MATH 1700

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Chapter 9C
Inferences Involving One Population

Department of Mathematical and Statistical Sciences

INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

Problems often arise that require us to make inferences about variability.

For example, a soft drink bottling company has a machine that fills 16-oz bottles. The company needs to control the standard deviation σ (or variance σ^2) in the amount of soft drink, x, put into each bottle.

The mean amount placed into each bottle is important, but a correct mean amount does not ensure that the filling machine is working correctly.

INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

If the variance is too large, many bottles will be overfilled and many underfilled. Thus, the bottling company wants to maintain as small a standard deviation (or variance) as possible.

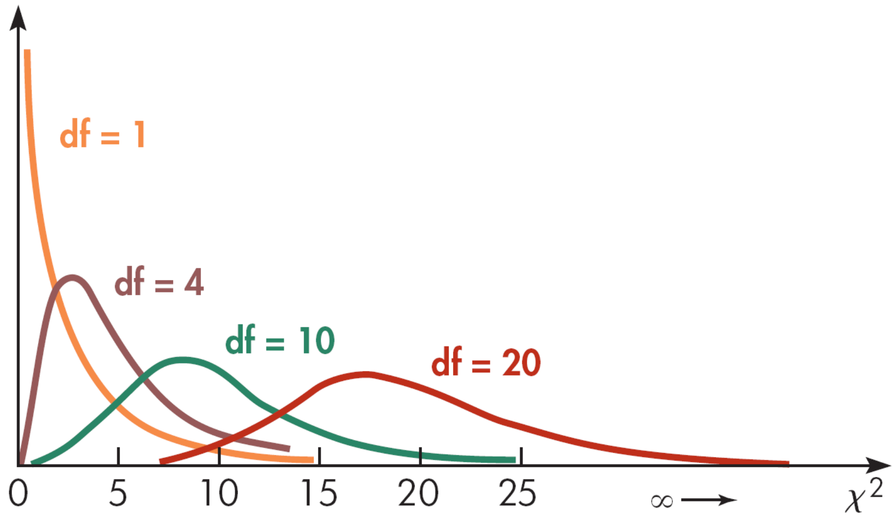
When discussing inferences about the spread of data, we usually talk about variance instead of standard deviation because the techniques (the formulas used) employ the sample variance rather than the standard deviation.

However, remember that the standard deviation is the square root of the variance; thus, talking about the variance of a population is comparable to talking about the standard deviation.

INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

Inferences about the variance of a normally distributed population use the **chi-square**, χ^2, distributions (*"ki-square"*, and χ is the Greek lowercase letter chi).

The chi-square distributions, like Student's *t*-distributions, are a family of probability distributions, each of which is identified by the **parameter** number of **degrees of freedom.**



INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

Properties of the chi-square distribution:

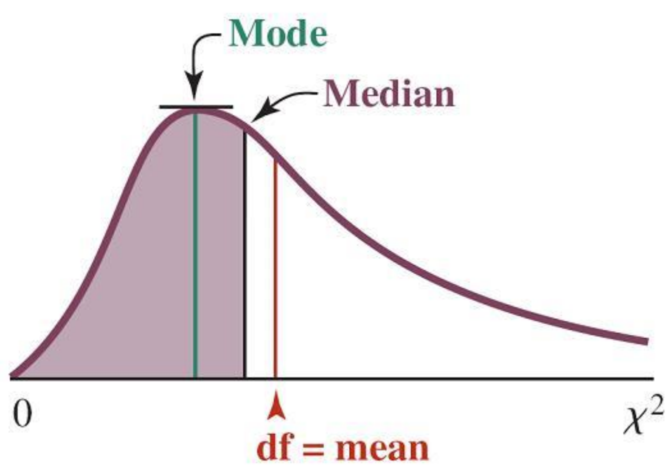
1. χ^2 is nonnegative in value; it is zero or positively valued.
2. χ^2 is not symmetrical; it is skewed to the right.
3. χ^2 is distributed so as to form a family of distributions, a separate distribution for each different number of degrees of freedom.

Note:

The mean value of the chi-square distribution is df. The mean is located to the right of the mode (the value where the curve reaches its high point) and just to the right of the median (the value that splits the distribution, 50% on each side).

INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

By locating zero at the left extreme and the value of df on your sketch of the χ^2 distribution, you will establish an approximate scale so that other values can be located in their respective positions. See Figure 9.8.

