Math 4720: Statistical Methods

4^{th} Week Summary (02/06/25)

- A **Normal** distribution is described by a Normal density curve. We will say $X \sim N(\mu, \sigma^2)$, and the normal density function is given by: $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$
- The 68-95-99.7 Rule (a.k.a **the Empirical Rule**)
- The standardized value is called a z-score. $z = \frac{x-\mu}{\sigma}$
- Finding Normal Probabilities :

  Less than: $P(X < x) = P\left(\frac{X-\mu}{\sigma} < \frac{x-\mu}{\sigma}\right) = P(Z < z)$

  Greater than: $P(X > x) = P(Z > z) = 1 - P(Z < z)$

  Between two numbers: $P(a < X < b) = P(z_a < Z < z_b) = P(Z < z_b) - P(Z < z_a)$

  Outside two numbers:

  $P(X < a \bigcup X > b) = P(Z < z_a \bigcup Z > z_b) = 1 - P(z_a < Z < z_b) = P(Z < z_a) + 1 - P(Z < z_b)$
- The **sampling distribution** of a statistic is the distribution of values taken by the statistic in all possible samples of the same size from the same population
- If *individual observations* have the $N(\mu, \sigma^2)$ distribution, then the *sample mean* of a random sample of size n has the $N(\mu, \frac{\sigma^2}{n})$ distribution.
- **CLT** (Central Limit Theorem): Draw a random sample of size n from any population with mean μ and finite standard deviation σ. When n is large, the sampling distribution of the sample mean is approximately $N(\mu, \frac{\sigma^2}{n})$.
- Means of *random samples* are **less variable** than *individual observations.*
- Means of *random samples* are **more normal** than *individual observations.*
- The probability that the sample mean $\bar{x}$ is close to μ increases as the sample size n becomes large. This is commonly referred to as the **Law of Large Numbers**.