9.48 Homes in a nearby college town have a mean value of \$88,950. It is assumed that homes in the vicinity of the college have a higher mean value. To test this theory, a random sample of 12 homes is chosen from the college area. Their mean valuation is \$92,460, and the standard deviation is \$5200. Complete a hypothesis test using $\alpha = 0.05$. Assume prices are normally distributed.

a. Solve using the *p*-value approach.

b. Solve using the classical approach.

Solution:

9.48 Step 1: a. The mean value of homes in a college town

b. H_o: $\mu = 88{,}950$ ($\leq$)

H_a: $\mu > 88{,}950$ (higher)

Step 2: a. normality indicated

b. t c. $\alpha = 0.05$

Step 3: a. $n = 12$, $\bar{x} = 92{,}460$, $s = 5{,}200$

b. $t = (\bar{x} - \mu)/(s/\sqrt{n})$

$t^* = (92{,}460 - 88{,}950)/(5200/\sqrt{12}) = 2.34$

Step 4: -- using p-value approach --------------------

a. $\mathbf{P} = P(t > 2.34 | df = 11)$;

Using Table 6, Appendix B, ES11-p719: $0.01 < \mathbf{P} < 0.025$

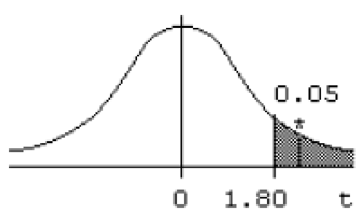
Using Table 7, Appendix B, ES11-p720: $0.017 < \mathbf{P} < 0.022$

Using computer: $\mathbf{P} = 0.0196$

b. $\mathbf{P} < \alpha$

-- using classical approach ------------------

a. $t(11, 0.05) = 1.80$



b. t* falls in the critical region, see * Step 4a

Step 5: a. Reject H_o

b. The sample does provide sufficient evidence to justify the contention that the mean value is higher than \$88,950, at the 0.05 level of significance.

9.106 The full-time student body of a college is composed of 50% males and 50% females. Does a random sample of students (30 male, 20 female) from an introductory chemistry course show sufficient evidence to reject the hypothesis that the proportion of male and of female students who take this course is the same as that of the whole student body? Use $\alpha = 0.05$.

a. Solve using the p-value approach.

b. Solve using the classical approach.

Solution:

9.106 Step 1: a. The proportion of male students in a chemistry course

b. H_o: p = P(male) = 0.50 [proportion of males taking chemistry is same as proportion of males in student body]

H_a: $p \neq 0.50$ [not same proportion]

Step 2: a. independence assumed

b. z; n = 50; n > 20, np = (50)(0.50) = 25, nq = (50)(0.50) = 25, both np and nq > 5

c. $\alpha = 0.05$

Step 3: a. n = 50, x = 30, p' = x/n = 30/50 = 0.600

b. $z = (p' - p)/\sqrt{pq/n}$

$z^* = (0.60 - 0.50)/\sqrt{(0.5)(0.5)/50} = 1.41$

Step 4: -- using p-value approach --------------------

a. **P** = 2·P(z > 1.41);

Using Table 3, Appendix B, ES11-pp716-717: **P** = 2·(1.0000 - 0.9207) = 0.1586

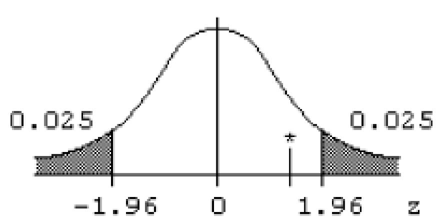
Using Table 5, Appendix B, ES11-p718: 0.0735 < ½**P** < 0.0808;

0.147 < **P** < 0.1616

b. **P** > α

-- using classical approach ------------------

a. $\pm z(0.025) = \pm 1.96$



b. z* falls in the noncritical region, see Step 4a

Step 5: a. Fail to reject H_o

b. The sample does not provide sufficient evidence to show that the proportion is different than 0.50, at the 0.05 level; the sample evidence does not indicate the proportion of males taking chemistry to be different than 50%.

