MATH 1700

Instructor: Mehdi Maadooliat

Chapter 6

Department of Mathematical and Statistical Sciences

CHAPTER 6

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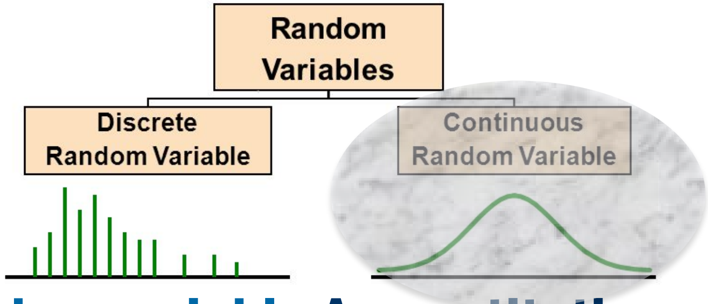
- **Continuous random variable**
- **Normal Probability Distributions**
  - **Applications of Normal Distributions**
  - **Standard Normal Distribution**
  - **Mean and Standard Deviation**
  - **How to Use z-table**
  - $z(\alpha)$ **Notation**
- **Normal Approximation of the Binomial**
  - **Continuity Correction Factor**
  - np **and** $n(1-p)$ **Rule of Thumb**

EXAMPLE 1 – RANDOM VARIABLES



- **Continuous random variable A quantitative random variable that can assume an uncountable (continuum of values) number of values. Example:**
  - **Let the "length of the cord" on an electrical appliance be a random variable. The random variable is a numerical value between 12 and 72 inches for most appliances.**

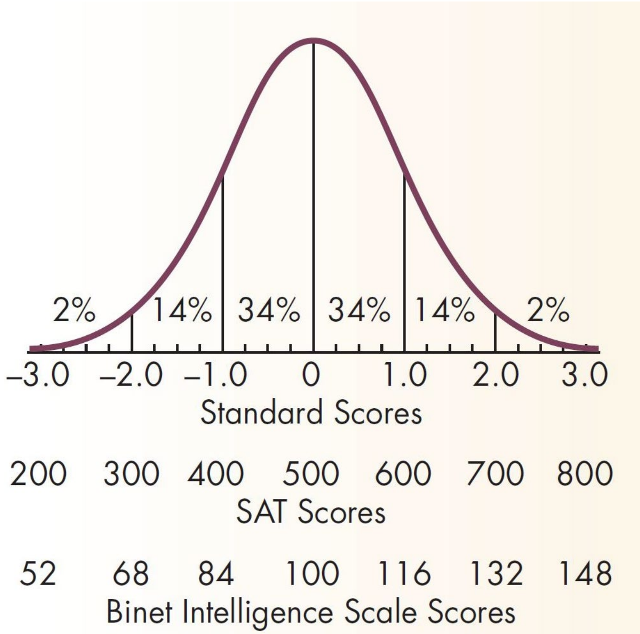
- **The normal probability distribution is considered the single most important probability distribution.**

- **An unlimited number of continuous random variables have either a normal or an approximately normal distribution.**

INTELLIGENCE SCORES

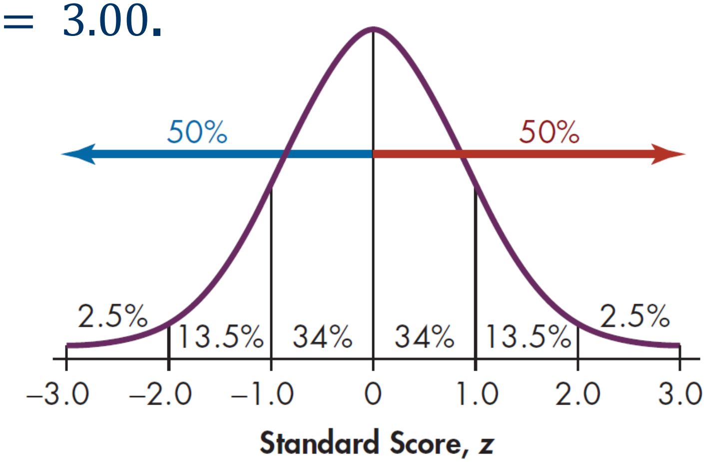
- We are all familiar with IQ (intelligence quotient) scores and/or SAT (Scholastic Aptitude Test) scores.
  - IQ scores have a mean of 100 and a standard deviation of 16.
  - SAT scores have a mean of 500 with a standard deviation of 100.

- But did you know that these continuous random variables also follow a normal distribution?



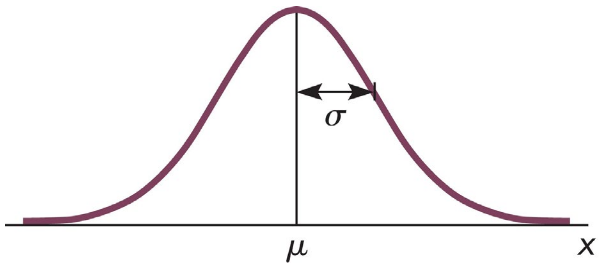
THE STANDARD NORMAL DISTRIBUTION

- All normal distributions are related to one distribution, called **standard normal distribution**.
- The standard normal distribution is the normal distribution of the standard variable z (called "**standard score**" or "z***-score***").
- **Properties of the Standard Normal Distribution**
  1. The total area under the **normal curve** is equal to 1.
  2. The distribution is mounded and symmetric; it extends indefinitely in both directions, approaching but never touching the horizontal axis.
  3. The distribution has a **mean** of 0 and a **standard deviation** of 1.
  4. The mean divides the area in half, 0.50 each side.
  5. Nearly all the area is between $z = -3.00$ and $z = 3.00$.
- **Probability distribution:** A **formula** or a **list** that provides the probability for a continuous random variable having a value falling within a specified **interval**.

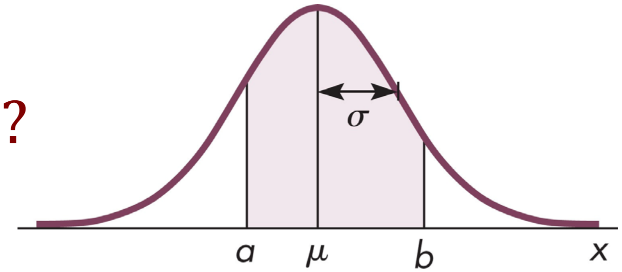


NORMAL DISTRIBUTION

- is denoted by $N(\mu, \sigma^2)$, and it has the following probability distribution function
- $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \qquad -\infty < x < \infty$
  - The normal distribution is always **bell shaped**.
  - The normal distribution is defined in terms of its **mean** and variance (or **standard deviation**).

