# MATH 1700

Instructor: Mehdi Maadooliat

## Chapter 4

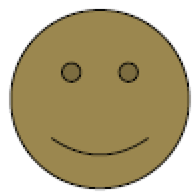
Department of Mathematical and Statistical Sciences

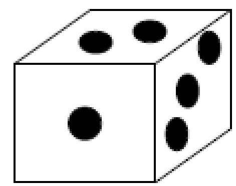
# CHAPTER 4

- **Probability**
  - **Experiment, Outcome, Sample space and Event**
- **Different type of Probability**
  - **Subjective, Empirical (experimental) and Theoretical**
- **Tree Diagram**
- **Properties of Probability**
- **Law of large numbers**
- **Odds**
- **Conditional probability**
- **Rules of probability**
  - **Complement Rule**
  - **General Addition Rule**
  - **General Multiplication Rule**
- **Mutually exclusive events**
  - **Special Addition Rule**
- **Independent and dependent events**
  - **Special Multiplication Rule**

# DEFINITIONS

- Let's start with some definitions.
- An **experiment** is a process by which a measurement is taken or observations is made.
  - i.e. *flip coin* or *roll die*



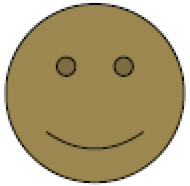


- An **outcome** is the result of an experiment.
  - i.e. *Heads* in flipping the coin, or *3* in rolling a die
  - Coin: $O_1 = H, O_2 = T$
  - Die: $O_1 = 1, O_2 = 2, O_3 = 3, O_4 = 4, O_5 = 5, O_6 = 6$
- **Sample space** is a listing of possible outcomes.
  - Coin: $S = \{H, T\}$
  - Die: $S = \{1, 2, 3, 4, 5, 6\}$

# DEFINITIONS

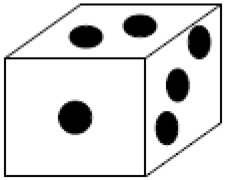
- An **event** $A$ is an outcome or a combination of outcomes.

- ***Flipping the coin***



  - $A$ = ***observing Head*** $= \{H\}$
  - **The probability of an event $A$ is written $P(A)$.**
  - **i.e.** $P(A) = P(\text{observing Head when tossing a coin})$
  - $= P(\{H\}) = 1/2$

- **Rolling a die**



  - $B$ $=$even number when rolling a die$= \{2,4,6\}$
  - **The probability of an event $B$ is written $P(B)$.**
  - **i.e.** $P(B) = P(\text{even number when rolling a die})$
  - $= P(\{2,4,6\}) = 3/6$

# DEFINITIONS

- **Probability of an event** The relative frequency with which that event can be expected to occur. It can be obtained in three different ways
  1. Theoretically,
  2. Empirically,
  3. Subjectively.

- A **subjective probability** generally results from personal judgment. Your local weather forecaster often assigns a probability to the event "precipitation."
  - For example, "There is a $20\%$ chance of rain today," or "There is a $70\%$ chance of snow tomorrow."

- **Empirical (experimental) probability**: This probability is the **observed relative frequency** with which an event occurs.

# DEFINITIONS

- **Empirical (Observed) probability: The value assigned to the probability of event $A$ as a result of experimentation can be found by means of the formula**

**Empirical (Observed) Probability $P'(A)$**

In words: $\text{empirical probability of A} = \dfrac{\text{number of times A occurred}}{\text{number of trials}}$

In algebra: $$P'(A) = \frac{n(A)}{n} \quad (4.1)$$

- **The " $'$ " in $P'(A)$ means an empirical probability.**

- **Example: Use computer simulation to flip a coin n (1000) times.**
  - Probability Applet

# DEFINITIONS

- The **theoretical** method for obtaining the probability of an event uses a *sample space*.

- When this method is used, the sample space must contain **equally likely** sample points. For example, the sample space for the rolling of one die is $S = \{1, 2, 3, 4, 5, 6\}$.

- Each **outcome** (i.e., number) is equally likely.

- An **event** is a subset of the sample space (denoted by a capital letter other than $S$; $A$ is commonly used for the first event).

# DEFINITIONS

- **Therefore, the *probability of an event* $A$, $P(A)$, is the ratio of the number of points that satisfy the definition of event $A$, $n(A)$, to the number of sample points in the entire sample space, $n(S)$.**

**Theoretical (Expected) Probability $P(A)$**

In words:

$$\textit{theoretical probability of A} = \frac{\textit{number of times A occurs in sample space}}{\textit{number of elements in sample space}}$$

In algebra: $P(A) = \dfrac{n(A)}{n(S)}$, when the elements of S are equally likely **(4.2)**

# DEFINITIONS

- **Notes**
- When the value assigned to the probability of an event results from a theoretical source, we will identify the probability of the event with the symbol $P(\cdot)$.