9.137 In the past the standard deviation of weights of certain 32.0-oz packages filled by a machine was 0.25 oz. A random sample of 20 packages showed a standard deviation of 0.35 oz. Is the apparent increase in variability significant at the 0.10 level of significance? Assume package weight is normally distributed.

a. Solve using the p-value approach.

b. Solve using the classical approach.

9.137 Step 1: a. The standard deviation of weights of 32.0 oz. packages

b. H_o: $\sigma = 0.25$ [no increase] ($\leq$)

H_a: $\sigma > 0.25$ [an increase]

Step 2: a. normality indicated

b. χ^2 c. $\alpha = 0.10$

Step 3: a. $n = 20$, $s = 0.35$

b. $\chi^{2*} = (n-1)s^2/\sigma^2 = (19)(0.35^2)/(0.25^2) = 37.24$

Step 4: -- using p-value approach --------------------

a. $\mathbf{P} = P(\chi^2 > 37.24 | df = 19)$

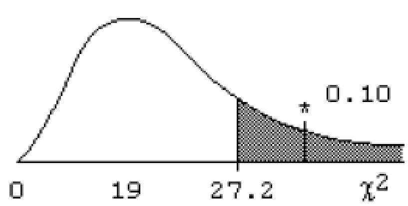
Using Table 8: $0.005 < \mathbf{P} < 0.010$

Using computer: $\mathbf{P} = 0.0074$

b. $\mathbf{P} < \alpha$

-- using classical approach ------------------

a. $\chi^2(19, 0.10) = 27.2$



b. χ^{2*} falls in the critical region, see Step 4a

Step 5: a. Reject H_o

b. There is sufficient reason to conclude that the apparent increase is significant, at the 0.10 level.

10.19 [EX10-019] An experiment was designed to estimate the mean difference in weight gain for pigs fed ration A as compared with those fed ration B. Eight pairs of pigs were used. The pigs within each pair were littermates. The rations were assigned at random to the two animals within each pair. The gains (in pounds) after 45 days are shown in the table at top of next column.

Litter	1	2	3	4	5	6	7	8
Ration A	65	37	40	47	49	65	53	59
Ration B	58	39	31	45	47	55	59	51

Assuming weight gain is normal, find the 95% confidence interval estimate for the mean of the differences μ_d, where d = ration A − ration B.

Solution:

10.19 Sample statistics: $d = A - B$ $\quad n = 8,\ \bar{d} = 3.75,\ s_d = 5.726$

Step 1: The mean difference in weight gain for pigs fed ration A as compared to those fed ration B

Step 2: a. normality indicated

b. t $\quad$ c. $1-\alpha = 0.95$

Step 3: $n = 8,\ \bar{d} = 3.75,\ s_d = 5.726$

Step 4: a. $\alpha/2 = 0.05/2 = 0.025$; df = 7; $t(7, 0.025) = 2.36$

b. $E = t(df,\alpha/2)\cdot(s_d/\sqrt{n}) = (2.36)(5.726/\sqrt{8})$

$= (2.36)(2.0244) = 4.78$

c. $\bar{d} \pm E = 3.75 \pm 4.78$

Step 5: <u>-1.03 to 8.53</u>, the 0.95 interval for μ_d

10.32 Complete the hypothesis test with alternative hypothesis $\mu_d \neq 0$ based on the paired data that follow and $d = O - Y$. Use $\alpha = 0.01$. Assume normality.

Oldest	199	162	174	159	173
Youngest	194	162	167	156	176

a. Solve using the p-value approach.

b. Solve using the classical approach.

Solution:

10.32 Data Summary: n = 5, $\Sigma d = 12$, $\Sigma d^2 = 92$

Step 1: a. The mean difference, μ_d

b. H_o: $\mu_d = 0$

H_a: $\mu_d \neq 0$

Step 2: a. normality indicated

b. t c. $\alpha = 0.01$

Step 3: a. $n = 5$, $\overline{d} = 2.4$, $s_d = 3.97$

b. $t^* = (\overline{d} - \mu_d)/(s_d/\sqrt{n}) = (2.4 - 0)/(3.97/\sqrt{5}) = 1.35$

Step 4: -- using p-value approach --------------------

a. $\mathbf{P} = 2P(t > 1.35|df = 4)$;