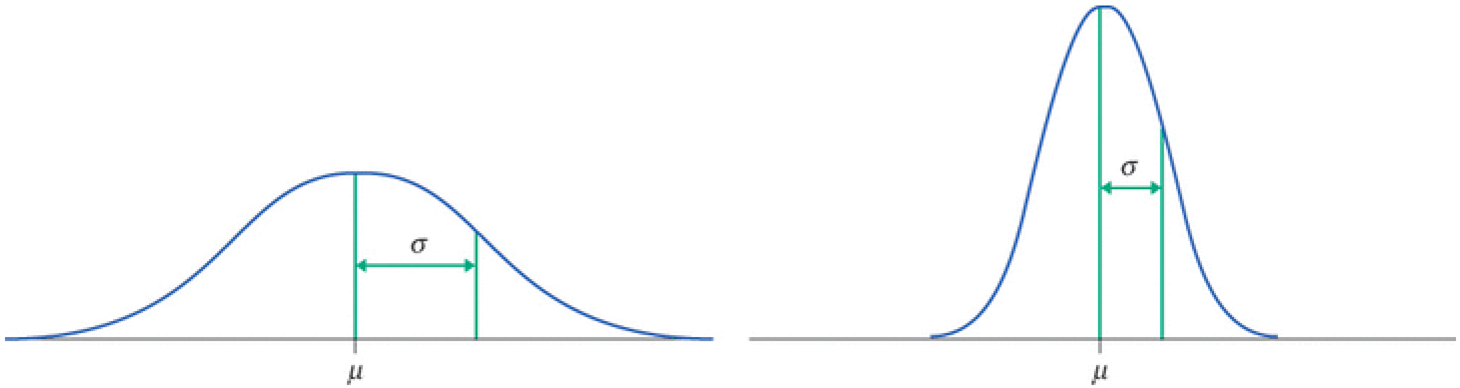


- What we need is to find $P(a \leq x \leq b) = ?$



- [Normal calculator]
- TI-84 Calculator:
  - normal**cdf**(a, b, μ, σ^2) = $P(a \leq x \leq b) = \int_a^b f(x)dx$
- Z-table ("D2L > Useful Links > Z, T and Chi^2 Tables")
  - $P(Z \leq z)$, where Z is a **standard** Normal, $Z \sim N(\mu = 0, \sigma^2 = 1)$.

THE NORMAL DISTRIBUTION



- Two Normal curves, showing the mean μ and standard deviation σ.

- The **mean** of a Normal distribution is at the **center** of the symmetric Normal curve.
- The **standard deviation** is the distance from the center to the **change-of-curvature points** on either side

- [Mean and Standard deviation of Normal distribution]()
- [Link 2]()

THE STANDARD NORMAL DISTRIBUTION

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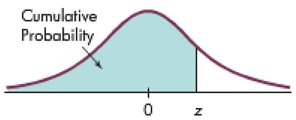
- **Table 3 in Appendix B lists the probabilities associated with the cumulative area to the left of a specified value of *z*.**

- **Probabilities associated with other intervals may be found by using the table entries along with the operations of addition and subtraction, in accordance with the preceding properties.**

TABLE 3

Cumulative Areas of the Standard Normal Distribution (*continued*)

The entries in this table are the cumulative probabilities for the standard normal distribution z (that is, the normal distribution with mean 0 and standard deviation 1). The shaded area under the curve of the standard normal distribution represents the cumulative probability to the left of a z-value in the **left-hand tail**.



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9980	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997
4.0	0.99997	0.99997	0.99997	0.99997	0.99997	0.99997	0.99998	0.99998	0.99998	0.99998
4.5	0.999997									
5.0	0.9999997									

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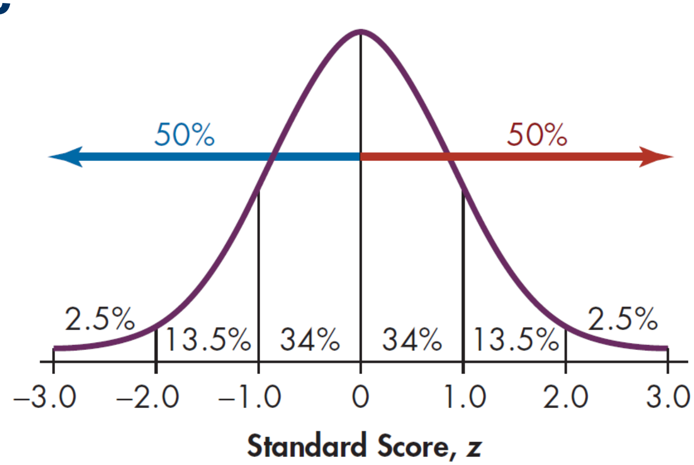
- **Notes**
  - Probabilities associated with positive z-values are greater than 0.5000 since they include the entire left half of the normal curve.
  - Always draw and label a sketch. It is most helpful.
  - Make it a habit to write the z-score with two decimal places, and the areas (probabilities, percentages) with four decimal places, as in Table 3. This will help with distinguishing between the two concepts.
  - "The area under the entire normal distribution curve is equal to 1" is the key factor in determining probabilities associated with the values to the right of a z-value.
- **Find the area under the normal curve**
  - $P(z < 1) = ?$
  - $P(z > -2) = ?$

    $= 1 - P(z < -2)$
  - $P\,(-2 < z < 1) = ?$



EXAMPLE 4 – *FINDING AREA*

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- $P(z < 2.14) = ?$

  $= 0.9838$

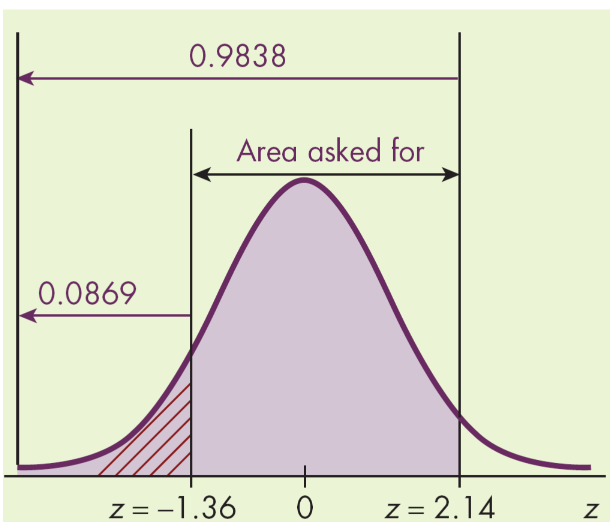
- $P(z < -1.36) = ?$

  $= 0.0869$

- $P(-1.36 < z < 2.14) = ?$

  $= 0.9838 - 0.0869$

  $= \mathbf{0.8969}$



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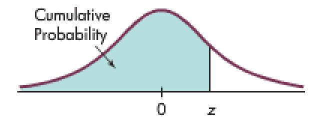

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2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

