Math 4720: Statistical Methods

5^{th} and 6^{th} Week Summary (2/20/25)

- **Confidence intervals (Estimation)**

  An interval calculated from the data, usually of the form: estimate $\pm$ margin of error.

  confidence interval for μ is $\bar{x} \pm z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$ if

  data is coming from normal population having unknown mean μ and known σ. or

  large random sample from a non-normal population having unknown mean μ and known σ.

- The confidence interval will have a specified **margin of error** m when the sample size is: $n = \left(\dfrac{z_{\alpha/2}\sigma}{m}\right)^2$

- **Hypothesis tests** (a.k.a., tests of significance) for *population parameter*

  H_0 is called the *null hypothesis* while H_a is called the *alternative hypothesis.*

  Usually the null hypothesis is a statement of *no effect* or *no difference.*

  The claim about the population that we are *trying to find evidence* for is the alternative hypothesis.

  A **test statistic** calculated from the sample data measures how far the data diverge from the null hypothesis H_0. Define the test statistic $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

  Decision Rule: Given fixed $\alpha = P(\text{Type-I Error})$.

  $H_a : \mu > \mu_0$, Reject H_0 if $z > z_\alpha$

  $H_a : \mu < \mu_0$, Reject H_0 if $z < -z_\alpha$

  $H_a : \mu \neq \mu_0$, Reject H_0 if $|z| > -z_{\alpha/2}$

- If we reject H_0 when in fact H_0 is true, this is a **Type I error**.

  The significance level α of any fixed-level test is the probability of a Type I error.

- If we fail to reject H_0 when in fact H_a is true, this is a **Type II error**, also called β.

  For one sided alternative test ($H_a : \mu > \mu_0$ or $H_a : \mu < \mu_0$): $\beta = P\Big(Z \leq z_\alpha - \dfrac{|\mu_0 - \mu_a|}{\sigma/\sqrt{n}}\Big)$

  For two sided alternative test ($H_a : \mu \neq \mu_0$): $\beta = P\Big(Z \leq z_{\alpha/2} - \dfrac{|\mu_0 - \mu_a|}{\sigma/\sqrt{n}}\Big)$

- The **power of a test against any alternative** is 1 minus the probability of a Type II error for that alternative: power $= 1 - \beta$.

- **Power analysis** (Sample size determination):

  For one sided alternative test ($H_a : \mu > \mu_0$ or $H_a : \mu < \mu_0$): $n = \sigma^2 \dfrac{(z_\alpha + z_\beta)^2}{\Delta^2}$

  For two sided alternative test ($H_a : \mu \neq \mu_0$): $n = \sigma^2 \dfrac{(z_{\alpha/2} + z_\beta)^2}{\Delta^2}$