MATH 1700

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Chapter 8C

Department of Mathematical and Statistical Sciences

CHAPTER 8C

- **Hypothesis Test Approaches**
  - P-value Approach
  - Classical Approach
- **The Classical Hypothesis Test: A 5-step Procedure**
  - Step 1 The Set-Up
  - Step 2 The Hypothesis Test Criteria
  - Step 3 The Sample Evidence
  - Step 4 The Probability Distribution
  - Step 5 The Results

EXAMPLE 17 – TWO-TAILED HYPOTHESIS TEST WITH SAMPLE DATA

MARQUETTE UNIVERSITY
Be The Difference.

- **Let's use computer simulation, to draw a sample of 40 single-digit numbers.**
- **Remember:**
  - $\mu = \sum_{x=0}^{9} xP(x) = 4.5$ **and**
  - $\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{x=0}^{9}[x^2P(x)] - \mu^2} = \sqrt{8.25} = 2.87$

x	$P(x)$
0	$1/10$
1	$1/10$
2	$1/10$
3	$1/10$
4	$1/10$
5	$1/10$
6	$1/10$
7	$1/10$
8	$1/10$
9	$1/10$

- **Use $\alpha = 0.10$ and test the hypothesis that**
- **"The mean of this distribution is different from 4.5."**

EXAMPLE 17 – *SOLUTION*

- **Step 1 The Set-Up:**
  - **a. Describe the population parameter of interest.**
    - The population parameter of interest is the mean μ of the population of single-digit numbers with uniform distribution.
  - **b. State the null hypothesis (H_0) and the alternative hypothesis (H_a).**
    - H_0: $\mu = 4.5$ (mean is 4.5)
    - H_a: $\mu \neq 4.5$ (mean is not 4.5)

- **Step 2 The Hypothesis Test Criteria:**
  - **a. Check the assumptions.**
    - ✓ σ is known. Samples of size 40 should be large enough to satisfy the **CLT**.
  - **b. Identify the probability distribution and the test statistic to be used.**
    - We use the standard normal probability distribution, and the test statistic is $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$; where $\sigma = 2.87$.
  - **c. Determine the level of significance, α.**
    - $\alpha = 0.10$ (given in the statement of the problem)

EXAMPLE 17 – *SOLUTION*

- **Step 3 The Sample Evidence:**

  a. Collect the sample information.

  - **This random sample was drawn using computer simulation:**

2	8	2	1	5	5	4	0	9	1	0	4	6	1
5	1	1	3	8	0	3	6	8	4	8	6	8	
9	5	0	1	4	1	2	1	7	1	7	9	3	

  - **From the sample $\bar{x} = 3.975$ and $n = 40$.**

  a. Calculate the value of the test statistic.

  - $z^* = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

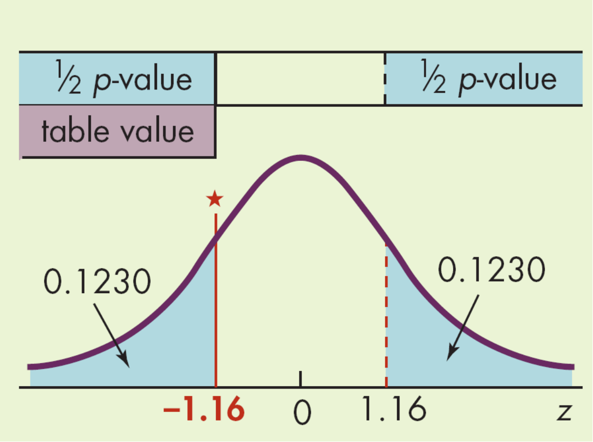
$$z\bigstar = \frac{3.975 - 4.50}{2.87/\sqrt{40}} = \frac{-0.525}{0.454}$$

  - $z^* = -1.156 = \mathbf{-1.16}$

EXAMPLE 17 – *SOLUTION*

- **Here is the main difference between the p-value approach and the classical approach**

- **Step 4 The Probability Distribution: (p-value approach)**
  - **a. Calculate the *p*-value for the test statistic.**
    - Because the alternative hypothesis indicates a two-tailed test, we must find the probability associated with both tails.
    - The $p-\text{value}$ is found by doubling the area of one tail.
    - $z^* = -1.16$.
    - $\textit{The}\ \text{p}-\text{value} \;=\; 2 \times P\,(z < -1.16)$
    - $= 2(0.1230)$
    - $= 0.2460$
  - **b. Determine whether or not the $\text{p}-\text{value}$ is smaller than α.**
    - The *p*-value (0.2460) is greater than $\alpha\,(0.10)$.