THE STANDARD NORMAL DISTRIBUTION

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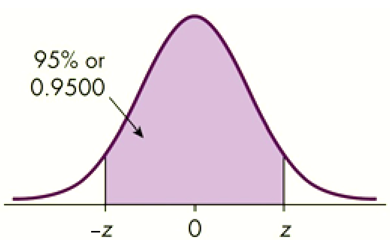
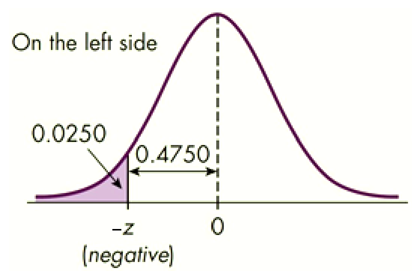
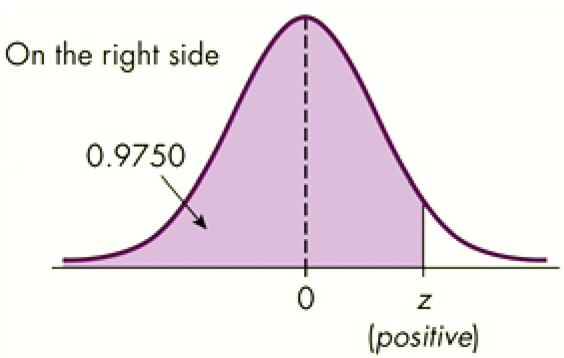
- Table 3 can also be used to find the z-score (s) that bound (s) a specified area.
- Example 7: What z-scores bound the middle 95% of a normal distribution?







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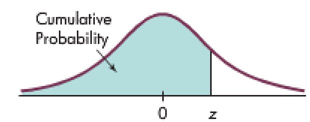

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936

- $z = 1.96$
- $P\,(-1.96 < z < 1.96) = 0.95$

z		0.06
⋮		
−1.9	0.0250	
⋮		

z	0.06
⋮	
1.9	0.9750
⋮	

APPLICATIONS OF NORMAL DISTRIBUTIONS

- We have learned how to use Table 3 in Appendix B to convert information about the standard normal variable z into probability, or the opposite, to convert probability information about the standard normal distribution into z-scores.

- Now we are ready to apply this methodology to all normal distributions. The key is the standard score, z.

- In general, the information associated with a normal distribution will be in terms of x values or probabilities.

Standard Score, z

$$z = \frac{x - (\text{mean of } x)}{(\text{standard deviation of } x)}$$

$$z = \frac{x - \mu}{\sigma} \qquad (6.3)$$

EXAMPLE 8 – CONVERTING TO A STANDARD NORMAL CURVE TO FIND PROBABILITIES

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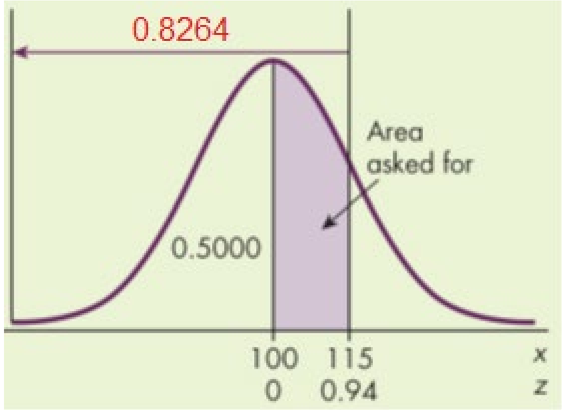
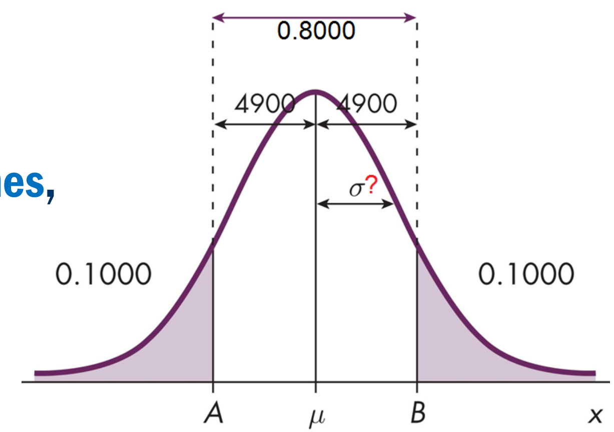
- **Consider the intelligence quotient (IQ) scores for people. IQ scores are normally distributed, with a mean of 100 and a standard deviation of 16. If a person is picked at random, what is the probability that his or her IQ is between 100 and 115?**
- **Solution:**
- $P(100 < x < 115) = ?$
  - **The variable x must be standardized using formula (6.3).**
  - $z = \frac{x-\mu}{\sigma}$
  - **When** $x = 100$: $z = \frac{100-100}{16} = 0$
  - **When** $x = 115$: $z = \frac{115-100}{16} = 0.94$
  - $P(100 < x < 115) = P(0.00 < z < 0.94)$
  - $= 0.8264 - 0.5000$
  - $= \mathbf{0.3264}$
  - **Thus, the probability is** 0.3264 **that a person picked at random has an IQ between** 100 **and** 115**.**



APPLICATIONS OF NORMAL DISTRIBUTIONS

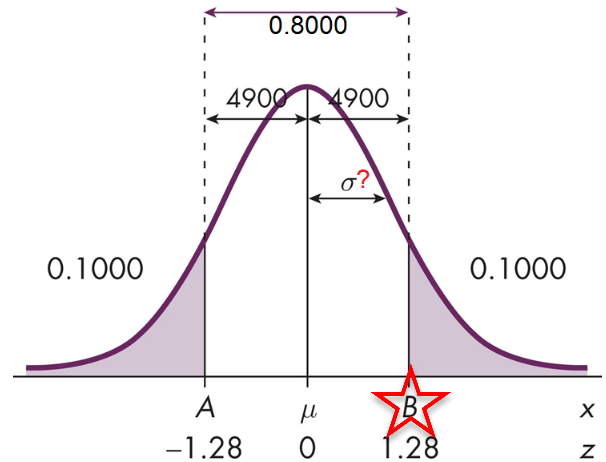
- The normal table, Table 3, can be used to answer many kinds of questions that involve a **normal distribution**.
- Next example concerns a normal distribution in which you are asked to find the **standard deviation** σ when given the related information.

- **Example 12**: The incomes of junior executives in a large corporation are approximately normally distributed. A pending cutback will not discharge those junior executives with earnings **within** $\$4900$ of the mean.
  - If this represents the **middle** 80% **of the incomes**, what is the standard deviation for the salaries of this group of junior executives?