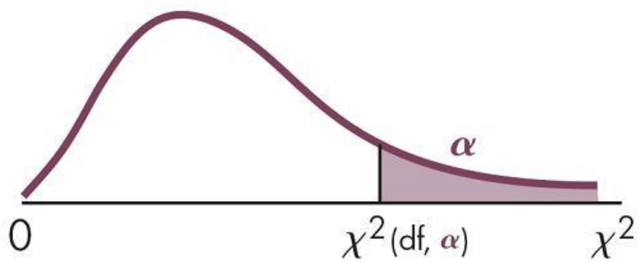


Location of Mean, Median, and Mode for χ^2 Distribution

Figure 9.8

INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

Thus, $\chi^2(\mathrm{df}, \alpha)$ (read "chi-square of df, alpha") is the symbol used to identify the critical value of chi-square with df degrees of freedom and with α area to the right, as shown in Figure 9.9.



Chi-Square Distribution Showing χ^2(df, α)

Figure 9.9

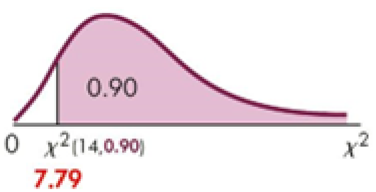
Since the chi-square distribution is not symmetrical, the critical values associated with the right and left tails are given separately in Table 8.

EXAMPLE 16 – χ^2 ASSOCIATED WITH THE LEFT TAIL

Find $\chi^2(\text{df} = 14, \alpha = 0.90)$

Solution:

At the intersection of row df = 14 and the column for an area of 0.90 to the right:

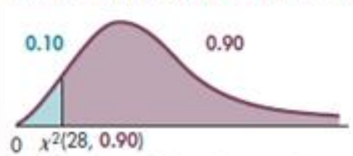


$$\chi^2(14, 0.90) = 7.79$$

a) Area to the Right

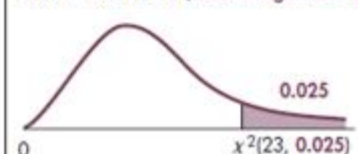
	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005
b) Area to the Left (the Cumulative Area)							Median						
df	0.005	0.01	0.025	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.975	0.99	0.995
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.34	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.34	18.2	22.3	25.0	27.5	30.6	32.8
16	5.14	5.81	6.91	7.96	9.31	11.9	15.34	19.4	23.5	26.3	28.8	32.0	34.3
17	5.70	6.41	7.56	8.67	10.1	12.8	16.34	20.5	24.8	27.6	30.2	33.4	35.7
18	6.26	7.01	8.23	9.39	10.9	13.7	17.34	21.6	26.0	28.9	31.5	34.8	37.2
19	6.84	7.63	8.91	10.1	11.7	14.6	18.34	22.7	27.2	30.1	32.9	36.2	38.6
20	7.43	8.26	9.59	10.9	12.4	15.5	19.34	23.8	28.4	31.4	34.2	37.6	40.0
21	8.03	8.90	10.3	11.6	13.2	16.3	20.34	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.34	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.34	27.1	32.0	35.2	$\boxed{38.1}$	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.34	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.34	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.34	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.34	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	$\boxed{18.9}$	22.7	27.34	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.34	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.34	34.8	40.3	43.8	47.0	50.9	53.7

Left-tail example:
Find χ^2 with df = 28; area in left-tail = 0.10.



χ^2(df, area to right) = $\chi^2(28, 0.90) = 18.9$

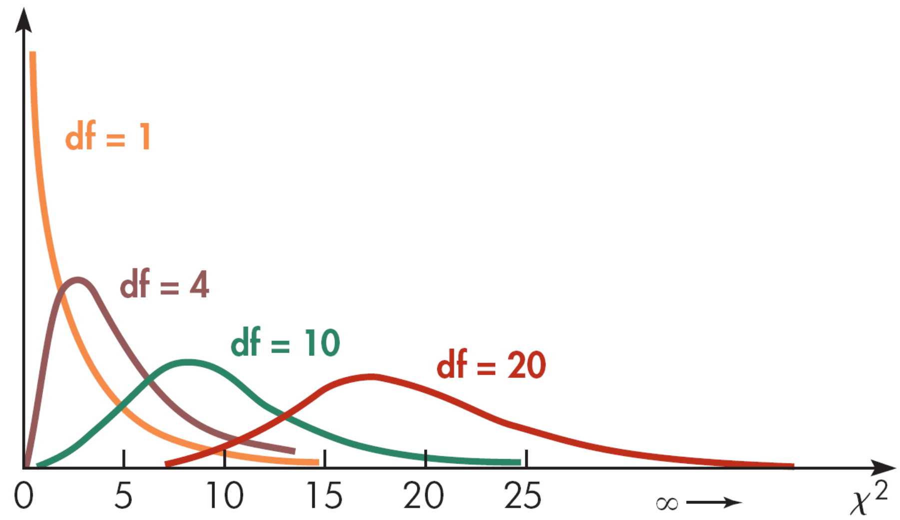
Right-tail example:
Find χ^2 with df = 23; area in right-tail = 0.025

