Math 4720: Statistical Methods

3^{rd} Week Summary (01/31/25)

- Probability Rules

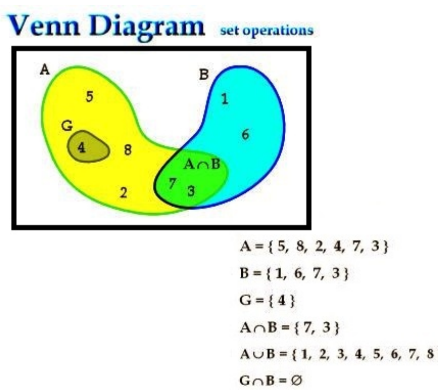
  The probability $P(A)$ of any event A satisfies $0 < P(A) < 1$.

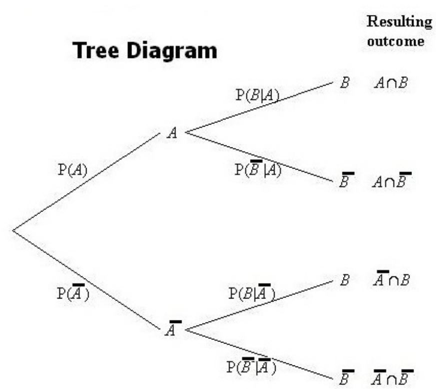
  If S is the sample space, then $P(S) = 1$.

  Two events A and B are **disjoint** (mutually exclusive) if they have no outcomes in common and so can never occur together. If A and B are disjoint, $P(A \text{ or } B) = P(A \bigcup B) = P(A) + P(B)$

  For any event A, $P(A \text{ does not occur}) = P(\overline{A}) = 1 - P(A)$.

- Addition rule in general : $P(A \text{ or } B) = P(A \bigcup B) = P(A) + P(B) - P(A \bigcap B)$





- $P(A|B)$, the **conditional probability** of A given that B has occurred, can be thought of an adjusted version of the probability of A in light of the additional information that B has occurred.

  When $P(B) > 0$, the conditional probability of A given B is: $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{P(A \bigcap B)}{P(B)}$

- General **multiplication** rule : $P(A \text{ and } B) = P(A \bigcap B) = P(A|B)P(B) = P(B|A)P(A)$.

- Two events A and B that both have positive probability are **independent** if: $P(A|B) = P(A)$ or $P(A \bigcap B) = P(A)P(B)$.

- Bayes' Rule : $P(A|B) = \dfrac{P(B|A)P(A)}{P(B)} = \dfrac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\overline{A})P(\overline{A})}$

- Law of total probability: If $A_1, A_2, \ldots, A_k$ are disjoint events whose probabilities are not 0 and add to exactly 1, then:
$$P(B) = P(B|A_1)P(A_1) + \ldots + P(B|A_k)P(A_k)$$

- General Bayes' Rule : Suppose that $A_1, A_2, \ldots, A_k$ are disjoint events whose probabilities are not 0 and add to exactly 1, then :
$$P(A_i|B) = \frac{P(B|A_i)P(A_i)}{P(B|A_1)P(A_1) + \ldots + P(B|A_k)P(A_k)}$$

- A **random variable**(*discrete* or *continuous*) is a variable whose value is a numerical outcome of a random phenomenon.

  A probability model with a sample space made up of a list of individual outcomes is called **discrete**.

  Binomial(n, π): $P(X = k) = \dfrac{n!}{k!(n-k)!}\pi^k(1-\pi)^{n-k}$, $k = 0, 1, 2, \cdots, n$

  Poisson(μ): $P(Y = y) = \dfrac{\mu^y e^{-\mu}}{y!}$, $y = 1, 2, \cdots$