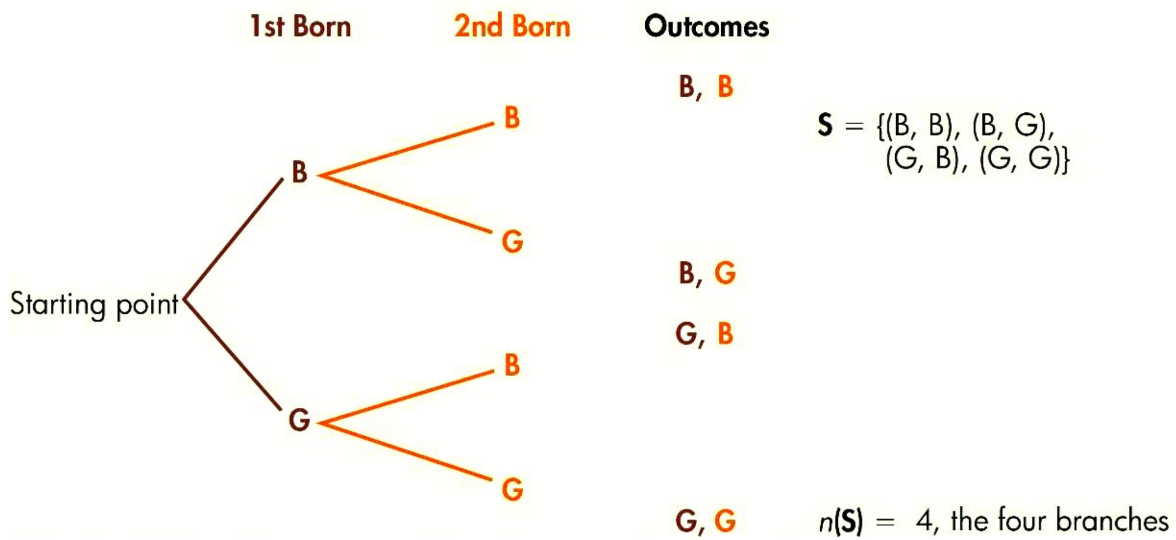
- The prime symbol is *not used* with theoretical probabilities; it is used only for empirical probabilities.

- When a probability experiment can be thought of as a sequence of events, a **tree diagram** often is a very helpful way to picture the sample space.

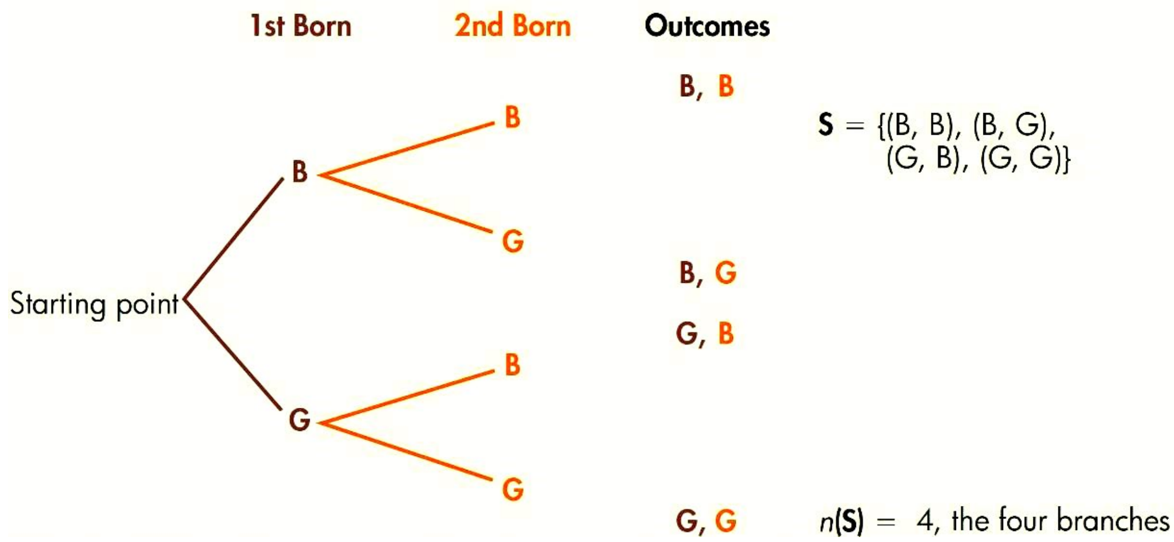
# EXAMPLE 4 – USING TREE DIAGRAMS

- A family with **two** children is to be selected at random, and we want to find the probability that the family selected has **one child of each gender**.

