6.48 Given that x is a normally distributed random variable with a mean of 28 and a standard deviation of 7, find the following probabilities.

a. $P(x < 28)$ b. $P(28 < x < 38)$

Solution:

Use formula $z = (x - \mu)/\sigma$:

a. $P[x < 28] = P[z < (28 - 28)/7]$
$= P[z < 0.00] = \underline{0.5000}$

b. $P[28 < x < 38] = P[(28 - 28)/7 < z < (38 - 28)/7]$
$= P[0.00 < z < 1.43] = 0.9236 - 0.5000 = \underline{0.4236}$

6.96 Find the normal approximation for the binomial probability $P(x \le 8)$, where $n = 14$ and $p = 0.4$. Compare this to the value of $P(x \le 8)$ obtained from Table 2.

Solution:

6.96 $P(x \le 8) = P(x < 8.5) = P[z < (8.5 - 5.6)/\sqrt{3.36}]$
$= P[z < 1.58] = \underline{0.9430}$

$P[x \le 8 | B(n = 14, p = 0.4)] = P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6) + P(7) + P(8)$
$= 0.001+0.007+0.032+0.085+0.155+0.207+0.207+0.157+0.092$
$= \underline{0.943}$ from Table 2 (Appendix B, ES11)

7.35 Consider the approximately normal population of heights of male college students with mean $\mu = 69$ inches and standard deviation $\sigma = 4$ inches. A random sample of 16 heights is obtained.

a. Describe the distribution of x, height of male college students.

b. Find the proportion of male college students whose height is greater than 70 inches.

c. Describe the distribution of $\bar{x}$, the mean of samples of size 16.

d. Find the mean and standard error of the $\bar{x}$ distribution.

e. Find $P(\bar{x} > 70)$.

Solution:

7.35 a. Heights are approximately normally distributed with a $\mu = 69$ and $\sigma = 4$.

b. $P(x > 70) = P[z > (70 - 69)/4]$
$= P[z > 0.25]$
$= 1.0000 - 0.5987 = \underline{0.4013}$

c. The distribution of $\overline{x}$'s will be approximately normally distributed.

d. $\mu_{\overline{x}} = \underline{69}$; $\sigma_{\overline{x}} = 4/\sqrt{16} = \underline{1.0}$

e. $P(\overline{x} > 70) = P[z > (70 - 69)/1.0]$
$= P[z > +1.00]$
$= 1.0000 - 0.8413 = \underline{0.1587}$

8.36 A sample of 60 night-school students' ages is obtained in order to estimate the mean age of night-school students. $\bar{x} = 25.3$ years. The population variance is 16.

a. Give a point estimate for μ.

b. Find the 95% confidence interval for μ.

Solution:

8.36 a. 25.3

b. Step 1: The mean age of night school students

Step 2: a. normality assumed, CLT with n = 60.

b. z , $\sigma^2 = 16$ or $\sigma = 4$ c. $1-\alpha = 0.95$

Step 3: $n = 60$, $\bar{x} = 25.3$

Step 4: a. $\alpha/2 = 0.05/2 = 0.025$; $z(0.025) = 1.96$

b. $E = z(\alpha/2)\cdot\sigma/\sqrt{n} = (1.96)(4/\sqrt{60})$

$= (1.96)(0.516) = 1.01$

c. $\bar{x} \pm E = 25.3 \pm 1.01$

Step 5: 24.29 to 26.31, the 0.95 confidence interval for μ

8.106 Find the test statistic $z\bigstar$ and the p-value for each of the following situations.

b. H_o: $\mu = 200$, H_a: $\mu < 200$; $\bar{x} = 192.5$, $\sigma = 40$, $n = 50$

Solution:

b. $z^* = (\bar{x} - \mu)/(\sigma/\sqrt{n}) = (192.5 - 200)/(40/\sqrt{50}) = -1.33$

p-value = $P(z < -1.33) = \underline{0.0918}$

8.158 According to the Center on Budget and Policy Priorities' article "Curbing Flexible Spending Accounts Could Help Pay for Health Care Reform" (revised June 10, 2009), flexible-spending accounts encourage the overconsumption of health care. People buy things they do not need; otherwise they lose the money. In 2007, for those who did not use all of their account (about one out of every seven), the average amount lost was \$723.

Source: http://www.cbpp.org/

Suppose a random sample of 150 employees who did not use all of their funds in 2009 is taken and an average amount of \$683 was lost. Test the hypothesis that there is no significant difference in the average amount forfeited. Assume that σ = \$307 per year. Use $\alpha = 0.05$.

a. Define the parameter.

b. State the null and alternative hypotheses.

c. Specify the hypothesis test criteria.

d. Present the sample evidence.

e. Find the probability distribution information.

f. Determine the results.

Solution:

8.158 a. The average amount forfeited in flexible spending accounts.

b. H_o: μ = \$723

H_a: $\mu \neq$ \$723

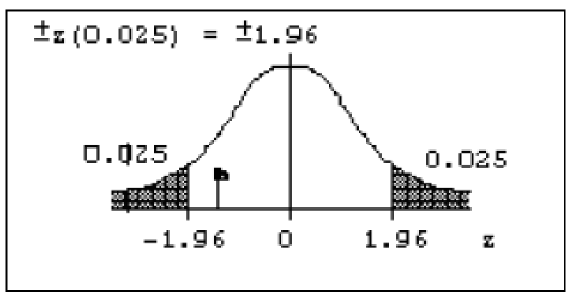
c. normality is assumed, CLT with n = 150

use z with σ = \$307; an α = 0.05 is given

d. n = 150, $\bar{x}$ = \$683

e. $z = (\bar{x} - \mu)/(\sigma/\sqrt{n})$

$z^* = (683 - 723)/(307/\sqrt{150}) = -1.60$



f. z* is in the noncritical region; Fail to reject H_o

At the 0.05 level of significance, there is sufficient evidence to support the contention that there is no significant difference in average amount forfeited.