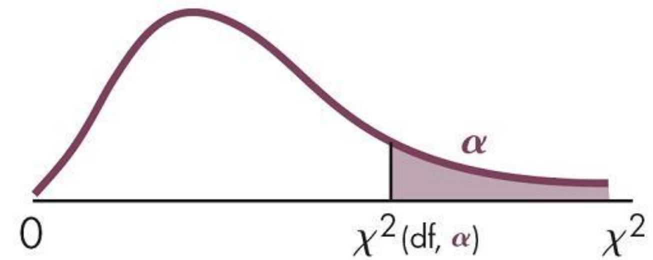


χ^2(df, area to right) = $\chi^2(23, 0.025) = 38.1$

CHI-SQUARED (χ^2) DISTRIBUTION

- **Right skewed distribution**
- **Defined over positive numbers**
- **Parameter: df**
- **How to write:**
  - ➢ $\chi^2(\mathrm{df})$
- **How to find probabilities?**
  - – Chi-Squared Calculator
  - – χ^2-table ("D2L > Useful Links > Z, T and χ^2 Tables")
    - ➢ $P(\chi^2 \geq c_\alpha)$
  - • χ^2-table (from book, next slide)

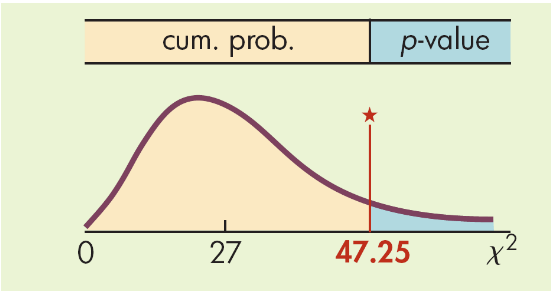




χ^2 TABLE

- χ^2(df, area to right)
  - $df = 28$, **area to right=0.9** $\rightarrow$ **area to left=0.1**
  - $\chi^2(df = 28, \text{area to right=0.9}) = 18.9$

- **Example in a Hypothesis Test:**
- **Lets say** $\chi^2_* = 47.25$ and you want to find:
- $\text{p} - \text{value} = P(\chi^2 > \chi^2_*, \text{with df} = 27)$



- $0.005 < \text{p} - \text{value} < 0.01$
- **So if** $\alpha = 0.05$, **we have**
- $\text{p} - \text{value} < \alpha$, **so we Reject H$_0$**

a) Area to the Right

b) Area to the Left (the Cumulative Area)