- We will use a tree diagram to show the possible arrangements of gender



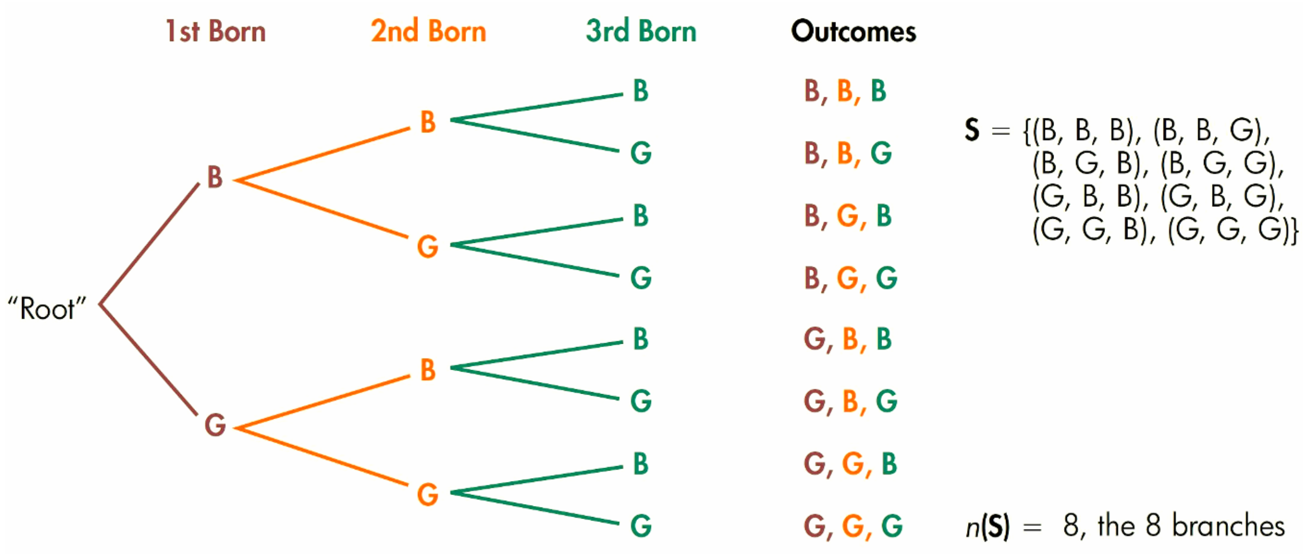
# EXAMPLE 4 – USING TREE DIAGRAMS




- **The two middle branches, $(B, G)$ and $(G, B)$, represent the event of interest, so $n(A) = n(\text{one of each}) = 2$, whereas $n(S) = 4$ because there are a total of four branches.**
- **Thus,**
  - $P$ (one of each gender in family of two children) $= 2/4$
  - **Or we write:** $P(A) = 0.5$

# EXAMPLE 4 – USING TREE DIAGRAMS

- **Now let's consider selecting a family of three children and finding the probability of "at least one boy" in that family.**

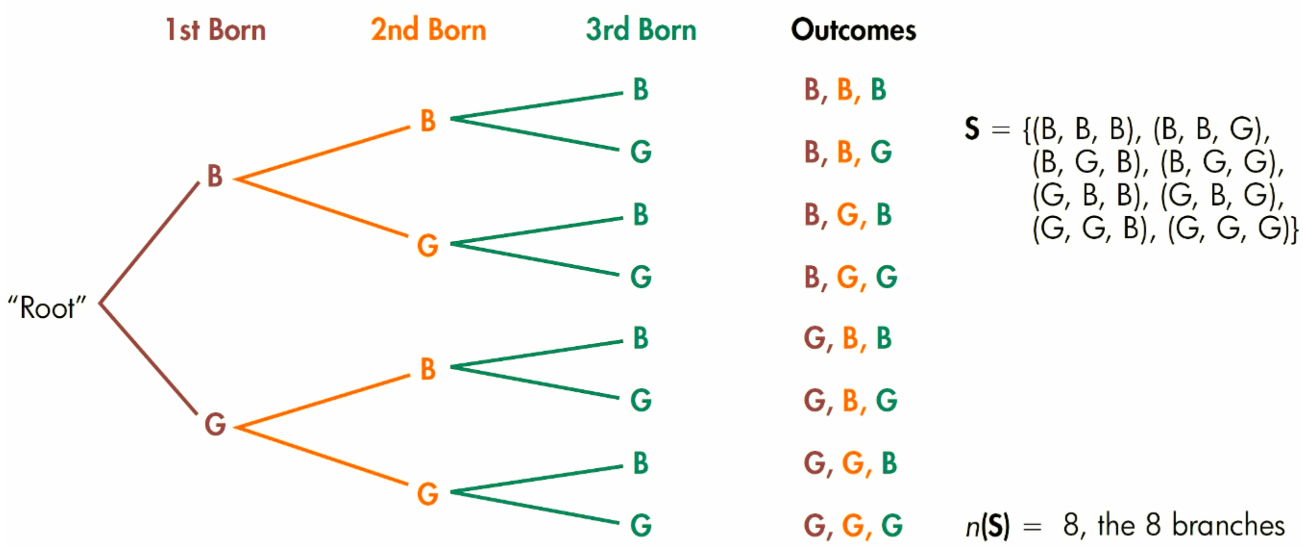



- $P\,(\text{at least one boy in a family of three children}) = \frac{7}{8}$

$$= 0.875$$

# EXAMPLE 4 – USING TREE DIAGRAMS

- **Let's consider one other question before we leave this example. What is the probability that the third child in this family of three children is a girl?**




- **The question is actually an easy one; the answer is $0.5$, because we have assumed equal likelihood of either gender.**

# NOTES

- The **empirical** approach may be off in the short run.
  - Suppose you get on a streak and out of $10$ flips all $10$ are heads?
  - By the empirical method we would say that $P'(H) = 1$.