EXAMPLE 12 – *SOLUTION*

- **Table 3 indicates that the middle 80%, or 0.8000, of a standard normal distribution is bounded by -1.28 and 1.28. How?**



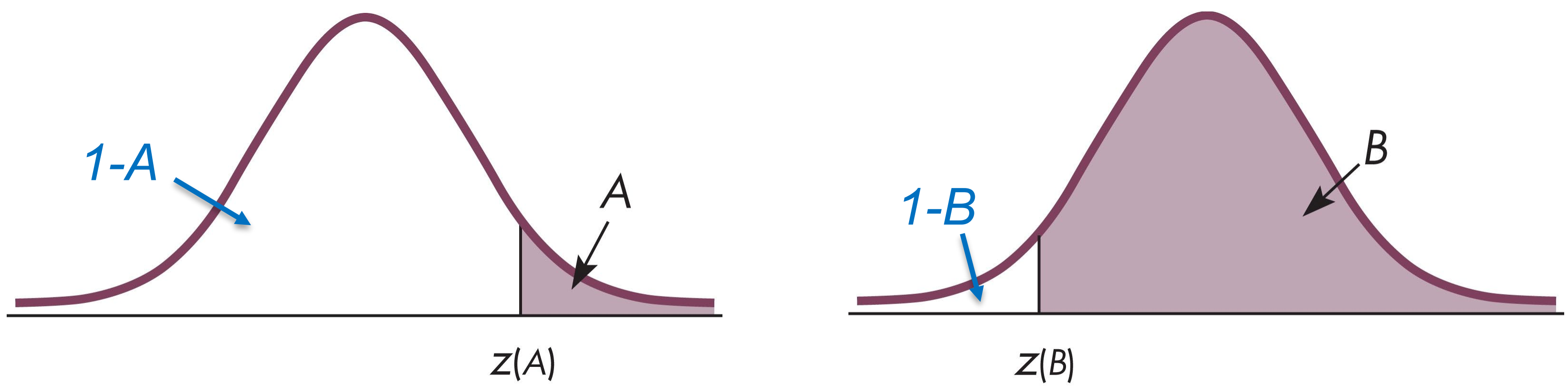
- **Consider point B shown in the figure.**
  - $P(z < z_B) = 0.9000$
  - $z_B = 1.28$
- **Note that**
  - $z_B = \frac{B-\mu}{\sigma}$ **and**
  - $B - \mu = 4900$
- **Therefore:**
  - $1.28 = \frac{4900}{\sigma}$
  - $\sigma = \frac{4900}{1.28} = 3828$
  - **the current standard deviation for the salaries of junior executives is $3828**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936

NOTATION

- The z-score is used throughout statistics in a variety of ways; however, the relationship between the numerical value of z and the area under the standard normal distribution curve is **unique**.

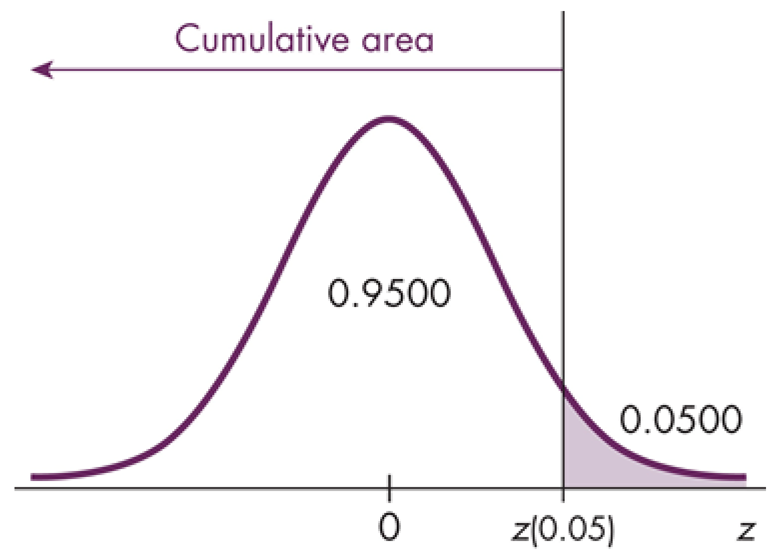
- The convention that we will use is notation $z(\alpha)$ for a specific z-score, where
  - α represents the "**area to the right**" of the z being named.



EXAMPLE 16 – DETERMINING CORRESPONDING Z-VALUES FOR Z(α)

- **Find the numerical value of $z(0.05)$.**
- **Solution:**
  - **Remember that the area under the entire normal curve is 1.**
  - **Therefore, subtracting 0.05 from 1 gives 0.95, the area to the left of the $z(0.05)$.**
  - **We use the z that corresponds to the area closest in value.**
  - **When the value happens to be exactly halfway between the table entries as above, always use the larger value of z.**
  - **Therefore, $z(0.05) = \mathbf{1.65}$**

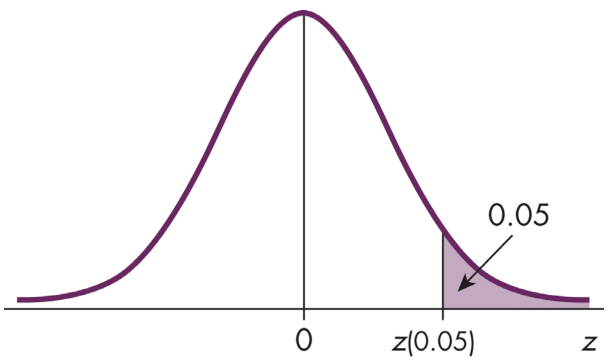
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9430	0.9441
1.6	0.9452	0.9463	0.9474	0.9485	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9700	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9762	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9865	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936

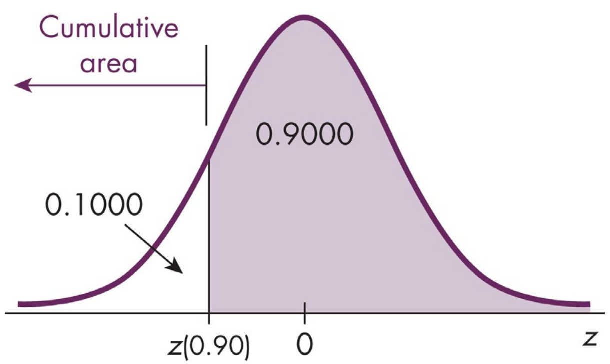


NOTATION

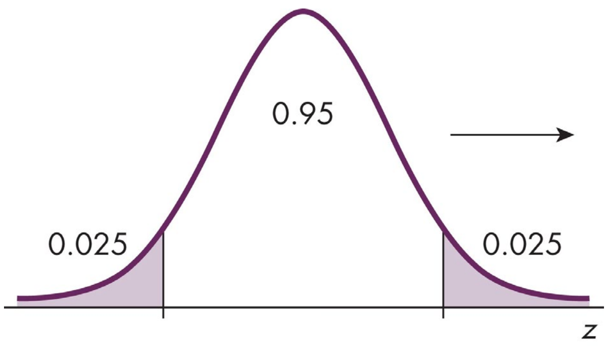
- **The values of z that will be used regularly come from one of the following situations:**