a) Area to the Right	0.995	0.99	0.975	0.95	0.90	0.75	0.50 (Median)	0.25	0.10	0.05	0.025	0.01	0.005
df	**0.005**	**0.01**	**0.025**	**0.05**	**0.10**	**0.25**	**0.50**	**0.75**	**0.90**	**0.95**	**0.975**	**0.99**	**0.995**
1	0.0000393	0.000157	0.000982	0.00393	0.0158	0.102	0.455	1.32	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	0.575	1.39	2.77	4.61	5.99	7.38	9.21	10.6
3	0.0717	0.115	0.216	0.352	0.584	1.21	2.37	4.11	6.25	7.81	9.35	11.3	12.8
4	0.207	0.297	0.484	0.711	1.06	1.92	3.36	5.39	7.78	9.49	11.1	13.3	14.9
5	0.412	0.554	0.831	1.15	1.61	2.67	4.35	6.63	9.24	11.1	12.8	15.1	16.7
6	0.676	0.872	1.24	1.64	2.20	3.45	5.35	7.84	10.6	12.6	14.4	16.8	18.5
7	0.989	1.24	1.69	2.17	2.83	4.25	6.35	9.04	12.0	14.1	16.0	18.5	20.3
8	1.34	1.65	2.18	2.73	3.49	5.07	7.34	10.2	13.4	15.5	17.5	20.1	22.0
9	1.73	2.09	2.70	3.33	4.17	5.90	8.34	11.4	14.7	16.9	19.0	21.7	23.6
10	2.16	2.56	3.25	3.94	4.87	6.74	9.34	12.5	16.0	18.3	20.5	23.2	25.2
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.8	18.5	21.0	23.3	26.2	28.3
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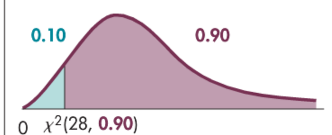
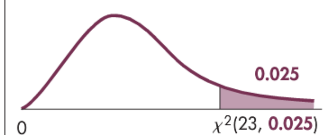
26	11.2	12.2	13.8	15.4	17.3	20.8	25.34	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.34	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.34	32.6	37.9	41.3	44.5	48.3	51.0
29	13.1	14.3	16.0	17.7	19.8	23.6	28.34	33.7	39.1	42.6	45.7	49.6	52.3
30	13.8	15.0	16.8	18.5	20.6	24.5	29.34	34.8	40.3	43.8	47.0	50.9	53.7
40	20.7	22.2	24.4	26.5	29.1	33.7	39.34	45.6	51.8	55.8	59.3	63.7	66.8
50	28.0	29.7	32.4	34.8	37.7	42.9	49.33	56.3	63.2	67.5	71.4	76.2	79.5
60	35.5	37.5	40.5	43.2	46.5	52.3	59.33	67.0	74.4	79.1	83.3	88.4	92.0
70	43.3	45.4	48.8	51.7	55.3	61.7	69.33	77.6	85.5	90.5	95.0	100.4	104.2
80	51.2	53.5	57.2	60.4	64.3	71.1	79.33	88.1	96.6	101.9	106.6	112.3	116.3
90	59.2	61.8	65.6	69.1	73.3	80.6	89.33	98.6	107.6	113.1	118.1	124.1	128.3
100	67.3	70.1	74.2	77.9	82.4	90.1	99.33	109.1	118.5	124.3	129.6	135.8	140.2

Left-tail example:
Find χ^2 with df = 28; area in left-tail = 0.10.



χ^2(df, area to right) = χ^2(28, 0.90) = 18.9

Right-tail example:
Find χ^2 with df = 23; area in right-tail = 0.025



χ^2(df, area to right) = χ^2(23, 0.025) = 38.1

INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

The assumptions for inferences about the variance σ^2 or standard deviation σ : The sampled population is normally distributed.

The t procedures for inferences about the mean were based on the assumption of normality, but the t procedures are generally useful even when the sampled population is nonnormal, especially for larger samples.

However, the same is not true about the inference procedures for the standard deviation. Therefore, the only inference procedure to be presented here is the hypothesis test for the standard deviation of a normal population.

INFERENCES ABOUT THE VARIANCE AND STANDARD DEVIATION

The **test statistic** that will be used in testing hypotheses about the population variance or standard deviation is obtained by using the following formula:

Test Statistic for Variance and Standard Deviation

$$\chi^2\bigstar = \frac{(n-1)s^2}{\sigma^2}, \text{ with df} = n - 1 \qquad (9.10)$$

When random samples are drawn from a normal population with a known variance σ^2, the quantity $\frac{(n-1)s^2}{\sigma^2}$ possesses a probability distribution that is known as the chi-square distribution with $n-1$ degrees of freedom.

Hypothesis-Testing Procedure

HYPOTHESIS-TESTING PROCEDURE

Let's return to the example about the bottling company that wishes to detect when the variability in the amount of soft drink placed into each bottle gets out of control.

A variance of 0.0004 is considered acceptable, and the company wants to adjust the bottle-filling machine when the variance, σ^2, becomes larger than this value.

In other words, the soft drink bottling company wants to control the variability in the amount of fill by not allowing the variance to exceed 0.0004.

EXAMPLE 17 – ONE-TAILED HYPOTHESIS TEST FOR VARIANCE, σ^2

Does a sample of size 28 with a variance of 0.0007 indicate that the bottling process is out of control (with regard to variance) at the 0.05 level of significance?

Solution:

Step 1 The Set-Up:

a. Describe the population parameter of interest.

σ^2, the variance in the amount of fill of a soft drink during a bottling process

b. State the null hypothesis (H_o) and the alternative hypothesis (H_a).

