MATH 1700

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Chapter 8B

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CHAPTER 8B

- Statistical hypothesis test
  - Null Hypothesis: H_0
  - Alternative Hypotheses: H_a
- Types of Error
  - Type I error or Level of Significance (α)
  - Type II Probability (β)
- Test Statistic
- The Assumptions for Hypothesis Test
- Hypothesis Test Approaches
  - P-value Approach
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- The Probability-value Hypothesis Test: A 5-step Procedure
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  - Step 2 The Hypothesis Test Criteria
  - Step 3 The Sample Evidence
  - Step 4 The Probability Distribution
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HYPOTHESIS TEST OF MEAN μ (σ KNOWN):

- We make decisions every day in our lives.

- How to decide?

- **Statistical hypothesis test**: A process by which a decision is made between two opposing hypotheses.

- There are two type of hypothesis:

- We call the proposed hypothesis the **null hypothesis** and the opposing hypothesis the **alternative hypothesis**.

HYPOTHESIS TEST OF MEAN μ (σ KNOWN):

- **Null Hypothesis**, H_0: The hypothesis that we will test. Generally a statement that a parameter has a specific value.

- **Alternative Hypothesis**, H_a: A statement about the same parameter that is used in the null hypothesis. ... parameter has a value different ... from the value in the null hypothesis.

- Example: A person comes into court charged with a crime. A jury must decide whether the person is innocent (null hypothesis) or guilty (alternative hypothesis). Even though the person is charged with the crime, at the beginning of the trial (and until the jury declares otherwise) the accused is assumed to be innocent.
  - H_0: The person is innocent
  - H_a: The person is guilty

HYPOTHESIS TEST OF MEAN μ (σ KNOWN):

- H_0: **Null Hypothesis**
  H_a: **Alternative Hypotheses (Research Hypothesis)**
- **Based on the evidence from the data, either we reject H_0 in favor of H_a or we fail to reject (accept) H_0.**

Decision	H_0 is True	H_a is true
Reject H_0	Type-I Error	Correct Decision
Accept H_0	Correct Decision	Type-II Error

- **Back to Example 1:**

Decision	Truth is Person Innocent	Truth is Person Guilty
Jury Decides Person Guilty	Type-I Error	Correct Decision
Jury Decides Person Innocent	Correct Decision	Type-II Error

HYPOTHESIS TEST OF MEAN μ (σ KNOWN):

- **Level of Significance (α): The probability of committing a Type I error**
  - **Sometimes α is called the false positive rate.**

	H_0 True	H_0 False
Fail to Reject H_0	Type A Correct Decision $(1-\alpha)$	Type II Error (β)
Reject H_0	Type I Error (α)	Type B Correct Decision $(1-\beta)$

- **Type II Probability (β): The probability of committing a Type II error.**
  - **Sometimes β is called the false negative rate.**

HYPOTHESIS TEST OF MEAN μ (σ KNOWN):

- We need to determine a measure that will quantify what we should believe.

- **Test Statistic**: A random variable whose value is calculated from the sample data and is used in making the decision "**reject** H_0: or "**fail to reject** H_0."

- Court example: The test statistics is:
  - Jury's decision

- In general, if we are testing about population parameter μ:
- We take a sample and it is reasonable to expect that test statistics is somehow related to
  - the sample mean: $\bar{x}$

HYPOTHESIS TEST OF MEAN μ (σ KNOWN):

- **The hypothesis test is a well-organized, step-by-step procedure used to make a decision.**

- **Should I do *A* or should I do *B* (not *A*)?**

- **The assumption for hypothesis tests about mean μ using a known σ:**
- **The sampling distribution of $\bar{x}$ has a normal distribution. Therefore, we can satisfy the required assumption by either**
  1. **knowing that the sampled population is normally distributed or**
  2. **using a random sample that contains a sufficiently large amount of data.**

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- Two different formats are commonly used for hypothesis testing.
  - P-value approach
  - Classical approach (Next Lecture)

- The **probability-value** approach, or simply **p-value** approach, is the hypothesis test process that has gained popularity in recent years, largely as a result of the convenience and the "number-crunching" ability of the computer.

- This approach is organized as a five-step procedure.