- In the ***empirical*** method you actually have to perform the experiment of flipping the coin. But in the ***theoretical*** method you do NOT have to perform the experiment of flipping the coin.

- If each of the events are equally likely, then the **theoretical** approach is correct from the start.

- If the events are NOT equally likely, then the **theoretical** method is not correct and we should use a different approach.

# PROPERTIES OF PROBABILITY

- **Property 1:**
- **Whether the probability is *empirical*, *theoretical*, or *subjective*, the following properties must hold.**

**Property 1**

In words: "A probability is always a numerical value between zero and one."

In algebra: $0 \leq \text{each } P(A) \leq 1$ or $0 \leq \text{each } P'(A) \leq 1$

- **Notes about Property 1:**
  1. **The probability is 0 if the event cannot occur.**
  2. **The probability is 1 if the event occurs every time.**
  3. **Otherwise, the probability is a fractional number between 0 and 1.**

# PROPERTIES OF PROBABILITY

- **Property 2:**
- **Whether the probability is *empirical*, *theoretical*, or *subjective*, the following properties must hold.**

**Property 2**

In words: "The sum of the probabilities for all outcomes of an experiment is equal to exactly one."

In algebra: $\sum_{\text{all outcomes}} P(A) = 1$ or $\sum_{\text{all outcomes}} P'(A) = 1$

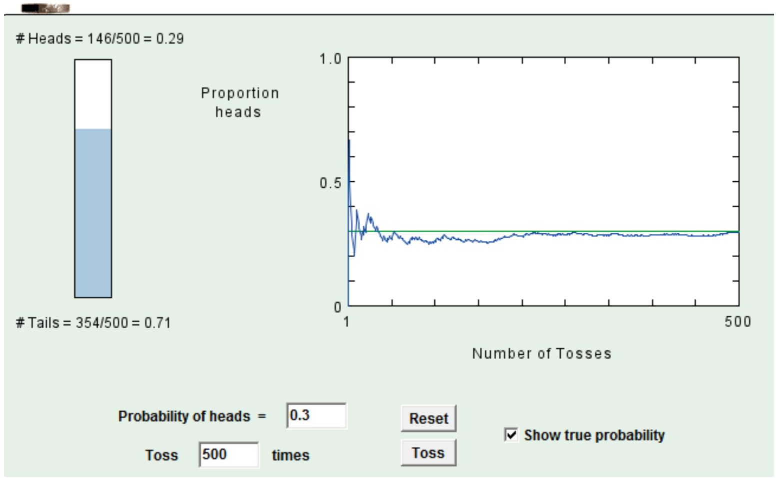
- **Note about Property 2: The list of "all outcomes" must be a non-overlapping set of events that includes all the possibilities (all-inclusive).**

# HOW ARE EMPIRICAL AND THEORETICAL PROBABILITIES RELATED?

- **Law of large numbers** As the number of times an ***experiment*** is repeated increases, the **ratio of the number of successful occurrences to the number of trials** will tend to **approach** the ***theoretical probability*** of the outcome for an individual trial.

- **The law of large numbers is telling us that the larger the number of experimental trials, $n$, the closer the empirical probability, $P'(A)$, to the theoretical probability, $P(A)$.**

- **Probability Applet**



# PROBABILITIES AS ODDS

- **Odds** are a way of expressing probabilities by expressing the number of ways an event can happen compared to the number of ways it can't happen.

- The statement "It is four times more likely to rain tomorrow (R) than not rain (NR)" is a probability statement that can be expressed as odds:
  - "The odds are $4$ to $1$ in favor of rain tomorrow" (also written $4{:}\,1$).

- Therefore, the probability of rain tomorrow is $\frac{4}{4+1}$, or $= 0.8$.

- The odds against rain tomorrow are $1$ to $4$ (or $1{:}\,4$), and the probability that there will be no rain tomorrow is $\frac{1}{4+1}$, or $= 0.2$.

# PROBABILITIES AS ODDS

- **The relationship between odds and probability is shown here.**

- **If the odds in favor of an event $A$ are $a$ to $b$ (or $a\!:b$), then**

1. **The odds against event $A$ are $b$ to $a$ (or $b\!:a$).**

2. **The probability of event $A$ is $P(A) = \frac{a}{a+b}$**

3. **The probability that event A will not occur is**

$$P\ (\text{not } A) = \frac{b}{a+b}.$$

# CONDITIONAL PROBABILITY OF EVENTS

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- **Conditional probability an event will occur** A conditional probability is the relative frequency with which an event can be expected to occur under the condition that additional, preexisting information is known about some other event.