$H_o: \sigma^2 = 0.0004(\leq)$ (variance is not larger than 0.0004)

$H_a: \sigma^2 > 0.0004$ (variance is larger than 0.0004)

EXAMPLE 17 – SOLUTION CONT'D …

Step 2 The Hypothesis Test Criteria:

a. Check the assumptions.

The amount of fill put into a bottle is generally normally distributed. By checking the distribution of the sample, we could verify this.

b. Identify the probability distribution and the test statistic to be used.

The chi-square distribution and formula (9.10), with $\text{df} = n - 1 = 28 - 1 = 27$, will be used.

c. Determine the level of significance: $\alpha = 0.05$.

EXAMPLE 17 – SOLUTION CONT'D …

Step 3 The Sample Evidence:

a. Collect the sample information: $n = 28$ **and** $s^2 = 0.0007$.

b. Calculate the value of the test statistic.

Use formula (9.10):

$$\chi^2\bigstar = \frac{(n-1)s^2}{\sigma^2} : \chi^2\bigstar = \frac{(28-1)(0.0007)}{0.0004}$$

$$= \frac{(27)(0.0007)}{0.0004}$$

$$= \mathbf{47.25}$$

EXAMPLE 17 – SOLUTION CONT'D …

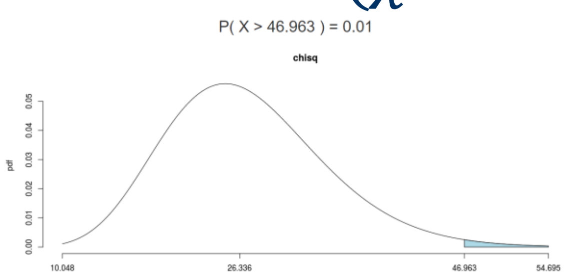
Step 4 The Probability Distribution:

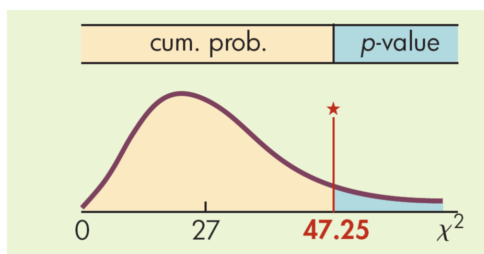
Using the *p*-value procedure:

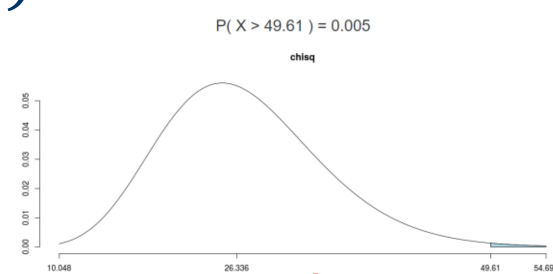
a. Calculate the *p*-value for the test statistic.

Use the right-hand tail because H_a expresses concern for values related to "larger than."

$\mathrm{P} = P(\chi^{2*} > 47.25, \text{with df} = 27)$ **as shown below.**







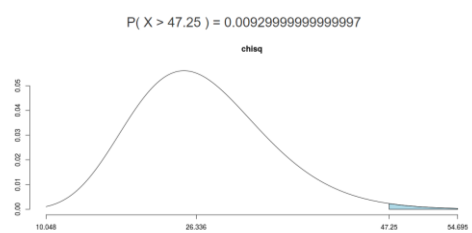
a) Area to the Right

	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005
b) Area to the Left (the Cumulative Area)							Median						
df	0.005	0.01	0.025	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.975	0.99	0.995
21	8.03	8.90	10.3	11.6	13.2	16.3	20.34	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.34	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.34	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.34	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.34	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.34	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.34	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.34	32.6	37.9	41.3	44.5	48.3	51.0

EXAMPLE 17 – SOLUTION CONT'D …

To find the $p - value$, use one of two methods:

1. **Use Table 8 in Appendix B to place bounds on the** $p - value$: $0.005 < \mathbf{P} < 0.01$.
2. **Use a computer or calculator to calculate the $p - value$:** $P = 0.0093$.



b. Determine whether or not the $p - value$ is smaller than α.

