R-13-EcoTouch-PINK-Kraft-Faced-Insulation-Fiberglass-Insulation-Batt-15-in-x-93-in-BF10/202676829`.

[5] *price list wood fiber 2019*. 2019.

[6] *Stardust - Frequently Asked Questions*. URL: `https://stardust.jpl.nasa.gov/overview/faq.html`.

[7] *Wool Insulation Suppliers: Where to Buy Wool Insulation - Havelock Wool*. June 2020. URL: `https://havelockwool.com/buy/`.