Using Table 6, Appendix B, ES11-p719: $0.10 < ½\mathbf{P} < 0.25$; $0.20 < \mathbf{P} < 0.50$

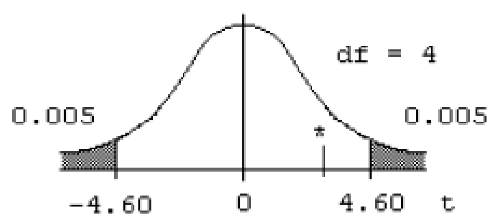
Using Table 7, Appendix B, ES11-p720: $0.117 < ½\mathbf{P} < 0.132$; $0.234 < \mathbf{P} < 0.264$

Using computer: $\mathbf{P} = 0.2484$

b. $\mathbf{P} > \alpha$

-- using classical approach ------------------

a. $\pm t(4, 0.005) = \pm 4.60$



b. t* falls in the noncritical region, see Step 4a

Step 5: a. Fail to reject H_o

b. At the 0.01 level of significance, there is not sufficient evidence that the mean difference is different than zero.

10.62 Is having a long, more complex first name more dignified for a girl? Are girls' names longer than boys' names? With current names like "Alexandra," "Madeleine," and "Savannah," it certainly appears so. To test this theory, random samples of seventh-grade girls and boys were taken. Let x be the number of letters in each seventh grader's first name.

Boys' Names	$n = 30$	$\bar{x} = 5.767$	$s = 1.870$
Girls' Names	$n = 30$	$\bar{x} = 6.133$	$s = 1.456$

At the 0.05 level of significance, do the data support the contention that the mean length of girls' names is longer than the mean length of boys' names?

Solution:

10.62 Step 1: a. The difference in mean length of boys' and girls' names, $\mu_G - \mu_B$

b. H_o: $\mu_G - \mu_B = 0$

H_a: $\mu_G - \mu_B > 0$

Step 2: a. normality assumed

b. t c. $\alpha = 0.05$

Step 3: a. sample information given in exercise

b. $t = [(\bar{x}_G - \bar{x}_B) - (\mu_G - \mu_B)] / \sqrt{(s_G^2/n_G) + (s_B^2/n_B)}$

$t^* = [(6.133 - 5.767) - 0] / [\sqrt{(1.456^2/30) + (1.870^2/30)}\,] = 0.85$

Step 4: -- using p-value approach --------------------

a. **P** = P(t > 0.85|df = 29);

Using computer/calculator, **P** = 0.201

Using Table 6, Appendix B, ES11-p719: 0.10 < **P** < 0.25

Using Table 7, Appendix B, ES11-p720: 0.188 < **P** < 0.215

OR P = P(t > 0.85|df = 54);

Using computer/calculator, **P** = 0.201

b. **P** > α

-- using classical approach ------------------

a. critical region: $t \geq 1.70$

b. t* is in the noncritical region

Step 5: a. Fail to reject H_o

b. There is insufficient evidence to support the contention that mean length of girls' names is longer than the mean length of boys' names, at the 0.05 level of significance.

10.88 Find the 95% confidence interval for $p_A - p_B$.

Sample	*n*	*x*
A	125	45
B	150	48

Solution:

10.88 Step 1: The difference between proportions, p_A-p_B

Step 2: a. n's > 20, np's and nq's all > 5

b. z   c. $1-\alpha = 0.95$

Step 3: sample information given in exercise

$p'_A = x_A / n_A = 45/125 = 0.36, \; q'_A = 1 - 0.36 = 0.64$

$p'_B = x_B / n_B = 48/150 = 0.32, \; q'_B = 1 - 0.32 = 0.68$

$p'_A - p'_B = 0.36 - 0.32 = 0.04$

Step 4: a. $\alpha/2 = 0.05/2 = 0.025; \; z(0.025) = 1.96$

b. $E = z(\alpha/2)\cdot\sqrt{(p_A q_A)/n_A + (p_B q_B)/n_B}$

$= (1.96)\cdot\sqrt{(0.36)(0.64)/125 + (0.32)(0.68)/150}$

$= (1.96)(0.057) = 0.11$

c. $(p'_A - p'_B) \pm E = 0.04 \pm 0.11$

Step 5: <u>-0.07 to 0.15</u>, the 0.95 confidence interval for p_A-p_B

10.108 Adverse side effects are always a concern when testing and trying new medicines. Placebo-controlled clinical studies were conducted in patients 12 years of age and older who were receiving "once-a-day" doses of Allegra, a seasonal allergy drug. The following results were published in the April 2005 edition of *Reader's Digest*.

