4.33 Let F = event fire occurs and T_i = event a type i furnace is in the home for i = 1, 2, 3, 4, where T₄ represents other types.

$$P(T_1|F) = \frac{P(F|T_1)P(T_1)}{P(F|T_1)P(T_1) + P(F|T_2)P(T_2) + P(F|T_3)P(T_3) + P(F|T_4)P(T_4)}$$

$$= \frac{(0.05)(0.30)}{(0.05)(0.30) + (0.03)(0.25) + (0.02)(0.15) + (0.04)(0.30)} = 0.40$$

4.41

a.
$$P(y > 3) = 0.067 + 0.021 + 0.014 = 0.102$$

b.
$$P(2 \le y < 5) = 0.354 + 0.161 + 0.067 = 0.582$$

c.
$$P(y > 4) = 0.021 + 0.014 = 0.035$$

4.45 Binomial experiment with y = number that over the legal limit, n = 15 and $\pi = 0.20$.

a.
$$P(y = 15) = 0.2^{15} = 3.28 \times 10^{-11} \approx 0$$

b.
$$P(y=6) = {15 \choose 0.2^6 (1-0.2)^{15-6}} = 0.043$$

c.
$$P(y \ge 6) = \sum_{y=0}^{15} 0.2^{y} (1 - 0.2)^{15 - y} = 0.061$$

d. $P(y = 0) = 0.8^{15} = 0.035$

d.
$$P(y = 0) = 0.815 = 0.035$$

4.48 Binomial experiment with y = number contracting infection, n = 50 and $\pi = 0.1$.

a.
$$P(y \ge 5) = \sum_{x=5}^{50} {50 \choose y} 0.1^y (1 - 0.1)^{50-y} = 0.5688$$
 (using technology)

- b. Infection after a one-week stay occurs with the same probability (0.1) for each individual and the occurrences of the infection are independent among individuals.
- 4.50 Assume y is Poisson with $\mu = 6$

a.
$$P(y = 6) = 0.0025$$

b.
$$P(y > 6) = 0.3937$$

c.
$$P(y \le 3) = 0.1512$$

4.52
$$P(y \ge 1) = 1 - P(y = 0) = 1 - (0.9999)^{5000} = 0.3935$$

We need to assume that outcomes of births are independent.

4.66 y is Normally distributed with μ =250 and σ =10

a.
$$y < 260 \Rightarrow \frac{y-250}{10} < \frac{260-250}{10} \Rightarrow z < 1$$

b. $y > 230 \Rightarrow \frac{y-230}{10} > \frac{230-250}{10} \Rightarrow z > -2$

c.
$$P(y < 260) = P(z < 1) = 0.8413$$
 and $P(y > 230) = P(z > -2) = 0.9972$

c.
$$P(y < 260) = P(z < 1) = 0.8413$$
 and $P(y > 230) = P(z > -2) = 0.9972$
d. $P(y > 265) = P\left(z > \frac{265 - 250}{10}\right) = P(z > 1.5) = 0.0668$
 $P(y < 242) = P\left(z < \frac{242 - 250}{10}\right) = P(z < -0.8) = 0.2119$
 $P(242 < y < 265) = 1 - P(y > 265) - P(y < 242)$
 $= 1 - 0.0668 - 0.2119 = 0.7213$

4.70

a.
$$P(y > 600) = P\left(z > \frac{600 - 500}{100}\right) = P(z > 1.0) = 0.1587$$

b.
$$P(y > 700) = P\left(z > \frac{700 - 500}{100}\right) = P(z > 2.0) = 0.0228$$

c.
$$P(y < 450) = P\left(z < \frac{450 - 500}{100}\right) = P(z < -0.50) = 0.3085$$

d.
$$P(450 < y < 600) = P\left(\frac{450 - 500}{100} < z < \frac{600 - 500}{100}\right) = P(-0.5 < z < 1.0) = 0.5328$$