- $P(A|B)$ is used to symbolize the probability of event $A$ occurring under the condition that event $B$ is known to already exist. But How to express the conditional probability:
  - 1st: The "*probability of* $A$, *given* $B$"
  - 2nd: The "*probability of* $A$, *knowing* $B$"
  - 3rd: The "*probability of* $A$ *happening, knowing* $B$ *has already occurred*"

# EXAMPLE 10 – FINDING PROBABILITIES FROM A TABLE OF COUNT DATA

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- **From a nationwide exit poll of $1000$ voters in $25$ precincts across the country during the $2008$ presidential election, we have the following:**

| Education | Number for Obama | Number for McCain | Number for Others | Number of Voters |
|---|---|---|---|---|
| No high school | 19 | 20 | 1 | 40 |
| HS graduate | 114 | 103 | 3 | 220 |
| Some college | 172 | 147 | 1 | 320 |
| College grad | 135 | 119 | 6 | 260 |
| Postgraduate | 70 | 88 | 2 | 160 |
| | 510 | 477 | 13 | 1000 |

1. **What is the probability that the person selected voted for McCain?**
   - **Answer: 477/1000 = 0.477=0.48.**
   - **Expressed in equation form:**

$$P(\text{McCain}) = \frac{477}{1000} = \underline{\mathbf{0.48}}$$

# EXAMPLE 10 – FINDING PROBABILITIES FROM A TABLE OF COUNT DATA

| Education | Number for Obama | Number for McCain | Number for Others | Number of Voters |
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2. **What is the probability that the person selected voted for McCain, knowing that the voter is a high school graduate?**
   - **Answer: 103/220 = 0.46818 = 0.47.**
   - **Expressed in equation form:**
     $P(\text{McCain} \mid \text{HS graduate})$ **=103/220 = 0.46818 = 0.47**

3. **What is the probability that the person selected voted for Obama, given that the voter has some college education?**
   - **Answer: 172/320 = 0.5375 = 0.54.**
   - **Expressed in equation form:**
     $P(\text{Obama} \mid \text{some college})$ **= 172/320 = 0.5375 = 0.54**

# EXAMPLE 10 – FINDING PROBABILITIES FROM A TABLE OF COUNT DATA

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| Education | Number for Obama | Number for McCain | Number for Others | Number of Voters |
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| College grad | 135 | 119 | 6 | 260 |
| Postgraduate | 70 | 88 | 2 | 160 |
| | 510 | 477 | 13 | 1000 |

4. **Knowing the selected person voted for McCain, what is the probability that the voter has a postgraduate education?**
   - **Answer: 88/477 = 0.1844 = 0.18.**
   - **Expressed in equation form:**
     $P(\text{postgraduate} \mid \text{McCain})$ **= 88/477 = 0.1844 = <u>0.18</u>**

5. **Given that the selected person voted for Obama, what is the probability that the voter does not have a high school education?**
   - **Answer: 19/510 = 0.0372 = 0.04.**
   - **Expressed in equation form:**
     $P(\text{no high school} \mid \text{Obama})$ **= 19/510 = 0.0372 = <u>0.04</u>**

# EXAMPLE – DECK OF CARD

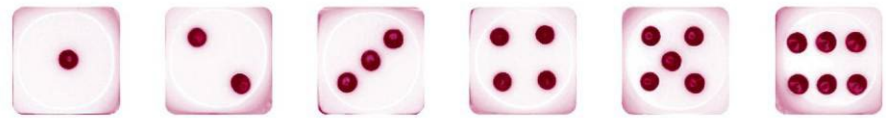
- **Example:** Draw card from deck.
- Let $A$ = red card.
- Let $B$ = heart.
- What is $P(A)$ and $P(B)$?
- $P(A) = \frac{n(A)}{n(S)} = \frac{26}{52} = \frac{1}{2}$
- $P(B) = \frac{13}{52} = \frac{1}{4}$