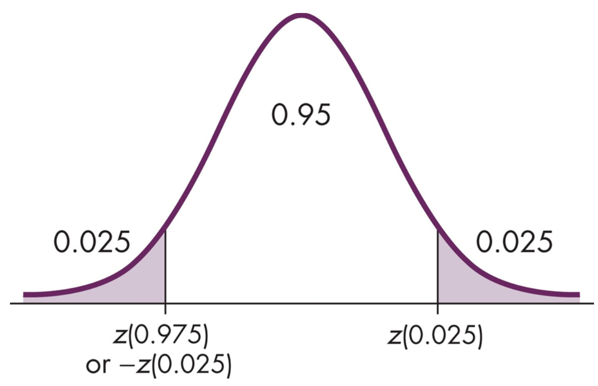
1. **the z-score such that there is a specified area in one tail of the normal distribution**





2. **the z-scores that bound a specified middle proportion of the normal distribution.**





NORMAL APPROXIMATION OF THE BINOMIAL

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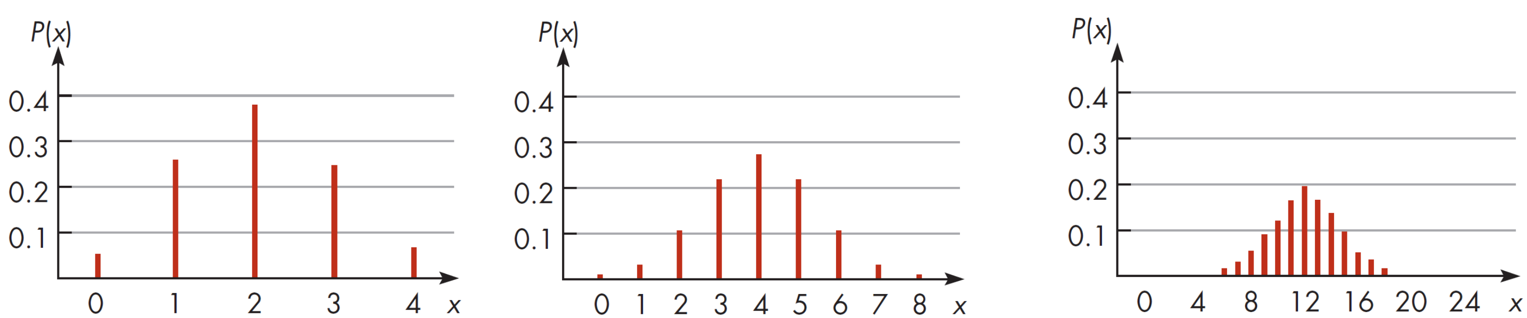
- **As we know that the binomial distribution is a probability distribution of the discrete random variable x, the number of successes observed in n repeated independent trials.**

- **We will now see how binomial probabilities—that is, probabilities associated with a binomial distribution—can be reasonably approximated by using the normal probability distribution.**

- **Let's look first at a few specific binomial distributions.**
  - **Probabilities of x for 0 to n in three situations: $n = 4, n = 8$, and $n = 24$.**

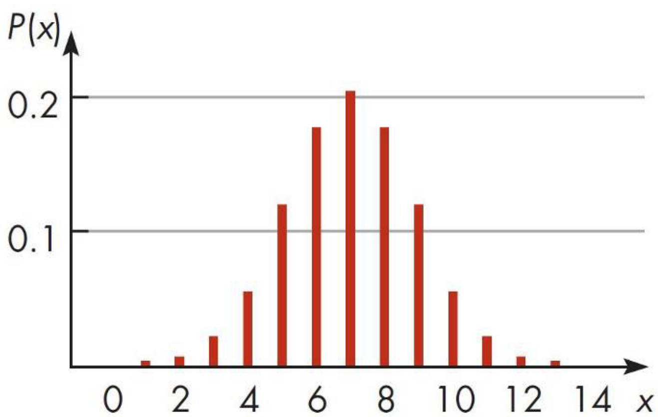


(a) Distribution for $n = 4, p = 0.5$ (b) Distribution for $n = 8, p = 0.5$ (c) Distribution for $n = 24, p = 0.5$

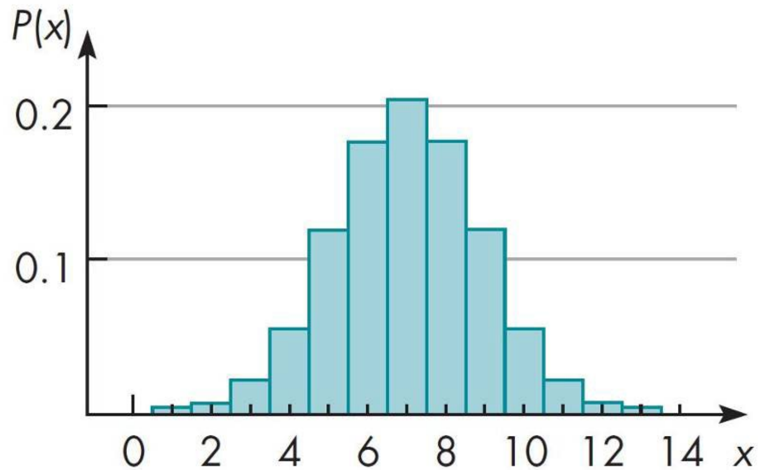
NORMAL APPROXIMATION OF THE BINOMIAL

- **To make the desired approximation, we need to take into account one major difference between the binomial and the normal probability distribution.**
  - **The binomial random variable is discrete, whereas the normal random variable is continuous.**

- **Let's look at the distribution of the binomial variable x, when $n = 14$ and $p = 0.5$.**



The Distribution of *x* when *n* = 14, *p* = 0.5



Histogram for the Distribution of *x* when *n* = 14, *p* = 0.5

NORMAL APPROXIMATION OF THE BINOMIAL

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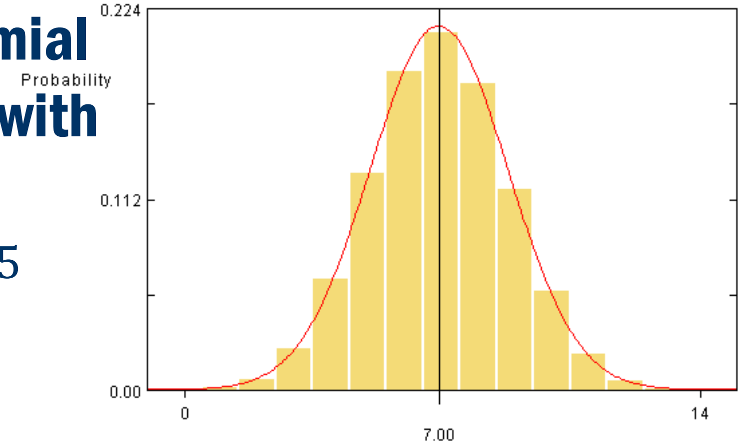
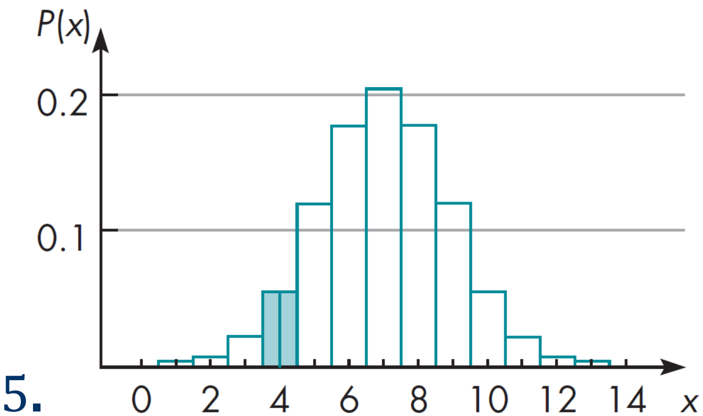
- **Let's compare the histogram of binomial distribution and Normal distribution with**
  - **mean:** $\mu = np = 14(0.5) = 7$,
  - **variance:** $\sigma^2 = npq = 14(0.5)(0.5) = 3.5$