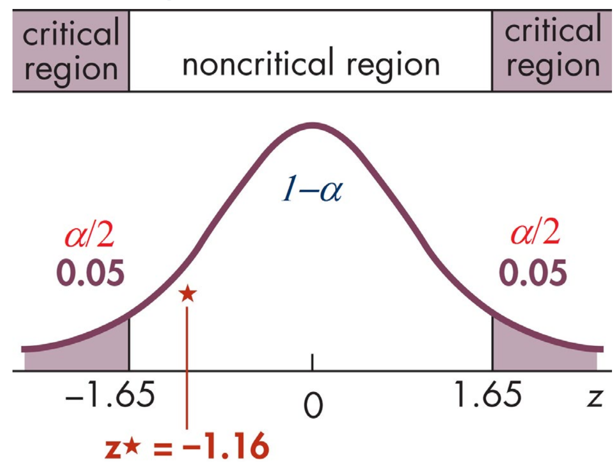
HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A CLASSICAL APPROACH

- **Step 4 The Probability Distribution:**

  a. **Determine the critical region and critical value (s).**
  The standard normal variable z is our test statistic for this hypothesis test.

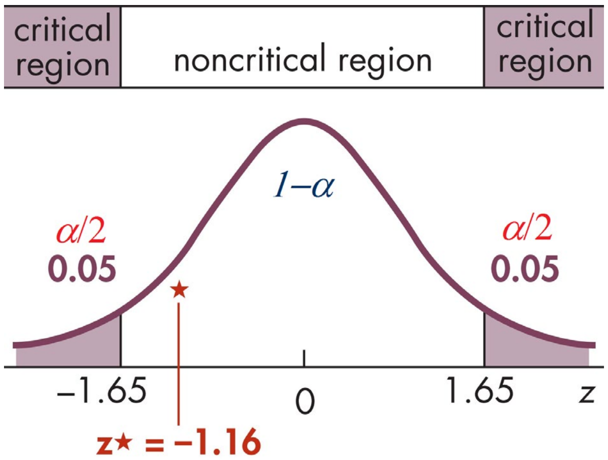
  - **Critical region** The set of values for the test statistic that will cause us to reject the null hypothesis. The set of values that are not in the critical region is called the **noncritical region** (sometimes called the *acceptance region*).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681



  - **Critical value(s)** The "first" or "boundary" value(s) of the critical region(s).

EXAMPLE 17 – *SOLUTION*



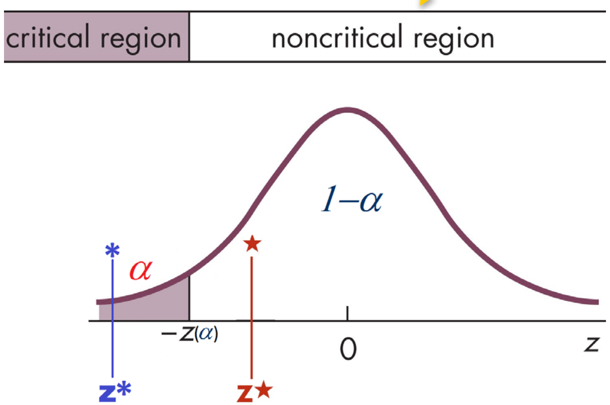
- **Step 5 The Results:**
  - z^* **is in noncritical region:**
    - $-z(\alpha/2) < z^* < z(\alpha/2)$

a. **State the decision about H_0:**
   - **Fail to reject H_0.**

b. **State the conclusion about H_a:**
   - **The observed sample mean is not significantly different from 4.5 at the 0.10 level of significance.**

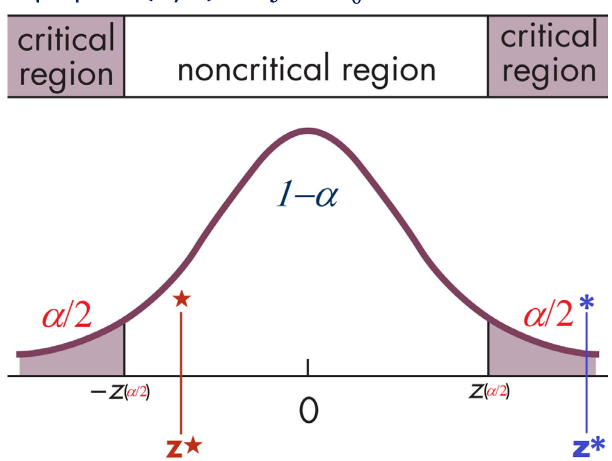
HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A CLASSICAL APPROACH