- What is $P(A|B)$?
- $P(A|B) = \frac{13}{13} = 1$

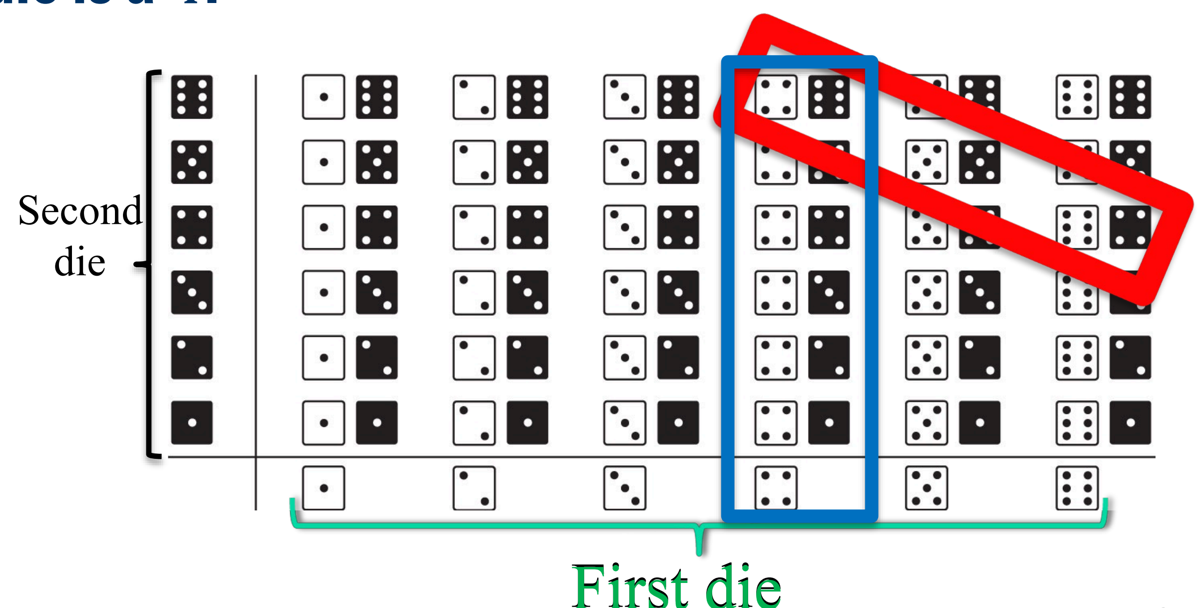
- $P(A) = 1/2$ vs. $P(A|B) = 1$

# EXAMPLE – ROLLING A PAIR OF DICE

- **Example: Roll two dice.**



- **Let $A$ be that $10$ is the sum of the two dice.**
- $P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$
- **Let $B$ that the first die is a $4$.**
- $P(B) = \frac{1}{6}$
- **What is $P(A|B)$?**
- $P(A|B) = \frac{1}{6}$



# RULES OF PROBABILITY

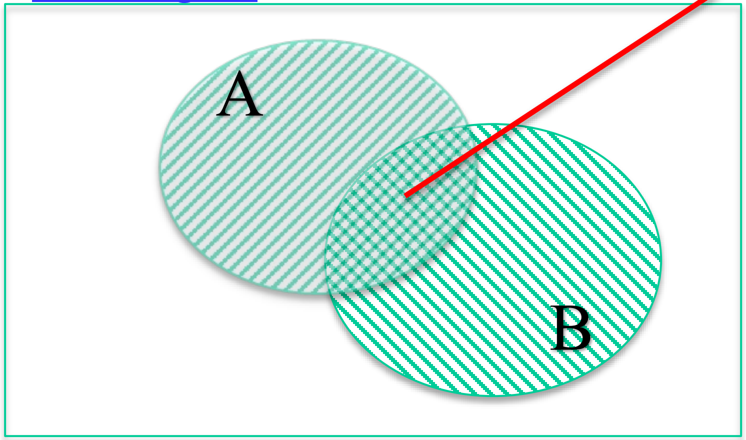
- **Complementary events:** The complement of an event $A$ is the set of all sample points in the sample space that do not belong to event $A$.
  - The complement of event $A$ is denoted by $\bar{A}$ (read "$A$ complement").
- A few examples of complementary events are
  - the complement of the event "success" is "failure,"
  - the complement of "selected voter is Republican" is "selected voter is not Republican," and
  - the complement of "no heads" on $10$ tosses of a coin is "at least one head."
- Using Property 2:
  - $P(A) + P(\bar{A}) = 1.0$ for any event $A$
- **Complement Rule:**
  - In words: *probability of A complement = one – probability of A*
  - In algebra: $P\,(\bar{A}) = 1 - P\,(A)$

# RULES OF PROBABILITY

- **Finding the Probability of "$A$ or $B$":**
  - **Let $A$ and $B$ be two events defined in a sample space, $S$.**
- **General Addition Rule**
  - **In words:**
    ***probability of A or B =probability of A + probability of B – probability of A and B***
  - **In algebra:** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

**Double Count, so we subtract one off**

- Venn Diagram




  - **Unioin:** $A$ or $B$
  - **Intersection:** $A$ and $B$

# EXAMPLE 12 – UNDERSTANDING THE ADDITION RULE

- **A statewide poll of 800 registered voters in 25 precincts from across New York State was taken. Each voter was identified as being registered as Republican, Democrat, or other and then asked, "Are you in favor of or against the current budget proposal awaiting the governor's signature?" The resulting tallies are shown here.**

| | Number in Favor | Number Against | Number of Voters |
|---|---|---|---|
| Republican | 136 | 88 | 224 |
| Democrat | 314 | 212 | 526 |
| Other | 14 | 36 | 50 |
| Totals | 464 | 336 | 800 |

  - **Suppose one voter is to be selected at random from the 800 voters summarized in the preceding table.**
  - **Let's consider the two events "The voter selected is in favor" and "The voter is a Republican." Find the four probabilities:**
  - ***P*(in favor), *P*(Republican), *P*(in favor or Republican), and *P*(in favor and Republican).**

# EXAMPLE 12 – SOLUTION

| | Number in Favor | Number Against | Number of Voters |
|---|---|---|---|
| Republican | 136 | 88 | 224 |
| Democrat | 314 | 212 | 526 |
| Other | 14 | 36 | 50 |
| Totals | 464 | 336 | 800 |

- Probability the voter selected is "in favor" =
- $P$(in favor)
- = 464/800 = <u>0.58.</u>
- $P$(Republican)
- = 224/800 = <u>0.28.</u>
- $P$(in favor or Republican)
- = (136 + 314 + 14 + 88)/800
- = 552/800 = <u>0.69.</u>
- $P$(in favor and Republican)
- = 136/800 = <u>0.17</u>.