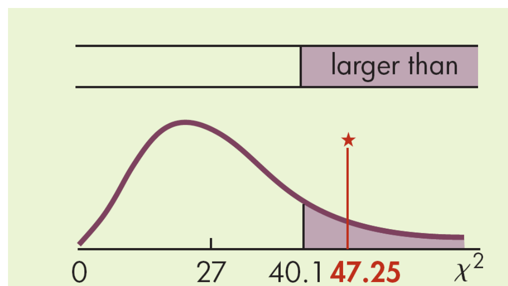
The *p*-value is smaller than the level of significance, $\alpha(0.05)$.

EXAMPLE 17 – SOLUTION CONT’D …

Using the classical procedure:

a. Determine the critical region and critical value(s).

- The critical region is the right-hand tail because H_a expresses concern for values related to “larger than.” The critical value is obtained from Table 8, at the intersection of row $\text{df} = 27$ and column $\alpha = 0.05$: $\chi^2(27, 0.05) = 40.1$.



a) Area to the Right

	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005
b) Area to the Left (the Cumulative Area)							Median						
df	0.005	0.01	0.025	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.975	0.99	0.995
21	8.03	8.90	10.3	11.6	13.2	16.3	20.34	24.9	29.6	32.7	35.5	38.9	41.4
22	8.64	9.54	11.0	12.3	14.0	17.2	21.34	26.0	30.8	33.9	36.8	40.3	42.8
23	9.26	10.2	11.7	13.1	14.8	18.1	22.34	27.1	32.0	35.2	38.1	41.6	44.2
24	9.89	10.9	12.4	13.8	15.7	19.0	23.34	28.2	33.2	36.4	39.4	43.0	45.6
25	10.5	11.5	13.1	14.6	16.5	19.9	24.34	29.3	34.4	37.7	40.6	44.3	46.9
26	11.2	12.2	13.8	15.4	17.3	20.8	25.34	30.4	35.6	38.9	41.9	45.6	48.3
27	11.8	12.9	14.6	16.2	18.1	21.7	26.34	31.5	36.7	40.1	43.2	47.0	49.6
28	12.5	13.6	15.3	16.9	18.9	22.7	27.34	32.6	37.9	41.3	44.5	48.3	51.0

EXAMPLE 17 – SOLUTION CONT'D …

Step 5 The Results:

a. State the decision about H_o: Reject H_o.

b. State the conclusion about H_a.

At the 0.05 level of significance, we conclude that the bottling process is out of control with regard to the variance.

EXAMPLE 19 – TWO-TAILED HYPOTHESIS TEST FOR STANDARD DEVIATION, σ

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A manufacturer claims that a photographic chemical has a shelf life that is normally distributed about a mean of 180 days with a standard deviation of no more than 10 days.

As a user of this chemical, Fast Photo is concerned that the standard deviation might be different from 10 days; otherwise, it will buy a larger quantity while the chemical is part of a special promotion.

Twelve random samples were selected and tested, with a standard deviation of 14 days resulting. At the 0.05 level of significance, does this sample present sufficient evidence to show that the standard deviation is different from 10 days?

EXAMPLE 19 – SOLUTION

Step 1 The Set-Up:

a. Describe the population parameter of interest.

σ , the standard deviation for the shelf life of the chemical

b. State the null hypothesis (H_o) and the alternative hypothesis (H_a).

H_a: $\sigma = 10$ (standard deviation is 10 days)

H_o: $\sigma \neq 10$ (standard deviation is different from 10 days).

EXAMPLE 19 – SOLUTION CONT'D …

Step 2 The Hypothesis Test Criteria:

a. Check the assumptions.

The manufacturer claims shelf life is normally distributed; this could be verified by checking the distribution of the sample.

b. Identify the probability distribution and the test statistic to be used.

The chi-square distribution and formula (9.10), with $\mathrm{df} = n - 1 = 12 - 1 = 11$, will be used.

c. Determine the level of significance: $\alpha = 0.05$.

EXAMPLE 19 – SOLUTION CONT'D …

Step 3 The Sample Evidence:

a. Collect the sample information: $n = 12$ and $s = 14$.

b. Calculate the value of the test statistic.

