MATH 1700 **Exam II (Fall 2025)** **Section 101**

Time Limit: 70 minutes

Form A

Print Your Name:___SOLUTION____ **Seat #:______**

Notes:

1 - DO NOT OPEN THE EXAM UNTIL YOU ARE TOLD TO DO SO.

2 - GIVE ALL THE NECESSARY DETAILS TO GET FULL CREDITS.

3 - IF YOU USE CALCULATOR FOR A PROBLEM, GIVE THE MODEL NAME OF THE CALCULATOR AND THE FUNCTIONS USED HERE: …………………

4 - NO ELECTRONIC DEVICES OTHER THAN A CALCULATOR MAY BE USED.

25 pts

1. Suppose z is a standard normal random variable. Compute the following (with the help of z-table)

a) $P(z < 1.5) = ?$

$$P(z < 1.5) = 0.9332$$

b) $P(z < -0.38) = ?$

$$P(z < -0.38) = 0.3520$$

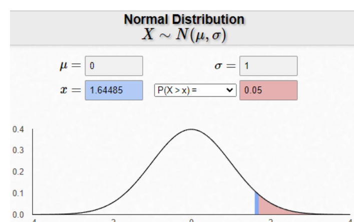
c) $P(-0.38 < z < 1.5) = ?$

$$P(z < 1.5) - P(z < -0.38) = 0.9332 - 0.3520 = 0.5812$$

d) $P(z > -0.38) = ?$

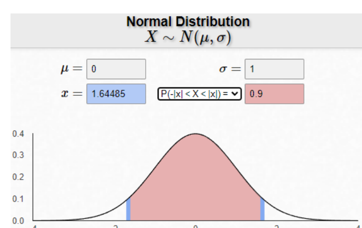
$$1 - P(z < -0.38) = 1 - 0.3520 = 0.6480$$

e) Find the $z(\alpha)$ for $\alpha = 0.05$.



$$z(\alpha) = 1.65$$

f) Find the z^* such that $P(-z^* \leq z \leq z^*) = 0.90$.



$$z^* = 1.65$$

PLEASE DO NOT WRITE IN THE FOLLOWING SPACE.

1	**2**	**3**	**4**	**5**	***Total***
25	*15*	*15*	*25*	*25*	***105***

15 pts

2. Suppose x is a binomial random variable with parameter $n = 36$ and $p = 0.50$. Use normal approximation to binomial (if possible) to compute:

$$P(x < 23) = ?$$

(First, check the assumptions)

$$\begin{cases} np > 5 & \rightarrow 36 \times 0.50 = 18 > 5 \\ n(1-p) > 5 & \rightarrow 36 \times 0.50 = 18 > 5 \end{cases}$$

$$\mu = np = 36 \times 0.50 = 18$$
$$\sigma = \sqrt{np(1-p)} = \sqrt{36 \times 0.50 \times 0.50} = \sqrt{9} = 3$$

$$P(x < 23) = P(x \leq 22.5) = P\left(z \leq \frac{22.5 - \mu}{\sigma}\right) = P\left(z \leq \frac{22.5 - 18}{3}\right)$$
$$= P(z \leq 1.5) = 0.9332$$

15 pts

3. From previous semesters we know that the mid-term grade for MATH-1700 classes (denoted with x) follows a normal distribution with population mean 77 ($\mu = 77$), and standard deviation 4 ($\sigma = 4$).

a) Compute $P(x \geq 78.5) = ?$

$$P\left(z \geq \frac{78.5 - \mu}{\sigma}\right) = P\left(z \geq \frac{78.5 - 77}{4}\right) = P(z \geq 0.38) = 0.3520$$

b) We took a random sample of 16 students from MATH-1700 class this semester. Describe the sampling distribution of the sample mean $\bar{x}$?

$\bar{x}$ is following normal with mean ($\mu = 77$) and standard deviation$\left(\frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{16}} = 1\right)$

c) Compute $P(\bar{x} \geq 76.5) = ?$

$$P(\bar{x} \geq 78.5) = P\left(z \geq \frac{78.5-\mu}{\sigma/\sqrt{n}}\right) = P\left(z \geq \frac{78.5-77}{1}\right) = P(z \geq 1.5) = 0.0688$$

25 pts

4. Soroush (our TA) didn't know the population mean of the mid-term grade for MATH-1700 classes (μ). I already told him that the population standard deviation 4 ($\sigma = 4$). He took a random sample of 16 students from MATH-1700 classes, resulted $\bar{x} = 78.5$ and $s^2 = 15$. Help him to obtain the followings:

a) Give a point estimate of μ.

$$\bar{x} = 78.5$$

b) Construct a 90% confidence interval for μ.