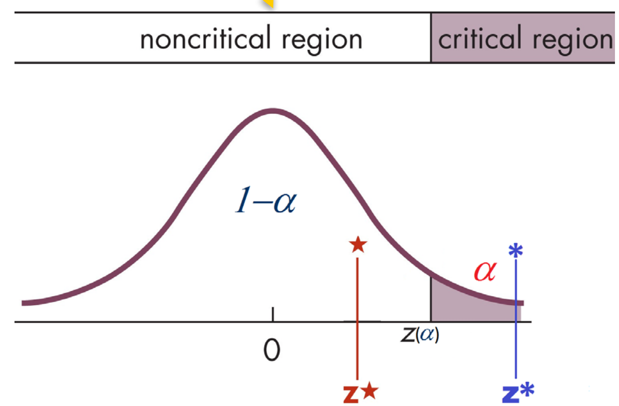
	Sign in the Alternative Hypothesis		
	$<$	$\neq$	$>$
Critical Region	One region Left side One-tailed test	Two regions Half on each side Two-tailed test	One region Right side One-tailed test



- H_0: $\mu \geq 4.5$
- H_a: $\mu < 4.5$
- $z^* > -z(\alpha)$: Fail to Reject H_0
- $z^* \leq -z(\alpha)$: Reject H_0

- H_0: $\mu = 4.5$
- H_a: $\mu \neq 4.5$
- $-z(\alpha/2) < z^* < z(\alpha/2)$: Fail to Reject H_0
- $|z^*| \geq z(\alpha/2)$: Reject H_0





- H_0: $\mu \leq 4.5$
- H_a: $\mu > 4.5$
- $z^* < z(\alpha)$: Fail to Reject H_0
- $z^* \geq z(\alpha)$: Reject H_0

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): ANOTHER EXAMPLE ON P-VALUE & CLASSICAL APPROACHES

- **Assumptions:**
  1. σ is known
  2. $n \geq 30$ or the sample is drawn from a normal population.

- **Example:**
  Let $\{y_1, y_2, \dots, y_{100}\}$ be sample of blood pressures of 100 patients with Cardiovascular disease. We want to investigate that the population of patients with this disease have high blood pressure. Suppose that the mean normal blood pressure is 120. Assume that $\sigma = 7.5$.

- **Sample Information:** $\bar{y} = 121.5$

- **Do we have sufficient evidence to conclude that this population has high blood pressure?** $\alpha = 0.05$.

EXAMPLE – *SOLUTION*

- **Step 1 The Set-Up:**
  - a. **Describe the population parameter of interest.**
    - The population parameter of interest is the mean blood pressure μ of the population of patients with Cardiovascular disease.
  - b. **State the null hypothesis (H_0) and the alternative hypothesis (H_a).**
    - H_0: $\mu = 120$ (normal blood pressure is 120)
    - H_a: $\mu > 120$ (Wondering if patients with this disease have high blood pressure?)

- **Step 2 The Hypothesis Test Criteria:**
  - a. **Check the assumptions.**
    - ✓ σ is known. Samples of size 100 should be large enough to satisfy the **CLT**.
  - b. **Identify the probability distribution and the test statistic to be used.**
    - We use the standard normal probability distribution, and the test statistic is $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$; where $\sigma = 7.5$.
  - c. **Determine the level of significance, α.**
    - $\alpha = 0.05$ (given in the statement of the problem)

EXAMPLE – *SOLUTION*

- **Step 3 The Sample Evidence:**

a. Collect the sample information.

- $\{y_1, y_2, \dots, y_{100}\}$ is a sample of blood pressures of 100 patients with Cardiovascular disease

- From the sample $\bar{x} = 121.5$ and $n = 100$.

a. Calculate the value of the test statistic.