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

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- **The Probability-value Hypothesis Test: A 5-step Procedure**

- **Step 1 The Set-Up:**
  a. Describe the population parameter of interest.
  b. State the null hypothesis (H_0) and the alternative hypotheses (H_a).
- **Step 2 The Hypothesis Test Criteria:**
  a. Check the assumptions.
  b. Identify the probability distribution and the test statistic to be used.
  c. Determine the level of significance, α.
- **Step 3 The Sample Evidence:**
  a. Collect the sample information.
  b. Calculate the value of the test statistic.
- **Step 4 The Probability Distribution:**
  a. Calculate the $\mathrm{p-value}$ for the test statistic.
  b. Determine whether or not $\mathrm{p-value}$ is smaller than α:
- **Step 5 The Results:**
  a. State the decision about H_0
  b. State the decision about H_a

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **Step 1 The Set-Up:**

a. **Describe the population parameter of interest.**

- Population mean (μ),
- Population proportion (p), or
- Population standard deviation (σ)

b. **State the null hypothesis (H_0) and the alternative hypotheses (H_a).**

Null Hypothesis	Alternative Hypothesis
1. Greater than or equal to (≥)	Less than (<)
2. Less than or equal to (≤)	Greater than (>)
3. Equal to (=)	Not equal to (≠)

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **Examples:**
  - **A teacher claims her method of teaching will increase test scores more than 5 points on average. You randomly sample 35 students to receive her method of teaching and find their test scores. You plan to test her claim that the method of teaching she proposes is better.**

Notice the Null Hypothesis ALWAYS has equality associated with it.

Null Hypothesis: $H_0\colon\ \mu \le 5$

Alternative Hypothesis: $H_a\colon\ \mu > 5$

One-sided alternative hypothesis.

  - **Commercial aircraft manufacturer concern: "The mean shearing strength of all rivets should be at least 925 lb." Each time they buy rivets, it is concerned that the mean strength might be less than 925 lb specification.**

Null Hypothesis: $H_0\colon\ \mu \ge 925 \text{ lb}$

Alternative Hypothesis: $H_a\colon\ \mu < 925 \text{ lb}$

  - **The milk price of a gallon of 2% milk is normally distributed with standard deviation of \$0.10. Last week the milk price was \$2.78. We want to determine if this week the price is different.**

We only wanted to see if the number of years had "changed." We are not looking for a direction of change.

Null Hypothesis: $\mathbf{H_0} : \mu = \$2.78$

Alternative Hypothesis: $\mathbf{H_a} : \mu \ne \$2.78$

Two-sided alternative hypothesis.

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

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- **Step 2 The Hypothesis Test Criteria:**

a. Check the assumptions.

- Assume we know σ from past experience.
- Assume that n is "large" so that by the CLT $\bar{x}$ is normally distributed.
  - ✓ $\mu_{\bar{x}} = \mu_0$
  - ✓ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

b. Identify the probability distribution and the test statistic to be used.

- The standard normal probability distribution is used. Because $\bar{x}$ is expected to have a normal distribution.
- Test Statistic for Mean:
  - $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$

c. Determine the level of significance, α.

- The type I error occurs when a true null hypothesis is rejected.

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **Step 3 The Sample Evidence:**

a. Collect the sample information.

- **Take a random sample from the population whose mean μ is being questioned.**
  - **Obtain** $x_1, x_2, \ldots, x_n$
  - **Obtain** $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$

b. Calculate the value of the test statistic.

- $z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **Step 3 (Example: the aircraft manufacturer):**

a. Collect the sample information.

- A random sample of 50 rivets is selected, each rivet is tested, and the sample mean shearing strength is calculated:
- $\bar{x} = 921.18$ **and** $n = 50.$

b. Calculate the value of the test statistic.

- The sample evidence ($\bar{x}$ and n found in Step 3a) is next converted into the **calculated value of the test statistic,** z^*
- **Remember:** (μ is 925 from H_o, and $\sigma = 18$ is a known quantity.)
  - H_0: $\mu \geq 925$ lb
  - H_a: $\mu < 925$ lb
- We have
  - $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}$:

$$z\bigstar = \frac{921.18 - 925.0}{18/\sqrt{50}} = \frac{-3.82}{2.5456}$$

$$= -1.50$$

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

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- **Step 4 The Probability Distribution:**

a. Calculate the $\mathrm{p-value}$ for the test statistic.