- **For example,** $P\ (x\ =\ 4)$ **is equal to** 0.061 **(see Table 2 in Appendix B):**
  - $P(x = 4) = \dfrac{14!}{4!(14-4)!}(0.5)^4(0.5)^{14-4}$
    $= \mathbf{0.061}$
  - **which is the area of the bar above** $x\ =\ 4$



Area of Bar above x = 4 is 0.061, for $B(n$ = 14, p = 0.5)

- **For** $x = 4$**, the bar starts at** 3.5 **and ends at** 4.5**.**
- **The addition and subtraction of** 0.5 **to the** x **value is called the continuity correction factor.**
- **This is how we convert a discrete variable into a continuous variable.**

NORMAL APPROXIMATION OF THE BINOMIAL

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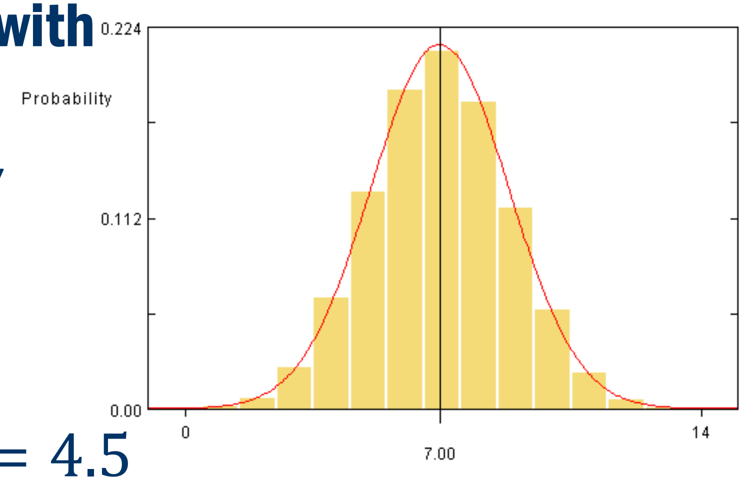
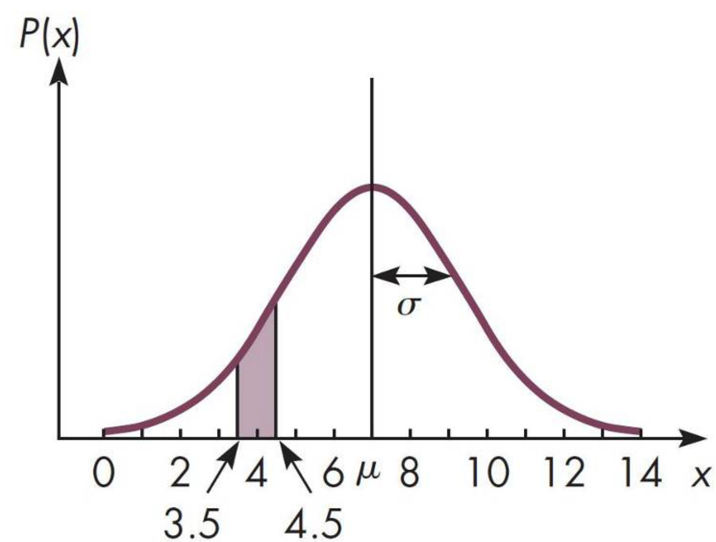
- **Let's consider the Normal distribution with**
  - **mean:** $\mu = np = 14(0.5) = 7,$
  - **st. dev.** $\sigma = \sqrt{npq} = \sqrt{14(0.5)(0.5)} = 1.87$



- **The probability that** $x = 4$ **is approximated by the area under the normal curve between** $x = 3.5$ **and** $x = 4.5$

- **Using the** z**-score formula:** $z = \dfrac{x-\mu}{\sigma}$
  - $P(3.5 < x < 4.5) = P\left(\dfrac{3.5-7}{1.87} < z < \dfrac{4.5-7}{1.87}\right)$
  - $= P(-1.87 < z < -1.34)$
  - $= 0.0901 - 0.0307$
  - $= \mathbf{0.0594}$

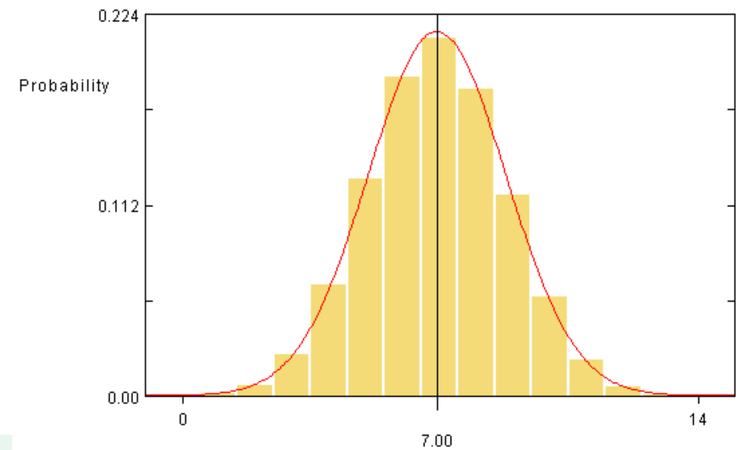
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	[illegible]	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1094	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1563	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379



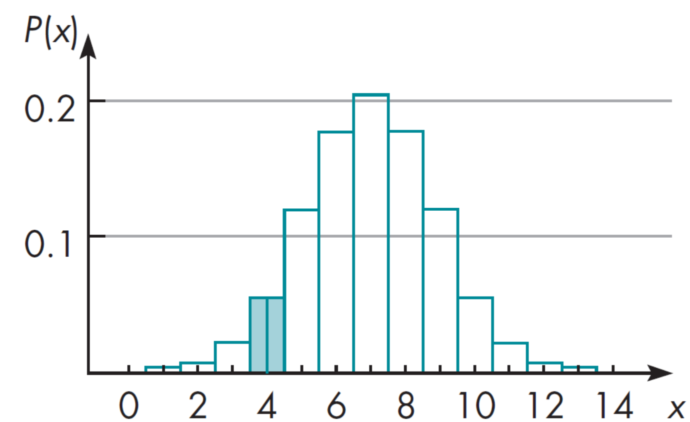
Probability that $x = 4$ is Approximated by Shaded Area