- $z^* = \frac{\bar{y}-\mu_0}{\sigma/\sqrt{n}}$

- $z^* = \frac{\bar{y}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{121.5-120}{\frac{7.5}{\sqrt{100}}}$

- $z^* = \mathbf{2.0}$

EXAMPLE – *SOLUTION*

- **Here is the main difference between the p-value approach and the classical approach**

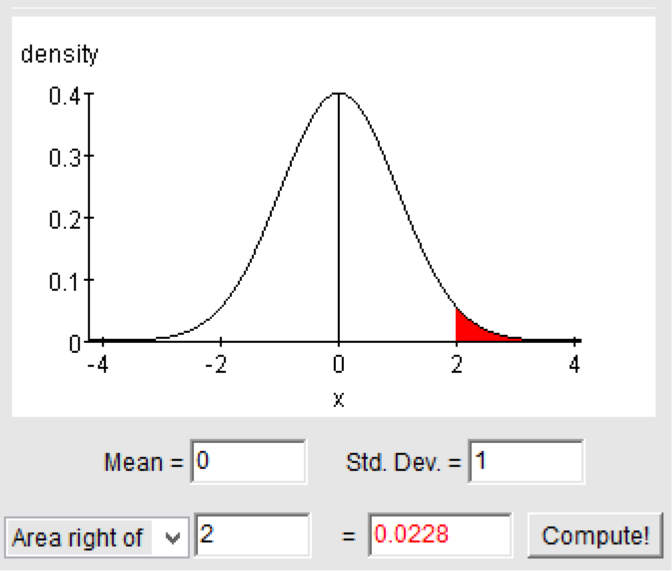
- **Step 4 The Probability Distribution: (p-value approach)**

  a. Calculate the *p*-value for the test statistic.

  - **Because the alternative hypothesis indicates a one-tailed test, we must find the probability associated with one tail.**
  - **The $p-\text{value}$ is found by doubling the area of one tail.**
  - $z^* = 2.0.$
  - $The\ \mathrm{p-value}\ = P\,(z > \mathbf{2.0})$
  - $= 1 - P\,(z < \mathbf{2.0})$
  - $= 0.0228$

  b. Determine whether or not the $\mathrm{p-value}$ is smaller than α.

  - **The *p*-value (0.0228) is smaller than $\alpha\,(0.05)$.**



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- **Step 4 The Probability Distribution:**
  - **a. Determine the critical region and critical value (s).**
    The standard normal variable z is our test statistic for this hypothesis test.
  - **Critical region** The set of values for the test statistic that will cause us to reject the null hypothesis. The set of values that are not in the critical region is called the **noncritical region** (sometimes called the ***acceptance region***).

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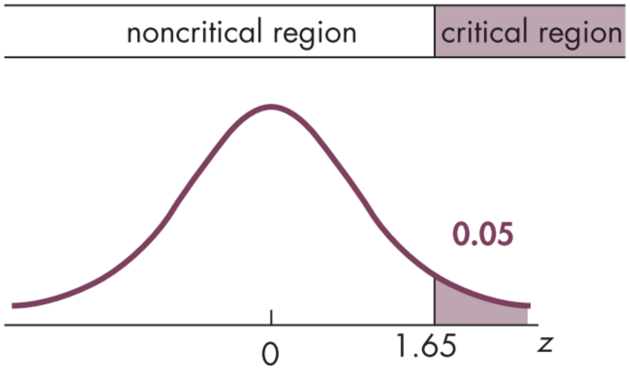
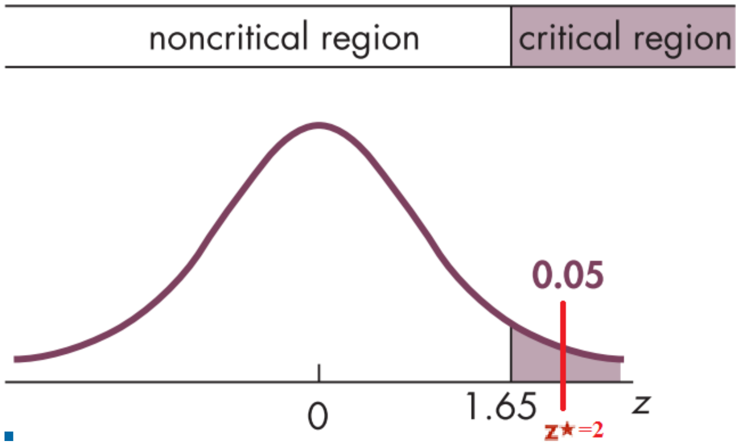


  - **Critical value(s)** The "first" or "boundary" value(s) of the critical region(s).

EXAMPLE 17 – *SOLUTION*



- **Step 5 The Results:**
  - z^* **is in the critical region:**
    - ➢ $z^* > z(\alpha/2)$

a. State the decision about H_0:

➢ **Reject H_0.**

b. State the conclusion about H_a:

➢ **The observed sample mean is significantly larger than 120.0 at the 0.05 level of significance and conclude that the population of patients with the disease have high blood pressure.**

QUESTIONS?

- ANY QUESTION?