

12th Week Summary (04/10/25)

- **Multiple Comparisons**

- The problem of **multiplicity** is serious when we are testing many hypotheses

- Instead of using Type-I error rate, we use familywise error rate (FEW):

$$\alpha_F = P(\text{Falsely reject at least one hypotheses})$$

- Bonferroni Method:

If there are m hypotheses, then Reject $H_0^{ij} : \mu_i = \mu_j$ if $|\bar{y}_i - \bar{y}_j| > t_{\alpha/2m} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$

Pros: This will guarantee that $\alpha_F \leq 0.05$

Cons: The chance of rejecting H_0 is small. In other word, the power of Bonferroni is very poor

- Fisher's Least Significant Difference (LSD):

Reject $H_0^{ij} : \mu_i = \mu_j$ if $|\bar{y}_i - \bar{y}_j| > t_{\alpha/2} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$

Pros: Easy to discover differences (reject H_0^{ij}). In other word, high power

Cons: This DOES NOT control the familywise error

- Tukey's Method:

Reject $H_0^{ij} : \mu_i = \mu_j$ if $|\bar{y}_i - \bar{y}_j| > \frac{q_\alpha(t, df_E)}{\sqrt{2}} \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$

Pros: Control the familywise error rate

Cons: Lower power comparing to Fisher LSD and Dunnett methods

- Dunnett's Method:

Reject $H_0^{ij} : \mu_i = \mu_j$ if $|\bar{y}_i - \bar{y}_j| > d_\alpha(t-1, df_E) \sqrt{\frac{2MSE}{n}}$

Pros: Control the familywise error rate. Higher power comparing to Bonferroni and Tukey's

Cons: It's just for comparing with a control group.

- **Population proportion**

- Draw a large random sample of size n from a large population having unknown proportion p of successes.

To test the hypothesis $H_0 : \pi = \pi_0$, compute the Z statistic: $z = \frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0(1 - \pi_0)}{n}}}$

- Use this test when the sample size n is so large that both $n\pi_0$ and $n(1 - \pi_0)$ are 5 or more.

- The p-value for a test of $H_0 : \pi = \pi_0$ against

$$H_a : \pi > \pi_0 \text{ is p-value} = P(Z \geq z)$$

$$H_a : \pi < \pi_0 \text{ is p-value} = P(Z \leq z)$$

$$H_a : \pi \neq \pi_0 \text{ is p-value} = 2P(|Z| \geq |z|)$$

- Confidence interval for π can be obtained by $\hat{\pi} \pm z_{\alpha/2v} \sqrt{\frac{\hat{\pi}(1 - \hat{\pi})}{n}}$. This interval should not be used unless $n\hat{\pi} \geq 5$ and $n(1 - \hat{\pi}) \geq 5$.