Use formula (9.10):

$$\chi^2\bigstar = \frac{(n-1)s^2}{\sigma^2} : \chi^2\bigstar = \frac{(12-1)(14)^2}{(10)^2}$$

$$= \frac{2156}{100}$$

$$= \mathbf{21.56}$$

EXAMPLE 19 – SOLUTION CONT'D …

Step 4 The Probability Distribution:

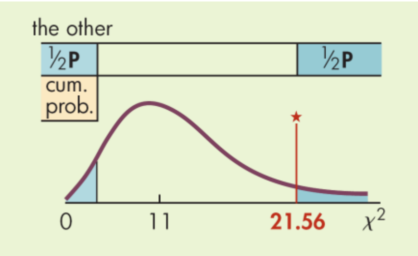
Using the *p*-value procedure:

a. Calculate the $p - value$ for the test statistic.

Since the concern is for values "different from" 10, the $p - value$ is the area of both tails.

The area of each tail will represent ½P. Since $\chi^{2*} = 21.56$ is in the right tail, the area of the right tail is ½ $\boldsymbol{P}$:

½ $\mathbf{P} = P(\chi^2 > 21.56, \text{with df} = 11)$, **as shown below.**



a) Area to the Right

	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005
b) Area to the Left (the Cumulative Area)							Median						
df	0.005	0.01	0.025	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.975	0.99	0.995
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.34	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.34	18.2	22.3	25.0	27.5	30.6	32.8

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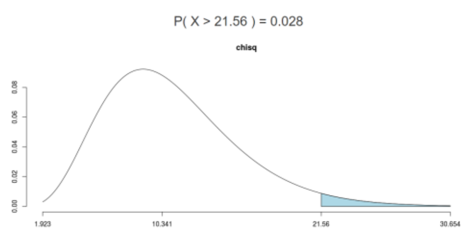
EXAMPLE 19 – SOLUTION CONT'D …

To find ½ P, use one of two methods:

1. **Use Table 8 in Appendix B to place bounds on ½P:** $0.025 < ½\mathbf{P} < 0.05$. **Double both bounds to find the bounds for P:** $2 \times (0.025 < ½\mathbf{P} < 0.05)$ **becomes** $0.05 < \mathbf{P} < 0.10$.
2. **Use a computer or calculator to find** $½\mathbf{P} = 0.0280$**; therefore,** $\mathbf{P} = 0.0560$.



b. Determine whether the $p - value$ is smaller than α.

The $p - value$ is not smaller than the level of significance, $\alpha(0.05)$.

EXAMPLE 19 – SOLUTION CONT'D …

Using the classical procedure:

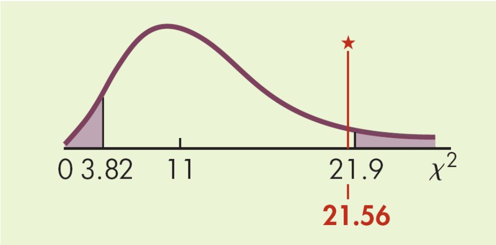
a. Determine the critical region and critical value(s).

The critical region is split into two equal parts because H_a expresses concern for values related to "different from."

The critical values are obtained from Table 8 are

$\chi^2(11, 0.0975) = 3.82$ and $\chi^2(11, 0.025) = 21.9$.

b. Determine whether or not the calculated test statistic is in the critical region.



a) Area to the Right

	0.995	0.99	0.975	0.95	0.90	0.75	0.50	0.25	0.10	0.05	0.025	0.01	0.005
b) Area to the Left (the Cumulative Area)							Median						
df	0.005	0.01	0.025	0.05	0.10	0.25	0.50	0.75	0.90	0.95	0.975	0.99	0.995
11	2.60	3.05	3.82	4.57	5.58	7.58	10.34	13.7	17.3	19.7	21.9	24.7	26.8
12	3.07	3.57	4.40	5.23	6.30	8.44	11.34	14.8	18.5	21.0	23.3	26.2	28.3
13	3.57	4.11	5.01	5.89	7.04	9.30	12.34	16.0	19.8	22.4	24.7	27.7	29.8
14	4.07	4.66	5.63	6.57	7.79	10.2	13.34	17.1	21.1	23.7	26.1	29.1	31.3
15	4.60	5.23	6.26	7.26	8.55	11.0	14.34	18.2	22.3	25.0	27.5	30.6	32.8

EXAMPLE 19 – SOLUTION CONT'D …

Step 5 The Results:

a. State the decision about H_o: Fail to reject H_o.

b. State the conclusion about H_a.

There is not sufficient evidence at the 0.05 significance level to conclude that the shelf life of this chemical has a standard deviation different from 10 days. Therefore, Fast Photo should purchase the chemical accordingly.

QUESTIONS?

- ANY QUESTION?