- **Now let's use the results to check the truth of the addition rule.**

# EXAMPLE 12 – SOLUTION

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| | Number in Favor | Number Against | Number of Voters |
|---|---|---|---|
| Republican | 136 | 88 | 224 |
| Democrat | 314 | 212 | 526 |
| Other | 14 | 36 | 50 |
| Totals | 464 | 336 | 800 |

- **$P$(in favor)=0.58,** **$P$(Republican)=0.28,**
- **$P$(in favor or Republican)=0.69,** **$P$(in favor and Republican)=0.17**

- **General Addition Rule:** $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
  - $P$ (in favor or Republican)
  - $= P$ (in favor) $+$
    $P$ (Republican) $-$ $P$ (in favor and Republican)
  - **= 0.58 + 0.28 – 0.17 = 0.69**

- **Notes about finding the preceding probabilities:**
  - **The connective "or" means "one or the other or both"; thus, "in favor or Republican" means all voters who satisfy either event.**
  - **The connective "and" means "both" or "in common"; thus, "in favor and Republican" means all voters who satisfy both events.**

# RULES OF PROBABILITY

- **Finding the Probability of "$A$ and $B$":**
  - **Let $A$ and $B$ be two events defined in a sample space, $S$.**

- **General Multiplication Rule**
  - **In words:**
  - probability of A and B =
  - probability of A × probability of B, knowing A
  - **In algebra:** $P(A \text{ and } B) = P(A).P(B|A)$

- **Alternatively:**
  - **In words:**
  - probability of A and B =
  - probability of B × probability of A, knowing B
  - **In algebra:** $P(A \text{ and } B) = P(B).P(A|B)$

# EXAMPLE 13 – UNDERSTANDING THE MULTIPLICATION RULE

| | Number in Favor | Number Against | Number of Voters |
|---|---|---|---|
| Republican | 136 | 88 | 224 |
| Democrat | 314 | 212 | 526 |
| Other | 14 | 36 | 50 |
| Totals | 464 | 336 | 800 |

- $P$ (in favor) $= \frac{464}{800} = 0.58$, $P$ (in favor and Republican) $= \frac{136}{800} = 0.17$
- **Probability the voter selected is "Republican, given in favor" =**
- $P$ (Republican | in favor)
- $= \frac{136}{464}$
- $= 0.29$

- **Now let's use the previous probabilities to demonstrate the truth of the multiplication rule.**
- **General Multiplication Rule:** $P(A \text{ and } B) = P(A).P(B|A)$
  - $P$ (in favor and Republican) = $P$(in favor) × $P$(Republican | in favor)
  - $= \frac{464}{800} \cdot \frac{136}{464}$
  - $= \frac{136}{800} = 0.17$

# NOTES ON FINDING THE PROBABILITY OF "A AND B"

| | Number in Favor | Number Against | Number of Voters |
|---|---|---|---|
| Republican | 136 | 88 | 224 |
| Democrat | 314 | 212 | 526 |
| Other | 14 | 36 | 50 |
| Totals | 464 | 336 | 800 |

- The conditional "given" means there is a restriction; thus, "Republican, given in favor" means we start with only those voters who are "in favor." In this case, this means we are looking only at 464 voters when determining this probability.

- You typically do not have the option of finding $P\ (A \text{ and } B)$ two ways, as we did here. When you are asked to find $P(A \text{ and } B)$, you will often be given $P(A)$ and $P(B)$.

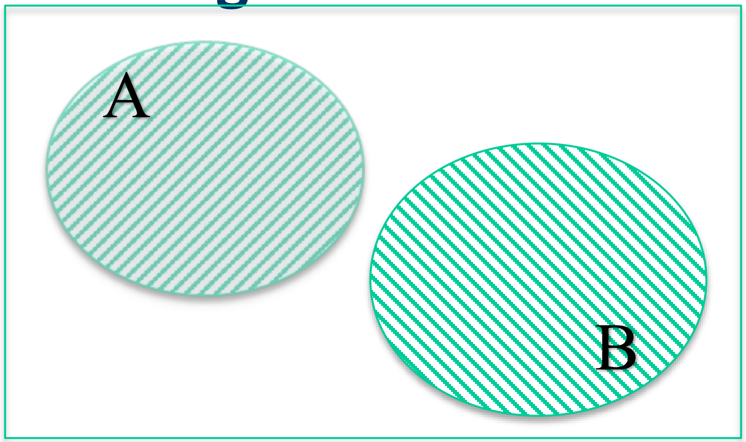
- However, you will not always get the correct answer by just multiplying those two probabilities together.

- You will need a third piece of information: the conditional probability of one of the two events or information that will allow you to find it.

# MUTUALLY EXCLUSIVE EVENTS

- **Mutually exclusive events** **Nonempty events defined on the same sample space with each event excluding the occurrence of the other. In other words, they are events that share no common elements.**

- **Venn Diagram**




- **In algebra:** $P(A \text{ and } B) = 0$
- **In words: both, $A$ and $B$, cannot happen together.**

# EXAMPLE 17 – MUTUALLY EXCLUSIVE CARD EVENTS

- **Consider a regular deck of playing cards and the two events $A$="card drawn is a queen" and $B$="card drawn is an ace." The deck is to be shuffled and one card randomly drawn.**

  - **Probability of the event "card drawn is a queen"**
  - $P(A) = \frac{n(A)}{n(S)} = \frac{4}{52}$
  - **Probability of the event "card drawn is an ace"**
  - $P(B) = \frac{n(A)}{n(S)} = \frac{4}{52}$
  - **Notice that there is no card that is both a queen and an ace. Therefore, these two events, "card drawn is a queen" and "card drawn is an ace," are mutually exclusive events.**
  - **In equation form:** $P\ (A \text{ and } B)\ =\ 0$.