	Allegra (dose once a day)	Placebo (dose once a day)
Side effects	$n = 283$	$n = 293$
Number reporting headaches	30	22

Determine, at the 0.05 level of significance, whether there is a difference in the proportion of patients reporting headaches between the two groups.

Solution:

10.108 Step 1: a. The difference in the proportion of patients reporting headaches between the two groups, Allegra and Placebo.

b. H_o: $p_A - p_P = 0$

H_a: $p_A - p_P \neq 0$

Step 2: a. n's > 20, np's and nq's all > 5

b. z c. $\alpha = 0.05$

Step 3: a. $n_A = 283$, $p'_A = 30/283 = 0.1060$, $n_P = 293$, $p'_P = 22/293 = 0.0751$

$p'_p = (x_A + x_P)/(n_A + n_P) = (30+22)/(283+293) = 0.0903$

$q'_p = 1 - p'_p = 1.000 - 0.0903 = 0.9097$

b. $z = [(p'_A - p'_P) - (p_A - p_P)] / \sqrt{(p'_p)(q'_p)[(1/n_A) + (1/n_P)]}$

$z^* = (0.1060 - 0.0751) / \sqrt{(0.0903)(0.9097)[(1/283) + (1/293)]}$

$= 0.0309/0.023887 = 1.29$

Step 4: -- using p-value approach --------------------

a. $\mathbf{P} = 2P(z > 1.29)$;

Using Table 3, Appendix B, ES11-pp716-717: $\mathbf{P} = 2(1.0000 - 0.9015)$
$= 2(0.0985) = 0.1970$

Using Table 5, Appendix B, ES11-pp718: $(0.0968 < \frac{1}{2}\mathbf{P} < 0.1056)$;
$0.1936 < \mathbf{P} < 0.2112$

b. $\mathbf{P} > \alpha$

-- using classical approach ------------------

a. Critical region: $z \leq -1.96$ and $z \geq 1.96$

b. z^* falls in the noncritical region

Step 5: a. Fail to reject H_o

b. There is not sufficient evidence to show a difference, at the 0.05 level of significance.

11.19 A certain type of flower seed will produce magenta, chartreuse, and ochre flowers in the ratio 6:3:1 (one flower per seed). A total of 100 seeds are planted and all germinate, yielding the following results.

Magenta	Chartreuse	Ochre
52	36	12

Solve the following using the p-value approach and the classical approach:

a. If the null hypothesis (6:3:1) is true, what is the expected number of magenta flowers?

b. How many degrees of freedom are associated with chi-square?

c. Complete the hypothesis test using $\alpha = 0.10$.

11.19 a. E(magenta) = n·p = 100[6/(6+3+1)] = 60
b. 2
c. (1) & (2)
Step 1: a. The proportions: P(magenta), P(chartreuse), P(ochre)
b. H_o: 6:3:1 ratio vs. H_a: ratio other than 6:3:1
Step 2: a. Assume that the 100 seeds represent a random sample.
b. χ^2 with df = 2   c. $\alpha = 0.10$
Step 3: a. sample information given in exercise
b. $\chi^2 = \Sigma[(O-E)^2/E]$ (as found on accompanying table)
E(magenta) = n·P(m) = 100(0.6) = 60
E(chartreuse) = n·P(c) = 100(0.3) = 30
E(orche) = n·P(o) = 100(0.1) = 10

Color	magenta	chartreuse	orche	Total
Observed	52	36	12	100
Expected	60	30	10	100
$(O-E)^2/E$	64/60	36/30	4/10	160/60

$\chi^{2*} = 160/60 = 2.67$

Step 4: -- using p-value approach -------------------
a. **P** = $P(\chi^2 > 2.67|df=2)$;
Using Table 8: $0.25 < \mathbf{P} < 0.50$
Using computer/calculator: **P** = 0.263
b. **P** > α
-- using classical approach -----------------
a. critical region: $\chi^2 \geq 4.61$
b. χ^{2*} falls in the noncritical region

Step 5: a. Fail to reject H_o
b. At the 0.10 level of significance, there is not sufficient evidence to show the ratio is other than 6:3:1.