CONTINUITY CORRECTION FACTOR

- **The correction is to either add or subtract 0.5 of a unit from each discrete x-value. This fills in the gaps to make it continuous.**

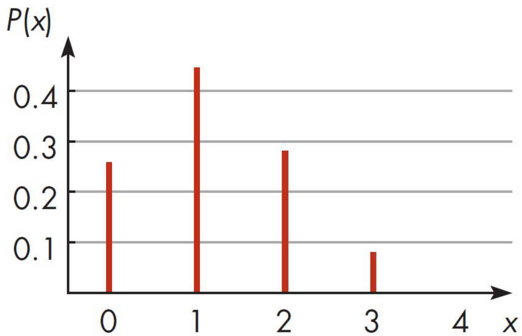


Binomial Probability (Discrete)	Normal Approximation (Continuous)
$P(x = 4)$	$P(3.5 < x < 4.5)$
$P(x > 4)$	$P(x > 4.5)$
$P(x \geq 4)$	$P(x > 3.5)$
$P(x < 4)$	$P(x < 3.5)$
$P(x \leq 4)$	$P(x < 4.5)$

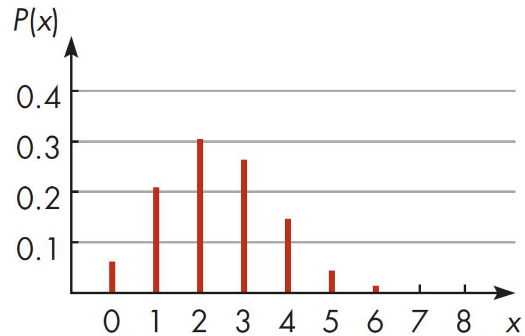


NORMAL APPROXIMATION OF THE BINOMIAL

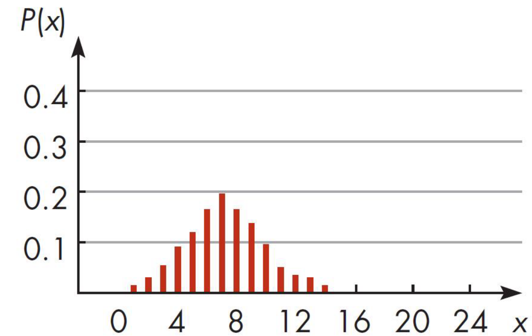
- **The normal approximation of the binomial distribution is also useful for values of p that are not close to 0.5.**



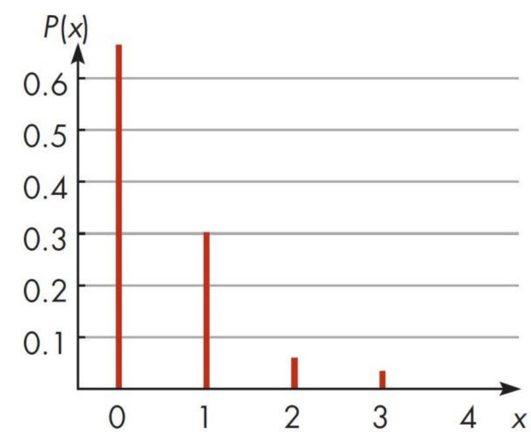
(a) Distribution for $n = 4$, $p = 0.3$



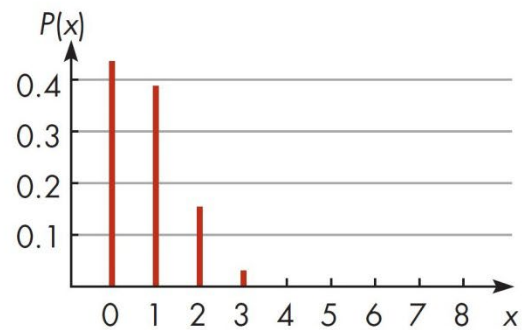
(b) Distribution for $n = 8$, $p = 0.3$



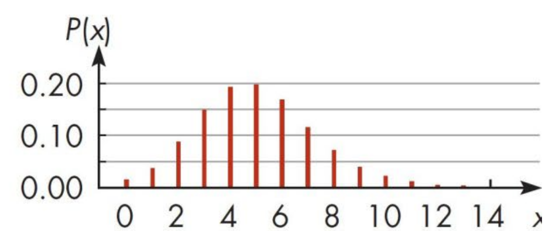
(c) Distribution for $n = 24$, $p = 0.3$



(d) Distribution for $n = 4$, $p = 0.1$



(e) Distribution for $n = 8$, $p = 0.1$



(f) Distribution for $n = 50$, $p = 0.1$

- **Normal Approximation to Binomial Applet (Link #2)**

NORMAL APPROXIMATION OF THE BINOMIAL

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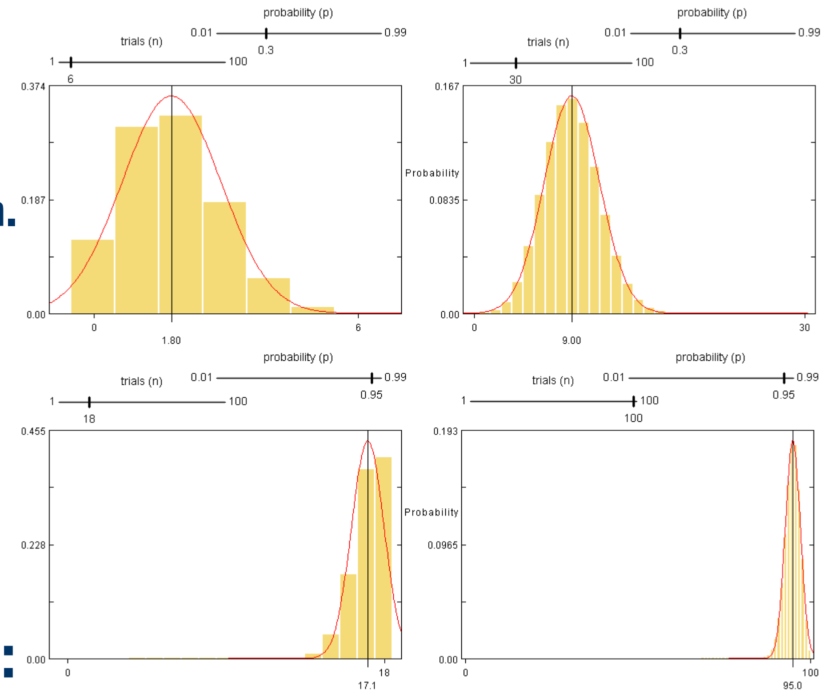
- **Notice that:**
  - As n **increases**, the binomial distribution begins to look like the normal distribution.