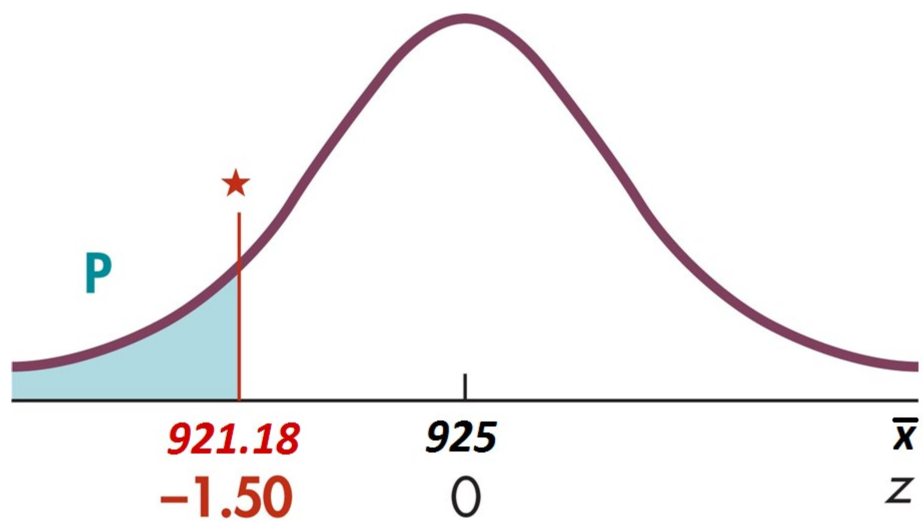
– **Probability value, or $\mathrm{p-value}$ The probability that the test statistic could be the value it is or a more extreme value (in the direction of the alternative hypothesis) when the null hypothesis is true.**

b. Determine whether or not $\mathrm{p-value}$ is smaller than α:

– **Remember:**
  - α stands for probability of type I error
  - Type I error: Probability of falsely rejecting H_0

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **Step 4 (Example: the aircraft manufacturer):**
- **Remember:**
  - **(μ is 925 from H_0, and $\sigma = 18$ is a known quantity.)**
  - ➢ H_0: $\mu \geq 925$ lb
  - ➢ H_a: $\mu < 925$ lb
  - ➢ $z^* = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = -1.50$
- $\text{p} - \text{value} = P(\bar{x} < 921.18)$
- $\text{p} - \text{value} = P(z < z^*)$
- $\text{p} - \text{value} = P(z < -1.50)$
- $= \mathbf{0.0668}$



- **The** $\text{p} - \text{value}$ **(0.0668) is not smaller than** α **(0.05).**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **Step 5 The Results:**

a. State the decision about H_0.

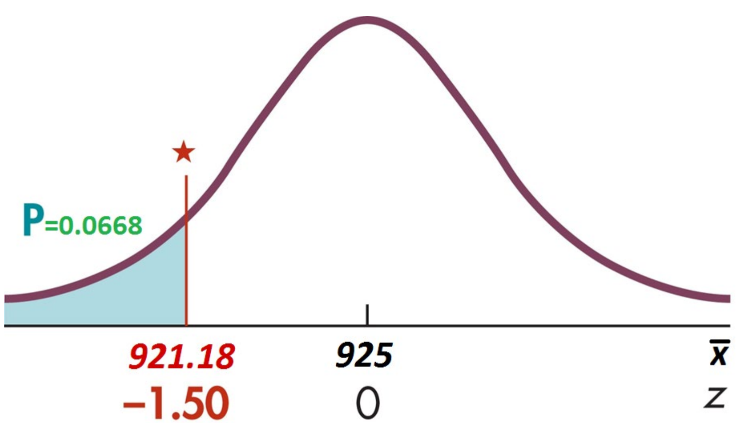
- **Is the $\mathrm{p-value}$ small enough to indicate that the sample evidence is highly unlikely in the event that the null hypothesis is true?**
- **In order to make the decision, we need to know the *decision rule*.**

- **Decision rule**

a. **If the $\mathrm{p-value}$ is *less than or equal to* the level of significance α, then the decision must be reject H_0.**

b. **If the $\mathrm{p-value}$ is *greater than* the level of significance α, then the decision must be fail to reject H_0.**

- **Back to the aircraft manufacturer Example:**
  - $\mathrm{p-value} = 0.0668$
  - $\alpha = 0.05.$
  - **Decision about H_0: Fail to reject H_0.**



HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **Step 5 The Results:**

b. State the decision about H_a.