11.38 A survey of randomly selected travelers who visited the service station restrooms of a large U.S. petroleum distributor showed the following results:

Gender of Respondent	Quality of Restroom Facilities: Above Average	Average	Below Average	Totals
Female	7	24	28	59
Male	8	26	7	41
Total	15	50	35	100

Using $\alpha = 0.05$, does the sample present sufficient evidence to reject the hypothesis "Quality of responses is independent of the gender of the respondent"?

a. Solve using the p-value approach.

b. Solve using the classical approach.

11.38 Step 1: a. The proportions of gender:P(female for quality of restroom facilities category), P(male for quality of restroom facilities category).

b. H_o: The quality rating of restroom facilities is independent of the gender of the respondent.
H_a: The quality rating of restroom facilities is not independent of the gender of the respondent.

Step 2: a. Given a random sample.

b. χ^2 with df = 2

c. $\alpha = 0.05$

Step 3: a. sample information given in exercise

b. $\chi^2 = \Sigma[(O-E)^2/E]$ (as found on accompanying table)
Expected values:

	Above	Average	Below
Female	8.85	29.50	20.65
Male	6.15	20.50	14.35

$\chi^{2*} = 0.387 + 1.025 + 2.616 + 0.557 + 1.476 + 3.765 = 9.825$

Step 4: -- using p-value approach -------------------

a. $\mathbf{P} = P(\chi^2 > 9.825|df=2)$;
Using Table 8: $0.005 < \mathbf{P} < 0.01$
Using computer/calculator: $\mathbf{P} = 0.007$

b. $\mathbf{P} < \alpha$

-- using classical approach -----------------

a. critical region: $\chi^2 \geq 5.99$

b. χ^{2*} falls in the critical region

Step 5: a. Reject H_o

b. At the 0.05 level of significance, there is sufficient evidence to reject the hypothesis that the quality of the responses are independent of the gender of the respondent.

12.31 [EX12-31] Random samples of 2009 pickup trucks with 4-cylinder, 5-cylinder, 6-cylinder, and 8-cylinder engines were obtained. Each pickup truck was tested for miles per gallons in highway driving.

4 Cyl (H)	5 Cyl (H)	6 Cyl (H)	8 Cyl (H)
24	21	19	20
23	21	19	19
22	23	19	19
24	21	18	20
24	18	21	16
23	22	20	18
23	23	19	15
24	18	20	21
24	20	19	
23	20	19	

Is there significant evidence to show that the mpg for pickup trucks is not the same for all four engine sizes? Use $\alpha = 0.01$.

12.31 Step 1: a. The mean level of mpg for a 4 Cyl engine, the mean level of mpg for a 5 Cyl engine, the mean level of mpg for a 6 Cyl engine, the mean level of mpg for a 8 Cyl engine.

b. H_o: The mean mpg values for engines are all equal.

H_a: The mean mpg values for engines are not all equal.

Step 2: a. Assume the data were randomly collected and are independent, and the effects due to chance and untested factors are normally distributed.

b. F c. $\alpha = 0.01$

Step 3: a. $n = 38$, $C_1 = 234$, $C_2 = 207$, $C_3 = 193$, $C_4 = 148$, $T = 782$, $\Sigma x^2 = 16292$

Source	**df**	**SS**	**MS**	**F***
Factor	3	130.66	43.55	21.59
Error	34	68.60	2.02	
Total	37	199.26		

$F^* = 43.55/2.02 = 21.59$

Step 4: -- using p-value approach ---------------

a. $\mathbf{P} = P(F > 21.59 | df_n = 3, df_d = 34)$;

Using Table 9: $\mathbf{P} < 0.01$

Using computer: $\mathbf{P} = 0.000$

b. $\mathbf{P} < \alpha$

-- using classical approach -------------

a. critical region: $F \geq 4.51$

b. F* falls in the critical region

Step 5: a. Reject H_o.

b. There is significant evidence to show that the mpg for pickup trucks is not the same for all four engine sizes, at the 0.01 level of significance.