  - As the value of p **moves away** from 0.5, a larger n is needed in order for the normal approximation to be reasonable.

- **The following *rule of thumb* is used:**
  - **Rule:** The normal distribution provides a reasonable approximation to a binomial probability distribution whenever the values of np and $n(1-p)$ both **equal or exceed** 5.

- **By now you may be thinking,**
  - "So what? I will just use the binomial table and find the probabilities directly and avoid all the extra work."
  - **This is not always the case**

EXAMPLE: OVERWEIGHT AMERICANS

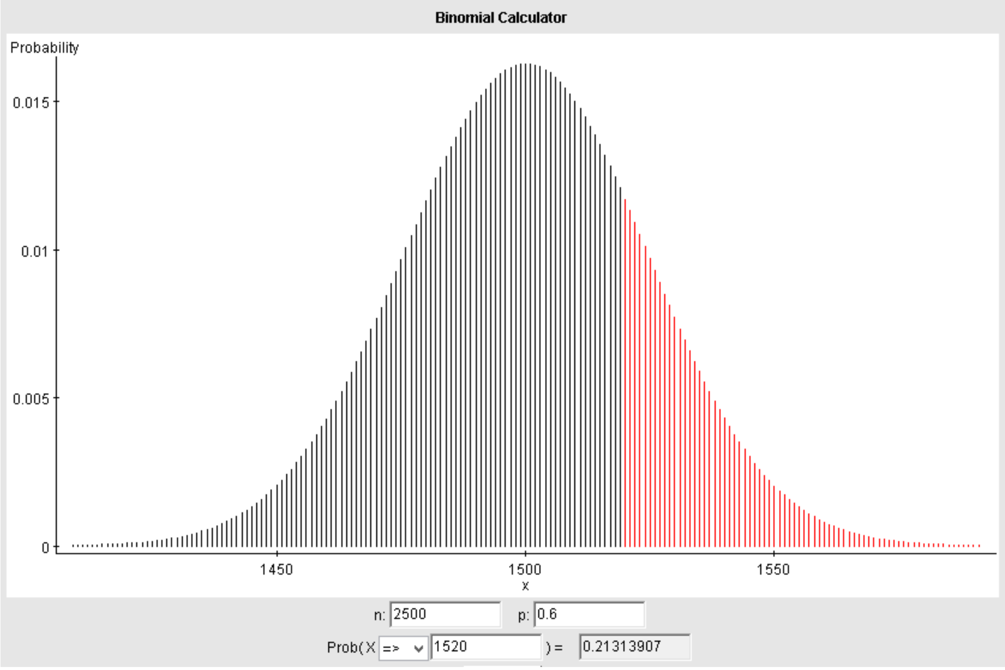
- **Nearly 60% of American adults are either overweight or obese, according to the U.S. National Center for Health Statistics. Suppose that we take a random sample of 2500 adults. What is the probability that 1520 or more of the sample are overweight or obese?**

- **Solution:**
- **In this example of a binomial experiment:**
  - x **is number of adults who are either overweight or obese,**
  - $n = 2500$**, and** $p = P(\text{overweight}) = 0.6$**.**

- **To find the probability that at least 1520 of the people in the sample are overweight, we must add the binomial probabilities of all outcomes from $x = 1520$ to $x = 2500$, where:**
  - $P(x) = \frac{2500!}{x!(2500-x)!}(0.6)^x(0.4)^{2500-x}$
  - $P(x = 1520) + P(x = 1521) + \cdots + P(x = 2499) + P(x = 2500) = ?$

OVERWEIGHT EXAMPLE

- **[Binomial Calculator]**
- **Probability distribution for the binomial model $n = 2500$ and $p = 0.6$, displayed graphically.**



- **Notice how the shape of this binomial probability distribution closely resembles a Normal curve.**

$$\mu = np = (2500)(0.6) = 1500$$
$$\sigma = \sqrt{npq} = \sqrt{(2500)(0.6)(0.4)} = 24.49$$

OVERWEIGHT EXAMPLE

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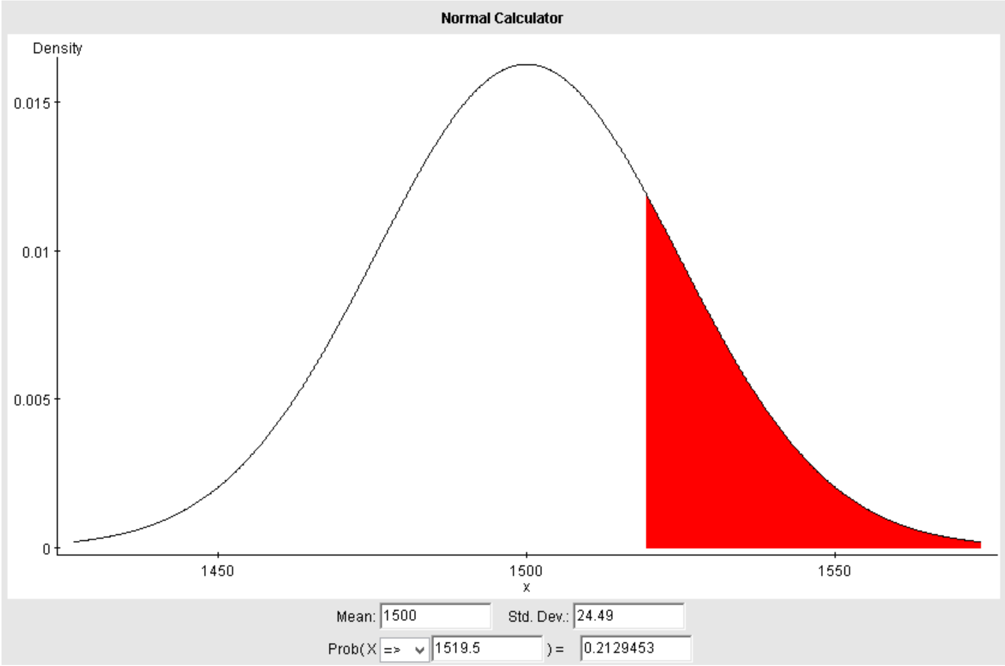
- **Normal Calculator**
  - $\mu = 1500$
  - $\sigma = 24.49$



- **The area of the shaded region ($x > 1519.5$) represents the probability of *interest*.**

- **Remember that $x = 1520$, the discrete binomial variable, covers the continuous interval from 1519.5 to 1520.**

- $P(x > 1519.5) = 1 - P(x < 1519.5)$
- $= 1 - P\left(z < \frac{1519.5-1500}{24.49}\right)$
- $= 1 - P(z < 0.80) = 1 - 0.7881 = \mathbf{0.2119}$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5754
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7258	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7518	0.7549
0.7	0.7580	0.7612	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7996	0.8023	0.8051	0.8079	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389

QUESTIONS?

- **ANY QUESTION?**