- **There is not sufficient evidence at the 0.05 level of significance to show that the mean shearing strength of the rivets is less than 925.**

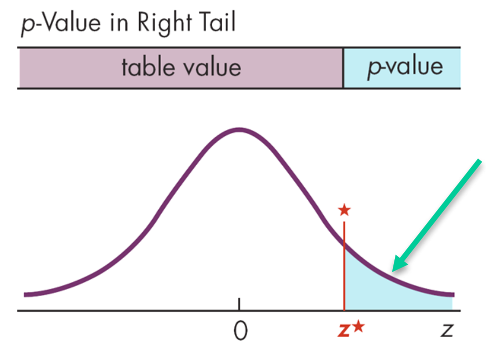
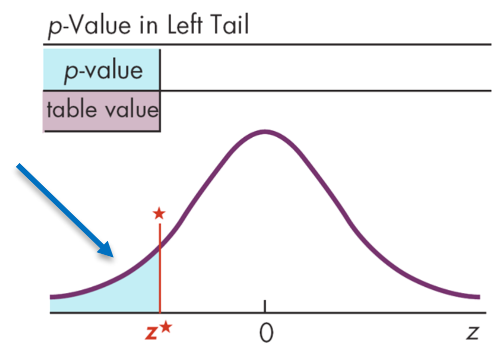
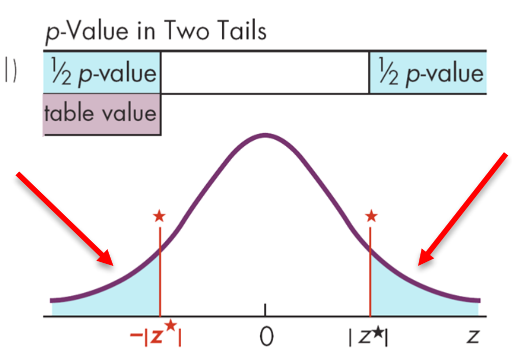
- **"We failed to convict" the null hypothesis.**

- **In other words, a sample mean as small as 921.18 is likely to occur (as defined by α) when the true population mean value is 925.0 and $\bar{x}$ is normally distributed.**

- **The resulting action by the manager would be to buy the rivets.**

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- **The outlines the procedure for all three cases of $\mathrm{p-value's}$:**

Case 1 H_a contains ">" "Right tail"	*p*-value is the *area to right of* $z\bigstar$ *p*-value $= P(z > z\bigstar)$	
Case 2 H_a contains "<" "Left tail"	*p*-value is the *area to left of* $z\bigstar$ *p*-value $= P(z < z\bigstar)$	
Case 3 H_a contains "$\neq$" "Two-tailed"	*p*-value is the *total area of both tails* *p*-value $= P(z < -\lvert z\bigstar\rvert) + P(z > \lvert z\bigstar\rvert)$ $z\bigstar$ may be in either tail, and since both areas are equal, find the probability of one tail and double it. Thus, *p*-value $= 2 \times P(z < -\lvert z\bigstar\rvert)$	

HYPOTHESIS TEST OF MEAN μ (σ KNOWN): A PROBABILITY-VALUE APPROACH

- The ***fundamental idea of the*** $\mathrm{p-value}$ **is to express the degree of belief in the null hypothesis:**
  - **When the** $\mathrm{p-value}$ **is minuscule (something like** 0.0003**), the null hypothesis would be rejected by everybody because the sample results are very unlikely for a true** H_0**.**
  - **When the** $\mathrm{p-value}$ **is fairly small (like** 0.012**), the evidence against** H_0 **is quite strong and** H_0 **will be rejected by many.**
  - **When the** $\mathrm{p-value}$ **gets large (like** 0.15 **or more), the data are not at all unlikely if the** H_0 **is true, and no one will reject** H_0**.**
- **The *advantages of the*** $\mathrm{p-value}$ ***approach* are as follows:**
  1) **A** $\mathrm{p-value}$ **can be reported and the user of the information can decide on the strength of the evidence as it applies to his or her own situation.**
  2) **Computers can do all the calculations and report the** $\mathrm{p-value}$**, thus eliminating the need for tables.**
- **The *disadvantage of the*** $\mathrm{p-value}$ ***approach* is the tendency for people to put off determining the level of significance.**