# SPECIAL ADDITION RULE

- **If we know two events are mutually exclusive, then by applying $P(A \text{ and } B) = 0$ to the addition rule for probabilities, it follows that**
  - $P\,(A \text{ or } B) \;=\; P\,(A) \;+\; P\,(B) - P\,(A \text{ and } B)$ **becomes**
  - $P\,(A \text{ or } B) \;=\; P\,(A) \;+\; P\,(B).$
- **This formula can be expanded to consider more than two mutually exclusive events:**
  - $P\,(A \text{ or } B \text{ or } C \text{ or } \cdots \text{ or } E) \;=\; P\,(A) \;+\; P\,(B) \;+\; P\,(C) \;+ \cdots +\; P\,(E)$
- **Reconsider Example 17,**
  - **Two events $A$="card drawn is a queen" and $B$="card drawn is an ace."**
  - **That one card cannot be both a queen and an ace at the same time, thereby making these two events mutually exclusive. The special addition rule therefore applies to the situation of**

$$P(\text{queen or ace}) = P(\text{queen}) + P(\text{ace}) = \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

# INDEPENDENT AND DEPENDENT EVENTS

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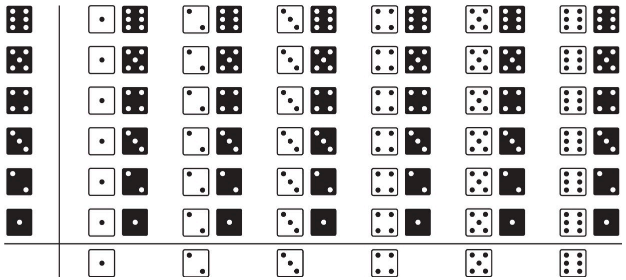
- **Independent events** **Two events are *independent* if the occurrence (or nonoccurrence) of one gives us no information about the likeliness of occurrence of the other.**
  - **In other words, if the probability of $A$ remains unchanged after we know that $B$ has happened (or has not happened), the events are independent.**
- **In algebra:**
  - $P\,(A) \;=\; P\,(A\,|\,B) \;=\; P\,(A\,|\,not\;B)$
- **Two Events $A$ and $B$ are independent if**
  - $P\,(A) \;=\; P\,(A\,|\,B)$
  - $P\,(B) \;=\; P\,(B\,|\,A)$
  - $P\,(A \text{ and } B) \;=\; P\,(A).\,P\,(B)$
- **Dependent events:** **Events that are not independent. That is, the occurrence of one event does have an effect on the probability for occurrence of the other event.**
  - **In algebra:** $P\,(A) \;\neq P\,(A\,|\,B)$

# EXAMPLES OF INDEPENDENT EVENTS

- **Roll two dice:**
  - $P(\text{observing two } 6's) = ?$
  - **Two dice are independent, therefore**
  - $P(\text{observing two } 6's) = P(\{6\}) \cdot P(\{6\})$
  - $P(\text{observing two } 6's) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$



- **Toss a coin five times:**
  - $P(\text{observing all heads}) = ?$
  - **coins are independent of each other, therefore**
  - $P(\{\mathrm{HHHHH}\}) = P(\{\mathrm{H}\}) \cdot P(\{\mathrm{H}\}) \cdot P(\{\mathrm{H}\}) \cdot P(\{\mathrm{H}\}) \cdot P(\{\mathrm{H}\})$
  - $P(\{\mathrm{HHHHH}\}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$
  - $P(\{\mathrm{HHHHH}\}) = \left(\frac{1}{2}\right)^5$
  - $P(\{\mathrm{HHHHH}\}) = \frac{1}{32}$

# SPECIAL MULTIPLICATION RULE

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- **The multiplication rule simplifies when the events involved are independent. Independence means $P(B \mid A) = P(B)$.**
  - **Therefore,** $P(A \text{ and } B) = P(A) \cdot P(B|A)$ **becomes**
  - $P(A \text{ and } B) = P(A) \cdot P(B)$

- **This formula can be expanded to consider more than two independent events:**
  - $P\,(A \text{ and } B \text{ and } C \text{ and } \cdots \text{ and } E) = P(A) \cdot P(B) \cdot P(C) \cdot \cdots \cdot P(E)$

- **Mutual Exclusiveness and Independence**
  - **Mutual Exclusivene:**
  - $P(A \text{ and } B) = 0$

  - **Independence:**
  - **Given** $P(B \mid A) = P(B)$**, therefore:**
  - $P(A \text{ and } B) = P(A) \cdot P(B)$

# QUESTIONS?

- **ANY QUESTION?**