

7.6

- a. The boxplot is symmetric with whiskers of approximately the same length. There are no obvious violations of the assumptions.

b. 95% CI on σ : $\left(\sqrt{\frac{(50-1)(8.77)^2}{70.22}}, \sqrt{\frac{(50-1)(8.77)^2}{31.55}} \right) \Rightarrow (7.32, 10.92)$

- c. $H_0: \sigma \leq 10$ versus $H_a: \sigma > 10$

$$\text{Reject } H_0 \text{ if } \frac{(n-1)s^2}{10^2} \geq 66.34; \quad \frac{(50-1)8.77^2}{10^2} = 37.687 < 66.34 \Rightarrow$$

Fail to Reject H_0 and conclude the data does not support σ greater than 10.

(similarly, one could have noticed the sample variance is less than 10 which couldn't possibly lead to sufficient evidence in the other direction).

- d. We can apply this inference to all cars on that section of interstate at comparable times of day.

```
> library("EnvStats")  
> sqrt(varTest(ex7.6$amount)$conf)
```

```
> varTest(ex7.6$amount, sigma.squared = 10^2, alternative = "greater")
```

7.14 $n_1 = 25, n_2 = 20, s_1 = 5.2, s_2 = 6.8$

- a. $H_0: \sigma \neq 10$ versus $H_a: \sigma = 10$

$$\text{Let } F = \frac{\max(s_1^2, s_2^2)}{\min(s_1^2, s_2^2)} = \frac{6.8^2}{5.2^2} = 1.71. \quad \text{Under } H_0, F \sim F(df_1 = 19, df_2 = 24)$$

$$\text{Given } F_{\frac{0.05}{2}}(v_1 = 19, v_2 = 24) = 2.345 \quad \text{and } F_{1-\frac{0.05}{2}}(v_1 = 9, v_2 = 14) = 0.408$$

With $\alpha = 0.05$, reject H_0 if $F > 2.345$ or $F < 0.408$

Thus, we fail to reject H_0 and conclude there is not sufficient evidence that there is a difference in the two standard deviations.

- b. 95% CI on $\frac{\sigma_1^2}{\sigma_2^2}$:

$$\frac{s_1^2}{s_2^2} F_L < \frac{\sigma_1^2}{\sigma_2^2} < \frac{s_1^2}{s_2^2} F_U \Rightarrow \frac{5.2^2}{6.8^2} \left(\frac{1}{2.45} \right) < \frac{\sigma_1^2}{\sigma_2^2} < \frac{5.2^2}{6.8^2} (2.345) \Rightarrow 0.238 < \frac{\sigma_1^2}{\sigma_2^2} < 1.371$$

- c. We need to assume that the two samples were independent random samples from normally distributed populations.

7.18 The data is summarized in the following table:

Environment	n	Mean	Standard Deviation
Wild	8	122.86	8.2331
Ranch	8	118.40	30.4326
Zoo	8	102.90	36.8529

- a. Test $H_0 : \sigma_W = \sigma_R = \sigma_Z$ versus H_a : population standard deviations are not all equal

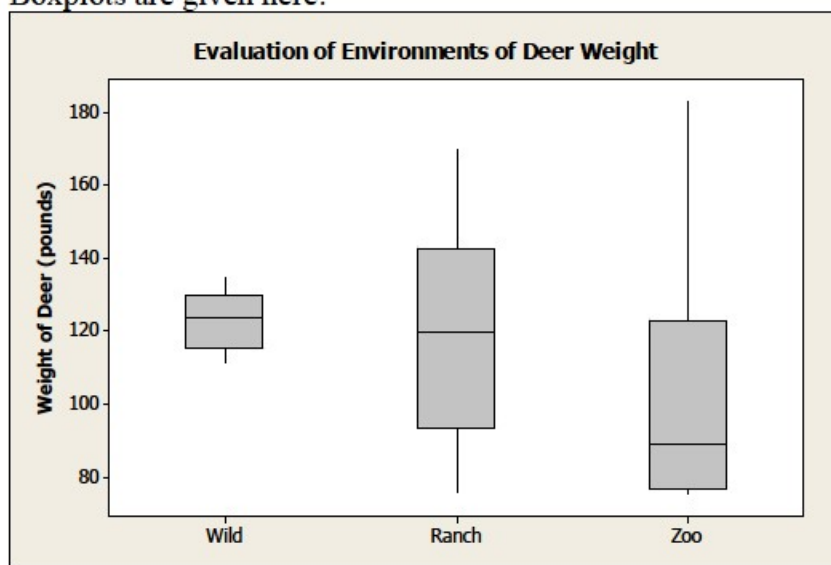
Reject H_0 at level $\alpha = 0.01$ if $L \geq F_{0.01,2,21} = 5.78$

From the data, $L = 1.439 < 5.78 \Rightarrow$

Thus we fail to reject H_0 and conclude there is not significant evidence of a difference in variability of deer raised in captive environments.

Using the F table, we have p-value > 0.25 , which indicates the total lack of evidence of a difference in variability.

- b. Boxplots are given here:



From the boxplots, we observe that the data from the Wild and Ranch appear to be from a normal distribution since the boxplots have symmetric boxes with whiskers of approximately equal length and no outliers. The boxplot for the Zoo data is skewed to the right. Thus, using the BFL test rather than the Hartley test was the appropriate choice.

Test for Equal Variances: Weight versus Environment

Null hypothesis All variances are equal
 Alternative hypothesis At least one variance is different
 Significance level $\alpha = 0.05$

Method	Test Statistic	P-Value
Levene	1.44	0.260

```
> lawstat::levene.test(ex7.18$wt, ex7.18$Env)
```

8.6

- a. Yes, the mean for Device A is considerably (relative to the standard deviations) smaller than the mean for Device D.
- b. $H_0: \mu_A = \mu_B = \mu_C = \mu_D$ versus H_a : At least one of the four population means is different from the rest.

Reject H_0 if $F \geq F_{0.05, 3, 20} = 3.10$

$$SSW = 5[(0.1767)^2 + (0.2091)^2 + (0.1532)^2 + (0.2492)^2] = 0.8026$$

$$\bar{y}_{..} = 0.0826 \Rightarrow$$

$$SSB = 6[(-0.1605 - 0.0826)^2 + (0.0947 - 0.0826)^2 + (0.1227 - 0.0826)^2 + (0.2735 - 0.0826)^2] \\ = 0.5838$$

$$F = \frac{0.5838/3}{0.8026/20} = 4.85 > 3.10$$

Thus, we reject H_0 and conclude there is a significant difference among the mean difference in pH readings for the four devices.

- c. p-value = $P(F_{3,20} \geq 4.85) = 0.0107$
- d. The data must be independently selected random samples from normal populations having the same value for σ .
- e. Suppose the devices are more accurate at higher levels of pH in the soil, and if by chance all soil samples with high levels of pH are assigned to a particular device, then that device may be evaluated as more accurate based just on the chance selection of soil samples and not on a true comparison with the other devices.

> summary(aov(pH ~ Device, data=ex8.6))

8.7

- a. Based on the ANOVA table below, there is not a significance difference in soil density among the grazing regimens at the $\alpha = 0.05$ level.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Factor	2	2.692	1.3460	2.06	0.156
Error	18	11.738	0.6521		
Total	20	14.430			

- b. The p-value is 0.156.
- c. The standard deviation of the two-week grazing, two-week no grazing group is smaller (about half) of the other two. This may call into question our assumption of equal variance, but with the small sample size, it's likely ok to proceed.

> summary(aov(Density ~ Regimen, data=ex8.7))