$$\alpha = 0.1 \Rightarrow z(\alpha/2) = z(0.05) = 1.65$$

$$\left(\bar{x} - z(\alpha/2)\frac{\sigma}{\sqrt{n}}, \bar{x} + z(\alpha/2)\frac{\sigma}{\sqrt{n}}\right) \Rightarrow \left(78.5 - 1.65\frac{4}{\sqrt{16}}, 78.5 + 1.65\frac{4}{\sqrt{16}}\right) \Rightarrow (76.85, 80.15)$$

c) What is your maximum error in part (b) with confidence level of 0.90?

$$E = z(\alpha/2)\frac{\sigma}{\sqrt{n}} = 1.65$$

d) Finally I want to have confidence interval with maximum error not more than 1.00 unit. Choose the sample size n so that your maximum error be less than 1.00 with a confidence level 0.90.

$$n = \left(\frac{z(\alpha/2) \cdot \sigma}{E}\right)^2 = \left(\frac{1.65 \cdot 4}{1}\right)^2 = 43.56 \uparrow$$

$$n = 44$$

25 pts

5. Again Soroush didn't know the population mean of the mid-term grade for MATH-1700 classes (μ). I already told him that the population standard deviation 4 ($\sigma = 4$). He took a random sample of 16 students from MATH-1700 classes, resulted $\bar{x} = 78.5$ and $s^2 = 15$. I asked Soroush to test if the mid-term grade has increased comparing to the previous semesters.

a) Use **p-value** approach to test $H_0: \mu \leq 77$ versus $H_a: \mu > 77$ at the level of significance $= 0.1$

1. $H_0: \mu \leq 77$ versus $H_a: \mu > 77$
2. Population is normal $\Rightarrow$ $\bar{x}$ is following normal (assumption holds)
3. T.S. $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{78.5-77}{4/\sqrt{16}} = \frac{3}{2} = 1.5$
4. p-value $= P(z > 1.5) = 0.0668 < 0.1$ $\Rightarrow p$-value $< \alpha$ $\Rightarrow$ Reject H_0
5. There is a significant evidence that mid-term grade has increased.

b) Use **classicial** approach to test $H_0: \mu \leq 77$ versus $H_a: \mu > 77$ at the level of significance $= 0.1$.

1. $H_0: \mu \leq 77$ versus $H_a: \mu > 77$
2. Population is normal $\Rightarrow$ $\bar{x}$ is following normal (assumption holds)
3. T.S. $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{78.5-77}{4/\sqrt{16}} = \frac{3}{2} = 1.5$
4. $C.R$: $[z > z(\alpha)] = [z > 1.28]$ $\Rightarrow$ Since $[z^* > 1.28]$ we Reject H_0
5. There is a significant evidence that mid-term grade has increased.

c) Do the conclusions from (a) align with those from (b)?

Yes. In both approaches (a) and (b) we reject H_0

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25 pts

1. Suppose z is a standard normal random variable. Compute the following probabilities (with the help of z-table)

   a) $P(z < 0.38) = ?$

   $$P(z < 0.38) = 0.6480$$

   b) $P(z < -1.5) = ?$

   $$P(z < -1.5) = 0.0668$$

   c) $P(-1.5 < z < 0.38) = ?$

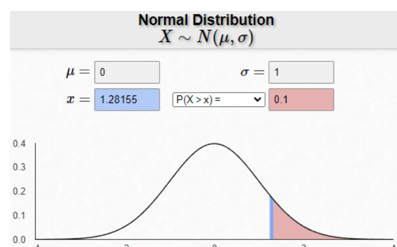
   $$P(z < 0.38) - P(z < -1.5) = 0.6480 - 0.0668 = 0.5812$$

   d) $P(z > -1.5) = ?$

   $$1 - P(z < -1.5) = 0.9332$$

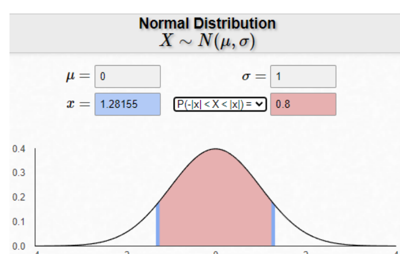
   e) Find the $z(\alpha)$ for $\alpha = 0.10$.

   $$z(\alpha) = 1.28$$



   f) Find the z^* such that $P(-z^* \leq z \leq z^*) = 0.80$.

   $$z^* = 1.28$$



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25	*15*	*15*	*25*	*25*	***105***

15 pts

2. Suppose x is a binomial random variable with parameter $n = 36$ and $p = 0.50$. Use normal approximation to binomial (if possible) to compute:

$$P(x > 13) = ?$$

(First, check the assumptions)

$$\begin{cases} np > 5 & \rightarrow 36 \times 0.50 = 18 > 5 \\ n(1-p) > 5 & \rightarrow 36 \times 0.50 = 18 > 5 \end{cases}$$

$$\mu = np = 36 \times 0.50 = 18$$
$$\sigma = \sqrt{np(1-p)} = \sqrt{36 \times 0.50 \times 0.50} = \sqrt{9} = 3$$

$$P(x > 13) = P(x \geq 13.5) = P\left(z \geq \frac{13.5 - \mu}{\sigma}\right) = P\left(z \geq \frac{13.5 - 18}{3}\right)$$
$$= P(z \geq -1.5) = 1 - P(z \leq -1.5) = 0.9332$$