EXAMPLE 17 – TWO-TAILED HYPOTHESIS TEST WITH SAMPLE DATA

- **Let's use computer simulation, to draw a sample of 40 single-digit numbers.**
- **Remember:**
  - $\mu = \sum_{x=0}^{9} xP(x) = 4.5$ **and**
  - $\sigma = \sqrt{\sigma^2} = \sqrt{\sum_{x=0}^{9}[x^2P(x)] - \mu^2} = \sqrt{8.25} = 2.87$

x	$P(x)$
0	$1/10$
1	$1/10$
2	$1/10$
3	$1/10$
4	$1/10$
5	$1/10$
6	$1/10$
7	$1/10$
8	$1/10$
9	$1/10$

- **Use $\alpha = 0.10$ and test the hypothesis that**
- **"The mean of this distribution is different from 4.5."**

EXAMPLE 17 – *SOLUTION*

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- **Step 1 The Set-Up:**
  - **a.** **Describe the population parameter of interest.**
    - The population parameter of interest is the mean μ of the population of single-digit numbers with uniform distribution.
  - **b.** **State the null hypothesis (H_0) and the alternative hypothesis (H_a).**
    - ➢ H_0: $\mu = 4.5$ (mean is 4.5)
    - ➢ H_a: $\mu \neq 4.5$ (mean is not 4.5)

- **Step 2 The Hypothesis Test Criteria:**
  - **a.** **Check the assumptions.**
    - ✓ σ is known. Samples of size 40 should be large enough to satisfy the **CLT**.
  - **b.** **Identify the probability distribution and the test statistic to be used.**
    - ➢ We use the standard normal probability distribution, and the test statistic is $z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$; where $\sigma = 2.87$.
  - **c.** **Determine the level of significance, α.**
    - ➢ $\alpha = 0.10$ (given in the statement of the problem)

EXAMPLE 17 – *SOLUTION*

- **Step 3 The Sample Evidence:**

  a. **Collect the sample information.**

  - **This random sample was drawn using computer simulation:**

2	8	2	1	5	5	4	0	9	1	0	4	6	1
5	1	1	3	8	0	3	6	8	4	8	6	8	
9	5	0	1	4	1	2	1	7	1	7	9	3	

  - **From the sample $\bar{x} = 3.975$ and $n = 40$.**

  a. **Calculate the value of the test statistic.**

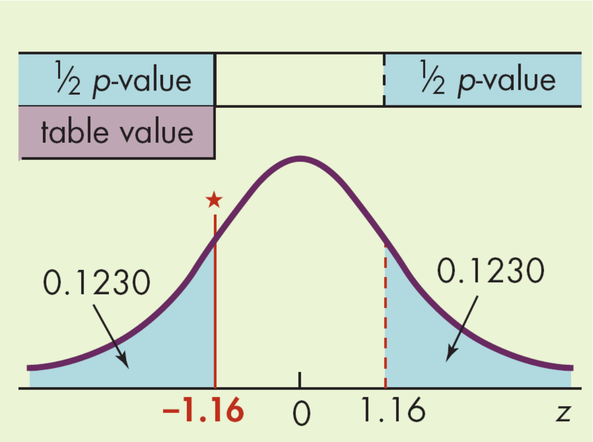
  - $z^* = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

$$z\star = \frac{3.975 - 4.50}{2.87/\sqrt{40}} = \frac{-0.525}{0.454}$$

  - $z^* = -1.156 = \mathbf{-1.16}$

EXAMPLE 17 – *SOLUTION*

- ## Step 4 The Probability Distribution:
  - **a. Calculate the *p*-value for the test statistic.**
    - ➢ **Because the alternative hypothesis indicates a two-tailed test, we must find the probability associated with both tails.**
    - ➢ **The** $p-\text{value}$ **is found by doubling the area of one tail.**
    - ➢ $z^* = -1.16$.
    - ➢ $The\ \text{p}-\text{value} \ = \ 2 \times P\ (z < -1.16)$
    - ➢ $\qquad\qquad\qquad = \ 2(0.1230)$
    - ➢ $\qquad\qquad\qquad = \ 0.2460$



  - **b. Determine whether or not the** $\text{p}-\text{value}$ **is smaller than** α**.**
    - ➢ **The *p*-value** (0.2460) **is greater than** $\alpha\,(0.10)$**.**

- ## Step 5 The Results:
  - **a. State the decision about** H_0**: Fail to reject** H_0**.**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
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−1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379

  - **b. State the conclusion about** H_a**:**
    - ➢ **The observed sample mean is not significantly different from** 4.5 **at the** 0.10 **level of significance.**

QUESTIONS?

- ANY QUESTION?