15 pts

3. From previous semesters we know that the mid-term grade for MATH-1700 classes (denoted with x) follows a normal distribution with population mean 75 ($\mu = 75$), and standard deviation 4 ($\sigma = 4$).

a) Compute $P(x \geq 76.5) = ?$

$$P\left(z \geq \frac{76.5 - \mu}{\sigma}\right) = P\left(z \geq \frac{76.5 - 75}{4}\right) = P(z \geq 0.38) = 0.3520$$

b) We took a random sample of 16 students from MATH-1700 class this semester. Describe the sampling distribution of the sample mean $\bar{x}$?

$\bar{x}$ is following normal with mean ($\mu = 75$) and standard deviation$\left(\frac{\sigma}{\sqrt{n}} = \frac{4}{\sqrt{16}} = 1\right)$

c) Compute $P(\bar{x} \geq 76.5) = ?$

$$P(\bar{x} \geq 76.5) = P\left(z \geq \frac{76.5 - \mu}{\sigma/\sqrt{n}}\right) = P\left(z \geq \frac{76.5 - 75}{1}\right) = P(z \geq 1.5) = 0.0688$$

25 pts

4. Soroush (our TA) didn't know the population mean of the mid-term grade for MATH-1700 classes (μ). I already told him that the population standard deviation 4 ($\sigma = 4$). He took a random sample of 16 students from MATH-1700 classes, resulted $\bar{x} = 76.5$ and $s^2 = 15$. Help him to obtain the followings:

a) Give a point estimate of μ.

$$\bar{x} = 76.5$$

b) Construct a 90% confidence interval for μ.

$$\alpha = 0.1 \Rightarrow z(\alpha/2) = z(0.05) = 1.65$$

$$\left(\bar{x} - z(\alpha/2)\frac{\sigma}{\sqrt{n}}, \bar{x} + z(\alpha/2)\frac{\sigma}{\sqrt{n}}\right) \Rightarrow \left(76.5 - 1.65\frac{4}{\sqrt{16}}, 76.5 + 1.65\frac{4}{\sqrt{16}}\right) \Rightarrow (74.85, 78.15)$$

c) What is your maximum error in part (b) with confidence level of 0.90?

$$E = z(\alpha/2)\frac{\sigma}{\sqrt{n}} = 1.65$$

d) Finally I want to have confidence interval with maximum error not more than 1.00 unit. Choose the sample size n so that your maximum error be less than 1.00 with a confidence level 0.90.

$$n = \left(\frac{z(\alpha/2) \cdot \sigma}{E}\right)^2 = \left(\frac{1.65 \cdot 4}{1}\right)^2 = 43.56 \uparrow$$

$$n = 44$$

25 pts

5. Again Soroush didn't know the population mean of the mid-term grade for MATH-1700 classes (μ). I already told him that the population standard deviation 4 ($\sigma = 4$). He took a random sample of 16 students from MATH-1700 classes, resulted $\bar{x} = 76.5$ and $s^2 = 15$. I asked Soroush to test if the mid-term grade has increased comparing to the previous semesters.

a) Use **p-value** approach to test $H_0: \mu \leq 75$ versus $H_a: \mu > 75$ at the level of significance = 0.1

1. $H_0: \mu \leq 75$ versus $H_a: \mu > 75$
2. Population is normal $\Rightarrow$ $\bar{x}$ is following normal (assumption holds)
3. T.S. $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{76.5-75}{4/\sqrt{16}} = \frac{3}{2} = 1.5$
4. p-value $= P(z > 1.5) = 0.0668 < 0.1 \Rightarrow p\text{-value} < \alpha \Rightarrow$ Reject H_0
5. There is a significant evidence that mid-term grade has increased.

b) Use **classicial** approach to test $H_0: \mu \leq 75$ versus $H_a: \mu > 75$ at the level of significance = 0.1.

1. $H_0: \mu \leq 75$ versus $H_a: \mu > 75$
2. Population is normal $\Rightarrow$ $\bar{x}$ is following normal (assumption holds)
3. T.S. $z^* = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{76.5-75}{4/\sqrt{16}} = \frac{3}{2} = 1.5$
4. $C.R: [z > z(\alpha)] = [z > 1.28] \Rightarrow$ Since $[z^* > 1.28]$ we Reject H_0
5. There is a significant evidence that mid-term grade has increased.

c) Do the conclusions from (a) align with those from (b)?

Yes. In both approaches (a) and (b) we reject H_0